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BARGAINING WITH RENEGOTIATION IN MODELS WITH ON-THE-JOB SEARCH

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Latest version

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Abstract

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1 Introduction

Search models have become the standard tool for modeling unemployment in macroeconomics. Due to the empirical prevalence of job-to-job transitions, on-the-job search has been included in many model extensions (Burdett and Mortensen, 1998; Bontemps et al., 2000; Postel-Vinay and Robin, 2002). However, while bargaining is a standard feature of models without on-the-job search (Mortensen and Pissarides, 1994), it is still an open question how bargaining should be modeled when workers search on the job, and the turnover depends on the wage (Shimer, 2006). In this paper, I provide a solution to this problem.

The challenge of modeling bargaining in on-the-job search models was highlighted by Shimer (2006). He pointed out that a worker's quit rate should depend on the wage, and that this can lead to a non-convex bargaining set, violating a key condition underlying the Nash bargaining solution concept (Nash, 1950). Shimer (2006) was a response to earlier papers in the literature that modeled bargaining with on-the-job search using the Nash bargaining solution. Pissarides (1994) did have a convex bargaining set, but assumed that the worker turnover was independent of the wage. Mortensen (2003, Section 4.3.4), in contrast, allowed for the turnover depending on the wage, but faced the potential problem of a non-convex bargaining set. Shimer (2006) shows that the non-convexity problem can be solved using a non-cooperative bargaining game in the spirit of Binmore et al. (1986), which coincides with the Nash bargaining solution concept when the bargaining set is convex. However, he also shows that this solution is not unique. In response to this problem, the literature has when modelling on-the-job search, in general, avoided bargaining and/or turnover which is wage dependent. Instead, it has been assumed that firms give a take-it-or leave it offers to new workers (i.e., no bargaining),[1] or that firms can make counteroffers to workers that receive an offer on the job (making turnover independent of the wage).[2]

Pissarides (1994) had used the cooperative Nash bargaining solution, and does not allow the wage to have an effect on worker turnover. Later, Mortensen (2003, Section 4.3.4) notes that worker turnover should depend on the wage and extends the model of Pissarides (1994) in such a way. However, Shimer (2006) points out that if the worker's quit rate depends on the wage, this leads to a potentially non-convex bargaining set, violating a key condition underlying the Nash bargaining solution (Nash, 1950). To address this, he assumes that there is non-cooperative bargaining, and shows that there exists a multiplicity of equilibria.

I set up a model with (i) on-the-job search (OJS), (ii) non-cooperative bargaining, and (iii) renegotiation. With renegotiation, the equilibrium is unique, and the payoffs from the models of Pissarides (1994), Mortensen (2003, Section 4.3.4), and Shimer (2006) can all be nested, under parametric assumptions, as the frequencies of renegotiation goes to zero or infinity. In the process,

[1] See, for example, Gautier et al. (2010) and Coles and Mortensen (2016).

[2] See, for example, Dey and Flinn (2005) and Cahuc et al. (2006).

I show that with more frequent renegotiation, the share of surplus captures by workers and the spillovers from minimum wages are both less.

In the model, job offers arrive at a Poisson rate. Wages are determined in a non-cooperative bargaining game with alternating offers made by the firm and the worker in the spirit of Rubinstein (1982) and Binmore et al. (1986). Between offers, there is an exogenous probability that the bargaining process breaks down. In the event of a breakdown, the worker becomes unemployed, and the firm gets an empty vacancy. If an agreement is reached, the wage remains fixed until the end of the wage contract at which time they once more bargain over a new wage. The end of the contract arrives at a Poisson rate, and the Poisson intensity represents the inability of agents to commit.[3] A match differs in a contracted wage and expected outcome of future negotiations. In equilibrium, the contracted wage is always equal to the expected wage from renegotiation. However, renegotiation still plays a big role as it governs how a worker values an increase in the contracted wage.

A consequence of the setup is that the worker's share of the surplus will generally be larger than her bargaining power. The reason is that the reduced turnover from a higher wage implies that it will be relatively "cheap" for the firm to transfer value to the worker, since part of the increase in worker value is recouped through a longer duration of the match. Hence, higher wage offers by the worker will be accepted by the firm and, in this way, the dependence of the surplus on the wage acts as an extra source of worker bargaining power.

The change in surplus with the wage depends on the response of turnover to the wage and the level of profits. Whereas workers always gain from transitions, firms lose their profits from a transition. Thus, transitions are bilaterally inefficient whenever the gain in worker value is less than the loss in profits and the marginal transition reduces surplus by the level of the profits.[4] A higher wage will increase the worker value which decreases the quit rate of the worker. The decrease in the quit rate with the wage is high if similar jobs arrive often and the expected proportion of the duration of the match that will be covered by the contracted wage is high. The proportion covered by an initally agreed wage captures how the worker values the current wage, versus the wage expectation (which regulates the wage that will be paid after the renegotiation). The model nests several earlier models in the literature. With no renegotiation, i.e., perfect commitment, this proportion is one, and the wage expectation plays no role conditional on the agreed wage. The equilibrium corresponds to one of the equilibria in Shimer (2006) and the values are the same as those in Mortensen (2003, Section 4.3.4). With continuous renegotiation, i.e., no commitment, this share is zero, and the worker places no value on a higher contracted wage, since this will only last an infinitesimal amount of time. Since the workers transitions decisions are independent of the wage,

[3] The model treats commitment as a parameter, but the same specification also occurs if there are productivity shocks and each party can choose whether or not to renegotiate at a small cost.

[4] In equilibrium, all transitions are socially efficient once the value of the future employer is accounted for.

the surplus is also independent of the wage. Wages therefore solve the Nash bargaining solution when values are perfectly transferable comparable to Pissarides (1994).

The role of wage expectations, introduced via renegotiation, results in a unique equilibrium in contrast to Shimer (2006). The bargaining game implies that the wage outcome maximizes the Nash product in a region in which the offers are considered. A higher wage expectation is associated with lower turnover of the worker. The match surplus is higher for matches with higher wage expectations as the marginal transitions are inefficient. The Nash product cannot be maximized at the same wage for two firms with different wage expectations.[5] This results in an unique bargaining outcome within each match. The bargaining outcome in the match with the lowest wage expectation pins down the lowest wage. In the model of Shimer, there is no role for wage expectations conditional on the agreed wage. With homogeneous firms, the Nash product must be the same evaluated at all the bargaining outcomes. This implies that the bargaining outcome within a match is not unique and the lowest wage, i.e., the initial condition, is not pinned down.[6]

When the model is used to interpret moments in the data, the assumed frequency of renegotiation has important implications for the conclusions drawn. Since a higher frequency of renegotiation reduces the worker's share of the surplus, matching the labor share in a calibration requires a higher bargaining power when renegotiation is more frequent. I illustrate this by setting the job offer arrival rates and the separation rates to match transition moments from the literature. I calibrate the productivity distribution and the bargaining power of the worker to match a labor share of 2/3 and a lognormal wage offer distribution with a scale parameter from the literature. When wages are continuously renegotiated, the bargaining power of workers is calibrated to be 0.46 and, when wages are never renegotiated, it is calibrated to 0.02.

One stark manifestation of the role of renegotiation is the difference in the model's implications following an introduction of a minimum wage. The spillover from the minimum wage is lower when renegotiation is more frequent. When the minimum wage is increased, the partial equilibrium response will lead to a mass point at the minimum wage. If there is a high density, firms are willing to agree on a higher wage, as the lower turnover partially compensates for the reduced markup. As firms accepts these higher wages, the increase in the minimum wage spills over into firms previously paying above the minimum wage. The extent to which there is such a positive spillover above the minimum wage depends on the extent to which firms are willing to accept higher wages due to

[5]In equilibrium, all matches must have a different wage expectation.

[6]Formally, the bargaining game implies that the Nash product must be the same evaluated at the offer by the worker and firm. Withut renegotiation and homogeneous productivities, the Nash product is constant on the support of wages. Hence, the offer by the worker and firm are both on interior of the support. This implies that the lowest wage, or alternatively the level of the Nash product, is not pinned down. Whereas, with renegotiation, the Nash product is a increasing prior to the bargianing outcome and decreasing thereafter. Since the firms offer is below the outside the support, this gives the inital condition. Alternative refinements are also provided that imply the same unique equilibrium.

the higher density. When there is frequent renegotiation, the turnover of the worker responds little to the wage, and firms are then unwilling to accept higher wages. Thus, with more frequent renegotiation, the positive spillovers from the minimum wage are less. In the limit, as renegotiation becomes continuous, the spillover disappears and there is a mass point at the minimum wage. In contrast, in the wage posting model, firms are able to commit to a wage forever resulting in a strong response of worker turnover to the wage and therefore high minimum wage spillovers.

Lastly, I endogenize the choice of the frequency of renegotiation by allowing each firm to pick the contract length after meeting a worker. The firm can then influence the wage that results from a renegotiation by setting the contract length. I show that there are two cases. Either, the firm does not profit from increasing the wage permanently above the wage obtained under a Nash bargaining solution with perfectly transferable values. This happens if the bargaining power of the worker is sufficiently high and when profits are small compared to the increased productivity from a better match. In this case, the firm will choose continuous renegotiation. When productivities are homogeneous, this case only occurs when the worker has all the bargaining power. Alternatively, the lower turnover means that it is profitable to raise the wage above the continuous renegotiation level, in which case the firm will choose a contract length leading to an equilibrium wage that maximizes the firm value (taking turnover into account). This leads to a finite renegotiation rate, with no renegotiation (infinite contract length) being a limiting case which is optimal when the firm has all the bargaining power. Thus, when frequency of renegotiation is endogenous, the standard assumptions in the literature of full or no commitment only arise under particular parameter values.

Related literature. In wage posting models, firms have all the bargaining power and can commit to a wage at the time of vacancy posting, i.e., full ex-ante commitment; see, for instance, Burdett and Mortensen (1998).[7] The wage is chosen so that the marginal gain from hiring and retaining workers exactly matches the increased wage cost. After hiring a worker, the firm has an incentive to change the agreed wage as it no longer affects the probability of hiring a worker. Hence, wages are not time consistent and a large cost is potentially needed to prevent ex-post deviations by the firm. In the model in this paper, the same agreement is reached each time when wages are renegotiated. The commitment is only needed to evaluate small deviations away from the equilibrium wage. Commitment in an ex-post setting is thus a much weaker condition.

Postel-Vinay and Robin (2002) also consider a model in which firms have all the bargaining power, but where firms can observe outside offers and make counteroffers. When the worker receives an offer, the firm employing the worker and the other firm engage in a second price auction for the worker. In the equilibrium, the worker moves to the most productive firm at a wage such that

[7] Gautier et al. (2010), like Shimer (2006), consider a model in which wages are set after the match has been formed. The increased retention is then the only reason to set a wage higher than the reservation wage. Coles and Mortensen (2016) consider a model in which the hiring cost is independent of the wage. The increased retention is then the only reason for increased pay irrespective of the timing of wage setting.

the worker is indifferent to receiving the full surplus in the less productive firm. The wage thereby increases as counteroffers arrive during a match. This model is extended to include bargaining with a threat point equal to the value of the worst match by Dey and Flinn (2005) and Cahuc et al. (2006).[8] These models also require firms to commit to keeping the wage (or value) forever after the counteroffer has expired. In these models, there is no bilateral inefficiency, as the less productive firm "surrenders" all its profits to the worker. Values are therefore perfectly transferable. However, if the search effort was endogenous, the worker would search an excessive amount to bid up her wage. In fact, when the firm has all the bargaining power the bilaterally efficient contract entails no search effort by the worker as the worker receives the surplus of the current jobs in any new match. Firms might optimally ex-ante commit not to match outside offers to decrease the worker's search effort (ex post the firm would always want to match the offer if it makes the worker stay at positive profits) (Postel-Vinay and Robin, 2004).

