Abstract
This paper examines UK business cycle fluctuations in twenty-two complete cycles since 1871. We catalogue the incidence and typical duration of UK business cycles. The summary statistics of the economy are explored, including the spectral density, sample covariation and cross spectrum of macroeconomic time series. The real exchange rate, total factor productivity, consumption and investment show systematic pro-cyclicality whereas the current account and long-term interest rates show countercyclicality. Real wages are broadly countercyclical *pre-war* and procyclical *post-war*; conversely prices become countercyclical *post-war*. Consumption, investment and the current account show important *leads* over the business cycle and there is some evidence to suggest that real interest rates, real exchange rates and prices *lag* the business cycle. Finally there is some evidence to suggest that post-war fluctuations have displayed greater coherence and reduced volatility.

JEL Classification: E20, E32, and E40.

Keywords: Business cycles; quantitative theory; spectral analysis.

An Examination of UK Business Cycle Fluctuations: 1871-1997[1]

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'There is…no need to qualify [business cycle] observations by restricting them to particular countries or time periods; they appear to be regularities common to all decentralised market economies.'

R E Lucas (1977).

1. Introduction

Business cycles are a familiar phenomenon in market economies.[5] And the study of such fluctuations has become one of the defining characteristics of modern macroeconomics.[6] They can be characterised as involving periodic fluctuations across a range of macroeconomic variables. The aim of this paper is to document the nature of those business cycle fluctuations over the past century and a quarter in the United Kingdom, arguably the best recorded industrialised economy over this long period.[7] We are therefore able to present empirical macroeconomic regularities that comprise some twenty-two complete business cycles.[8]

The analysis of this dataset contributes to business cycle research in two important respects. First, the availability of a long-run of high quality data allows us to examine the robustness through time of many key business cycle facts (see a related exercise for the US by Stock and Watson, 1999). The small number of long-run studies of

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1 Acknowledgements: Versions of this paper have been presented at the Bank of England, University of Cambridge and the Royal Economic Society conference in St Andrews. We are particularly grateful for comments from Sumru Altuğ, Philip Arestis, Laurence Ball, Charles Feinstein, Robin Matthews, Guy Meredith, Anton Muscatelli, Adrian Pagan, Peter Sinclair and Martin Weale. Any views expressed or errors that in the paper are solely the responsibility of the authors. We are grateful for research assistance from Wesley Fogel. This research has been partly funded by Leverhulme Grant No. F/09567/A.

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5 See Moore and Zarnowitz (1986) for a comprehensive statement of their existence.

6 King and Rebelo (1999) provide a valuable overview of this research effort.

7 See Feinstein (1972), Matthews *et al.* (1982) and Mitchell (1988) for the sources of the UK series.

8 A Supplementary Paper including statistical annexes is available on http://www.econ.cam.ac.uk/dae/people/chadha/index.htm.

business cycles have so far tended to focus on only a few key variables, although they have generally considered a cross-section of countries.[9] Our study, which takes a closer look at an individual country, complements those analyses. A second contribution of this paper is to aid in the development of quantitative general equilibrium (GE) macromodels, on which business cycle research has concentrated over the recent past.[10] The paper, therefore, offers a tentative assessment of what key attributes such models might have in order to capture important characteristics of an industrialised small open economy such as the UK.[11]

It has become standard to assess the quantitative success of GE macromodels with respect to their ability to track key moments in post-war quarterly data.[12] This reflects, in large measure, practical necessity since it is only for this period that reliable national accounts data exist for most industrialised economies. However, such an analysis may suffer from potential small sample bias: Cooley and Prescott (1995), amongst others, argue that an arbitrarily short post-war period may not necessarily provide the ideal benchmark for GE models. Basu and Taylor (1999) who emphasise that the openness of an economy should be a crucial issue in the specification of GE models raise a similar concern. In the UK for example, exchange controls were progressively introduced in the immediate post-war period and only finally abolished in 1979.[13] Indeed, much of the post-war period has been characterised by restrictions on international trade and capital flows, a situation that is often contrasted sharply with the period to 1914.[14]

The present analysis of this long-run of data from the perspective of the business cycle is an attempt to assess the concern raised by Cooley and Prescott (*op. cit.*) by simply asking: 'How stable are the stylised facts through time?' The long-run UK data seems to be a good place to start in this exercise. Many economists view the pre-war UK data as being of a high quality and to rank very highly in international

[9] See most notably, Backus and Kehoe (1992) who particularly consider the cyclical and temporal behaviour of the aggregate price level across countries.

[10] See Cooley (1995) for a comprehensive statement of quantitative theory based on business cycle facts.

[11] King and Watson (1996) explore a post-war US dataset to understand better the usefulness of some key extensions to the 'classic' real business cycle model; sticky prices, financial frictions and/or endogenous money. We will also consider in some detail some open-economy aspects.

[12] We use post-war to describe the period since the Second World War, in which there have only been some nine complete cycles.

[13] See Dow (1998) on this point. Cairncross (1973) points to some 'hysteretic' properties in capital controls. He argues that the comprehensive system of exchange controls that evolved during the Second World War became increasingly anachronistic and poorly targeted in the post-war years.

[14] See the overview of developments in capital account openness over the past 150 years in Taylor (1996) and a balanced assessment of the extent of openness through time and excellent description of the layering of global financial and trade integration in Bordo, Eichengreen and Irwin (1999).

comparison (see the discussions in Englund *et al.*. (1992), Backus and Kehoe (*op. cit*) and Sheffrin (1988)).[15]

Our analysis is straightforward and is based on two simple nonparametric tools, the spectral density and auto- and cross-correlation functions, respectively. It has become common practice to examine variate dynamics with the autocorrelation function. However, we also examine the dynamics in the frequency domain using the spectral density function – which represents the Fourier transformation of the autocovariance function. This function examines the unconditional variation in a given variable by frequency. We can also transform the real and imaginary parts of the cross-spectral density in order to analyse both correlation and the lead or lag of the variables with output by frequency.

As Granger (1969), Watson (1993) and King and Watson (1996) demonstrate, the power spectrum of an economic variable provides an insightful first pass through the data; it demonstrates the importance of business cycles, which we take to be cyclical variations in the data lasting between 2 and 8 years, in keeping with much of the literature.[16] Analysis in the frequency domain also leads naturally to the use of the bandpass filter to extract the business cycle component of each time series in our dataset. We employ therefore the bandpass filters recently developed by Baxter and King (1999) and by Christiano and Fitzgerald (1999), as well as the more widely used filter recommended by Hodrick and Prescott (1997). In an appendix, we exposit the construction of the bandpass filters and we also document some of the differences in the obtained cyclical series, as this may be of separate interest to some readers.

We compare the relative variability of each macroeconomic time series and analyse which variables are significantly pro- or countercyclical. An important part of the analysis is what Blackburn and Ravn (1992) refer to as the dynamic stability of the stylised facts. This assessment will provide a useful first pass at whether the concerns of Cooley and Prescott and Basu and Taylor look to be important. We split our data into pre- and post-war samples. Basu and Taylor (*op. cit.*) argue that their full sample (which corresponds fairly closely to ours) should be divided into four separate samples (corresponding to four distinct nominal regimes). And indeed, it would be fairly easy to suggest an even finer classification along these lines. Our aim here, however, is only to document systematic business cycle 'stylised facts'.[17]

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[15] We discuss in Appendix B of the Supplementary Paper the issue of data quality.

[16] As well as providing a pass through the data, the spectral density results provide additional criteria for testing the artificial economy that is generated by GE models. Note also that the results from Table 1 are broadly consistent with this definition.

