The goal of this research is to trigger the devolution of the system balancing responsibility entirely belonging to the transmission system operator (TSO) to several local distribution system operators by fairly allocating system balancing cost based on a cost-causality principle. Within the devolved system balancing scheme, distribution system operators (DSOs) have appropriate motivation for reducing the variability and uncertainty caused by units in their own area. As the number of renewable electricity sources (RES) being connected to the local distribution system increases, it would be advantageous for the TSO to share the increasing burden of the system balancing responsibility with multiple DSOs. To achieve this, we suggest that, first DSOs be designated as the representatives of their own jurisdictions with primary economic responsibility for balancing payments that are originally charged to each energy market participant. Second, this research proves that a cost-causality based cost allocation scheme (CC-CAS) is superior to an energy-amount based cost allocation scheme (currently widely used) in terms of economic efficiency. Additionally, to avoid the side effect that a DSO with a large amount of RES may face a high and risky balancing payment under the CC-CAS, this research also proposes an optimal balancing payment insurance (BPI) contract which helps the DSO hedge the risks associated with uncertain balancing payments.
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Keywords  System Balancing Responsibility, Devolution Principle, Cost-causality Principle, System Balancing Cost Allocation, Risk Hedging Contract

JEL Classification  D21, G22, L94

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Contractual Framework for the Devolution of System Balancing Responsibility from the Transmission System Operator to Distribution System Operators¹

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1. Introduction

Authorities in many jurisdictions, especially in Europe, California, and New York, have been eagerly trying to integrate more renewable electricity sources (RES) into the power grid to realize their dream of a low carbon society in the near future (Zibelmen, 2016; Wyns et al., 2014; Martinot, 2016).

The future power system with a high penetration of RES can be regarded as a system which is decentralized in terms of energy production. The increase in the number of distributed RES facilities in terms of production may cause some local problems in the distribution system such as over/under voltage phenomena and thermal overload on certain distribution lines (Kim et al., 2016). To manage these problems, the distribution system operator (DSO) needs to evolve into a more active entity with regards to governing its distribution system (DECC, 2014; LBNL, 2015), which acts in some way analogous to a local autonomous entity. This gives rise to an inevitable coexistence with the transmission system operator (TSO) who currently takes the balancing actions for system reliability and the DSO who undertakes the local management of distribution system. System balancing actions could include, for example, the procurement of local frequency response services and voltage support within the distribution system. In what follows we focus on frequency control actions.

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In the future power system, there may therefore be some conflicts between traditional TSO balancing actions and a DSO’s local management actions. It is suggested that the activation of flexibility from RES located in the distribution system for system balancing may cause other constraints in the distribution system, and similarly constraint management actions of DSOs will also affect the TSO’s balancing actions (Eurelectric, 2014). Given this potential for conflict, CERRE (2016) suggests that explicit frameworks are needed to encourage appropriate TSO-DSO cooperation and to allow for regulatory and contractual arrangements clearly defining the responsibility of each system operator regarding system balancing. It is emphasized that system balancing cannot easily be managed centrally anymore within a future environment with high penetration of RES (EDSO, 2015). Moreover, in the same report, an updated principle where each DSO is responsible for its own distribution system and system users is outlined. This would also mean that a TSO can only interact indirectly with the users connected to the distribution system via the DSO. Under this principle, named as the devolution principle in this research, each DSO with high penetrations of renewable distributed generators (RDGs) manages and tries to reduce its variability and uncertainty caused by itself through various kinds of efforts such as more advanced forecasting, active scheduling and control of generation/demand, using energy storage systems or voltage control equipment actively in short-term while encouraging efficient long-term investment and not disrupting system balancing at the transmission level.

