This study nests historical evidence for credit growth-fuelled financial instability in a 2-state non-homogeneous Markov chain with logistic crisis incidence. A long-run frequency measure is defined and calibrated for 17 advanced economies from 1870-2016. It is found that historical (implied) crisis frequencies display a V (J)-pattern over time. A key implication is that policies strengthening capital adequacy contribute more to systemic stability than expanding deposit insurance or curbing credit booms.
Abstract

This study nests historical evidence for credit growth-fuelled financial instability in a 2-state non-homogeneous Markov chain with logistic crisis incidence. A long-run frequency measure is defined and calibrated for 17 advanced economies from 1870-2016. It is found that historical (implied) crisis frequencies display a V (J)-pattern over time. A key implication is that policies strengthening capital adequacy contribute more to systemic stability than expanding deposit insurance or curbing credit booms.

JEL Classification: C15, E30, E58, G01

Keywords: Credit cycle; Systemic banking crises; Markov chain

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1 Introduction

What are the long-term implications of the view that excessive credit growth precipitates financial crises? The short-term link between credit cycles and systemic banking crisis likelihood in advanced economies has been modeled with a convex (logistic) function by Ajello et al. (2019), Jordà et al. (2017b), Schularick and Taylor (2012) and Taylor (2015), among others, but the implications for banking crises’ long-run frequency have not been explored.¹

In this study I embed three key determinants of crisis frequency in a two-state Markov chain with time-varying convex crisis incidence. Each state’s implied long-run frequency (ergodic probability) is driven by credit fundamentals (their mean, persistence and volatility); crises’ expected duration, a key input to severity indices (Reinhart and Rogoff (2014)); and the elasticity of the crisis incidence rate to credit conditions, that has declined since the post-war adoption of deposit insurance (Bordo and Meissner (2016), Schularick and Taylor (2012)). The resulting long-run frequency measure consists of a certainty-equivalent component—increasing in average credit growth, the elasticity of crisis incidence and average crisis duration—and a positive uncertainty wedge.

¹To determine historical crisis frequency (share of years in a crisis), researchers have focused on binary crisis indicators based on financial events, including bank runs and official interventions (Bordo et al. (2001), Reinhart and Rogoff (2009)), events of systemic importance (Jordà et al. (2017a), Taylor (2015)), and fluctuations in bank share prices (Baron et al. (2019)). Bordo and Meissner (2016) contrast the alternative long-run perspectives. The duration of banking crises since 1970 has been rigorously documented by Laeven and Valencia (2013, 2018). Recently, Romer and Romer (2017) have proposed a narrative-based, non-binary financial distress index.
Calibrating the model for three periods, excluding wars, over which credit data are available for 17 advanced economies accords with a V-shape for historical crisis frequency: banking crises since 1970 have tended to be as frequent as in the pre-war era, consistent with leading crisis chronologies. By contrast, a J-pattern emerges for model-implied frequency: as low as 5-7 percent from 1947-2008, but around 10 percent since 1970, or one crisis episode per decade on average. With the “ascent of credit” (Schularick and Taylor (2012)) having moderated in the globalization era, the post-1970 increase is likely due to longer crisis expected duration. Both frequency measures concur the 1947-2008 period was a “remarkable exception” (Admati and Hellwig (2013)) from a long-term perspective.

An advantage of the Markov-switching framework is that it directly computes the long-run crisis frequency implied by credit-driven, short-run crisis incidence, facilitating comparison with the historical measures listed above. Further, the present approach complements research on Early Warning Indicators (EWI) of banking crises, which employs a range of leading indicators to detect the short-term build-up of financial vulnerability. A limitation of the implied frequency measure is that it excludes demand-side drivers of instability, such as swings in bank deposits; it also abstracts from multiple crises triggered by cross-market contagion.

There are two contributions. Mapping fundamental and policy-influenced crisis drivers to ergodic probabilities extends the influential rare disaster literature, where these probabilities are assumed fixed (Rietz (1988), Barro (2006), Tambakis

\footnote{Aldasoro et al. (2018) review the EWI literature, including indicators of household and cross-border debt build-up. Simorangkir (2012) finds a univariate model of Markov-switching bank deposit growth serves as an effective EWI for Indonesia.}
frequency measure can thus inform estimates of welfare losses (Barro (2009)) or bailout costs of systemic events (Haldane (2010)). Concerning crisis prevention, the implied frequency’s sensitivity to key parameters informs the macro-prudential debate on the desired bank capital ratio for long-term financial stability.\footnote{Extensive theoretical and empirical research suggests that higher bank equity capital counters the risk-taking incentives created by limited liability; in addition to Admati and Hellwig (2013), see Merton (1974), Collard et al. (2017) and Hanson et al. (2011).} Specifically, I find shorter average durations yield a greater decline in crisis frequency than less excessive credit cycles and/or more extensive liability guarantees. With crises lasting considerably less for better capitalized banking systems (Cerutti et al. (2015), Jordà et al. (2017b)), the policy implication is that Basel III-type capital build-up (BIS (2017)) is more effective at containing systemic instability.