[17] Actually, we face a practical constraint here. Even if we wanted to analyse shorter time periods our computations, particularly in the frequency domain, would become increasingly unreliable. See Priestley (1981) for some discussion of these practical issues. Basu and Taylor work with a cross-sectional dataset.

A key finding of our analysis is that consumption, investment and the current account have significant leads over output fluctuations at business cycle frequencies. Furthermore, the current account is found to be consistently countercyclical and with important leads over output. Real wages are acyclical over the full sample period but this masks pre-war countercyclicality and post-war procyclicality.[18] Similarly, the countercyclicality of prices would seem to be a post-war phenomenon; prior to 1914 prices seem procyclical.[19]

Nominal and real interest rates tend not to contain any leading information about output in the pre-war period, though real rates are negatively (and robustly) related to output. In the post-war period nominal rates have a significant negative leading relationship over output. Measured total factor productivity has significant explanatory power over output fluctuations at all frequencies.[20] Overall, the conformity in variables over business cycles has increased markedly in the post-war period, a result that may be closely related to the familiar finding of increased price persistence in most industrialised countries since World War II (see Backus and Kehoe, *op. cit.*).

With respect to the second motivating factor behind this paper - the tentative lessons for GE modelling - our finding that many 'stylised facts' do not appear robust to sample period leaves a number of issues open. This seems to support those who are sceptical as to whether the post-war data do provide reliable guidance when designing macroeconomic models. Whether the differences across our sample periods reflect changes in the structural relationships of the economy, differences in the shocks hitting the economy and the effects of stabilisation policy remain important and largely open questions for future research (see King and Rebelo (*op. cit.*)).

1.1. Related Literature

The two studies closest to ours, in the nature of the dataset analysed, are Englund *et al.* (*op. cit.*) and Backus and Kehoe (*op. cit.*). The first paper in particular is similar to ours in that it examines the business cycle over a long-run of Swedish data and employs similar techniques and uncovers some similar findings. For instance, they find that the variability of most macro-aggregates has probably fallen in the post-war period, although the ranking of the variability of individual series has remained broadly stable. They also point out that for a number of series the finding of high pre-1939 volatility is, at least to some extent, driven by the macroeconomic instability that arguably characterises the inter-war years.[21] The data presented at the end of our

[18] This finding is consistent with Blackburn and Ravn's (1992) examination of UK quarterly data and we note that if we corrected the real wage index to measure real wage per unit of efficiency the procyclicality is likely to be stronger (see Solon *et al.* (1994) on this point).
[19] There is also some evidence of price stickiness, in the sense that both money and prices display similar spectral densities, we return to this point below.
[20] Though note that there is clear lead and lag information for output from TFP.
[21] See Dow (1998) on this point.

paper in Appendix C and Annex A indicates that this finding is echoed in the UK context.

The second paper contains a comparative analysis of business cycles in ten countries. Our analysis adds to this paper in three respects. First, we update the data, ending the sample period in 1997 and analyse a wider range of variables. Second, we estimate the business cycle component of each variable employing more theoretically appealing filters. Third, our paper analyses the correlations between output and the other macroeconomic variables both in the time and the frequency domains.

In terms of documenting the UK business cycle, our paper is complementary to the analyses of Holland and Scott (1998) and Blackburn and Ravn (*op. cit.*). These studies focus on post-war (quarterly) UK data and uncover important regularities, often similar to findings for other industrialised countries (in particular the US). Our post-war findings seem reasonably consistent with these authors'. For instance, we are able to confirm their findings with respect to the volatility of investment and the procyclicality of measures of expenditure. There also appears to be substantial volatility in the current account balance. However, we question the robustness of some of the post-war findings, such as the counter-cyclicality of prices and pro-cyclicality of the real wage. The most robust findings for our dataset involve procyclical real exchange rates, counter-cyclical real interest rates and current account balance, and procyclical expenditure and productivity measures. We also, through the course of the paper, present tentative evidence that the post-war business cycle has become 'compressed' in the sense that although business cycles have remained within their typical frequency of up to 32 quarters,[22] these later business cycle tends to account for a larger portion of a variable's unconditional variance, while the degree of co-movement across macro-aggregates has increased.

The structure of the paper is as follows. Section 2, in conjunction with appendix B, describes the data sources and presents summary statistics relating to the UK business cycle. We analyse whether business cycle frequencies can account for a significant proportion of the (unconditional) variance of our data. Section 3 discusses robustness issues for both the underlying dataset and the filtered data and examines the dynamic interactions between each variable and output. Section 4 extends the frequency domain analysis and focuses on each series' cross-spectrum with output, analysing, by frequency, the significance of the co-movement with output (the coherence), the strength of this relationship (the gain), and whether the relationship is leading or lagging (the phase). Section 5 brings together some of our key results and offers a tentative assessment of what might be the key attributes of a GE macro-model of the UK economy and provides some concluding remarks.

[22] Although expansions and contractions have arguably shortened and become more equal in length.

2. The UK Business Cycle

2.1. The typical duration of business cycles

Business cycles are typically represented in industrialised economies as periodic fluctuations across a range of macro-aggregates with duration of some 8 to 32 quarters. Quantitative measurement of the business cycle (or cyclical time unit) therefore requires the isolation of this component of aggregate fluctuations.[23] But first we establish whether the UK evidence suggests that business cycles can be characterised by a similar duration of periodic fluctuations. Table 1 shows averages for the duration of expansions, contractions and complete UK business cycles since 1871 and clearly shows that business cycle durations are entirely consistent with that definition.[24] The average UK business cycle, from peak to peak or from trough to trough, lasts some 62 months, with a standard error of some 28 months.[25]

Through time there seems to be little significant difference in the duration of the period from peak to peak or from trough to trough. There is some evidence to support the notion that the average cycle, whether measured peak to peak or trough to trough, seems to have become somewhat shorter in the post-war period to some 60 months, particularly compared with the cyclical period prior to 1914 of some 80 months.[26] And over the whole sample period, expansions have lasted a little longer than contractions; 35 to 28 months respectively. However, in the post-war period, expansions and contractions have become similar in average duration, at around 30 months.

There have been many attempts to explore likely differences in business cycle duration across periods. For example, Diebold *et al.* (1993) present a cross-country analysis (which includes the UK) and find that *expansions* in the pre-war period exhibit positive duration dependence, whereas post-war *contractions* show positive duration dependence. The latter finding might help provide some explanation for post-war contractions seeming to be somewhat shorter in the UK. Of course, significant changes in governments' macroeconomic policy in the post-war period may offer a further rationale for the analysis of different sample periods (see, for example, Cooley and Ohanian (1997) who explain low UK growth with reference to the (non) operation of tax smoothing).[27]

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[23] See Neftci (1986) for a provocative introduction to this issue.

[24] The sources of the dates are given in Appendix A.

[25] The resulting period encompassing one standard deviation of the business cycle, 11 to 30 quarters, corresponds almost exactly to our chosen frequency for annual data of 2 to 8 years.

[26] Note that our detrending procedures encompass both measures of the business cycle duration.

[27] We are grateful to Charles Feinstein for suggesting that the reduction in volatility may be related to greater diversification in production patterns, an issue we will take up in future research.

2.2. Summary Statistics

Tables 2A-C outline the summary statistics of our basic annual data series. Quantity series are expressed in per capita growth rates (see notes to table). Over our full sample period (1871-1997) four variables have sample standard deviations sufficiently high to render their sample means insignificantly different from zero: investment; real narrow money balances; changes in the real exchange rate and the current account balance as a proportion of GDP. The other variables, except nominal interest rates, seem well characterised by similar first and second moments, with the coefficient of variation in the range of 1.4-3.4. Nominal interest rates do appear particularly 'sticky' with less than half the variability, as measured by the coefficient of variation, of the next least variable aggregate, broad money.

