Testing for Changing Persistence in U.S. Treasury On/Off Spreads Under Weighted-Symmetric Estimation

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SUMMARY

The debt management policy changes of 1998-2001 and subsequent reversal of the U.S. government's fiscal position have prompted research on the dynamics of the U.S. Treasury bond market. We extend the recursive break test procedure of Leybourne et al. by using weighted-symmetric estimation to detect a single change in persistence in U.S. Treasury on/off spreads. It is found that a significant change from $I(0)$ to $I(1)$ occurred in the late 1990s, which appears to be linked to changes in the U.S. Treasury's debt management policy. Monte Carlo evidence shows that correcting for conditional heteroskedasticity in the data can successfully deal with the tests being oversized, albeit at a considerable loss in power for smaller sample sizes and large short-run variation in volatility. It is therefore advisable mainly for large sample sizes.

Keywords: Persistence; unit root test; break point; Monte Carlo simulation; U.S. Treasury bonds, liquidity

JEL classification codes: C15; C22; G10

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1. INTRODUCTION

Commentators have noted that U.S. budget surpluses in the late 1990s led to a staged contraction in the supply of Treasury bonds with a series of debt management policy changes since 1998, notably a reduction in their issuance frequencies; see Boni and Leach (2002), Fleming (2000, 2002) and Furfine and Remolona (2002). These were adopted against the background of a sustained upbeat fiscal environment, leading to the Treasury's debt buyback program in March 2000.[1]

The U.S. budget position reverted sharply after 2000—the initial deterioration was likely triggered by the mild recession of 2001 and the impact of the terrorism acts in September of that year. Indeed, the months of September and October 2001 have been identified as a *monetary* policy regime switch associated with very high residual variance in money demand; see Sims and Zha (2006). The U.S government's tax cuts coupled with increased defense spending and the "war on terrorism" have caused further deterioration in the fiscal outlook; see Gale and Orszag (2004) and Auerbach, Gale and Orszag (2006). More recently, the 30-year bond was brought back in the first quarter of 2006.

The above developments have meant a higher profile for policies related to managing the U.S government's borrowing needs over time. They also suggest that yield curve

[1] Changes included: January 1998, when the 3-year note was discontinued; May 1998 and February 2000, when the 5-year note and 1-year bill's auction frequencies were reduced from monthly to quarterly; and February and October 2001, when the 1-year bill and 30-year bond were discontinued. The Treasury also increased issue sizes, leading to more liquidity through lower inventory costs. The reduction in the supply of traded securities as a result of the budget surpluses is particularly noteworthy in the case of the 5-year note. Although awarded amounts per auction increased, maturing 5-year notes decreased from $162 billion in 2001 to $61 billion in 2005 based on U.S. General Accounting Office figures. The issuance frequency of the 5-year note was restored to monthly in mid-2003.

liquidity continues to be an important factor affecting the dynamics of the Treasury market. In principle, Treasury bonds whose remaining time to maturity and other characteristics are similar should trade at approximately the same price. However, less liquid (older, or *off-the-run*) bond yields are often higher than their more liquid (most recent, or *on-the-run*) counterparts, especially at the longer end of the term structure. Researchers have interpreted this yield differential, typically between the first off-the-run and the on-the-run issues at each maturity, as a time-varying liquidity premium which is expected to be mean-reverting by market efficiency. Krishnamurthy (2002) and Longstaff (2004), among others, have documented the significance of the on/off spread across the term structure. It is worth noting the different classes of investors that are likely to be holders of the on versus off-the run issues; the off-the run issues though illiquid are likely to be held by longer term investors such as central banks and insurance companies, whereas the more recent on the run issues are likely to be traded.

While Treasury securities' on/off spreads to a large extent capture investors' time-varying liquidity preferences, another component of the on/off spread involves interest rate risk. This arises because the on-the-run and first off-the-run securities need not lie on the same point on the yield curve, despite being very close. As pointed out by Goldreich, Hanke and Nath (2005), if the yield curve is sloping we would expect them to have different yields even in the absence of any liquidity effect. Any yield curve effects would tend to have a greater impact on shorter than longer maturities. Thus, lowering the issuance frequency of maturities at the shorter end of the term structure would cause greater exposure to interest rate risk, and potentially affect the time series properties of their on/off spreads.

