Matching Markets and Market Design (Reflections of a designing economist)

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I. **Labor market clearinghouses for doctors in the U.S. and U.K.**
II. Kidney exchange (and repugnant transactions)

Markets are familiar…

I’ll try to expand your idea of what is a marketplace, by telling you about some less familiar markets

Maybe the simplest way to begin is to ask the simplest question…

- What is the role of money in markets?

Commodity markets

Fruit market

NY Stock Exchange

Small trades in large commodity markets can be arms-length and anonymous

- To buy 100 shares of AT&T on the New York Stock Exchange, you don’t need to worry about whether the seller will pick you—**you don’t have to submit an application or engage in any kind of courtship**. Likewise, the seller doesn’t have to pitch himself to you.
- The price does all the work, bringing the two of you together at the price at which supply equals demand. **On the NYSE, the price decides who gets what.**
- The market helps do “price discovery” to find prices that work.

But in many markets prices don’t do all the work

- **Stanford and Cambridge don’t raise tuition until just enough applicants remain to fill the entering class.**
- They keep tuition low enough so that *many* students would like to attend, and then they admit a fraction of those who apply.
- Universities don’t rely on prices alone to equate supply and demand
- **Labor markets and college/university admissions are more than a little like courtship and marriage**: each is a two-sided matching market that involves searching and wooing on both sides.

Matching markets

- *Matching markets* are markets in which **you can't just choose what you want (even if you can afford it), you also have to be chosen.**
- You can't just inform Stanford or Cambridge that you're enrolling, or Google or Facebook that you're showing up for work. You also have to be *admitted* or *hired*. Neither can Stanford simply choose who will come, any more than one spouse can simply choose another: each also has to be *chosen*.

Market design:

- **Medical labor markets in the U.S. and U.K.**
  - Medical Residents: in the U.S.: NRMP in 1995
  - Couples
- **Kidney exchange**
  - Regional networks (2004)
  - National US (2010-?)
  - UK
  - Canada (2010)
- School choice systems:
  - New York City since Sept. 2004 (high schools only)
  - Boston since Sept. 2006
  - Denver, New Orleans—Sept. 2012
  - In discussion with Newark , Philadelphia, DC
- American labor market for new Ph.D. economists
  - Scramble March 2006
  - Signaling December 2007

Four papers…

1. Roth, A.E. "The Evolution of the Labor Market for Medical Interns and Residents: A Case Study in Game Theory", *Journal of Political Economy,* Vol. 92, **1984**, 991-1016.
2. Roth, A.E. "A Natural Experiment in the Organization of Entry Level Labor Markets: Regional Markets for New Physicians and Surgeons in the U.K.", *American Economic Review*, vol. 81, June **1991**, 415-440.
3. Roth, A.E. and E. Peranson, "The Redesign of the Matching Market for American Physicians: Some Engineering Aspects of Economic Design," *American Economic Review*, 89, 4, September, **1999**, 748-780.
4. Kojima, Fuhito, Parag A. Pathak, and Alvin E. Roth, "Matching with Couples: Stability and Incentives in Large Markets," *Quarterly Journal of Economics*, **2013** 128 (4): 1585-1632.

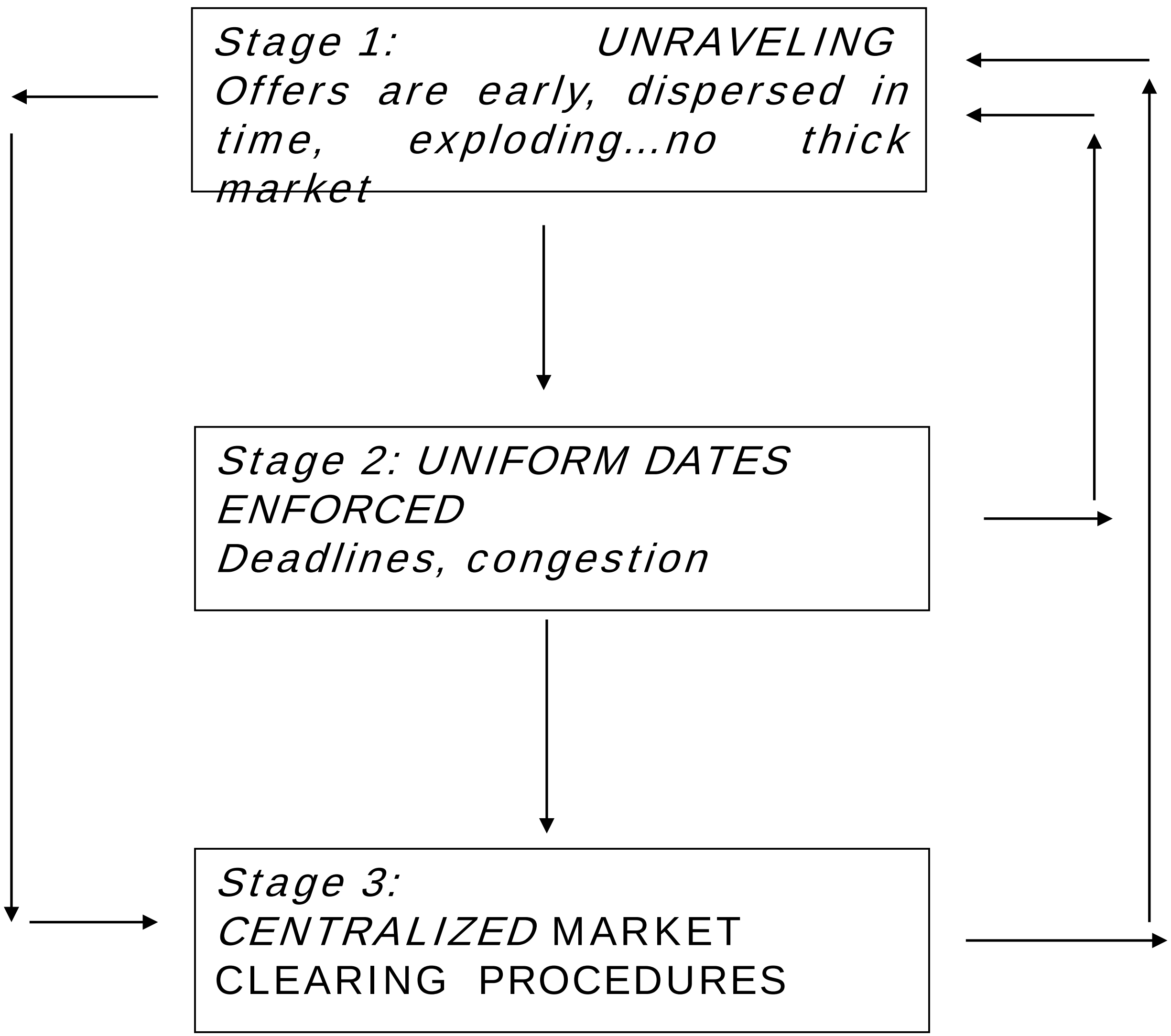
What have we learned from market design?

