Global carbon price asymmetry

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Abstract

This paper studies a social planner who chooses countries' carbon prices so as to maximize global welfare. Product markets are characterized by firm heterogeneity, market power, and international trade. Because of the market-power distortion, the planner's optimal policy is second-best. The main insight is that optimal carbon prices may be highly asymmetric: zero in some countries and above the social cost of carbon in countries with relatively dirty production. This result obtains even though a uniform global carbon price is always successful at reducing countries' emissions. Competition policy that mitigates market power may enable stronger climate action.

Keywords: Carbon leakage, carbon pricing, imperfect competition, international trade, second best

JEL codes: H23 (externalities), L11 (market structure), Q54 (climate)

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1 Introduction

Carbon pricing is increasingly used as a key policy instrument to combat climate change. Yet carbon prices around the world remain low and uneven: above \$50 per ton of CO_2 in Europe's flagship cap-and-trade system—and even higher for some national carbon taxes—but much lower in most other jurisdictions (World Bank 2021). This picture contrasts markedly with the Pigouvian ideal of a uniform global carbon price set at the social cost of carbon (SCC).

So far, carbon pricing has focused on power generation and emissions-intensive industrial sectors like aluminium, cement and steel. Three characteristics of these industries are striking. First, firms within each industry often have widely varying carbon intensities of production. This enhances the potential for market-based regulation to enhance abatement-cost efficiency. Second, emissions-intensive industries are often highly concentrated with long-standing concerns about the exercise of market power. This makes relevant the theory of the second best. Third, international trade is important as the scope of the product market in which regulated firms compete is often wider than that of the carbon price they face. This has led to concerns about leakage of emissions to less regulated jurisdictions.

This paper studies the optimal design of carbon prices in a model in which these three characteristics are crucial. The model considers a social planner who chooses countries' carbon prices so as to maximize global welfare. Because of a market-power distortion in the product market, the planner's optimal policy is second-best. The central trade-off is that a higher carbon price reduces a country's domestic emissions but also increases deadweight losses in the product market (due to pass-through of carbon costs to consumers) and leads to a degree of carbon leakage to the other country.[1] Thereby, the country with relatively clean firms is more vulnerable to carbon leakage as a policy-induced loss in production to the dirtier country translates into a larger increase in emissions.

The main insight is that second-best carbon prices can be extremely asymmetric across countries. Market power, on its own, pushes countries' optimal carbon prices downwards as the planner seeks to cushion the increase in consumer prices. The presence of international trade introduces a further effect: if carbon leakage for the country with relatively clean firms is sufficiently pronounced, its optimal carbon price is zero. This, in turn, limits deadweight losses in the product market and enables the planner to choose a higher carbon price for the dirtier country—which creates additional climate benefits as it reshuffles production to cleaner firms. As long as market power is not too pronounced, the dirtier country's optimal carbon price may lie above the SCC. This finding obtains even though a uniform global carbon price is always successful in reducing countries' emissions.

The result should not be overplayed given the model's very simple welfare function.[2] The more general point is that, while carbon prices around the world today are almost certainly far too low, failing to implement a global carbon price does not necessarily imply the wrong

[1]The leakage channel in the model arises from the market-share losses of more tightly regulated firms.

[2]The model is partial equilibrium without further distortions in factor markets or wider tax interactions. The social planner does not have additional policy instruments to directly address the market-power distortion.

response to climate change. Moreover, competition policy to mitigate market power may enable stronger and more balanced climate action.

2 Model

Consider a global industry in which $n_k \geq 1$ firm(s) are based in country $k = i, j$. Country k's firms face a linear demand curve $p_k(X) = \alpha_k - X$ where $X \equiv X_i + X_j$ is total industry output ($X_k \equiv \sum_m x_k^m$ for $k = i, j$), and α_k is a measure of k's product quality.

Firm m from country k needs to produce $y_k^m = \xi_k x_k^m$ units to get x_k^m units of output to market, where $\xi_k \geq 1$ is a trade cost that takes an "iceberg" form. Its emissions are $e_k^m = z_k y_k^m - a_k^m$ where z_k is its baseline emissions intensity (emissions per unit of production) and a_k^m is abatement.

Faced with a carbon price τ_k in its country, firm m of k's profits are $\Pi_k^m = p_k(X)x_k^m - C_k^m(y_k^m, a_k^m; \tau_k)$, where its total costs $C_k^m(y_k^m, a_k^m; \tau_k) = c_k y_k^m + \tau_k e_k^m + \phi_k(a_k^m)$ consist of a production cost c_k per unit of y_k^m, carbon costs, and an abatement cost $\phi_k(a_k^m) = \frac{\gamma_k}{2}(a_k^m)^2$.

The product market features a generalized version of Cournot competition with a conduct parameter $\theta \in (0, 1]$. Formally, firms' equilibrium outputs $(\widehat{x}_k^m)_{k=i,j}$ satisfy:

$$\widehat{x}_k^m = \arg\max_{x_k^m \geq 0} \left\{ \left[p_k \left(\theta(x_k^m - \widehat{x}_k^m) + \sum\nolimits_m \widehat{x}_i^m + \sum\nolimits_m \widehat{x}_j^m \right) x_k^m - C_k^m(y_k^m(x_k^m), a_k^m) \right] \right\} \tag{1}$$

Firm m in country k, in deviating its output by $(x_k^m - \widehat{x}_k^m)$, conjectures that industry output will change by $\theta(x_k^m - \widehat{x}_k^m)$ as a result. In this "conduct equilibrium" (Weyl & Fabinger 2013), a lower θ corresponds to more intense rivalry while competition is imperfect with $\theta > 0$. The Cournot-Nash equilibrium occurs where $\theta = 1$.

The firm's first-order conditions for output and abatement are thus:

$$p_k - \theta x_k^m - c_k \xi_k + \gamma_k z_k \xi_k a_k^m = 0 \text{ and } -\gamma_k a_k^m + \tau_k = 0 \tag{2}$$

so a generalized version of marginal revenue equals the marginal cost of output while the marginal cost of abatement is equal to the carbon price. These conditions together imply:

$$p_k - \theta x_k^m = (c_k + \tau_k z_k)\xi_k. \tag{3}$$

Given separability of production and abatement costs, the product-market equilibrium does not depend on the extent of abatement. Let $X_k(\tau_i, \tau_j)$, $p_k(\tau_i, \tau_j)$, and $E_k(\tau_i, \tau_j)$ denote equilibrium outputs, prices and emissions (with $E \equiv E_i + E_j$).

Global welfare $W = U - \sum_k c_k \xi_k X_k - sE - \Phi$ reflects consumer utility $U = \sum_k \alpha_k X_k - \frac{1}{2}X^2$ (with $\frac{\partial U}{\partial X_k} = p_k$), production and trade costs, the global SCC s, and total abatement costs $\Phi \equiv \sum_k \sum_m \phi_k(a_k^m)$.[3] The social planner's problem is to $\max_{\tau_i, \tau_j} W(\tau_i, \tau_j)$ subject to the constraint

[3]Product-market revenues are a transfer from consumers to firms and carbon-pricing revenues are a transfer from firms to governments.

that, at equilibrium, firms make non-negative profits, $\Pi_k^m \geq 0$. Assume that $W(0,0) \geq 0$ so the market is socially viable without carbon pricing—and the planner therefore never shuts it down. A necessary condition is that consumers' willingness-to-pay exceeds social costs, $\min_k \{\alpha_k - (c_k + sz_k)\xi_k\} > 0$.

For conciseness, the main text focuses on the case with symmetric product qualities ($\alpha_i = \alpha_j = \alpha$) and non-carbon costs ($c_i = c_j = c$, $\xi_i = \xi_j = \xi$) and without abatement ($\gamma_i \to \infty$, $\gamma_j \to \infty$).

3 Carbon prices and global emissions

The first results characterize basic properties of carbon pricing in an international context. The rate of carbon leakage associated with carbon pricing by country i is:

$$L_i^C \equiv \frac{dE_j(\tau_i, \tau_j)/d\tau_i}{-dE_i(\tau_i, \tau_j)/d\tau_i}. \tag{4}$$

This measures the fraction of i's emissions reduction that leaks to j. Similarly, output leakage $L_i^O \equiv (dX_j/d\tau_i)/(-dX_i/d\tau_i)$.

Lemma 1 *An increase in country i's carbon price τ_i reduces its domestic production, $dX_i/d\tau_i < 0$ and its domestic emissions, $dE_i/d\tau_i < 0$, where:*
(a) the rate of output leakage $L_i^O = n_j/(n_j + \theta) > 0$;
(b) the rate of carbon leakage $L_i^C = (z_j/z_i)[n_j/(n_j + \theta)] > 0$;
(c) the rate of carbon cost pass-through $dp(\tau_i, \tau_j)/d\tau_i = [n_i/(n_i + n_j + \theta)]\, z_i\xi > 0$.