The main aim of this paper is to study the persistence of the U.S. Treasury's 1-year bill and 5-year note weekly on/off spreads. We are interested in these particular maturities because both were significantly affected by the debt management policy changes of the late 1990s and the subsequent monetary policy changes. This raises the question of whether there was a change in persistence of Treasury bond liquidity premia from a stationary, $I(0)$, to a nonstationary, $I(1)$, process. Determining the location and direction of such changes is a key issue for policy makers and market forecasters alike; see Kim (2000) and Newbold, Leybourne, Sollis and Womar (2001). Additionally, wrongly characterising the behaviour of economic time series has profound implications for econometric modelling strategies and forecasting accuracy.

The null hypothesis is that the data is $I(1)$ throughout, and the alternative is a change from $I(0)$ to $I(1)$ at some point in the series. The recursive procedure of Leybourne, Kim, Smith and Newbold (2003)—henceforth LKSN—is extended by adopting weighted-symmetric (WS) estimation of the unit root coefficient. Under stationary alternatives and OLS detrending, this estimation method yields a more powerful unit root t-test than standard Dickey-Fuller and its Generalised Least Squares (GLS)-detrended version proposed by Elliott, Rothenberg and Stock (1996).[2] LKSN develop GLS-based recursive and sequential unit root tests for detecting a single possible change in persistence under the alternative. The tests allow for an unknown breakpoint and, in their general form, unknown direction of change in persistence. Based on Monte Carlo

[2] On related power gains see Leybourne, Kim and Newbold (2005) and Pantula,Gonzalez-Farias and Fuller (1994).

evidence, they find the recursive tests to be the most favourable.

Our results suggest that the single-break methodology lends itself well to time series affected by low-frequency events. We find evidence of a significant change in the persistence of U.S. Treasury bond on/off spreads in the late 1990s. This is likely related to financial market uncertainty prompted by Russia's default and LTCM's near-collapse in autumn 1998, as well as to the debt management policy changes implemented by the U.S. Treasury in 1998-2001. The subsequent macroeconomic uncertainty regarding the deterioration and likely future course of the Federal budget deficit may be contributing to the higher persistence.

To assess these empirical findings, we evaluate the size and power properties of the recursive WS tests and the accuracy of the break point estimator in a Monte Carlo study. As persistent conditional heteroscedasticity and excess kurtosis are pervasive in financial time series, including those under investigation, the properties of the test statistics need to be examined within this context. In doing so, we also shed light on the usefulness of White-correcting in the presence of GARCH and non-normality. The simulation evidence suggests that recursive WS tests display more size distortions than their non-recursive counterparts. The overrejections arise mainly for GARCH parameters involving near-integration and significant short-run variation in volatility, as documented by Kim and Schmidt (1993) and Seo (1999). Correcting for GARCH using White's (1980) heteroskedasticity-consistent covariance matrix effectively deals with the overrejections, although in certain cases power loss is considerable.

The remainder of this paper is arranged as follows. Section 2 presents the model.

Section 3 contains an application to U.S. Treasury bond on/off spreads. In Section 4 we carry out a Monte Carlo study on the performance of the proposed WS-based statistics. Section 5 concludes.

2. THE MODEL

Assume that the true data generating process for T observations on y_t is

$$\begin{aligned} y_t &= d_t + u_t \ , \quad d_t = z_t'\beta \\ u_t &= \alpha u_{t-1} + \phi(L)\Delta u_{t-1} + \epsilon_t \ , \end{aligned} \tag{1}$$

where $z_t = [1, t]'$ and $\beta = [\beta_0, \beta_1]'$. We restrict attention to $\beta_1 = 0$, without loss of generality. Lag polynomial $\phi(L)$ is of known order $p-1$, where the roots of $1-\phi(L) = 0$ lie outside the unit circle. The errors follow a martingale difference sequence and the first $p-1$ values of y_t are assumed to exist.

The null hypothesis H^{11} is that y_t is $I(1)$ throughout, or $\alpha = 1$. The alternative is that y_t undergoes a change in persistence from $I(0)$ to $I(1)$ at observation τ^*T in forward time,

$$\begin{aligned} |\alpha| &< 1 \ , \qquad t \leq \tau^*T \\ \alpha &= 1 \ , \qquad t > \tau^*T \end{aligned} \tag{2}$$

or from $I(1)$ to $I(0)$, implying the time-reversed series $\widetilde{y}_t = y_{T-t+1}$, $t = 1, 2, ..., T$ changes from $I(0)$ to $I(1)$ at observation $(1-\tau^*)T$, where the break fraction τ^* is unknown.

The respective alternative hypotheses are denoted H^{01} and H^{10}.