Two related papers have considered changes to the standard model that result in wages which do not affect the quit rate of the worker. In these papers, the wage solves the Nash bargaining solution with perfectly transferable values. Krause and Lubik (2007) analyze a model in discrete time in which transition decisions are made at the beginning of the period. Wages, on the other hand, are set at the end of each period. The timing restriction means that the wage is not a state variable when the worker makes the transition decision. Similarly, in Moscarini (2005) the two competing firms decide whether to enter an auction for the worker. An equilibrium is analyzed in which only the most productive firm shows up. The worker turnover is therefore independent of the previously agreed wage.

The bilateral inefficiency comes from the forgone profits of the firm when the worker changes jobs. If a worker pays an upfront fee and is subsequently paid the full productivity of the match, the profits are zero and the transitions are bilaterally efficient. An optimal dynamic contract offered by the firm would entails a decrease in profits over time, thereby creating a "tenure profile"; see Burdett and Coles (2003) and Stevens (2004). The same is true in models with counteroffers if search effort by the workers is non-contractable (Lentz, 2014).

A number of papers have used the wage posting model to assess the impact of a change in the minimum wage. See, for instance, Van den Berg and Ridder (1998), Bontemps et al. (1999, 2000) and recently Engbom and Moser (2017). In the wage posting model, firms can commit to a wage forever and workers have no bargaining power. Thus, the model implies, by construction, that the spillover effects are strong. Flinn (2011, Chapter 10) studies the role of minimum wages in models in which wages do not affect the allocation of workers to jobs. This can either be due to counteroffers or that worker turnover is assumed to be independent of the agreed wage. Recently, Flinn et al. (2017) study the role of minimum wages in a model in which some firms that post wages and some

[8] Cai (2017) considers the model of Cahuc et al. (2006) but with a cost of delay instead of a probability of breakdown.

firms match outside offers. The firms using wage posting only pay a higher wage due to the turnover motive whereas firms making counteroffers only pay to satisfy the participation constraint. Thus, in the model, the turnover motive in the economy can be different due to differences between firms, whereas, in the model in this paper, there is only a partial turnover motive for a given firm and the degree can also vary between firms.

Infrequent renegotiation of wages is common in models without OJS; see, for example, Gertler and Trigari (2009). A few papers model infrequent renegotiation in models with OJS by, similar to Pissarides (1994), assuming that the acceptance decisions of workers are independent of the wage (Carlsson and Westermark, 2016; Gertler et al., 2016). Brügemann et al. (2015) show that if a firm employs many workers and the firm successively bargains with each worker, there is a similar mechanism to the one in this paper. If agreement with a worker is reached at a high wage then the cost, for the firm, will be partly recouped from lower wage agreements with subsequent workers. Lastly, search is random in the model. Unlike papers with directed search, wages only (indirectly) affect search frictions via the entry decisions of firms. With directed search, the worker can trade off the promised wage (value or job type) against the job finding rate, thereby alleviating some of the search friction. In this way, firms must either be able to (Moen, 1997) or have an incentive to (Menzio, 2007) commit to deliver that which the worker directs her search towards; see, for instance, the recent survey by Wright et al. (2017).

Outline. Section 2 defines the general model and expands on the contributions of the paper discussed above. Section 3 provides a closed form solution in the case of homogeneous productivities. Section 4 provides a quantitative evaluation of the model. Section 5 analyzes the impact of a change in the minimum wage. Section 6 provides an extension of the model to a case in which the frequency of renegotiation is endogenous. Section 7 concludes the paper. The proofs are collected in the Appendix.

2 Model

Environment. There is a frictional labor market with a continuum of two types of risk neutral and infinitely lived agents, firms and workers. Time is continuous and discounted at a rate ρ. Matches between workers and firms differ in the type F determining the wage expectation. The type may also be associated with the quality of the match and is observable by the agents. When a worker meets a firm, the type is drawn from the standard uniform distribution. A firm matched with a worker produces a flow output of $x(F)$, where the function $x(\cdot)$ is differentiable and weakly increasing. The flow profit is given by production $x(F)$ minus the agreed wage. There is a minimum wage w_{min}, less than $x(0)$, which the agreed wage must be weakly higher than. Workers are homogeneous but differ in their employment state (unemployed or employed), the wage w, and the match type F. An unemployed worker receives a flow benefit b and job offers at rate λ_u. An employed worker receives a wage w and job offers at rate λ_e. The job is destroyed at rate δ in which case the worker becomes unemployed and the firm gets an empty vacancy (which has no value by the free entry condition).

In contrast to Burdett and Mortensen (1998), Mortensen (2003, Section 4.3.4), and Shimer (2006), I assume that wage contracts do not last forever but are instead occasionally renegotiated. The wage in a match remains fixed until renegotiation which occurs at a Poisson rate $\gamma(F)$. $\gamma(F)$ is a strictly positive and weakly decreasing differentiable function.[9] At the time of renegotiation, a new wage is determined in a bargaining game with alternating offers between the worker and the firm. I consider equilibria in Markov strategies in the bargaining game in which the wage is weakly increasing in the type. Lastly, I assume that if the worker is indifferent between an offer and the current job, she moves to the type which is higher. The tie breaking rule ensures that the turnover is the same in any equilibrium. In general, all that is needed is that workers move with some probability if the value is the same in order to rule out equilibria in which a positive mass of firm types pay the same wage. The restrictions on the set of equilibria are discussed at the end of this section.

Bargaining game. The set-up closely follows Binmore et al. (1986). The players alternate in making offers. After the proposer has made an offer, the responder chooses to accept or reject the offer. If the offer is accepted, the agents get the payoffs associated with the agreed wage. If the offer is rejected, we move to the next bargaining period and there is a probability that the bargaining process breaks down. If the process breaks down, the parties get their outside option. The probability that there is no breakdown is $(1-\Delta)^{\beta}$ after the worker makes an offer and $(1-\Delta)^{1-\beta}$ after an offer by the firm; $\beta \in (0,1)$ determines the relative bargaining power of the worker. Rejecting an offer causes a delay and a potential a breakdown of the relation. I assume that the times it takes to formulate an offer is very short such that the only relevant friction is the probability of breakdown. In this limit case when the time between offer goes to zero, bargianing occurs in discrete artificial time as in Shimer (2006).[10]

During the bargaining process, the firm and the worker take the outcome of future wage negotiations as fixed. This means that the value function of a job to the worker and firm is treated as fixed. The bargaining game consists of two players: a firm with excess payoff function Π and a worker with (excess) payoff function V. The action set is $\mathbb{R}_+$ for the proposer and $\{\text{Accept}, \text{ Reject}\}$ for the responder. A Markov strategy is such that the offer and acceptance rules only depend on the type and not on the previous history. I define $w_\Delta(F)$ to be a bargaining outcome associated with an Markov-perfect equilibrium (MPE) when the friction is Δ. Definition 1 defines the outcome of the bargaining game.

[9]When I allow firms to choose the contract length in section 6, different types optimally pick different contract lengths.

[10]The artificial time ensures that the model payoffs correspond to the standard Nash bargaining solution in a stochastic environment without OJS; see Coles and Muthoo (2003).

Definition 1 *w is an outcome of the bargaining game for type F if $\lim_{\Delta \to 0} w_{f,\Delta}(F) = w$.*

For an equilibrium, I require that the wage function $w(\cdot)$ is an outcome of an MPE in the bargaining game as the friction goes to zero.

Definition 2 *The wage function $w(F)$ is an equilibrium wage function if for all $F \in [0,1]$, $w(F)$ is an outcome of the bargaining game.*

Value functions. Given an equilibrium wage function $w(\cdot)$, the value function for an employed worker, $W(F, w)$, is given by the expression

$$(\delta+\rho+\gamma(F))W(F,w) = w + \lambda_e \int_0^1 \max\left\{W(\tilde{F}, w(\tilde{F})) - W(F,w), 0\right\} d\tilde{F} + \delta U + \gamma(F) W(F, w(F)). \tag{1}$$

The value function of the worker thus depends on the wage, the search option, and the value of renegotiating. Importantly, in the model, the worker observes the type F and forms rational beliefs about the wage in renegotiation. The value function for an unemployed worker, U, is given by

$$\rho U = b + \lambda_u \int_0^1 \max\left\{W(\tilde{F}, w(\tilde{F})) - U, 0\right\} d\tilde{F}, \tag{2}$$

where $\tilde{F}$ denotes the type from whom an offer is received. The value function of the firm, $\Pi(F, w)$, is given by

$$(\delta + \rho + \gamma(F) + \lambda_e(1 - G(V(F,w))))\Pi(F,w) \quad = \quad x(F) - w + \gamma(F)\Pi(F, w(F)), \tag{3}$$

where $G(V)$ is the fraction of match qualities with an implied value that is strictly less than V or where the value is the same and the type is less. Note that the firms' expected profits are different even if the current wage is the same due to differences in (i) productivity, (ii) turnover, and (iii) the renegotiation value.

It is useful to work with the excess values of the match for the two types of agents. The excess value of the match to the firm is just the profits. The excess value to the worker, $V(F, w)$, is given by the value to the worker of the job, less the value of unemployment. That is,

$$V(F,w) = W(F,w) - U. \tag{4}$$

The reservation wage (bargaining position) of the worker, w_r, is similarly defined as the constant wage that makes a worker indifferent to unemployment. At such a wage, the difference in flow income exactly matches the difference in search option. w_r is thus defined by the following equation

$$w_r = b + (\lambda_u - \lambda_e) \int_0^1 \max\left\{V(\tilde{F}, w(\tilde{F})), 0\right\} d\tilde{F}. \tag{5}$$

The measure F is normalized to only include productivity levels that lead to matches. That is, the productivity satisfies $x(0) > w_r$. The surplus, $S(F, w)$, of the match is the sum of the profits and the excess value to the worker. We can express the match surplus as

$$\begin{aligned}(\delta+\rho)S(F,w) &= x(F) - w_r - \lambda_e \int_0^1 \max\left\{V(\tilde{F}, w(\tilde{F})), 0\right\} d\tilde{F} + \gamma(F)\left(S(F, w(F)) - S(F,w)\right) \\ &+ \lambda_e \int_0^1 1\left(V(\tilde{F}, w(\tilde{F})) \geq V(F,w)\right)\left(V(\tilde{F}, w(\tilde{F})) - V(F,w) - \Pi(F,w)\right) d\tilde{F}. \quad (6)\end{aligned}$$

The surplus of the match depends on the excess productivity, $x(F) - w_r$, the renegotiation option, the search option as well as the duration of the match. The last term in the expression reveals a bilateral inefficiency in the relation that will play a key role in the model. The worker moves whenever the value to the worker is higher in the new firm. But when the worker quits, the profits are lost to the pair. If the profits are positive then the surplus of the match is higher than the worker value. The transition is bilaterally inefficient when $V(\tilde{F}, w(\tilde{F})) < S(F, w)$. Importantly, by agreeing on a higher wage, the worker will quit less often, and since these marginal transitions are bilaterally inefficient, the surplus of the match will be higher. Note, however, that from the planner's perspective, the transitions are efficient as the worker keeps moving to more productive jobs.

