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DEBT SUSTAINABILITY AND THE TERMS OF OFFICIAL SUPPORT

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Abstract

We study theoretically and quantitatively how official lending regimes affect a government's decision to raise saving as opposed to defaulting, and its implication for sovereign bond pricing by investors. We reconsider debt sustainability in the face of both output and rollover risk under two types of institutional bailouts: one based on long-maturity, low-spread loans similar to the ones offered by the euro area official lenders; the other, on shorter maturity and high-spread loans, close to the International Monetary Fund standards. We show that official lending regimes raise the stock of safe debt and facilitate consumption smoothing through debt reduction. However, to the extent that bailouts translates into higher future debt stocks and countercyclical deficits in persistent recessions, they also have countervailing effects on sustainability. Quantitatively, the model is able to replicate Portuguese debt and spread dynamics in the years of the bailout after 2011. We show that, depending on the composition of debt by maturity and official lending, sustainable debt levels can vary between 50% of GDP and 180% of GDP depending on the state of the economy and the conditions for market access. Longer maturities have a stronger effect on sustainability than lower spreads.

Keywords: Sovereign debt, default, maturity, spread, rollover, bailout

1 Introduction

In response to the sovereign debt crises that shook the euro area in the aftermath of the Great Recession, the governments of Cyprus, Greece, Ireland and Portugal received funds from both the International Monetary Fund and newly created European institutions: the European Financial Stability Facility at first, then the European Stability Mechanism (ESM). The scale and modalities of the official assistance re-ignited the long-standing debate on debt sustainability and the design of international financial rescues (see Conesa and Kehoe (2014); Tirole (2015); D'Erasmo et al. (2016); Collard et al. (2015); Gourinchas and Martin (2017); Roch and Uhlig (2018) among others).

The euro area crisis indeed resulted in a large-scale experiment with international bailouts, creating opportunities for reconsidering critical and understudied issues in the providing international financial assistance. One of the most striking features of the country programs in the euro area is that the terms of official loans deviated significantly from consolidated international practices. Relative to International Monetary Fund standards, ESM loans were issued with much longer maturities (15/22 years versus 7 years), lower interest rates (200-300 basis points, vs, 300-400 basis points) and larger size (far exceeding the amount permissible under IMF rules)——for a detailed account of the evolution of official lending in Europe, see Corsetti et al. (2017).

Motivated by this evidence, this paper studies the effects of varying the terms of official lending on debt sustainability, market access conditions, and a country's vulnerability to crises. Our point of departure is the notion that official loans affect a government's incentives to issue, repay and default on debt, hence they matter for how much debt a country can sustain just like tax capacity, spending and inflation. Yet, the extent to which official loans impinge on a government fiscal decision may differ, depending on their maturity, spreads and size, raising questions concerning the underlying economic mechanisms, their quantitative relevance and potential welfare-relevant trade-offs.

To investigate these issues, we introduce official lending institutions in an otherwise standard model of debt default, after Conesa and Kehoe (2017). We use this framework to gain analytical insight on how official lending may restore debt sustainability, i.e. induce a government to choose higher surpluses over default, in an economic environment with both non-fundamental (rollover) risk in the debt market, and fundamental (output) risk. In a quantitative exercise, we show how official lending may have contributed to address the sovereign risk crisis in Portugal in 2011. Bases on this exercise, we carry out counter-factual exercises to explore the sensitivity of our results to changing maturity versus prices of loans.

Our main results are as follows. First, we show that the availability of official bailout funds

raises the threshold below which debt is default free, and helps high-debt countries facing rollover and fundamental risk to smooth consumption in the transition from the 'crisis' to the 'safe zone.' However, higher levels of safe debt translates into lower long-run consumption and thus welfare. Because of these opposing effects, the effects of official lending on debt sustainability may be non linear. Larger bailout may actually lower the debt threshold at which the government chooses default when in the crisis zone.

Second, quantitatively, we show that the model is able to replicate the dynamic evolution of debt and spreads in Portugal around the access of this country to the ESM program. In our exercises, the stock of debt and spreads move in opposite directions as the country substitutes official for market lending—the same pattern as in the data (although, relative to the evidence, our model under-predicts debt growth). In short, by containing borrowing costs and reducing the period-by-period financing need of the government, official financial programs of the kind Portugal received during the crisis reduced the need for strong deleveraging during a deep recession.

Third, through counterfactuals based on the parameters of IMF and ESM lending, we find that reducing the period-by-period flow payments through lengthening maturities appears to have much stronger effects on sustainability, than reducing spreads. The counterfactual exercises also suggests that the fundamental trade-offs unveiled by the model—between raising the threshold below which debt is traded as a safe asset, and reducing the threshold at which the country defaults when facing rollover risk—is quantitatively relevant.

On methodological grounds, our main contribution consists of a comprehensive model of official lending addressing both rollover and fundamental sources of sovereign risk. We lay out our theory in the framework developed by Conesa and Kehoe (2017), where both types of risk may cause a government to default on its obligations to market creditors. Output risk reflects the fact that business cycles introduce uncertainty in the government's ability and willingness to raise taxes and generate surpluses when in a recession. Rollover risk manifests itself through market lenders' beliefs. For a high enough initial level of debt, the country is in the 'crisis zone:' if market lenders coordinate their expectations on the belief that the government will not repay, this belief becomes self-fulfilling. When in the crisis zone, a welfare-maximizing government has an incentive to reduce its debt, to obtain a better price for newly issued debt; however, this incentive may be overturned in a temporary recession, when the government may prefer to run up debt in anticipation of better times, so as to smooth consumption over time. This may explain why welfare-maximizing governments may fail to bring down debt to its safe, no-default zone during a recession, and may prefer 'gambling' on the recovery.

Drawing on the empirical evidence and policy practice, in the model the government

can access official loans that may differ along three key dimensions: interest rates, loan maturities, and availability/scope of the bailout. The way in which official debt impinges on debt sustainability vary depending on these terms as well as the source of risk.

Consider first a model environment with rollover risk only. It is quite intuitive that a low (below market) spread on official loans can help a country keep consumption smooth while reducing debt to safe level. Similarly, by alleviating the period-by-period payment flows that the government faces in the event of a confidence crisis, long-term official debt can induce policymakers to avoid default when they would otherwise opt for it, and keep deleveraging, for *higher initial stocks of public debt*. The economics is straightforward. With one period debt, the period-by-period government financing need coincides with the entire debt stock. With long debt maturity, what matters for repayment and default decision (and as a result, sustainability) is the primary deficit and the debt coming due every period.

The main trade-off here is that official assistance also raises the threshold of the safe zone, such that a country may end up with larger amounts of debt in the long run—implying lower long-run consumption. Hence generous bailout terms that widen the 'safe zone' (and thus lower long-run consumption) may also lower debt sustainability; i.e., the 'crisis zone' shrinks both from below (a larger safe zone) and from above (a lower debt threshold). However, the model unveils a subtle, but crucial, consideration: bailouts affect the path chosen by the government to run debt down to the safe zone. Since, once debt levels are within the safe zone, the country regains access to market funding with certainty, a faster transition translates into bail-in of private lenders.

Now bring in output risk. For simplicity, consider the case of two states, high output (normal times) and low output (recession), and assume that the high state is an absorbing state (i.e. the economy is not expected to be in a recession again)—so that we can focus on the effect of official debt during recessionary periods. Without official lending, the maximum stock of debt that is sustainable during recessions is at the level that would induce the country to default, should the recession persists. With single-period debt, at the threshold the entire debt stock must be repaid and the government must pay a premium for newly issued debt to compensate market lenders for default risk. Official debt contributes to sustainability along two fronts: first, with longer maturities, countries do not have to repay the amount owed to creditor in full in each period; second, lower spreads can ease the financing burden exacerbated by default risk. Those are the key mechanisms by which official lending enhances sustainability vis-á-vis output uncertainty.

To be effective in enhancing sustainability, the terms of the bailout have to be such that the government is indifferent between generating surpluses or default, *should the recession persist*. The size and terms of official loans can set generous enough as to induce governments

to repay even through recessions that are deep and persistent. Nonetheless, theory sheds light on a key trade-off in doing so (mirroring the one already discussed for the case of rollover risk only). When the economy is hit by several negative output shocks in succession, the optimal debt policy is to increase debt to smooth consumption in response to each shock. But this means that current debt cannot be too high, lest it default in the future.

We take theory to the data combining both environments, so that the model can trace the implications of official debt for both the endogenous paths of the country's stock of liabilities, and repayment and default policies under rollover risk and different realizations of output uncertainty. We focus on the case of Portugal in 2011-2015. At the height of the debt crisis, the Portuguese debt to GDP ratio was close to 100%. In the second quarter of 201, borrowing costs were over 700 basis points (bps) higher than the German Bund, with an average maturity of market debt of six years. Default risk arose from a combination of fears of serial default across the euro area arguably reflecting both fundamentals and non-fundamentals factors and the persistence of the ongoing recession (creating output risk in terms of our model). Portugal received loans from the IMF, and the ESM, different in terms of spread and maturity.[1] In July 2011, the IMF loan had a spread of 300 bps and a maturity of seven years. By contrast, the euro area loan had virtually no spread and a maturity of 15 years. Shortly after receiving these loans, market spreads on Portuguese bonds started to come down, to the point that, after a couple of years, Portugal started to repay its IMF debt by issuing market bonds at better terms.

We use our model to replicate the dynamics of Portuguese debt, starting out from an initial situation in which debt is well inside the 'crisis zone,' and the government only borrows from markets at a high spread. Official loans, up to about 25% of total Portuguese debt, induce market spreads to fall by the same extent as in the data. The substitution of risky high-spread, short-maturity market instruments with safer low-spread, long-maturity instruments allows Portugal to extend the terms of engagement thereby reducing default risk and the premia that comes with it. We model official loan disbursements staggering them as in the data: without this, the government will want to substitute market debt for official debt to a much larger extent.

Having replicated the Portuguese experience, we then use the model as a laboratory to assess sustainability under different counterfactual policies: we allow the ESM and IMF either to charge the same rate, or to have the same maturity. One quantitative finding is that

[1]The Portuguese official loans was funded the European Financial Stability Facility (EFSF) and the European Financial Stability Mechanism (EFSM), the prequels of the European Stability Mechanism (see Corsetti et al. (2017)). As the three Funds operate using analogous maturity and pricing terms, and the ESM is the one with a permanent character, to ease the exposition we refer to the Portuguese European bail-out as an ESM one.

differences in maturity matter more than differences in the spread. The largest quantitative difference in terms of sustainability is observed when the IMF adopts ESM-type maturities.

The rest of the paper is organized as follows. Section 2 reviews the literature, Section 3 review stylized facts of the sovereign crises in the euro area that motivate our study. Section 4 specify the model, and derives analytical insights from a simplified version of it. Section 5 discusses our quantitative exercise. Section 6 concludes.

2 Literature review

Our paper contributes to the vast and rich body of literature on debt sustainability—see D'Erasmo et al. (2016); Aguiar and Amador (2014) for recent overviews—on at least two dimensions.[2]

First, our paper relates to contributions concerned with the role of official sector interventions during crises. Close to our analysis, Arellano et al. (2012) argue that policies that result in lower interest rates or contain default costs lead governments to run up more debt during recessions. Conesa and Kehoe (2014) argues that a an official lender of last resort charging penalty rates would not be able to resolve the euro area crisis. The literature has also specifically focused on catalytic effects of official lending, enhancing the ability of sovereigns to re-access capital markets.[3] Corsetti et al. (2006); Morris and Shin (2006) defines conditions under which official sector loans can prevent runs on sovereign debt markets. Broner et al. (2014) models sovereign risk when debt is financed by domestic investors, foreign private creditors and the official sector, and shows that the official loans can influence private portfolios, fostering investment and growth. Sandri (2015) focuses on the role of official support in preventing spillovers from sovereign defaults. Dellas and Niepelt (2016) present a model of a sovereign that can obtain financing from heterogeneous private lenders and the official sector, which rationalizes the change in the composition of debt towards official sources as countries approach default.

[2]In this literature, a variety of theoretical analyses discuss issues specific to Monetary Unions (Aguiar et al., 2015, 2016), interaction with monetary policy and inflation (Aguiar et al., 2013) as well as structural reforms accompanying official sector programs (Muller et al., 2015). While all these issues are at least indirectly relevant for our analysis, our modeling abstracts from them.

[3]A number of empirical contributions have focused on this issue. Mina and Martinez-Vazquez (2002); Saravia (2013), study the relation between sovereign debt maturity and official sector lending. Even more recently, concerned with the safety of the official sector resources, Reinhart and Trebesch (2016) argue that in providing longer maturity loans to increasingly stressed sovereigns, the IMF's role as a lender of last resort is becoming at risk. Our findings qualify Reinhart and Trebesch (2016) view, as we argue that, adequately designed, longer official loans can turn an unsustainable situation into a sustainable one.

Relative to Conesa and Kehoe (2014); Arellano et al. (2012), our specific contribution consists of modelling official lenders offering different terms on their loans. Also, our interest is to understand which combination of terms and conditions (among those observed during the euro area crisis) can enhance the effectiveness of official lending to avoid defaults for a significant range of economic fundamentals. Our analysis also contributes to improve our understanding of when and how official loans have a catalytic effect. The impact of official bailouts on market dynamics critically depend on the lending terms offered to the sovereign.