\section{The model}

Let \{\(Z_t\)\}_{t \geq 0} be a discrete, homogeneous 2-state Markov chain such that \(Z_t = 0\) when the economy is in state \(s_t = 0\) (normal) and \(Z_t = 1\) if \(s_t = 1\) (crisis). The transition probabilities are \(p^{ij} \equiv \Pr\{s_{t+1} = j|s_t = i\}\) with \(p^{ii} + p^{ij} = 1\) and \(0 < p^{ij} < 1, i, j \in \{0, 1\}\). Denoting crisis incidence and exit rates by \(p\) and \(q\), with crisis expected duration \(T_1 \equiv q^{-1}\),

\[
\mathcal{M} = \begin{pmatrix}
1 - p & p \\
q & 1 - q
\end{pmatrix}
\]
(stationary) distribution is the 2x1 vector \( \pi \equiv [\pi_0, \pi_1]' \) solving \( \pi = \mathcal{M}\pi \).\(^4\)

\[
\lim_{T \to \infty} \prod_{t=1}^{T} \mathcal{M} = \begin{pmatrix} \pi_0 & \pi_1 \\ \pi_0 & \pi_1 \end{pmatrix}
\]

\( \pi_1 = \frac{p}{q + p} \in (0, 1) \), \( \pi_0 = 1 - \pi_1 \) \( \text{(2)} \)

As is well known (Wachter (2013)), the incidence rate and ergodic probability only coincide for transient crisis events \( (q \to 1, p \to 0) \), else \( \pi_1 > p \). Against this benchmark, let crisis incidence be a logistic function of state-independent stochastic fundamental, \( L_t \), with unconditional moments \( E[L_t] = \mu_L \) and \( \text{var}[L_t] = \sigma_L^2 \):

\[
p_{t+1} \equiv p(L_t) = \Pr\{s_{t+1} = j|s_t = i, L_t\} = \frac{e^{h_0 + h_1 L_t}}{1 + e^{h_0 + h_1 L_t}} \in (0, 1) \Rightarrow \quad (3)
\]

\[
p \equiv p(E[L_t]) = \Pr\{s_{t+1} = j|s_t = i, E[L_t]\} = \frac{e^{h_0 + h_1 \mu_L}}{1 + e^{h_0 + h_1 \mu_L}} \quad (4)
\]

where \( h_0 \) is a scaling constant and \( h_1 > 0 \) is the elasticity of crisis incidence to \( L_t \).\(^5\)

As \( p_t(\cdot) \) is strictly convex left of its unique inflexion point, expected crisis likelihood exceeds its certainty-equivalent by Jensen’s inequality:

\[
E[p_{t+1}] - p > 0 \quad (5)
\]

\( ^4 \)\( \sum_{t=1}^{T} Z_t \) and \( Z \sim N(\pi, \frac{1}{T} \frac{p(q-1)}{p+q}) \) by a central limit theorem for non-i.i.d. variables (Kelbert and Suhov (2008)). Convergence follows \( \mathcal{M}^T = \begin{pmatrix} \pi_0 & \pi_1 \\ \pi_0 & \pi_1 \end{pmatrix} + \frac{\lambda^T}{p+q} \begin{pmatrix} p & -p \\ -q & q \end{pmatrix} \), where \( \lambda = 1 - p - q < 1 \) is the real-valued eigenvalue.

\( ^5 \)By controlling the steepness of the logistic function, \( h_1 \) determines the sensitivity of the probability of switching state to the fundamental. This is a simple version of Benigno et al.’s (2020) medium-scale structural model; these authors also endogenize the crisis exit rate.
where $E[p_{t+1}] = \frac{1}{T} \sum_{t=1}^{T} p(L_t)$ and $p = \left[1 + e^{-h_0-h_1\sum_{t=1}^{T}_L L_t}\right]^{-1}$. The Markov transition matrix $\mathcal{M}_{t+1} = \begin{pmatrix} 1-p_{t+1} & p_{t+1} \\ q & 1-q \end{pmatrix}$ is now non-homogeneous, and its ergodic distribution is given by the $\infty$-step-ahead transition, defined as $[\pi_0^\infty, \pi_1^\infty]' \equiv \lim_{T \to \infty} [\prod_{t=1}^{T} \mathcal{M}_t]$. Equalizing the expected probabilities of entering and exiting each state yields:

$$
\pi_1^\infty q = (1-\pi_1^\infty)E[p_t] \Rightarrow \\
\pi_1^\infty = \frac{E[p_t]}{q + E[p_t]}, \quad \pi_0^\infty = 1 - \pi_1^\infty (6)
$$