However, the most important challenge in implementing the devolution principle in a real system is the lack of incentives facing the DSO to follow the principle. In the current power system, the cost of system balancing is allocated to energy market participants in proportion to their contracted amount of energy, not to the DSO. For instance, in South Korea, the system balancing cost including reserve cost for frequency control and corresponding opportunity cost is contained in the cost for energy consumption(MWh) paid by demand-side of wholesale electricity market. It is estimated at 8~14% of the total cost for energy consumption. In PJM of United States, the rates of the balancing payment are calculated as the total cost for system balancing divided by the total load in MWh (PJM, 2016). The value of the rate is generally in the range of $0.7 ~ $1.1/MWh. Great Britain’s National Grid charges the system balancing cost to market participants in the name of balancing service use of system (BSUoS) per MWh. BSUoS charges for entire system in GB were almost £1billion in 2015. This number is not negligible in the view of individual DSO.
ORNL (2000) have shown the economic inefficiency of the described current cost allocation method by reporting the fact that some participants, such as industrial consumers can account for 93% of the frequency regulation and 58% of the load-following requirements even though they compose only 34% of the total amount of load in United States. This is because system balancing cost is affected by variability and uncertainty in loads, not their contracted amount of energy. Some articles such as (Chu and Chen, 2009; Isemonger, 2009; Milligan et al., 2011) mention that the cost allocation scheme (CAS) based on the cost-causality principle is more rational in terms of delivering a transparent economic signal than the current energy related CAS. Nonetheless, even though the CAS based on the cost-causality principle is better than the current scheme, it is hard to measure the variability and uncertainty of each market participant one by one, especially on the demand-side, in a real power system.

For the above reason, this research suggests designating DSOs as the entities who primarily have responsibility for system balancing cost as the representatives of energy market participants in their jurisdictions. Under a CAS based on the cost-causality principle, DSOs would be motivated to reduce and manage imbalances on interface flows caused by variability and uncertainty from units in their own jurisdictions. However, if the DSO can pass all its balancing payment costs directly onto final customers, the devolution principle will not work. This situation can happen under cost-of-service regulation which guarantees cost-recovery for the regulated distribution entity. Therefore, an incentive regulation scheme would be needed, where the DSO can only partly pass the incremental balancing cost onto final customers and hence can increase its own profitability through technological innovation and strategic action to manage variability and uncertainty. Under the incentive regulation scheme, this research assumes the degree of pass-through of DSO onto final customers is less than 100% but more than 0%. This kind of incentive regulation scheme could be designed by benchmarking against the incentive regulation scheme for the TSO’s balancing service as described in Ofgem (2017).

Under the devolution principle, this research seeks to mathematically prove that the CAS based on cost-causality principle is more economically efficient than the current energy related CAS. Additionally, we aim to prove that the CAS based on the cost-causality principle can reduce the system balancing cost compared to the current CAS.

Nevertheless, there is another emerging problem even if the current CAS is changed to a more economically efficient scheme based on cost-causality principle. If this scheme is adopted, the DSO with a high
portion of RES would face risky system balancing costs. Because this expected consequence is not desirable for DSOs with many RES in their distribution systems, the change in CAS could be in conflict with national policies for promoting more RES, even though the cost-causality principle is more socially economic efficient. It could also potentially drive up the cost of capital for low risk DSOs by significantly increasing the volatility of their expected profits. Therefore, in order to increase the acceptability to stakeholders of the proposed CAS, a well-designed risk hedging instrument for each DSO should be provided together with the change in charging basis for system balancing.

In this context, this research additionally suggests the concept of balancing payment insurance (BPI). This is a kind of option contract between the DSO and the TSO. The DSO as buyer of this contract is able to choose whether to be fully exposed to uncertain balancing payments based on the cost-causality principle without a BPI, or instead to hedge the uncertainty in the balancing payment via making an ex-ante arrangement for a BPI with the TSO as seller of the contract. We need to note that the proposed scheme in this research does not work well if the DSO is allowed to sufficiently hedge its exposure to uncertain balancing payments and the cost of the hedging contract can be passed through to end-customers 100%. Therefore, the precondition that the DSO cannot pass through 100% of incremental cost caused by newly introduced regulatory instrument should be established for successful implementation of the proposed scheme. Such a contract should leave the DSO better off and the TSO no worse off. Therefore, this research proposes an optimal design framework for the BPI by adopting the idea from classical optimal insurance theory in Raviv (1979).