We might also consider whether the basic data provide any clues as to the likelihood of finding different numbers of cycles in our dataset. A variable with a low mean relative to its standard deviation (see column four in Table 2) is likely to have more (negative) turning points.[28] Assuming business cycle symmetry, the probability of reaching a peak or trough in the growth rate of per capita output in any one year is 0.38. On the basis of these crude statistics there appears to be considerable cyclical congruence in the aggregate data. Apart from the variables already identified as displaying extremities of variability, the data lie in a relatively narrow range of turning point probability of 23-38%. We might therefore expect, on the basis of the sample variability of aggregate data that business cycles operate on a five to eight year frequency,[29] which is, of course, broadly what was found in section 2.1.

Over the full and pre-war period, there is evidence of significant skewness and/or kurtosis in all the variables except real broad money balances. An indication of asymmetric behaviour is given by the evidence of significant skewness: output, real long rates, real wages and the current account balance are negatively skewed and investment, narrow money, broad money, short and long nominal rates are positively skewed. The absence of persistent macroeconomic behaviour at turning points is captured by the kurtosis: no variable displays platykurtic behaviour and investment appears particularly inclined to spend considerable time near its expected value.[30]

There are some differences between the pre- and post-war samples but insufficient to threaten our view of a conformable set of macroeconomic variables consistently

[28] Consider a variable following a Wiener process, the area under the standard normal distribution, $\Phi(-\mu/\sigma)$, gives the probability of a negative turning point. See Harding and Pagan (1998) on this point. Note also that the number in parentheses is the p-value associated with a negative draw in any one year, we assume normality but given the statistics reported in columns four and five this may be a little misleading.

[29] Given that a full business cycle requires two turning points.

[30] The development of GE models that generate skewness in macroeconomic variables and, relatedly, kurtotic behaviour has yet to be fully considered.

oscillating at frequencies of 2-8 years. For example, the post-war mean per capita growth rate of output, consumption and investment is higher than pre-war, and while the variability of the first two series is lower, investment has become more variable in the post-war period. The mean growth rate of total factor productivity (TFP) has increased in the post-war period and is, as expected, similar to that of output, but somewhat less variable. Mean nominal interest rates are both substantially higher and more variable in the post-war period, while the growth in prices is higher but *less* variable, as is the case for real wages. The current account has been insignificantly negative on average over the post-war period (around –2.4%, compared with +18% in the pre-war period).[31] It has also been less variable post-war (14.6 compared with 18.8), although the current account has generally been more volatile than other variables. The next most volatile series (in both samples) are investment and the real exchange rate (with standard deviations of around 9 in the pre-war sample and 13 and 11, respectively, in the post-war period). A corollary is that there is a somewhat lower probability of turning points in most of the individual variables over the post-war period.[32] Only nominal interest rates, real short rates and the current account display a higher probability of turning points after WWII.

2.3. Are Business Cycles Important? The Power Spectrum

The data presented so far need not indicate that the business cycle is quantitatively important in terms of explaining a significant proportion of the variability of a given macro variable. We assess the importance of the business cycle by investigating how much of a variable's unconditional variance is accounted for by variation at the business cycle frequencies. To do this we are required to work with stationary series and so we work with the variables as defined for Table 2. We estimate the standardised power spectrum for each variable $\{y_t\}$ multiplied by π using:

$$(1) \qquad \hat{f}(\varpi_i) = \left(\lambda_0 + 2\sum_{k=1}^{m} \lambda_k R_k \cos(k\varpi_i) \right),$$

where $\varpi_i = i\pi / m$; $i = 0,1,2\ldots m$; m is the window size of 24 and R_k is the autocorrelation coefficient of order k. We employ each of the Barlett, Tukey and Parzen representations of the lag window. Asymptotic standard errors are calculated according to:

$$(2) \qquad \hat{f}(\varpi_i)_{SE} = \sqrt{\frac{x}{v}}\, \hat{f}(\varpi_i),$$

[31] Generating the long-run pattern of current account balances in the UK has proved a particular problem, see Bergin and Sheffrin (2000) and Chadha *et al.* (2000).
[32] The change in the sample estimate of turning points is not large enough for us to suggest that cycles should have become longer.

with x=2 for $i=1,2,\ldots m-1$ and x=4 for $i=0,m$ and where $v=2n\Big/\sum_{k=-m}^{m}\left(\lambda_k^2\right)$. See Priestley (*op. cit.*) for further details. Having estimated the power spectrum we can then isolate the frequencies normally considered as representing the business cycle and calculate what proportion of the variables' unconditional variance lies in those frequencies.

2.3.1 The Full Sample

Table 3A shows that the trend frequency (a measure of persistence corresponding to zero cycles per year in the table) provides significant information on the unconditional variance for short and long nominal interest rates – a point estimate of 49.6% and 54.4%, respectively. The trend component explains between 20% and 30% of the unconditional variance of narrow and broad money, the price level and the current account, whereas only 10% or less of the unconditional variance of the other variables can be explained by their trend components. The frequencies between the trend component and 8 years explain some 30% of the unconditional variance of nominal interest rates, nominal money, prices, the current account and real interest rates. Most of the variation of real macro variables is found in the business cycle frequencies of 2-8 years: output (47.0%), consumption (55.8%), investment (61.9%), real narrow money balances (50.7%), real broad money balances (57.0%). A similar story can be told for relative prices; real short interest rates (46.5%), real long rates (48.8%), the real exchange rate (47.0%) and real wages (58.2%). Only for the real exchange rate (47.3%) and TFP (50.3%) is the modal unconditional variance found in the frequencies higher than 2 years, though the variance of real quantities can also largely be attributed to these frequencies as well.

Turning to the peak frequency estimates, over the full sample we find that nominal rates seem to operate on a slower burn that the other variables – all of which, apart from investment (1.2) and M3 (10.7) have peaks within the classic business cycle of 18 months to 32 quarters. How do we interpret these results? Recall first that, although the power of a series may be within the business cycle frequencies, other frequencies may also have significant influences on the endogenous variables that constitute the business cycle.[33] What does emerge, though, is that there is some dislocation between important frequencies in this set of macroeconomic variables. Both real quantities (other than the current account and TFP) and relative prices display similar spectra at the higher frequencies but nominal magnitudes in the money market consistently display greater variation at longer horizons. The finding for nominal magnitudes carries equally for money as well as the aggregate price level and so constitutes some evidence against price stickiness.[34]

[33] In other words we are not necessarily inferring any causality between variables with similar spectral densities.

[34] Where price stickiness would imply lower frequency peaks for money compared with the price level.

2.3.2 The Sub-samples

Again the differences between the pre- and post-war periods are revealing. They show that, in the post-war period, output and consumption have become more volatile at frequencies higher than eight years, whereas investment variability has become somewhat lower at these same frequencies. Real narrow money and the current account balance also display more variability at these higher frequencies: the other series have become less volatile at these frequencies in the post-war period. Over the post-war period nominal money and prices display substantially higher power at the zero frequency, the frequency domain analogue to the finding by, amongst others, Backus and Kehoe (*op. cit.*) of increased price persistence. There is some increase in the power of TFP at lower frequencies post-war but the series is still dominated by the higher frequencies. The power spectrum for real interest rates has shifted somewhat towards the lower frequencies of less than 8 years.