A second strand of the literature to which we closely relate consists of studies focusing on the role of debt maturity in enhancing debt sustainability. These contributions calls attention on the the need to trade off refinancing risks with borrowing costs—a trade-off that is also center-stage in our analysis. In Cole and Kehoe (2000), lengthening debt maturity reduces the region where debt is exposed to self-fulling roll-over crisis (see also Aguiar et al. (2016); Hatchondo and Martinez (2009); Hatchondo et al. (2016); Bai et al. (2015); Mihalache (2016, 2017); Sanchez et al. (2016)). Angeletos (2002) assumes that governments issue long-term debt to invest in short-term reserves, which help smoothing refinancing needs. In Niepelt (2014), sovereign risk leads sovereigns to issue short-maturity debt when debt issuance is high, output is low and cross-default is more likely. Relatedly, Arellano and Ramanarayanan (2012) and Broner et al. (2013) show that when interest rates rise, maturity shortens. Aguiar and Amador (2013) show that during a debt crisis it is optimal to switch to short-term financing and only payback longer debt as it matures. This occurs because when default risk rises, shorter debt provides better incentives to repay, with positive feedback effects on borrowing costs. Chatterjee and Eyigungor (2012, 2015) show that to overcome the dilution problem that makes long-term debt more expensive, sovereigns should include an absolute priority rule clause on their bonds, giving seniority to earlier lenders.[4]

Relative to the outstanding literature, we specifically model the 'tension' between stock and flows, showing how the terms of official support can substantially alter the stock of debt that is sustainable by affecting the cash flow from a country liabilities. A notable result from our analysis is that, if official creditors are willing to increase the maturity of their exposure, this may result in an increase in the likelihood of repayment—effectively counteracting debt dilution as discussed in Arellano and Ramanarayanan (2012); Chatterjee and Eyigungor (2015); Hatchondo et al. (2016).

[4]From an empirical perspective, Dias et al. (2014) shows that understanding sovereign risk requires looking beyond the debt stock variable and into fails to recognize the cash-flows stream associated with such stock. Relatedly, focusing on the Greek debt stock, Weder di Mauro and Schumaker (2015); Zettelmeyer et al. (2017) show that different amortization schedules and varying interest rates can make a given stock of debt look very different. Gabriele et al. (2017) shows the importance of jointly considering debt stock and refinancing needs to understand borrowing costs, and Bassanetti et al. (2016) show that changes on the debt stock are an important driver of countries' capacity to tap financial markets at sustainable rates.

3 Official Lending in the Euro Area: Stylized Facts

In this section, we provide a a synthetic account of the creation and evolution of euro area crisis resolution framework, stressing a set of stylized facts that we use to motivate and discipline our theoretical model. First, we briefly describe the creation this framework and the key elements of the various programs. Second, we compare and contrast the approach to official support followed by the IMF and the euro area official lenders, with a focus on their lending terms. Finally, we provide stylized facts connecting the sovereign debt dynamics in the euro area to the terms of the official loans provide.

3.1 A Brief Review of Euro Area Official Lending

When in 2009 the Greek authorities admitted they had fiddled with the fiscal deficit figures and progressively lost market access, the first reaction by European authorities was to demand a significant fiscal adjustment. As this failed and the situation spun out of control, in March 2010, euro area governments, together with the IMF, agreed to provide financial assistance, setting up the Greek Loan Facility. The first program consisted of IMF credit and bilateral loans by other euro area members, for a total of 110 billion euros including a 30 billion euros IMF loan with a 3-years duration and a maturity of five years. Following IMF practice, the pricing of this loans was a step-wise function of their duration.

When financial stress did spread to Ireland and Portugal, the reaction was to move away from a bilateral approach and create jointly managed institutions. In June 2010, the European Financial Stability Mechanism (EFSM) and the European Financial Stability Facility (EFSF) were created. The EFSM was designed as an emergency funding program, managed by the European Commission, with the authority to borrow up to 60 billion euros. In turn, the EFSF was created as a temporary rescue mechanism to provide financial assistance within the framework of an adjustment program.

In December 2010, Ireland became the first country to seek assistance of the new institutions. The Irish program provided a financing package of EUR 85 bill, including contributions from the EFSM (22.5 billion) and EFSF (17.7 billion), and bilateral loans from UK, Sweden and Denmark (3.8, 0.6 and 0.4 billion euros, respectively). In addition, Ireland signed a 7 years Extended Fund Facility (EFF) agreement with the IMF for 22.5 billion. A few months later, in April 2011, it was the turn of Portugal to seek support. In this case the financing of the 78 billion euros program fell on equal parts on the EFSM, EFSF and IMF. The IMF loan to Portugal was disbursed through the EFF program.

In June 2011, the European authorities agreed to set-up a permanent crisis-management

institution, the European Stability Mechanism (ESM), to become operative by 2014. As the euro area problems did not abate, the authorities however brought its inauguration forward in time, in 2012. With 500 billion euros lending capacity supported by 700 billion in capital, so far the ESM has provided assistance to Spain (July 2012), Cyprus (June 2013) and Greece (September 2015).

3.2 The Terms of Official Support: IMF-style versus ESM-style

As is well-known, the International Monetary Fund relies on two crisis-resolution credit lines, the Stand-By Arrangement (SBA) and the Extended Fund Facility (EFF): SBA programs aim to help members address short-term balance of payments (BoP) problems; EFF programs instead aim to help countries overcoming medium-term BoP problems. SBA programs are typically structured over 3 years, with a repayment horizon up to 5 years; EFF programs are structured over 4 years, with a repayment horizon of up to 10 years. SBA and EFF programs apply identical borrowing limits and pricing structure. The lending rate is linked to the special drawing rights (SDR) interest rate. For loans below 187.5 percent of the member's quota, the IMF charges 100 bps. This increases to 200 bps for credit above 187.5 percent of the quota. Moreover, to discourage large use of IMF resources, the spread over the SDR rate is increasing in the time over which credit is outstanding. Additional 100 bps are charged on loans outstanding over 36 months, provided the loan size remains above 187.5 percent of quota, or if credit remains outstanding in excess of 51 months.[5]

Relative to the IMF, euro area official lending through the European Stability Mechanism has a larger concessional element. As described in detailed in a companion paper (Corsetti et al., 2017), the European framework evolved significantly in time. Initially, euro area official loans were designed after the IMF blueprint. As new institutions were created, the terms of official lending changed. Euro area official lenders do not apply fixed loan maturity standards, and stand ready to extend maturities even beyond three decades. This is in contrast to IMF practice, where loans horizons are officially limited to 10 years. Similarly, while IMF spreads are set to grow both with the size of the loan and the repayment period and can vary between 100 and 300 basis points, euro area official lenders charge a lower margin between 10 bps for standard loans and 30 bps for loans directed to the banking system. As a result of its narrower margins, the interest rate paid for euro area official loans can be significantly lower than that charged for an IMF loan.

[5]Borrowing limits were modified in 2016. A country has access up to 145 percent of its quota for any 12-month period, and cumulative access up to 435 percent of quota over a program. The IMF can, on a case-by-case basis, lend above normal limits under the exceptional access policy framework.

While European institutions and the IMF operated in coordination in each country program, the figures just described document a significant divergence in the approach to official lending. Some implications of these different lending terms for sovereign debt dynamics are discussed in the following subsection.

3.3 Key Facts

This section presents a set of stylized facts that we will use to discipline our model. We rely on three main data sources. Financial markets data (sovereign yields) is collected from Bloomberg. Data on the various official lending vehicles comes from Corsetti et al. (2017). Finally, quarterly data on debt stocks and refinancing needs are from the European Central Bank.

We focus on five stylized facts.

Fact 1: Euro area official loans are larger in size, feature longer maturities, and imply lower borrowing costs that loans from the International Monetary Fund

Table 1 here

As illustrated by Table 1 (and described in detail by Corsetti et al. (2017)), the financial terms offered by the euro area official lenders in the programs signed by Spain, Cyprus or Greece with the European Stability Mechanism were substantially more concessional than IMF standards. In Cyprus and Greece, the original maturity of the loans stood above 20 and 30 years, with 87 bps and 107 bps interest rates, respectively. This contrasts with the 7 year maturity loan Cyprus received from the IMF with an interest rate 20 bps higher, and the 8-year maturity EFF loan to Greece with a 406 bps interest rate.

Fact 2: Despite the sharp increase in sovereign spreads at the onset of the crisis, public debt stocks kept increasing. Countries financed further debt accumulation by switching from market financing to official sources, to a large extent euro area official loans.

Public debt in Europe began to increase in 2008, but it took a while before spreads reacted negatively, especially in Peripheral countries. Despite a significant worsening in market access conditions, however public debt accumulation proceeded unabated into the crisis. Official creditors play a key role in supporting debt expansion.

Figure 1 here

Figure 1 shows that while in an initial phase governments could finance their increasing debt by relying on sovereign bond markets, starting in 2010, when market access conditions significantly worsened, the importance of official loans rose markedly, especially that of euro area official lending.

Fact 3: With improving market conditions, sovereigns return to bond financing.

Figure 2 compares the evolution of market and official borrowing costs of each country, over time. As market yields fell significantly (also by virtue of the ECB policies), Ireland and Portugal started to replace the expensive IMF debt, with cheaper bond issuance.[6]

Figure 2 here

Concessional loans shifted repayments into the future, containing the risk of roll-over crises and easing the constraints on further debt accumulation.

Fact 4: Official loans, especially those from the euro area, smoothened the repayment structure of public debt in program countries.

Figure 3 shows visually the importance of the ESM/EFSF repayment schedule in smoothing debt repayments in the four program countries. The repayment of ESM loans kicks in only once the loans provided by the IMF have been repaid in full.

Figure 3 here

From a different angle, Figure 4 plots one-year ahead roll-over needs—measured as debt maturing in the next 12 months as percentage of total debt—against different share of official debt in total debt. A negative relation between the two is apparent. Official loans by the ESM significantly smoothed the repayment flows over time, reducing the period-by-period refinancing needs of crisis countries and their vulnerability to rollover risk (see also Corsetti et al. 2018).

Figure 4 here

[6]In January 2014, the cost of the IMF credit to Portugal stood above 4 percent. Given that Portuguese market rates during 2014 were consistently below the IMF rate, the authorities decided to embark on an early repayment of the IMF loan, financed issuing marketable securities. Similarly, in the Irish case, following large IMF disbursements since January 2011, borrowing from the Fund exceeded the 300 percent of the quota in early 2014. This implied that Ireland faced marginal interest payments of 4.05 percent on its IMF credit. In contrast, during the summer of 2014, prevailing market interest rates and the rates on the longer maturity ESM loans were far lower. This created an opportunity for Ireland to lower interest expenses by replacing the portion of IMF credit subject to surcharges with newly issued cheaper bonds.

Finally, we discuss evidence regarding the extent to which euro area and IMF official lending terms affected the interest rate bill from the existing debt stocks (see also ESM, 2017).

Fact 5: Official loans, especially those from the euro area, significantly reduced the interest rate bill for the sovereign under programs.

Figure 5 reports ESM calculations of the 'saving' on interest costs made possible by the low spreads charged by euro area official lenders in the period 2011-2016, relative to both IMF and market financing (see ESM 2017). The figure plots a rough measure of "savings on the interest bill" in percentage of GDP.[7]

Figure 5 here

The effects of euro area official loans on interest payment flows are quite significant, relative to market conditions. Even for Spain, whose program was the smallest in percentage of GDP relative to other crisis countries, official assistance is estimated to have lowered the interest bill by a full percentage point of GDP.[8] While an order of magnitude smaller, savings are also non-negligible relative to the IMF lending conditions.[9]

4 The Model

In this section we describe our model. Building on Conesa and Kehoe (2017), henceforth CK), we specify an environment with both rollover and fundamental (output) risk where the country can be in one of three zones, labeled 'safe', 'crisis' and 'default' zone. At low enough levels of debt, the country is in the 'safe zone' where it never defaults, not even if it suffers a debt rollover crisis and lose market access. For high enough levels of debt, the country is in the 'default zone,' where it defaults for fundamental reasons (a persistent recession), regardless of the availability of market funding. At intermediate levels of debt, the country is in a 'crisis zone,' where it services its debt if market funding is available, but defaults

[7]In the Figure 5, "savings" are calculated by comparing each country's average sovereign market spread (or IMF rate corresponding to a loan with size and maturity as that of the ESM loan), matching the ESM maturity profile, with the equivalent ESM funding cost, and applying that difference to the actual loan by the ESM. Following ESM (2017), a cap of 6.4 percent is applied to the market rate

[8]We note that Figure 5 does not include savings from the EFSM, GLF and other bilateral official loans. Given that the conditions of the vehicles were analogous to those applied to the EFSF loans, overall savings might be significantly larger.

[9]As Figure 5 does not to include the hedging costs of borrowing on SDR nor different fees the IMF charges, but it does include the fees charged by the ESM, we see these figures as a lower bound on the amount of extra savings.

if funding dries up per effect of a rollover crisis. The domestic government and consumers have concave utility, since the crisis zone would disappears with linear utility. International investors are instead risk neutral.

In this framework, we model the possibility that sovereigns obtain financing from international bailout agencies, which dictate the terms of the official loans. We specify two types of bailout agencies: a IMF-like agency which lends short at relatively high (still below market) rates; and a ESM-like agency which lends long on more generous terms.