Examining (6) reveals that the joint surplus is maximized if the worker moves, if and only if the value to the worker is higher at the new job than the surplus in the current job. Depending on the type of commitment available to the agents, a number of contracts can implement efficient transitions. A simple contract that implements bilaterally efficient transitions entails the worker paying an upfront fee and subsequently being paid the productivity of the match. The profit of the firm is then zero so the worker value is equal to the full surplus. Alternatively, in models with counteroffers, see for example Postel-Vinay and Robin (2002), the outside option (bargaining position) of the worker is given by the full value of the previous match. The worker is thus able to extract the full profits of the current firm from the new firm. At the time of the transitions, the profits associated with staying at the less productive job are zero and the worker value once more captures the full value of the match so that the transition is bilaterally efficient.

Equilibrium. In the bargaining game, the firm and the worker make offers such that the value function of the responder evaluated at the offer is equal to their continuation value. A wage offer associated with a value less than the continuation value is not accepted and, given the costly delay, such an offer is not optimal. Similarly, an offer higher than the continuation value is accepted, but results in a smaller payoff for the proposer. $w_{i,\Delta}(F)$ denotes the wage offer by agent i in the bargaining game with friction Δ and the type is F, for $i \in \{\mathrm{w}, \mathrm{f}\}$, where w and f refer to the workers and the firms, respectively. The following theorem summarizes these results.

Theorem 1 *There exists a unique equilibrium for which the wage function satisfies the differential* *equation*

$$\beta\Pi(F,w(F))\frac{\partial V(F,w)}{\partial w}|_{w=w(F)} + (1-\beta)V(F,w(F))\frac{\partial \Pi(F,w)}{\partial w}|_{w=w(F)} = 0, \tag{7}$$

with the initial condition

$$w(0) = \max\{\beta x(0) + (1-\beta)w_r, w_{min}\}. \tag{8}$$

For all $F \in [0,1]$ *and a sufficiently small* Δ*, the firm's and worker's offer solve*

$$V(F, w_{\mathrm{f},\Delta}(F)) = \max\{(1-\Delta)^{(1-\beta)}\, V(F, w_{\mathrm{w},\Delta}(F)), V(F, w_{min})\}, \tag{9}$$

$$\Pi(F, w_{\mathrm{w},\Delta}(F)) = (1-\Delta)^{\beta}\, \Pi(F, w_{\mathrm{f},\Delta}(F)), \tag{10}$$

with $\lim_{\Delta\to 0} w_{\mathrm{f},\Delta}(F) = \lim_{\Delta\to 0} w_{\mathrm{w},\Delta}(F) = w(F)$*.*

At the end of this section, I discuss why the equilibrium is unique and which assumptions are necessary.

Discussion of the equilibrium. Using the derivative of the value functions, we can rewrite the bargaining equation, using the definition of match surplus ($S(F,w(F)) = \Pi(F,w(F)) + V(F,w(F))$), as[11]

$$\Pi(F,w(F)) = \frac{(1-\beta)\left[1 - \frac{\lambda_e}{w'(F)}\frac{\delta+\rho+\lambda_e(1-F)}{\delta+\rho+\gamma(F)+\lambda_e(1-F)}\Pi(F,w(F))\right]}{\beta + (1-\beta)\left[1 - \frac{\lambda_e}{w'(F)}\frac{\delta+\rho+\lambda_e(1-F)}{\delta+\rho+\gamma(F)+\lambda_e(1-F)}\Pi(F,w(F))\right]} S(F,w(F)), \tag{11}$$

$$V(F,w(F)) = \frac{\beta}{\beta + (1-\beta)\left[1 - \frac{\lambda_e}{w'(F)}\frac{\delta+\rho+\lambda_e(1-F)}{\delta+\rho+\gamma(F)+\lambda_e(1-F)}\Pi(F,w(F))\right]} S(F,w(F)). \tag{12}$$

Compared to bargaining without OJS, there is an extra term given by $\frac{\lambda_e}{w'(F)}\frac{\delta+\rho+\lambda_e(1-F)}{\delta+\rho+\gamma(F)+\lambda_e(1-F)}\Pi(F,w(F))$ which results in the worker receiving a higher share of the surplus. The term exactly captures the increase in the surplus with a higher wage. If the worker leaves for a marginally better firm, the profits are lost. The marginal quits are therefore bilaterally inefficient and a small reduction in turnover increases the match surplus by the change in turnover multiplied by the level of profits. The change in turnover is given by the density of incoming wage offers $\lambda_e/w'(F)$ multiplied by the fraction of the duration of the match that the wage remains fixed for $\frac{\delta+\rho+\lambda_e(1-F)}{\delta+\rho+\gamma(F)+\lambda_e(1-F)}$, reflecting how the worker trades off a higher wage against the match type. As the length of the contract decreases, the wage becomes less important, as compared to the match quality, for the worker. Thus, as the length of the contract decreases, the turnover becomes less responsive to the wage and workers capture a smaller share of the surplus.

In the limit, as the contract length goes to zero ($\gamma(F) \to \infty$), the surplus becomes unresponsive and the wage then solves the standard Nash bargaining solution with perfectly transferable values,

[11] Where we use that $G'(V(F,w(F))) = (\delta + \rho + \lambda_e(1-F))/w'(F)$ and $\frac{\partial V(F,w)}{\partial w}|_{w=w(F)} = (\delta+\rho+\gamma(F)+\lambda_e(1-F))^{-1}$

given by (13) and (14) below:

$$\Pi(F, w(F)) = (1-\beta)S(F, w(F)), \tag{13}$$

$$V(F, w(F)) = \beta S(F, w(F)). \tag{14}$$

The model thus provides a justification, based on continuous renegotiation, for using the Nash bargaining solution with perfectly transferable values, as in Pissarides (1994).[12] Intuitively, it is only future wages that reduce the turnover and as renegotiation becomes very frequent, the contracted wage becomes irrelevant for future wages. Similarly, as the length of the contract goes to infinity ($\gamma(F) \to 0$) and the bargaining power is made symmetric, the differential equation corresponds to that in Shimer (2006) but with a unique initial condition. This limit, as the length of the contract goes to infinity, also corresponds to the values from Mortensen (2003, Section 4.3.4). Since wages are never renegotiated, the worker only cares about the wage and not about the match quality. Importantly, if wages are only sometimes renegotiated, the solution corresponds to none of Pissarides (1994), Mortensen (2003, Section 4.3.4) and Shimer (2006).

Discussion of the types and uniqueness. Compared to the existing models, the notion of type is somewhat different in this model. In the wage posting model of Burdett and Mortensen (1998), firms can pick and commit to any wage. When productivities are homogeneous, firms must be indifferent between all wages. The types can thus be seen as the outcome of a mixed strategy. The types in this paper should not be seen as the outcome of a mixed strategy; in fact, a firm will generally not be indifferent between types even when productivities are homogeneous. Two aspects are important. First, the type coordinates the beliefs of both agents. Thus, unlike the model of Burdett and Mortensen (1998), the firm cannot unilaterally deviate and set a different wage. Second, the expectations about the outcome of future wage renegotiations cannot be changed by agreeing to a different wage today. These expectations about future wages are payoff relevant. In the model of Shimer (2006), firms and workers coordinate on different wages with firms preferring lower wages. Since Shimer does not model renegotiation, the type is not payoff relevant.

It is now useful to discuss why the inclusion of renegotiation, in Shimer (2006), results in a unique equilibrium wage distribution. In the model of Shimer (2006), the types are not payoff relevant as there is no renegotiation and productivities are homogeneous. Without payoff relevant types, (7) implies that the Nash product is constant on the support of wages. For a (sufficiently) small probability of breakdown and an arbitrary initial condition, an offer by the worker and the firm can then be found, on the support of wages, such that (9) and (10) hold. Letting the friction go to zero, the offers can then converge to an arbitrary point on the distribution. Shimer's

[12]The equilibrium corresponds to Pissarides (1994) in the sense that (i) the worker moves to better matches independent of the wage and (ii) values solve the Nash bargaining solution with perfectly transferable values (i.e., treating turnover as fixed). The payoffs are the same once distinct types are defined. However, Pissarides does not define distinct types; he considers only two types of jobs.

model therefore gives a differential equation, implying a constant Nash product, but not an initial condition.[13] When types are distinct, the offer by the firm falls outside the support of wages which gives the initial condition. Without a minimum wage, the bargaining outcomes correspond to the global maximum of the Nash product. Shimer actually conjectured that since there is a unique initial condition for his differential equation such that the Nash product is a local maximum for all wages, the limit of a model with heterogeneous productivities as firms become homogeneous might result in a unique equilibrium. In this paper, I show that introducing an arbitrarily small probability of renegotiation results in Shimer's conjectured equilibrium. The proof of uniqueness goes through if productivities are heterogeneous but there is no renegotiation, thereby verifying Shimer's conjecture.

There might be some concern about a discontinuity in the number of equilibria in the limit as the frequency of renegotiation goes to zero. There are alternative refinements to the model in Shimer (2006) that result in the same unique equilibrium. For example, considering bargaining outcomes that correspond to the limit from an arbitrary large initial friction results in the same unique equilibrium. With a sufficiently large (initial) friction, the two offers converge to the global maximum of the Nash product. which corresponds to the bargaining outcome only if the initial condition is given by (8). Moreover, if $\lambda_e = 0$, only one of the equilibria in Shimer (2006) gives the correct solution. Similarly, if firms have all the bargaining power, the firm can unilaterally decide to deviate below the support. Thus, considering equilibria with the correct limit as the worker's bargaining power or the efficiency of OJS goes to zero is yet another refinement.

Discussion of assumptions. I make two restrictions on the set of equilibria that I consider. First, I restrict my attention to equilibria in which the wage function is weakly increasing in type. To see why this matters, consider the case of continuous renegotiation. Assume that workers move whenever the type is lower than the current type. If the reduced turnover at a lower type more than offsets the lower productivity, the match surplus decreases in the type. The wages will then also decrease in the type which implies that the worker will, in fact, choose to quit for less productive jobs. The literature has, to my knowledge, not analyzed the equilibrium as it has been assumed that the worker moves whenever the job is more productive.

Second, I assume that the worker moves some time if she is indifferent which rules out mass points on the distribution. Shimer (2006) pointed out that assuming that workers never move when they are indifferent results in a larger set of equilibria with different mass points. There are two alternative refinements that result in the unique equilibrium analyzed in this paper. First, if workers move with some probability, which is the approach taken in this paper, there cannot be

[13]Similarly, in wage posting models, the firm's optimal choice of the wage, on the support, implies a differential equation for the wage function. In order to get the initial condition for the wage function, a deviation below the support of the wage distribution must be considered. Such an offer must not be accepted by the worker and it must therefore be that the worker is indifferent between being employed at the lowest wage and unemployment.

a mass point on the support of wages. Alternatively, introducing a firm heterogeneity also breaks the indeterminacy.

Lastly, the model treats the frequency of renegotiation as a parameter. Imagine instead that match productivity and unemployment benefits are linear in a worker's human capital and there is a small cost c to initiate renegotiation. Assume that at a rate $\gamma(F)$, there is a shock which either increases or decreases human capital by κ percent with equal probability. If the change in human capital is sufficiently large such that renegotiation occurs after any shock, we get exactly the wage function in this paper if c and κ are arbitrarily small.