Over the pre-war sample the peak frequencies in narrow money and prices correspond to around four years, implying that there is little evidence of price stickiness. But if we compare the pre and post-war periods, some important changes occur. We find a significant increase in the persistence (lower frequency variability) of TFP, real wages, nominal money and real quantities but only a moderate increase in price persistence. That the increase in price persistence seems unrelated to price stickiness is an important observation. We return to this point below.

3. Business Cycle Coherence

3.1. Filtering the Data

Quantitative macroeconomic models are often constructed to capture key aspects of business cycle fluctuations. We have mapped the extent to which the business cycle impulse provides an important dimension to aggregate UK fluctuations since 1871. The qualitative measurement of business cycles, the summary statistics and the spectral density functions discussed in Section 2 all point to important fluctuations at this horizon. We now, therefore, analyse how the cyclical components of these variables interact contemporaneously with one another and with output (Tables 4A-C), as well as with output at (four) leads and lags (Tables 5A-C).[35] We use two recently developed versions of the bandpass filter, a natural compliment to our frequency-domain characterisation of the data, and arguably more theoretically appealing than the HP filter. In Appendix C we review in detail the construction of these filters.

First, we use the Baxter-King bandpass filter. This is a two-sided symmetric filter where the lag/lead length needs to be chosen. As Baxter and King note there is a trade-off here, since a longer lag length approximates the optimal filter more closely,

[35] Tables 4 and 5 present the results for the Baxter-King bandpass filter.

while it shortens the cyclical series obtained. We experimented with a number of lag/lead lengths and found that the results were virtually identical after a lag/lead of 6 years. With respect to the Christiano and Fitzgerald filter we use their recommended filter which assumes the underlying data is well-characterised by a pure random walk. We have also extracted the cyclical components of the variables using the Hodrick-Prescott filter with the smoothing parameter set to 7 and 100 (see Harvey and Jaeger (1993)) and present the results for the latter case only in Appendix C, Tables C1 and C2.[36] Annex A plots all the filtered data, against our reference cycles, with peak to trough periods marked by shaded areas.

We find no significant differences in the results between variables using any of the three filters nor for the contemporaneous correlation with output either in the full sample or the period 1956-1990,[37] we therefore only present the results for the Baxter-King filter throughout the paper. Three related points emerge: (i) the strength of the business cycle associations increases substantially in the post-war period, (ii) expenditure components, narrow money, real interest rates, real wages, prices and TFP exhibit considerably less overall variability post-war, (iii) the variability in broad money, the real exchange rate, nominal interest rates and the current account balance does not decrease in the post-war period. Finally, the quantitative measure of the business cycle used for the rest of the paper - the filtered output per head series - returns in each case a high Pearson contingency coefficient with the qualitative business cycle dates given in Appendix A: each filter reports a statistic in excess of 85.[38]

3.2 Business Cycle Facts

3.2.1 The correllogram

Tables 4A-C present the correllogram of the sixteen filtered variables over the full, pre- and post-war samples, respectively. We find consistent correlations across samples between output and investment (positive), long real rate (negative), real exchange rate (positive), current account (negative) and TFP (positive). Table 4B shows that nominal and real long interest rates, and the current account balance are significantly countercyclical, whereas the real exchange rate is procyclical.[39] We note that the standard deviation of the current account is higher in the post-war period at business cycle frequencies than in the pre-war period, in contrast to the results for the

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[36] The results with lambda set to 7 are available on request, as are all other results using the alternative filters.

[37] Note though that for the full sample the HP filter reports (mildly) countercyclical broad nominal money and real money balances and (mildly) procyclical short rates and prices, whereas the two bandpass filters report precisely the opposite. However, these point estimates are insignificantly different from zero and there is no disagreement in the post-war sample.

[38] The χ^2 statistic is corrected to lie between 0 and 100.

[39] Figures 12A-C suggest that the insignificant real exchange rate correlation, post-war results from high volatility in the bilateral real exchange rate that we use.

pre-filtered data. Importantly, consumption (positive), prices (negative) and real wages (negative) appear contemporaneously insignificantly correlated with output. Importantly, if we end our pre-war sample in 1913 (thus excluding the volatile inter-war years) we find that consumption is now significantly pro-cyclical, however prices and wages remain insignificantly related to output. The situation changes drastically in the post-war period; consumption is significantly procyclical, as are real narrow and broad money and real short rates and real wages. Even though we confirm the findings of Blackburn and Ravn (*op. cit.*) and King and Watson (*op. cit.*) in respect of a counter-cyclical (long) real rate, we note an important procyclical short real rate in the post-war period. Prices are now strongly countercyclical (see Chadha and Prasad (1994), as is the current account.

The current account balance is more strongly related with other macroeconomic variables over the post-war period than in the pre-war period, whereas the position is the reverse for the real exchange rate. Consumption (column three in Table 4C) is significantly related to a larger number of macroeconomic variables than is output – this finding may be consistent with the observation that there is an important temporary component to output fluctuations.

3.2.2. The Dynamics

We examine the dynamic interrelations of the filtered variables with output (Tables 5A-C) and again we find important distinctions between the two 'benchmark' sample periods. Consumption consistently and significantly leads output in both samples, although in the post-war period the relationship is significantly positive contemporaneously. Investment leads output significantly in the pre-war sample (output shows no leading properties over investment). In contrast, in the post-war period there is no significant investment relationship, in either direction. The volatility of investment is a well-known stylised fact but why the post-war reduction in business cycle volatility (and corresponding increase in trend volatility) should reduce the business cycle coherence presents something of a puzzle.

Nominal and real money balances show consistently significant leads and lags with output, with arguably stronger evidence of leads, although over the post-war period broad nominal money balances do not significantly lag output. Nominal interest rates tell a similar story but in the full sample real interest rates show considerably less evidence of leading output. There appears to be an important break in the behaviour of real wages post-war. Whereas real wages are approximately acyclical or even countercyclical in the pre-war period, with a significant lead over output (Table 5B(1)), they become strongly procyclical in the post-war era, with lead and lag information for output, a change that is fairly clear upon visual inspection of the data (see Figures 1A-C in Annex A). The dynamic correlations suggest that the interactions between rising output and rising wages became substantially stronger and more persistence post-war. TFP is robustly positively related to output across all samples and is found to have significant lead and lag relationships – we return to this point in Section 4.

Turning to the open economy variables we see that in the earlier sample the real exchange rate showed significant leads over output. In the post-war period there seems to be little significant link between output and the real exchange rate. Figures 12A, B and C suggest that much of the reason for this finding lies in the notably more volatile time path for the exchange rate in the post-war period. The relationship between output and the current account balance is contemporaneously negative in the post-war period, and turns positive with the current account leading output. In the pre-war period the contemporaneous correlation was also negative as was the leading relationship (although it was marginally less significant). The countercyclicality of the external balance and its robust lead for output is consistent with recent findings in the theoretical literature (see, for example, Baxter and Crucini (1993) and Chadha *et al.* (2000)).

4. Cross-Spectral Analysis

A natural extension of the analysis in the previous sections is to ask whether the significant time series correlations we uncover remain significant when we examine the correlations among the raw data in the frequency domain. We might reasonably expect this to be the case since we established the conformity among many of our filtered series. And indeed, by and large, that is what we find. In Section 2 we established that business cycles were important in that they capture an (increasingly) important portion of many of our variables' unconditional variance. The cross-spectra indicate the importance of the relationship between each variable and output across *all* frequencies. In fact the cross-spectral analysis extends the previous analysis in two specific ways. First, it provides new evidence on the significance level of our business cycle correlations, and second, it provides additional evidence on the correlations in the data (specifically, correlations in the data that we removed for the purposes of Tables 4 and 5).

