UNIVERSITY OF CAMBRIDGE
Faculty of Economics

Institute for
New Economic Thinking

CAMBRIDGE WORKING PAPERS IN ECONOMICS
CAMBRIDGE-INET WORKING PAPERS

Rating the Competition: Seller Ratings and Intra-Platform Competition

George
Charlson
University of
Cambridge

Abstract

Product ratings are commonplace on large online platforms, like Airbnb and Amazon Marketplace. One use for these ratings is to order search results. Platform owners are able to choose the extent to which ratings can be used to determine the probability a given seller is observed by a sets of buyers. Since demand is higher for high quality products, there is an incentive to increase the probability that highly-rated sellers are observed by biasing search results towards them. However, biasing search results in this way results in competition being more concentrated, reducing prices. The extent to which it is profitable to use ratings as a means of ordering search results depends on the properties of the market(s) the platform operates in.

Reference Details

2106	Cambridge Working Papers in Economics
2021/03	Cambridge-INET Working Paper Series
Published	13 January 2021
Key Words	Networks, strategic interaction, network games, interventions, industrial organisation, platforms
JEL Codes	D4, L1, L4
Websites	www.econ.cam.ac.uk/cwpe www.inet.econ.cam.ac.uk/working-papers

Rating the Competition: Seller Ratings and Intra-Platform Competition

George Charlson*

January 13, 2021

Abstract

Product ratings are commonplace on large online platforms, like Airbnb and Amazon Marketplace. One use for these ratings is to order search results. Platform owners are able to choose the extent to which ratings can be used to determine the probability a given seller is observed by a sets of buyers. Since demand is higher for high quality products, there is an incentive to increase the probability that highly-rated sellers are observed by biasing search results towards them. However, biasing search results in this way results in competition being more concentrated, reducing prices. The extent to which it is profitable to use ratings as a means of ordering search results depends on the properties of the market(s) the platform operates in.

KEYWORDS: Networks, strategic interaction, network games, interventions, industrial organisation, platforms. JEL classification: D4, L1, L4.

*Cambridge INET, Austin Robinson Building, Sidgwick Ave, Cambridge CB3 9DD, gc556@cam.ac.uk

1 Introduction

Many online platforms allow users to rate products on their platform. These ratings have many potential uses, including revealing to buyers their own valuation for products on offer. Another way in which product ratings are used is to determine the likelihood that sellers are observed by buyers. Given that buyers do not tend to observe every seller on a platform (see, for example, Ringer and Skiera, 2016 and Kim, Albuquerque and Bronnenberg, 2010), the choice of which sellers buyers observe will affect platform profits. Many online platforms use ratings as one of the inputs that determine how search results are ordered (Dinerstein et al, 2018).[1] We analyse the incentive of a profit-maximising, monopolistic platform owner to use ratings as a means of determining the prominence of a seller on a platform.

To give an example of the problem under consideration, suppose there is a market with three products, X, $Y$and Z. Prior to the ratings of these products being realised, the platform owner chooses the relative likelihood that products are observed given their quality by choosing a observation function that maps quality into observation probabilities. If X is more highly-rated than Y and Y is more highly-rated than Z, then the platform's choice of observation function will determine how much more prominent X is than Y and Z.[2]

To keep track of the price-setting behaviour of an arbitrarily large number of sellers in which buyers observe sellers with some observation probability and the matrix of

[1] For an empirical assessment of the effect of ratings on consumer demand, see Chevalier and Mayzlin (2006), Vana and Lambrecht (2020), Fradkin, Grewal and Holtz (2017) Luca (2016) and Ögut and Tas (2009)

[2] Prominence here refers to the probability that buyers observe a given seller. This approach contrasts with Armstrong, Vickers and Zhou (2009) and Armstrong and Zhou (2011), where prominent sellers are observed first and either accepted or rejected in a search framework.

such observation probabilities generates a network which in turn determines prices. Our analysis therefore utilises results from the games on networks literature Ballester, Calvò-Armengol and Zenou, 2006, and Bramoullé, Kranton and D'Amours, 2014) in a setting where a platform can influence (but not directly set) the links between sellers.[3] Hence, this study fits into the growing interventions in networks literature.[4]

Using our framework, we find that there is a high-level trade-off inherent with choosing to positively weight the observation probabilities according to a product's rating. On the one hand, there is an incentive to bias the search process towards highly-rated sellers because doing so increases the probability that buyers observe high quality sellers. Buyers also demand more of the products sold by highly-rated sellers for a given set of prices, which implies that matching buyers to these sellers increases expected profits, if prices were independent of the search process chosen by the platform owner.

However, making it more likely that highly-rated sellers are observed causes competition to become more concentrated across the network, in the sense that highly-rated sellers are more likely to compete with each other than in the case where matches are random. Fierce competition between a subset of the sellers in the network more than offsets the fact that sellers with a low rating are less likely to compete with one another. Highly-rated sellers have a lower price than is the case where matching is random, and all sellers on platform, including those with low ratings, respond by reducing their prices as well.

[3]See Bimpikis, Ehsani and Ilkiliç, 2018 and Elliott and Galeotti, 2019 for applications of this literature to an IO context.

[4] Contributions to the wider intervention in economic networks literature include: Galeotti and Goyal (2009), Candogan, Bimpikis, and Ozdaglar (2012), Banerjee, Chandrasekhar, Duflo, and Duflo (2013), Bloch and Querou (2013), Leduc, Jackson, and Johari (2017), Li (2019), Belhaj, Deroïan and Safi (2020), Galeotti, Golub and Goyal (2020) and Akbarpour, Malladi, and Saberi (2020).

Our setting also makes it possible to characterise the type of markets in which biasing search results more towards high quality products is beneficial. In markets where product substitutability is relatively low or where product quality is relatively important, the concentration of competition effect described above becomes less important, driving up the optimal prominence of high quality

Similarly, as the variance of the component of the buyers' valuation that is idiosyncratic increases, the less the platform owner has the incentive to bias search results. Making high quality sellers prominent increases the probability that buyers miss out on high quality matches with a surplus largely composed of the buyer's idiosyncratic preference towards the product. Our analysis therefore suggests that the nature of the market or markets that the platform hosts is crucial for determining the extent to which ratings are used to rank products.

Our result that there are potential ex-ante gains for the platform owner generated by consumers observing high quality products shares features with the results found in work analysing the incentive of monopolists to reveal information relating to consumer valuations, like Lewis and Sappington (1994) and Myatt and Johnson (2006).

By focusing on a monopolistic platform, we identify a trade-off that becomes increasingly important as individual platforms become dominant. As such platforms gain significant market share, their ability to shape intra-platform competition becomes more important, something in which regulators are increasingly taking an interest.[5]Our focus on intra-platform competition then differs from the more common approach of examining inter-platform competition, leading to a more thorough consideration of the fact

[5]See, for example, the European Commission's recent preliminary finding that Amazon breached EU antitrust rules by distorting competition on its own platform by using privately held data to the benefit of its own retail offering and its investigation in to Amazon's "buy box" (European Commission, 2020)

that modern platforms are hosts to a market which they can influence. While there have been some recent analyses to intra-platform competition from a management perspective (Zhu and Liu (2018 and Nambisan and Baron, 2019), there has been less focus on this aspect of platforms in economics. An exception to this is Choi, Dai and Kim (2018), who analyse the case in which oligopolists compete on prices when prices partly determine the probability of being observed, but do not explicitly examine the question of platform design.[6]

Charlson (2020) also examines internal competition on platforms, but examines the case where the platform owner has full control over which buyers observe which sellers, and seller ratings do not play an explicit role. Here, we examine the case where ratings are the only means of the platform being able to determine relative observability, identifying a trade-off between profitable matches and competition that real-life platforms face.

Recent work by Armstrong and Zhou (2020) and Elliott, Galleotti and Koh (2020) has examined how platform owners can increase profits by segmenting the market via limiting the information sellers have about consumer valuations.[7] Our framework analyses an alternative method of segmentation achieved via changing the probability that buyers observe sellers.

Due to the concentration of competition effect described above, the platform prefers to show consumers high quality sellers less than a consumer surplus maximising central planner. Competition authorities should pay close attention what determines prominence in search on online platforms. In cases where they are concerns of monopolistic

[6]For canonical approaches to inter-platform competition, see Tirole and Rochet (2003) Armstrong (2006) and Tan and Zhou (2019).

[7]These approaches our part of a wider literature on Bayesian persuasion Kamenica and Gentzkow (2011), which has been applied to IO contexts in e.g. Bergemann, Brooks, and Morris (2015) and Chen and Zhang (2020)

behaviour, requiring transparency regarding the inputs of the algorithm used to determine the results of internal search may be required.

2 The model

Suppose that there are a finite number of consumer segments, each composed of potentially a large but finite number of buyers. Each buyer within a consumer segment shares some trait, such as geographical location, age demographic, occupation etc. Call the set of these consumer segments B and suppose that $|B| = n$. Let S be a finite set of sellers and where $|S| = m$. Sellers each sell a single type of completely divisible good, and each seller's good is an imperfect substitute for each of the goods.

Sellers and consumer segments interact on a platform. A consumer segment, i, observes a seller j with probability w_{ij}, which is generated by an observation function discussed below. The observation process generates a weighted network $G = (B \cup S, E)$, where E is the set of weighted edges from S to B, where each consumer segment can be thought of as a node in the graph G. G has an adjacency matrix, $\boldsymbol{R}$, which is a zero-diagonal matrix with components w_{ij}.

Let $\boldsymbol{p}$ denote a $m \times 1$ vector whose jth entry is $p_j \in \mathbb{R}_+$, the price of j's good. We will assume that the ex-post demand function, $x_{ij}(\boldsymbol{p}) : \mathbb{R}^m \to \mathbb{R}_+$, captures the demand of a group of buyers i for a product j condition on i observing j:

$$x_{ij}(\boldsymbol{p}) = b(\gamma_{ij} - p_j) + \sum_{k \neq j} \mu_{ik} c(p_k - \gamma_{ik}).$$

where $b, c \in \mathbb{R}_+$ and $\mu_{ik} = 1$ if i observes k and 0 otherwise. We will suppose throughout that each γ_{ij} is stochastic. More specifically, assume that:

$$\gamma_{ij} = \theta\gamma_j + \epsilon_{ij},$$

where both γ_j and ϵ_{ij} are random iid variables with continuous and symmetric distribution supported on bounded intervals $[\gamma_L, \gamma_H]$ and $[\epsilon_L, \epsilon_H]$, with means $\bar{\gamma}$ and 0 and variances $\sigma_\gamma^2 > 0$ and $\sigma_\epsilon^2 \geq 0$ respectively. The first of these terms can be thought of capturing the quality of product j, whereas the second captures the idiosyncratic value consumer i derives from j.

The parameter $\theta \in \mathbb{R}_+$ then captures the extent to which consumer segments value quality relative to their idiosyncratic assessment of the good. We will assume that all of the elements of the buyer's demand function, which are non-stochastic, including θ, are common knowledge. We also assume that $x_{ij} > 0$, which is guaranteed when b is sufficiently large, the conditions for which we discuss in more detail below.