- To achieve efficient outcomes, marketplaces need make markets sufficiently
  - **Thick**
    - Enough potential transactions available at one time
  - **Uncongested**
    - Enough time for offers to be made, accepted, rejected, transactions carried out…
  - **Safe**
    - Safe to participate, and to reveal relevant information
- Some kinds of transactions are **repugnant**…and this can constrain market design.
  - The repugnance of some markets may give us a new viewpoint on regulation of familiar markets—markets often require a lot of social buy-in to work well.

Background to redesign of the medical clearinghouses

- 1900-1945 UNRAVELLING OF APPOINTMENT DATES
- 1945-1950 CHAOTIC RECONTRACTING--Congestion
- 1950-197x HIGH RATES OF ORDERLY PARTICIPATION (95%) in centralized clearinghouse
- 197x-198x DECLINING RATES OF PARTICIPATION (85%) particularly among the growing number of MARRIED COUPLES
- 1995-98 Market experienced a crisis of confidence with fears of substantial decline in orderly participation;
  - Design effort commissioned—to design and compare alternative matching algorithms capable of handling modern requirements: couples, specialty positions, etc.
  - Roth-Peranson clearinghouse algorithm adopted, and employed

Stages and transitions observed in various markets



How new doctors get their first jobs: the NRMP

In the Fall of their senior year of medical school, students apply to residency programs and go on interviews.

- GME References
- Ensuring Match Integrity
- Match Algorithm
- Impact of ROL Length

DATA AND REPORTS

SCHEDULE OF DATES

POLICIES

- Match Agreements
- Violations Policy
- Waiver Policy
- Case Summaries
- Statement on Professionalism

HOW TO LOG IN

January 15, 2010	**Rank order list entry begins.** Applicants and programs may start entering their rank order lists at 12:00 noon eastern time.
January 31, 2010	**Quota change deadline** Programs must submit final information on quotas and withdrawals by 11:59 p.m. eastern time.
February 24, 2010	**Late registration deadline** **Rank order list certification deadline** Applicants and programs must certify their rank order lists by 9:00 p.m. eastern time. Staff will be available to answer your questions during the final deadline hours. CERTIFIED applicant and program rank order lists and any other information pertinent to the Match must be entered in the R3 System by this date and time. **Withdraw Deadline** Independent applicants who have accepted a position through another national matching plan or by agreement outside the Matching Program must withdraw by 9:00 p.m. eastern time.
March 15, 2010	Applicant matched and unmatched information posted to the Web site at 12:00 noon eastern time.
March 16, 2010	Filled and unfilled results for individual programs posted to the Web site at 11:30 a.m. eastern time. Locations of all unfilled positions are released at 12:00 noon eastern time. Unmatched applicants may begin contacting unfilled programs at 12:00 noon eastern time.
March 18, 2010	Match Day! Match results for applicants are posted to Web site at 1:00 pm eastern time.
March 19, 2010	Hospitals send letters of appointment to matched applicants

"Scramble"

Single students fill out rank order lists of programs that have interviewed them.

Couples fill out rank order lists of *pairs* of programs that have interviewed them.

Married couples looking for two residencies weren't an issue in the medical job market until the 1970's

Table 13.—Women in US Medical Schools

Academic Year*	Women Applicants†	Women in Entering Class	Total Women Enrolled	Graduates
	No. (%)			
1967-1968	1951 (10.4)	934 (9.9)	3003 (8.7)	641 (8.0)
1977-1978	10 195 (25.1)	4149 (25.7)	14 373 (23.8)	3086 (21.4)
1980-1981	10 644 (29.5)	4970 (28.9)	17 373 (26.5)	3892 (24.8)
1981-1982	11 673 (31.8)	5343 (30.8)	18 555 (27.9)	3991 (25.0)
1982-1983	11 685 (32.7)	5445 (31.6)	19 627 (29.3)	4229 (26.7)
1983-1984	11 961 (33.9)	5659 (32.9)	20 685 (30.7)	4617 (28.3)
1984-1985	12 476 (34.7)	5705 (33.6)	21 287 (31.7)	4898 (30.0)
1985-1986	11 562 (35.1)	5788 (34.2)	21 624 (32.5)	4930 (30.8)
1986-1987	11 267 (36.0)	5866 (35.0)	22 082 (33.4)	5092 (32.1)
1987-1988	10 411 (37.0)	6087 (36.5)	22 539 (34.3)	5219 (32.7)‡