Our test statistics are constructed as follows. After detrending the series by OLS, $y_t^d = y_t - \hat{\beta}_0(\tau),\ t = 1, 2, ..., T$, an ADF regression with no deterministic trend is ran on Δy_t^d using only the first $[\tau T]$ observations for varying break fraction τ,

$$\Delta y_t^d = \widehat{\rho}(\tau) y_{t-1}^d + \Sigma_{j=1}^{p-1}\ \hat{\phi}_j(\tau)\Delta y_{t-j}^d + \hat{\epsilon}_t\ , \qquad t = 1, 2, ..., [\tau T] \tag{3}$$

where $[\cdot]$ is the integer part of τT and τ belongs to a non-empty closed interval in $(0, 1)$, denoted Λ.

In this setting, weighted-symmetric estimation of $\rho(\tau)$—proposed originally in Fuller (1976)—minimises

$$\begin{aligned} Q(\boldsymbol{\theta}) \quad &= \quad \sum_{t=p+1}^{[\tau T]} w_t \left(\Delta y_t^d - \rho(\tau) y_{t-1}^d - \sum_{j=1}^{p-1} \phi_j(\tau) \Delta y_{t-j}^d \right)^2 & (4) \\ &+ \sum_{t=1}^{[\tau T]-p} (1 - w_{t+1}) \left(\Delta y_t^d - \rho(\tau) y_{t+1}^d + \sum_{j=1}^{p-1} \phi_j(\tau) \Delta y_{t+j+1}^d \right)^2 & (5) \end{aligned}$$

over all τ, and $\boldsymbol{\theta} = (\rho, \boldsymbol{\phi})$, $\boldsymbol{\phi} = \{\phi_1, \phi_2, ..., \phi_{p-1}\}$ with w_t defined as

$$w_t = \begin{cases} 0, & 1 \le t < p + 1 \\ (t - p)/([\tau T] - 2p + 2), & p + 1 \le t < [\tau T] - p + 2 \\ 1, & [\tau T] - p + 2 \le t \le [\tau T]\ . \end{cases}$$

The t-statistic associated with $\widehat{\rho}(\tau)$ under the null hypothesis is $WS(\tau) = \frac{\widehat{\rho}(\tau)}{\sqrt{\widehat{var}(\widehat{\rho}(\tau))}}$, where $\widehat{var}(\widehat{\rho}(\tau)) = \widehat{\sigma}^2(\tau) h_{PP}$, the estimated error standard deviation is $\widehat{\sigma}(\tau) = \frac{Q(\widehat{\theta})}{[\tau T]-p-2}$

and h_{PP} is the $[1,1]$ element of the $\left(\frac{\partial^2 Q(\boldsymbol{\theta})}{\partial\theta\partial\theta'}\right)^{-1}$ matrix.[3]

The statistic for testing the alternative H^{01} is given by

$$WS^{f\ inf}(\tau) = inf_{\ \tau\in\ \Lambda}\ WS^{f}(\tau), \tag{6}$$

where f denotes the recursive test in forward time and τ^* is the break fraction minimising equation (6). When the alternative hypothesis is a switch from $I(1)$ to $I(0)$, this test statistic can be applied to the first-difference of the time-reversed series $\widetilde{y}_t^d$

$$\Delta\widetilde{y}_t^d = \widetilde{\rho}(\tau)\widetilde{y}_{t-1}^d + \Sigma_{j=1}^{p-1}\,\widetilde{\phi}_j(\tau)\Delta\widetilde{y}_{t-j}^d + \widetilde{\epsilon}_t\ , \qquad t = 1, 2, ..., (1-\tau)T. \tag{7}$$

Let the t-ratio for $\widetilde{\rho}(\tau)$ be $WS^r(\tau)$. The statistic for testing H^{11} against H^{10} is then given by

$$WS^{r\ inf}(\tau) = inf_{\ \tau\in\ \Lambda}\ WS^{r}(\tau)\ , \tag{8}$$

with r denoting the test on the time-reversed series.

If one is *a priori* uncertain about the direction of change in persistence, a "two-sided" test can be constructed whose null is $I(1)$ throughout against the alternative of a change from $I(0)$ to $I(1)$ or *vice versa* at break fraction τ^*. The statistic is then the pairwise minimum of $WS^{f\ inf}$ and $WS^{r\ inf}$

$$\min(WS^{f\,\mathrm{inf}}, WS^{r\,\mathrm{inf}}). \tag{9}$$

[3]Note that if a trend is included in the regression the denominator for $\widehat{\sigma}(\tau)$ becomes $[\tau T] - p - 3$.