2. System Balancing Cost Allocation based on the Cost-Causality Principle

2.1 Simple example for economic inefficiency of current system balancing CAS

Let’s assume that there are only two DSOs, 1 and 2, (under a single TSO) that are responsible for the system balancing cost with same contracted energy volume by 10MWh per one operating time window in the electricity market, but the interface flow of DSO 1 is relatively constant and the DSO 2’s interface flow is relatively fluctuating due to the variability and uncertainty caused from many RES in its own distribution system. For a certain operating time window, the total system balancing cost for stable and reliable system
balancing by the TSO is assumed to be $200/one time window, and the contribution ratios of DSO 1 and 2 to the system balancing cost are assumed 10% and 90%, respectively. In the current energy related cost allocation scheme (EA-CAS), DSO 1 and 2 would be charged by same allocated cost $100/one time window, because they have equal contracted energy volume. However, the real values of balancing cost caused by each DSO are $20/one time window and $180/one time window, respectively. In other words, it implies that DSO 1 subsidizes DSO 2 by $80/one time window and DSO 2 is subsidized by DSO 1 by $80/one time window. This EA-CAS is unfair and does not give efficient economic signals to the two DSOs to reduce their variability and uncertainty by themselves.

2.2 Proof of economic efficiency of cost-causality based system balancing cost allocation scheme

Note that this proof draws on the polluter-pays principle as being the most efficient way to regulate pollution (as in Ambec and Ehlers, 2016). The polluter-pays principle in environmental economics and the cost allocation scheme based on cost-causality principle(CC-CAS) in the power industry have much in common.

Consider a set \( D = \{1, 2, \ldots, i, \ldots, n\} \) of DSOs. Each DSO \( i \) imports or exports energy \( e_i \) and gives rise to variability and uncertainty \( v_i \) via the interface flow to the transmission system at a certain time window as depicted in Figure 1. Precisely, variability and uncertainty \( v_i \) can be defined as the ex-post measured contribution of DSO \( i \)’s imbalance to the aggregated imbalance of entire system.

Figure 1. Conceptual figure for interface flow between TSO and DSO

Note that this proof draws on the polluter-pays principle as being the most efficient way to regulate pollution (as in Ambec and Ehlers, 2016). The polluter-pays principle in environmental economics and the cost allocation scheme based on cost-causality principle(CC-CAS) in the power industry have much in common.
This study assumes that DSO $i$ has the ability to manage variability and uncertainty $v_i$ through various distributed control options with smart grid technologies. Additionally, simply assume that energy flow is given a constant value, denoted as $\bar{e}_i$, which is independent of its variability and uncertainty $v_i$, and the system balancing cost is only affected by $v_i$, not $\bar{e}_i$. Because the whole grid is interconnected via a transmission system and thus frequency of the whole grid is synchronized, each DSO $i$ is affecting the system balancing cost and is affected by the system balancing cost simultaneously. Assume that each DSO $i$ has a strictly concave and differentiable benefit function $b_i(\bar{e}_i, v_i)$ which depends on the energy flow $\bar{e}_i$ and the variability and uncertainty $v_i$ which spills out on to the transmission system but is a somewhat controllable variable through smart grid technologies. Additionally, the marginal increase $c \geq 0$ on the total system balancing cost $C_B$ caused by $v_i$ from each DSO $i$ can be modeled as follows:

$$C_B = c \sum_{i \in D} v_i$$  \(1)\)

Individual utility $w_i(\bar{e}_i, v_i)$ of DSO $i$ with $\bar{e}_i$ by spilling out $v_i$, and social utility $W(\bar{e}, v)$ of all DSOs are as follows:

$$w_i(\bar{e}_i, v_i) = b_i(\bar{e}_i, v_i) - x_i$$  \(2)\)

$$W(\bar{e}, v) = \sum_{i \in D} w_i(\bar{e}_i, v_i) = \sum_{i \in D} b_i(\bar{e}_i, v_i) - c \sum_{i \in D} v_i$$  \(3)\)

where $x_i$ is the allocated payment for DSO $i$ for system balancing cost.