4.1. Cross-Spectrum

We can define the cross-spectrum between two series as:

(3) $$\varpi_{12}(\alpha) = \sum_{s=-\infty}^{\infty} \rho_{(12)s} e^{i\alpha s},$$

with the corresponding integrated spectral function $W(\alpha)$ defined over the range 0 to π. Solving for the cross correlation we find that

(4) $$\rho_{(12)s} = \frac{1}{\pi}\int_{-\pi}^{\pi} \varpi_{12}(\alpha)e^{-i\alpha s}\partial\alpha.$$

In a univariate setting the sin terms cancel as $\rho_k = \rho_{-k}$ and the spectral density is real. But we now have:

(5) $$\varpi_{12}(\alpha) = 1 + \sum_{1}^{\infty}\{\rho_{(12)s}\cos s\alpha + \rho_{(12)-s}\cos s\alpha\} + i\left\{\sum_{1}^{\infty}\rho_{(12)s}\sin s\alpha + \rho_{(12)-s}\sin s\alpha\right\}$$

From (5) we can see that the cross-spectra has an imaginary and real component. The first two terms on the right hand side, $c(\alpha)$, are the co-spectra and the final term, $q(\alpha)$, is the spectral density. The sum of the squares of these two terms is the amplitude, which when standardised by the spectral densities of each separate series, $\varpi_i(\alpha)$, is called the coherence:

(6) $$C(\alpha) = \frac{c^2(\alpha) + q^2(\alpha)}{\varpi_1(\alpha)\varpi_2(\alpha)}$$

The coherence measures the degree to which the series vary together and can be thought of as the squared correlation coefficient. The gain diagram plots ordinate $R_{12}^2(\alpha)$ against α as abscissa, where

(7) $$R_{12}^2(\alpha) = \frac{\varpi_1(\alpha)}{\varpi_2(\alpha)}C(\alpha).$$

The gain is analogous to a regression coefficient. Finally, the phase diagram plots $\psi(\alpha)$ as against α as abscissa, with the phase measuring the lead or lag in the relationship at each frequency:

(8) $$\psi(\alpha) = \arctan\frac{q(\alpha)}{c(\alpha)}.$$

Table 6 tabulates the interpretation of the Cross-Spectra charts (the Annex presents a small selection) over the full sample. This analysis provides a useful representation of the association found in macroeconomic aggregates with the reference cycle. Figure 1 from the Annex illustrates the high cross-spectra of consumption with output across the frequency range but also suggests important consumption leads up to around 4 years in frequency after which output leads consumption. Figure 2 shows very clearly why TFP shocks perform an admirable role in explaining the business cycle in the modern literature. Figure 3 demonstrates both the long-run neutrality of money, with both coherence and gain heading for 1 at the trend frequency, and the existence of some price stickiness, with money leading to some extent prices at the business cycle frequencies of 1-3 years.

In general, though, real quantities display significant relationships at business cycle frequencies with some evidence of output lagging consumption, investment and the current account balance, and of output leading real M3. Relative prices display similar coherences but real rates tend to lead output, whereas real wages lag output. Nominal money balances, short rates and prices each display important business cycle relationships but essentially with output leading. Long rates display some contemporaneous relationship at the trend frequency only. There is considerable evidence of conformity at business cycles but there is tentative evidence of some difference in phase between nominal and real magnitudes. Specifically, nominal financial indicators tend to lag output, but real rates and real quantities display some lead over output.

The two sets of results seem to imply that variables that display more fluctuations at short horizons – real quantities and relative prices - may have important leading implications for output at that frequency. Variables characterised by fluctuations at longer horizons – nominal interest rates and narrow and broad money - are more likely to respond to output developments at that frequency. Real narrow money, long nominal rates and the real exchange rate only move contemporaneously with output. One possible interpretation is that shocks most closely related to innovations in consumption, investment, current account positions and real rates are more likely to drive the cyclical frequencies in output than shocks related to nominal monetary phenomena, which themselves seem largely driven by the output fluctuations.

The cross-spectral results over our sample confirm much of what we said in our analysis of the dynamic correlations in the data. A few results stand out. The changing profile of real wages shows up very clearly, with the phase flattening off sharply in the post-war period (consistent with a high contemporaneous correlation with output), but otherwise there is little obvious lead-lag (by frequency) relationship with output. This appears a significant relationship as indicated by both the coherence and the gain. In the pre-war sample the real exchange rate leads output significantly, but this relationship appears unstable, with output leading over the post-war sample, although the coherence indicates that the relationship may not be significant (which is what the Ljung-Box statistic indicates).

5. Concluding Remarks

This paper presents a benchmark description of the UK economy. We have analysed a range of data and performed a number of summary statistical analyses. The UK business cycle accounts for a large, and perhaps increasing, portion of the unconditional variation in the growth rate of many important macroeconomic variables. There appears to be a pronounced shift in the behaviour of business cycles in both conformity (higher) and amplitude (lower) since WW2 – our ideal model would provide an explanation in terms of changes in model parameters, technology and the constellation of shocks.[40] More specifically, we find that the strength of the business cycle associations increases substantially post-war, while the expenditure components, narrow money, real interest rates, real wages, prices and TFP exhibit considerably less overall variability post-war. However, that is not the case for broad money, the real exchange rate, nominal interest rates and the current account.

We find evidence of some dislocation at important frequencies in this set of macroeconomic variables. Both real quantities (other than the current account and TFP) and relative prices display similar spectra at the higher frequencies. In contrast, nominal magnitudes, money as well as the aggregate price level, consistently display greater variation at longer horizons. This may constitute some evidence against price stickiness,[41] suggesting that increasing post-war price persistence is *not* accompanied by great price stickiness. Consumption is found to be only marginally smoother than output – a result in the flavour of liquidity constraints or precautionary savings models.[42]

Models of sticky wages imply, in the absence of offsetting changes in the mark-up, that real wages ought to move counter-cyclically (Christiano and Eichenbaum (1992)). Over the full sample period we find some mild evidence for such counter-cyclicality – although the literature normally states categorically that real wages are pro-cyclical.[43] Output is positive in the real exchange rate and negative in the current account balance.[44] The current account balance leads output at long business cycles and at trend frequencies, but the real exchange rate shows little lead or lag relationship with output.

Overall, the results in this paper seem to imply that variables that display more fluctuations at short horizons – real quantities and relative prices - may have important leading implications for output at that frequency. Variables characterised by fluctuations at longer horizons – nominal interest rates as well as narrow and broad money - are more likely to respond to output developments at that frequency. Real narrow money, long nominal rates or the real exchange rate only tend to move contemporaneously with output.

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40 Rotemberg and Woodford (1996) place this particular concern about the current generation of models high in their priorities for future research.

41 Where price stickiness would imply lower frequency peaks for money compared with the price level.

42 King and Watson (*op. cit.*) find that output is 2.7 times as variable as consumption our ratio is 1.2: essentially consumption is twice as volatile in the UK. Employing unpublished series on non-durable consumption, kindly made available by Nicholas Dimsdale, does not substantially alter these ratios.

43 Blackburn and Ravn write 'the pro-cyclicality of the real wage is now an established stylised fact' (p388).

44 That the real exchange rate and output are positively correlated tells us that there are important productivity shocks to traded goods. In an inter-temporal current account model a permanent country-specific shock is likely to cause the current account to deteriorate because investment increases and current consumption increases by more than current output.