In any given period, the state of the economy is given by the vector $s = (b, b_i, b_e, a, z_{-1}, \zeta)$. Here b is the level of government debt owed to international creditors, b_i is debt owed to the IMF, b_e is debt owed to the ESM; a is an indicator variable recording whether the economy is in a recession, $a = 0$, or in normal times, $a = 1$; z is an indicator variable recording whether default has occurred in the past $z_{-1} = 0$ or has not (yet) occurred $z_{-1} = 1$; finally, ζ is a sunspot coordinating agents' belief on the possibility of a rollover crisis. As in Conesa and Kehoe (2017), the country GDP is given by $GDP(a, z) = A^{1-a}Z^{1-z}y$.

In our analysis, we assume that the economy starts out with $a = 1$ and $z = 1$ but is hit by a recessionary shock in period 0, $a = 0$. Every period thereafter, the economy recovers with probability $p < 1$, and, once recovered, never falls into recession again. For simplicity, if and when the government chooses to default, the economy stays in default forever, $z = 0$. We posit a constant tax rate θ (calibrated to match the data) so that consumption is $c(a, z) = (1 - \theta)y(a, z)$. The government can sell new bonds b' at the price $q(b', s)$ to international investors, or seek a bailout either from the IMF b'_i at the price q_i, or from the ESM b'_e at price q_e. The country's government takes the official prices q_e and q_i as given. In accordance with the data we will set $q_e \leq q_i$, i.e. the ESM lends at more generous terms. Furthermore, to derive transparent analytical solutions, we will posit that loans by the IMF-type agency are one-period, while loans by the ESM-type agency have long maturity (so $\delta < 1$). Denoting government expenditure with g, the government's budget constraint is

$$g + z(b + b_i + \delta b_e) = \theta y(a, z) + q(b', s)b' + q_i b'_i + q_e\left[b'_e - (1 - \delta)b_e\right]$$

As in Cole and Kehoe (2000) and Conesa and Kehoe (2017), rollover risk is modeled as a sunspot ζ drawn from a uniform distribution on $[0, 1]$. If $\zeta > 1 - \pi$, international creditors develop beliefs that a rollover crisis may occur, and, for debt levels high enough that a crisis is self-validating in equilibrium, refuse to lend to the government. The probability π determines the probability that a self-fulfilling rollover crisis materializes, for debt levels that are high enough for a speculative run on debt to induce the government to default.

As regards timing, we adopt the same sequence as in Conesa and Kehoe (2017), save

for the fact that in our model the sovereigns can appeal to international bailout agencies if international creditors refuse to lend. In particular, in each period, the time line is as follows. First, the shocks a and ζ are realized, and given the aggregate state $s = (b, b_i, b_e, a, z_{-1}, \zeta)$, the government chooses how much to borrow from international creditors b', the IMF b'_i, and the ESM b'_e. Second, each of a continuum of measure-one international bankers choose how much debt B' to purchase, and the IMF and ESM provide the funds $B'_i = b'_i$ and $B'_e = b'_e$ according to the sovereign's request but within the constraint of their standard loans. Lastly the government decides to repay or default z, thereby generating y, c, and g.

Formally, the problem of the government is as follows:

$$
\begin{aligned}
V(s) = \max u(c, g) + \beta EV(s') \\
c = (1-\theta)y(a, z) \\
g + z(b + b_i + rb_e) = \theta y(a, z) + q(b', s)b' + q_e b'_e + q_i b'_i \\
z = 0 \text{ if } z_{-1} = 0
\end{aligned}
$$

As in CK we assume that, for any feasible (b, b_i, b_e), the following condition holds: $u_g((1-\theta)Ay, \theta Ay - b - b_i - \delta b_e) > u_g((1-\theta)y, \theta y - b - b_i - \delta b_e)$. The government has an incentive to raise debt and "gamble for redemption" during a recession, as the marginal benefit of government spending is higher in a recession than in normal times. This assumption is satisfied by standard concave utility functions like $\log(c + g - \bar{c} - \bar{g})$.

International creditors are risk neutral with discount factor β, so bond prices $q(b', s)$ are determined by probability of default next period. There is a continuum of such creditors, each solving

$$
\begin{aligned}
W(b, b', s) = \max x + \beta EW(b', b'', s') \\
x + q(b', s)b' = w + z(b', s, q(b', s))b \\
x \geq 0, \ b \geq -A
\end{aligned}
$$

whereas we assume that investors have 'deep enough pocket', i.e., x is large enough to rule out corner solutions, and the condition on A rules out Ponzi schemes. The bailout agencies, IMF and ESM, solve similar problems, except that the prices q_i and q_e at which they lend are exogenously set and while the entire stock of IMF debt comes due in the next period, only a fraction δ of the ESM debt comes due every period.

Finally, again following Conesa and Kehoe (2017), we consider equilibria with a simple

Markov structure. In an environment where output and rollover risk can only take on two values, the three zones (safe, crisis and default) can then be characterized by four debt thresholds: two above which default occurs in a recession, without market financing $b(0)$ and with market financing $B(0)$; two in normal times, without market financing $b(1)$ and with market financing $B(1)$. Since $b(0) < B(0)$ and $b(1) < B(1)$, the intervals between these thresholds define the crisis zone in normal times and in a recession, respectively. The safe zone, where there is no default, is for level of debt below $b(0)$ in a recession, and $b(1)$ in normal times. In the original contribution by Conesa and Kehoe, these thresholds are points on the real line. In our model, these are three dimensional objects, as they depend on the composition of debt, i.e. the share of b_i and b_e in total debt. A qualifying feature of our analysis is indeed that sustainability will be assessed in relation to these four thresholds, hence conditional on the state of the economy and the investors' 'sentiment.'

Finally, as in the CK model, our model also features multiple equilibria. Given our interest in understanding the effect of official bailouts on sovereign incentives to run up or run down its debt due to output and rollover risk, we turn to the characterization of the equilibrium under these two types of risk next.

5 How does official lending address rollover and recession risks?

To gain analytical insight, in this section we consider a simplified version of the model. Namely, we let the country to have access to only one official lending instrument, characterized by two parameters: maturity δ and price q_e. Hence we model bailouts indexing official loans by δ and q_e, with a one-for-one exchange from (short-term) market debt to official debt. We assume that a government in default suffer a sunk loss in output equal to τ and recessions are associated by a sunk loss in output equal to a (rather than fractions of output $1-Z$ and $1-A$ as in the full model).[10] We also assume no minimum consumption spending, and an initial state with positive outstanding market debt, but zero official debt.

To focus sharply on the different economic forces at work, we will now consider two separate model environments, one with rollover risk only, the other with output risk only. As a matter of notation, when appropriate, we will denote (sustainable) debt conditional on official lending with the superscript 'l', and 'nl' for the case of no official lending.

[10]For analytical tractability, without loss of generality we posit that governments do to abscond with current period borrowing (the proofs will go through even if they do, but with added complications).

5.1 Rollover Risk

Absent output risk, the government only problem is to choose the path bringing the country out of the crisis zone, that is, how fast it should reduce the debt stock to safe levels. The trade off is between smoother consumption (a longer period in the crisis zone) and better borrowing costs (from reaching the safe zone earlier). We will see below that official lending can substantially ameliorate this trade-off, fostering consumption smoothing in the process of deleveraging from the crisis to the safe zone. Official lending indeed allows the government to sustain higher consumption along the transition and can (be structured to) ensure an early exit from the crisis zone. However, to the extent that it ends up raising average debt in the economy, a bailout regime also reduces long-run consumption, with potential consequences for sustainability.

When the only source of risk is the possibility of a rollover crisis, the equilibrium in our model can be characterized in terms of two debt thresholds only: a lower threshold $b(1)$ beyond which the government defaults conditional on a rollover crisis, i.e., if market funding becomes unavailable; an upper threshold $B(1)$ conditional beyond which the government would default even if market funding were available. Note that there is no recession risk, hence our assessment is conditional on normal times.

5.1.1 Official lending and sustainability with rollover risk

The key question of interest is how the debt threshold responds to the availability of official financial assistance in the presence of rollover risk. We find it useful to introduce our analysis by studying the effects of extending the country access to official loans from one period to two or more periods.

Notation-wise, denote with b_{nl} the limit of sustainable (one-period) debt with no market financing and no official debt available, while $b_{l(1)}$, $b_{l(2)}$ etc. denote the debt limit when one-period bailout funding is available for one period only, two periods and so on. To save on notation and enhance transparency, in this subsection we write debt omitting the state of the economy, that is, we write b_{nl} instead of $b_{nl}(1)$.

In an economic environment with rollover risk, but no official financial assistance, the debt threshold delimiting the safe zone solves the following conditions (same as in CK):

$$u(y - b_{nl}) + \beta \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta}$$

Consider now the availability of one-period bailout, in the form of short loans B' at the price q. The sustainability condition for $b_{l(1)}$—the debt limit when bailout funding are available

for one period only—is:

$$u(y - b_{l(1)} + qB') + \beta u(y - B') + \beta^2 \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta}, \quad qu'(y - b_{l(1)} + qB') = \beta u'(y - B').$$

where the latter condition defines a mapping $B' = G(b_{l(1)}, q)$, which essentially ensures that B' is chosen optimally to satisfy the government's Euler equation.

Similarly, for a bailout available for two periods, we can write the sustainability condition as follows:

$$u(y - b_{l(2)} + qB') + \beta u(y - B' + qB'') + \beta^2 u(y - B'') + \beta^3 \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta},$$
$$qu'(y - b_{l(2)} + qB') = \beta u'(y - B' + qB''), \quad qu'(y - B' + qB'') = \beta u'(y - B''')$$

For the sake of transparency, focus on the case of log preferences and posit $q = \beta$, such that official lending is at the risk free rate. With single period bailout, the optimal policy function is $B' = \frac{b_{l(1)}}{1+\beta}$. So, the sustainability condition simplifies to:

$$u\left(y - b_{l(1)} + \frac{\beta b_{l(1)}}{1+\beta}\right) + \beta u\left(y - \frac{b_{l(1)}}{1+\beta}\right) + \beta^2 \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta}$$

By concavity, it follows that $b_{l(1)} > b_{nl}$ since

$$(1+\beta)u\left(y - \frac{b_{l(1)}}{1+\beta}\right) = u\left(y - b_{l(1)} + \frac{\beta b_{l(1)}}{1+\beta}\right) + \beta u\left(y - \frac{b_{l(1)}}{1+\beta}\right) = u(y - b_{nl}) + \beta u(y)$$

By the same token, when the bailout is available for two periods, the sustainability condition becomes

$$(1+\beta+\beta^2)u\left(y - \frac{b_{l(2)}}{1+\beta+\beta^2}\right) + \beta^3 \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta}$$

Again, by concavity, $b_{nl} < b_{l(1)} < b_{l(2)}$. The logic of the argument and the proof extends to more periods and holds for general preferences.

The sequence of sustainability conditions above illustrates how lengthening the time horizon over which the official loans are available widens the safe zone, de facto inducing private sector bail-in. We should stress that, if official lending widens the boundary of the safe zone as to include the initial debt stock, sustainability is restored without any need for actual disbursement of official funds. With rollover risk only, it is well known that the mere availability of official loans creates guarantees that calms market fears and allows countries to sustain market financing for higher levels of debt.[11]

[11] See Corsetti and Dedola (2016) for a detailed analysis of the mechanism by which credible backstops

A simple example clarifies this point. Let $\tau = 5$, $Y = 100$, $\beta = 0.95$. With log preferences, in the absence of official lending, sustainable debt is up to $b_{l(1)} = 64.15\%$ of GDP. Now let official lending become available for one period, at the price $q_e = 0.95$, up to 15% of GDP. The availability of official funds (not subject to rollover risk) for one period at the equilibrium price raises the threshold to $b_{l(1)} = 72.4\%$ of GDP, so any stock of debt between 64.15 and 72.4% of GDP is safe. For any debt level below the latter threshold, private investors know that, even if they coordinated on not rolling over their credit to the country, access to official fund would allow the government to (optimally) avoid default. Hence no rollover crisis will occur in equilibrium. The difference between 72.4 and 64.15% of GDP is the private sector bail-in generated by the one-period bailout.

When the country debt exceeds the threshold 72.4%, default is possible despite official financial assistance, and rollover risk is priced by the markets. The mere availability of official debt is not enough to bail-in market investors. However, as explained above, official loans still generate significant benefits when they have either longer maturity or lower spreads than the market.

5.1.2 The debt threshold conditional on a rollover crisis, $b(1)$

In what follows we will dig deeper on the effects of bailouts on the thresholds, focusing specifically on their effects on the transition path from the crisis to the safe zone. For the sake of tractability and analytical transparency, we will develop our arguments imposing a regime of permanent official lending—in other words, we proceed by assuming that the country actually uses official loans every period. Official loans thus change the composition of the country debt in both the short and the long run.