Let’s assume that the optimal strategy vector of $v^* = (v^*_i)_{i \in D}$ for all DSOs in spilling out variability and uncertainty via the interface flow into transmission system and the vector maximizes social utility $W(\bar{e}, v)$. This optimal strategy vector satisfies the following first-order condition for every $i$:

$$\frac{\partial b_i(\bar{e}_i, v^*_i)}{\partial v_i} = c$$  \(4)\)

In current scheme, the total system balancing cost $C_B$ is allocated in accordance with the ratios of contracted volume in the energy market. Therefore, $x^E_i$, the balancing payment for DSO $i$ in the EA-CAS, can be calculated as follows:
\begin{align*}
x_i^E &= \frac{\bar{e}_i}{\sum_{i \in D} \bar{e}_i} \times C_B = \frac{\bar{e}_i}{c} \sum_{i \in D} c(v_1 + v_2 + \cdots + v_n) \tag{5)

\text{Even if the optimal strategy vector of } v^* = (v_i^*)_{i \in N} \text{ ideally exists, under the current EA-CAS, each DSO}
\text{naturally considers the impact of } v_i \text{ only on its own individual utility, not social utility, by satisfying the first-}
\text{order condition of each individual utility in Nash-equilibrium state as follows for every } i:
\frac{\partial b_i(\bar{e}_i,v_i^E)}{\partial v_i} &= \frac{\bar{e}_i}{\sum_{i \in D} \bar{e}_i} c \tag{6)}

\text{Comparing Eq. (4) and the first-order condition in Eq. (6), it can be seen that the Nash-equilibrium state under}
\text{the current EA-CAS does not guarantee the maximization of social utility of all DSOs.}
\text{If the CC-CAS is adopted, the balancing payment } x_i^{CC} \text{ can be calculated as follows:}
\begin{align*}
x_i^{CC} &= \frac{v_i}{\sum_{i \in D} v_i} \times C_B = \frac{v_i}{\sum_{i \in D} v_i} c(v_1 + v_2 + \cdots + v_n) = cv_i \tag{7)}
\end{align*}

\text{With regard to this case, maximizing the individual utility of each DSO } i \text{ leads to the same first-order condition}
\text{of ideal efficient strategy as follows:}
\frac{\partial b_i(\bar{e}_i,v_i^{CC})}{\partial v_i} &= \frac{\partial b_i(\bar{e}_i,v_i^*)}{\partial v_i} = c \tag{8)}

\text{In other words, the CC-CAS can deliver an economically efficient and transparent price signal to DSOs}
\text{regarding system balancing cost.}
\text{Additionally, the cost-causality based scheme can reduce the total system balancing cost compared to the}
\text{case in the current scheme. To prove the mentioned statement, let’s define } h_i = \left(\frac{\partial b_i(\bar{e}_i,v_i)}{\partial v_i}\right)^{-1} \text{ first, noting that}
\text{the inverse exists since the marginal benefit } \frac{\partial b_i(\bar{e}_i,v_i)}{\partial v_i} \text{ is strictly decreasing, due to the assumption of a strictly}
\text{concave benefit function. Then the total system balancing cost at the Nash-equilibrium state in the case of EA-}
\text{CAS and CC-CAS can be represented respectively as follows:}
\begin{align*}
C_B^{EA} &= c \sum_{i \in D} v_i^{E^*} = c \sum_{i \in D} h_i \left(\frac{\bar{e}_i}{\sum_{i \in D} \bar{e}_i} c\right) \tag{9)}
\end{align*}
\[ C_{cc}^E = c \sum_{i \in D} v_{ci}^c = c \sum h_i(c) \] (10)

Under the assumption of strictly decreasing \( h_i \), it can be easily known from the fact that \( C_{cc}^E \) is always larger than \( C_{cc}^C \), the CC-CAS is more effective than the current EA-CAS for reducing the total system balancing cost by inducing DSOs to manage their variability and uncertainty by themselves following the devolution principle. This is because the DSOs should invest to reduce variability and uncertainty on their system if incentives to do so exist.