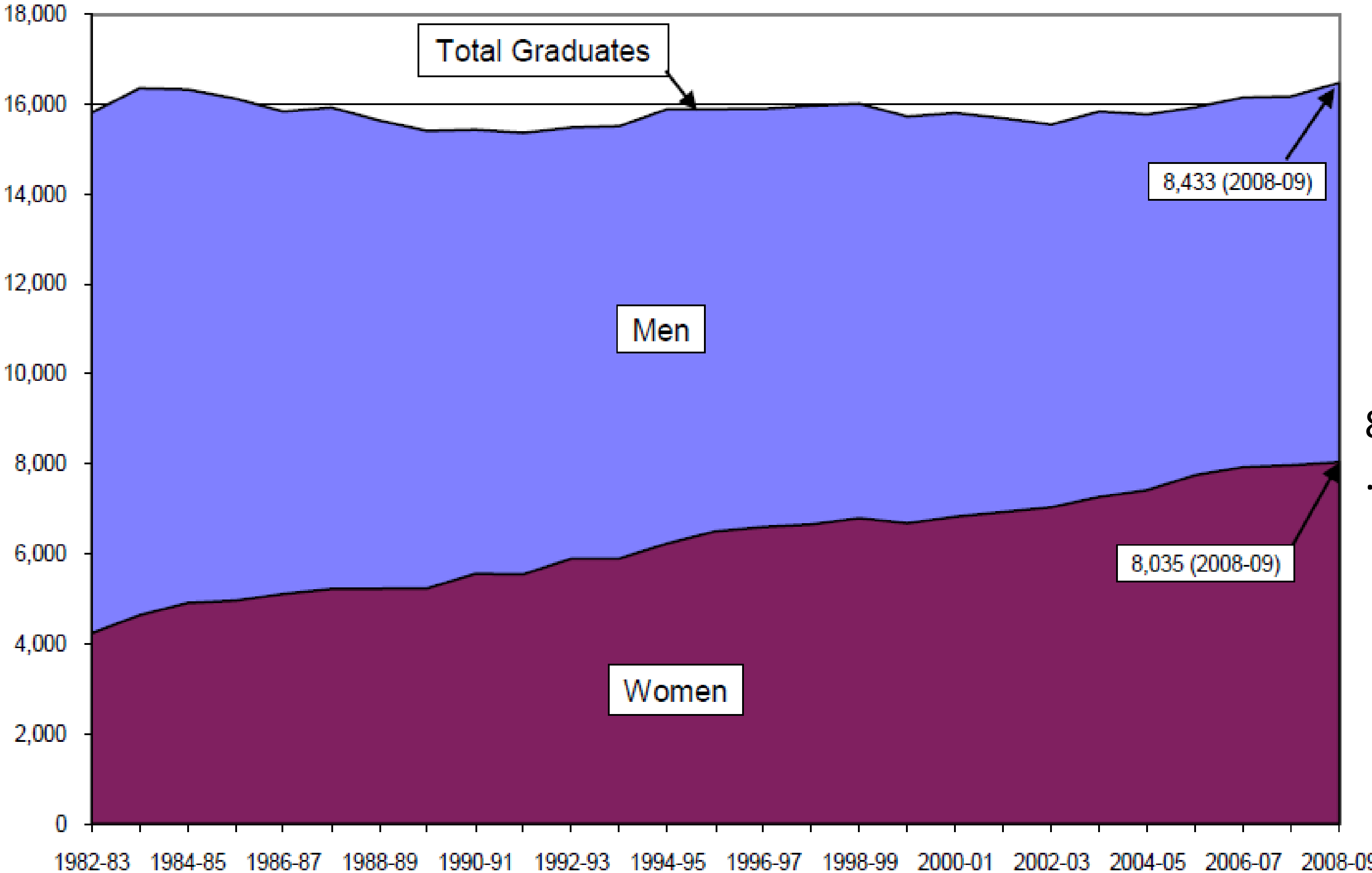
*Ponce (Puerto Rico) School of Medicine and the University of South Dakota, Sioux Falls, did not provide information for the 1980-1981 academic year, so 1979-1980 enrollment data were used for these schools. Similarly, Howard University, Washington, DC, did not provide information for the 1987-1988 academic year, so 1986-1987 enrollment data were used for this school.

†Source of data: *Medical School Admission Requirements*, Association of American Medical Colleges, Section for Student Services.

‡Data estimated in April 1988.

Harry S. Jonas, MD, Sylvia I. Etzel, "Undergraduate Medical Education," JAMA, Aug 26, 1988, vol. 260, 8 1063-1071, 3008.

U.S. Medical School Graduates 1982-83 to 2008-09



8035/16,468=
.49

Chart 4, http://www.aamc.org/data/facts/charts1982to2010.pdf

What makes a clearinghouse successful or unsuccessful?

- A matching is “stable” if there aren’t a doctor and residency program, not matched to each other, who would both prefer to be.
- Hypothesis: successful clearinghouses produce stable matchings.
- How to test this?

Market	Stable	Still in use (halted unraveling)
• NRMP	yes	yes (new design in ’98)
• *Edinburgh ('69)*	*yes*	*yes*
• *Cardiff*	*yes*	*yes*
• *Birmingham*	*no*	*no*
• *Edinburgh ('67)*	*no*	*no*
• *Newcastle*	*no*	*no*
• Sheffield	no	no
• Cambridge	no	yes
• London Hospital	no	yes
• Medical Specialties	yes	yes (~30 markets, **1 failure**)
• Canadian Lawyers	yes	yes (Alberta, no BC, Ontario)
• Dental Residencies	yes	yes (5) (no 2)
• Osteopaths (< '94)	no	no
• Osteopaths ($\geq$ '94)	yes	yes
• Pharmacists	yes	yes
• Reform rabbis	yes (first used in ‘97-98)	yes
• Clinical psych	yes (first used in ‘99)	yes
• Lab experiments	yes	yes.
(Kagel&Roth *QJE* 2000)	no	no

Stability is an important criterion for a successful clearinghouse.