Once the value of each γ_{ij} is realised, consumer segment i's ex-ante expected demand function for a good j can be represented by $\mathrm{E}[x_{ij}(\boldsymbol{p}, \gamma_{ij})] : \mathbb{R}^{m+1} \to \mathbb{R}$:

$$\mathrm{E}[x_{ij}(\boldsymbol{p}, \gamma_{ij}, \gamma_{ik})] = w_{ij}(b(\gamma_{ij} - p_j) + \sum_{k \neq j} c(p_k - \gamma_{ik})).$$

Define as j's expected demand function, $\mathrm{E}[x_j(\boldsymbol{p})] : \mathbb{R}^m \to \mathbb{R}_+$ as $\mathrm{E}[x_j(\boldsymbol{p})] = \sum_{i=1}^n \mathrm{E}[x_{ij}(\boldsymbol{p})]$. Then, assuming the marginal cost of each seller is zero, the profit function for seller j is $\mathrm{E}[\pi_j(\boldsymbol{p})] = p_j \mathrm{E}[x_j(\boldsymbol{p})]$.

Sellers are assumed to observe the entire graph G: they observe the matrix of observation probabilities. On the basis of this observation, sellers choose their prices. A seller, j, is assumed not to be able to price discriminate across buyers, and hence each sets a single price $p_j \in \mathbb{R}_+$. Sellers compete with one another on price, and set prices simultaneously. Therefore each seller's maximisation problem can be expressed as:

$$\max_{p_j} \pi_j(p_j,\ \boldsymbol{p_{-j}};\ \boldsymbol{G}).$$

Having characterised the game played by the sellers given the graph G, it is necessary to set out how G is determined. Define the rating of a product j as follows:

$$\hat{\gamma}_j := \frac{\sum_i \gamma_{ij}}{\theta n}.$$

We assume that the rating of a product is determined by previous buyers of that product truthfully revealing their intercept parameters. By the law of large numbers, as $n \to \infty$, $\hat{\gamma}_j \to^p \gamma_j$. We will assume throughout that n is sufficiently large such that $\hat{\gamma}_j$ is arbitrarily close to γ_j.

The platform and the observation function

Having set out the demand function of the buyers, it is necessary to outline the observation process. Let $\hat{\gamma}_{-j}$ be a $(m-1)\times 1$ vector that contains every seller rating except for j. We will assume that the probability that a buyer i observes a seller j, w_{ij}, is determined as follows:

$$w_{ij}(\hat{\boldsymbol{\gamma}}) = \phi(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{\boldsymbol{-j}})\,\forall i, j \tag{1}$$

where $\phi(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{\boldsymbol{-j}}) : \mathbb{R}^m \to [0,1]$ is a observation function that maps the rating of each seller to an observation probability for the seller j. Given the equilibrium price setting behaviour and a set of conditions which we outline below the platform owner is assumed to choose a observation function $\phi(.) \in \mathit{\Phi}$, where $\mathit{\Phi}$ is the set of all feasible observation functions, $\phi(.,.)$ to solve the following problem:[8]

$$\max_{\phi(.)\in\mathit{\Phi}}\{\mathrm{E}_{\phi}[\pi_P^*(\boldsymbol{p})]\} = \max_{\phi(.)\in\mathit{\Phi}}\{\chi\sum_{j=1}^{m}\mathrm{E}_{\phi}[\pi_j^*(\boldsymbol{p})]\},$$

where $\mathrm{E}_{\phi}[\pi_P^*(\boldsymbol{p})]$ is the expected platform profit when the observation function is $\phi(.)$ and $0 \leq \chi \leq 1$. We will assume throughout that the platform chooses $\phi(\hat{\gamma}_j, \boldsymbol{\hat{\gamma}_{-j}})$ prior to the realisation of the ratings vector. Hence, the timing of the model can be summarised as follows: the platform owner chooses $\phi(.)$; the ratings vector $\boldsymbol{\hat{\gamma}}$ is realised, generating a graph G; sellers set $\boldsymbol{p^*}$; and finally the observation graph is realised and consumer segments purchase products according to their ex-post demand function.

3 Determining observation probabilities

Equation (1) defines an observation function that determines the probability that each consumer segment observes each seller. Let $\mathit{\Phi}$ represent the set of all feasible observation functions. Throughout, we will assume that any $\phi(.) \in \mathit{\Phi}$ obeys the following conditions:

(C1) Competitor symmetry. For any i, j, k,with $j \neq i, k$, $\hat{\gamma}_i$ and $\hat{\gamma}_k$ enter $\phi(\hat{\gamma}_j, \boldsymbol{\hat{\gamma}_{-j}})$ identically for all j.

(C2) Distributional symmetry. Suppose $\hat{\gamma}_b = \bar{\gamma} + z$ and $\hat{\gamma}_s = \bar{\gamma} - z$. For any $\phi(\hat{\gamma}_j, \boldsymbol{\hat{\gamma}_{-j}}) \in \mathit{\Phi}$, it must be the case that if $|\phi(\hat{\gamma}_b, \boldsymbol{\hat{\gamma}_{-j}}) - \phi(\bar{\gamma}, \boldsymbol{\hat{\gamma}_{-j}})| = |\phi(\hat{\gamma}_s, \boldsymbol{\hat{\gamma}_{-j}}) - \phi(\bar{\gamma}, \boldsymbol{\hat{\gamma}_{-j}})|$.

[8] Throughout, we use $\mathrm{E}_{\phi}[]$ to refer to the expectation of some function when the observation function is $\phi(.)$. Any expectations without this notation refer to the expectation of some variable or function after the realisation of the ratings vector but prior to the realisation of the observation graph.

(C3) Continuity and differentibility. $\phi(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})$ is continuous and differentiable on its entire domain.

Of these three conditions, (C2) is the most restrictive. Intuitively, this condition reduces the platform owner's ability to exclude products with low ratings from being observed at all. While such a condition likely does meaningfully bind the platform owner's decision, it is a simplification that allows us to understand the key trade-offs between competition and matching concerns.

When (C1)-(C3) hold, noting that each γ_j is distributed symmetrically, any $\phi(.) \in \Phi$ generates a set of random variables $\tilde{w}_{ij}$ as follows:

$$\tilde{w}_{ij} =_d \upsilon + \varepsilon_j,$$

where $\upsilon \in [0, 1]$ and each ε_j is identically and independently distributed according to some bounded, symmetric and continuous distribution with mean 0. The scalar υ can be thought of as the "baseline" probability with which each segment observes each seller, which is adjusted up or downwards depending on a segment's quality.

Loosely, the extent to which ratings are used to determine observation probabilities can be used to categorise different functions within the set $\varPhi$ as generating "ratings-based" matching environments, where the probability that high-quality sellers are observed with a higher probability, or "random match" environments, where the matching probabilities are even or near even. These two environments generate different structures for a given realisation of the rankings vector $\hat{\boldsymbol{\gamma}}$, as Figure 1 shows.

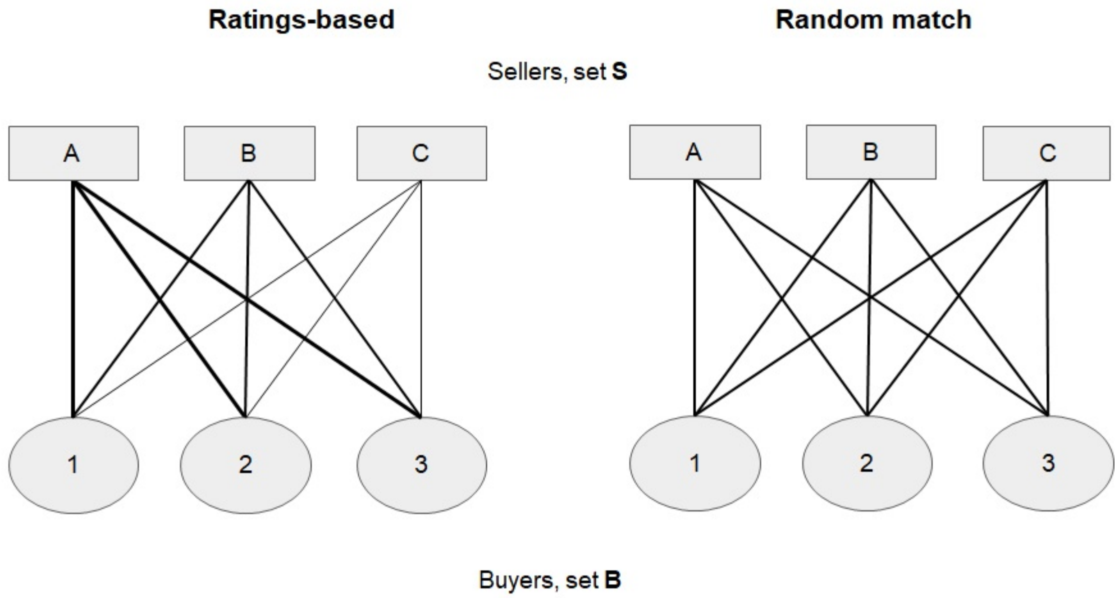

Figure 1: Ratings-based and random match environments where the product quality parameters are as follows: $\hat{\gamma}_A > \hat{\gamma}_B > \hat{\gamma}_C$. The thicker a line, the higher the probability of observation.

4 The price setting equilibrium

The competition graph

To characterise the equilibrium price vector of the sellers competing in G, it should be noted that buyers are not strategic agents in the game, and therefore it is possible to define a network which is strategically equivalent to an original network G, but only includes seller nodes. Such a network can be used to characterise the equilibrium of the pricing game. Define:

$$\alpha_j := \beta_j \gamma_{ij} - \sum_{k \neq j}^{m} \hat{c}_{jk},$$

where $\hat{c}_{jk} := c \sum_{k=1}^{n} \sum_{i=1}^{n} [w_{ij} w_{ik}] \gamma_{ik}$ and $\beta_j := b \sum_{i=1}^{n} w_{ij}$.

Transforming the above profit function to put it in terms of a seller-only network yields the following payoff function for seller j:

$$\pi_j(\boldsymbol{p}) = p_j(\alpha_j - \beta_j p_j + \sum_{k \neq j} \hat{c}_{jk} p_k),$$

Rescaling (6) by $1/\beta_i$ and multiplying by $\frac{1}{2}$ generates $\tilde{\pi}_j(\boldsymbol{p}) = p_j \tilde{\alpha}_j - \frac{1}{2} p_j^2 + \sum_{k \neq j} \tilde{c}_{jk} p_j p_k$, where $\tilde{\alpha}_j = \frac{\frac{1}{2}\alpha_j}{\beta_j}$ and $\tilde{c}_{jk} = \frac{\frac{1}{2}\hat{c}_{jk}}{\beta_j}$. The maximisation problem $\max_{p_i} \tilde{\pi}_i(\boldsymbol{p_i})$ has the same first-order condition as $\max_{p_i} \pi_i(\boldsymbol{p_i})$. Define R_S as a symmetric zero diagonal matrix of a network G_S with entries $\widetilde{c}_{jk}$. This transformation yields a competition network, G_S, which is a projection of G.[9]

The links in G_S represent the relative importance of the connection between two sellers. Hence, this representation allows us to track the competition faced by each seller tractably, even if the number of sellers is arbitrarily large.

The structure of the competition network is of course shaped by the observation function. When the observation function is relatively insensitive to ratings, the competition graph generated is such that the links between each seller are of near-equal value.

When the observation function is relatively sensitive to rating, G_S is such that the links between highly-rated sellers are larger than the links between lowly-rated sellers. Furthermore, the directed link from a highly-rated seller to a lowly-rated seller is smaller

[9] Charlson (2020) also relies on the transformation between a bipartite buyer-seller graph and a competition network. However, here the competition network is determined by an observation, rather than directly by the platform owner.

than the equivalent link in the other direction. Highly-rated sellers face relatively less competition from lowly-rated sellers than the reverse, because lowly-rated sellers are less likely to have captive or near-captive buyers.