3. Contract for Hedging Uncertainty in Balancing Payment

3.1 The need for optimal designing framework of balancing payment insurance

The proposed BPI between DSO \( i \) and the TSO is composed of a contract price \( P_i \) and coverage function \( I(x_i) \). The contract price is the purchase paid by the DSO to the TSO and the coverage function is defined by the form of the monetary rebate being applied to the balancing payment of the DSO together with the threshold of applying the rebate.

Let’s assume that \( v_i^0 \) is the contribution of DSO \( i \)’s imbalance to the aggregated imbalance of the entire system without any control effort by the DSO, and it is a random variable. Meanwhile, a DSO with active network management infrastructure as described in (Kim et al., 2016) has the ability to indirectly manage balancing payment \( x_i \) to some degree by reducing \( v_i^0 \) through some control action \( a_i \), by for example, power factor control of RES, curtailment of RES, an energy storage facility, voltage control equipment and/or utilizing more advanced forecasting techniques. The balancing payment \( x_i \) can be represented as \( x_i = cv_i = c(v_i^0 - a_i) \), and \( x_i \) is also a random variable. Yet, making \( x_i \) be zero by completely reducing \( v_i^0 \) leaked via the interface flow may not the rational choice for DSO because it can cause dramatic increases in other operating cost factors in distribution system operation. Therefore, the DSO should find the optimal strategy for minimizing its net operating cost \( NC_{DOS} \) of the distribution system by optimally taking out a BPI contract with the TSO. \( NC_{DOS} \) can be represented as the sum of the original operating cost \( C_{DOS} \), \( P_i \), \( x_i \), and \( -I(x_i) \). It can be also assumed
that $C_{DSO}^D$ is strictly decreasing with respect to $a_i$ and $\frac{\partial^2 C_{DSO}^D}{\partial a_i^2} > 0$. To implement the optimal BPI, the contract should be designed in order that the DSO can maximize its utility and the TSO can avoid a decrease in its utility. Therefore, this research proposes the optimal design framework for the BPI by adopting an idea from classical optimal insurance theory in Raviv (1979).

### 3.2 Designing balancing payment insurance (BPI)

The BPI is characterized by the contract price $P_i$ and the coverage function $I(x_i)$ for DSO $i$. Any admissible coverage function $I(x_i)$ satisfies

\begin{align}
0 &\leq I(x_i) \leq x_i \quad \text{for all } x_i \\
I(0) &= 0
\end{align}

(11)

(12)

The constraint (11) means that the covered amount for payment $x_i$ is necessarily non-negative and cannot exceed the size of the original payment $x_i$, and Eq. (12) implies basic condition that there is no monetary rebate if there is no payment.

The TSO as the seller of the BPI is assumed to maximize the expected value of its utility from selling the contract, which is a concave function of the revenue $R$ within a certain operation time window. $V(R)$ denotes the utility function of the TSO with conditions of $\frac{\partial^2 V(R)}{\partial R^2} > 0$ and $\frac{\partial^2 V(R)}{\partial R^2} < 0$ for all values of $R$. This assumption means that the seller of BPI is a risk-averse agent. After selling the BPI and receiving the contract price $P_i$ from a DSO $i$, TSO’s final revenue can be represented as $R_{INI} - C_B + x_i + P_i - I(x_i)$ where $R_{INI}$ denotes the initial revenue of the TSO. The TSO offers such a BPI when the following necessary condition is satisfied by selling the contract.\(^3\)

\[
E[V[R_{INI} - C_B + x_i + P_i - I(x_i)]] \geq E[V[R_{INI} - C_B + x_i]]
\]