Ergodic probability (long-run frequency) $\pi_s^\infty$ measures the share of time an economy is in state $s_t \in \{0, 1\}$. By inequality (5), $\pi_s^\infty$ and its certainty-equivalent in eq. (2) evaluated at $p$, denoted $\pi_s(p)$ are monotonically ordered such that, for given $q$, the crisis (normal) state probability includes a strictly positive (negative) uncertainty wedge:

$$
\pi_1^\infty(h_1, \mu_L, \sigma_L, q) > \pi_1(p(h_1, \mu_L, q)) (7) \\
\pi_0^\infty(h_1, \mu_L, \sigma_L, q) < \pi_0(p(h_1, \mu_L, q))
$$

Note that fundamental uncertainty only impacts $\pi_s^\infty$, while model uncertainty about $h_1$ influences both ergodic probability measures.
3 Empirical application

3.1 Baseline calibration

In this section I calibrate the ergodic probability measure to the historical credit cycles of 17 advanced economies (ADV) and the United States. Following Ajello et al.’s (2019) credit dynamics and restricting their inflation and output gap response coefficients to zero, wlog, yields a stationary AR(1) process: 6

\[ L_t = \phi_0 + \rho_L L_{t-1} + \xi_t , \quad \rho_L < 1 \]  \tag{8}

\( L_t \) is the real bank loan growth rate (annualized average) with unconditional mean and variance \( \mu_L = \frac{\phi_0}{1-\rho_L}, \sigma_L = \frac{\sigma_\xi}{\sqrt{1-\rho_L^2}}; \xi_t \sim N(0, \sigma_\xi) \) are i.i.d. Gaussian credit shocks. Table 1 summarizes annual real loan growth statistics (real growth of total bank loans to the non-financial sector) and calibrated parameter values for the pre-World War II, post-war through 2008, and 1970-2016 periods:

TABLE 1 HERE — Credit cycle descriptive statistics with calibrated and estimated parameter values, by period

Post-war credit growth in advanced economies rose over 50 percent above its pre-war average (columns 2-3), but subsided in the globalization period since 1970 (column 4); these trends are also evident in the U.S. (columns 5-7). Coupling the

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6 Allowing feedback from the target variables to credit growth does not affect the main insight. I adopt these authors’ baseline parameter values \( \rho_L = 0.80, h_0 = -3.396, h_1 = 1.88 (\sigma_h = 1.14) \) for ADV, and estimate the AR(1) persistence coefficient for the U.S.
calibrated and estimated parameters with average crisis durations generates model-implied ergodic probabilities, $\pi_1(p)$ and $\pi_1^\infty$. Table 2 compares them to historical frequencies imputed from three leading crisis chronologies:

**TABLE 2 HERE — Historical and implied frequencies of systemic banking crises, by period**

There are two lessons from the calibration exercise. *First*, for the application at hand implied crisis frequency is dominated by its certainty-equivalent component, $\pi_1(p)$. The uncertainty wedge (column 7 minus column 6) does not exceed 0.1 percent of the time, echoing Ajello et al.’s (2019) results in their optimal policy setting under uncertainty. *Second*, historical crisis frequencies display a V-pattern across all three periods, documented also by Bordo and Meissner (2016) and Taylor (2015). Further, the implied frequencies lie broadly within their post-war historical ranges but well below the corresponding pre-war ones, reflecting Schularick and Taylor’s (2012) “two eras of finance”—1870-1939 and 1945-2008—with the latter marked by secular credit expansion. Hence, a $J$-shape emerges for $\pi_1(p)$: a slight post-war decline followed by a steep rise since 1970. With comparatively subdued credit growth (cf. Table 1), Table 2 indicates this is driven by longer average crisis duration in the financial globalization era. This finding validates earlier suggestions

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7The certainty-equivalent crisis incidence rate, $p$, ranges from 3.5 to 3.8 percent annually; Schularick and Taylor’s (2012) annual unconditional crisis likelihood is nearly 4 percent.