Figure 2 shows the competition graphs generated in these two cases.

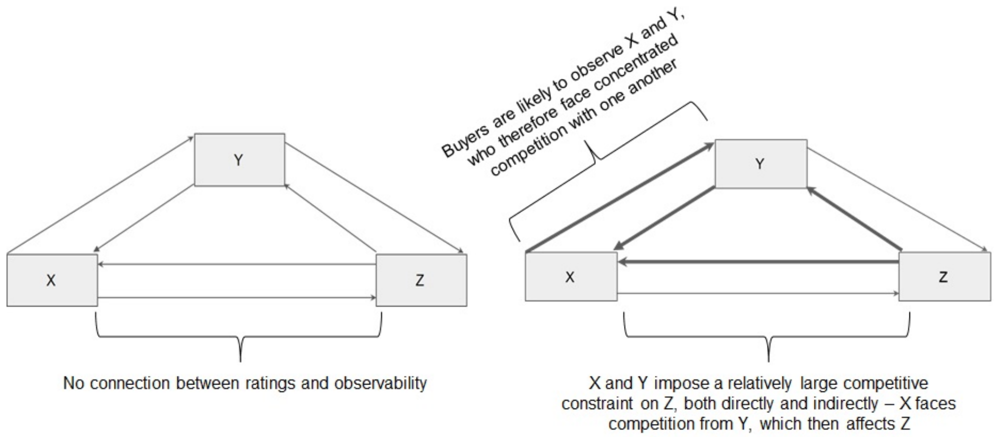

Figure 2: The effect on the competition graph when the observation function is insensitive (left) and sensitive (right) to ratings, where the product quality parameters are as follows: $\hat{\gamma}_X = \hat{\gamma}_Y > \hat{\gamma}_Z$. The thicker a line, the higher the competitive overlap between the two sellers.

Equilibrium

Let $\gamma_l := \theta\gamma_L + \epsilon_L$ and $\gamma_h := \theta\gamma_H + \epsilon_H$. To ensure that there exists a unique equilibrium in this setting, we assume the following throughout:

$$(A1) : b > c\frac{(m-1)\gamma_h}{\gamma_l}.$$

(A1) guarantees both that the substitutability parameter c is not too large relative to the own-price elasticity parameter b and that $x_{ij} > 0$ for any realisation of the quality parameter vector $\boldsymbol{\gamma}$ and the random bipartite graph G.

Let $\tilde{\boldsymbol{\alpha}}$ represent a $m \times 1$ vector with element j equal to $\tilde{\alpha_j}$ and, abusing notation slightly, let $\boldsymbol{C} = \boldsymbol{C}(\boldsymbol{R_S},1)$,where $\boldsymbol{C}(\boldsymbol{R_S},1) = [\boldsymbol{I} - R_S]^{-1}$ is the Bonacich centrality of $\boldsymbol{G_S}$ with a decay factor of 1. The following result holds:[10]

Proposition 1. *If (A1) holds, then the above pricing game has a unique Nash equilibrium in pure strategies, which is the equilibrium price vector:*

$$E_{\boldsymbol{\phi}}[\boldsymbol{p}^*] = \theta(E[\boldsymbol{\gamma}] - \frac{1}{2}E_{\boldsymbol{\phi}}[\boldsymbol{C}\boldsymbol{\gamma}]).$$

As $\mathrm{E}[\boldsymbol{C}\boldsymbol{\gamma}] = \mathrm{cov}_{\phi}(\boldsymbol{C}, \boldsymbol{\gamma}) + \mathrm{E}_{\phi}[\boldsymbol{C}]\mathrm{E}[\boldsymbol{\gamma}]$, and $\mathrm{cov}_{\phi}(\boldsymbol{C}, \boldsymbol{\gamma}) \geq 0$ when $\hat{\gamma}_j > \hat{\gamma}_k$ then $w_{ij} \geq w_{ik}$ (which, as we will see, is optimal for the platform), seller j's expected price is decreasing in their weighted expected Bonacich centrality for all j.

Network structure and concentrated competition

The equilibrium behaviour of sellers implied by Proposition 1 implies that the structure of the network affects profits. The observation probabilities can be thought of as a measure of "prominence" in the sense that they capture the likelihood that the seller is observed by a given buyer. Increasing a seller j's prominence in the network potentially increases profits as a result of increasing the probability of sales, but at the same time it imposes a cost on the rest of the network by increasing competition, reducing prices of every seller, including for the more prominent seller.

[10] All proofs can be found in the appendix.

As the number of sellers and the centrality of those sellers in the in the network G_S increases, $w_{ij} > w_{ik}$ has an increasingly large effect on prices. The centrality vector in G_S can be expressed:

$$C(R_S, 1)\mathbf{1} = \sum_{k=0}^{\infty} R_S^k \mathbf{1}.$$

It follows that $\frac{\partial^2 C_k}{\partial^2 w_{ij}} > 0$. Recall that seller prices are falling in the centrality of the sellers in this setting. Hence, setting $w_{ij} > w_{ik}$ imposes a cost upon the platform owner because the centrality measure has a feedback effect such that increasing an observation probability w_{ij} (weakly) reduces j's price, which reduces k's price which then reduces j's price and so on. This feedback effect, which is a feature of the Bonacich centrality measure, is increasing as the centralities of the sellers in G_S become larger.

Furthermore, the fact that the second derivative of the centrality vector is positive implies that two or more sellers also has a disproportionately large effect on competition. As the prominence of seller X and a seller Y increase, then the size of paths of length $l \geq 2$ that run from X to Y and then Y to X (or vice versa) increases more than linearly. The increase in the size of these paths then increases the centrality of every other seller in the network, decreasing prices.

The above analysis indicates that two or more sellers being in relatively intense competition with one another is disproportionately costly for the platform owner. We refer to the case where there is relatively intense competition between a subset of the sellers on the network as one in which competition is concentrated. There being a larger probability that high quality sellers compete with one another drives the prices of those sellers down, which propagates across the entire network.

In terms of the observation function, as $\frac{\partial \phi_j(.)}{\partial \hat{\gamma}_j}$ increases, the concentration of competition also increases, as the probability that two highly-rated sellers compete with one another increases. This has implications for the seller in terms of profit, which in turn shapes the optimal observation function $\phi(.)$ from the platform owner's perspective.

5 Weighting ratings

We will consider the case where the platform owner is chooses optimal observation function from a set of such functions that all adhere to (C1)-(C3). Notice that under the specified observation probability formation process, $w_{ij} = w_{lj}$ for all i, l pairs, and it is therefore not possible for the platform to differentiate between different buyers.

To understand some of the trade-offs inherent in the platform owner's maximisation problem, it is useful to suppose at first that the platform owner can only choose υ, such that $\varepsilon_j = 0$ for all j with probability 1. The platform owner's problem can then be re-stated as follows:

$$\max_{\upsilon} \mathrm{E}_{\upsilon}[\pi_P(\boldsymbol{p})],$$

where $\mathrm{E}_{\upsilon}[.]$ is the expectation function when the baseline probability is υ and $\varepsilon_j = 0$ for all j with probability 1. An increase in υ has two effects on profits. Increasing υ increases the probability that a given buyer is observed increases expected sales, which increases profits. However, there is a cost to increasing υ: it results in there being a higher probability that buyers observe more sellers, which creates more competition across the network. More competition reduces demand for any one good and reduces prices, decreasing profits as well.

To see the trade-off the platform owner faces more formally, we will split the marginal effect of υ into two parts. At the equilibrium value of υ, the marginal increase in profits associated with an increase in the probability that each buyer observes each seller can, by the envelope theorem (Milgrom and Segal, 2002) be expressed as follows:

$$\mathrm{MB}_{\upsilon} = n\chi \mathrm{E}_{\upsilon}[\sum_{j} bp_j^*(\theta\gamma_j - p_j^*].$$

The expected marginal cost in terms of increased competition can similarly be defined:

$$\mathrm{MC}_{\upsilon} = \chi \mathrm{E}_{\upsilon}[\sum_{j}\sum_{k\neq j} 2\upsilon ncp_j^*(p_k^* - \theta\gamma_k) + \sum_{l}\sum_{k} \hat{c}_{kl}\frac{\partial p_k^*}{\partial \upsilon}p_l^*].$$

When the solution is interior, the platform owner's profit is maximised when the marginal benefit and marginal cost sum to zero. Letting $\bar{\upsilon}$ denote a solution to the optimisation problem above, Proposition 2 sets out the solution to that problem:

Proposition 2. *There exists a unique solution to $max_{\upsilon}E_{\upsilon}[\pi_P(\boldsymbol{p})]$, $0 < \bar{\upsilon} \leq 1$. Furthermore, $\bar{\upsilon}$ is decreasing in c, and hence there exists a $\bar{c} \in \mathbb{R}_+$ such that if $c > \bar{c}$, then $\bar{\upsilon} < 1$.*

Seller, j's equilibrium price is decreasing in their Bonacich centrality, which is convex in υ. Hence, the marginal cost associated with increasing υ is convex, while the equivalent marginal benefit is linear. Hence, either the two meet at some $\bar{\upsilon} < 1$ or there is a corner solution such that $\bar{\upsilon} = 1$; either way, there exists is a unique solution to the seller's problem.

Increasing υ increases the level of competition in the network. If c is sufficiently high, then products are so substitutable that the platform owner is willing to forgo some of the profit associated with buyers observing sellers for certain in order to increase prices. The platform owner faces a trade-off between expected sales on the one hand and competition and lower prices on the other.

Now, consider the case where the platform owner maximises their profit by choosing an element of the set Φ. As discussed above, doing so involves generating a distribution of observation weights centered around a mean, υ. It is useful to compare the mean, υ, of a solution to the platform owner's optimisation problem in the case where they can choose any element of the set Φ and $\bar{\upsilon}$:

Theorem 1. *Suppose $c > \bar{c}$ and hence $\bar{\upsilon} < 1$, and that every element in Φ satisfies (C1)-(C3). Then, for any optimal observation function, $\phi^*(.)$, it must be the case that: (i) if $\hat{\gamma}_j > \hat{\gamma}_k$ then $w_{ij} \geq w_{ik}$ for all i, j, k with the inequality strict for some $\hat{\gamma}_j$ and $\hat{\gamma}_k$ and (ii) $\upsilon^* < \bar{\upsilon}$.*

The platform owner has an incentive to ensure that there is a greater probability that highly-rated products are observed for a given level of υ and $\hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}$. Buyers demand more high quality products for a given price level. Hence, if they are also more likely to observe those products, then the platform owner's profit is increasing in the variance of the quality parameter, holding prices constant.

At the same time, setting $w_{ij} \geq w_{ik}$ has implications for the structure of competition on the network. As discussed above, making a seller more prominent concentrates competition. Highly-rated sellers are more likely to compete with one another, as a result of the fact that they are more likely to be observed. Concentrated competition decreases expected prices and therefore expected profit.

Hence, the platform owner faces a trade-off between the increased revenue (holding prices constant) resulting from buyers observing high quality sellers and the fact that increasing the probability highly-rated sellers are observed increases competition. Assuming the optimal network structure when observation probabilities are equal is not just the complete graph (i.e. assuming $\bar{\upsilon} < 1$),then this trade-off is resolved such that at least some more highly-rated sellers are more likely to be observed than relatively low quality sellers.