We start once again from the condition defining the debt limit in the absence of any official financing, rewritten below for convenience:

$$u(y - b_{nl}(1)) + \beta \frac{u(y)}{1-\beta} = \frac{u(y-\tau)}{1-\beta} \tag{1}$$

With official lending, parameterized by (q_e, δ) and made available to the country in the event of a crisis, the above condition becomes

$$u(y - b(1) + q_e b'_e) + \beta u(y - \delta b'_e + q_e[b''_e - (1-\delta)b'_e]) + \ldots = \frac{u(y-\tau)}{1-\beta}$$

To derive an analytical expression for the threshold, we posit $b'_e = b''_e = \ldots$, as required by

can raise sustainable debt in the context of a model after Calvo (1988).

optimality, so that:

$$
\begin{aligned}
b(1) + q_e b'_e &= -\delta b'_e + q_e[b'_e - (1-\delta)b'_e] \\
b(1) &= b'_e[\delta(1-q_e) + q_e] \\
b'_e &= \frac{b(1)}{[\delta(1-q_e)+q_e]} \\
-b(1) + q_e b'_e &= \frac{[-\delta(1-q_e) - q_e + q_e]b(1)}{[\delta(1-q_e)+q_e]} = \frac{[-\delta + q_e\delta]b(1)}{[\delta(1-q_e)+q_e]}
\end{aligned}
$$

Given that the threshold condition implies $-b+q_e b'_e = -\tau$, we can write $b(1)$ as a function of price and maturity of official loans, as follows:

$$
b(1) = \frac{\delta(1-q_e)+q_e}{\delta(1-q_e)}\tau
$$

Note that, as $q_e \nearrow 1$, official loans are interest-free and $b(1) \nearrow \infty$, i.e., any level of debt is sustainable.[12] By contrast, if $q_e \searrow 0$ and $\delta > 0$, we have $b(1) \searrow \tau$ and $\delta b_e \searrow \tau$. Even if in the limit official lending does not carry a positive price, it is still affecting the repayment structure. Instead of repaying debt in full every period, only a fraction needs to be repaid hence allowing for potentially larger amounts of outstanding debt (to official, not to market lenders).

In general we have that

$$
\frac{db(1)}{dq_e} = \frac{\delta(1-q_e)+q_e\delta}{\delta^2(1-q_e)^2} > 0
$$

The higher the price (lower the spread) attached to official lending, the higher the level of sustainable debt absent market funding.

Next consider the maturity parameter δ. As $\delta \nearrow 1$, debt has to be repaid in full the following period, and $b(1) \searrow \frac{1}{1-q_e}\tau$. Note that we still have a positive correlation between debt and its price q_e. By contrast we have that, as $\delta \searrow 0$, $b(1) \nearrow \infty$—as before. This is quite intuitive: as the fraction that needs to be repaid in each period approaches zero, the amount that can be borrowed tends to infinity. In general

$$
\frac{db(1)}{d\delta} = -\frac{q_e}{\delta^2(1-q_e)^2} < 0
$$

[12] As $q_e \nearrow 1$, future borrowing from the official lenders is exactly equal the the amount coming due each period, i.e. $\delta b'_e = q_e[b''_e - (1-\delta)b'_e]$. So, for optimality, we also have $-b(1) = q_e b'_e = b'_e$ which means that if the government wants to repay $b(1)$ this period, it can simply borrow $b(1)$ from the official lenders at maturity δ and keep debt levels at $b(1)$ every period henceforth.

This establishes that longer maturities (lower δ) and lower spreads (higher q_e) both serve to raise the level of debt sustainable without market financing.

5.1.3 The debt threshold conditional on market financing, $B(1)$.

In the absence of official lending, the level of debt that is sustainable when investors are willing to finance the government $B_{nl}(1)$ is determined by the following conditions:

$$\max\{V^1(B_{nl}(1)), V^2(B_{nl}(1)), \ldots, V^\infty(B_{nl}(1))\} = \frac{u(y-\tau)}{1-\beta}$$
$$V^T(B) = \frac{1-[\beta(1-\pi)]^T}{1-\beta(1-\pi)}u(g^T) + \frac{1-[\beta(1-\pi)]^{T-1}}{1-\beta(1-\pi)}\frac{\beta\pi}{1-\beta}u(y-\tau)+$$
$$+[\beta(1-\pi)]^{T-2}\frac{\beta u(y-[1-\beta]b_{nl}(1))}{1-\beta}$$
$$g_{nl}^T = y - \frac{1-\beta(1-\pi)}{1-[\beta(1-\pi)]^T}\left(B - [\beta(1-\pi)]^{T-1}b_{nl}(1)\right)$$

where T denotes the time of exit from the crisis zone, when debt is brought to the safe zone, and the last equation in the system above is the result of the series of equations given by

$$g_{nl}^T + B_0 = y + \beta(1-\pi)B_1$$
$$g_{nl}^T + B_1 = y + \beta(1-\pi)B_2$$
$$\ldots$$
$$g_{nl}^T + B_{T-2} = y + \beta(1-\pi)B_{T-1}$$
$$g_{nl}^T + B_{T-1} = y + \beta b_{nl}(1)$$

Now, introduce official lending, and suppose that, in the transition back to the safe zone from the crisis zone, the country relies exclusively on official loans. The relevant budget constraints become

$$g_l^T + B_0 = y + q_{e,T-1}B_1$$
$$g_l^T + \delta B_1 = y + q_{e,T-2}[B_2 - (1-\delta)B_1]$$
$$\ldots$$
$$g_l^T + \delta B_{T-2} = y + q_{e,1}[B_{T-1} - (1-\delta)B_{T-2}]$$
$$g_l^T + \delta B_{T-1} = y + q_{e,0}[b_l(1) - (1-\delta)B_{T-1}]$$

This results in the the following key equation characterizing g_l^T

$$\left[1+\frac{q_{e,T-1}}{\delta+q_{e,T-2}(1-\delta)}+\frac{\Pi_{t=T-2}^{T-1}q_{e,t}}{\Pi_{t=T-3}^{T-2}\delta+q_{e,2}(1-\delta)}+\ldots+\frac{\Pi_{t=1}^{T-1}q_{e,t}}{\Pi_{t=0}^{T-2}\delta+q_{e,2}(1-\delta)}\right]g_l^T+B_0=$$
$$\left[1+\frac{q_{e,T-1}}{\delta+q_{e,T-2}(1-\delta)}+\frac{\Pi_{t=T-2}^{T-1}q_{e,t}}{\Pi_{t=T-3}^{T-2}\delta+q_{e,2}(1-\delta)}+\ldots+\frac{\Pi_{t=1}^{T-1}q_{e,t}}{\Pi_{t=0}^{T-2}\delta+q_{e,2}(1-\delta)}\right]y+$$
$$\frac{\Pi_{t=1}^{T-1}q_{e,t}}{\Pi_{t=0}^{T-2}[\delta+q_{e,t}(1-\delta)]}q_{e,0}b_l(1)$$

Note that in the special case where $q_{e,0}=\beta$, $q_{e,1}=q_{e,1}=\ldots=q_{e,T-1}=\beta(1-\pi)$ and $\delta=1$ we get back the equation that obtains in the absence of official lending. The functional specification of the value function V^T is unchanged save for the g^T argument—as by definition $B(1)$ is the maximum amount that can be sustained with market lending, so the country still defaults with probability π. [13]

As before, we gain insight on the role of official lending by splitting the analysis into the case when official loans differ from market loans in maturity only, and in the price only. First consider official lending with maturity $\delta<1$, while setting $q_{e,0}=\beta$, $q_{e,1}=q_{e,1}=\ldots=q_{e,T-1}=\beta(1-\pi)$. In this case the equation for g_l^T specializes to

$$g_l^T=y-\frac{1-x}{1-x^T}\left[B-x^{T-1}\beta b_l(1)\right],\quad x=\frac{\beta(1-\pi)}{\delta+\beta(1-\pi)(1-\delta)}$$

Recall that we are interested by assessing the amount of additional debt B that can be supported by increasing the maturity of official lending (a lower δ). The key equation stipulates that $B(1)$ also depends on T since, by making its current debt decision, the country chooses how long it remains in the crisis zone, i.e.

$$\max\{V^1(B_{nl}(1)),V^2(B_{nl}(1)),\ldots,V^\infty(B_{nl}(1))\}=\frac{u(y-\tau)}{1-\beta}$$

It is certainly possible that $dT/d\delta\neq 0$, i.e., a change in maturity would also affect the optimal time to exit. However, it is convenient to proceed at first abstracting from any endogenous change in the time of exit; and then extending our arguments and results to the general case.

Sustainability for a given time of exit from the crisis zone, T.

For the time being, suppose that the optimal time to exit is unaffected by the change in δ.

[13] In this stylized example, the budget constraint is derived under the implicit assumption that official lending terms are more generous than market lending.

Then we can write:

$$\frac{dV^T}{d\delta} = \underbrace{\frac{1-[\beta(1-\pi)]^T}{1-\beta(1-\pi)} u'(g^T) \underbrace{\frac{dg^T}{d\delta}}_{<0}}_{\uparrow V^T \text{ as } \downarrow \delta} + \underbrace{[\beta(1-\pi)]^{T-2} \frac{\beta u'(y-[1-\beta]b_l(1))}{1-\beta} \cdot -(1-\beta) \underbrace{\frac{db_l(1)}{d\delta}}_{<0}}_{\downarrow V^T \text{ as } \downarrow \delta}$$

where

$$\frac{dg^T}{d\delta} = \underbrace{-\underbrace{\frac{d\frac{1}{1+x+x^2+\ldots+x^{T-1}}}{dx}}_{<0} \underbrace{\frac{dx}{d\delta}}_{<0} B - \beta \underbrace{\frac{d\frac{x^{T-1}}{1+x+x^2+\ldots+x^{T-1}}}{dx}}_{<0} \underbrace{\frac{dx}{d\delta}}_{<0} b_l(1)}_{\uparrow \text{ consumption for } \downarrow \delta} + \underbrace{\beta \frac{x^{T-1}}{1+x+x^2+\ldots+x^{T-1}} \underbrace{\frac{db_l(1)}{d\delta}}_{<0}}_{\uparrow \text{ consumption for } \downarrow \delta}$$

There are at least three important results that can be read from these expressions. The first concerns the effects of maturity on government consumption. Focus on the decomposition on the right hand side of $\frac{dg^T}{d\delta}$: the first two terms show that, in the transition, a longer maturity ($d\delta < 0$) raises consumption by increasing the effective discount rate on the amount to be repaid—a lower discount factor lowers it in present value terms. The last terms highlights an additional positive effect of longer loans maturity: it raises the level of debt that the government can sustain in the long run $b_l(1)$. To the extent that the government has less debt to run down or repay, it can consume more *in the transition*. However, and this is key to our argument below, if the steady-state debt (in the safe zone) is higher, long-run consumption will correspondingly be lower.

Having established that, with longer maturity, government consumption rises in the transition but falls in the long run, we can then discuss whether the value function V^T rises or falls with the maturity parameter δ. There are two opposing forces at play. The value function rises with higher transitional government consumption—this is shown in the first term in the decomposition of $\frac{dV^T}{d\delta}$. It falls with a lower long-term consumption (associated with a higher steady-state level of debt in the safe zone $b_l(1)$).

Which of these two forces dominates depends on the probability that market financing dries up, π, and the time to exit the crisis zone T. To appreciate these points, note that the second term in the value function expression disappears, so that the value function unambiguously rises with longer maturity, both when $\pi \to 1$ and $\beta(1-\pi) \to 0$, and when $T \to \infty$. In the first case, a rollover crisis and hence the default are almost sure; in the second case the country stays in the crisis zone indefinitely, as long as market lenders are willing to lend (so that, again, a crisis will occur almost surely at some point). The value function goes up because longer maturity allows the government to consume more while in a crisis.

We have seen that, conditional on $\pi \to 1$ or $T \to \infty$, $V^T(B_l(1))$ increases as δ decreases. We are now ready to discuss our third, and key, result, concerning the level of sustainable debt. Consider the indifference condition:

$$\max\{V^1(B_l(1)), V^2(B_l(1)), \ldots, V^\infty(B_l(1))\} = \frac{u(y-\tau)}{1-\beta}$$

The key observation is that, while the left hand side of this condition rises, the right hand side remains unchanged. Since we know that $V^T(B_l(1))$ is decreasing in $B_l(1)$ for all T—for the equality above to hold. it must be the case that $B_l(1)$ rises as δ falls.

Just as we have shown how debt maturity δ affects $B_l(1)$, we can examine the effect lowering the spread on (i.e. raising the price of) official debt, q_e. As above, we start by writing out the expressions for both $\frac{dV^T}{dq_e}$ and $\frac{dg^T}{dq_e}$:

$$\frac{dV^T}{dq_e} = \underbrace{\frac{1-[\beta(1-\pi)]^T}{1-\beta(1-\pi)} u'(g^T) \underbrace{\frac{dg^T}{dq_e}}_{>0}}_{\uparrow V^T \text{ as } \uparrow q_e} + \underbrace{[\beta(1-\pi)]^{T-2} \frac{\beta u'(y-[1-\beta]b_l(1))}{1-\beta} \cdot -(1-\beta) \underbrace{\frac{db_l(1)}{dq_e}}_{>0}}_{\downarrow V^T \text{ as } \uparrow q_e}$$

and

$$\frac{dg^T}{dq_e} = \underbrace{-\underbrace{\frac{d\frac{1}{1+x+x^2+\ldots+x^{T-1}}}{dx}}_{<0} \underbrace{\frac{dx}{dq_e}}_{>0} B - \beta \underbrace{\frac{d\frac{x^{T-1}}{1+x+x^2+\ldots+x^{T-1}}}{dx}}_{<0} \underbrace{\frac{dx}{dq_e}}_{>0} b_l(1)}_{\uparrow \text{ consumption for } \uparrow q_e} + \underbrace{\beta \frac{x^{T-1}}{1+x+x^2+\ldots+x^{T-1}} \underbrace{\frac{db_l(1)}{dq_e}}_{>0}}_{\uparrow \text{ consumption for } \uparrow q_e}$$

The impact of the higher price on government consumption and the value function can be appreciated following the same steps as for loan maturity. Namely, we first observe that a higher debt price increases both the discount rate and the debt limit absent market funding—both serve to increase government consumption in the transition. Higher transitional government consumption g^T in turn raises the value function for a given level of $B_l(1)$. This positive effect is however countered by the higher steady state level of $b_l(1)$, which lowers the value function. With $\pi \to 1$ or $T \to \infty$, the second force vanishes and $V^T(B_l(1))$ increases with higher q_e. Given that the optimal time to exit remains unchanged, this in turn implies that $B_l(1)$ rises with higher q_e.