Output leakage is more pronounced with (i) more rivals in j engaging in "business stealing" from those in i as a result of the unilateral cost increase (higher n_j); and (ii) more competitive conduct (lower θ).

Carbon leakage equals output leakage scaled by the relative emissions intensity z_j/z_i. A higher carbon price by i increases in global emissions if its carbon leakage exceeds 100%. This is ruled out by symmetry but occurs if j's production is sufficiently more polluting.[4]

Carbon pricing reduces i's profit margin as less than 100% of its carbon cost is passed on to consumers; pass-through decreases with market power and with more rivals in j.

Global action "works" in the following sense:

Lemma 2 *An increase in a uniform global carbon price ($\tau_k = \tau$ for $k = i, j$):*
(a) reduces global emissions, $dE(\tau, \tau)/d\tau < 0$;
(b) reduces country k's emissions, $dE_k(\tau, \tau)/d\tau \leq 0$, if and only if $L_k^C \leq 1$.

A uniform tightening in carbon prices is always successful at reducing aggregate emissions—even if it may induce higher emissions by an individual country. Intuitively, if unilateral action

[4]Large intra-industry heterogeneity is borne out in practice (Lyubich, Shapiro & Walker 2018). Babiker (2005) finds carbon leakage rates up to 130% in a general-equilibrium model with similar ingredients to the present model.

by i has carbon leakage above 100%, then i's firms are significantly cleaner than j's so a higher global carbon price improves their competitiveness and they expand production and emissions.

4 Carbon prices and global welfare

Now consider the second-best carbon prices chosen by a social planner. At a global level, carbon pricing involves a trade-off between lower consumer utility and the potential for lower environmental damages. Letting $\widehat{\alpha} \equiv (\alpha - c\xi)/\xi > 0$, the former dominates where:

Lemma 3 *If country i's rate of carbon leakage is sufficiently high,*

$$L_i^C \geq 1 - \frac{\theta}{(n_j + \theta)} \frac{\theta}{(n_i + n_j + \theta)} \frac{\widehat{\alpha}/s}{z_i} \equiv \underline{L}_i^C,$$

then a zero carbon price is welfare-dominant, $W(0, \tau_j) \geq W(\tau_i, \tau_j)$ *for all* $\tau_i, \tau_j \geq 0$.

The result is immediate if $L_i^C > 1$. Then a "reverse leakage" argument applies: a reduction in i's carbon price raises its own emissions but this is outweighed by the induced reduction in j's emissions. As consumers also gain, global welfare rises. Given the linear-quadratic model structure, its leakage rate is a constant (Lemma 1) so this logic holds at any level of countries' carbon prices. Put simply, the extent of i's carbon leakage precludes effective climate action.

This conclusion applies as long as i's leakage rate is sufficiently high, $L_i^C \geq \underline{L}_i^C$, where $\underline{L}_i^C < 1$ because $\theta > 0$. The critical value $\underline{L}_i^C$ declines with the ratio $\widehat{\alpha}/s$, which is a measure of the size of market-power distortion (via $\widehat{\alpha}$) relative to the climate problem (via s). If the former is sufficiently important, $\underline{L}_i^C$ turns negative.

The main interest of the paper lies in global carbon price asymmetry, so suppose that i's firms are cleaner with $z_i/z_j < 1$. The problem is then resolved by the three industry characteristics described in the introduction:

Lemma 4 *Suppose that country i's carbon price* $\tau_i = 0$*. Then an interior solution* $\tau_j^* > 0$ *for country j that maximizes* $W(0, \tau_j)$ *satisfies:*

$$\frac{\tau_j^*}{s} = 1 - \underbrace{\frac{\theta}{n_j}\left(\frac{\widehat{\alpha}/s - z_j}{z_j}\right)}_{\text{market power}} + \underbrace{\frac{n_i}{n_j}\left[1 + \left(\frac{n_i + n_j + \theta}{\theta}\right)\left(1 - \frac{z_i}{z_j}\right)\right]}_{\text{international competition \& firm heterogeneity}}.$$

The first deviation of τ_j^* from the SCC is driven by market power. The standard result for a second-best domestic emissions tax is nested where $\tau_j^*\big|_{n_i=0} = [s - (\theta/n_j)(\widehat{\alpha}/z_j - s)] < s$ (recalling $\min_k \{\alpha - (c + sz_k)\xi\} > 0$). With perfect competition, $\tau_j^*\big|_{n_i=0,\theta=0} = s$ is Pigouvian.

The second deviation from the SCC instead pushes τ_j^* upwards—driven by firm heterogeneity and cross-border competition. An increase in j's carbon price shifts production to i's cleaner firms. This has two implications. First, output leakage to i limits the contraction in industry output due to j's carbon price, mitigating the incremental product-market distortion. Second, the contraction in industry output leads to a greater reduction in global emissions precisely because i's firms are cleaner. These factors limit deadweight losses and amplify environmental benefits, pushing upwards j's optimal carbon price.

A related observation is that the social planner regards countries' carbon prices as strategic substitutes.[5] A higher carbon price by j raises the product price and so exacerbates the market-power distortion. This sharpens the planner's trade-off against emissions cuts by i, and reduces the welfare gain from i's own carbon price.

The main result shows how this international-competition effect can dominate the planner's calculus and yield extreme asymmetry in global carbon prices:

Proposition 1 *Suppose that country i's firms are sufficiently cleaner than j's, with*

$$\frac{z_i}{z_j} \leq 1 - \frac{\theta n_j}{[(n_i+\theta)(n_i+n_j+\theta)+n_j(n_j+\theta)]} \equiv \delta < 1.$$

Then, for the range of parameter values given by

$$\frac{\widehat{\alpha}}{s} \in \left[\Psi, \Psi + \frac{n_i}{\theta}\frac{n_j}{\theta}(z_j - z_i)\right] \text{ where } \Psi \equiv \left(1+\frac{n_i}{\theta}\right)\left[z_j + \frac{n_i}{\theta}(z_j - z_i)\right],$$

welfare-optimal carbon prices are $\tau_i^* = 0$ while $\tau_j^* \geq s$.

Proposition 1 establishes in equilibrium the logic underlying Lemmas 3 and 4. The range on $\widehat{\alpha}/s$ ensures that the market-power distortion is small enough for τ_j^* to exceed the SCC by Lemma 4 but also large enough for j's firms to remain profitable. The condition $z_i/z_j \leq \delta$ ensures that indeed $\tau_i^* = 0$ because i's leakage is sufficiently pronounced as per Lemma 3.

Illustrations. Figure 1 illustrates how Proposition 1 applies to a significant "chunk" of the parameter space. It sets $s = 50$, $z_j = 1$, and $n_i/\theta = n_j/\theta = 6$—corresponding, e.g., to a relatively concentrated market $n_i = n_j = 3$ and competition "halfway" between perfect and Cournot ($\theta = \frac{1}{2}$). The result holds notably where i is much cleaner and $\widehat{\alpha}/s$ is not too large.

For example, if i's firms are modestly cleaner with $z_i = 0.9$, Proposition 1's condition $\frac{z_i}{z_j} \leq \delta = \frac{127}{133}$ is met. With $\widehat{\alpha} = 600$, Lemma 4 gives $\tau_j^* = 73\frac{1}{3}$—almost 50% above the SCC. If instead $\widehat{\alpha} = 560$, $\tau_j^* = 80$ makes j's firms just indifferent about being active ($\Pi_j^* = 0$) while $\tau_j^* \geq s$ as long as $\widehat{\alpha} \leq 740$. For these parameter values, $L_i^C = .952$ and $L_j^C = .771$ by Lemma 1, confirming that global action "works" as per Lemma 2.[6]

Extensions. Proposition 1's insight obtains in the generalized model (see Appendix) with heterogeneity in product qualities and non-carbon costs, plus abatement by firms. These heterogeneities have an ambiguous impact: if j has a lower-quality product or higher costs, this strengthens the planner's case for setting a relatively higher carbon price (and vice versa).

[5]Global welfare, $W(\tau_i,\tau_j) = U(\tau_i,\tau_j) - \sum_k c\xi X_k(\tau_i,\tau_j) - sE(\tau_i,\tau_j)$ is submodular in countries' carbon prices:

$$\frac{d}{d\tau_j}\left[\frac{dW(\tau_i,\tau_j)}{d\tau_i}\right] = \frac{d}{d\tau_j}\left[\sum\nolimits_k [p(\tau_i,\tau_j) - c\xi]\frac{dX_k}{d\tau_i} - s\frac{dE_i}{d\tau_i}(1-L_i^C)\right] = \frac{dp}{d\tau_j}\frac{dX}{d\tau_i} < 0,$$

since $dX_k/d\tau_i$, $dE_i/d\tau_i$ and L_i^C are all constants, $dp/d\tau_j > 0$, and $dX/d\tau_i < 0$ (Lemma 1).