Gale, David and Lloyd Shapley [**1962**], The college admissions model without couples

PLAYERS: Firms = $\{f_1,\ldots, f_n\}$ Workers= $\{w_1,\ldots,w_p\}$

 # positions $\mathbf{q_1,\ldots,q_n}$

Synonyms: F=Firms = C=Colleges = H=Hospitals;

W=Workers = S=Students=Doctors

PREFERENCES *over individuals* (complete and transitive)

$P(f_i) = w_3, w_2, \ldots f_i \ldots$ $[w_3 >_{fi} w_2]$

$P(w_j) = f_2, f_4, \ldots w_j \ldots$

An OUTCOME of the game is a *MATCHING*:

μ: $F \cup W \rightarrow F \cup W$

such that $\mu(f)$ contains w iff $\mu(w)$ = f, and for all f and w

$|\mu(f)|$ is less than or equal to q_f

either $\mu(w)$ is in F or $\mu(w)$ = w. so f is matched to the *set* of workers $\mu(f)$.

We need to specify how firms' preferences over matchings, are related to their preferences over individual workers, since they hire groups of workers. The simplest model is

Responsive preferences: for any set of workers $S \subset W$ with $|S| < q_i$, and any workers w and w' in W/S,

$S \cup w >_{fi} S \cup w'$ if and only if $w >_{fi} w'$, and

$S \cup w >_{fi} S$ if and only if w is acceptable to f_i.

A matching μ is *individually irrational* if $\mu(w) = f$ for some worker w and firm f such that either the worker is unacceptable to the firm or the firm is unacceptable to the student. An individually irrational matching is said to be blocked by the relevant individual. (**Note the modeling assumption here**.)

A matching μ is *BLOCKED BY A PAIR OF AGENTS* (f,w) if they each prefer each other to μ:

[$w >_f w'$ for some w' in $\mu(f)$ or $w >_f f$ if $|\mu(f)| < q_f$]

and $f >_w \mu(w)$

A matching is (pairwise) *stable* if it isn't blocked by any individual or pair of agents.

GS Deferred Acceptance Algorithm, with firms offering

- 0. *If some preferences are not strict*, arbitrarily break ties
- 1 a. Each firm f with q positions makes offers to its top q choices (if it has any acceptable choices).
- b. Each worker rejects any unacceptable offers and, if more than one acceptable offer is received, "holds" the most preferred and rejects all others.
- k a. Any firm rejected at step k-1 makes a new offer, for each rejection, to its most preferred acceptable worker who hasn't yet rejected it. (If no acceptable choices remain, it makes no further offers.)
- b. Each worker holds her most preferred acceptable offer to date, and rejects the rest.
- STOP: when no further proposals are made, and match each worker to the firm (if any) whose offer she is holding.

GS Deferred Acceptance Algorithm, with **workers applying**

- 0. *If some preferences are not strict*, arbitrarily break ties
- 1 a. Each worker applies to his/her top choice firm.
- b. Each firm f with q positions holds the top q applications among the acceptable applications it receives, and rejects all others.
- k a. Any worker rejected at step k-1 makes a new application, to its most preferred acceptable firm that hasn't yet rejected him/her. (If no acceptable choices remain, he/she makes no further offers.)
- b. Each firm holds its q most preferred acceptable applications to date, and rejects the rest.
- STOP: when no further proposals are made, and match each firm to the workers (if any) whose applications it is holding.

GS's 2 Remarkable Theorems

- **Theorem 1 (GS):** A stable matching exists for every marriage market.
- **Theorem 2 (GS):** When all firms and workers have **strict preferences**, there always exists an F-optimal stable matching (that every firm likes at least as well as any other stable matching), and a W-optimal stable matching. Furthermore, the matching μ_F produced by the deferred acceptance algorithm with firms proposing is the F-optimal stable matching. The W-optimal stable matching is the matching μ_W produced by the algorithm when the workers propose.

Stability and the medical match

- Roth (1984) showed that the algorithm adopted by the NRMP in 1951 was equivalent to the hospital-proposing deferred acceptance algorithm.
- But when couples started being a noticeable part of the market, they often made arrangements outside of the match.

An initial “couples algorithm” in the 1970’s

- Couples (after being certified by their dean) could register for the match as a couple.
  - They had to specify one member of the couple as the “leading member.”
  - They submitted a separate rank order list of positions for each member of the couple
- The leading member went through the match as if single.
- The other member then had his/her rank order list edited to remove positions not in the ‘same community’ as the one the leading member had matched to.
  - Initially the NRMP determined communities; in a later version, when couples were still defecting, couples could specify this themselves.

A similar algorithm is used in Scotland today

- **“Linked applicants**
- To accommodate linked applicants, a *joint* preference list is formed for each such pair, using their individual preference lists and the programme compatibility information. If such a pair, *a* and *b*, have individual preferences *p*1, *p*2, . . . , *p*10 and *q*1, *q*2, . . . , *q*10 respectively (with *a* the higher scoring applicant), then the joint preference list of the pair (*a*,*b*) is (*p*1,*q*1), (*p*1,*q*2), (*p*2,*q*1), (*p*2,*q*2), (*p*1,*q*3), (*p*3,*q*1), (*p*2,*q*3), (*p*3,*q*2), . . ., (*p*9,*q*10), (*p*10,*q*9), (*p*10,*q*10) (except that incompatible pairs of programmes are omitted) “
- http://www.nes.scot.nhs.uk/sfas/About/default.asp

But this didn’t work well for couples

- Why?
- The iron law of marriage: You can’t be happier than your spouse.
- Couples consume *pairs* of jobs. So an algorithm that only asks for their preference orderings over *individual* jobs can’t hope to avoid instabilities (appropriately redefined to include couples’ preferences)
- But even if we ask couples for their preferences over pairs of jobs, we may still have a problem: Roth (1984) observed that the set of stable matchings may be *empty* when couples are present.

Example--market with one couple and no stable matchings (motivated by Klaus and Klijn, and Nakamura (JET corrigendum 2009 to K&K JET 2005):

Let c=(s1,s2) be a couple, and suppose there is another single student s3, and two hospitals h1 and h2. Suppose that the acceptable matches for each agent, in order of preference, are given by

c: (h1,h2);　　　　s3: h1, h2,
h1: s1, s3;　　　　h2: s3, s2

Then no individually rational matching μ (i.e. no μ that matches agents only to acceptable mates) is stable. We consider two cases, depending on whether the couple is matched or unmatched.