\(^3\) For now, we ignore the covariance between the contracts that the TSO might sign with multiple DSOs. This is later incorporated into our assumptions about the relative risk aversion of the TSO and the DSOs. We also ignore the impact of the TSOs regulatory regime on its incentives to sign optimal contracts, in order to focus on the DSO.
On the other hand, the DSO as the buyer of the BPI is assumed to maximize the expected utility in terms of buying the BPI, which is a concave function of revenue $R_{DSO}$ for a certain operation time window; $U(R_{DSO})$ denotes the utility function of the DSO with the conditions that $\frac{\partial U(R_{DSO})}{\partial R_{DSO}} > 0$ and $\frac{\partial^2 U(R_{DSO})}{(\partial R_{DSO})^2} < 0$ for all values of $R_{DSO}$. This function says that the DSO is risk-averse. After buying the BPI, being covered in accordance with $I(x_i)$ and paying contract price $P_i$ to the TSO, the DSO’s final revenue at a certain operating time window can be represented as $R_{INI} - (1 - \rho)(C_{OP} + P_i + x_i - I(x_i))$. $R_{INI}$ denotes the initial revenue of DSO at a certain time window, $C_{OP}$ denotes the operating cost of DSO at certain time window, and $\rho$ is a certain degree of pass-through rate of DSO’s costs onto final customers. The pass-through rate $\rho$ is specific to each DSO and can be assumed to be enforced by its regulatory regime. The DSO accepts such a BPI when the following necessary condition is satisfied by buying the contract.

$$E[U[R_{INI}^D - (1 - \rho)(C_{DSO} + P_i + x_i - I(x_i))]] \geq E[U[R_{INI}^D - (1 - \rho)(C_{DSO} + x_i)]]$$

(14)

### 3.3 Optimal contract design given a fixed contract price

To find the optimal design of the BPI in terms of the relationship between the coverage function $I(x_i)$ and the given contract price $P_i$, the optimization model is required to maximize the DSO’s expected utility with final revenue for a certain time window, subject to the constraint that the TSO’s expected utility with final revenue is not less than the expected utility with initial revenue. The optimization model can be formulated as follows:

$$\text{Max}_{x_i} E[U(R_{DSO})] \equiv \int_0^\infty U[R_{INI}^D - (1 - \rho)(C_{DSO} + P_i + x_i - I(x_i))] f(x_i)dx$$

(15)

where $f(x_i)$ is the probability density function of $x_i$. Eq. (15) is subject to Eq. (11), (12) and the inequality constraint (14) which has the same meaning with the constraint (13) as follows;

$$E[V(W)] = E[V[R_{INI} - C_B + x_i + P_i - I(x_i)]] \equiv \int_0^\infty V[R_{INI} - C_{B,-1} + P_i - I(x_i)] f(x)dx$$

$$\geq E[V[R_{INI} - C_{B,-1}]] = K$$

(16)
where $x_i + C_{B,-i}$ is substituted into $C_B$. $C_{B,-i}$ is the system balancing cost excluding the DSO $i$’s portion.

This optimization problem can be solved via optimal control theory. The integrand of constraint (16) can be rewritten using $I(x_i)$ as the control variable and $s(x_i)$ as the state variable with the boundary conditions as follows:

$$s(x_i) = V[R_{INI} - C_{B,-i} + P_i - I(x_i)]f(x_i)$$

(17)

$$s(0) = 0$$

(18)

$$s(\infty) = K$$

(19)

The Hamiltonian for this optimization problem can be formulated as follows:

$$H = \left[U[R_{INI}^{DSO} - (1 - \rho)(C_{OP}^{DSO} + P_i + x_i - I(x_i)) + \lambda \cdot V[R_{INI} - C_{B,-i} + P_i - I(x_i)]f(x_i)\right]$$

(20)

where $\lambda$ is the Lagrange multiplier.