Endogenous exit time, T.

The argument so far abstracts from the effect of varying the terms of the official loans on the

time of exit from the crisis zone. We now complete our argument by examining the general case, discussing how the optimal time to exit T changes with a (sufficiently large) fall in δ. To do so, observe that $\frac{dV^T}{d\delta} < 0$ as $T \to \infty$, since the steady-state term disappears, and $\frac{dV^T}{d\delta} > 0$ as $T \to 0$, since the transitional term disappears. Thinking of T as a continuous variable and using the fact that $\frac{dV^T}{d\delta}$ is continuous and decreasing in T, we have that $\exists T^*$ s.t. $\frac{dV^T}{d\delta} > 0$ for all $T < T^*$ and $\frac{dV^T}{d\delta} \leq 0$ for all $T \leq T^*$. Suppose T_1 solves

$$V^{T_1}(B_{nl}(1)) = \max\{V^1(B_{nl}(1)), V^2(B_{nl}(1)), \ldots, V^\infty(B_{nl}(1))\} = \frac{u(y-\tau)}{1-\beta}$$

so that T_1 is the optimal time to exit absent official lending, with market debt indexed by maturity paramter $\delta_1 = 1$. Define T_2 to be the optimal time to exit when there is official lending characterized by maturity parameter $\delta_2 < 1$ and q_e. Suppose $T_1 \leq T^* - 1$. Then we know that, as maturity lengthens and $\delta_2 \searrow 0$, we have $\frac{dV^T}{d\delta} > 0$ for all $T \leq T_1$ and $\frac{dV^T}{d\delta} \leq 0$ for all $T > T^*$. So for a sufficiently large drop in δ_2 it is possible that the optimal time to exit switches to $T_2 > T_1$. If this is the case, denote by $B_l(1) = B_{T_2}(\delta_2)$ the level of sustainable debt with both official lending and time to exit T_2, with $B_{T_1}(\delta_2)$ the level of sustainable debt with both official lending and time to exit T_1, and with $B_{nl}(1) = B_{T_1}(\delta_1)$ the level of sustainable debt with no official lending and time to exit T_1. We then have that $B_l(1) = B_{T_2}(\delta_2) > B_{T_1}(\delta_2) > B_{T_1}(\delta_1) = B_{nl}(1)$, where the first inequality follows from the value function being decreasing in debt and T_2 being the optimal time to exit (so $V^{T_2}(\delta_2) > V^{T_1}(\delta_2)$);[14] and the second inequality follows from the earlier observation that the level sustainable rises with longer maturity provided π or T is sufficiently large. As the change in maturity transpires into a lengthening of the time spent in the crisis zone, this only serves to increase the equilibrium value function (and hence the level of sustainable debt)—since the utility gains from higher government consumption while in the crisis zone outweigh the losses in the safe zone.

A comment is in order concerning the asymmetric effect of official lending on the government's choice sets in the presence of rollover risk. In particular, official lending introduces an asset that raises utility in a rollover crisis, but not necessarily so with market lending. In some cases, official lending narrows the 'crisis zone' both from below (higher level of safe debt) and above (a lower level of initial debt that can be sustained even absent rollover crisis). Namely, we know that for sufficiently low T, an increase in the safe zone (a higher $b(1)$) only serves to lower the value function (as official lending has limited or no impact on the discount factor).[15] This in turn results in a drop of the upper threshold $B(1)$, and

[14]The result follows from equating these values with the value of default.

[15]Recall our earlier observation, that a credible regime of financial assistance widens the safe zone inde-

thus a smaller crisis zone $B(1) - b(1)$. Official lending can actually eliminate the crisis zone altogether, for sufficiently high q_e and low δ—as shown earlier, $b(1) \to \infty$. But to the extent that a longer maturity of official loans transpires into a delay in exit, a key conclusion of our analysis is that they will raise the country's welfare—essentially by improving consumption smoothing in the transition.

5.2 Output Risk

In a model environment with fundamental output risk, the two relevant thresholds define the maximum level of sustainable debt in a recession and in normal times, denoted by, respectively, $B(0)$ and $B(1)$.

5.2.1 The debt threshold in normal times, $B(1)$.

To analyze this threshold, it is convenient to start considering two extreme lending cases, one in which the government borrows short term, the other—long term. Without official financing, if the government borrows short term only, the condition defining $B_{nl}(1)$ takes the form:

$$\frac{u(y - [1-\beta]B_{nl}(1))}{1-\beta} = \frac{u(y-\tau)}{1-\beta} \Rightarrow B_{nl}(1) = \frac{\tau}{1-\beta}.$$

Denote with B_1 the debt threshold when the country borrows long term. With the country issuing bonds with maturity $\delta < 1$ and sold at risk-neutral prices, this threshold satisfies:

$$\frac{u(y - \delta B_1 + q[B_1 - (1-\delta)B_1])}{1-\beta} = \frac{u(y-\tau)}{1-\beta},$$

where

$$q = \frac{\beta\delta}{1-\beta(1-\delta)} \Rightarrow B_1 = \frac{\tau}{\delta(1-q)}$$

It is easy to verify that that $B_1 > B_{nl}(1)$, since

$$\delta(1-q) = \frac{\delta(1-\beta)}{1-\beta(1-\delta)} < \frac{[1-\beta(1-\delta)](1-\beta)}{1-\beta(1-\delta)} = 1-\beta$$

We can in fact decompose the ratio $B/B_{nl}(1) > 1$ into a maturity and price components:

$$\frac{B_1}{B_{nl}(1)} = \underbrace{\frac{1}{\delta}}_{\text{maturity effect} > 1} \cdot \underbrace{\frac{1-\beta}{1-q}}_{\text{price effect} > 1}$$

pendently of effective disbursement of funds.

The maturity effect is intuitive. Suppose that with one period debt (all debt had to be repaid in each period), the maximum sustainable level is 100. If only a quarter has to be repaid in each period, a given cash flow can sustain 400. The price effect is subtler, in that it relies on discounting. Essentially, the difference between β and q lies in the fact that q splits the bond return over time so that with future proceeds discounting, $q < \beta$. To wit: if there was no discounting and $\beta = 1$, we would have $q = \beta = 1$ and there would be no price effect. The mechanism works in the following way: for any given level of debt issuance, a lower price for debt allows the government to consume more (and hence have a higher value function when it repays); so, to make the government indifferent between repaying and defaulting, it must be allowed to borrow more.

When we introduce official lending (once again, assuming that official debt is chosen in perpetuity), the relevant condition for the threshold becomes:

$$u(y-B_l(1)+q_eB_e)+\beta u(y-\delta B_e+q_e[B'_e-(1-\delta)B_e])+\beta^2\frac{u(y-\delta B'_e+q_e[B'_e-(1-\delta)B'_e])}{1-\beta}=\frac{u(y-\tau)}{1-\beta}$$

We are interested in understanding the implications of a portfolio change, from market to official debt. Assuming $q_e \in (q, \beta))$, we can think of the effects of this change essentially as a convex combination of two cases analyzed above: one has only Short Term (ST) lending and price β, the other Long-Term (LT) lending and price q. In other words, we will have $B_{nl}(1) < B_l(1) \leq B_1$.[16]

5.2.2 The debt threshold in recessions, B(0).

The last threshold is for the case of an economy currently in a recession, with output equal to $y - a$, with a probability of recovery equal to p. Absent official lending, the indifference condition is given by

$$u(y-B_{nl}(0)-a+\beta pB_{nl}(1))+\beta p\frac{u(y-[1-\beta]B_{nl}(1))}{1-\beta}+\beta(1-p)\frac{u(y-\tau-a)}{1-\beta}=$$
$$u(y-\tau-a)+\beta p\frac{u(y-\tau)}{1-\beta}+\beta(1-p)\frac{u(y-\tau-a)}{1-\beta}$$

[16]Setting $B_e = B'_e = B_1$, and letting $q_e = q$ we obtain $B_l(1) = [q + \delta(1-q)]B_1 = \frac{\delta}{1-\beta(1-\delta)}B_1 < B_1$. Now imagine that $q_e = q = 0$. In this case we have $B_{q=0} = (1-q)B_1$ and $B_{l,q=0}(1) = \delta B_{q=0} = \delta(1-q)B_1 = \frac{\delta(1-\beta)}{1-\beta(1-\delta)}B_1 < B_l(1)$. In fact, $B_{l,q=0}(1)/B_l(1) = (1-\beta)$. On the other hand, consider the case where $q_e = q = 1$. In this case, we have $B_{l,q=1}(1) \to \infty$ and any level of debt is sustainable as one simply has to borrow the same amount every period ad infinitum.

We know from the discussion above that $B_{nl}(1) = \frac{\tau}{1-\beta}$. Hence we have

$$B_{nl}(0) + a - \beta p B_{nl}(1) = \tau + a \Rightarrow B_{nl}(0) = \tau + \beta p \frac{\tau}{1-\beta} = \frac{\tau(1-\beta+\beta p)}{1-\beta} < B_{nl}(1)$$

It is useful to clarify from the start that default risk in future recession is what drives the thresholds $B_{nl}(0)$ and $B_{nl}(1)$ apart. To see this most clearly, take the ratio of the two:

$$\frac{B_{nl}(0)}{B_{nl}(1)} = 1 - \beta + \beta p$$

Under our assumption, once in a recession, the economy only recovers with finite probability p and thus remains exposed to risk of default. The debt price at which investors are willing to lend is therefore βp and not β, as it would be if the economy were expected to recover for sure. Indeed, as $p \nearrow 1$, debt becomes less risky and the two thresholds converge—they actually coincide in the limiting case in which the probability of remaining in a recession (hence default risk) vanishes.

As above, consider the case in which the government borrows long term, by issuing bonds with maturity δ at the price q. We have

$$u(y - \delta B_0 - a + q_0[B_1 - (1-\delta)B_0]) + \beta p \frac{u(y - \delta B_1 + q[B_1 - (1-\delta)B_1])}{1-\beta} + \beta(1-p)\frac{u(y-\tau-a)}{1-\beta} =$$
$$u(y-\tau-a) + \beta p \frac{u(y-\tau)}{1-\beta} + \beta(1-p)\frac{u(y-\tau-a)}{1-\beta}.$$

where from risk-neutral pricing and the previous analysis we know that

$$q_0 = \frac{\beta p \delta}{1 - \beta p(1-\delta)}, \quad B_1 = \frac{\tau}{\delta(1-q)}, \quad q = \frac{\beta\delta}{1-\beta(1-\delta)}$$

After some algebraic manipulation, we obtain the following expression for B_0

$$\delta B_0 - q_0 B_1 + q_0(1-\delta)B_0 = \tau \Rightarrow B_0 = \frac{\tau + q_0 B_1}{\delta + q_0(1-\delta)} \Rightarrow B_0 = \frac{1-\beta+\beta p\delta}{\delta(1-\beta)}\tau$$

Taking once again the ratio between the two thresholds (for the safe and crisis zone), we obtain:

$$\frac{B_0}{B_1} = \frac{1-\beta+\beta p\delta}{1-\beta+\beta\delta}.$$

This ratio is also converging to 1 as $p \nearrow 1$. However, the overall effect is now tempered by the fact that only a fraction δ of debt is repaid every period.

In order to fully appreciate why the threshold B_0 is different from $B_{nl}(0)$, we find it useful

to decompose the ratio between the two as follows:

$$\frac{B_0}{B_{nl}(0)} = \underbrace{\frac{B_0}{B_1}}_{\text{default}\cdot\text{maturity effect}} \cdot \underbrace{\frac{B_1}{B_{nl}(1)}}_{\text{pure maturity+price effect}} \cdot \underbrace{\frac{B_{nl}(1)}{B_{nl}(0)}}_{\text{pure default price effect}} = \frac{1-\beta+\beta p\delta}{\delta(1-\beta+\beta p)} > 1$$

The first component is what we have just discussed: it tells us that the two thresholds are different because the price effect induced by default differs when interacted with longer maturity debt. The second component reflects our earlier discussion, for the case of no recessions and hence no default. In this case, long-term debt has both maturity and price effects. The third component is a pure 'default price effect', that account for the change in the sustainable level of debt brought about by the mere possibility of default in future recessions, with no interaction with debt maturity (see the beginning of this subsection). Of these three components, only the first one is smaller than one. In aggregate, the ratio is greater than 1.