[6]First-best would be restored with a global carbon price $\tau^* = s$ plus a discriminatory output subsidy of $(\theta/n_i)[\alpha - (c + sz_i)\xi]$ to i's cleaner firms that pushes j out of the market. Here, the planner attempts to mimic this policy by instead skewing carbon pricing towards the dirtier country.

Abatement pushes optimal carbon prices towards the SCC so the result is less likely—but still applies over a significant parameter range.

5 Conclusions and related literature

The finding of extreme global asymmetry in equilibrium carbon prices—with $\tau_i^* = 0$ but simultaneously $\tau_j^* \geq s$—differs from prior literature in several respects. First, a classic literature (Buchanan 1969; Requate 2006) studies local environmental policy with imperfect competition where the planner chooses a single *domestic* emissions price. This second-best emissions price is typically less than Pigouvian, with $\tau_i^* <$ social marginal damage. By contrast, this paper has studied global welfare with multiple carbon prices.

Second, the literature on international climate policy (e.g., Babiker 2005; Fowlie, Reguant & Ryan 2016) typically examines models where a unilateral actor/coalition (e.g., OECD) pursues carbon pricing, often with $\tau_j^* < s$, while other countries (e.g., non-OECD) *exogenously* have $\tau_i \equiv 0$.[7] For example, Fowlie, Reguant & Ryan (2016) also focus on impacts of market power and international trade but from the perspective of domestic US welfare. By contrast, this paper has studied a global planner where all carbon prices are endogenous and extreme asymmetry with $\tau_i^* = 0$, $\tau_j^* \geq s$ is optimal.[8]

Third, it is known that cross-country differences in marginal abatement costs can be optimal due to equity concerns—a less rich country may have a higher marginal utility of income—and restrictions on financial transfers (Chichilnisky & Heal 1994). By contrast, this paper has obtained an extreme version of non-uniform pricing in a model without equity concerns.

Future research could incorporate this paper's approach—global-welfare maximization with imperfect competition and endogenous carbon prices—into detailed simulation models that are calibrated to global market data. This may help understand the extent to which observed asymmetries in carbon prices around the world represent second-best policy; the present analysis suggests that more carbon-intensive countries should have (much) higher carbon prices.

[7] When climate action is exogenously restricted to a subset of countries, it is second-best to set lower carbon prices for sectors with internationally-traded products—unless corrective trade tariffs are available (Hoel 1996).

[8] While results with $\tau_i^* \neq \tau_j^*$ are not surprising, the extent of the equilibrium asymmetry shown in this paper seems much less obvious.

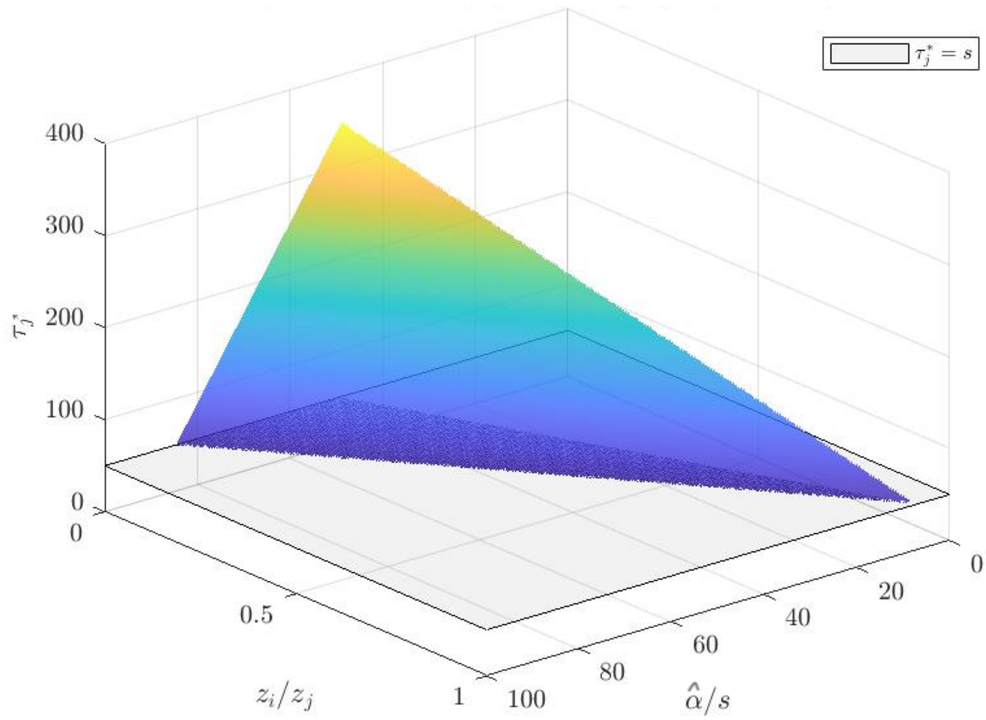

Figure 1: Parameter region for Proposition 1's result $\tau_i^* = 0$ and $\tau_j^* \geq s$

Notes: Fixes $s = 50$, $n_i/\theta = n_j/\theta = 6$, $z_j = 1$; varies z_i and $\widehat{\alpha} \equiv (\alpha - c\xi)/\xi$

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Online Appendix

This appendix solves the generalized version of the model with asymmetries in product qualities, production costs and trade costs as well as abatement by firms. It derives generalized results, Lemmas 1A–4A and Proposition 1A, which nest the results of the main text as a special case. It shows how the key insight of the main text—extreme asymmetry in socially-optimal carbon prices—is robust in this richer setting.

Define $\lambda_k(\tau_k) \equiv [\alpha_k - (c_k + \tau_k z_k)\xi_k] > 0$ as a measure of the per-unit "value-added" of country k's firms, that is, the wedge between consumers' willingness-to-pay for its product and the costs (carbon and non-carbon) of bringing it to market. A necessary condition for the viability of the market is that $\min_k \lambda_k(s) > 0$.

It will be useful to have an intuitive metric of the potential for abatement by firms. Write country k's emissions as $E_k = \overline{E}_k - A_k$ where $A_k = \sum_m a_k^m$ is total abatement and $\overline{E}_k \equiv z_k \xi_k X_k$ are "baseline" emissions. Now define, for $\tau_k > 0$, the following metric:

$$\Gamma_k \equiv \frac{A_k'(\tau_k)}{-\frac{d\overline{E}_k}{d\tau_k}} \geq 0 \tag{5}$$

as a measure of the extent to which emissions reductions come about by abatement relative to cuts in baseline emissions (via production cuts). The first-order condition for abatement from (2) implies $A_k'(\tau_k) = \frac{n_k}{\gamma_k} \geq 0$. So the metric is driven by the convexity of abatement costs; in particular, there is zero abatement $\Gamma_k \to 0$ as $\gamma_k \to \infty$. If, for example, half of a k's overall emission reduction comes by way of abatement, then $\Gamma_k = 1$.

Lemma 1A *A unilateral increase in country i's carbon price τ_i reduces its domestic production, $dX_i/d\tau_i < 0$ and its domestic emissions, $dE_i/d\tau_i < 0$, where:*
(a) the rate of output leakage $L_i^O = \frac{n_j}{(n_j+\theta)} > 0$;
(b) the rate of carbon leakage $L_i^C = \frac{1}{(1+\Gamma_i)} \frac{z_j \xi_j}{z_i \xi_i} \frac{n_j}{(n_j+\theta)} > 0$;
(c) the rate of carbon cost pass-through $\frac{dp}{d\tau_i} = \frac{n_i}{(n_i+n_j+\theta)} z_i \xi_i > 0$.

The qualitative features of the generalized model in terms of output, emissions and price responses are the same as for Lemma 1 in the main text. Abatement mitigates carbon leakage, as it leads to a stronger emissions reduction by i for any given output reduction, but not does not alter the rates of output leakage or carbon cost pass-through. All three rates remain constants with respect to carbon prices.

Asymmetry in trade costs has a comparable effect to asymmetry in emissions intensities. For example, i's rate of carbon leakage is higher if j's trade cost is relatively higher; then j's induced increase in sales is associated with relatively greater production (due to higher iceberg costs) and hence greater emissions. Asymmetries in product qualities and production costs have no direct impact on Lemma 1A.