8Baron et al. (2019) report higher historical frequencies because their bank equity-based crisis identification reveals more ‘quiet’ episodes. Similarly, da Rocha and Solomou (2015) find that non-systemic crises had a long-lasting impact on inter-war industrial production.
that “[...] when the recent wave of crises is fully factored in, the apparent drop will likely be even less pronounced” (Reinhart and Rogoff (2009), p.151). 9

3.2 Sensitivity analysis

Long-run crisis frequency $\pi_1(p)$ is sensitive to uncertainty about fundamentals ($\sigma_L$) and model parameters ($h_1$), as well as episodes’ expected duration ($T_1$). On the latter, Jordà et al. (2017b) show that, conditional on the crisis state, countries with above-average capitalized banking sectors experience milder recessions (over 13 percent cumulatively lower GDP per capita) and swifter recoveries (up to 3 years); see also Cerutti et al. (2015). Regarding $h_1$, Schularick and Taylor (2012) estimate significantly lower credit elasticities post-war, with deposit insurance and lender of last resort facilities preventing banking crises from becoming panics.

Against these stylized facts, Fig. 1 displays $\pi_1(p)$ as crisis duration varies from $1 \leq T_1 \leq 10$ years ($1 \geq q \geq 0.1$) for the $\sigma_L$ and $h_1$ estimates of the 1947-2008 U.S. credit cycle, along with 95 percent confidence intervals:

FIGURE 1 HERE

Implied crisis frequency and confidence intervals: U.S. 1947-2008

Ceteris paribus, less (more) volatile financial conditions (Panel A) result in

---

9To gauge the robustness of post-1970 ADV (pre-war U.S.) implied frequencies to the Global Financial Crisis and Great Recession (Great Depression) outliers—lasting 5 or more years for many economies—in column 3 I also report $\pi_1(p)$ and $\pi_1^\infty$ for median duration. The mean-median gaps are comparable to Reinhart and Rogoff’s (2014) peak-to-trough measure. Crisis frequencies fall by 1.5-2 percent, but the J-shape remains as the 1947-2008 decline appears less pronounced.
marginally lower (higher) long-run crisis frequency than a stronger (weaker) policy backstop (Panel B). However, as $\pi_1(p)$ declines at hyperbolic rate with $q$, by eq. (2), it is far more responsive to expected duration than $\sigma_L$ or $h_1$. For example, crisis frequency would drop from 10 to nearly 5 percent if the average episode lasted 1.5 rather than 3 years, a plausible shift given crisis aftermath heterogeneity. In turn, this would halve a tax charge on Globally Systemically-Important Banks ($GSI-B$) to US$5 billion per year; see BIS (2017) and Haldane (2010).

4 Concluding observations

This study nested systematic evidence for credit growth-fuelled financial instability in a 2-state non-homogeneous Markov chain with logistic crisis incidence. It was shown that a $V$ ($J$)-pattern characterizes historical (implied) crisis frequencies over time. Insofar as crisis episodes are less protracted when banks are better capitalized, sensitivity analysis suggested that bolstering capital ratios—which declined strongly from 1870 until the mid-20th century, remaining low but stable thereafter—yields sharply lower crisis frequencies. The policy lesson is that Basel III-type measures strengthening bank capital adequacy enhance long-term systemic resilience more than expanding deposit insurance or curbing sustained credit booms.

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10 Reinhart and Rogoff’s (2014) study of 63 financial crises in advanced economies (1857-2013) reports a mean (median) duration of 2.9 (2) years for peak-to-trough contractions of per-capita GDP; Romer and Romer’s (2017) real-time narrative index for OECD countries (1967-2012) also reveals wide dispersion in recovery times.
References


Table 1. Credit cycle descriptive statistics with calibrated and estimated parameter values, by period

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td></td>
<td>ADV</td>
<td>U.S.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean $\mu_L$</td>
<td>4.5</td>
<td>6.5</td>
<td>4.3</td>
<td>3.9</td>
<td>5.3</td>
<td>3.1</td>
</tr>
<tr>
<td>S.D. $\sigma_L$</td>
<td>10.6</td>
<td>6.9</td>
<td>5.8</td>
<td>6.4</td>
<td>5.6</td>
<td>4.7</td>
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<td>Parameters</td>
<td></td>
<td></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Intercept $\phi_0$</td>
<td>0.9</td>
<td>1.3</td>
<td>0.9</td>
<td>1.4</td>
<td>2.8</td>
<td>0.7</td>
</tr>
<tr>
<td>Persistence $\rho_L$</td>
<td>0.80</td>
<td>0.80</td>
<td>0.80</td>
<td>0.64***</td>
<td>0.48***</td>
<td>0.78***</td>
</tr>
<tr>
<td>Adj. $R^2$</td>
<td>–</td>
<td>–</td>
<td>–</td>
<td>0.41</td>
<td>0.22</td>
<td>0.61</td>
</tr>
<tr>
<td>S.E. $\sigma_\xi$</td>
<td>6.4</td>
<td>4.1</td>
<td>3.5</td>
<td>4.9</td>
<td>4.9</td>
<td>2.9</td>
</tr>
<tr>
<td>Obs.</td>
<td>1007</td>
<td>1044</td>
<td>793</td>
<td>69</td>
<td>62</td>
<td>46</td>
</tr>
</tbody>
</table>