3 Homogeneous productivities

In this section, I impose some restrictions on the general model. First, I assume homogeneous productivity of firms (i.e., $x(F) = x > w_r$). Second, I assume that the expected duration of a wage contract is a fixed fraction, θ, of the expected discounted duration of the job. Under the specification, a small increase in the wage by $w'(F)dw$ decreases the turnover by θdw. θ captures the (marginal) relative importance of the wage and the type. A constant θ is a useful benchmark for varying the amount of commitment.[14] When θ is one, the model corresponds to the case of no renegotiation, as in Shimer (2006), and the limit as θ goes to zero corresponds to continuous renegotiation. The arrival rate of renegotiation solves

$$\gamma(F) = \frac{1-\theta}{\theta}(\delta + \rho + \lambda_e(1-F)).$$

For this specific model, we have a closed-form analytical solution. Theorem 1 presents the distribution function, the inverse of the wage function, and the value functions.

Proposition 1 *The wage offer distribution is given by*

$$F(w) = \frac{(\delta+\rho+\lambda_e)}{\lambda_e}\left(1 - \left[\frac{x-w}{x-\underline{w}}\right]^{1/\theta}\left[1 - \frac{(x-\underline{w})^{1/\theta}}{\underline{w}-w_r}\left(\frac{1-\beta+\beta/\theta}{1-\beta}\right)\frac{(x-w)^{1-1/\theta}-(x-\underline{w})^{1-1/\theta}}{1-1/\theta}\right]^{\frac{\beta}{\theta(1-\beta)+\beta}}\right),$$

and the value functions are given by

$$\begin{aligned}
\Pi(F(w),w) &= \frac{x-w}{(\delta+\rho+\lambda_e)\left[\frac{x-w}{x-\underline{w}}\right]^{1/\theta}\left[1 - \frac{(x-\underline{w})^{1/\theta}}{\underline{w}-w_r}\frac{1-\beta(1-1/\theta)}{(1-\beta)}\frac{(x-w)^{1-1/\theta}-(x-\underline{w})^{1-1/\theta}}{(1-1/\theta)}\right]^{\frac{\beta}{\theta(1-\beta)+\beta}}},\\
V(F(w),w) &= (\underline{w}-w_r)\frac{\left[1 - \frac{(x-\underline{w})^{1/\theta}}{\underline{w}-w_r}\left(\frac{1-\beta+\beta/\theta}{1-\beta}\right)\frac{(x-w)^{1-1/\theta}-(x-w)^{1-1/\theta}}{1-1/\theta}\right]^{1-\frac{\beta}{\theta(1-\beta)+\beta}}}{\delta+\rho+\lambda_e},
\end{aligned}$$

where $\underline{w} = \max\{\beta(x-w_r)+w_r, w_{min}\}$.

Proposition 1 extends the solution in Shimer (2006) to incorporate renegotiation and asymmetric bargaining powers. In order to illustrate how the model behaves, I pick parameter values that are

[14] If the contract length, $\gamma(F)$, was constant across types, then the (marginal) relative importance of the type compared to the wage would be higher for higher types.

Table 1: Parameters and moments

	Value	Moment	Reference
λ_u	0.45	45% Monthly job finding rate	Shimer (2012)
λ_e	0.181	3.2% Monthly job-to-job transition rate	Moscarini and Thomsson (2007)
δ	0.024	5% unemployment rate	–
ρ	0.004	5% annual discount rate	–
β	0.139	2/3 labor share	–

Source: The job finding rate refers to the average over the period 1948Q1-2007Q1, as calculated by Shimer (2012).

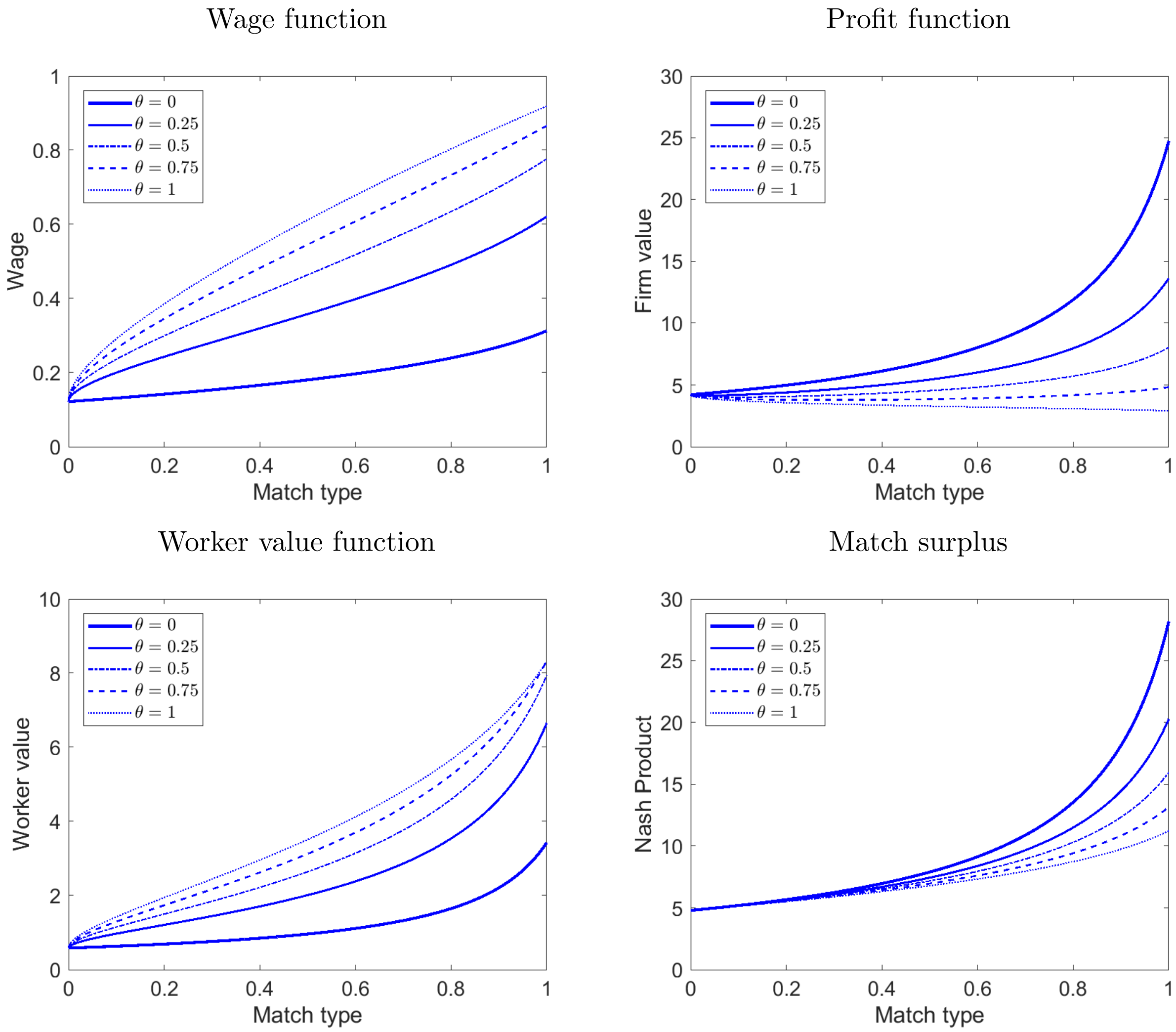
typically used in the literature to model the US labor market. The job arrival rate and the job destruction rate are picked to match the job finding rate and the unemployment rate. Similarly, the job offer arrival rate in employment is picked to match the rate at which workers change employers. I pick an annual discount rate of 5%. I pick the bargaining power, β, such that the worker captures 2/3 of the difference between x and w_r when θ is equal to a half. Furthermore, x and w_r are normalized to one and zero, respectively. The parameter values are presented in Table 1. Figure 1 shows the wage function and the worker's and firm's value as a function of the type for different frequencies of renegotiation. The counterfactuals with different frequencies of renegotiation are using the same parameter values, including the reservation wage (w_r) of the worker.

When renegotiation is continuous, and productivities are homogeneous, the match surplus still increases with the type due to the lower turnover. The higher surplus results in an increasing wage function, unlike the case of no OJS in which the function would be flat. Due to the additional turnover motive, the wage is higher with more infrequent renegotiation. The worker's value function exhibits the same behavior. Worker turnover does not, in equilibrium, depend on the frequency of renegotiation; however, wages decrease if renegotiation becomes more frequent. Thus, profits are increasing in the frequency of renegotiation with the exception of the lowest type. For frequent renegotiation, profits increase in type; however, if renegotiation becomes sufficiently infrequent, profits decrease in type. The level of the match surplus is given by

$$S(F, w(F)) = \int_0^F \frac{x'(\tilde{F}) + \lambda_e \Pi(\tilde{F}, w(\tilde{F}))}{\delta + \rho + \lambda_e(1 - \tilde{F})} d\tilde{F} + \frac{x(0) - w_r}{\delta + \rho + \lambda_e}. \tag{15}$$

With more frequent renegotiation, the level of profits are higher which implies that the level of surplus is higher as well.

Figure 1: Functions for homogeneous productivities



Note: θ refers to the expected fraction of the discounted duration of the match an agreed wage last for. The other parameters are presented in Table 1.

4 Calibration

The previous sections analyzed how the wage distribution and the firm's and worker's values depend on the frequency of renegotiation for a given productivity distribution. In this section, I aim at quantitatively assessing how, if the model is calibrated, the primitives depend on the assumed frequency of renegotiation. The key parameters of interest will be the bargaining power of the worker and the productivity distribution. I use the same transition parameters as in the previous section. I take the estimate of the scale parameter of the wage offer distribution from Gottfries and Teulings (2017), assuming that log wages are Normally distributed, which they find to provide

Table 2: Wage distribution

	Normal	Moment	Reference
σ	0.16	scale parameter	Gottfries and Teulings (2017)
$\bar{F}$	0.019	1.7 mean-min ratio	Hornstein et al. (2007)

a good fit for the data. Their method predominantly uses information from the upper tail of the wage distribution and is therefore less suitable for assessing the lowest wage. Assessing the lowest wage is instead the focus of Hornstein et al. (2007) who find that the mean wage is about 1.5 to 2 times larger than the lowest wage. I use a truncation parameter to target a mean-min ratio of 1.7. The targeted wage offer, $\hat{w}(F)$, is then given by

$$\hat{w}(F) = \exp\left[\sigma \mathcal{N}^{-1}(F(1-\bar{F}) + \bar{F})\right], \tag{16}$$

where $\mathcal{N}$ refers to the CDF of the standard Normal distribution and $\bar{F}$ and σ denote the truncation and scale parameter, respectively. Table 2 presents the parameters of the wage distribution.

Using only wage data, it is not possible to identify the bargaining power separately from the productivity distribution. As in the previous section, I also target a labor share of 2/3 to determine the bargaining power of the worker. Note that in so far as there is capital that is not specific to the worker, and therefore not lost in case of a bargaining breakdown, the bargaining power of the worker would have to be higher to match the same income share. Therefore, we have a conservatively calibrated bargaining power of the worker. I target the 1-99th percentile in the log wage distribution with equal weights. I use a beta distribution for the log productivity with a scale and location parameter. That is the productivity of a match quality F satisfies

$$x(F) = \exp[\mu_x + \sigma_x \mathrm{B}^{-1}\left(F; \alpha_x, \beta_x\right)], \tag{17}$$

where $\mathrm{B}^{-1}\left(F; \alpha_x, \beta_x\right)$ is the inverse of the beta distribution with parameters α_x and β_x. The transition parameters are the same as in the previous section. In the calibration, the reservation wage w_r is set to exactly match the lowest wage. I calibrate the model assuming that wages are (i) continuously, (ii) annually, and (iii) never renegotiated.