Case 1: μ(c)=(h1,h2). Then s3 is unmatched, and s/he and h2 can block μ, because h2 prefers s3 to μ (h2)=s2.

Case 2: μ (c)=c (unmatched). If μ (s3)=h1, then (c, h1,h2) blocks μ. If μ (s3)=h2 or μ (s3)=s3 (unmatched), then (s3,h1) blocks μ.

And there are couples: Descriptive Statistics, NRMP

	1987	1993	1994	1995	1996
APPLICANTS					
Primary ROL's	**20071**	**20916**	**22353**	**22937**	**24749**
Applicants with Supplemental ROL's	**1572**	**2515**	**2312**	**2098**	**2436**
Couples					
Applicants who are Coupled	**694**	**854**	**892**	**998**	**1008**
PROGRAMS					
Active Programs	**3225**	**3677**	**3715**	**3800**	**3830**
Active Programs with ROL Returned	3170	3622	3662	3745	3758
Potential Reversions of Unfilled Positions					
Programs Specifying Reversion	**69**	**247**	**276**	**285**	**282**
Positions to be Reverted if Unfilled	225	1329	1467	1291	1272
Programs Requesting Even Matching	**4**	**2**	**6**	**7**	**8**
Total Quota Before Match	**19973**	**22737**	**22801**	**22806**	**22578**

Why is the couples problem hard?

- Note first that the ordinary deferred acceptance algorithm won't in general produce a stable matching (even when one exists, and even when couples state preferences over pairs of positions)
  - In the worker proposing algorithm, if my wife and I apply to a pair of firms in Boston, and our offers are held, and I am later displaced by another worker, my wife will want to withdraw from the position in which she is being held (and the firm will regret having rejected other applications to hold hers)
  - In the firm proposing algorithm, it may be hard for a couple to determine which offers to hold.

The Market for Clinical Psychology Interns

In 1998, this market converted to a centralized match using the Roth-Peranson algorithm (run for the first time in the academic year ‘98-99 for jobs beginning in June 1999.) For approximately 25 years prior to that, a decentralized market was run, under a changing set of rules.

In the early 1990’s, transactions in this market were supposed to all be made by telephone on "Selection Day," the second Monday in February, from 9:00 AM to 4:00 PM Central Standard Time. That is, the market was supposed to operate for seven hours.

Roth, A.E. and X. Xing "Turnaround Time and Bottlenecks in Market Clearing: Decentralized Matching in the Market for Clinical Psychologists," *Journal of Political Economy*, 105, April 1997, 284-329.

Rules 5+6: “Deferred acceptance by telephone”

5. **An applicant must respond immediately to each offer tendered in one of three ways. The offer may be accepted, rejected or "held."**
    a. *Accepting* the offer constitutes a binding agreement between applicant and internship program.
    b. *Refusing* the offer terminates all obligations on either side and frees the internship program to offer the position to another applicant.
    c. *Holding* the offer means that the offer remains valid until the applicant notifies the program of rejection or acceptance, or until the end of selection day.

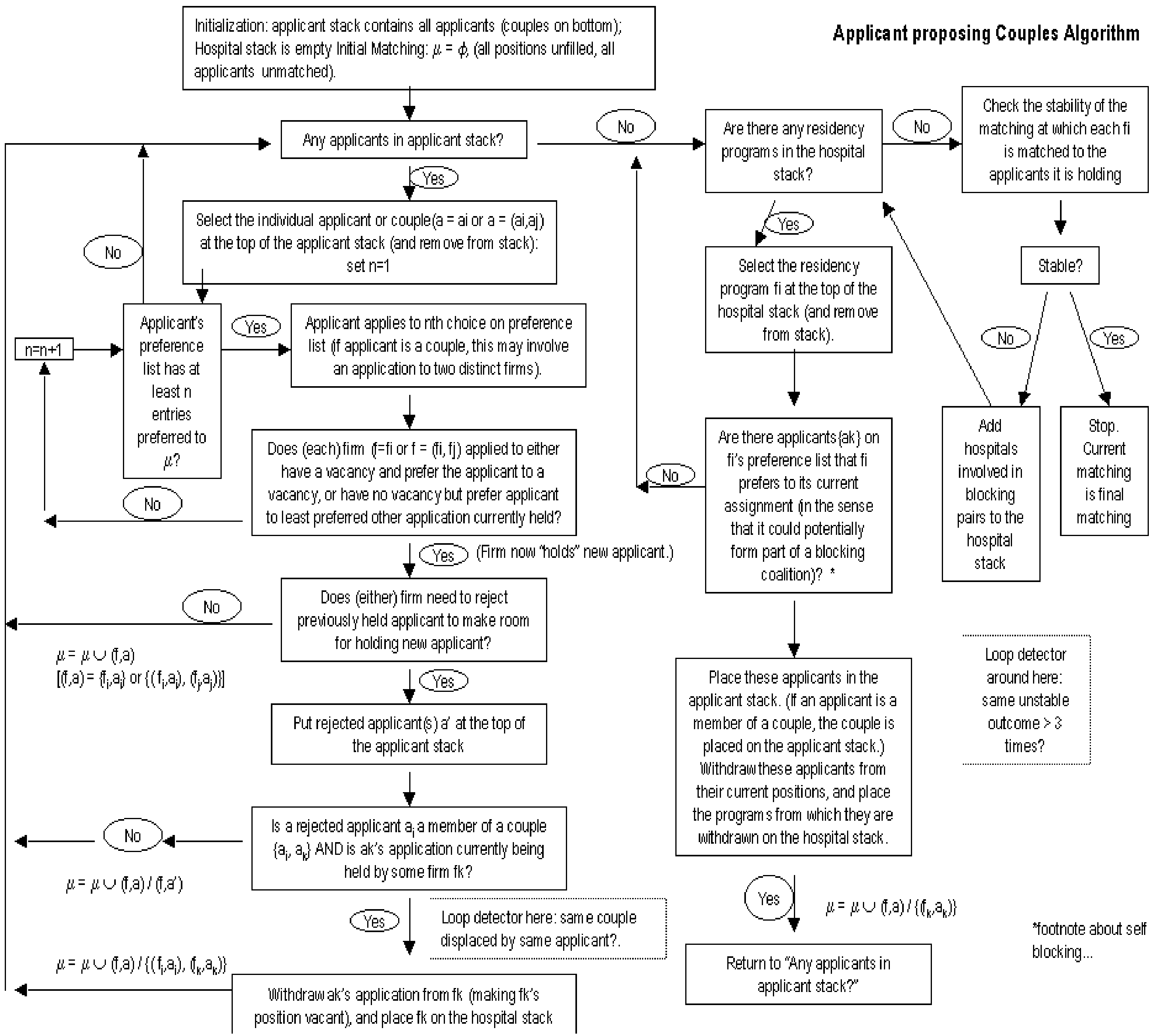
6. **Applicants may "HOLD" no more than one active offer at a time.**
    a. If an applicant is holding an offer from one program and receives an offer from a more preferred program, s/he may accept or "hold" the second offer provided that the less preferred program is notified *immediately* that the applicant is rejecting the previously held offer.
    b. **If a program confirms that an applicant is holding more than one offer, the program is free to withdraw their previously tendered offer of acceptance**, and to offer that position to another applicant *after* the offending applicant is notified of that decision.