Following LKSN and existing asymptotic results in Pantula,Gonzalez-Farias and Fuller (1994) and Park and Fuller (1995), the $WS^{f\inf}$ and $WS^{r\inf}$ tests will be consistent only under the alternative hypothesis for which they are designed. Thus, the $\min(WS^{f\inf}, WS^{r\inf})$ test will also be consistent under H^{01} or H^{10}. Moreover, all test statistics can be shown to estimate the break fraction consistently against the true alternative. From these results it further follows that the ADF and non-recursive WS tests are inconsistent under a break in persistence, as the random walk component of the series will dominate these statistics and render them $O_p(1)$. In the sequel, WS refers to the statistic using the non-recursive weighted-symmetric estimation procedure, and WS-based tests refer to both the recursive and the non-recursive statistics.

3. APPLICATION TO U.S. TREASURY ON/OFF SPREADS

Our sample period extends from 17.6.1991 to 31.12.2002, i.e. 504 weekly observations on the levels of the 1-year Treasury bill on/off spread—the yield differential between the first off-the-run and the on-the-run issues—and 592 for the 5-year note on/off spread.[4] Figure 1 shows the levels and first-differences of the two series in basis points and Table 1 Panel A summarises their distributional properties.

FIGURE 1 AND TABLE 1 HERE

Both spreads are tightly distributed around their mean until the late 1990s, when they become more volatile, and there is significant excess kurtosis and GARCH effects.

[4] We select Wednesday observations from the daily data to address day-of-the-week effects. Inflation-indexed and callable bond issues are excluded, as are holidays and observations more than 30 basis points, reflecting posting errors. Data source: GovPX.

From early 1999 the volatility of both on/off spreads increases sharply. This is reflected both in the levels and first-differences of the series and may relate as much to the reduction in the maturities' issuance frequency, implying more interest rate risk, as to investors' increasingly uncertain outlook. Note that in 1999 and early 2000 the 1-year on/off spread displays a positive trend, related to the Fed's "preemptive" tightening in early 1999 that brought the federal funds rate to 6.5 percent by May 2000. Subsequently, however, and until October 2001 when it was discontinued, the 1-year spread becomes much more volatile, reflecting financial market uncertainty in the aftermath of the dot.com bubble. This is consistent with the timing of the monetary policy regime switch located by Sims and Zha (2006). Also note that from mid-2001 to late 2002 the 5-year note on/off spread has a negative trend, likely due to the sharp inversion in the U.S. yield curve following the bursting of the bubble and the events of September 11, 2001. Against that background, a *flight-to-liquidity* (Longstaff, 2004) likely occurred, whereby market participants expressed a strong preference for the highly liquid (on-the-run) Treasury securities. The federal funds rate was lowered by 4.75 percent during 2001 and another 0.75 percent by June 2003, bringing it to a 45-year low of about 1 percent; see Greenspan (2004).

Table 1 Panel B reports estimates of an AR-GARCH(1,1) process $y_t = c + ay_{t-1} + \epsilon_t$, where $\epsilon_t = h_t^{1/2} v_t$, $h_t = \phi_0 + \phi_1 \epsilon_{t-1}^2 + \phi_2 \ h_{t-1}$, with Student's-t innovations. The ϕ_1 coefficient estimate, capturing short-run variation in volatility, is approximately 0.12 for both series, and $\phi_1 + \phi_2$ (persistence in volatility) is around 0.98. While one could experiment with alternative specifications, these results should be treated with caution as in the presence of a break the estimate of a could be biased, affecting GARCH parameter estimates. Table 1 Panel C reports the results of ADF and WS tests for the whole sample. Critical values for these and subsequent tests are given in Appendix A. These are based on 20,000 replications.

The lag order p is selected using the sequential 0.10 level t-tests for the longest lag significance, recommended by Ng and Perron (1995). We use the same p for WS-based tests as selected from the standard ADF regressions in equations (6) and (7).[5] Nonstationarity is not rejected for the 5-year on/off spread, while it is for the 1-year spread. However, when the WS statistic is corrected for GARCH, nonstationarity is not rejected for either series; WS_w denotes the White-corrected statistic.

In general, standard unit root tests are asymptotically valid in the presence of conditional heteroskedasticity. The robustness of unit root limit theory to conditional heterogeneity was noted by, among others, Phillips (1987) and Phillips and Perron (1988). However, simulations reported in Kim and Schmidt (1993) and Seo (1999) indicate that DF statistics tend to overreject the null hypothesis when GARCH errors are persistent, and to decrease towards nominal size at a very slow rate as T increases. The former authors also consider DF t-ratios using White's (1980) heteroskedasticity-consistent covariance matrix estimator, and find that the observed size distortions can be eliminated. We quantify these findings for WS-based tests in the Monte Carlo study of Section 4.