Proof of Lemma 1A. Faced with a carbon price τ_k, the first-order condition for firm m in country k for output satisfies (3):

$$p_k - \theta x_k^m = \alpha_k - X - \theta x_k^m = (c_k + \tau_k z_k)\xi_k \tag{6}$$

Summing over all $n_i + n_j$ firms shows that the industry output and product prices, respectively, are equal to:

$$X(\tau_i, \tau_j) = \frac{n_i\left[\alpha_i - (c_i + z_i\tau_i)\xi_i\right] + n_j\left[\alpha_j - (c_j + z_j\tau_j)\xi_j\right]}{(n_i + n_j + \theta)} = \frac{n_i\lambda_i(\tau_i) + n_j\lambda_j(\tau_j)}{(n_i + n_j + \theta)} \tag{7}$$

$$p_i(\tau_i, \tau_j) = \alpha_i - \frac{n_i\lambda_i(\tau_i) + n_j\lambda_j(\tau_j)}{(n_i + n_j + \theta)}. \tag{8}$$

The optimality conditions (3) imply that $\theta X_i = n_i(\lambda_i - X)$ for country i and so:

$$X_i(\tau_i, \tau_j) = \frac{n_i}{(n_i + n_j + \theta)}\left[\lambda_i(\tau_i) + \frac{n_j}{\theta}[\lambda_i(\tau_i) - \lambda_j(\tau_j)]\right]. \tag{9}$$

For part (a), this pins down the output responses to i's own carbon price as well as to j's:

$$\frac{dX_i}{d\tau_i} = \frac{n_i}{(n_i + n_j + \theta)}\left[\frac{d\lambda_i}{d\tau_i} + \frac{n_j}{\theta}\left(\frac{d\lambda_i}{d\tau_i} - \frac{d\lambda_j}{d\tau_i}\right)\right] = -\frac{n_i(n_j + \theta)}{\theta(n_i + n_j + \theta)}z_i\xi_i < 0 \tag{10}$$

$$\frac{dX_i}{d\tau_j} = -\frac{n_i}{(n_i + n_j + \theta)}\frac{n_j}{\theta}\frac{d\lambda_j}{d\tau_j} = \frac{n_i n_j}{\theta(n_i + n_j + \theta)}z_j\xi_j > 0 \tag{11}$$

So output leakage equals $L_i^O \equiv (dX_j/d\tau_i)/(-dX_i/d\tau_i) = n_j/(n_j + \theta)$ as claimed.

For part (b), in terms of emissions, using the definition $E_k = z_k\xi_k X_k - A_k$ $(k = i, j)$, it follows that:

$$\frac{dE_i}{d\tau_i} = z_i\xi_i\frac{dX_i}{d\tau_i} - \frac{dA_i}{d\tau_i} = -\frac{n_i(n_j + \theta)}{\theta(n_i + n_j + \theta)}z_i^2\xi_i^2 - \frac{n_i}{\gamma_i} < 0 \tag{12}$$

$$\frac{dE_i}{d\tau_j} = z_i\xi_i\frac{dX_i}{d\tau_j} = \frac{n_i n_j}{\theta(n_i + n_j + \theta)}z_i\xi_i z_j\xi_j > 0 \tag{13}$$

So the rate of carbon leakage can be written as:

$$L_i^C \equiv \frac{dE_j/d\tau_i}{-dE_i/d\tau_i} = \frac{z_j\xi_j\frac{dX_j}{d\tau_i}}{-z_i\xi_i\frac{dX_i}{d\tau_i}\left(1 + \frac{\frac{dA_i}{d\tau_i}}{-z_i\xi_i\frac{dX_i}{d\tau_i}}\right)} = \frac{z_j\xi_j}{z_i\xi_i}\frac{n_j}{(n_j + \theta)}\frac{1}{(1 + \Gamma_i)}, \tag{14}$$

as claimed, where the abatement effect

$$\Gamma_i \equiv \frac{\frac{dA_i}{d\tau_i}}{-z_i\xi_i\frac{dX_i}{d\tau_i}} = \frac{\frac{n_i}{\gamma_i}}{\frac{n_i(n_j+\theta)}{\theta(n_i+n_j+\theta)}z_i^2\xi_i^2} \geq 0 \tag{15}$$

is a constant with respect to carbon prices.

Finally, for part (c), carbon cost pass-through follows directly as:

$$\frac{dp_i}{d\tau_i} = -\frac{n_i}{(n_i + n_j + \theta)}\frac{d\lambda_i}{d\tau_i} = \frac{n_i}{(n_i + n_j + \theta)}z_i\xi_i > 0, \tag{16}$$

as claimed. ■

Lemma 2A *An increase in a uniform global carbon price ($\tau_k = \tau$ for $k = i, j$):*
(a) reduces global emissions, $dE(\tau,\tau)/d\tau < 0$;
(b) reduces country k's emissions, $dE_k(\tau,\tau)/d\tau \leq 0$, if and only if carbon leakage for its unilateral carbon price $L_k^C \leq 1$.

Again, the qualitative features of the special model (Lemma 2) and the generalized model (Lemma 2A) are the same.

Proof of Lemma 2A. For part (a), global emissions in terms of carbon prices are $E(\tau_i,\tau_j) = \sum_k z_k\xi_k X_k(\tau_i,\tau_j) - \sum_k A_k(\tau_k)$, so with a uniform global carbon price $\tau_i = \tau_j = \tau$:

$$E(\tau) \equiv E(\tau,\tau) = \sum\nolimits_k z_k\xi_k X_k(\tau,\tau) - \sum\nolimits_k A_k(\tau) = \sum\nolimits_k \overline{E}_k(\tau,\tau) - \sum\nolimits_k A_k(\tau) \qquad (17)$$

As abatement is non-decreasing in the carbon price, $A_k'(\tau) \geq 0$ ($k = i, j$), a sufficient condition for global emissions to fall is that baseline emissions fall (due to output cuts), that is:

$$\frac{d}{d\tau}\sum\nolimits_k \overline{E}_k(\tau,\tau) < 0 \Longrightarrow E'(\tau) < 0. \qquad (18)$$

Using (9), country i's output faced with a global carbon price responds according to:

$$\frac{dX_i(\tau,\tau)}{d\tau} = -\frac{n_i}{(n_i+n_j+\theta)}\left[\frac{d\lambda_i}{d\tau} + \frac{n_j}{\theta}\left(\frac{d\lambda_i}{d\tau} - \frac{d\lambda_j}{d\tau}\right)\right] = -\frac{n_i}{(n_i+n_j+\theta)}\left[z_i\xi_i + \frac{n_j}{\theta}(z_i\xi_i - z_j\xi_j)\right]. \qquad (19)$$

So the response of global baseline emissions is given by:

$$\begin{aligned}\frac{d}{d\tau}\sum\nolimits_k \overline{E}_k &= z_i\xi_i\frac{dX_i(\tau,\tau)}{d\tau} + z_j\xi_j\frac{dX_j(\tau,\tau)}{d\tau} \\ &= -\frac{n_i z_i\xi_i}{(n_i+n_j+\theta)}\left[z_i\xi_i + \frac{n_j}{\theta}[z_i\xi_i - z_j\xi_j]\right] - \frac{n_j z_j\xi_j}{(n_i+n_j+\theta)}\left[z_j\xi_j + \frac{n_i}{\theta}[z_j\xi_j - z_i\xi_i]\right] \qquad (20) \\ &= -\frac{1}{\theta(n_i+n_j+\theta)}\left[n_i n_j(z_i\xi_i - z_j\xi_j)^2 + \theta(n_i z_i^2\xi_i^2 + n_j z_j^2\xi_j^2)\right] < 0 \qquad (21)\end{aligned}$$

which is always negative, as claimed.
For part (b), also noting that $\frac{dA_i(\tau)}{d\tau} = \frac{dA_i(\tau_i)}{d\tau_i}$, shows that country i's emissions respond according to:

$$\frac{d}{d\tau}E_i(\tau,\tau) = z_i\xi_i\frac{dX_i(\tau,\tau)}{d\tau} - \frac{dA_i(\tau)}{d\tau} = z_i\xi_i\frac{dX_i(\tau,\tau)}{d\tau} + z_i\xi_i\frac{dX_i}{d\tau_i}\left(\frac{\frac{dA_i(\tau_i)}{d\tau_i}}{-z_i\xi_i\frac{dX_i}{d\tau_i}}\right) = z_i\xi_i\left[\frac{dX_i}{d\tau} + \frac{dX_i}{d\tau_i}\Gamma_i\right] \qquad (22)$$

where the abatement effect $\Gamma_i \geq 0$ is constant with respect to carbon prices by (15). From above and (9), respectively, the output changes are given by:

$$\frac{dX_i(\tau,\tau)}{d\tau} = -\frac{n_i}{(n_i+n_j+\theta)}\left[z_i\xi_i + \frac{n_j}{\theta}(z_i\xi_i - z_j\xi_j)\right] \text{ and } \frac{dX_i}{d\tau_i} = -\frac{n_i(n_j+\theta)}{\theta(n_i+n_j+\theta)}z_i\xi_i < 0. \qquad (23)$$

Using these two results in the previous expression yields:

$$\frac{d}{d\tau}E_i(\tau,\tau) = -\frac{n_i}{(n_i+n_j+\theta)} z_i\xi_i \left[z_i\xi_i + \frac{n_j}{\theta}(z_i\xi_i - z_j\xi_j) + \frac{(n_j+\theta)}{\theta} z_i\xi_i\Gamma_i \right]. \tag{24}$$

It follows that i's emissions decline with a higher global carbon price whenever:

$$\frac{d}{d\tau}E_i(\tau,\tau) \leq 0 \Longleftrightarrow L_i^C = \frac{z_j\xi_j}{z_i\xi_i}\frac{n_j}{(n_j+\theta)}\frac{1}{(1+\Gamma_i)} \leq 1 \tag{25}$$

recalling the result for L_i^C for i's unilateral carbon price from Lemma 1A. ■

Lemma 3A *If country i's rate of carbon leakage is sufficiently high,*

$$L_i^C \geq 1 - \frac{1}{(1+\Gamma_i)}\frac{\theta}{(n_j+\theta)}\left[\frac{\theta}{(n_i+n_j+\theta)}\frac{\lambda_i(0)}{sz_i\xi_i} + \left[\frac{n_j}{\theta} + \frac{n_j}{(n_i+n_j+\theta)}\right]\frac{\Delta\lambda_i(0)}{sz_i\xi_i}\right] \equiv \underline{L}_i^C,$$

then a zero carbon price is welfare-dominant, $W(0,\tau_j) \geq W(\tau_i,\tau_j)$ for all $\tau_i,\tau_j \geq 0$.