Table 3 presents the calibrated parameters. With less frequent renegotiation, the calibrated bargaining power of the worker is lower. The bargaining power is lower because with infrequently renegotiated wages, workers capture a higher share of the surplus due to the response of turnover to the wage. Except in the case when there is no renegotiation, the bargaining power of workers needs to be significantly positive for the model to match a labor share of 2/3.

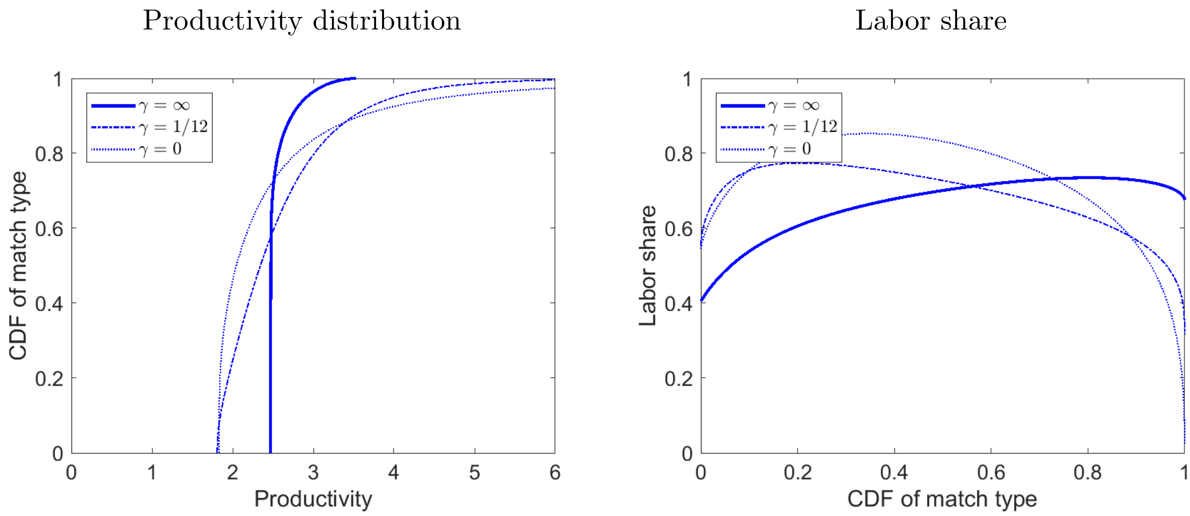
The difference in bargaining power has important implications for the calibrated productivity distribution. Figure 2 shows the implied firm productivity distribution for different frequencies of

Table 3: Parameters

	Normal		
	$\gamma = \infty$	$\gamma = 1/12$	$\gamma = 0$
α_x	0.02	0.3	0.09
β_x	1.69	7.79	3.1×10^6
μ_x	0.9	0.59	0.61
σ_x	0.36	2.36	1.5×10^6
β	0.46	0.19	0.02
$\bar{b}$	-0.24	0.81	0.98

Note: The table shows the productivity parameters and the bargaining power of the worker. γ refers to the assumed Poisson rate by which wages are renegotiated and the value of the transition parameters are presented in Table 2.

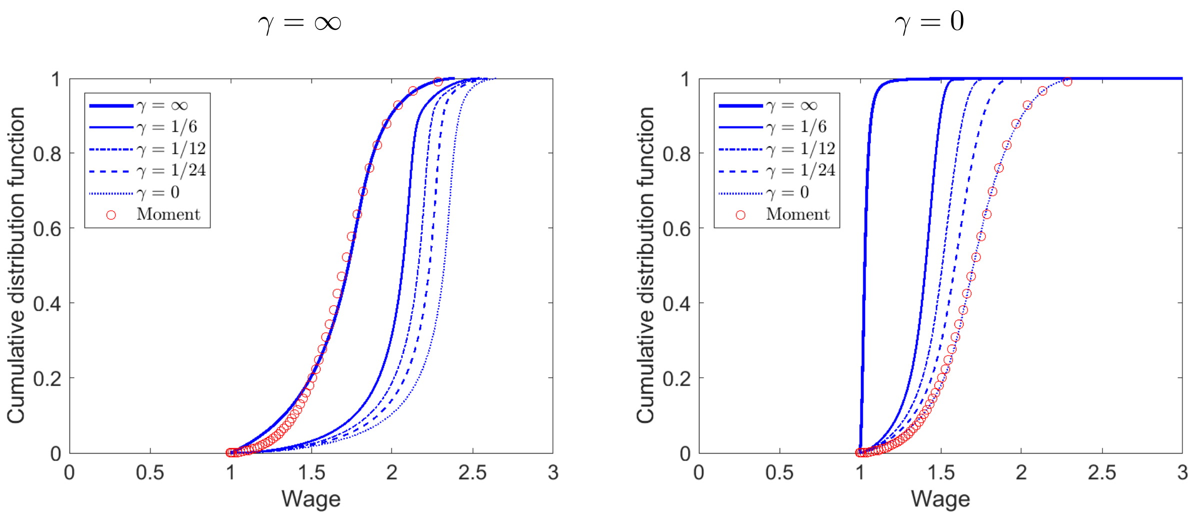
Figure 2: Productivity distribution and labor share



Note: The left-hand panel shows the steady state CDF of the productivity distribution of outstanding matches. The right-hand panel shows the labor share as a function of the rank of match quality in the steady-state distribution. γ refers to the Poisson rate by which wages are renegotiated and the value of the other parameters is presented in Tables 1 and 3.

renegotiation and also the labor share at different firms. The labor share falls more in the tail of the wage distribution when wages are infrequently renegotiated. When the bargaining power of the worker is low and wages are very infrequently renegotiated, much of the wage increase with a better type comes from the endogenous response of turnover to the wage. In the tail of the distribution there are few firms paying the same wage, so the turnover responds very little to the wage. This means that if the worker's bargaining power is low then the worker captures a very small share of the surplus. Indeed, if the density goes to zero, the labor share goes to the worker's bargaining power. For the model with infrequently renegotiated wages to match the same wage distribution, it must then be that the productivity distribution has a fatter right tail; see the left-hand panel. Mortensen (2003) pointed out that the labor share is too small in the tail of the distribution in an estimated wage posting model. I show that allowing for some worker bargaining power resolves this issue.

Figure 3: Wage distributions



Note: The sub titles refer to the contract length that is assumed when the model is calibrated. Moment refers to the targeted wage distribution. The figures show the counterfactual wage distribution that is implied by a change in the contract length holding all parameters fixed. γ refers to the Poisson rate by which wages are renegotiated and the value of the other parameters is presented in Tables 1 and 3.

I now illustrate the response of the wage distribution to a change in the contract length. Figure 3 provides the wage distribution and the counterfactual wage distribution if the contract length is changed. In the counterfactuals, the parameter values, as well as the reservation wage of the worker, are held fixed. This is a useful comparison since the government has separate instruments for these, e.g. adjusting the benefit level and subsidizing hiring. The wage distribution with infrequently renegotiated wages stochastically dominates the wage distribution with more frequent renegotiation. Further, a change in the contract length has a large impact on the wage distribution, and particularly so if there is initially no wage renegotiation.

5 The impact of minimum wages

In this section, I analyze how the wage distribution changes in the model when a minimum wage is introduced. The introduction of a minimum wage has a mechanical effect on impact of moving all mass from below the minimum wage to a point mass at the minimum wage. With OJS, this point mass is spread upwards in the wage distribution due to firms accepting higher wages to reduce turnover, a so-called spillover effect. These two effects are the main focus of my analysis. In addition, there can also be a change in the reservation wage and in the amount of firm entry. The entry effect depends on the functional form of the vacancy posting cost function, and is difficult to identify, and the impact on the reservation wage is, in general, ambiguous.[15] Thus, my main analysis treats firm entry as constant, and adjusts the flow value of unemployment so that the reservation wage is constant.

My analysis focuses on two dimensions: first, on the extent of spillovers across the wage distribution, and, second, on whether there is a spike at the minimum wage. I study the case when the minimum wages is such that all matches are formed (i.e., $w_{min} < x(0)$).

Spillovers. Beginning with spillovers, it can be shown that, if the reservation wage is constant, the minimum wage unambiguously increases wages. This is stated formally in the following proposition.

Proposition 2 *If firm entry is constant, and the value of unemployment is adjusted to keep the reservation wage constant, the wage distribution is increasing in the minimum wage in the sense of first order stochastic dominance.*

To analyze how these spillovers are affected by the frequency of renegotiation, consider two economies, L and H, which have the same productivity distributions, reservation wages, and entry of firms, as well as the same observed wage distributions in a region $[0, \bar{F}]$. Let H have a higher commitment than L, i.e., a lower rate of renegotiation, $\gamma_H(F) < \gamma_L(F)$. Note that, since H and L have the same wage distributions, this implies that H has a lower bargaining power of workers, i.e., $\beta_H < \beta_L$. The following proposition states that the high-commitment economy H will have a higher degree of spillover from a minimum wage increase than the low-commitment economy L.

Proposition 3 *Given a small increase in the minimum wage, $w_H(F) > w_L(F)$ for all $F \in (0, \bar{F}]$.*

Intuitively, without an equilibrium response, an increase in the minimum wage would result in a mass point at the minimum wage. However, given a point mass or a high density, firms would be willing to accept higher wage offers to reduce turnover. Thus, in equilibrium the point mass will spread upwards in the wage distribution. A lower frequency of renegotiation strengthens this mechanism, since turnover then responds more strongly to a wage increase.

To illustrate this mechanism, it is instructive to consider how the slope of the wage function changes as the minimum wage increases. Focusing on the slope at the type paying the minimum wage, $F = 0$, it can be shown that the slope of the wage function unambiguously decreases as the minimum wage increases. However, it falls *more* the higher the worker's bargaining power is, and

[15] This is because the amount of firm entry can either be too high or too low.

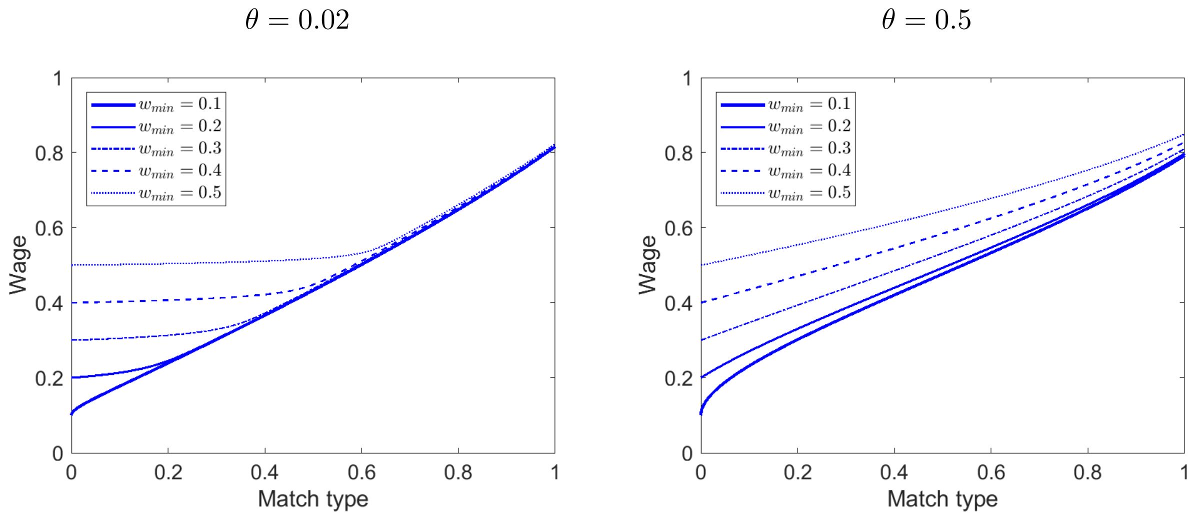
hence it falls *less* for the type H-economy. Formally, the elasticity of the slope with respect to the minimum wage, evaluated at $F = 0$, is

$$\frac{w_{min}}{w'(0; w_{min})} \frac{\partial w'(0; w_{min})}{\partial w_{min}} = -\left[\left(\frac{x(0)}{w_{min}} - 1 \right)^{-1} + (1 - w_r/w_{min})^{-1} \left(\frac{w_{min} - w_r}{\beta(x(0) - w_r)} - 1 \right)^{-1} \right]. \tag{18}$$

The term $\left(\frac{x(0)}{w_{min}} - 1\right)^{-1}$ captures the effect of changing profits and is the same across both economies. The second term is larger when β is large, meaning that there is a larger reduction in the slope when β is large.[16] In the proof of Proposition 3, I show that the same reasoning can be applied for the whole interval $F \in [0, \bar{F})$, and since $w_H(0) = w_L(0)$, the result follows.