We are now ready to assess the effects of a change in the composition of debt, from market to official lenders. The indifference condition for this case is:[17]

$$u(y - B_l(0) - a + q_{e,0}[B_1 - (1-\delta)B_l(0)]) + \beta p \frac{u(y - \delta B_1 + q_e[B_1 - (1-\delta)B_1])}{1-\beta}$$
$$+ \beta(1-p)\frac{u(y-\tau-a)}{1-\beta} = u(y-\tau-a) + \beta p \frac{u(y-\tau)}{1-\beta} + \beta(1-p)\frac{u(y-\tau-a)}{1-\beta}$$

Setting $q_e = q$ and $q_{e,0} = q_0$, this yields the threshold

$$B_l(0) = \frac{\tau + q_0 B}{1 + q_0(1-\delta)} < \frac{\tau + q_0 B}{\delta + q_0(1-\delta)} = B_0$$

where the inequality is strict because market debt is short-term—less debt $B_l(0)$ is sustainable relative to the case where the original debt was already long-term, B_0.[18]

[17] Observe that the debt policy adopted in the indifference condition is one where the government chooses B_1 and not $B_l(1)$; it might seem more natural to pick $B_l(1)$ but $B_l(1)$ would not be appropriate as it entails another portfolio shift while the official lending terms are already LT, hence the optimal choice being B_1 instead.

[18] Now consider what happens when $q_{e,0} = 0$. In this case we get $B_{l,q_{e,0}=0} = \tau < B_0$. The converse is true if we set $q_{e,0} = 1$. In this case, we obtain $B_{l,q_{e,0}=1} = \frac{\tau+B_1}{2-\delta} \to \infty$ as $B_1 \to \infty$. One key assumption underlying this result is the availability of official lending at all times: this result of infinitely sustainable debt goes away once this assumption is relaxed and official lending is stopped at some point, as then it is impossible to refinance debt every period in perpetuity the way it is done here.

5.3 Discussion

The analytical characterization presented so far is meant to illustrate the forces at work in a simple environment. Three comments are in order concerning our simplifying assumptions. First, for tractability, we have focused on the case in which bailout loans are available at all times and for an infinite number of periods. Not all our results can be expected to go through if this is not the case—i.e., if the availability of bailout funds is on a temporary basis. As official loans affect exit times and repayment/default decisions, it is possible that bailouts over finite horizons can paradoxically have adverse effects on sustainability. In particular, to the extent that a few years with official lending induce countries to service debt in a persistent recession, the eventual withdrawal of support may actually result in a lower threshold for the initial stock of debt that the country is willing to sustain—relative to the case where the default and repayment states are unaffected by the bailout terms.

Second, to ease exposition, we have assumed that the right-hand side of the indifference conditions are unchanged following the introduction of the bailout. This need not be the case: a country may default on official loans, implying that its utility in default could well be a function of the funds obtained from official lenders.

Finally, while we have analyzed rollover and output risk separately, they have significant interactions in the model (as well as in the data). Similarly, while we have considered only one type of bailouts, debt dynamics can potentially be very different if various types of official debt are used in conjunction with market lending. We will allow for this possibility in the quantitative analysis to follow.

6 Quantitative Analysis

In this section, we use the model in its full specification described at the beginning of Section 4 for a quantitative exercise focusing on the case of Portugal in July 2011—preferring it over Ireland, who did not face a recession but a banking crisis, and over Greece, as Portugal did not restructure its debt the way Greece did.[19] We then use the model to carry out some counterfactual exercises, to gain insight on the sensitivity of our results to key features of official lending—maturity and price.[20]

Calibration

We show the list of parameters and targets in Table 2. We normalize output Y to 100 so

[19] The Appendix contains a brief summary of the Portuguese program.

[20] In Corsetti et al. (2017) we study the effect of the terms of the official loans on Ireland's market access conditions, and find results in line with the ones here.

that the units in our model can be interpreted as percentage of GDP: e.g. $B = 120$ means debt that is 120% of GDP. We set the default cost at 5%, consistent with Cole and Kehoe (1996). Our default cost is lower relative to the literature (e.g. Mendoza and Yue (2012)), on the grounds that we assume this cost to be permanent—while others assume this to be temporary. We set the probability of recovery p equal to 0.33, in line with evidence of recovery in Portugal (and other euro area countries that received official support) where the economy bounced back after 3 years. Similarly, we set the fraction of output lost in a recession in line with the realized output drop in the 2011 recession, equal to seven percent. The target for the level of 'essential' government expenditure is average government consumption (21% of GDP). The presence of this non-homothetic term allows us to have a discount factor closer to standard business cycle or growth models than in standard quantitative sovereign default models (where the discount factor can be as low as 0.8; see also Bocola and Dovis 2015). To match model with data, the probability of a market frenzy π is calibrated in conjunction with the discount factor and the probability of recovery to target the spread in Portuguese bonds in July 2011. Government revenue as a fraction of output is used to parameterize the tax variable θ. We follow Conesa and Kehoe (2017) in setting the relative weight of c and g in the utility function equal to 0.5; sensitivity analysis shows that this particular parameter is unimportant for our results.

The parameters discussed thus far are relatively standard and found in other models in this literature; we depart from the literature by introducing two types of official debt instruments into the government's decision problem. In accordance with their empirical counterparts, these two instruments have different maturities and spreads. To capture debt maturity in a parsimonious way, we model long-term debt in the same way as most models in this literature (Chatterjee and Eyigungor, 2012; Hatchondo and Martinez, 2009) where the borrowing country repays a fraction δ of debt each period, and old and new debt are treated alike. Given this assumption, ESM debt is parameterized to reflect initial lending conditions to Portugal: 15 years maturity and no spread over the Bund rate and likewise for IMF debt (7 years maturity and a spread of 300 bps over the SDR rate). The market spread is endogenously determined; its maturity is set to 6 years, consistent with the average maturity of Portuguese debt.[21]

The Portugal case study

In our calibration, the model economy is initially well within the crisis zone, conditional

[21]We note that our stylized specification of maturity captures the key difference between IMF- and ESM-style lending, but falls short of accounting for the specific management of repayment flows that ESM programs feature in reality.

on the level of debt and recessionary state of the economy. The country can only issues market debt at the high spread implied by both rollover and output uncertainty; as the cost of borrowing from the market is prohibitive, it then chooses to borrow from the IMF and ESM instead.

We find that, with the parameterization of Table 2, our model is able to replicate both the initial state of the Portuguese economy and the dynamic evolution of debt and spreads following the access to euro area official lending. In particular, as debt to the IMF and ESM accumulates in the model as it does in the data, consistent with empirical evidence, market spreads fall after the economy recovers in the model—holding constant the sequence of shocks to output and for given market financing. Remarkably, in the model, total debt as a fraction of output is upward sloping, but only mildly so. In this dimension, the model is quantitatively less effective: the total-debt-to-GDP ratio rises by more than 20% in the data. In other words, while our model and parameterization replicate the shifts in debt composition towards official loans, the substitution into (cheaper) official debt and away from market debt hinders the model's ability to replicate the overall rising trajectory observed in the data (where market debt stays almost flat even as official debt rises).

It is worth reiterating, nonetheless, that the model is able to capture most of the change in dynamics of official debt, the associated evolution of spreads (especially the endogenous response of market spreads), as well as the Portugal's transition from the 'crisis' to the 'safe' zone. This is remarkable: all these are generated endogenously and not targeted in the calibration. If anything, the model achieves too much, in that our calibration does not factor in key policy initiatives that, especially after 2012, weighed on spreads—such as the introduction of the Outright Monetary Transaction program by the ECB.

Counterfactuals

We now use the model as a lab to shed light on how the terms of official lending may affect debt sustainability. In particularly, theory suggests that we can think of sustainability in terms of four debt limits or thresholds—separating a safe from a crisis zone for debt, conditional on the economy being in a recession or in normal time. To gain insight on sustainability, we can evaluate quantitatively how these four limits respond to changing the terms of official assistance in three dimensions: size, price and maturity.

We report our results in Figure 6 through 10. Each figure includes 4 panels, one for each debt limit. Each panel shows 16 histograms, that we present as static representations of the full dynamic model. Debt thresholds can be read on the y-axis, corresponding to the height of each histogram. The color pattern of the histogram represents the composition of borrowing. This changes from market only (the first block to the left of each figure) to

markets plus official lending—allowing the IMF- and ESM-type loans to increase in steps, each with size 5% of GDP. To be clear: Block 1 is the no-assistance case. Block 2 through 4 shows that happens to the debt limit when the model economy receives IMF-type assistances in tickets of 5, 10 or 15% of GDP. In block 4 through 8, we consider the effect of an official loan with ESM characteristics equal to 5% of GDP—first on its own, then added to the sequence of tickets from the IMF. In blocks 9 through 12, we repeat the same experiment with ESM loans up to 10% of GDP. Blocks 13 through 16 repeat the same with an ESM ticket up to 15% of GDP.

Consider Figure 6 first, referring to our benchmark calibration. Focus on b(0), on the upper left-hand side of the figure. This threshold measures how much debt the country finds it optimal to sustain free of default risk when the economy is in a recession and suffers a market rollover crisis. A key result suggested by the panel is clearly in line with our analytical result—that the size of the safe zone, where there is no vulnerability to either rollover or fundamental risk, is increasing in official assistance. Our quantitative analysis confirms that this goes true whether assistance comes in the form of either IMF or ESM loans, or both. Quantitatively, the first panel suggests that, in a recession, the economy could only sustain about 80% GDP of debt, had it to rely exclusively on borrowing from the market. But sustainable debt can go up to 90% when the country hold a portfolio of ESM and IMF loans, each measuring up to 15% of GDP. Again, this is consistent with our theory where official debt unambiguously raises the level that is sustainable without market financing.

Comparing the two graphs in the first line further suggests that the effect is similar for b(1)—the debt limit when the economy is not in a recession. This limit is higher than b(0), but is also monotonically rising in assistance—even more than b(0).

The picture is however quite different, and much richer, for B(0) and B(1), the debt limit in a recession or in normal times, respectively, beyond which debt is not sustainable for fundamental reasons (the government would default whether or not subject to a rollover crisis). Consider the case in which the country has access to market financing while in a recession B(0). Here, a moderate amount of official loans is good—as it can raise debt levels from 175% to 180%. But higher levels of official debt turn out to be counterproductive, in the sense that it ends up *decreasing* the total amount of sustainable debt. This is consistent with the theory as well: we have shown that, to the extent that a wider safe zone translate into an increase in long-term steady state debt (shown in the first panel) lowers consumption in the future; this raises the incentives to default, hence lowers the amount of sustainable debt. Similar conclusions can be drawn looking at the panel for B(1).

There are four key variables underlying our benchmark result: ESM debt maturity, ESM lending rate, IMF debt maturity, and IMF lending rate. To examine the role of each of these

variables separately, we run four counterfactuals: a counterfactual where both the ESM and IMF lend at the ESM rate, keeping everything else constant; a counterfactual where both ESM and IMF lend at the same maturity, ceteris paribus; and two counterfactual switching maturity and rates across lending institutions.

The results for these counterfactuals are shown in Figures 7 through 10, using the same format as Figure 6. First consider Figure 7, which illustrates the effects of these four types of policies on the level of sustainable debt for a country in a recession with no access to market financing—the b(0) threshold. The panel on the top left is the result of having both the IMF and ESM lend at the same maturity as ESM debt (15 years), the panel on the top right has both institutions lending at the maturity of IMF debt (7 years), the bottom left is when both ESM and IMF lend at the ESM rate (150 bps above risk free), and the bottom right is when both institutions adopt the IMF rate (350 bps over risk free) ceteris paribus. A key result from this figure is that debt maturity has a stronger impact on the threshold than spreads: holding constant debt composition, one can see that sustainability is highest in the case when both lending institutions structure their bailouts with ESM maturity, and are lowest when both bailouts have IMF maturity. The cases in which maturities are as in the data but spreads are set as in IMF and ESM loans lie within these two extremes.

The result that debt maturity has a stronger effect on thresholds than spread (consistent with our analytical derivation) is confirmed in the other counterfactuals. These however also lend quantitative support to another key theoretical prediction. Namely, debt limits are increasing in official debt conditional on no market financing, but not necessarily so conditional on market financing. In particular, Figures 9 and 10 illustrate our finding, that the debt limits with financing at first increase with more official debt, but eventually drop. They actually drop to levels even lower than conditional on no official debt at all.

Key to this result is the fact that the the upper limit with market financing is a function of the lower limit without market financing—this is so, because a higher lower limit also raises the average (steady state) level of debt that the government finds it optimal to pursue. As we have seen in our analytical section, a rise in the stock of steady-state debt translates into lower long-run consumption. This effect may more than offset any gains from a rise in consumption in the transition from the crisis to the safe zone, reducing the overall value function for any given initial level of debt. To the extent that the value of default is unaffected by these changes—as it is in our model specification—the level of initial debt that is sustainable must fall to bring the value function back in line with the value of default.

This is indeed a key message of our paper: depending on how official debt is structured, the stock of debt that is sustainable with official ending can be higher than or lower than the amount that is sustainable without official lending—the way in which public support is

structured and made available to country matters significantly. In our exercises, sustainable debt ranges from low (80% GDP) to very high (180% GDP) levels, depending on both the state of the economy (output and market access) and the availability, size spreads and maturities (debt composition) of official loans. These effects can be moderated by modeling (political) uncertainty in the access to official lenders, something that we abstract from but would be straightforward to model.

7 Conclusion

In this paper we have explored the extent to (and modalities with) which the terms of official lending affect debt sustainability, by impinging on the behavior of both governments and investors. Official support modifies the incentives for a country to borrow, repay, and eventually reduce its debt to a sufficiently low level that its bonds are issued at default-risk free prices. Using our model, we have explored the mechanisms by which official loans may raise the debt limit for issuing safe bonds, improve risk-sharing in period of market turbulences and recessions—gaining insight into various countervailing forces that official lenders may want to consider when designing a bailout package. If the official lending is structured as to induce higher levels of steady state debt, or repayment in states of the world where the economy has been subject to a sufficiently long sequence of negative output shocks, the threshold for sustainable debt can actually fall.