Note: All entries except $\phi_0$ and $\rho_L$ are in percent. Columns 1-7 display the average and standard deviation of annual real loan growth rates for 1870-1939, excluding war years, 1947-2008 and 1970-2016 periods for Australia, Belgium, Canada, Denmark, Finland, France, Germany, Italy, Holland, Norway, Japan, Portugal, Spain, Sweden, Switzerland, the United Kingdom and the United States, from Jordà et al. (2017a). The intercept and standard error of eq. (8) are calibrated as $\phi_0 = (1 - \rho_L)\mu_L$ and $\sigma_\xi = \sqrt{1 - \rho_L^2}\sigma_L$. *** denotes significance at the 1 percent level.

Data: Mean and st.dev. of total bank loans to non-financial sector (annualized) from Jordà, Schularick and Taylor (2017a), www.macrohistory.net/data consulted on 09.02.2020.
Table 2. Historical and implied frequencies of systemic banking crises, by period

<table>
<thead>
<tr>
<th>Period</th>
<th>Historical</th>
<th>Implied</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ave. Duration</td>
<td>Frequency</td>
</tr>
<tr>
<td></td>
<td>$T_1$ (years)</td>
<td>$\pi_1$ (percent)</td>
</tr>
<tr>
<td>1870-1939</td>
<td>ADV</td>
<td>2.3</td>
</tr>
<tr>
<td></td>
<td>U.S.</td>
<td>2.5 mean</td>
</tr>
<tr>
<td></td>
<td>U.S.</td>
<td>2.0 median</td>
</tr>
<tr>
<td>1947-2008</td>
<td>ADV</td>
<td>1.9</td>
</tr>
<tr>
<td></td>
<td>U.S.</td>
<td>1.5</td>
</tr>
<tr>
<td>1970-2016</td>
<td>ADV</td>
<td>3.7 mean</td>
</tr>
<tr>
<td></td>
<td>U.S.</td>
<td>3.0 median</td>
</tr>
<tr>
<td>1970-2016</td>
<td>U.S.</td>
<td>3.0</td>
</tr>
</tbody>
</table>

Note: The crisis frequencies are reported in percent of time (share of country-years spent in crisis state in each period). Given average duration $T_1$ in column 3, the historical crisis frequencies in columns 4-5 are computed as $\pi_1 = \frac{T_1 \times \text{Episodes}}{\text{Country-years}}$. $\pi_1(p)$ in column 6 is eq. (2) evaluated at $p$ in eq. (4) and $q = T_1^{-1}$. The model-implied frequency in column 7 is computed as $\pi_1^\infty = \frac{1}{K} \left( \prod_{t=1}^{T} M_t \right) (1, 2)$ for $K = 1000$ sample paths, each of length 1000 credit growth realizations drawn from $L_t \sim N(\mu_L, \sigma_L)$ such that $EL_t = \frac{\sum_{k=1}^{K} \left[ \sum_{t=1}^{T} L_t \right]}{K}$ converges to $\mu_L$. The parameter values are in Table 1.

Data: All crisis average durations are from Laeven and Valencia (2018) except 1870-1939 from Bordo et al. (2001), Table 1 (ADV) and Reinhart and Rogoff (2009), Table A.4.1 (U.S.) Historical frequencies consistent with the crisis indices of Baron et al. (2019), Laeven and Valencia (2018) and Taylor (2015) are denoted BVX, LV and $T$, respectively.
FIGURE 1. Implied crisis frequencies and confidence intervals: U.S. 1947-2008

A. Fundamental uncertainty ($\sigma_L$)  

B. Model uncertainty ($h_1$)

Note: The black schedule in Panel A (B) shows the certainty-equivalent ergodic probability for $\mu_L=5.3$ ($h_1=1.88$). The red schedules show 95 (91) percent symmetric (asymmetric) confidence intervals around the respective baseline, $\mu_L \pm 2\sigma_L (h_1 - 1.5 \sigma_h, h_1 + 2\sigma_h)$ for $\sigma_L=5.6$ ($\sigma_h=1.14$). Other parameter values are in Table 1.