The problems facing couples:

- Suppose my wife and I have a particular pair of jobs in Boston as our first choice, in St. Louis as our second choice, SF third, Pittsburgh fourth, etc.
- Suppose I get an offer from my choices in Boston, and then SF, while my wife gets offers from St. Louis and Pittsburgh.
  - We can’t safely reject any of them…although we may have to…(and if we don’t, the process may grind to a halt and new offers will be difficult to come by).
  - So couples had a tough decision problem in the telephone algorithm with firms proposing (and it’s a tough design problem even in a computerized algorithm).

Current NRMP match
(Roth/Peranson algorithm)

- The algorithm starts as a **student (and couple)-proposing deferred acceptance algorithm**,
- then resolves instabilities with an algorithm modeled on the Roth-Vande Vate (1990) blocking-pair-satisfying algorithm
- Deals with major match complications
  - Married couples
    - They can submit preferences over pairs of positions
  - Applicants can match to pairs of jobs, PGY1&2
    - They can submit supplementary preference lists
  - Reversions of positions from one program to another


Applicant proposing Couples Algorithm
Initialization: applicant stack contains all applicants (couples on bottom); Hospital stack is empty Initial Matching: $\mu = \phi$, (all positions unfilled, all applicants unmatched).
Any applicants in applicant stack?
Yes
No
Select the individual applicant or couple(a = ai or a = (ai,aj) at the top of the applicant stack (and remove from stack): set n=1
Applicant's preference list has at least n entries preferred to μ?
No
Yes
n=n+1
Applicant applies to nth choice on preference list (if applicant is a couple, this may involve an application to two distinct firms).
Does (each) firm (f=fi or f = (fi, fj) applied to either have a vacancy and prefer the applicant to a vacancy, or have no vacancy but prefer applicant to least preferred other application currently held?
No
Yes
(Firm now "holds" new applicant.)
Does (either) firm need to reject previously held applicant to make room for holding new applicant?
No
$\mu = \mu \cup (f,a)$
$[(f,a) = (f_i,a_j)$ or $\{(f_i,a_i), (f_j,a_j)\}]$
Yes
Put rejected applicant(s) a' at the top of the applicant stack
Is a rejected applicant a_j a member of a couple $\{a_j, a_k\}$ AND is ak's application currently being held by some firm fk?
No
$\mu = \mu \cup (f,a) / (f,a')$
Yes
Loop detector here: same couple displaced by same applicant?.
$\mu = \mu \cup (f,a) / \{(f_j,a_j), (f_k,a_k)\}$
Withdraw ak's application from fk (making fk's position vacant), and place fk on the hospital stack
No
Are there any residency programs in the hospital stack?
No
Check the stability of the matching at which each fi is matched to the applicants it is holding
Yes
Select the residency program fi at the top of the hospital stack (and remove from stack).
Are there applicants{ak} on fi's preference list that fi prefers to its current assignment (in the sense that it could potentially form part of a blocking coalition)? *
No
Place these applicants in the applicant stack. (If an applicant is a member of a couple, the couple is placed on the applicant stack.) Withdraw these applicants from their current positions, and place the programs from which they are withdrawn on the hospital stack.
Yes
$\mu = \mu \cup (f,a) / \{(f_k,a_k)\}$
Return to "Any applicants in applicant stack?"
Stable?
No
Yes
Add hospitals involved in blocking pairs to the hospital stack
Stop. Current matching is final matching
Loop detector around here: same unstable outcome > 3 times?
*footnote about self blocking...

Stable Clearinghouses (those now using the Roth Peranson Algorithm)

NRMP / SMS:

Medical Residencies in the U.S. (NRMP) (1952)
Abdominal Transplant Surgery (2005)
Child & Adolescent Psychiatry (1995)
Colon & Rectal Surgery (1984)
Combined Musculoskeletal Matching Program (CMMP)

- Hand Surgery (1990)

Medical Specialties Matching Program (MSMP)

- Cardiovascular Disease (1986)
- **Gastroenterology (1986-1999; rejoined in 2006)**
- Hematology (2006)
- Hematology/Oncology (2006)
- Infectious Disease (1986-1990; rejoined in 1994)
- Oncology (2006)
- Pulmonary and Critical Medicine (1986)
- Rheumatology (2005)

Minimally Invasive and Gastrointestinal Surgery (2003)
Obstetrics/Gynecology

- Reproductive Endocrinology (1991)
- Gynecologic Oncology (1993)
- Maternal-Fetal Medicine (1994)
- Female Pelvic Medicine & Reconstructive Surgery (2001)