Figure 4 illustrates the results when productivities are homogeneous. I consider two values for θ of 0.02 and 0.5, which means that wages are renegotiated, on average, fifty and two times over the discounted duration of the match, respectively. The reservation wage w_r and the bargaining power β are chosen such that the labor share is 2/3 and the lowest wage is equal to 0.1 without a minimum wage.

Figure 4: Impact of a minimum wage policy



Note: θ refer to the length of the wage contract as a fraction of the expected discounted duration of the match.

Prior to the introduction of the minimum wage, the implied wage functions are very similar in the two calibrations (compare the thick lines in Figure 4). However, the impact of an introduction of the minimum wage differs starkly between the calibrations. Comparing the two figures reveals that there is much less spillover above the minimum wage in the calibration with a more frequent renegotiation.

Spikes. So far, I have retained the assumptions from Section 2, guaranteeing a unique equi-

[16] Under the wage posting model, i.e., $\beta = 0$, the second term is zero, in which case the slope of the wage function stays high, and spillovers become especially large.

librium. In this equilibrium, there is no mass point, since workers move when they are indifferent, which means that firms have a strong incentive to increase wages slightly above the mass point. Without a minimum wage the restriction ensuring no mass point is appealing since firm types will generally separate if there is some heterogeneity.

I now relax the assumption that workers move when they are indifferent. As Shimer (2006) has pointed out, such a model can generate mass points on the wage distribution. I will analyze one particular type of equilibrium which has a number of intuitive properties. In particular, I consider equilibria in which there is a positive mass point at the minimum wage and firms above the minimum wage separate.

I introduce a free parameter ϕ that governs how much mass there is at the minimum wage compared to density just above the minimum wage. This parameter regulates how much the mass at the minimum wage increases as the minimum wage increases. This can, for example, be motivated by a very large mass point necessitating a certain density just above the minimum wage due to some randomness in bargaining.

The mass point at the minimum wage satisfies

$$F = \frac{\phi}{\lambda_e} \frac{\delta + \rho + \gamma(F) + \lambda_e(1 - F)}{x(F) - w_{min}} \frac{(1 - \beta)(w_{min} - w_r) - \beta(x(F) - w_r)}{(1 - \beta)(w_{min} - w_r)}, \tag{19}$$

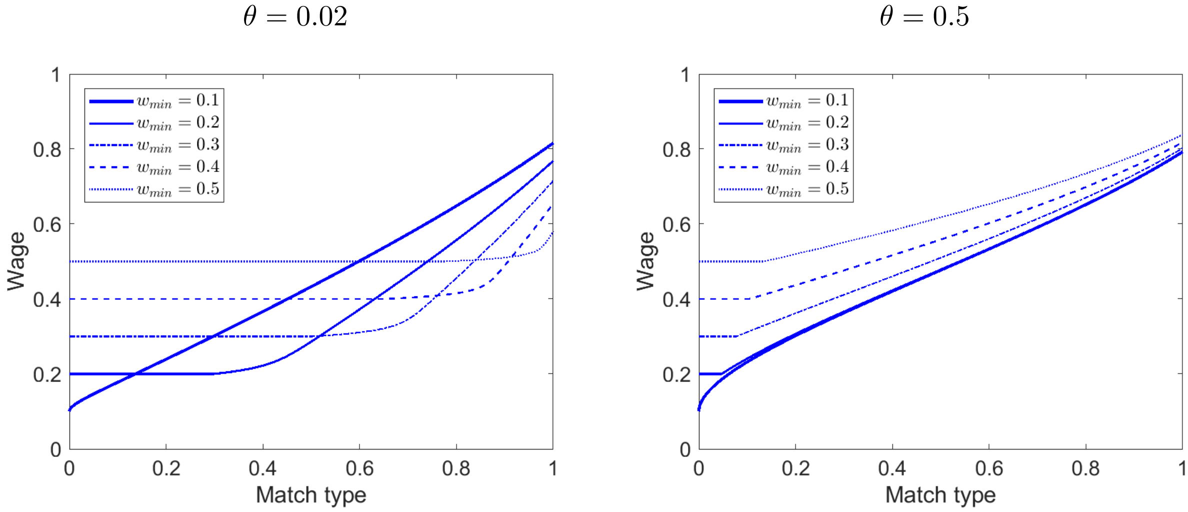
where ϕ measures the ratio of the mass at the minimum wage and density just above the minimum wage. If there is no minimum wage, there is no mass point. (19) implies directly that the spike is higher when wages are renegotiated more frequently. This occurs as the density just above the minimum wage is much higher if the frequency of renegotiation is higher. When wages are renegotiated frequently, the spillover from an increase are less which implies that there is a high density just above the minimum wage. This implies that the spike at the minimum wage is high as well.

To illustrate the effect of increasing the renegotiation frequency, I will again rely on a model with homogeneous productivities. The mass point at the lowest wage is then given by

$$F = \frac{\phi}{\lambda} \frac{\delta + \rho + \lambda_e}{\theta \frac{(1-\beta)(x-w_{min})(w_{min}-w_r)}{(1-\beta)(w_{min}-w_r)-\beta(x-w_r)} + \phi}. \tag{20}$$

I will consider a value of $\phi = 0.04$, so that a mass point is created following the increase in the minimum wage. The equilibrium is then analyzed when the minimum wage is increased in Figure 5. With frequent renegotiation, the mass point at the minimum wage is much higher. In particular, with very frequent renegotiation the mass point at the minimum wage can be *higher* than the mass of firms previously paying a wage at or below the minimum wage.

Figure 5: Impact of a minimum wage policy with mass point ($\phi = 0.04$)



Note: θ refer to the length of the wage contract as a fraction of the expected discounted duration of the match. ϕ refers to the ratio of the density and the mass point at the minimum wage and the density just above the minimum wage.

6 Endogenous contracts

In this section, I extend the model by allowing firms to optimally pick the length of the contract after meeting the worker and having seen the type of the match. After the firm has decided on the contract length that will last throughout the match, the firm and the worker bargain over a wage. When deciding on the length of the contract, the firm takes the wage function at other firms as given. The firm, on the other hand, internalizes that the bargaining outcome will change with the contract length. Unlike the model in the previous sections, the firm can affect the wage outcome in future renegotiations by changing the contract length. At the margin, an increase in the contract length will increase the wage. The firm may prefer a higher wage than the wage resulting from continuous renegotiation, in order to reduce the turnover. The firm will, in that case, pick a positive contract length, and the profits will satisfy the standard envelope condition. In effect, the firm sets the wage to maximize the profits. Alternatively, it could be that the firms would optimally set a lower wage than the wage resulting from continuous renegotiation. Wages will then be continuously renegotiated and solve the Nash bargaining solution with perfectly transferable values. In equilibrium, profits satisfy the lower of: (i) the standard envelop condition or (ii) the Nash bargaining solution with perfectly transferable values. The change in profits are given by

$$\frac{\partial \bar{\Pi}(F)}{\partial F} = \begin{cases} \min\left\{\frac{x'(F)}{\delta+\rho+\lambda_e(1-F)}, (1-\beta)\frac{x'(F)+\lambda_e\bar{\Pi}(F)}{\delta+\rho+\lambda_e(1-F)}\right\}, & \text{if } \bar{\Pi}(F) = \beta\bar{S}(F), \\ \frac{x'(F)}{\delta+\rho+\lambda_e(1-F)}, & \text{otherwise.} \end{cases} \tag{21}$$

Firms will opt for a continuous renegotiation if the productivities increase quickly with the type as compared to the level of profits, or workers have a high bargaining power. Similarly, the initial condition is given by

$$\bar{\Pi}(0) = (1-\beta)\frac{x(0) - w_r}{\delta + \rho + \lambda_e}. \tag{22}$$

Using the expression for the bargaining solution, (11), we then get an expression for the contract length that would rationalize $\bar{\Pi}(F)$.

Theorem 2 *There exists an equilibrium in which the contract length solves*

$$\gamma(F) = (\delta + \rho + \lambda_e(1-F)))\, \frac{\beta\Pi(F, w(F))}{(1-\beta)V(F, w(F)) - \beta\Pi(F, w(F))}, \tag{23}$$

where profits are given by the (21) *and* (22).

The proof is presented in Appendix A.4. A few aspects are interesting to note. First, the lowest match quality always continuously renegotiates. When firms have all the bargaining power, the case analyzed in Coles (2001), Gautier et al. (2010), and Coles and Mortensen (2016), the optimal contract from the firm's perspective is to never renegotiate the wage (or, alternatively, that all future wage expectations respond to any wage change). Except in these cases, there will be an intermediate value for the frequency of renegotiation. Conditional on the asset values, the optimal contract length decreases in the bargaining power of the worker. This occurs because when the worker bargaining power is lower, the contract must be longer in order to result in the same wage outcome.

In the case of homogeneous productivities, the solution is particularly easy as all contract lengths will imply the same profits. The equilibrium profits of the lowest type firm equal

$$\bar{\Pi}(0) \quad = \quad (1-\beta)\frac{x - w_r}{\delta + \rho + \lambda_e}. \tag{24}$$

Since the profits for the other match types are the same, we get the simple expression for the surplus

$$\bar{S}(F) \quad = \quad \frac{x - w_r}{\delta + \rho + \lambda_e}\left(1 + (1-\beta)\ln\left(\frac{\delta + \rho + \lambda_e}{\delta + \rho + \lambda_e(1-F)}\right)\right). \tag{25}$$

Using the expression for the bargaining solution, (11), we get that the contract length must satisfy

$$\gamma(F) = \frac{\beta}{1-\beta}\frac{(\delta + \rho + \lambda_e(1-F))}{\ln\left(\frac{\delta+\rho+\lambda_e}{\delta+\rho+\lambda_e(1-F)}\right)}. \tag{26}$$

For the case of homogeneous productivities, there is always an intermediate region for the frequency of renegotiation unless one of the parties has all the bargaining power. The optimal contract length increases in the bargaining power of the firm and also in the type.

Discussion. The presence of OJS rationalizes firms' use of *wage contracts*. Further, the model also endogenously implies that the lowest type renegotiates continuously, thereby giving the smallest share of the surplus to the worker. Hence, the worst jobs do not pay to retain the worker but instead only pay according to the worker's bargaining power.