Compared to Lemma 3 in the main text, there are two additional effects. First, asymmetries product qualities, production costs and trade costs—as captured by $\Delta\lambda_i(0)$—have an ambiguous effect on $\underline{L}_i^C$; if country i's value-added is higher, with $\Delta\lambda_i(0) > 0$, then this pushes $\underline{L}_i^C$ downwards as the planner then cares more about avoiding production cuts from i (and vice versa). Second, abatement—as captured by $\Gamma_i > 0$—pushes $\underline{L}_i^C$ towards 100% and thus often makes the condition more difficult to meet.

The condition of Lemma 3 that $L_i^C \geq \underline{L}_i^C$ is grossly sufficient for $W(0,\tau_j) \geq W(\tau_i,\tau_j)$. Defining the social value of i's abatement $V_i(\tau_i) \equiv n_i\left[sa_i^m(\tau_i) - \frac{\gamma_i}{2}(a_i^m(\tau_i))^2\right]$, it uses the upper bound on the marginal social value that $V_i'(\tau_i) \leq sA_i'(\tau_i) = \frac{n_i}{\gamma_i}s$ holds for all $\tau_i,\tau_j \geq 0$.

Proof of Lemma 3A. Global welfare can be expressed as:

$$W(\tau_i,\tau_j) = \overline{W}(\tau_i,\tau_j) + \sum\nolimits_k V_k(\tau_k) \tag{26}$$

where "baseline" global welfare (covering baseline emissions, without abatement) is:

$$\overline{W}(\tau_i,\tau_j) \equiv U - \sum\nolimits_k c_k\xi_k X_k(\tau_i,\tau_j) - s\overline{E}(\tau_i,\tau_j) = U - \sum\nolimits_k \left[(c_k + sz_k)\xi_k\right] X_k(\tau_i,\tau_j) \tag{27}$$

while the social value of abatement by country k's (symmetric) firms is:

$$V_k(\tau_k) \equiv n_k\left[sa_k^m(\tau_k) - \frac{\gamma_k}{2}(a_k^m(\tau_k))^2\right]. \tag{28}$$

The proofs proceeds in three steps: first, derive and bound baseline global welfare; second, derive and bound the social value of abatement; third, bring the results together to obtain a condition in terms of carbon leakage.

Step 1. Recalling that $\frac{\partial U}{\partial X_k} = p_k$, the change in baseline global welfare due to i's carbon price is given by:

$$\frac{d\overline{W}}{d\tau_i}(\tau_i, \tau_j) = \sum\nolimits_k [p_k - (c_k + sz_k)\xi_k] \frac{dX_k}{d\tau_i} \quad (29)$$

$$= \frac{dX}{d\tau_i}\left[\sum\nolimits_k [p_k - (c_k + sz_k)\xi_k] \left(\frac{dX_k}{d\tau_i} \Big/ \frac{dX}{d\tau_i}\right)\right]. \quad (30)$$

Using the results from (9) in Lemma 1A, the relative output changes are:

$$\frac{dX_i}{d\tau_i} \Big/ \frac{dX}{d\tau_i} = \frac{(n_j + \theta)}{\theta} > 0 \text{ and } \frac{dX_j}{d\tau_i} \Big/ \frac{dX}{d\tau_i} = -\frac{n_j}{\theta} < 0 \quad (31)$$

while the marginal surplus of k's output satisfies:

$$p_k - (c_k + sz_k)\xi_k = \lambda_k(s) - \frac{n_i\lambda_i(\tau_i) + n_j\lambda_j(\tau_j)}{(n_i + n_j + \theta)}. \quad (32)$$

Using these results in the expression for baseline welfare yields:

$$\frac{d\overline{W}}{d\tau_i}(\tau_i, \tau_j) = \frac{dX}{d\tau_i}\left[\sum\nolimits_k \lambda_k(s) \left(\frac{dX_k}{d\tau_i} \Big/ \frac{dX}{d\tau_i}\right) - \frac{[n_i\lambda_i(\tau_i) + n_j\lambda_j(\tau_j)]}{(n_i + n_j + \theta)}\right] \quad (33)$$

$$= \frac{dX}{d\tau_i}\left[\lambda_i(s) + \frac{n_j}{\theta}\Delta\lambda_i(s) - \frac{[n_i\lambda_i(\tau_i) + n_j\lambda_j(\tau_j)]}{(n_i + n_j + \theta)}\right] \quad (34)$$

where $\Delta\lambda_i(s) \equiv [\lambda_i(s) - \lambda_j(s)]$ is the difference in countries' value-added (at the SCC) and also

$$\frac{d^2\overline{W}}{d\tau_i^2}(\tau_i, \tau_j) = \frac{dX}{d\tau_i}\left[-\frac{n_i}{(n_i + n_j + \theta)}\frac{d\lambda_i}{d\tau_i}\right] = \frac{dX}{d\tau_i}\frac{n_i}{(n_i + n_j + \theta)}z_i\xi_i = \frac{dX}{d\tau_i}\frac{dp_i}{d\tau_i} < 0 \quad (35)$$

$$\frac{d^2\overline{W}}{d\tau_i d\tau_j}(\tau_i, \tau_j) = \frac{dX}{d\tau_i}\left[-\frac{n_j}{(n_i + n_j + \theta)}\frac{d\lambda_j}{d\tau_j}\right] = \frac{dX}{d\tau_i}\frac{n_j}{(n_i + n_j + \theta)}z_j\xi_j = \frac{dX}{d\tau_i}\frac{dp_j}{d\tau_j} < 0 \quad (36)$$

since $\frac{dX}{d\tau_i}$ is constant with respect to carbon prices by Lemma 1A. It follows that the change in baseline welfare is bounded above, for all $\tau_i, \tau_j \geq 0$, by:

$$\frac{d\overline{W}}{d\tau_i}(\tau_i, \tau_j) \leq \frac{d\overline{W}}{d\tau_i}(0, 0) = \frac{dX}{d\tau_i}\left[\lambda_i(s) + \frac{n_j}{\theta}\Delta\lambda_i(s) - \frac{[n_i\lambda_i(0) + n_j\lambda_j(0)]}{(n_i + n_j + \theta)}\right]. \quad (37)$$

Recalling that $\lambda_k(s) = \lambda_k(0) - sz_k\xi_k$, this bound can also be written in terms of emissions intensities as:

$$\frac{d\overline{W}}{d\tau_i} \leq \frac{dX}{d\tau_i}\left[\lambda_i(0) + \frac{n_j}{\theta}\Delta\lambda_i(0) - \frac{[n_i\lambda_i(0) + n_j\lambda_j(0)]}{(n_i + n_j + \theta)} - \frac{(n_j + \theta)}{\theta}sz_i\xi_i + \frac{n_j}{\theta}sz_j\xi_j\right] \quad (38)$$

$$= \frac{dX}{d\tau_i}\left[\frac{\theta}{(n_i + n_j + \theta)}\lambda_i(0) + \left[\frac{n_j}{\theta} + \frac{n_j}{(n_i + n_j + \theta)}\right]\Delta\lambda_i(0) - \frac{(n_j + \theta)}{\theta}sz_i\xi_i + \frac{n_j}{\theta}sz_j\xi_j\right]. \quad (39)$$

Step 2. As $a_k^m = \tau_k/\gamma_k$ by the first-order condition from (15), the marginal impact of a higher carbon price on the social value of abatement by country i is:

$$V_i'(\tau_i) = n_i\left(s - \gamma_i a_i^m\right)\frac{da_i^m(\tau_i)}{d\tau_i} = (s-\tau_i)A_i'(\tau_i) = \frac{n_i}{\gamma_i}(s-\tau_i), \tag{40}$$

where $V_i''(\tau_i) = -\frac{n_i}{\gamma_i} < 0$ while $V_i'(0) > 0$ but $V_i'(\cdot) < 0$ for a carbon price above the SCC. It follows that the marginal social value of abatement is bounded above, for all $\tau_i, \tau_j \geq 0$, by:

$$V_i'(\tau_i) \leq sA_i'(\tau_i) = \frac{n_i}{\gamma_i}s. \tag{41}$$

It will be useful to rewrite this, in terms of the abatement effect $\Gamma_i \equiv \frac{A_i'(\tau_i)}{-\frac{d\bar{E}_i}{d\tau_i}} \geq 0$, as follows:

$$V_i'(\tau_i) \leq sA_i'(\tau_i) = s\frac{A_i'(\tau_i)}{\frac{dX_i}{d\tau_i}}\frac{\frac{dX_i}{d\tau_i}}{\frac{dX}{d\tau_i}}\frac{dX}{d\tau_i} = sz_i\xi_i\frac{A_i'(\tau_i)}{\frac{d\bar{E}_i}{d\tau_i}}\frac{\frac{dX_i}{d\tau_i}}{\frac{dX}{d\tau_i}}\frac{dX}{d\tau_i} = -sz_i\xi_i\Gamma_i\frac{(n_j+\theta)}{\theta}\frac{dX}{d\tau_i}. \tag{42}$$

which uses the formulae of Lemma 1A.