Ophthalmic Plastic & Reconstructive Surgery (1991)
Pediatric Cardiology (1999)
Pediatric Critical Care Medicine (2000)
Pediatric Emergency Medicine (1994)
Pediatric Hematology/Oncology (2001)
Pediatric Rheumatology (2004)
Pediatric Surgery (1992)
Primary Care Sports Medicine (1994)
Radiology

- Interventional Radiology (2002)
- Neuroradiology (2001)
- Pediatric Radiology (2003)

Surgical Critical Care (2004)
Thoracic Surgery (1988)
Vascular Surgery (1988)

Postdoctoral Dental Residencies in the United States

- Oral and Maxillofacial Surgery (1985)
- General Practice Residency (1986)
- Advanced Education in General Dentistry (1986)
- Pediatric Dentistry (1989)
- Orthodontics (1996)

Psychology Internships (1999)

Neuropsychology Residencies in the U.S. & CA (2001)
Osteopathic Internships in the U.S. (before 1995)
Pharmacy Practice Residencies in the U.S. (1994)
Articling Positions with Law Firms in Alberta, CA(1993)
Medical Residencies in CA (CaRMS) (before 1970)

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British (medical) house officer positions

- Edinburgh (1969)
- Cardiff (197x)

New York City High Schools (2003)
Boston Public Schools (2006)

Empirical puzzle

- Why do these algorithms virtually always find stable matchings, even though couples are present (and so the set of stable matchings *could* be empty)?

Some clues

- Empirically: Roth and Peranson (1999): as markets get large, # of interviews (and hence length of rank order lists) doesn't grow too much, and the set of stable matchings gets small.
- Theoretically: the set of stable matchings gets small and hard to manipulate,
  - Immorlica and Mahdian (2005), 1-1 matching
  - Kojima and Pathak (2009), many to one matching

Table 1. Summary Statistics for Market for Clinical Psychologists

	Total	Mean	Min	25th	Median	75th	Max
		A. Length of Rank-Order List (ROL)					
Single Applicants	3,010	7.6	1.0	4.0	7.1	10.4	73.1
Couples	19	81.2	7.3	29.4	52.3	115.0	249.9
Distinct Programs Ranked		10.2	2.0	6.4	9.9	13.0	20.9
Programs	1,094	16.7	1.0	7.6	14.3	23.9	80.9
		B. Program Capacities					
Capacity	2,721	2.5	1.0	1.0	2.0	3.0	21.4
		C. Geographic Similarity of Preferences					
Single Applicants							
# Regions Ranked		2.5	1.0	1.0	2.0	3.3	9.4
Couples							
# Regions Ranked		4.0	1.1	2.6	4.0	4.9	6.9
Fraction of ROL where both Members Rank Same Region		72.7%	29.2%	46.6%	77.3%	97.9%	100.0%

Notes: This table reports descriptive information from the Association of Psychology Postdoctoral and Internship Centers match, averaged over 1999-2007. Single applicants' rank order lists consist of a ranking over hospitals, while couples indicate rankings over program pairs. Distinct programs ranked are the set of distinct programs ranked by each couple member. Programs include only those which have positive capacity. There are 11 regions, corresponding to the first digit of US Zipcodes and Canada.

Stylized facts

1. Applicants who participate as couples constitute a small fraction of all participating applicants.
2. The length of single applicants' rank order lists is small relative to the number of possible programs.
3. Applicants who participate as couples rank more programs than single applicants. However, the number of distinct programs ranked by a couple member is small relative to the number of possible programs.
4. The most popular programs are ranked as a top choice by a small number of applicants.
5. A pair of hospital programs ranked by doctors who participate as a couple tend to be in the same region.
6. Even though there are more applicants than positions, many programs still have unfilled positions at the end of the centralized match.
7. A stable matching exists in all nine years in the market for clinical psychologists.

Random markets

- A random market is a tuple Γ=(H,S,C, $\succcurlyeq_{\{H^n\}}$, k,P,Q,ρ), where k is a positive integer (max length of ROL's), $P=(p_{\{h\}})_{\{h \in H\}}$ and $Q=(q_{\{h\}})_{\{h \in H\}}$ are probability distributions on H, and ρ is a function which maps two preferences over H to a preference list for couples.
- Hospitals' preference orderings are essentially arbitrary, and take account of their capacities, and couples preferences are formed from their individual preferences (drawn from probability distribution Q, different than P for singles), via an essentially arbitrary function ρ .

Random large markets

- A sequence of random markets is $(\Gamma^1,\Gamma^2,\ldots)$, where $\Gamma^n=(H^n,S^n,C^n,\succcurlyeq_{\{H^n\}},k^n,P^n,Q^n,\rho^n)$ is a random market in which $|H^n|=n$ is the number of hospitals.

Definition: A sequence of random markets $(\Gamma^1, \Gamma^2, \ldots)$ is **regular** if there exist $\lambda>0$, $a \in[0,(1/2))$, $b>0$, $r\geq1$, and positive integers k and κ such that for all n,

1. $k^n=k$, (constant max ROL length, doesn't grow with n)
2. $|S^n| \leq\lambda n$, $|C^n| \leq bn^a$, (singles grow no more than proportionally to positions—e.g. $\lambda>1$, and couples grow slower than root n)
3. $\kappa_h\leq\kappa$ for all hospitals h in H^n (hospital capacity is bounded)
4. $(p_h/p_{h'})\in[(1/r),r]$ and $(q_h/q_{h'})\in[(1/r),r]$ for all hospitals h,h′ in H^n. (The popularity of hospitals as measured by the prob of being acceptable to docs does not vary too much as the market grows, i.e. no hospital is everyone's favorite (in after-interview preferences)

A (really simple) sequential couples algorithm (like left side of RP flow chart)

1. run a deferred acceptance algorithm for a sub-market composed of all hospitals and single doctors, but without couples,
2. one by one, place couples by allowing each couple to apply to pairs of hospitals in order of their preferences (possibly displacing some doctors from their tentative matches), and
3. one by one, place singles who were displaced by couples by allowing each of them to apply to a hospital in order of her preferences.