The model with endogenous contract lengths generates a justification for using the wages associated with the wage posting model, even if workers have some bargaining power. A marginal change in the wage is perfectly offset by the change in the turnover for the firm types that pick an *interior* level of renegotiation. The wage thus satisfies the conditions associated with the standard wage posting model (if the timing contract length is decided prior to meeting the worker, an extra incentive to set a higher wage in order to increase hiring is included). Thus, interestingly, even though the model has different bargaining powers and infrequent renegotiation, the wages can still (sometimes) be described by the simple differential equation from the wage posting model.

In the models of Coles (2001) and Coles and Mortensen (2016) there are multiple equilibria.[17] Since firms have all the bargaining power in these models, I provide an justification for the analyzed equilibrium based on the fact that it corresponds to the firms' optimal choice of renegotiation.

7 Conclusion

In this paper, I study bargaining in an environment with search on the job and renegotiation. I find weak sufficient conditions for a unique equilibrium. I study the role of the frequency of renegotiation and I show that it plays an important role in the model. The less frequent is renegotiation, the more turnover responds to the wage. The more turnover responds to the wage, the higher will be the share of the surplus captured by the worker. As renegotiation becomes very frequent, the impact of the wage on turnover disappears and the equilibrium can be described by the Nash bargaining solution with perfectly transferable values. Further, following an increase in the minimum wage, wages previously above the minimum wage also increase, and the more so the less frequent is the renegotiation. Lastly, if firms were to set the frequency of renegotiation, we only get the corner cases analyzed in the literature for particular parameter values.

[17] In the wage posting models of Coles (2001) and Coles and Mortensen (2016), firms have all the bargaining power and can continuously adjust the wage. Still, in the equilibrium analyzed, all future wage expectations respond if the firm changes the wage. In the model of Coles (2001), all future wage expectations change to the reservation wages if the firm deviates and sets a lower wage. Given these wage expectations, the worker leaves for any better offer if the firm previously deviated. This implies that, if the firm has previously set a wage lower than the equilibrium wage, it is optimal for the firm to set a wage equal to the reservation wage forever after. In Coles and Mortensen (2016), the match quality is not observable, and an equilibrium where beliefs only depend on the current wage is analyzed.

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A Proofs

A.1 Theorem 1

Proof. The proof consists of the following steps. First, I show that in any equilibrium, the offers must satisfy (9) and (10). Second, I show that this implies that the wage distribution satisfies the differential equation (7) with initial condition (8). I show that this differential equation has a unique solution, which means that the equilibrium is unique if it exists. Finally, I demonstrate the existence of an equilibrium by construction.

Standard arguments imply that all offers made must be accepted, and at a wage such that the excess value is positive for both parties. If either of these conditions is not met, then an agent without a positive excess value could make an acceptable offer which extracts a small amount.[18]

[18] It is assumed that $x(0) > w_r$ which implies that all matches have a positive surplus.

I first show that (9) and (10) hold. (9) gives the condition on the firm's offer such that the worker is indifferent regarding accepting or rejecting the offer and (10) is the analogous equation for the worker's offer. To prove that these hold, note that, since there is agreement after both offers, the value of acceptance, the left-hand side, has to be weakly preferred to rejection, implying a weak inequality for (9) and (10). Thus, it suffices to rule out a strict inequality.

Suppose that (10) would be a strict inequality, (i.e., that the firm strictly prefers acceptance to rejection). Then, it is possible for the worker to choose a higher wage offer that will still be accepted by the firm.[19] The worker would always choose to make such an offer which implies that the firm must be indifferent between accepting and rejecting the worker's offer (i.e., (10) must hold).

Suppose instead that (9) does not hold with equality. By a symmetric argument, whenever the profit function is strictly decreasing in the wage, the firm would always offer a wage such that (9) holds with equality. Thus, it is sufficient to show that the profit function is strictly decreasing in the wage. The distribution of worker values cannot have any mass points, as there would otherwise be an incentive for a firm to offer a slightly higher wage to reduce the turnover.

Consider a type F' for which the distribution function is differentiable at the bargaining outcome. Suppose that the profit function would be weakly increasing in the wage at this point. Combining (10) with a weak inequality of (9), gives

$$\begin{aligned} & -\frac{V(F,w_{\mathrm{w},\Delta}(F))^{1/(1-\beta)}-V(F,w_{\mathrm{f},\Delta}(F))^{1/(1-\beta)}}{w_{\mathrm{w},\Delta}(F)-w_{\mathrm{f},\Delta}(F)}\Pi(F,w_{\mathrm{f},\Delta}(F))^{1/\beta} \\ \geq \quad & V(F,w_{\mathrm{w},\Delta}(F))^{1/(1-\beta)}\frac{\Pi(F,w_{\mathrm{w},\Delta}(F))^{1/\beta}-\Pi(F,w_{\mathrm{f},\Delta}(F))^{1/\beta}}{w_{\mathrm{w},\Delta}(F)-w_{\mathrm{f},\Delta}(F)} \end{aligned} \tag{27}$$

taking the limit as $\Delta \to 0$, gives

$$0 \geq \frac{\partial \Pi(F',w(F'))}{\partial w}\frac{1-\beta}{\Pi(F',w(F'))}+\frac{\partial V(F',w(F'))}{\partial w}\frac{\beta}{V(F',w(F'))}.$$

Since the profit functions is weakly increasing in the wage, and the value function is strictly increasing, the left-hand side expression is strictly positive, implying a contradiction. Therefore, at every point F' where the cumulative distribution function is differentiable, the offers solve (9) and (10) with the limit (7) as that probability of breakdown goes to zero. The distribution function is continuous since it has no mass points, and, as a monotone function, differentiable in worker values (and the wage) almost everywhere. Thus, the profit function, being a function of the distribution function, is differentiable at the bargaining outcome for almost all types.

The derivative of the profit function with respect to the wage is given by

$$(\delta+\rho+\gamma(F)+\lambda(1-G(V)))\frac{\partial \Pi(F,w(F,V))}{\partial w}=-1+\lambda\frac{G'(V)\Pi(F,w(F,V))}{(\delta+\rho+\gamma(F)+\lambda(1-G(V)))} \tag{28}$$

[19] The only requirement is a technical condition that the profit function does not discontinuously jump down as the wage increases. This is true since the local decrease in the profit function is bounded by the increased wage cost.

where $w(F,V)$ is defined such that $V(F,w)=V$. Combining (28) and (7) where the distribution function is differentiable we get

$$G'(V(F,w(F))) = \frac{(\delta+\rho+\gamma(F)+\lambda(1-G(V(F,w(F)))))}{\lambda}\left(\frac{1}{\Pi(F,w(F))} - \frac{\beta}{(1-\beta)V(F,w(F))}\right) \tag{29}$$

Moreover, whenever the wage function $w(F)$ is continuous around F', this implies that $V(F,w(F))$ and $\Pi(F,w(F))$ are as well.[20] (29) therefore implies that the density is continuous around F'. Since (28) is strictly negative for F and continuous in F, the derivative must also be negative for F' sufficiently close to F. Hence for all types the profit function is decreasing function of the wage for a small region around the bargaining outcome. This argument assumed that the distribution function is differentiable, which will leave out at most countably many points, and hence not affect the integral of the derivative of the profit function, which is the central object. (Whenever no firm offers a worker value close to that associated with F, the profits are clearly decreasing in the wage.) This means that the profit function must be decreasing in the wage in a region around the bargaining outcome. (9) and (10) must hold with equality for all types.

I will now show that, in the interior of the support of the type distribution, the Nash product is increasing in the wage to the left of the bargaining solution, and decreasing in the wage to the right of the bargaining solution. To analyze the derivative of the (log) Nash product for a type F with respect to the wage, it is helpful to index wages with types. Consequently, I write the derivative at a wage w as $D(F,F')$, where F' is defined such that $V(F,w)=V(F',w(F')))$, i.e. F' is the type where the worker receives the same equilibrium value as when getting w from type F. I assume again that the distribution function is differentiable at F', which since it will only leave out countably many points does not affect the integral of the derivative. The expression for $D(F,F')$ is:

$$\frac{D(F,F')}{\frac{\partial V(F,w)}{\partial w}} = \frac{\beta}{V(F',w(F')))} - (1-\beta)\left[\frac{1}{\Pi(F,w)} - \frac{\delta+\rho+\lambda_e(1-F')}{\delta+\rho+\gamma(F)+\lambda_e(1-F')}\frac{\lambda_e}{w'(F')}\right]. \tag{30}$$

By the definition of the differential equation (7), we have $D(F,F)=0$, i.e. the value is zero when $F=F'$. Now, suppose that $w<w(F)$, which means that $F'<F$, as the wage function is strictly increasing in the match quality. I want to show that $D(F,F')>0$, and for this purpose, I use that $D(F',F')=0$ and that $\frac{D(F,F')}{\frac{\partial V(F,w)}{\partial w}} > \frac{D(F',F')}{\frac{\partial V(F',w)}{\partial w}}$ when $F>F'$. Indeed, since $\gamma(F)\le\gamma(F')$ and $\Pi(F,w)>\Pi(F',w(F'))$, the whole expression in square brackets is smaller for F, which implies that the right hand side is larger for F than for F'.[21] By analogous reasoning, $D(F,F')<0$ whenever $F<F'$.

Combining (9) and (10) when the minimum wage does not bind, we see that the worker offer $w_{w,\Delta}(F)$ and the firm offer $w_{f,\Delta}(F)$ imply the same Nash product. Furthermore, since the

[20] This follows immediately from (1) and (3), as a continuous strictly increasing wage function implies that $G(V)$ is continuous.

[21] The firm profits are higher for a higher type conditional on the worker value. This is true since the worker quits less often after renegotiation and these quits are bilaterally inefficient.

Nash product is almost everywhere differentiable, it is also continuous, which implies that, for any sufficiently small Δ, there exist a unique combination $w_{\mathrm{f},\Delta}(F)$ and $w_{\mathrm{f},\Delta}(F)$ satisfying (9) and (10) with $w_{\mathrm{w},\Delta}(F) < w(F) < w_{\mathrm{f},\Delta}(F)$ on the interior support. Taking the limit of (27) we see that the left and right derivative of the profit function with respect to the wage exists as $w_{\mathrm{w},\Delta}(F) < w(F) < w_{\mathrm{f},\Delta}(F)$. Since $G'(V(F, w(F))$, by (29), is a continuous function of F the left and the right limit are the same which implies that the function is differentiable everywhere in the interior of the support.

Now consider why there is only one initial condition that is consistent with an equilibrium and similarly why there cannot be an gap in the support of wages. If the initial condition is greater than (8), then the Nash product is, for the lowest type, decreasing in the wage at the lowest wage. Hence, we get a contradiction, since it is not possible to find an offer by the worker and firm with equal Nash products arbitrarily close to the bargaining outcome, which is required by (9) and (10). Similarly, an initial condition lower than (8) is not consistent with an equilibrium as it implies that the Nash product is increasing in the wage at the initial wage, unless the lowest wage is lower than the minimum wage, but this is also excluded. A very similar argument rules out gaps on the support of wages. A worker in the highest type below the gap receives at least a share β of the surplus. Hence, a worker in the lowest match above the gap must receive a share of the surplus that is strictly higher than β. Therefore, the Nash product is, for the lowest type above the gap, decreasing in the wage at the bargaining outcome. This again implies that two offers around the lowest wage above the gap satisfying (9) and (10) cannot be found.