Step 3. The final step obtains a results in terms of i's rate of carbon leakage. Combining the welfare bounds from the previous two steps shows that global welfare change due to i's carbon price is bounded, for all $\tau_i, \tau_j \geq 0$, according to:

$$\frac{dW}{d\tau_i}(\tau_i,\tau_j) \leq \frac{dX}{d\tau_i} \times \left[\frac{\theta}{(n_i+n_j+\theta)}\lambda_i(0) + \left[\frac{n_j}{\theta} + \frac{n_j}{(n_i+n_j+\theta)}\right]\Delta\lambda_i(0)\right. \tag{43}$$

$$\left. -\frac{(n_j+\theta)}{\theta}sz_i\xi_i + \frac{n_j}{\theta}sz_j\xi_j - sz_i\xi_i\Gamma_i\frac{(n_j+\theta)}{\theta}\right] \tag{44}$$

So $\frac{dW}{d\tau_i}(\tau_i,\tau_j) \leq 0$ holds whenever:

$$\frac{\theta}{(n_i+n_j+\theta)}\lambda_i(0) + \left[\frac{n_j}{\theta} + \frac{n_j}{(n_i+n_j+\theta)}\right]\Delta\lambda_i(0) - \frac{(n_j+\theta)}{\theta}sz_i\xi_i\left[(1+\Gamma_i) - \frac{n_j}{(n_j+\theta)}\frac{sz_j\xi_j}{sz_i\xi_i}\right] \geq 0 \tag{45}$$

and using the expression for carbon leakage from Lemma 1A:

$$\frac{\theta}{(n_i+n_j+\theta)}\lambda_i(0) + \left[\frac{n_j}{\theta} + \frac{n_j}{(n_i+n_j+\theta)}\right]\Delta\lambda_i(0) - \frac{(n_j+\theta)}{\theta}sz_i\xi_i(1+\Gamma_i)(1-L_i^C) \geq 0 \tag{46}$$

from which the condition on L_i^C follows, as claimed. ■

Lemma 4A *Suppose that country i's carbon price $\tau_i = 0$. Then an interior solution $\tau_j^* > 0$ for country j that maximizes $W(0,\tau_j)$ satisfies:*

$$\frac{\tau_j^*}{s} = 1 - \underbrace{\frac{\frac{\theta}{n_j}\left[\frac{\lambda_j(0)-sz_j\xi_j}{sz_j\xi_j}\right]}{\left[1+\frac{(n_i+n_j+\theta)}{n_j}\frac{(n_i+\theta)}{\theta}\Gamma_j\right]}}_{\text{market power}} + \underbrace{\frac{\frac{n_i}{n_j}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right) - \left[1+\frac{(n_i+n_j+\theta)}{\theta}\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j}\right]}{\left[1+\frac{(n_i+n_j+\theta)}{n_j}\frac{(n_i+\theta)}{\theta}\Gamma_j\right]}}_{\text{international competition \& firm heterogeneity}}$$

The basic structure of Lemma 4 from the main text is preserved. First, a standard-market power effect again pushes τ_j^* below the SCC. Second, international competition and firm heterogeneity can have the opposite effect. In particular, they push τ_j^* upwards if j has (i) a higher "trade-adjusted" emissions intensity ($z_j\xi_j \geq z_i\xi_i$), and (ii) a lower "ex-carbon" value-added ($\Delta\lambda_j(0) \leq 0$). Finally, abatement by j's firms (via $\Gamma_j > 0$) always pushes τ_j^* towards the SCC but does *not* lead to a sign change in terms of $\tau_j^* \gtreqless s$.

As expected, the Pigouvian $\tau_j^* = s$ remains optimal without international trade ($n_i = 0$) and market power ($\theta = 0$).

Proof of Lemma 4A. By assumption, $\tau_i = 0$ for country i and the optimal $\tau_j^* > 0$ for country j is interior, and the second-order condition holds (with $\frac{d^2W}{d\tau_j^2}(\tau_i, \tau_j) < 0$), so it solves the analogous expression to (43)

$$\frac{dW}{d\tau_j}(0, \tau_j^*) = \frac{dX}{d\tau_j}\left[\lambda_j(s) - \frac{n_i}{\theta}\Delta\lambda_j(s) - \frac{[n_i\lambda_i(0) + n_j\lambda_j(\tau_j^*)]}{(n_i + n_j + \theta)} - \frac{(n_i + \theta)}{\theta}(s - \tau_j^*)z_j\xi_j\Gamma_j\right] = 0 \tag{47}$$

where $\Delta\lambda_j(s) \equiv [\lambda_j(s) - \lambda_i(s)]$. Recalling that $\lambda_j(\tau_j) = \lambda_j(0) - \tau_j z_j\xi_j$, it follows that τ_j^* satisfies:

$$\lambda_j(0) - sz_j\xi_j + \frac{n_i}{\theta}\Delta\lambda_j(s) - \frac{[n_i\lambda_i(0) + n_j\lambda_j(0)]}{(n_i + n_j + \theta)} + \frac{n_j}{(n_i + n_j + \theta)}\tau_j^* z_j\xi_j = \frac{(n_i + \theta)}{\theta}z_j\xi_j(s - \tau_j^*)\Gamma_j \tag{48}$$

and so

$$\frac{n_j}{(n_i + n_j + \theta)}\tau_j^* z_j\xi_j = sz_j\xi_j + \frac{n_i}{(n_i + n_j + \theta)}\lambda_i(0) - \frac{(n_i + \theta)}{(n_i + n_j + \theta)}\lambda_j(0) - \frac{n_i}{\theta}\Delta\lambda_j(s) + \frac{(n_i + \theta)}{\theta}z_j\xi_j(s - \tau_j^*)\Gamma_j. \tag{49}$$

Therefore the optimal carbon price relative to the SCC satisfies:

$$\frac{\tau_j^*}{s} = 1 + \frac{(n_i + \theta)}{n_j} + \frac{n_i}{n_j}\frac{\lambda_i(0)}{sz_j\xi_j} - \frac{(n_i + \theta)}{n_j}\frac{\lambda_j(0)}{sz_j\xi_j} - \frac{(n_i + n_j + \theta)}{n_j}\frac{n_i}{\theta}\frac{\Delta\lambda_j(s)}{sz_j\xi_j} + \frac{(n_i + n_j + \theta)}{\theta}\frac{(n_i + \theta)}{n_j}\left(1 - \frac{\tau_j^*}{s}\right)\Gamma_j \tag{50}$$