Far from being a theoretical construct, we take our model to the data and show that it can replicate the debt and spread dynamics in the recent debt crisis in Portugal (2011-2015). Consistent with our theory, we find in the quantitative analysis that official debt is a crucial component in spurring the recovery of the Portuguese economy and lowering market spreads. And, while both quantitatively significant, we find that longer maturities are more important to raising sustainability than lower spreads. We find that different terms of official lending give rise to significantly different thresholds for sustainable debt—quantitatively, we find that, per effect of the bailout, the safe debt threshold varies between 80 and about 100% of GDP for Portugal; the default threshold varies between about 150 to over 180% of GDP. While political uncertainty about the access to the program could moderate the effects of the bailout, the implications are bound to remain significant.

Among directions for future research, an important one concerns the problem official lenders solves when offering bailouts—raising issues in its objective function (possibly reflecting welfare relevant distortions and spillovers from a country default) and relevant constraints. Also, the analysis in this paper abstracts from possible adverse consequences of a bailout on the government incentives to undertake costly but beneficial reforms or policies.

As is well understood, these incentives may respond differently to bailouts addressing self-fulfilling risk as opposed to fundamental risk. Our framework could be developed to gain insight on policy trade-offs when they two risks cannot be completely separated, and interact significantly before and during crises.

Appendix: Official lending to Portugal

Portugal entered a program in April 2011. The financing of its 78 bill program fell on equal parts on the EFSM, EFSF and IMF. The maturity was initially set equal to 7.5 years, as in Ireland. The margin was however lower, about 210 bps. The decision to charge a lower spread might have reflected the fact that, by that time, the ongoing Greek and Irish programs were performing below expectations. Portugal signed a 26 billion euros EFF program with the IMF, with a maturity of seven years.

The program relied on the timely implementation of structural reforms. It was soon apparent that these reforms could not be expected to materialize over the relevant horizon. In reaction, the EFSF and the EFSM granted to the Portuguese government conditions similar to those offered Ireland in late 2010. In particular, in July 2011, the euro area authorities decided to eliminate the margin on both EFSM and EFSF loans, and extend their maturity to a maximum of 15 years.[22] In order to grant identical conditions in Portugal and Ireland, a final change in the terms of the EFSF and EFSM programs occurred in April 2013. On that date, authorities decided that EFSF and EFSM loan maturities would be extended by 7 and half years, to 22 years.

[22] As the initial EFSF loan to Portugal featured a lower margin, the June 2011 margin cut was 50 bps larger for Ireland than for Portugal.

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Tables and Figures

Table 1: IMF versus ESM Lending Terms

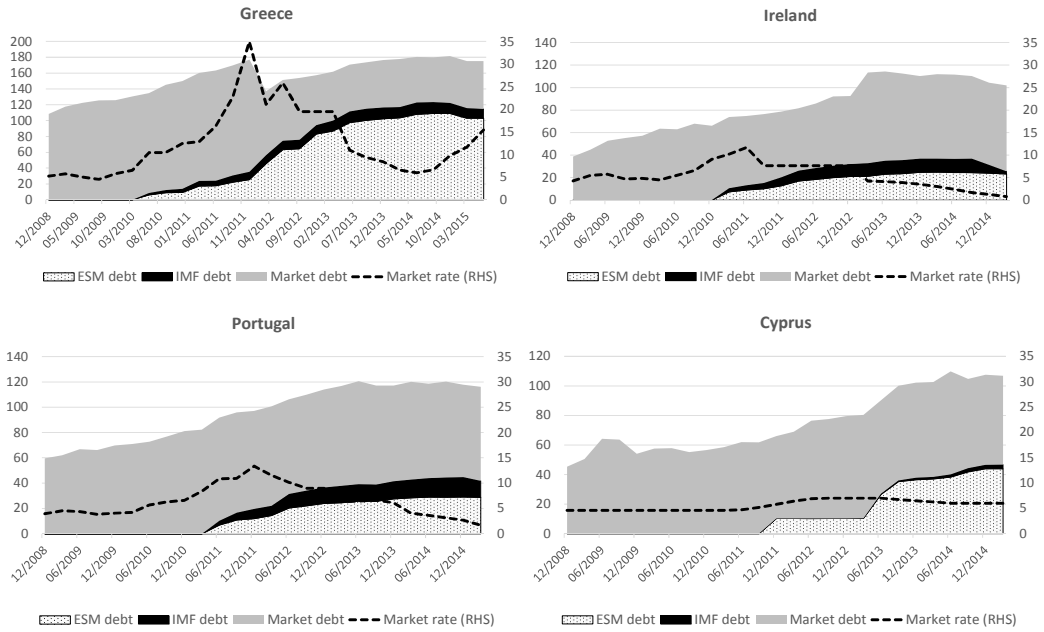
	EFSF/ESM Support		IMF Support	
	Maturity	**Interest rate**	**Maturity**	**Interest rate**
Greece	30 years	1.07	8 years	4.06
Ireland	22 years	2.45	7 years	3.07
Portugal	22 years	2.25	7 years	3.07
Spain	12.5 years	0.5	-	-
Cyprus	15 years	0.87	4 years	1.07

Sources: International Monetary Fund, European Commission, European Financial Stability Facility and European Stability Mechanism. Interest rates are computed as of June 2013.

Table 2: Calibration

Y	Output	100
Z_d	Default cost	0.95
A	Fraction of output during recession	0.93
β	Discount factor	0.98
π	Real interest rate in crisis	7%
θ	Goverment revenue as a share of output	0.4
$\overline{g}$	Level of essential government expenditure	25
γ	Relative weight of c and g in the utility function	0.5
p	Probability of leaving the recession	0.33
δ	Amortization of market borrowing	0.1667
δ_i	Amortization of IMF loan	0.1429
q_i	Interest on the IMF loan	0.9483
δ_e	Amortization of ESM loan	0.067
q_e	Interest on the ESM loan	0.9662

Figure 1: Market Spreads and Sovereigns' Creditor Structure



Sources: European Commission, European Stability Mechanism, Central Banks and Bloomberg. Debt is measured as percentage of GDP. The market rate, measured on the right hand side axis, refers to the spread on the benchmark 10 year sovereign bond. ESM debt refers to any debt issued by any of the various European vehicles (Greek Loan Facility, EFSF, EFSM, ESM) and to bilateral loans provided by European Governments.

Figure 2: Interest Rates on Market and Official Financing

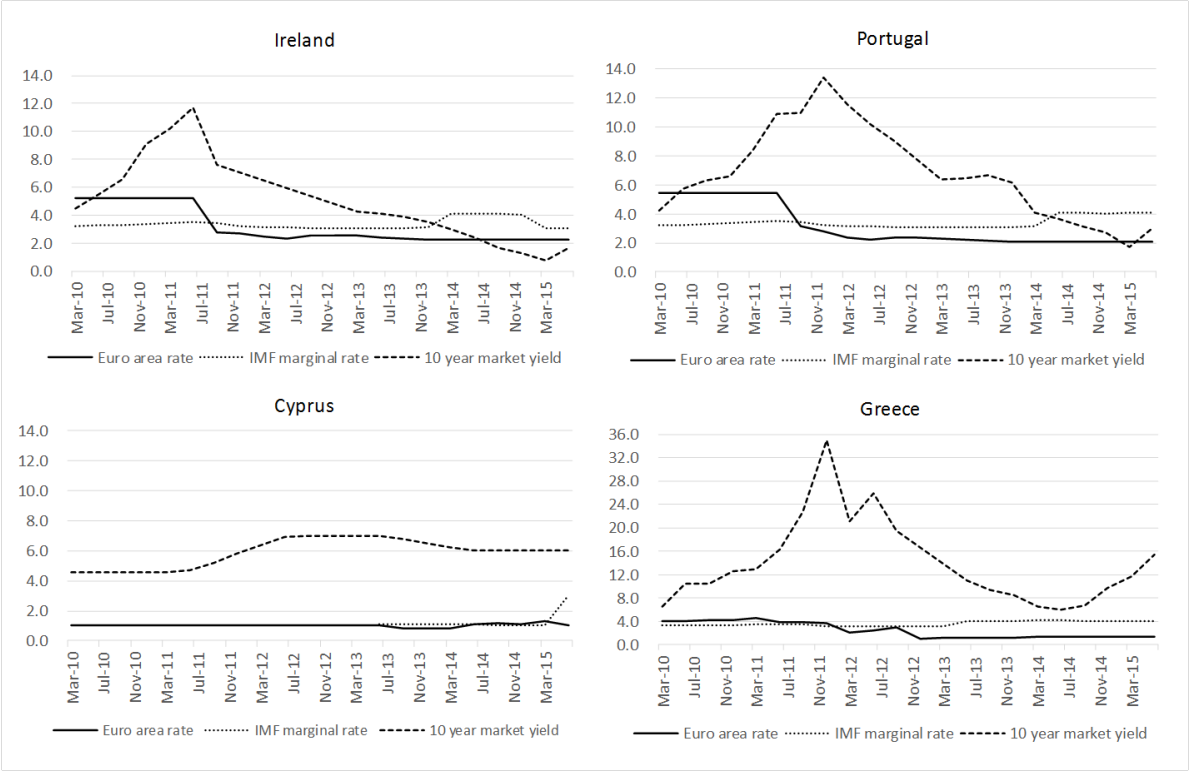
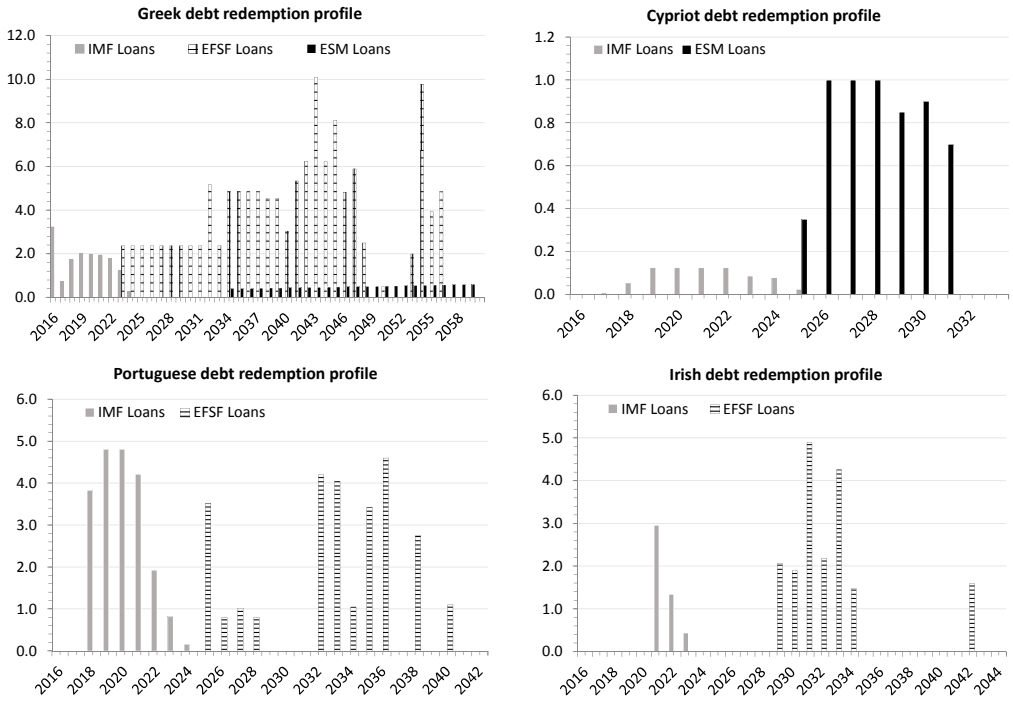


Figure 3: Repayment Profiles: IMF versus Euro-area Institutions



Sources: European Commission, European Stability Mechanism and International Monetary Fund. Debt repyaments measured in billion euros.