The results in this paper present researchers with a rich avenue of directions. What relationship is there between increased price persistence and greater business cycle coherence in macroeconomics aggregates? Can the findings of consumption volatility be reconciled with the robust countercyclicality of the current account? Can we accept at face value the seemingly unimportant systematic effects of money and nominal interest rate on the business cycle? Our future agenda will address these issues in turn.

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TABLES, FIGURE AND ANNEX

TABLE 1: Average Length in months of UK Business Cycles: (from 1871 onwards)

Average, all cycles:	Expansion	Contraction	Peak to Peak	Trough to Trough
1871-1997	34.67	28.23	61.48	62.81
1871-1913	44.57	35.14	79.71	78.43
1946-1997	29.5	29.67	60.75	55.63
Average peacetime cycles:				
1871-1913	44.57	35.14	79.71	78.43
1919-1939	26.2	20.4	45.4	46.6
1946-1997	29.5	29.67	60.75	55.63
Standard error, all cycles:				
1871-1997	17.90	18.54	28.80	27.80
1871-1913	14.57	24.94	32.93	33.47
1946-1997	14.23	15.61	26.54	10.68
Standard error, peacetime cycles:				
1871-1913	14.57	24.94	32.93	33.47
1919-1939	23.74	9.76	20.94	32.75
1946-1997	14.23	15.61	26.54	10.68

Source: Artis *et al*. (1995), Dow (1998) and Moore and Zarnowitz (1986).

TABLE 2: Summary Statistics of per capita growth rates 1871-1997[(a)]

	μ	σ	μ/σ	Skewness	Kurtosis
Output	1.155**	3.901	0.296 (0.384)	-1.023**	5.682**†
Consumption	1.096**	3.126	0.351 (0.363)	-0.069	6.051**‡
Investment	1.730	12.514	0.138(0.445)	2.548**‡	22.430**‡
M0	3.464**	5.491	0.631 (0.264)	0.883**‡	1.526**‡
M3	4.669**	6.383	0.731 (0.232)	0.811**‡	0.446‡
Real M0	0.549	5.674	0.097 (0.461)	0.356†	1.511**‡
Real M3	1.754**	5.594	0.314 (0.377)	0.205	0.096‡
Short	4.513**	3.488	1.294 (0.098)	1.257**	1.000*‡
Long	5.203**	3.159	1.647 (0.050)	1.344**‡	0.839‡
Real Short	1.600**	5.430	0.295 (0.384)	0.077‡	4.668**‡
Real Long	2.183**	4.278	0.510 (0.305)	-0.472*‡	3.392**‡
RER	-1.801	9.991	-0.180 (0.429)	-0.393‡	4.351**‡
Real wages	1.584**	3.100	0.511 (0.305)	-0.496*†	1.128*‡
Price level	2.915**	6.196	0.470 (0.319)	0.375	2.381**‡
TFP[d]	0.975**	3.320	0.294 (0.386)	-0.900**	4.581**
CA	6.244	24.582	0.254 (0.401)	-0.747**‡	1.158**‡

Notes: a) we use levels data for interest rates and current account balance relative to output; b) the numbers in parentheses in column four denote the probability of a negative draw ; c) * and ** indicates significant difference from zero at 5% and 1% respectively; † and ‡ indicate significance of the variable from zero at 5% and 1% level of the band pass filtered data (to be discussed later); d) .per employee.

TABLE 3: Power Spectrum - full sample

Variable	Cycles Per Year 0	<0.136	<0.5	<1	1	Durbin's Peak Freq.
Output	4.248 (2.140)	8.726	47.027	35.292	4.708 (2.373)	0.589 (5.334)
Cons.	7.278 (3.668)	8.425	55.768	25.801	2.729 (1.375)	1.669 (1.882)
Invest.	2.390 (1.204)	9.075	61.933	23.593	3.010 (1.517)	1.227 (2.560)
M0	16.251 (8.189)	32.347	41.552	9.293	0.557 (0.281)	0.589 (5.334)
M3	27.037 (13.63)	32.552	35.605	4.295	0.510 (0.257)	0.295 (10.668)
Real M0	6.068 (3.058)	9.857	50.672	31.433	1.970 (0.993)	1.669 (1.882)
Real M3	6.818 (3.436)	14.916	56.950	19.714	1.603 (0.808)	0.884 (3.555)
Short	49.555 (24.875)	35.649	12.427	2.255	0.115 (0.058)	0.147 (21.328)
Long	54.374 (27.294)	36.416	8.004	1.130	0.077 (0.039)	0.147 (21.328)
Real Short	8.984 (4.510)	27.197	46.536	16.481	0.803 (0.403)	0.884 (3.555)
Real Long	11.281 (5.663)	37.003	48.793	2.696	0.227 (0.114)	0.736 (4.267)
RER	1.192 (0.601)	4.501	47.032	45.495	1.780 (0.897)	0.589 (5.334)
Real wages	7.200 (3.629)	10.278	58.169	23.744	0.608 (0.307)	1.129 (2.783)
Prices	21.120 (10.644)	28.278	38.175	11.882	0.545 (0.275)	0.736 (4.267)
TFP	5.922 (2.985)	5.837	36.868	45.783	5.589 (2.817)	0.0982 (31.99)
CA	28.834 (14.474)	34.455	29.579	6.379	0.754 (0.378)	0.589 (5.334)

Notes: a) the power spectra are estimated for each of the Bartlett, Tukey and Parzen approximations; b) we show the results for the Bartlett, the others are available on request but no significant differences were found; c) the figures in parentheses in columns one and five are the asymptotic standard errors in percentages of the point estimates for the trend and Nyquist frequency respectively, d) Durbin's (1969) test defines the peak frequency as the largest gap between the estimated power spectra and hypothetical white noise, e) the second figures in each row of the final column corresponds to the peak frequency in years.

Table 4 : Correlation Matrix

	Output[a]	Output[b]	Output[c]	Output[d]
Output	2.23	2.72	2.19	1.41
Cons.	0.16	0.07	0.49**	0.56**
Invest.	0.23**	0.24	-0.02	0.11
M0	0.01	-0.02	-0.11	0.18
M3	-0.00	-0.12	0.05	0.25
Real M0	0.09	-0.01	-0.17	0.57**
Real M3	0.09	-0.11	-0.15	0.54**
Short	-0.04	-0.23	0.20	0.15
Long	-0.22**	-0.52**	0.09	-0.27
Real Sh.	-0.13	-0.23	-0.23	0.47**
Real Lo.	-0.47**	-0.46**	-0.35**	-0.64**
RER	0.20**	0.31*	-0.27	0.01
Wages	-0.05	-0.18	-0.01	0.39**
Prices	-0.11	-0.00	0.17	-0.67**
CA	-0.24**	-0.27*	-0.07	-0.45**
TFP	0.90**	0.92**	0.91**	0.83**

[a]Full Sample Correlation Matrix; [b]Pre-War Correlation Matrix 1871-1939; [c]Pre-War Correlation Matrix 1871-1913; [d]Postwar Correlation Matrix. * indicates significant at 5% and ** at 1%, using a Student's t-distribution where $t = r\sqrt{n-2}/\sqrt{1-r^2}$, with *r* is the sample correlation coefficient and *n* the number of observations.