Further rearranging now shows that:

$$\frac{\tau_j^*}{s} = 1 - \frac{\theta}{n_j}\left[\frac{\lambda_j(0)}{sz_j\xi_j} - 1\right] + \frac{n_i}{n_j}\left[1 - \frac{\Delta\lambda_j(0)}{sz_j\xi_j} - \frac{(n_i + n_j + \theta)}{\theta}\frac{\Delta\lambda_j(s)}{sz_j\xi_j}\right] + \frac{(n_i + n_j + \theta)}{n_j}\frac{(n_i + \theta)}{\theta}\left(1 - \frac{\tau_j^*}{s}\right)\Gamma_j \tag{51}$$

and noting that

$$\frac{\Delta\lambda_j(s)}{sz_j\xi_j} = \frac{\Delta\lambda_j(0)}{sz_j\xi_j} - \left(1 - \frac{z_i\xi_i}{z_j\xi_j}\right) \tag{52}$$

yields

$$\frac{\tau_j^*}{s} = 1 - \frac{\theta}{n_j}\left[\frac{\lambda_j(0) - sz_j\xi_j}{sz_j\xi_j}\right] + \frac{n_i}{n_j}\left[1 + \frac{(n_i + n_j + \theta)}{\theta}\left(1 - \frac{z_i\xi_i}{z_j\xi_j}\right) - \left[1 + \frac{(n_i + n_j + \theta)}{\theta}\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j}\right] \tag{53}$$

$$+\frac{(n_i + n_j + \theta)}{n_j}\frac{(n_i + \theta)}{\theta}\left(1 - \frac{\tau_j^*}{s}\right)\Gamma_j \tag{54}$$

from which the result follows as claimed. ■

Proposition 1A *Suppose that (i) country i has higher value-added, with $\Delta\lambda_i(0) \geq 0$, and (ii) country i's firms are sufficiently cleaner than j's according to "trade-adjusted" emissions intensities:*

$$\frac{z_i\xi_i}{z_j\xi_j} \leq \frac{\left[(n_i+\theta)(n_i+n_j+\theta)+n_j^2\right]+\theta^2 G(\Gamma_j)}{\left[(n_i+\theta)(n_i+n_j+\theta)+n_j(n_j+\theta)\right]+\theta^2 J(\Gamma_i,\Gamma_j)} \equiv \delta \in (0,1) \tag{55}$$

where

$$G(\Gamma_j) \equiv \frac{n_j}{\theta}\frac{(n_i+\theta)}{\theta}\frac{(n_i+n_j+\theta)}{\theta}\Gamma_j \tag{56}$$

$$J(\Gamma_i,\Gamma_j) \equiv \frac{(n_j+\theta)}{\theta}\frac{(n_i+n_j+\theta)}{\theta}\left\{\frac{(n_i+\theta)}{\theta}\Gamma_j+\left[1+\frac{(n_i+\theta)}{\theta}\Gamma_j\right]\Gamma_i\right\} \geq G(\Gamma_j) \tag{57}$$

with $G(0) = J(0,0) = 0$, $\frac{\partial}{\partial\Gamma_j}G(\Gamma_j) > 0$, $\frac{\partial}{\partial\Gamma_i}J(\Gamma_i,\Gamma_j) > 0$, $\frac{\partial}{\partial\Gamma_j}J(\Gamma_i,\Gamma_j) > 0$. Then, for the range of parameter values given by

$$\frac{\lambda_j(0)}{sz_j\xi_j} \leq 1+\frac{n_i}{\theta}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]-\frac{n_i}{\theta}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j} \equiv A \tag{58}$$

$$\frac{\lambda_j(0)}{sz_j\xi_j} \geq \frac{\frac{(n_i+\theta)}{\theta}\left[1+\frac{n_i}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]-\frac{n_i}{\theta}\left[1+\frac{(n_i+\theta)}{\theta}(1+\Gamma_j)\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j}}{\left[1+\frac{(n_i+\theta)}{\theta}\Gamma_j\right]} \equiv B \tag{59}$$

where $A > B$, welfare-optimal carbon prices are $\tau_i^* = 0$ while $\tau_j^* \geq s$.

The key point is that Proposition 1's insight of extreme asymmetry in socially-optimal carbon prices continues to hold in the generalized model. If country i has higher value-added—capturing relative product quality, production costs and trade costs—then, due to market power, the planner is "biased" against increasing its carbon price. Moreover, if i's firms are sufficiently cleaner, as per $\frac{z_i\xi_i}{z_j\xi_j} \leq \delta < 1$ (which, together with $\Delta\lambda_i(0) > 0$, implies that $\Delta\lambda_i(s) > 0$), then the planner wishes to levy a much higher carbon price on j.

To get a sense of magnitudes, without any abatement, $\Gamma_i = \Gamma_j = 0$ (as $\gamma_i \to \infty, \gamma_j \to \infty$), as in the main text, the condition has $\delta = \frac{127}{133}$ so only a small asymmetry in emissions intensities is needed. If, instead, $\Gamma_i = \Gamma_j = 1$ (by appropriate choice of γ_i, γ_j) then $\delta \simeq .4$ so the condition is considerably tighter.[9] However, regardless of i's and j's abatement, there always exists a relative emissions intensity that is "sufficiently low" for the result to obtain.

Moreover, if i has no potential for abatement but j does ($\Gamma_i = 0 < \Gamma_j$), this pushes δ towards 1 as $J(0,\Gamma_j) = G(\Gamma_j)$. That is, the planner then wishes to levy a relatively higher carbon price on j to also exploit its superior abatement opportunities.

Proof of Proposition 1A. The proof has four steps. First, to identify conditions under which $\tau_j^* \geq s$. Second, to identify conditions under which j's firms remain profitable under this τ_j^*.

[9] Given $\tau_i^* = 0$, country i's firms do not engage in any abatement in equilibrium but the sufficient condition from Lemma 3 still requires i's "potential" abatement (via Γ_i) to be sufficiently small such that $L_i^C \geq \underline{L}_i^C$.

Third, to obtain a condition under which $\tau_i^* = 0$ is indeed socially-optimal. Fourth, to derive overarching parameter conditions spanning those from the three previous steps.

Step 1. Suppose that $\tau_i^* = 0$ and that optimal $\tau_j^* > 0$. for country j is interior. If so, then τ_j^* satisfies the expression in Lemma 4A, and therefore $\tau_j^* \geq s$ holds if and only if:

$$\frac{\frac{n_i}{n_j}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)-\left[1+\frac{(n_i+n_j+\theta)}{\theta}\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j}\right]}{\left[1+\frac{(n_i+n_j+\theta)}{n_j}\frac{(n_i+\theta)}{\theta}\Gamma_j\right]} \geq \frac{\frac{\theta}{n_j}\left[\frac{\lambda_j(0)-sz_j\xi_j}{sz_j\xi_j}\right]}{\left[1+\frac{(n_i+n_j+\theta)}{n_j}\frac{(n_i+\theta)}{\theta}\Gamma_j\right]} \tag{60}$$

which rearranges as:

$$\frac{\lambda_j(0)}{sz_j\xi_j} \leq 1+\frac{n_i}{\theta}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]-\frac{n_i}{\theta}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j} \equiv A. \tag{61}$$

Step 2. By the first-order condition in (3), j's firms certainly remain profitable (with $\Pi_j^m \geq 0$) under this τ_j^* (regardless of the extent of abatement) as long as they have a positive profit margin, with $p_j(0,\tau_j^*) \geq (c_j + z_j\tau_j^*)\xi_j$. Using the equilibrium price from Lemma 1A and rearranging shows that this is equivalent to:

$$p_j(0,\tau_j^*) \geq (c_j+z_j\tau_j^*)\xi_j \iff \lambda_j(\tau_j^*) \geq \frac{\left[n_i\lambda_i(0)+n_j\lambda_j(\tau_j^*)\right]}{(n_i+n_j+\theta)} \tag{62}$$

which, recalling that $\lambda_j(\tau_j^*) = \lambda_j(0) - \tau_j^* z_j\xi_j$, can be written as:

$$p_j(0,\tau_j^*) \geq (c_j+z_j\tau_j^*)\xi_j \iff \frac{\tau_j^*}{s} \leq \frac{\theta}{(n_i+\theta)}\frac{\lambda_j(0)}{sz_j\xi_j}+\frac{n_i}{(n_i+\theta)}\frac{\Delta\lambda_j(0)}{sz_j\xi_j} \tag{63}$$

so j's carbon price cannot be too high. Now, using the expression for τ_j^* from Lemma 4A, this condition is equivalent to:

$$\frac{\theta}{(n_i+\theta)}\frac{\lambda_j(0)}{sz_j\xi_j}+\frac{n_i}{(n_i+\theta)}\frac{\Delta\lambda_j(0)}{sz_j\xi_j} \;\geq\; 1-\frac{\frac{\theta}{n_j}\left[\frac{\lambda_j(0)-sz_j\xi_j}{sz_j\xi_j}\right]}{\left[1+\frac{(n_i+n_j+\theta)}{n_j}\frac{(n_i+\theta)}{\theta}\Gamma_j\right]} \tag{64}$$

$$+\frac{\frac{n_i}{n_j}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)-\left[1+\frac{(n_i+n_j+\theta)}{\theta}\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j}\right]}{\left[1+\frac{(n_i+n_j+\theta)}{n_j}\frac{(n_i+\theta)}{\theta}\Gamma_j\right]} \tag{65}$$

or

$$\frac{\lambda_j(0)}{sz_j\xi_j}\left[\frac{\theta}{n_j}+\frac{\theta}{(n_i+\theta)}+\frac{(n_i+n_j+\theta)}{n_j}\Gamma_j\right] \;\geq\; \frac{(n_i+n_j+\theta)}{n_j}\left[1+\frac{n_i}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]$$
$$-\left[\frac{n_i}{n_j}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\right]+\frac{n_i}{(n_i+\theta)}+\frac{n_i}{\theta}\frac{(n_i+n_j+\theta)}{n_j}\Gamma_j\right]$$

which rearranges as:

$$\frac{\lambda_j(0)}{sz_j\xi_j}\left[\frac{\theta}{(n_i+\theta)}+\Gamma_j\right] \geq \left[1+\frac{n_i}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]-\left[\frac{n_i}{(n_i+\theta)}+\frac{n_i}{\theta}(1+\Gamma_j)\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j} \tag{68}$$

and so:

$$\frac{\lambda_j(0)}{sz_j\xi_j} \geq \frac{\frac{(n_i+\theta)}{\theta}\left[1+\frac{n_i}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]-\frac{n_i}{\theta}\left[1+\frac{(n_i+\theta)}{\theta}(1+\Gamma_j)\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j}}{\left[1+\frac{(n_i+\theta)}{\theta}\Gamma_j\right]} \equiv B. \tag{69}$$