The necessary conditions for the optimal coverage function $I^*(x_i)$ to maximize the Hamiltonian in (20) subject to constraint (11) and (12) are as follows:

$$I^*(x_i) = 0 \quad \text{if} \quad \frac{\partial H}{\partial I(x_i)} \leq 0$$

(21)

$$I^*(x_i) = x_i \quad \text{if} \quad \frac{\partial H}{\partial I(x_i)} \geq 0$$

(22)

$$\left. \frac{\partial H}{\partial I(x_i)} \right|_{I^*(x_i)} = (1 - \rho)U'[R_{INI}^{DSO} - (1 - \rho)(C_{OP}^{DSO} + P_i + x_i - I(x_i))] - \lambda \cdot V'[R_{INI} - C_{B,-i} + P_i - I^*(x_i)]$$

(23)

Because the conditions (21) and (22) cannot occur simultaneously, the optimal coverage function satisfies either (21) and (23), or (22) and (23). For each necessary condition (21) and (22), the extremum value (threshold value of contract) $\bar{x}_m^m, m = 1, 2$ can be uniquely defined with contract price with following equations:
\[
\frac{\partial H}{\partial I(x_i)}|_{I(x_i)=I^*(x_i)=0} = (1 - \rho) U'[R_{ini}^{DSO} - (1 - \rho)(C_{op}^{DSO} + P_i + \bar{x}_i^1)] - \lambda \cdot V'[R_{ini} - C_{B,-i} + P_i] = 0
\]

(24)

\[
\frac{\partial H}{\partial I(x_i)}|_{I(x_i)=I^*(x_i)=x_i} = (1 - \rho) U'[R_{ini}^{DSO} - (1 - \rho)(C_{op}^{DSO} + P_i)] - \lambda \cdot V'[W_{ini} - C_{B,-i} + P_i - \bar{x}_i^2] = 0
\]

(25)

For \( x_i > \bar{x}_i^m \), it is easily known that (23) is satisfied and the following equation (26) can be obtained by differentiating \( \frac{\partial H}{\partial I(x_i)}|_{I(x_i)=I^*(x_i)} \) with respect to \( x_i \):

\[
\frac{\partial}{\partial x_i}\left[ \frac{\partial H}{\partial I(x_i)}|_{I(x_i)=I^*(x_i)} \right] = (1 - \rho)^2 U''[A] \cdot (I^*(x_i) - 1) + \lambda \cdot V''[B] I^*(x_i) = 0
\]

(26)

for \( x_i > \bar{x}_i^m \)

where \( A = R_{ini}^{DSO} - (1 - \rho)(C_{op}^{DSO} + P_i + x_i - I(x_i)) \) and \( B = R_{ini} - C_{B,-i} + P_i - I^*(x_i) \).

From (23), \( \lambda \) can be determined as follows:

\[
\lambda = (1 - \rho) \frac{U'[A]}{V'[B]}
\]

(27)

Finally, the derivative of coverage function \( I^*(x) \) can be obtained by substituting \( \lambda \) into (26) as follows:

\[
(1 - \rho)^2 U''[A] \cdot (I^*(x_i) - 1) + (1 - \rho) \frac{U'[A]}{V'[B]} \cdot V''[B] I^*(x_i) = 0
\]

(28)

\[
I^*(x_i) = \frac{U''[A]U'[A]}{V''[B]V'[B] + U''[A]U'[A]}
\]

(29)

Using the Arrow-Pratt coefficient of absolute risk aversion, (30) can be more concisely presented as follows:

\[
I^*(x_i) = \frac{(1 - \rho) \Gamma_{DSO}[A]}{\Gamma_{Tso}[B] + (1 - \rho) \Gamma_{DSO}[A]} = \frac{1}{1 + \left( \frac{\Gamma_{Tso}[B]}{(1 - \rho) \Gamma_{DSO}[A]} \right)}
\]

(30)

where \( \Gamma_{DSO} = -U''[A]/U'[A] \) and \( \Gamma_{Tso} = -V''[B]/V'[B] \). It is generic risk-sharing formula which can be applied to any type of utility function.