In order to alleviate some of the decrease in price associated with making some sellers more prominent than others, the platform owner has an incentive to decrease the probability that every seller is observed by reducing υ. Hence, $\upsilon^* < \bar{\upsilon}$. The platform reduces the baseline probability that a given product is observed in order to reduce the price effects of biasing search towards high quality products.

Prices and comparative statics

The above analysis raises the question: what is the effect of biasing search results towards high quality sellers on prices? Let $\bar{\boldsymbol{\phi}}$ represent the optimal observation function where $\varepsilon_j = 0$ for all j with probability 1, and hence $\upsilon = \bar{\upsilon}$. The following proposition holds:

Proposition 3. *Suppose every element in Φ satisfies (C1)-(C3). If $\bar{\upsilon} < 1$, then there exists a $\tilde{c} \in \mathbb{R}_+$ such that if $c < \tilde{c}$, then for an optimal vector of observation functions $\boldsymbol{\phi}^*$, then $E_{\boldsymbol{\phi}^*}[\boldsymbol{p}] < E_{\bar{\boldsymbol{\phi}}}[\boldsymbol{p}]$.*

For a given baseline observation probability, υ, making some sellers more prominent than others results in a reduction in prices, as discussed above.

In response to this distortion in competition, the platform owner reduces υ such that it is below $\bar{\upsilon}$, which in turn increases expected prices. However, when substitutability between products is not too large, this effect is guaranteed not to fully compensate for the effect generated by the competition becoming more concentrated. Hence, prices fall in expectation; the platform owner is willing to lower expected prices in order to increase the probability of high-surplus matches.

It also useful to consider the effect of a change in c and θ on the weight the platform owner places on ratings. For ease of the exposition, we will amend (C2) as follows:

(C2$'$) $\quad \frac{\partial \phi(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-j})}{\partial \hat{\gamma}_j} = \kappa$ for all $\hat{\gamma}_j$ and $\hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}$.

(C2$'$) limits the platform owner to choosing linear observation functions. The previous analysis implies that $\kappa > 0$ and as κ increases, the greater the weight the observation function places on ratings. Limiting the effect ratings have on the observation function to a single parameter allows us to make more intuitive claims about the effect of changes to other parameters in the model.

To this end, it is useful to write the largest κ associated with any element of the set of optimal observation functions, Φ^*, as $\kappa^* = \kappa^*(c, \theta, \sigma_\epsilon)$. The following comparative statics hold:

Proposition 4. *Suppose every element in Φ satisfies (C1),(C2$'$) and (C3). If $\bar{\upsilon} < 1$, then: (i) $\frac{\partial \kappa^*(c,\theta,\sigma_\epsilon)}{\partial c} < 0$ and (ii) $\frac{\partial \kappa^*(c,\theta,\sigma_\epsilon)}{\partial \theta} > 0$.*

As the sellers' goods become more substitutable, the cost of a given level of competitive overlap between the sellers increases. Hence, as c increases, prices are lower for a given level of κ. An increase in c increases the effect of the concentration in competition implied by $\frac{\partial \phi_j(.)}{\partial \hat{\gamma}_j} > 0$. Highly-rated sellers compete with one another with higher probability when $\kappa > 0$, and the relative strength of this competition effect is increasing in product substitutability.

As θ increases, the marginal benefit of increasing κ also increases. This follows because (a) $\frac{\partial \mathrm{MB}_\kappa}{\partial \theta} = \chi \sum_j (\hat{\gamma}_j - \bar{\gamma})(\gamma_j) > 0$ and (b) $\frac{\partial p_j^*}{\partial \theta} > 0$. Hence, $\frac{\partial \kappa^*(c,\theta,\sigma_\epsilon)}{\partial \theta} > 0$. Intuitively, as the extent to which buyers value quality increases, the value of matching buyers with high quality products also increases, which implies that the optimal value of κ rises.

We can also use our framework to analyse the effect of an increase in σ_ϵ, the variance in idiosyncratic preferences. Theorem 2 sets this out:

Theorem 2. *Suppose (C1), (C2$'$) and (C3) hold. If $\bar{\upsilon} < 1$ and $\sigma_\epsilon' > \sigma_\epsilon$, then $\upsilon^*(\sigma_\epsilon') > \upsilon^*(\sigma_\epsilon)$ and $\kappa^*(\sigma_\epsilon') \geq \kappa^*(\sigma_\epsilon)$.*

As j's price and i's demand is increasing in ϵ_{ij}, j's expected profit is convex in ϵ_{ij}, a result consistent with Myatt and Johnson (2006). Thus, when the additional profit generated from a positive realisation of ϵ_{ij} is greater than the loss in profit associated with a negative realisation of ϵ_{ij}.

The result of this feature of the platform owner's profit function means that as σ_ϵ increases, so too does the platform owner's incentive to increase υ. Increasing the baseline probability that consumer segments observe sellers results in an increase in expected profit as it reduces the probability that a consumer segment does not observe a high-surplus idiosyncratic match, which in turn is more profitable than a equivalently low-surplus match. This is akin to the platform owner expanding the expected size of the consumer segments' information sets in the Myatt and Johnson framework.

Increasing the baseline probability υ is costly in terms of its effect on prices, and hence the platform owner is incentivised to reduce the extent to which observation is biased towards high quality sellers. Hence, as σ_ϵ increases, sellers are on average observed with higher probability and the difference between how likely it is high and low quality products are observed gets smaller.

6 Consumer welfare

Thus far we have analysed the extent to which biasing the search process towards high quality sellers increases profits. We now consider consumer welfare by assessing the effect of changes to the matching environment on consumer surplus. We define i's expected consumer surplus as follows:

$$\mathrm{E}_\phi[\mathrm{CS}_i(\boldsymbol{x_i^*}, \boldsymbol{p}^*)] = \sum_{j=1}^{m} \frac{1}{2}\mathrm{E}_\phi[x_{ij}^*(\gamma_{ij} + \sum_{k=1}^{m} \frac{c}{b} w_{ik}(p_k^* - \gamma_{ik})] - p_j^*)].$$

Define $\sum_i \mathrm{CS}_i(\boldsymbol{x_i^*}; \boldsymbol{p}^*) = \mathrm{CS}(\boldsymbol{x}^*; \boldsymbol{p}^*)$, where $\boldsymbol{x}^*$ is an $n \times m$ matrix whose ijth component is x_{ij}^*. Suppose that the central planner solves the following maximisation problem:

$$\max_\phi\{\mathrm{E}_\phi[\mathrm{CS}(\boldsymbol{x}^*, \boldsymbol{p}^*)]\}.$$

Let Φ_{CP} denote the set of solution to this maximisation problem. The following proposition holds:

Proposition 5. *Suppose every element in Φ satisfies (C1)-(C3). Any $\phi(.) \in \Phi_{CP}$ is such that $\upsilon = 1$ and hence $E_\phi[w_{ij}] = 1$ for all i, j pairs.*

Increasing an observation probability w_{ij} increases consumer surplus because it both increases the probability of surplus-increasing sales and reduces prices across the network. The platform owner therefore harms consumer surplus if they reduce υ below 1, which they may have an incentive to do in order to increase prices.

Now, consider the case where the central planner and/or platform cannot choose the baseline probability υ, supposing this is fixed by nature at $\hat{\upsilon}$. Under that condition, any element of the set Φ must result in each $\tilde{w}_{ij} =_d \hat{\upsilon} + \varepsilon_j$, for some symmetrically distributed random variable ε_j with mean 0. We term the condition (C4), under which we can compare the platform owner and central planner's preferences for observation functions.

Theorem 3. *Suppose every element in Φ satisfies (C1)-(C4) for some υ, $\hat{\upsilon}$. Take any*

$\phi(.) \in \Phi^*$. If there exists a $\phi'(.) \in \Phi^*$ under which: (a) $\hat{\gamma}_j > \hat{\gamma}_k$ implies that $w_{ij} \geq w_{ik}$ for all i, j, k and (b) the distribution of ε_j is a mean preserving spread of the distribution of ε_j generated by $\phi(.)$, then: $E_{\phi'}[CS(\boldsymbol{x^*}, \boldsymbol{p}^*)] > E_{\phi}[CS(\boldsymbol{x^*}, \boldsymbol{p}^*)]$.

If there exists a feasible observation function where high quality sellers are more likely to be observed than low quality sellers and that produces a mean-preserving spread of some other observation function, the central planner will always prefer the first observation function. This is because, for that observation function, prices are in expectation lower, and demand higher, due to the concentration in competition effect.

For the platform owner, as has been discussed, increasing the extent to which searches are biased towards high quality sellers is costly to profits due to this same price effect.The platform owner at least weakly prefers distributions of observation probabilities which are mean preserving contractions of those distributions preferred by the central planner. Such a preference comes at the cost of consumer welfare.

7 Discussion

Our analysis explains why online platform use ratings differently to determine which sellers are more likely to be observed by buyers. In general, the platform owner faces a trade-off between increased competition and lower expected prices on the one hand and an increase in profits due to matching on the other. The extent to which the platform owner has an incentive to bias their search process towards highly-rated sellers depends on the nature of the goods being sold on the platform.

To expand on this point, we compare the importance of product quality and the variance of the idiosyncratic preferences of different goods commonly sold on online platforms in Figure 3.

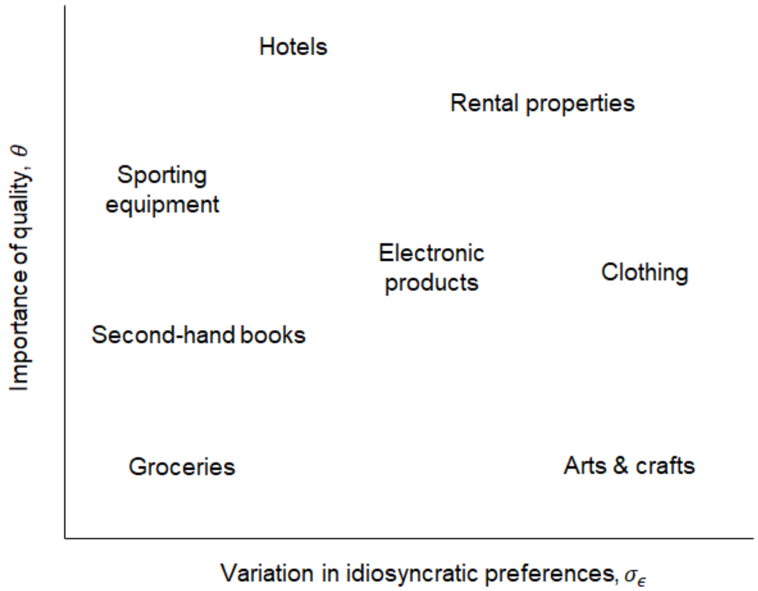

Figure 3: A comparison of different types of markets according to their values of importance of quality and the variance in idiosyncratic preferences

When platforms offer (or predominately offer) products with low values of θ, the concentrated competition effect identified in Section 6, whereby using ratings to determine observation probabilities increases the effective level of competition across the market, dominates. When the ex-post difference between the valuations of the sellers is low, the expected reduction in prices associated with biasing observation towards high-quality sellers outweighs any potential benefits associated with a ratings-based environment, which implies that it is relatively less profitable than a random match environment.