Step 3. By Lemma 3A, zero carbon price $\tau_i^* = 0$ is certainly socially-optimal if $L_i^C \geq \underline{L}_i^C$ while, by Lemma 1A, the rate of carbon leakage is given by $L_i^C = \frac{z_j\xi_j}{z_i\xi_i}\frac{n_j}{(n_j+\theta)}\frac{1}{(1+\Gamma_i)}$. Combining these two results therefore shows that $\tau_i^* = 0$ holds if:

$$\frac{z_j\xi_j}{z_i\xi_i}\frac{n_j}{(n_j+\theta)}\frac{1}{(1+\Gamma_i)} \geq 1-\frac{1}{(1+\Gamma_i)}\frac{\theta}{(n_j+\theta)}\left[\frac{\theta}{(n_i+n_j+\theta)}\frac{\lambda_i(0)}{sz_i\xi_i}+\left[\frac{n_j}{\theta}+\frac{n_j}{(n_i+n_j+\theta)}\right]\frac{\Delta\lambda_i(0)}{sz_i\xi_i}\right] \tag{70}$$

which can be rearranged as:

$$\frac{\theta}{(n_i+n_j+\theta)}\frac{\lambda_i(0)}{sz_i\xi_i}+\left[\frac{n_j}{\theta}+\frac{n_j}{(n_i+n_j+\theta)}\right]\frac{\Delta\lambda_i(0)}{sz_i\xi_i} \geq \left[1+\frac{n_j}{\theta}\left(1-\frac{z_j\xi_j}{z_i\xi_i}\right)\right]+\frac{(n_j+\theta)}{\theta}\Gamma_i. \tag{71}$$

To make this expression directly comparable with the condition from Step 2, note that $\lambda_i(0) = \lambda_j(0) - \Delta\lambda_j(0)$ and $\Delta\lambda_i(0) = -\Delta\lambda_j(0)$, and so the expression becomes:

$$\frac{\theta}{(n_i+n_j+\theta)}\frac{\lambda_j(0)}{sz_j\xi_j}\frac{z_j\xi_j}{z_i\xi_i} \geq \left[1+\frac{n_j}{\theta}\left(1-\frac{z_j\xi_j}{z_i\xi_i}\right)\right] \tag{72}$$

$$+\left[\frac{n_j}{\theta}+\frac{n_j}{(n_i+n_j+\theta)}+\frac{\theta}{(n_i+n_j+\theta)}\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j}\frac{z_j\xi_j}{z_i\xi_i}+\frac{(n_j+\theta)}{\theta}\Gamma_i \tag{73}$$

Further rearranging gives:

$$\frac{\lambda_j(0)}{sz_j\xi_j} \geq \frac{(n_i+n_j+\theta)}{\theta}\left[\frac{z_i\xi_i}{z_j\xi_j}-\frac{n_j}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]+\left[1+\frac{n_j}{\theta}\left[1+\frac{(n_i+n_j+\theta)}{\theta}\right]\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j} \tag{74}$$

$$+\frac{(n_i+n_j+\theta)}{\theta}\frac{(n_j+\theta)}{\theta}\frac{z_i\xi_i}{z_j\xi_j}\Gamma_i \equiv C. \tag{75}$$

Step 4. The final step involves (i) verifying that $A > B$ under the assumptions of Proposition 1A, and (ii) deriving a further parameter condition that ensures $B \geq C$. First, direct calculation shows that $A > B$ holds if and only if:

$$\frac{n_i}{\theta}\frac{n_j}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)+\frac{(n_i+\theta)}{\theta}\Gamma_j\left[\frac{(n_i+\theta)}{\theta}+\frac{n_i}{\theta}\frac{(n_i+n_j+\theta)}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right] \geq \tag{76}$$

$$\frac{n_i}{\theta}\left[\frac{n_j}{\theta}+\frac{(n_i+\theta)}{\theta}\frac{(n_i+n_j+\theta)}{\theta}\Gamma_j\right]\frac{\Delta\lambda_j(0)}{sz_j\xi_j} \tag{77}$$

which holds under the maintained assumptions that i's value-added is higher, $\Delta\lambda_i(0) \geq 0 \iff \Delta\lambda_j(0) \leq 0$, and that i's trade-adjusted emissions intensity is lower, $z_i\xi_i/z_j\xi_j < 1 \iff \Delta\lambda_j(s) <$

$\Delta\lambda_j(0)$.

Second, to obtain a sufficient condition for $B \geq C$, note that the maintained assumption $\Delta\lambda_i(0) \geq 0 \iff \Delta\lambda_j(0) \leq 0$ implies that:

$$B \geq \frac{\frac{(n_i+\theta)}{\theta}\left[1+\frac{n_i}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]}{\left[1+\frac{(n_i+\theta)}{\theta}\Gamma_j\right]} \tag{78}$$

$$C \leq \frac{(n_i+n_j+\theta)}{\theta}\left[\frac{z_i\xi_i}{z_j\xi_j}-\frac{n_j}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]+\frac{(n_i+n_j+\theta)}{\theta}\frac{(n_j+\theta)}{\theta}\frac{z_i\xi_i}{z_j\xi_j}\Gamma_i \tag{79}$$

Therefore a sufficient condition is that:

$$\frac{(n_i+\theta)}{\theta}\left[1+\frac{n_i}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right] \geq \frac{(n_i+n_j+\theta)}{\theta}\left[\frac{z_i\xi_i}{z_j\xi_j}-\frac{n_j}{\theta}\left(1-\frac{z_i\xi_i}{z_j\xi_j}\right)\right]-G(\Gamma_j)+\frac{z_i\xi_i}{z_j\xi_j}J(\Gamma_i,\Gamma_j) \tag{80}$$

where the abatement terms are defined as:

$$G(\Gamma_j) \equiv \frac{n_j}{\theta}\frac{(n_i+\theta)}{\theta}\frac{(n_i+n_j+\theta)}{\theta}\Gamma_j \tag{81}$$

$$J(\Gamma_i,\Gamma_j) \equiv \frac{(n_j+\theta)}{\theta}\frac{(n_i+n_j+\theta)}{\theta}\left\{\frac{(n_i+\theta)}{\theta}\Gamma_j+\left[1+\frac{(n_i+\theta)}{\theta}\Gamma_j\right]\Gamma_i\right\} \geq G(\Gamma_j) \tag{82}$$

with $G(0) = J(0,0) = 0$, $\frac{\partial}{\partial\Gamma_j}G(\Gamma_j) > 0$, $\frac{\partial}{\partial\Gamma_i}J(\Gamma_i,\Gamma_j) > 0$, $\frac{\partial}{\partial\Gamma_j}J(\Gamma_i,\Gamma_j) > 0$. Rearranging in terms emissions intensities now gives:

$$\frac{(n_i+\theta)}{\theta}\frac{(n_i+\theta)}{\theta}+\frac{n_j}{\theta}\frac{(n_i+n_j+\theta)}{\theta}+G(\Gamma_j) \geq \left[\frac{n_i}{\theta}\frac{(n_i+\theta)}{\theta}+\frac{(n_i+n_j+\theta)}{\theta}\frac{(n_j+\theta)}{\theta}+J(\Gamma_i,\Gamma_j)\right]\frac{z_i\xi_i}{z_j\xi_j} \tag{83}$$

and so:

$$\frac{z_i\xi_i}{z_j\xi_j} \leq \frac{(n_i+\theta)(n_i+\theta)+n_j(n_i+n_j+\theta)+\theta^2G(\Gamma_j)}{[n_i(n_i+\theta)+(n_j+\theta)(n_i+n_j+\theta)+\theta^2J(\Gamma_i,\Gamma_j)]} \equiv \delta \in (0,1) \tag{84}$$

as claimed. In sum, whenever $\lambda_j(0)/sz_j\xi_j$ exceeds B, and also exceeds C given that $z_i\xi_i/z_j\xi_j \leq \delta$, $\tau_i^* = 0$ is indeed optimal and j's firms remain profitable under the τ_j^* of Lemma 4A which, as long as $\lambda_j(0)/sz_j\xi_j$ is less than A, exceeds the SCC. ■