TABLE 5: Leads and Lags Correllogram with Output: Full Sample

Variable	Y_{t-4}	Y_{t-3}	Y_{t-2}	Y_{t-1}	Y_t	Y_{t+1}	Y_{t+2}	Y_{t+3}	Y_{t+4}	Ljung-Box Q Statistic		
										(-4,-1)	(1,4)	(-4,4)
Cons.	0.059	-0.105	-0.035	0.115	0.158	-0.369	-0.240	0.141	0.349	3.45	40.09***	46.45***
Invest.	-0.086	-0.137	-0.077	0.032	0.226	-0.110	-0.207	-0.021	0.196	3.99	11.26**	21.25**
M0	-0.065	0.046	0.164	0.364	0.012	-0.366	-0.395	0.055	0.407	19.60***	54.84***	74.46***
M3	-0.024	0.087	0.203	0.237	-0.005	-0.351	-0.293	-0.045	0.220	12.54**	30.91***	43.46***
Real M0	0.169	0.001	-0.221	0.153	0.093	-0.054	-0.270	-0.037	0.299	12.02**	20.02***	33.07***
Real M3	0.213	0.036	-0.200	0.087	0.089	-0.068	-0.236	-0.045	0.202	11.30**	12.38**	24.59***
Nom Sh.	-0.218	0.104	0.256	0.269	-0.043	-0.319	-0.143	0.163	0.166	23.39***	20.98***	44.58***
Nom Lo.	-0.141	0.041	0.295	0.170	-0.220	-0.291	-0.049	0.201	0.227	16.34***	21.39***	43.40***
Real Sh.	0.207	0.296	-0.188	-0.135	-0.127	0.119	0.066	-0.048	-0.088	22.11***	3.40	27.40***
Real Lo.	-0.080	0.395	0.315	-0.081	-0.470	-0.136	0.134	0.196	-0.055	32.13***	9.52**	67.53***
RER	0.002	-0.159	-0.184	0.102	0.204	0.107	-0.124	-0.168	-0.089	8.32*	7.53	20.72**
Real wage	-0.164	-0.031	0.275	0.248	-0.046	-0.289	-0.347	0.022	0.447	19.60***	48.47***	68.32***
Prices	-0.267	0.039	0.421	0.116	-0.108	-0.245	0.005	0.094	-0.032	31.66***	8.28*	41.32***
CA	0.014	0.047	0.070	-0.005	-0.235	-0.060	0.307	-0.05	-0.198	0.89	16.36***	23.73***
TFP	-0.045	-0.228	-0.400	-0.194	0.903	0.137	-0.125	-0.330	-0.246	30.00***	24.49***	49.88***

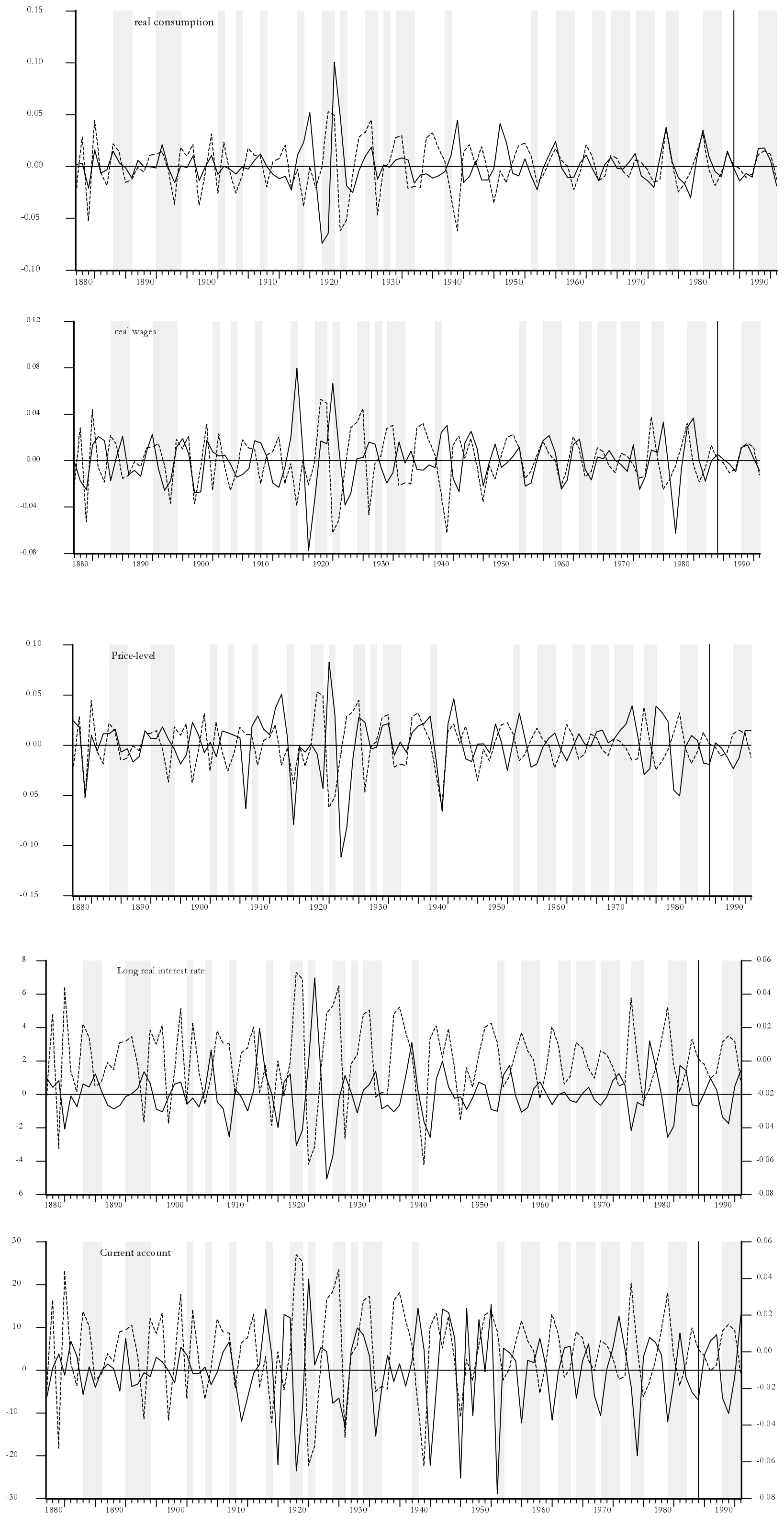
Notes: a) We calculate the correlation coefficient on the band pass filtered data and the b) Ljung-Box statistic is approximately χ^2.