For products that have high idiosyncratic variance but relatively low expected differences in quality, a random match (or near random match) environment is relatively more profitable than the ratings-based environment. This is because there is an increasing probability of high-surplus matches driven by the convexity of the profit function. On the other hand, the relative profit associated with high-quality sellers being observed with a high probability increases as θ increases, as a result of the same convexity in profit function. Hence, in such environments, the ratings-based environment would be more profitable.

To represent the above discussion graphically, suppose that (C2$'$) holds. If $\kappa > \bar{\kappa}$ for some arbitrary $\bar{\kappa} \in \mathbb{R}_+$, then we define the observation process as being "ratings based". If $\kappa^* \leq \bar{\kappa}$, let the search process be known as "random match". Figure 4 then depicts where ratings-based and random matching observation processes would be optimal in $\theta - \sigma_\epsilon$ space:

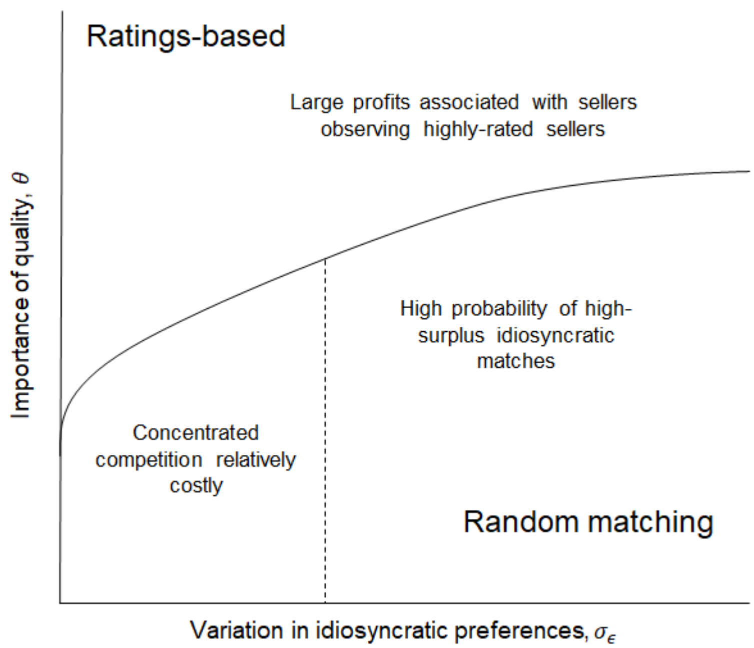

Figure 4: A comparison of the random match and ratings-based environments: in markets above the solid line, a ratings-based observation process is optimal, below it, random matching.

Figure 4 makes clear that platforms that (largely) host markets where product quality is less important to consumers should use an information environment that weights ratings less than platforms where product quality is more important to consumers. It is possible to approximate the positions of real-life platforms within the space shown in

Figure 4, which is shown in Figure 5.

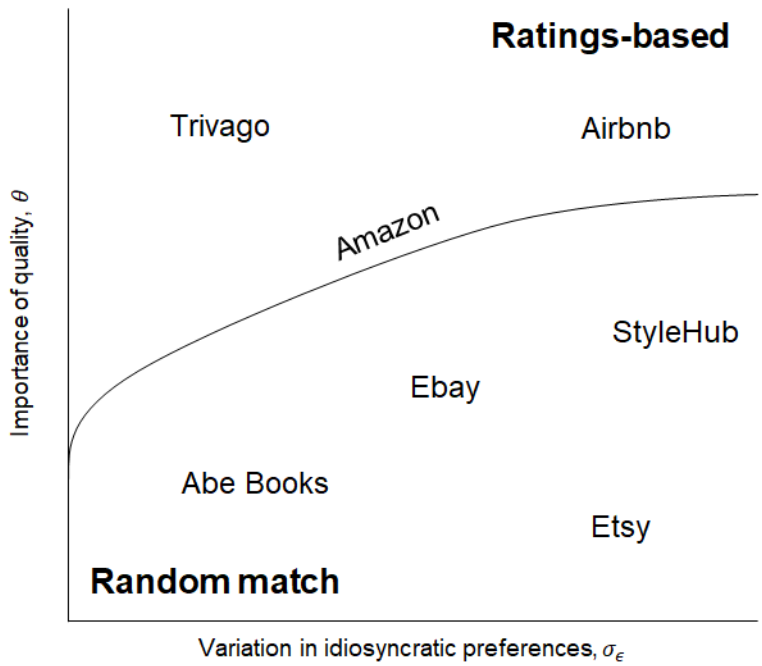

Figure 5: A comparison of the random match and ratings-based environments. For any market above the solid line, profits are higher in ratings-based environments.

Our analysis also makes it possible to assess some contemporary issues and debates within the regulation of platforms. For example, the European Commission has recently begun investigating the extent to which Amazon biases its search results towards its own products. Not only does this have clear implications for which sellers are successful on the platform, but it also has effect on competition more broadly.

In terms of the model here, if the platform sets probability that a preferred seller(s), j, is observed by all buyers to some higher than average value $w_{ij} = v'$, then the platform has an incentive to lower the baseline probability that other sellers are observed **and** reduces the extent to which high quality sellers are observed. This not only arises because this decreases the competition faced by the preferred seller, but because there being an especially prominent seller increases the concentration of competition across the network, which can be offset by reducing the bias placed on ratings by internal search algorithms.

8 Conclusion

Using seller ratings to order search shapes the structure of competition on a platform hosting a marketplace. Empirical evidence makes clear that there the extent to which seller ratings are used to order search results differs considerably between platforms, and the aim of our analysis has been to understand what drives these differences.

There is a trade-off that the platform owner faces when using seller ratings to order search results. Increasing the extent to which sellers that are highly-rated are observed by buyers is potentially profit increasing because buyers are more likely to be matched with a high quality product, increasing willingness to pay.

At the same time, biasing search towards highly-rated sellers has the effect of concentrating competition among those sellers, which results in lower expected prices across the network. The platform owner resolves this trade-off in the case where they can freely choose the extent to which they bias their search results by limiting the use of ratings in the search process. As products become more substitutable, or consumers become less sensitive to product quality, the platform owner has an incentive to reduce the extent to which the search process is biased towards highly-rated products.

Our analysis generates a number of predictions that would be worthwhile exploring empirically, including:

1. Platforms selling products where perceived or actual quality is more important will use ratings as a larger component of their search process;

2. Platforms selling products with high idiosyncratic variance will use ratings as a smaller component of their search process;

3. Revealing product rankings reduces prices within given platform or market;

4. Platforms selling relatively homogeneous products will use ratings as a smaller component of their search process.

Our results also indicate that competition authorities should pay close attention to what determines prominence within the search processes of online platforms. As platforms like Amazon become increasing dominant, the incentive and ability to influence competition by changing which buyers see which sellers increases. Greater transparency regarding the inputs of the algorithms that determine prominence would be needed for authorities to understand real-world platform incentives and reduce the potential for anti-competitive behaviour.

Appendix

Proof of Proposition 1

It can be readily shown that the first-order condition (and therefore the resulting optimisation problem) for the payoff vector $\tilde{\boldsymbol{\pi}}$ is equivalent to the first-order condition of the payoff vector associated with the original payoff vector $\boldsymbol{\pi}$. The first-order condition of the payoff vector is as follows:

$$\tilde{\boldsymbol{\alpha}} = [\boldsymbol{I} - R_S]\boldsymbol{p}.$$

The matrix $\boldsymbol{I} - R_S$ is positive definite (by (A1)), non-singular and the above first-order condition has a solution, which is denoted $\boldsymbol{p}^{*}$. Rearranging this first-order condition leads to the following: $[\boldsymbol{I} - R_S]^{-1}\tilde{\boldsymbol{\alpha}} = \boldsymbol{p}^{*}$.

As $\boldsymbol{I} - R_S$ is positive definite, the stated first-order condition above yields a unique interior solution. Furthermore, no corner solutions can exist, due to the fact that (A1) guarantees that $x_{ij} > 0$ for all i, j pairs. This implies that if $p_j^{*} = 0$ in some price vector $\boldsymbol{p}$,$\boldsymbol{p}$ is not optimal price vector. This is because there exists a $\varepsilon > 0$ such that $p_j^{'} = \varepsilon$ generates a strictly positive expected demand $\mathrm{E}[x_j^{'}(\boldsymbol{p'})] > 0$.

Recall that $\alpha_j = \sum_{i=1}^{n}[bw_{ij}\gamma_{ij}] - \sum_{k=1}^{m}\hat{c}_{jk}$. Noting that $\mathrm{E}[\epsilon_{ij}] = 0$ that each γ_j and ϵ_{ij} are independent, it follows that:

$$\mathrm{E}[\boldsymbol{C}\tilde{\boldsymbol{\alpha}}] = \mathrm{E}[\frac{1}{2}\sum_{k=0}^{\infty} R_S^k(\theta\boldsymbol{\gamma}) - \sum_{k=1}^{\infty} R_S^k\theta\gamma].$$

Implying that:

$$\mathrm{E}[C\tilde{\alpha}] = \mathrm{E}[\frac{1}{2}\theta\boldsymbol{\gamma} - \frac{1}{2}\sum_{k=0}^{\infty} R_S^k\theta\boldsymbol{\gamma} + \sum_{k=0}^{\infty} R_S^k\theta\boldsymbol{\gamma}].$$

Proof of Proposition 2

Note that the ijth entry in the centrality matrix $C(R_{S,}1)$ is equal to $\sum_l r_{ij}^{[l]}(R_S)$, where $r_{ij}^{[l]}(R_S)$ measures the weighted paths of length l that start at node i and end at node j. When $w_{ij} = v$ for all i, j pairs, then $C_j(R_{S,}1)$ can be written as follows:

$$\mathrm{E}[C_j(R_S, 1)] = \sum_{k=0}^{\infty}(\frac{c}{b}v(m-1))^k$$

Each expected centrality is convex in v, as $\frac{\partial^2 \mathrm{E}[C_j(R_S,1)}{\partial^2 v} > 0$. As the expected equilibrium price vector is decreasing in the expected centrality of each seller j, this implies that the expected price vector is strictly concave in υ. This implies that the platform owner's optimisation problem is also concave in υ as the following expression holds:

$$\frac{d^2\pi_k^*}{d^2\upsilon} = \frac{\partial^2\pi_k^*}{\partial^2\upsilon} + \sum_i [(\frac{\partial^2\pi_k^*}{\partial^2(p_i^*)}\frac{\partial p_i^*}{\partial\upsilon})(\frac{\partial p_i^*}{\partial\upsilon}) + \frac{\partial\pi_k^*}{\partial p_i^*}\frac{\partial^2 p_i^*}{\partial^2\upsilon}].$$

Noting that $\frac{\partial p_i^*}{\partial\upsilon} < 0$ and $\frac{\partial\pi_k^*}{\partial p_i^*} < 0$ for all p_i, all three of the terms on the right-hand side of this expression are negative, which implies that $\frac{d^2\pi_k^*}{d\upsilon^2} < 0$ for all k. Hence, if the solution of the first-order conditions of the platform owner's profit maximisation problem is interior, it represents a unique, global optimum, where $0 < \upsilon^* < 1$. Furthermore, if $\frac{d^2\pi_P^*}{d^2\upsilon} > 0$ when $\upsilon = 1$, then $\upsilon = 1$ is an optimal solution as $\upsilon \in [0, 1]$.