Consequently, it can be said that the optimal BPI takes one of the two forms as follows:

**Form I**

\[
I^*(x_i) = 0 \text{ for } x_i \leq \bar{x}_i^1
\]

(31)
\[
I^*(x_i) = \int_{\bar{x}_i}^{x_i} I'(t_i) \, dt \quad \text{for} \quad x_i > \bar{x}_i
\]

Form II

\[
I^*(x_i) = x_i \quad \text{for} \quad x_i \leq \bar{x}_i
\]

\[
I^*(x_i) = \bar{x}_i^2 + \int_{\bar{x}_i}^{x_i} I'(t_i) \, dt \quad \text{for} \quad x_i > \bar{x}_i^2
\]

(32)

For simply understanding the determinants of the risk-sharing, some explicit examples are shown as in Table 1 with the assumption of constant absolute risk averse (CARA) utility function and given fair contract price.
Table 1. Example Cases of Risk-sharing with BPI Contract

<table>
<thead>
<tr>
<th>Example Case</th>
<th>TSO’s risk-preference</th>
<th>DSO’s pass-through rate</th>
<th>Optimal Coverage Function</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Form I</td>
</tr>
<tr>
<td>Case I</td>
<td>Risk-neutral ((\Gamma_{TSO} = 0))</td>
<td>Any value</td>
<td>(I'(x_i) = 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case II</td>
<td>Risk-averse (but, (\frac{\Gamma_{TSO}}{\Gamma_{DSO}} &lt; 1))</td>
<td>(0 \leq \rho &lt; 1)</td>
<td>(0 &lt; I'(x_i) &lt; 1)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Case III</td>
<td>(\rho = 1)</td>
<td>(I'(x_i)) is not defined</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

As presented in Case I of Table 1, if the TSO is risk-neutral, the derivative of optimal coverage function is equal to unity. In this case, Form II is the dominant form of the BPI contract from the point of view of the DSO. This is because Form II has a zero threshold for the rebate from the TSO unlike the contract threshold in Form I has some positive value according to Eq. (24). If all DSOs choose the Form II BPI contract, they are fully exempted in balancing payment by paying BPI contract price to TSO. If the TSO is risk-averse and DSO is more risk-averse than TSO, the derivative of optimal coverage function is less than unity in Case II. Additionally, in this case, it can be shown that the coverage rate \(I'(x_i)\) decreases as the pass-through rate \(\rho\) increases. In the case of 100% pass-through rate as in Case III, the optimal coverage function cannot be defined. It can be
interpreted as DSO has no need for BPI contract in Case III because it can completely shift its incremental cost onto final customers.

4. Conclusion

This research suggests two things about cost allocation schemes (CASs) for balancing costs. First, in a world of rising distributed RES, DSOs should be designated as the entities who have primary economic responsibility for system balancing cost to give them the motivation to reduce or manage variability and uncertainty from their own distribution networks. Second, the system balancing cost allocation scheme should be transformed from the EA-CAS to the CC-CAS. The proof for the economic efficiency of the CC-CAS is provided in this research. To avoid the side effect that the DSO with a large amount of RES may face a high and risky balancing payment under the CC-CAS, this research also proposes an optimal design framework of the BPI which helps DSOs hedge the risks associated with uncertain balancing payments. The DSO can hedge the risk in its balancing payments by paying a BPI contract price to the TSO ex-ante. With this proposed hedging instrument, the DSO can achieve an optimal risk adjusted operating strategy minimizing its net operating cost. This cost includes the original operating cost of its distribution system, the balancing payment, the purchase price of the BPI, and the rebate amount through the BPI.

The proposed contractual framework can induce DSOs to reduce variability and uncertainty leaked on to the transmission system via interface flows. It is expected that a significant part of the system balancing responsibility of TSO can be handed over to a number of DSOs under the proposed contractual framework. In addition, the TSO can conduct system balancing actions more efficiently based on better partial ex-ante information on estimated variability and uncertainty arising from the local distribution system via its signed BPI.

In future work, the details of calculating the balancing payments for DSOs will be suggested under CC-CAS for various types of electricity markets over the world. Additionally, we plan to explore the optimal operating strategy of the DSO and TSO within the proposed contractual framework.
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