Figure 4: Roll-over needs and Official Lending

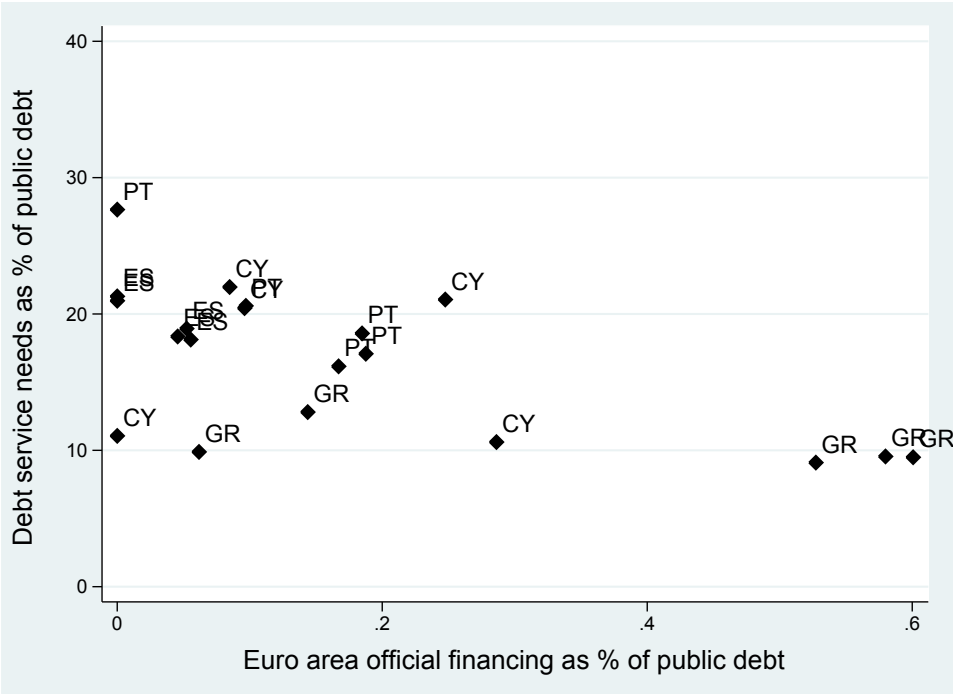


Figure 5: ESM vs. Market/IMF - Interest savings (as percentage of 2016 GDP)

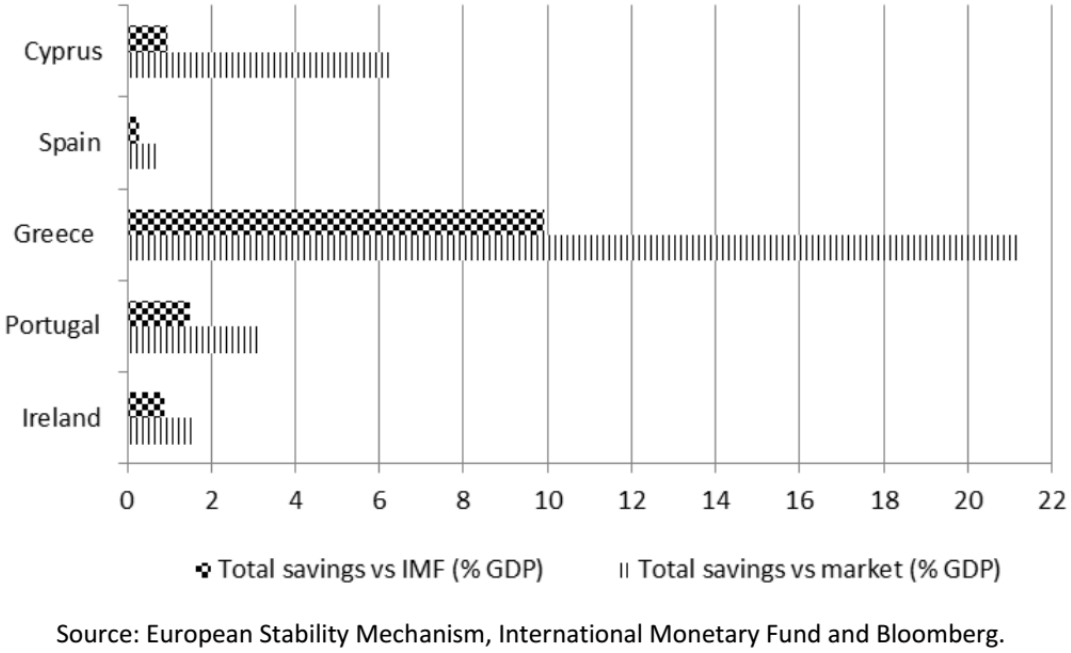


Figure 6: Benchmark

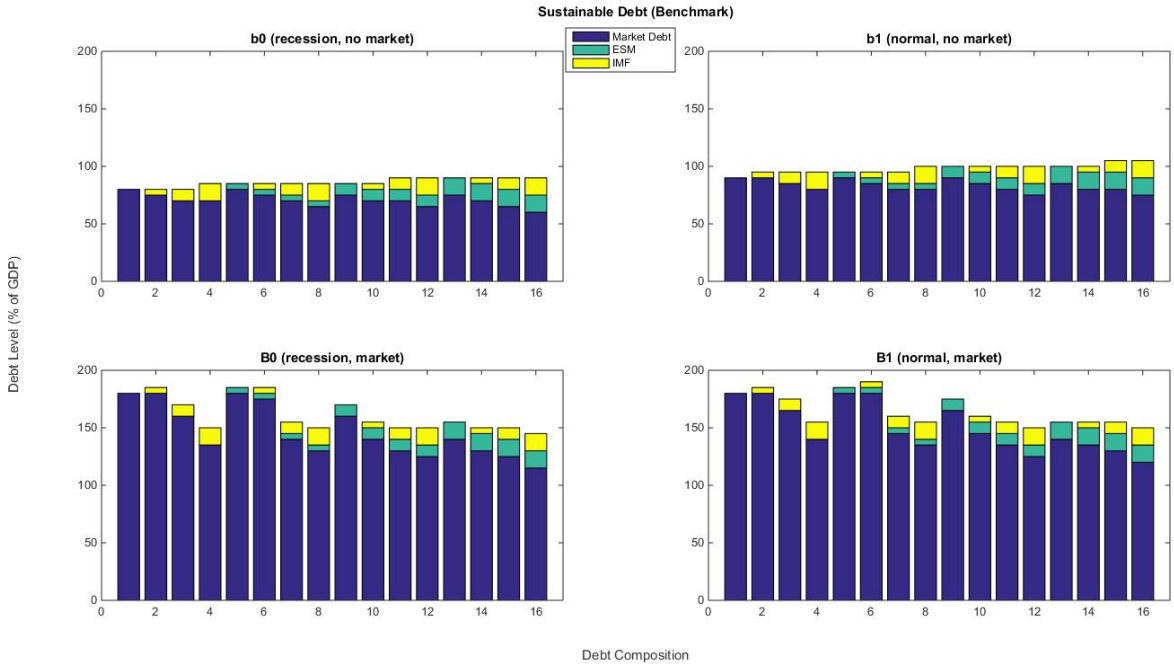


Figure 7: Safe zone threshold in a recession and no market financing $b(0)$

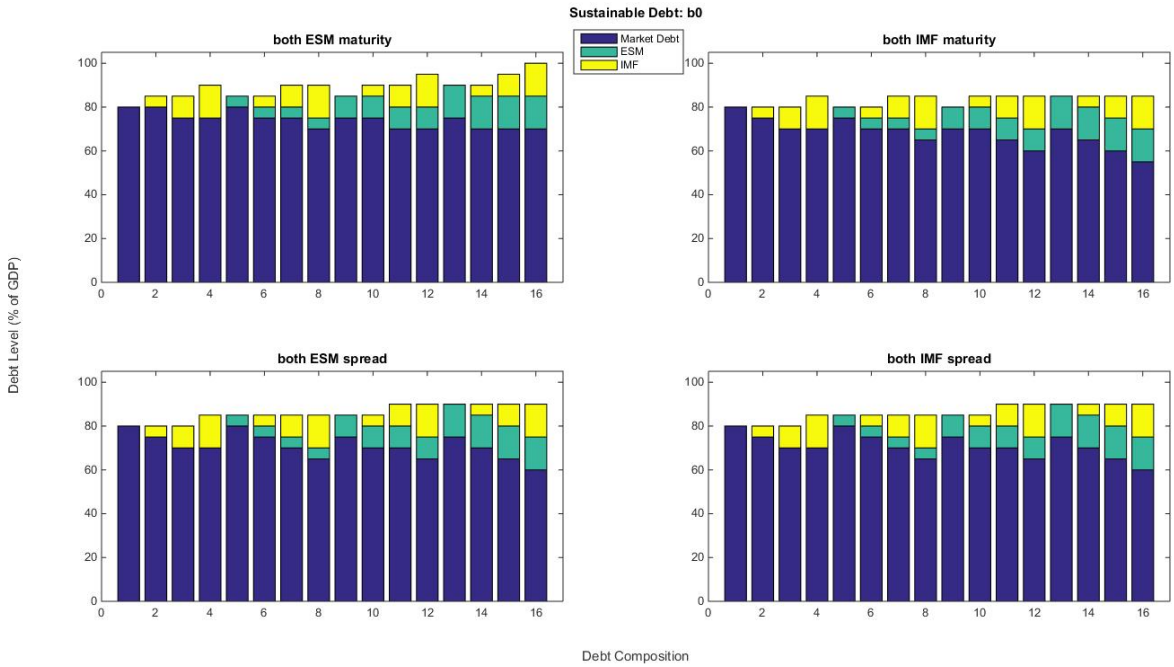


Figure 8: Safe zone threshold in normal time but no market financing $b(1)$

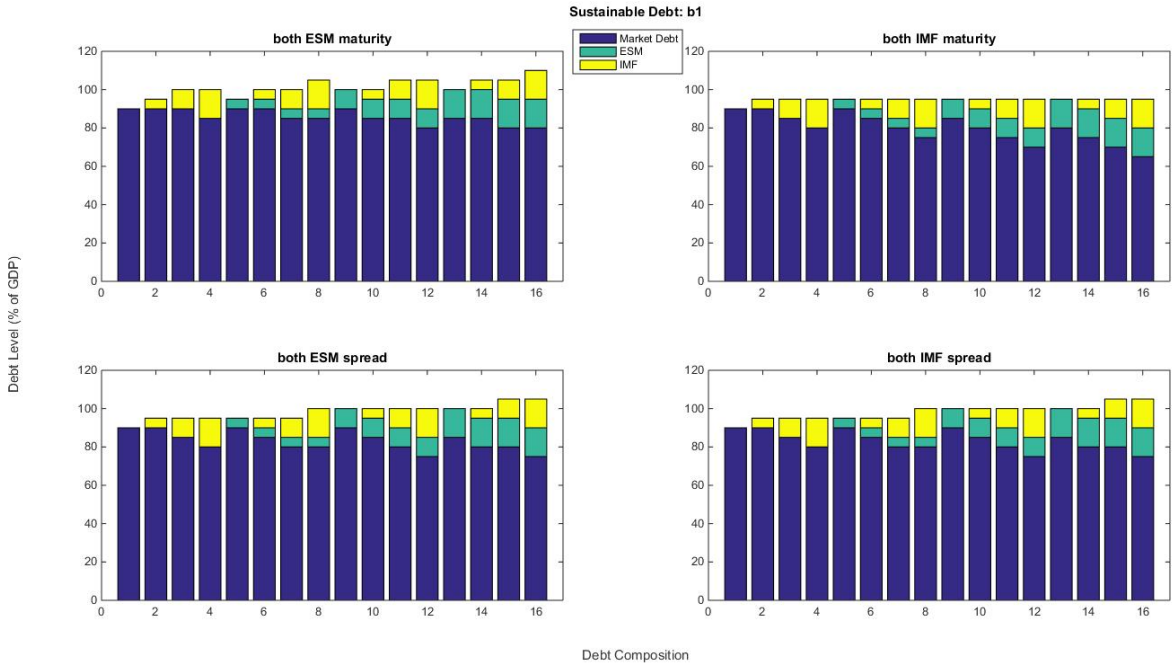


Figure 9: Upper crisis zone threshold in a recession $B(0)$

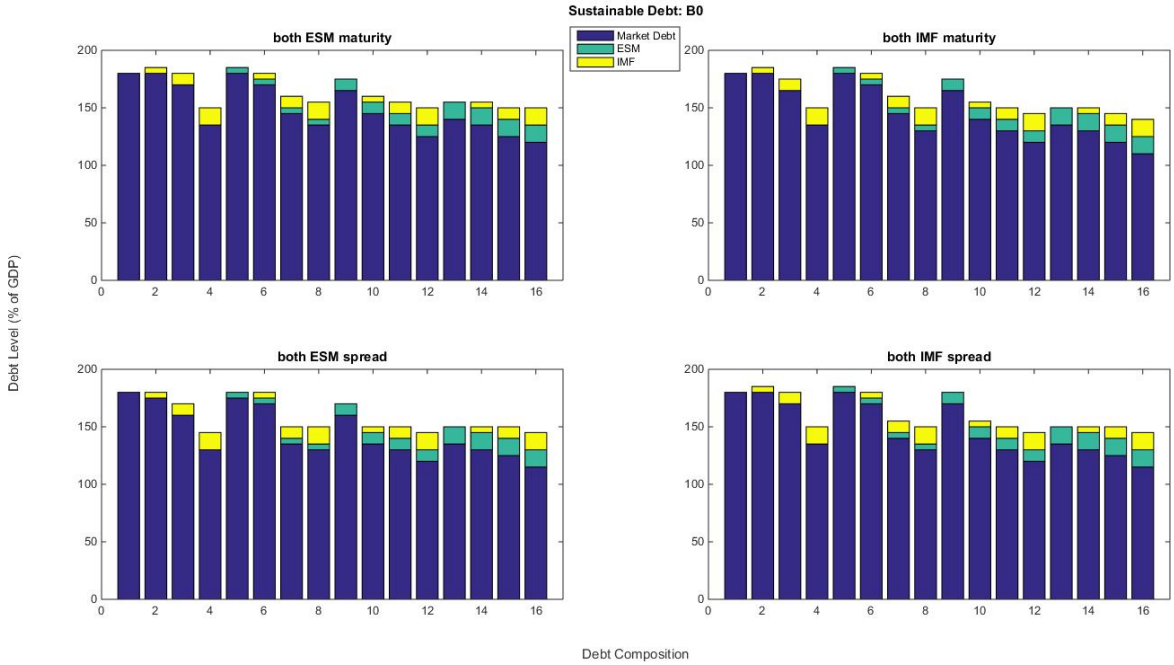


Figure 10: Upper crisis zone threshold in normal times $B(1)$