It is clear from the above that the absolute value of the derivative $\frac{\partial p_j}{\partial\upsilon}$ is increasing in c. Furthermore, the expression $\mathrm{E}[\sum_k 2\upsilon ncp_j^*(p_k^* - \theta\gamma_k)]$ features in MC_υ. (A1) dictates that $b\mathrm{E}[\gamma_j] > c\sum_k \mathrm{E}[\gamma_k]$, but at $\upsilon = 1$, there exists a c such that $2c\sum_k \mathrm{E}[\gamma_k] > b\mathrm{E}[\gamma_j] > c\sum_k \mathrm{E}[\gamma_k]$, holds. This is sufficient to guarantee that $\mathrm{MB}_\upsilon - \mathrm{MC}_\upsilon < 0$, when $\upsilon = 1$, and hence there exists a $\bar{c}$ such that if $c > \bar{c}$, $\upsilon^* < 1$.

Proof of Theorem 1

For (i), we consider the effect of a marginal increase in the probability that j is observed, by a consumer i, $w_{ij} = w_j$, once ratings have been realised. Note that each seller j's profit function is twice differentiable and continuous, and $\frac{\partial^2\pi_j}{\partial^2 p_j} < 0$. As per Milgrom and Segal (2002), these are jointly sufficient conditions for the envelope theorem to apply, and thus $\frac{d\pi_P(p^*)}{dw_j} = \frac{\partial\pi_P(p^*)}{\partial\gamma_j}$.

$$\frac{d\sum_i \pi_i(\boldsymbol{p})}{dw_j} = nbp_j^*(\gamma_j - p_j^*) + \sum_{i\neq j} w_i ncp_j^*(p_i^* - \theta\gamma_i) + \sum_i \sum_{k\neq i} (\hat{c}_{ik}\frac{\partial p_k^*}{\partial w_j})p_i^*.$$

By (A1) and the expression for $\boldsymbol{p^*}$, it follows that $\frac{\partial \sum_i \pi_i(\boldsymbol{p})}{\partial w_j \partial \gamma_j} > 0$ for $\gamma_j \in [\gamma_L, \gamma_H]$.

Now consider a observation function, $\tilde{\phi}(.)$ where $\frac{\partial \tilde{\phi}_j(\hat{\gamma}_j^{'}, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})}{\partial \hat{\gamma}_j} < 0$ for at least some $\hat{\gamma}_j^{'} \in [\gamma_L, \gamma_H]$ and some vector $\hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}$. The optimal function having this property is a necessary and sufficient condition for it to be the case that there exists a pair of ratings $\hat{\gamma}_j, \hat{\gamma}_k$ such that: $w_{ij} < w_{ik}$when $\hat{\gamma}_j > \hat{\gamma}_k$, violating the first statement in (i).

Recall that $\frac{\partial \sum_i \pi_i(\boldsymbol{p})}{\partial w_j \partial \gamma_j} > 0$. It follows that if $\frac{\partial \tilde{\phi}_j(\hat{\gamma}_j^{'}, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})}{\partial \hat{\gamma}_j} < 0$, then there always exists a $\hat{\phi}_j(.,.) \in \Phi$ where $\hat{\phi}_j(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}) < \tilde{\phi}_j(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})$ for all $\hat{\gamma}_j \leq \hat{\gamma}_j^{'}$, $\hat{\phi}_j(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}) \geq \tilde{\phi}_j(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})$ for all $\hat{\gamma}_j > \hat{\gamma}_j^{'}$ and $\mathrm{E}[\pi_P(p^*)|\tilde{\boldsymbol{\phi}}] < \mathrm{E}[\pi_P(p^*)|\hat{\boldsymbol{\phi}}]$. Hence the first statement in (i) is proved.

For the claim that for any optimal observation function it must be the case that for a given $\hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}$, $\frac{\partial \phi_j^*(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})}{\partial \hat{\gamma}_j} > 0$ for at least some $\hat{\gamma}_j \in [\gamma_L, \gamma_H]$, note first that $\upsilon^* < \bar{\upsilon} < 1$, as shown below. As we have ruled out the case where $\frac{\partial \phi_j^*(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})}{\partial \hat{\gamma}_j} < 0$, to show this we can show that $\frac{\partial \phi_j(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})}{\partial \hat{\gamma}_j} = 0$ for all $\hat{\gamma}_j \in [\gamma_L, \gamma_H]$ is suboptimal.

If for a given $\hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}$, $\frac{\partial \tilde{\phi}_j(\hat{\gamma}_j, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})}{\partial \hat{\gamma}_j} = 0$ for $\hat{\gamma}_j \in [\gamma_L, \gamma_H]$, then $w_{ij} = w$ for all i and j pairs. As $\upsilon^* < \bar{\upsilon} < 1$, it must be the case that for any candidate observation function, $\tilde{\phi}_j(.)$, $\tilde{\phi}_j(\hat{\gamma}_L, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}) < 1$ for any $\hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}$. Then, given $\frac{\partial \sum_i \pi_i(\boldsymbol{p})}{\partial w_j \partial \gamma_j} > 0$ for $\gamma_j \in [\gamma_L, \gamma_H]$ when $w_j < 1$, it follows that there exists a observation function where $\hat{\phi}_j(\hat{\gamma}_L, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}}) < w < \hat{\phi}_j(\hat{\gamma}_H, \hat{\boldsymbol{\gamma}}_{-\boldsymbol{j}})$ and $\mathrm{E}[\pi_P(p^*)|\tilde{\boldsymbol{\phi}}] < \mathrm{E}[\pi_P(p^*)|\hat{\boldsymbol{\phi}}]$.

For (ii), recall that for any $\boldsymbol{\phi} \in \Phi$, we can write observation probabilities as $w_{ij} =_d \upsilon + \epsilon_j$ for all i and j. Suppose that ε_j is distributed in accordance to some optimal vector $\boldsymbol{\phi}^*$ and has a distribution f. If f is non-degenerate, then for any given value of υ, $|\frac{\partial \mathrm{E}_{\upsilon,f}[p_i^*]}{\partial \upsilon}| > |\frac{\partial \mathrm{E}_{\upsilon,f_D}[p_i^*]}{\partial \upsilon}|$ where f_D is the degenerate distribution in which $\boldsymbol{\varepsilon_j} = 0$ with probability 1.

Furthermore, as increasing υ increases the probability that every seller is observed equally, the gross marginal benefit of increasing υ is independent of the distribution of ε_j. Hence, $\frac{\partial \mathrm{E}_{\upsilon,f}[\pi_P]}{\partial \upsilon} < \frac{\partial \mathrm{E}_{\upsilon,f_D}[\pi_P]}{\partial \upsilon}$ for all values of $\upsilon \in [\underline{\upsilon}, \bar{\upsilon}]$, where $\underline{\upsilon}$ and $\bar{\upsilon}$ are the lowest and highest values of υ consistent with $\boldsymbol{\varepsilon_j} \sim f$ and the conditions (C1)-(C3) being met. It follows that $\upsilon \leq \bar{\upsilon}$.

Proof of Proposition 3

As stated in the main text, any vector of observation functions $\boldsymbol{\phi}$ that obey (C1)-(C3) results in each $w_{ij} =_d \upsilon + \varepsilon_j$. Let ϑ^* denote the distribution of the vector of ε_js, $\boldsymbol{\varepsilon}$ induced by an optimal observation function $\phi^*(.)$ and ϑ_0 the distribution induced by the $\bar{\boldsymbol{\phi}}$: i.e. where $\boldsymbol{\varepsilon}_j = 0$ for all j. Let $\tilde{\upsilon}$ denote the υ that solves the following expression:

$$\mathrm{E}_{\upsilon,\vartheta_0}[p_j] = \mathrm{E}_{\tilde{\upsilon},\vartheta^*}[p_j].$$

By the envelope theorem, the marginal effect of increasing υ at $\upsilon = \upsilon'$ on j's profit is as follows:

$$\frac{d\mathrm{E}_{\upsilon',\vartheta}[\pi_j^*(\boldsymbol{p})]}{d\upsilon} = \frac{\partial \mathrm{E}_{\upsilon',\vartheta}[\pi_j(\boldsymbol{p})]}{\partial \upsilon} + \sum_{i \neq j} \frac{\partial \mathrm{E}_{\upsilon',\vartheta}[\pi_j(\boldsymbol{p})]}{\partial \mathrm{E}_{\upsilon',\vartheta}[p_i^*]} \frac{\partial \mathrm{E}_{\upsilon',\vartheta}[p_i^*]}{\partial \upsilon}$$

First, note that $\frac{\partial^2 E_{\upsilon',\vartheta}[\pi(\boldsymbol{p})]}{\partial^2 \upsilon} < 0$, as each seller j's profit function includes the expression $n\upsilon^2 \sum_{k \neq j} c(p_k - \gamma_{ik})$, which is negative by definition. As the conditional expectation of the term$\sum_{k \neq j} c(p_k - \gamma_{ik})$ is the same for both distribution and $\tilde{\upsilon} < \bar{\upsilon}$ it follows that:

$$|\frac{\partial \mathrm{E}_{\tilde{\upsilon},\vartheta^*}[\pi_j(\boldsymbol{p})]}{\partial \upsilon}| < |\frac{\mathrm{E}_{\bar{\upsilon},\vartheta_0}[\pi_j(\boldsymbol{p})]}{\partial \upsilon}|$$

Now consider the second term in the expression for $\frac{d\mathrm{E}_{\upsilon',\vartheta}[\pi_j^*(\boldsymbol{p})]}{d\upsilon}$ above. To assess this expression, we must consider $\frac{\partial \mathrm{E}_{\upsilon',\vartheta}[C_j]}{\partial \upsilon}$. Paths of length 2 in the expression $\mathrm{E}_{\upsilon',\vartheta}[C_j]$ are all functions of υ^2, $\mathrm{E}[\upsilon \varepsilon_i]$ for all i and $(\frac{c}{b})^2$. As $\mathrm{E}[\upsilon \varepsilon_i] = 0$, it follows that the derivative of the value of all paths of length $l \leq 2$ with respect to υ is strictly increasing in υ for all distributions of $\boldsymbol{\varepsilon}$.