TABLE 6: Cross-Spectra Analysis: Coherence, Gain and Phase with Output

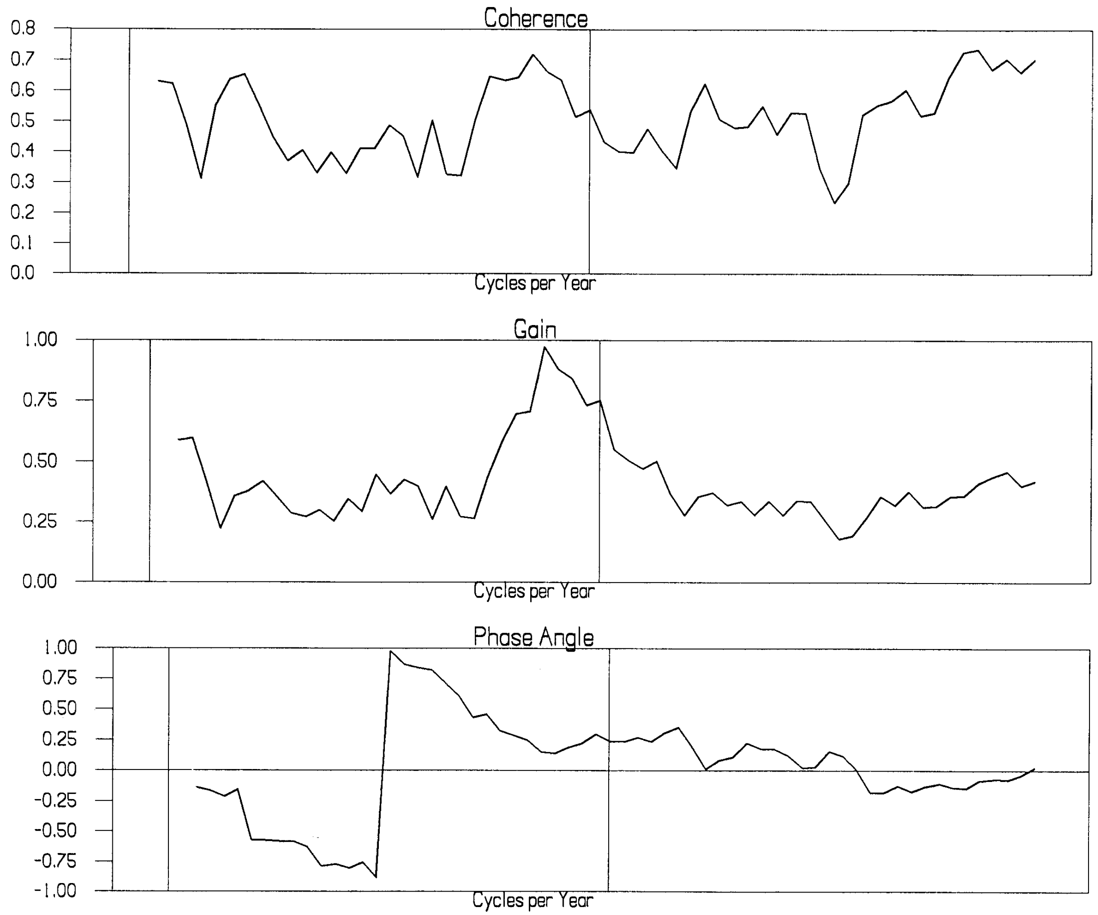
Variable	Frequency Domain
Cons.	Significant coherence across the set of cycles; maximum gain at the shorter end of the business cycle; some evidence of lead consumption at the low frequencies.
Invest.	Significant coherence in the business cycle and high frequencies; maximum gain at short end of business cycle; some evidence of lead investment at low frequencies.
M0	Significant coherences at business cycle frequencies; gain decreasing with frequency; some evidence of lead output over business cycle.
M3	Significant coherences at longer business cycle frequencies; gain decreasing with frequency; some evidence of lead output over business cycle.
Real M0	Significant coherence at short end of business cycle; maximum gain at same frequency; unlagged relationship at the significant frequencies.
Real M3	As real M0 but more evidence of longer horizon business cycle coherence where output tends to lead.
Short	Significant coherences across the business cycle; gain suggests relatively low response and strong evidence of output lead.
Long	Marginal significance at the trend and short business cycle frequencies, strong gain only at the trend; little evidence of any important lags.
Real Short	Significant business cycle coherences; gain falling with frequency; evidence of real short rate lead over business cycle and Nyquist frequency.
Real Long	As short real, arguable clearer evidence of lead relationship from long real rates.
RER	Significant coherences at longer business cycles; gain peaks at around three years; relationship is essentially unlagged over the business cycle.
Wages	Significant coherences at shorter business cycles; gain increase with frequency up to three years; output leads at important frequencies.
Inflation	Significant coherences across the business cycle and at the Nyquist frequency; falling gain with frequency, evidence for lead output relationship over the business cycle.
CA	Significant coherences at 4-5 years with marginal significances at higher frequencies; gain falls rapidly with frequency; CA lead over longer business cycles.
TFP	Highly significant coherences across all business cycle frequencies – contemporaneous cross-spectra throughout.

Note: The three statistics are derived from a polar decomposition of the complex-valued spectral density: a) see Annex A for plots of data; b) the x-axis corresponds to the cycles per year, where the first vertical line represent the trend, the second vertical line two years and the third one year, c) coherence is the squared correlation coefficient at each frequency, d) the gain is the analogous to the regression coefficient and e) the phase is the fraction of the cycle by which one series leads (lags) the other.

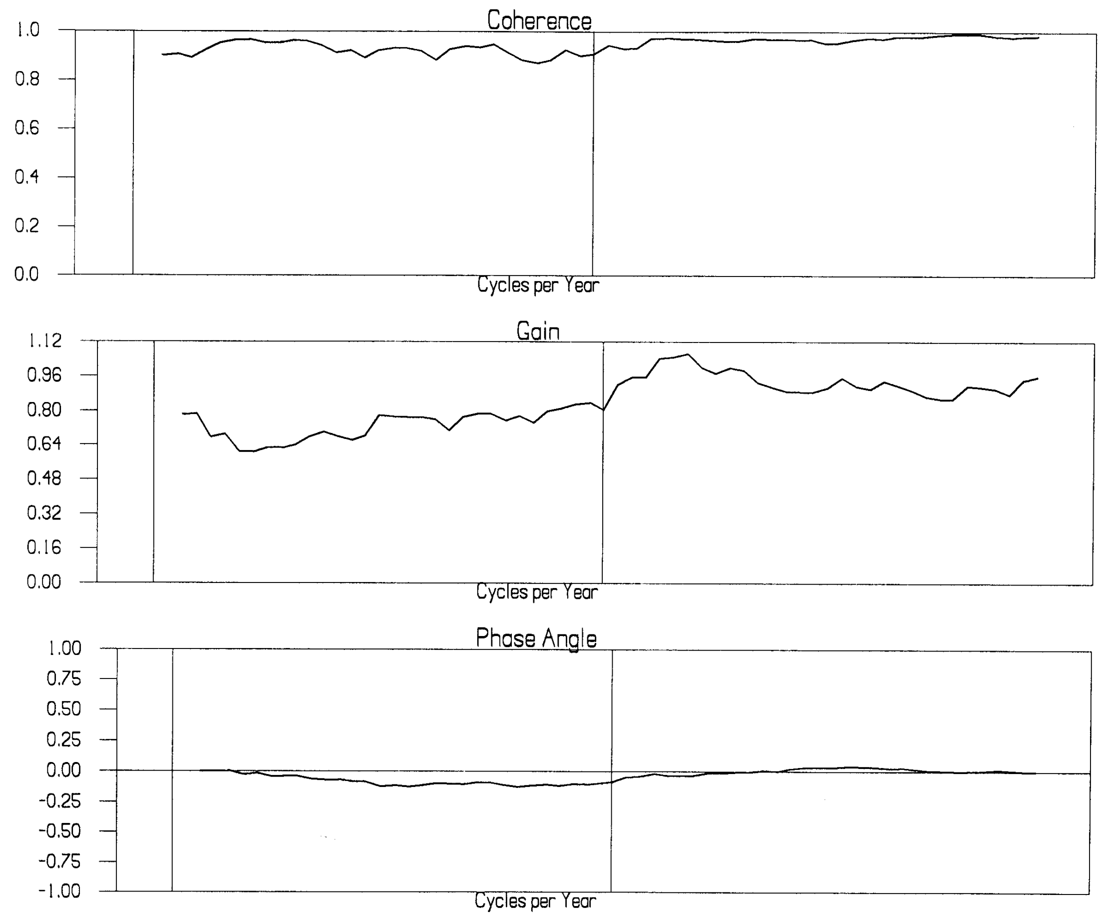
Figure 1 : Baxter-King filtered series



Annex B: Figure 1: Cross-Spectra of Output with Consumption



Annex B: Figure 2: Cross-Spectra of Output with TFP



Annex B: Figure 3: Cross-Spectra of Narrow Money with Prices