Lastly, it remains to show that the solution is unique. First, by Peano's existence theorem, a solution exists. Second, we prove that there is a unique solution by contradiction. Assume that there exist two solutions $w_1(F)$ and $w_2(F)$ such that $w_1(F) < w_2(F)$ for $F \in (\bar{F}, \bar{F} + \epsilon]$ and $w_1(F) = w_2(F)$ for $[0, \bar{F}]$. For $F \in (\bar{F}, \bar{F} + \epsilon]$, we thus have $\Pi_1(F, w_1(F)) > \Pi_2(F, w_2(F))$. Further, there exist an $\bar{\epsilon} > 0$ such that for all $F \in (\bar{F}, \bar{F} + \bar{\epsilon}]$ we have $V_1(F, w_1(F)) < V_2(F, w_2(F))$. Since

$$w'(F) = (1-\beta)\lambda_e \frac{\delta + \rho + \lambda_e(1-F)}{\delta + \rho + \gamma(F) + \lambda_e(1-F)} \frac{V(F, w(F)\Pi(F, w(F)))}{(1-\beta)V(F, w(F)) - \beta\Pi(F, w(F))}, \tag{31}$$

it must be that $w_1'(F) > w_2'(F)$ for $F \in (\bar{F}, \bar{F} + \bar{\epsilon}]$. The difference in wages $(w_2(F) - w_1(F))$ is non-increasing in F for all $F \in (\bar{F}, \bar{F} + \bar{\epsilon}]$. Since the wages are the same at $\bar{F}$ $(w_1(\bar{F}) = w_2(\bar{F}))$, and in the region $F \in [\bar{F}, \bar{F} + \bar{\epsilon}]$, the difference is non-increasing, it must be that $w_2(F) - w_1(F) \leq 0$ for $F \in [\bar{F}, \bar{F} + \bar{\epsilon}]$ which contradicts the initial conjecture.

(7) implies that the profit function is decreasing in the wage. Hence, given (7) and (8), the offer strategies specified in equations (9) and (10) together with the acceptance strategy in which workers (firms) accept weakly higher (lower) wages than the firms' (workers') offer, clearly constitute an MPE in the bargaining game for a sufficiently small Δ. Taking the limit as the probability of breakdown is going to zero ($\Delta \to 0$) of (9) and (10) gives (7) and (8). (7) and (8) are therefore the solution to a unique equilibrium. ■

A.2 Proposition 2

Rewriting (7), we get

$$w'(F) = \lambda_e \frac{\delta + \rho + \lambda_e(1-F)}{\delta + \rho + \gamma(F) + \lambda_e(1-F)} \frac{(1-\beta)\Pi(F, w(F))V(F, w(F))}{(1-\beta)V(F, w(F)) - \beta\Pi(F, w(F))}. \tag{32}$$

The proof will be by contradiction. As the wage function is continuous, if the wage function with the low minimum wage L is weakly higher, than the wage function with a high minimum wage H, at some point it must exist a first point when they are the same. Denote this point by $\tilde{F} \in (0, \bar{F}]$. At $\tilde{F}$ we have $w_H(\tilde{F}) = w_L(\tilde{F})$. The excess value function at a type F is given by

$$V(F, w(F)) = \frac{w_{min} - w_r}{\delta + \rho + \lambda} + \int_{w_{min}}^{w(F)} \frac{1}{\delta + \rho + \lambda(1 - w^{-1}(\tilde{w}))} d\tilde{w}, \tag{33}$$

Since the wage distribution with a high minimum wage H first order stochastically dominates the distribution for the low minimum wage L (i.e., $w_L^{-1}(\tilde{w}) < w_H^{-1}(\tilde{w})$) below the wage $w(\tilde{F})$, (33) implies that $V_H(\tilde{F}, w_H(\tilde{F})) < V_L(\tilde{F}, w_L(\tilde{F}))$. This in turn implies that at $\tilde{F}$

$$\frac{\Pi_L(\tilde{F}, w_L(\tilde{F}))}{V_L(\tilde{F}, w_L(\tilde{F}))} < \frac{\Pi_H(\tilde{F}, w_H(\tilde{F}))}{V_H(\tilde{F}, w_H(\tilde{F}))}, \tag{34}$$

as profits are the same. Combining (34) with (32) implies that the change in the derivative is greater for L which implies that $w'_H(\tilde{F}) > w'_L(\tilde{F})$. The derivative of the wage function must be higher for L ($w'_H(F) < w'_L(F)$), for some region below $\tilde{F}$, for the wage function to be the same at $\tilde{F}$. Since the derivative of the wage function is continuous, there must exist a point at which the derivatives are the same. Denote the last point for which the derivative is the same in $[0, \tilde{F}]$ by $\hat{F} < \tilde{F}$. As $w_H(\hat{F}) > w_L(\hat{F})$ and $w'_H(F) \geq w'_L(F)$ for $F \in [\hat{F}, \tilde{F}]$, we have that $w_H(\tilde{F}) > w_L(\tilde{F})$. This contradicts the initial conjecture that the wage functions are the same at $\tilde{F} \leq \bar{F}$.

A.3 Proposition 3

Proof. Define $w(F)$ as the wage function for the common region $[0, \bar{F}]$ prior to the introduction of the minimum wage and $w_i(F)\ \forall i \in \{L, H\}$ as the new wage function. Using (32), we can rewrite the ratio of the derivative of the wage function after and before the increase in the minimum wage as

$$\frac{w'_i(F)}{w'(F)} = \frac{x(F) - w_i(F)}{x(F) - w(F)} \frac{1 - \frac{\beta_i}{1-\beta_i}\frac{\Pi(F, w(F))}{V(F, w(F))}}{1 - \frac{\beta_i}{1-\beta_i}\frac{\Pi(F, w_i(F))}{V_i(F, w_i(F))}}. \tag{35}$$

At the minimum wage, for the lowest type, all terms are the same in L and H except for the bargaining power β_i. The second term is smaller whenever the bargaining power is higher. Thus, the change in the derivative evaluated at the minimum wage for the lowest type is higher in economy L than in economy H. Thus, for types F sufficiently close to 0, we have that $w_H(F) > w_L(F)$. The rest of the proof will be by contradiction. Denote the first point when the wage functions are the same by $\tilde{F} \in (0, \bar{F}]$ (such a point exists as the wage function is continuous). At $\tilde{F}$ we have $w_H(\tilde{F}) = w_L(\tilde{F})$. Since the wage distribution in calibration H first order stochastically dominates the distribution for L (i.e., $w_L^{-1}(\tilde{w}) < w_H^{-1}(\tilde{w})$) below the wage $w(\tilde{F})$, (33) implies that $V_H(\tilde{F}, w_H(\tilde{F})) < V_L(\tilde{F}, w_L(\tilde{F}))$. The first term in (35) is the same for L and H at $\tilde{F}$ whereas

$$\frac{\Pi(\tilde{F}, w_L(\tilde{F}))}{V_L(\tilde{F}, w_i(\tilde{F}))} < \frac{\Pi(\tilde{F}, w_H(\tilde{F}))}{V_H(\tilde{F}, w_i(\tilde{F}))}, \tag{36}$$

and, by (34) from Proposition 2 we get

$$\frac{\Pi(\tilde{F}, w_H(\tilde{F}))}{V_H(\tilde{F}, w_i(\tilde{F}))} < \frac{\Pi(\tilde{F}, w(\tilde{F}))}{V(\tilde{F}, w(\tilde{F}))}. \tag{37}$$

Combining this with the assumption that $\beta_H < \beta_L$ implies that the change in the derivative is greater for L which implies that $w'_H(\tilde{F}) > w'_L(\tilde{F})$. The derivative of the wage function must be higher for L ($w'_H(F) < w'_L(F)$), for some region below $\tilde{F}$, for the wage function to be the same at $\tilde{F}$. Since the derivative of the wage function is continuous, there must exist a point at which the derivatives are the same. Denote the last point for which the derivative is the same in $[0, \tilde{F}]$ by $\hat{F} < \tilde{F}$. As $w_H(\hat{F}) > w_L(\hat{F})$ and $w'_H(F) \geq w'_L(F)$ for $F \in [\hat{F}, \tilde{F}]$, we have that $w_H(\tilde{F}) > w_L(\tilde{F})$. This contradicts the initial conjecture that the wage functions are the same at $\tilde{F} \leq \bar{F}$. ■

A.4 Proposition 2

Proof. The proof consists of two steps. First, I show that no firm wants to change the contract length so that the bargaining outcome changes. Second, I show that given the contract length there is a bargaining outcome consistent with a MPE which rationalizes the value function.

Equation (21) implies that the profits increase weakly less with the type than the envelop condition. Hence, no firm would want to set a different contract length if that implied a higher wage. A firm would not want to set a lower wage that is on the interior where the envelop condition holds. Similarly, as the derivative of the Nash product is weakly higher for a higher type, the firm cannot set a contract length such that the wage is on the interior where the firms continuously renegotiates. Thus, no firm wants to change the contract length given the wage function implied by (21).

Second, it remains to show that the value functions can be rationalized as bargaining outcomes. We have to consider three cases: the interior of the region in which there is continuous renegotiation, infrequent renegotiation and for the types that are at the boundary. Within the region with infrequent renegotiation we have that

$$w'(F) = \lambda_e \bar{\Pi}(F). \tag{38}$$

(38) together with $\gamma(F)$ defined by (11) gives (23). This implies that the derivative of the Nash product is zero at the bargaining outcome. It remains to show that the Nash product is weakly

increasing prior to the bargaining outcome and weakly decreasing thereafter so that two offers around the bargaining outcome can be found. The derivative of the Nash product at w satisfying $V(F, w) = V(F', w(F'))$ is given by

$$\frac{\beta}{V(F', w(F'))} - \frac{(1-\beta)}{\Pi(F, w)} + (1-\beta)\frac{\lambda_e}{w'(F')}\frac{\delta + \rho + \lambda_e(1-F')}{\delta + \rho + \gamma(F) + \lambda_e(1-F')}. \tag{39}$$

We need to show that the Nash product is increasing prior to the bargaining outcome and decreasing thereafter. When wages are infrequently renegotiated, we can use the first-order condition from the firm's optimal choice of contract length, $\lambda_e \Pi(F, w(F)) = w'(F)$. Simplifying the derivative, with respect to F', of the first-order condition of the Nash product, we get

$$\begin{aligned}
&-\frac{\beta}{V(F', w(F'))^2}\frac{w'(F')}{\delta + \rho + \lambda_e(1-F')} + \frac{(1-\beta)}{\Pi(F, w)^2}\frac{\partial \Pi(F, w)}{\partial w}\frac{\delta + \rho + \lambda_e(1-F')}{w'(F')} \\
&-\frac{x'(F')}{\delta + \rho + \lambda_e(1-F')}\frac{(1-\beta)}{\Pi(F', w(F'))^2}\frac{\delta + \rho + \lambda_e(1-F')}{\delta + \rho + \gamma(F) + \lambda_e(1-F')} \\
&-\lambda_e(1-\beta)\frac{1}{\Pi(F', w(F'))}\frac{\gamma(F)}{(\delta + \rho + \gamma(F) + \lambda_e(1-F'))^2}.
\end{aligned} \tag{40}$$

For F' close to F, all terms must be negative. Further, we know that the derivative is zero for $F' = F$ which implies that for F' is sufficiently close to F, the derivative is positive (negative) for $F' < F$ ($F' > F$). Two offers around each point $w(F) > w_{min}$ can then be found which satisfy (9) and (10).

Lastly, it remains to show that the bargaining outcome is consistent at the boundary and for those that renegotiate continuously. The continuous contract lengths occur at the boundary as the types that renegotiate are a closed set of types. For types that renegotiation continuously, the surplus is fix as a function of the wage. The bargaining outcome therefore corresponds to the unique maximum of the Nash product. Two offers can then be found which satisfying (9) and (10) which converge two the bargaining outcome. ■