Furthermore, note that $\frac{c}{b} < 1$. So while paths of length $l > 2$ are a function of $\mathrm{E}[\varepsilon_i^k]$ with $k > 2$, any such path is a function of $(\frac{c}{b})^l$. Hence, there exists a $\tilde{c}$ such that if $c < \tilde{c}$, the following inequality must hold:

$$|\frac{\partial \mathrm{E}_{\tilde{\upsilon},\vartheta^*}[C_j]}{\partial \upsilon}| < |\frac{\partial \mathrm{E}_{\bar{\upsilon},\vartheta_0}[C_j]}{\partial \upsilon}| \ \ \forall j.$$

which then implies that: $|\frac{\mathrm{E}_{\tilde{\upsilon},\vartheta^*}[p_j^*]}{\partial \upsilon}| < |\frac{\partial \mathrm{E}_{\bar{\upsilon},\vartheta_0}[p_j]}{\partial \upsilon}| \ \ \forall j$. Hence:

$$\frac{d\mathrm{E}_{\tilde{\upsilon},\vartheta^*}[\pi_P^*(\boldsymbol{p})]}{d\upsilon} > \frac{d\mathrm{E}_{\bar{\upsilon},\vartheta_0}[\pi_P^*(\boldsymbol{p})]}{d\upsilon} = 0.$$

The last equality holds by definition as $\bar{\upsilon}$ maximises $\mathrm{E}_{\bar{\upsilon},\vartheta_0}[\pi_P^*(\boldsymbol{p})]$. It follows that the platform owner has an incentive to increase υ at $(\tilde{\upsilon}, \vartheta^*)$ which, as $\frac{d\mathrm{E}_{\upsilon,\vartheta}[\boldsymbol{p}^*]}{d\upsilon} < 0$, implies that:

$$\mathrm{E}_{\tilde{\upsilon},\vartheta^*}[p_j^*] < \mathrm{E}_{\bar{\upsilon},\vartheta_0}[p_j^*] \ \ \forall j.$$

Proof of Proposition 4

Consider the first-order conditions of the platform owner's problem $\max_{\kappa,\upsilon}\pi_P^*(\upsilon,\kappa)$, which we express as follows: $\pi_\kappa := \frac{\partial \pi_P(\upsilon,\kappa;\theta,c)}{\partial \kappa} = 0$ and $\pi_\upsilon := \frac{\partial \pi_P(\upsilon,\kappa;\theta,c)}{\partial \upsilon} = 0$. These two first-order conditions provide a mapping from $\mathbb{R}^4 \to \mathbb{R}^2$. Denote the Hessian of the platform owner's maximisation problem as follows:

$$H = \begin{bmatrix} \pi_{\kappa\kappa} & \pi_{\kappa\upsilon} \\ \pi_{\upsilon\kappa} & \pi_{\upsilon\upsilon} \end{bmatrix}.$$

It is clear that $\frac{\partial \pi_\kappa}{\partial \theta} > 0$ and $\frac{\partial \pi_\upsilon}{\partial \theta} > 0$. By the implicit function theorem:

$$-\frac{1}{|H|}\begin{bmatrix} \pi_{\upsilon\upsilon} & -\pi_{\upsilon\kappa} \\ -\pi_{\kappa\upsilon} & \pi_{\kappa\kappa} \end{bmatrix}\begin{bmatrix} \frac{\partial \pi_\kappa}{\partial \theta} \\ \frac{\partial \pi_\upsilon}{\partial \theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \kappa^*}{\partial \theta} \\ \frac{\partial \upsilon^*}{\partial \theta} \end{bmatrix}.$$

As $\pi_P(\kappa, \upsilon)$ is concave in (κ, υ), $|H| > 0$ and $\frac{\partial \kappa^*}{\partial \theta} > 0$.

A similar argument applies for c, with the difference being that $\frac{\partial \pi_\upsilon}{\partial c} < 0$ and $\frac{\partial \pi_\kappa}{\partial c} < 0$. It follows that $\frac{\partial \kappa^*}{\partial c} < 0$.

Proof of Theorem 2

For the purposes of our analysis, we rewrite the optimal price vector as a function of the relevant parameters of the model, $\boldsymbol{p}^* = \boldsymbol{p}^*(\upsilon, \kappa, \boldsymbol{\epsilon})$. Recall that $\gamma_{ij} = \theta\gamma_j + \epsilon_{ij}$. As ϵ_{ij} and γ_j are independent, ϵ_{ij} does not affect the centrality vector of the graph generated by υ, κ for all i and j. Hence:

$$\frac{\partial \boldsymbol{p}^*(\upsilon, \kappa, \boldsymbol{\epsilon})}{\partial \epsilon_{ij}} = \frac{\partial \sum_k \tilde{\alpha}_k}{\partial \epsilon_{ij}}.$$

Given the definition of $\sum_k \tilde{\alpha}_k$, it is clear that this expression is postie and does not include ϵ_{ij}, and hence $\boldsymbol{p}^*(\upsilon, \kappa, \boldsymbol{\epsilon})$ is linear and positive in each ϵ_{ij}. Consider the ex-post profits of a seller j:

$$\pi_j(\boldsymbol{p}^*, \boldsymbol{\epsilon}) = p_j(\upsilon, \kappa, \boldsymbol{\epsilon})(\alpha_j(\upsilon, \boldsymbol{\epsilon}) - \beta_j p_j(\upsilon, \kappa; \boldsymbol{\epsilon}) + \sum_{j \neq k} \hat{c}_{jk} p_k(\upsilon, \kappa; \boldsymbol{\epsilon}).$$

As p_j and $\alpha_j(\upsilon, \boldsymbol{\epsilon})$ are both increasing linear functions of ϵ_{ij}, $\pi_j(\boldsymbol{p}^*, \boldsymbol{\epsilon})$ is increasing in ϵ_{ij}^2. By implicitly limiting the size of the effect on j's profit of each ϵ_{ik}^2 $k \neq j$, (A1) then guarantees that platform owner's expected profit is increasing in $\sigma_\epsilon^2 = \mathrm{E}[\epsilon_{ij}^2]$.

Consider the expected effect of changing υ on j's profit at σ_ϵ and $\sigma_\epsilon^{'}$. The only element of seller profit functions that change as the result of a change in σ_ϵ is $s(\upsilon,\kappa) := \mathrm{E}_{\upsilon,\kappa}[\sum_j p_j(\upsilon,\kappa,\boldsymbol{\epsilon})\alpha_j(\upsilon,\boldsymbol{\epsilon})]$. This holds because neither β_j nor $\hat{c}_{jk}$ are functions of $\boldsymbol{\epsilon}$ and $\boldsymbol{p^*}(\upsilon,\kappa,\boldsymbol{\epsilon})$ is linear in $\boldsymbol{\epsilon}$.

The previous discussion surrounding j's profit function combined with the fact that $\alpha_j(\upsilon,\kappa,\boldsymbol{\epsilon})$ is an increasing function of υ, implies that if $\upsilon < \upsilon^{'}$, $s(\upsilon^{'},\kappa) - s(\upsilon,\kappa)$ is linearly increasing in σ_ϵ^2.

Recall that γ_j and ϵ_{ij} are independent and that, as (C2$^{'}$) holds, κ does not affect $\mathrm{E}[\alpha_j(\upsilon,\boldsymbol{\epsilon})]$. Hence, $s(\upsilon,\kappa^{'}) - s(\upsilon,\kappa)$ is independent of σ_ϵ^2 for all $\kappa^{'}$ and κ.

The above then directly implies that when $\bar{\upsilon}$, $\upsilon^*(\sigma_\epsilon) > \upsilon^*(\sigma_\epsilon^{'})$. As $\frac{\partial \mathrm{E}_{\upsilon,\kappa}[\pi_P(\boldsymbol{p}^*)]}{\partial\kappa}$ is strictly decreasing in υ for all $\kappa \geq 0$, it follows that the largest κ that solves the first order condition $\frac{\partial \mathrm{E}_{\upsilon^*(\sigma_\epsilon^{'}),\kappa^*(\sigma_\epsilon^{'})}[\pi_P(\boldsymbol{p}^*)]}{\partial\kappa} = 0$ must be less than or equal to the largest κ that solves the first order condition $\frac{\partial \mathrm{E}_{\upsilon^*(\sigma_\epsilon),\kappa^*(\sigma_\epsilon)}[\pi_P(\boldsymbol{p}^*)]}{\partial\kappa} = 0$. Hence, $\kappa^*(\sigma_\epsilon) \geq \kappa^*(\sigma_\epsilon^{'})$.

Proof of Proposition 5

As shown in the proof of Proposition 2,for a given observation function $\phi(.)$, $\frac{\partial \mathrm{E}_\phi[p_j]}{\partial\upsilon} < 0$ for all j. Furthermore, $\frac{\partial \mathrm{E}_\phi[x_{ij}]}{\partial\upsilon} > 0$. Defining the second part of the expression for CS_i in the main text, $y_{ij}(\boldsymbol{R}) := \tilde{\gamma}_{ij} + \sum_{k=1}^m \frac{c}{b} w_{ik}(p_k^*(\boldsymbol{R}) - \theta\tilde{\gamma}_{ik}) - p_j^*(\boldsymbol{R})$, it is also clear that $\frac{\partial \mathrm{E}_\phi[y_{ij}]}{\partial\upsilon} > 0$.

Given (A1) holds and noting that $\mathrm{E}[p_j^*] = \mathrm{E}[p_k^*] = \mathrm{E}[p^*]$, both x_{ij} and $y_{ij}(\boldsymbol{R})$ are decreasing in $\mathrm{E}[p^*]$. It must be the case that $\mathrm{E}[CS_i(\boldsymbol{x_i^*};\boldsymbol{p}^*)]$ is maximised when $\upsilon = 1$ for all i, which proves the result.

Proof of Theorem 3

Consider two distributions of ϵ_j, denoted ϑ and $\vartheta^{'}$, where ϑ is a mean-preserving spread of $\vartheta^{'}$ and both distributions are symmetric with mean 0. Suppose $\tilde{w}_{ij} \sim \vartheta$ and $\tilde{w}^{'}_{ij} \sim \vartheta^{'}$ for all i, j, implying it is possible to write $\tilde{w}_{ij} =_d \tilde{w}^{'}_{ij} + \zeta_{ij}$,where ζ_{ij} is symmetrically distributed random variable with mean 0. Recall that:

$$C(\boldsymbol{R}_S)\mathbf{1} = \sum_{k=0}^{\infty} R_S^k \mathbf{1}.$$

It follows then that $\mathrm{E}_{\vartheta}[C_j(\boldsymbol{R}_S)]$ is a weakly increasing function of $\mathrm{E}[\tilde{w}^{'}_{ij}]$ and $\mathrm{E}[\zeta^k_{ij}]$ for each $k \geq 1$ for some i, j pair. Noting that ϵ_{ij} is symmetric by definition, it must be the case that $\mathrm{E}[\zeta^k_{ij}] = 0$ when k is odd. Furthermore, $\mathrm{E}[\zeta^k_{ij}] > 0$ when k is even. It follows that $\mathrm{E}_{\vartheta^{'}}[C_j(\boldsymbol{R}_S)] > \mathrm{E}_{\vartheta}[C_j(\boldsymbol{R}_S)] \quad \forall j$.

$$\mathrm{E}_{\boldsymbol{\phi}}[\boldsymbol{p}^{*}] = \theta(\mathrm{E}[\boldsymbol{\gamma}] - \frac{1}{2}\mathrm{E}_{\phi}[\boldsymbol{C}\boldsymbol{\gamma}]).$$

Consider two observation functions, $\phi(,)$, $\phi^{'}(,) \in \Phi$, where the random observation probabilities generated by $\phi(,)$, $\tilde{w}_{ij} \sim \vartheta$ and the observation probabilities generated by $\phi^{'}(,)$ $\tilde{w}^{'}_{ij} \sim \vartheta^{'}$. Note that $\mathrm{cov}(\tilde{w}^{'}_{ij}, \gamma_j) = \mathrm{cov}(\tilde{w}_{ij}, \gamma_j) + \mathrm{cov}(\zeta_{ij}, \gamma_j)$. As $\phi^{'}(,)$ is such that $\hat{\gamma}_j > \hat{\gamma}_k$ then $w_{ij} \geq w_{ik}$ for all i, j, k then $\mathrm{cov}(\zeta_{ij}, \gamma_j) > 0$, and hence, $\mathrm{cov}(\tilde{w}^{'}_{ij}, \gamma_j) > \mathrm{cov}(\tilde{w}_{ij}, \gamma_j)$. Thus, $\mathrm{E}_{\phi^{'}}[\boldsymbol{C}\boldsymbol{\gamma}] > \mathrm{E}_{\phi}[\boldsymbol{C}\boldsymbol{\gamma}]$.