We say that the sequential couples algorithm *succeeds* if there is no instance in the algorithm in which an application is made to a hospital where an application has previously been made by a member (or both members) of a couple except for the couple who is currently applying. Otherwise, we declare a failure and terminate the algorithm.

Lemma: If the sequential couples algorithm succeeds, then the resulting matching is stable.

Stable matchings exist, in the limit

- Theorem: Suppose that $(\Gamma^1, \Gamma^2, \ldots)$ is a regular sequence of random markets. Then the probability that there exists a stable matching in the market induced by Γ^n converges to one as the number of hospitals n approaches infinity.

Key element of proof

- if the market is large, then it is a high probability event that there are a large number of hospitals with vacant positions at the end of Step 2 (even though there could be more applicants than positions) (i.e. the Scramble will remain important in large markets.)
- So chains of proposals beginning when a couple displaces a single doc are much more likely to terminate in an empty position than to lead to a proposal to a hospital holding the application of a couple member.

Formal statement

Proposition: many hospitals have vacancies in large markets

There exists a constant $\beta > 0$ such that

(1) the probability that, in a sub-market without couples, the doctor-proposing deferred acceptance algorithm produces a matching in which at least $\beta * n$ hospitals have at least one vacant position converges to one as n approaches infinity, and

(2) the probability that the sequential couples algorithm produces a stable matching and at least $\beta * n$ hospitals have at least one vacant position in the resulting matching converges to one as n approaches infinity.

Corollary

- Corollary: Suppose that $(\Gamma^1, \Gamma^2, \ldots)$ is a regular sequence of random markets. Then the probability that the Roth-Peranson algorithm produces a stable matching in the market induced by Γ^n converges to one as the number of hospitals n approaches infinity.

Incentives

- Positive and negative results for finite markets without couples.
  - Roth (1982, 85):There is no stable mechanism that makes it a dominant strategy for all agents to state their true preferences (and no mechanism that firms with multiple positions can't potentially manipulate)
  - Dubins and Friedman (81), Roth (82): The deferred acceptance algorithm with workers proposing makes it a dominant strategy for workers to state their true preferences.

Incentives in large markets with couples: truthtelling is an ε- equilibrium

- To address incentives, we need to specify what the mechanism produces when it does not find a stable matching (and we need an additional regularity condition that says cardinal utilities don’t become infinite as the market gets large).
- We consider a completion of the sequential couples algorithm in which displaced agents (including couples) continue proposing down their preference lists, as in the DA algorithm, and the algorithm stops at the possibly unstable matching that results.

Truthtelling* is an ε-Bayesian equilibrium

- Theorem: Consider a regular sequence of matching games. For any $\varepsilon > 0$, there exists n such that truth-telling is an ε-Bayes Nash equilibrium for every matching game in that sequence with more than n hospitals.

- * A truthtelling strategy is an ordinal ranking that corresponds to the underlying cardinal utility.

Open questions

- Limit theorems are blunt instruments, how about large finite markets?
  - What proportion of couples will give what probability of no stable matchings?
  - Possible path of attack: we didn't yet make any assumptions about couples' preferences.

Open questions

- Why has this form of labor organization been so important for doctors?
  - Why have unraveling and congestion been such devastating market failures in these markets?
  - Inelasticity of demand? On both sides?

Open questions

- Decentralized markets: rules of engagement?
  - In Gastro, we helped redesign the rules about (exploding/early) offers and acceptances
  - What would be good rules for couples?

What have we learned from market design?

- To achieve efficient outcomes, marketplaces need make markets sufficiently
  - **Thick**
    - Enough potential transactions available at one time
  - **Uncongested**
    - Enough time for offers to be made, accepted, rejected, transactions carried out…
  - **Safe**
    - Safe to participate, and to reveal relevant information
- Some kinds of transactions are **repugnant**…and this can constrain market design.
  - The repugnance of some markets may give us a new viewpoint on regulation of familiar markets—markets often require a lot of social buy-in to work well.
- In many markets, you can't just choose what you want, you also have to be chosen.

Tomorrow: II. Kidney exchange (and repugnant transactions)

Bayesian equilibria

- Consider a Bayesian game $\Gamma=(H,S,C,(U_i, F_i)_{\{i\in H\cup S\cup C\}})$.
- At strategy σ_i, a player i submits an ordinal preference relation $\sigma_i(u_i)$ upon observing her own type u_i, but not the realized types of the other players.
- Given $\varepsilon\geq 0$, a strategy profile σ^* is an ε-Bayes Nash equilibrium if there exists no $i\in H\cup S\cup C$, $u_i\in U_i$ and strategy σ_i such that

$$E[u_i(\varphi_i(\sigma_i(u_i),\sigma_{-i}*(u_{-i})))] > E[u_i(\varphi_i(\sigma*(u)))]+\varepsilon$$

A regular sequence of Bayesian games

- In addition to 1,…4, all cardinal utilities are bounded
- 5. $\sup_{n\in N, D'\subseteq D^n, h\in H^n, u\_\{h\in U\_\{h\}\}} u_h(D')$, $\sup_{n\in N, s\in S^n, h\in H^n, u\_\{s\}\in Us} u_s(h)$, and $\sup_{n\in N, c\in C^n, h, h'\in H^n, u\_\{c\}\in U\_\{c\}} u_c(h,h') < \infty$,

where D^n is the set of doctors in Γ^n

Sketch of proof: For large n…

- Single applicants: the applicant-applying DA is dominant strategy incentive compatible for single applicants, and there is a high probability that each applicant is matched to his outcome at the end of this phase (i.e. not displaced by a couple).
- Hospitals: can potentially manipulate by “strategically rejecting” a candidate and causing a rejection chain that results in an application by a more preferred candidate. But such a rejection chain is much more likely to end at a vacant position.
- Couples: are most likely to end up at their first choice pair of positions that isn’t filled with a preferable single applicant…