Given the result in Proposition 1, the above implies that $\mathrm{E}_{\phi^{'}}[\boldsymbol{p}^{*}] < \mathrm{E}_{\phi}[\boldsymbol{p}^{*}]$.

Now, consider $\mathrm{E}_{\phi}[x^{*}_{ij}(\boldsymbol{p})]$. Define the function:

$$x_{ij}(\boldsymbol{R}) = (b(\tilde{\gamma}_{ij} - p^{*}_j(\boldsymbol{R}) + \sum_{k \neq j} \hat{c}_{jk}(p^{*}_k(\boldsymbol{R}) - \theta\tilde{\gamma}_{ik})).$$

Where $\boldsymbol{R}$ is the adjacency matrix of the graph G when the observation function is $\phi(,)$. We can then write $\mathrm{E}_{\phi}[x^{*}_{ij}(\boldsymbol{p})] = \mathrm{E}[\tilde{w}_{ij}x_{ij}(\boldsymbol{R})]$ and $\mathrm{E}_{\phi^{'}}[x_{ij}(\boldsymbol{R}^{'})] = \mathrm{E}[\tilde{w}^{'}_{ij}x_{ij}(\boldsymbol{R}^{'})]$. Note that $\mathrm{E}[\tilde{w}_{ij}] = \mathrm{E}[\tilde{w}_{ij}^{'}]$ by definition. Due to $\mathrm{E}_{\phi'}[p_j] = \mathrm{E}_{\phi'}[p_k]$ and the fact that (A1) holds, $x_{ij}(\boldsymbol{R})$ is decreasing in the expected price vector $\boldsymbol{p}$. Thus, $\mathrm{E}[\tilde{x}_{ij}(\boldsymbol{R})] < E[\tilde{x}_{ij}(\boldsymbol{R}^{'})]$.

Furthermore, $\mathrm{cov}[\tilde{w}_{ij}^{'} x_{ij}(\boldsymbol{R}^{'})] > \mathrm{cov}[\tilde{w}_{ij} x_{ij}(\boldsymbol{R})]$ as $\mathrm{cov}(\zeta_{ij},\gamma_j) > 0$. Hence $\mathrm{E}[\tilde{w}_{ij} x_{ij}(\boldsymbol{R})] < \mathrm{E}[\tilde{w}_{ij}^{'} x_{ij}(\boldsymbol{R}^{'})]$. As $x_{ij}(\boldsymbol{R}^{'})$ is independent of $x_{lk}(\boldsymbol{R}^{'})$ for all i, j, l, k, Hence: $\mathrm{E}[\sum_j \tilde{w}_{ij} x_{ij}(\boldsymbol{R})] < \mathrm{E}[\sum_j \tilde{w}_{ij}^{'} x_{ij}(\boldsymbol{R}^{'})]$.

We can similarly analyse the second term in the expression for CS_i:

$$y_{ij}(\boldsymbol{R}) := \tilde{\gamma}_{ij} + \sum_{k=1}^{m} \frac{c}{b} w_{ik}(p_k^*(\boldsymbol{R}) - \theta\tilde{\gamma}_{ik}) - p_j^*(\boldsymbol{R}),$$

and conclude that $\mathrm{E}[\sum_j \tilde{w}_{ij} y_{ij}(\boldsymbol{R})] < \mathrm{E}[\sum_j \tilde{w}_{ij} y_{ij}(\boldsymbol{R}^{'})]$. Again, as $\mathrm{cov}(\zeta_{ij},\gamma_j) > 0$, the following result also holds:

$$\mathrm{cov}(\sum_j \tilde{w}_{ij}^{'} y_{ij}(\boldsymbol{R}^{'}), \sum_j \tilde{w}_{ij}^{'} x_{ij}(\boldsymbol{R}^{'})) > \mathrm{cov}(\sum_j \tilde{w}_{ij} y_{ij}(\boldsymbol{R}), \sum_j \tilde{w}_{ij} x_{ij}(\boldsymbol{R})).$$

Along with the results surrounding $\mathrm{E}[\sum_i \tilde{w}_{ij} x_{ij}(\boldsymbol{R}^{'})]$ and $\mathrm{E}[\sum_i \tilde{w}_{ij} y_{ij}(\boldsymbol{R})]$, this implies that:

$$\mathrm{E}[\sum_j \tilde{w}_{ij}^{'} y_{ij}(\boldsymbol{R}^{'})(\sum_j \tilde{w}_{ij}^{'} x_{ij}(\boldsymbol{R}^{'}))] > \mathrm{E}[\sum_j \tilde{w}_{ij}^{'} y_{ij}(\boldsymbol{R}^{'})(\sum_j \tilde{w}_{ij}^{'} x_{ij}(\boldsymbol{R}^{'})],$$

And so $\mathrm{E}_{\phi'}[\mathrm{CS}_i] > \mathrm{E}_{\phi}[\mathrm{CS}_i]$. The above analysis holds for all i, so $\mathrm{E}_{\phi'}[\mathrm{CS}] > \mathrm{E}_{\phi}[\mathrm{CS}]$.

Acknowledgements

I would like to thank Alex Teytelboym and Mark Armstrong for their invaluable contributions to this paper. Thank you also to Sanjeev Goyal, Matt Elliott, Meg Meyer and the participants at the Winter Meeting of the Econometric Society. This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] Akbarpour, M., Malladi, S., and Saberi, A. Just a few seeds more: Value of network information for diffusion. Tech. rep., Graduate School of Business, Stanford University, 2020.

[2] Armstrong, M. Competition in two-sided markets. *The RAND Journal of Economics 37*, 3 (2006), 668–691.

[3] Armstrong, M., Vickers, J., and Zhou, J. Prominence and consumer search. *The RAND Journal of Economics 40*, 2 (2009), 209–233.

[4] Armstrong, M., and Zhou, J. Paying for prominence. *The Economic Journal 121*, 556 (2011), F368–F395.

[5] Armstrong, M., and Zhou, J. Consumer Information and the Limits to Competition. Economics Series Working Papers 888, University of Oxford, Department of Economics, Nov. 2019.

[6] Ballester, C., Calvò-Armengol, A., and Zenou, Y. Who's who in networks. wanted: The key player. *Econometrica 74*, 5 (2006), 1403–1417.

[7] Banerjee, A., Chandrasekhar, A. G., Duflo, E., and Jackson, M. O. The diffusion of microfinance. *Science 341*, 6144 (2013).

[8] Belhaj, M., Derian, F., and Saf, S. Targeting in networks under costly agreements. Tech. rep., Aix-Marseille University, 2020.

[9] Bergemann, D., Brooks, B., and Morris, S. The limits of price discrimination. *American Economic Review 105*, 3 (March 2015), 921–57.

[10] Bimpikis, K., Ehsani, S., and Ilkiliç, R. Cournot competition in networked markets. *Management Science* (2018).

[11] Bloch, F., and Querou, N. Pricing in social networks. *Games and Economic Behavior 80* (2013), 243 – 261.

[12] Bramoullé, Y., Kranton, R., and D'Amours, M. Strategic interaction and networks. *American Economic Review 104*, 3 (March 2014), 898–930.

[13] Charlson, G. Rating the competition: Seller ratings and intra-platform competition. Tech. rep., Cambridge University, 2020.

[14] Chen, Y., and Zhang, J. Signalling by bayesian persuasion and pricing strategy. *The Economic Journal 130* (05 2020), 976–1007.

[15] Chevalier, J. A., and Mayzlin, D. The effect of word of mouth on sales: Online book reviews. *Journal of Marketing Research 43*, 3 (2006), 345–354.

[16] Choi, M., Dai, A. Y., and Kim, K. Consumer search and price competition. *Econometrica 86*, 4 (2018), 1257–1281.

[17] De Francesco, M., and Salvadori, N. Bertrand-edgeworth competition in an almost symmetric oligopoly. *Studies in Microeconomics 1*, 2 (2013), 213–219.

[18] Dinerstein, M., Einav, L., Levin, J., and Sundaresan, N. Consumer price search and platform design in internet commerce. *American Economic Review 108*, 7 (July 2018), 1820–59.

[19] EC. Antitrust: Commission sends statement of objections to amazon for the use of non-public independent seller data and

opens second investigation into its e-commerce business practices. https://ec.europa.eu/commission/presscorner/detail/en/ip$_2$0$_2$077, 2020. *Accessed* : 2021 − 08 − 01.

[20] Elliott, M., and Galeotti, A. The role of networks in antitrust investigations. *Oxford Review of Economic Policy 35*, 4 (12 2019), 614–637.

[21] Fradkin, A., Grewal, E., and Holtz, D. The determinants of online review informativeness: Evidence from field experiments on airbnb. Tech. rep., MIT Sloan School of Management, 2018.

[22] Galeotti, A., and Goyal, S. Influencing the influencers: A theory of strategic diffusion. *The RAND Journal of Economics* (2009).

[23] Galeotti, Andrea, G. B., and Goyal, S. Targeting interventions in networks. *Econometrica* (2020).

[24] Johnson, J. P., and Myatt, D. P. On the simple economics of advertising, marketing, and product design. *American Economic Review 96*, 3 (June 2006), 756–784.

[25] Kamenica, E., and Gentzkow, M. Bayesian persuasion. *American Economic Review 101*, 6 (October 2011), 2590–2615.

[26] Kim, J. B., Albuquerque, P., and Bronnenberg, B. J. Online demand under limited consumer search. *Marketing Science 29*, 6 (2010), 1001–1023.

[27] Leduc, M. V., Jackson, M. O., and Johari, R. Pricing and referrals in diffusion on networks. *Games and Economic Behavior 104* (2017), 568 – 594.

[28] Lewis, T., and Sappington, D. Supplying information to facilitate price discrimination. *International Economic Review 35*, 2 (1994), 309–27.

[29] Li, X. Designing weighted and directed networks under complementarities. Tech. rep., Economics Experimental Lab, Nanjing Audit University, 2019.

[30] Luca, M. Reviews, reputation, and revenue: The case of yelp.com. Harvard Business School Working Papers 12-016, Harvard Business School, 2016.

[31] Milgrom, P., and Segal, I. Envelope theorems for arbitrary choice sets. *Econometrica 70*, 2 (2002), 583–601.

[32] Nambisan, S., and Baron, R. A. On the costs of digital entrepreneurship: Role conflict, stress, and venture performance in digital platform-based ecosystems. *Journal of Business Research* (2019).

[33] Ogut, H., and Tas, B. K. O. The influence of internet customer reviews on the online sales and prices in hotel industry. *The Service Industries Journal 32*, 2 (2012), 197–214.

[34] Ringel, D. M., and Skiera, B. Visualizing asymmetric competition among more than 1,000 products using big search data. *Marketing Science 35*, 3 (2016), 511–534.

[35] Rochet, J.-C., and Tirole, J. Platform Competition in Two-Sided Markets. *Journal of the European Economic Association 1*, 4 (06 2003), 990–1029.

[36] Tan, G., and Zhou, J. The effects of competition and entry in multi-sided markets. Tech. rep., National University of Singapore, 2020.

[37] Vana, P., and Lambrecht, A. The effect of individual online reviews on purchase likelihood. Tech. rep., Tuck School of Business Working Paper No. 3108086., 2020.

[38] Zhu, F., and Liu, Q. Competing with complementors: An empirical look at amazon.com. *Strategic Management Journal 39*, 10 (2018), 2618–2642.