Turning to the recursive tests, the discussion in Section 1 suggested that the al-

[5] A trend term is not included in the regressions on market efficiency grounds.

ternative hypothesis is a change in persistence from $I(0)$ to $I(1)$ at observation τ^*T. Hence, the $WS^{f\,\inf}$ test in equation (6) is applied to the series, as it is expected to be more powerful than the two-sided $\min(WS^{f\,\inf}, WS^{r\,\inf})$ test.[6] The results in Table 2, Panels A and B contain both the non-White-corrected, $WS^{f\,\inf}$, and White-corrected, $WS_w^{f\,\inf}$, versions. Results for the reverse and "two-sided" test statistics, respectively $WS^{r\ inf}$ and $\min(WS^{f\,\inf}, WS^{r\,\inf})$, are also included.

TABLE 2 HERE

For the 1-year Treasury on/off spread, the $WS^{f\,\inf}$ and $\min(WS^{f\,\inf}, WS^{r\,\inf})$ tests both reject the unit root null at the 0.05 level. Supporting this outcome, the null is not rejected using $WS^{r\,\inf}$. The White-corrected statistics, $WS_w^{f\,\inf}$ and $WS_w^{r\,\inf}$, point in the same direction. For the 5-year spread, both $WS^{f\,\inf}$ and $\min(WS^{f\,\inf}, WS^{r\,\inf})$ reject the null at the 0.05 level. The rejections are less significant in the White-corrected case. The $WS_w^{f\,\inf}$ statistic rejects the null at the 0.10 level, marginally missing the 0.05 critical value, while $\min(WS_w^{f\,\inf}, WS_w^{r\,\inf})$ is narrowly not rejecting at 0.10.[7]

The switch from $I(0)$ to $I(1)$ according to the non-White-corrected statistics is found in July 1997 for the 5-year spread, and March 1999 for the 1-year spread. The corresponding dates identified by the White-corrected statistics are May 1998 and March 1999. It could be argued that the earlier date, under non-White correction, for the

[6] We let τ vary between 0.15 and 0.85 in 0.01 increments. Note that LKSN use GLS-detrending and trim at 0.20 employing the usual ADF statistics.

[7] Following a referee's suggestion we also performed a standard Chow test for structural breaks on the first-difference of the two series. The stability of the regression coefficients was rejected for both maturities using the break dates given in Table 2. These results, however, should be treated with caution as such a test is conducted conditional on equal variances in the pre- and post- break samples.

5-year spread coincides with the outbreak of the Asian financial crises (Thai baht devaluation). The later date, under White-correction, may reflect the 5-year spread's issuance frequency reduction from monthly to quarterly in May 1998. The switch for the 1-year spread in early 1999 occurs in the aftermath of the Russian/LTCM liquidity crises in 1998:Q3, prior to the debt management policy change affecting the series. Importantly, although this change was triggered against a background of sustained expectations of future Federal budget surpluses, the subsequent persistence was likely sustained by both financial (event-driven) and macroeconomic uncertainty concerning the inversion of the yield curve and the sharp deterioration of the U.S. fiscal position, discussed earlier.

Lastly, in Table 3 we report the results of WS tests for the pre- and post-break subsamples. The significant break dates are those given in Table 2, as determined by the forward-based recursive test $WS^{f\,\inf}$.

TABLE 3 HERE

The pre-break and post-break subsamples are respectively stationary and nonstationary at the 0.01 and 0.05 levels, both with and without the White-correction, supporting the hypothesis of a break in peristence.

Thus, our findings point towards a significant switch from $I(0)$ to $I(1)$ in U.S. Treasury's 1-year and 5-year bond on/off spreads in the late 1990s. This should caution analysts regressing on/off yield spreads of government bonds as a stationary explanatory variable in structural (factor) models of credit spreads; see Boss and Scheicher

(2002) and Collin-Dufresne, Goldstein and Martin (2001). In light of the distributional properties of the data in Table 1, in Section 4 the above empirical evidence is assessed using Monte Carlo simulation. The performance of WS-based tests and their White-corrected versions under conditional heteroskedasticity is investigated.

4. MONTE CARLO SIMULATIONS

Conditional heteroskedasticity is pervasive in financial time series. It is now common to conduct unit root tests when there is higher-order conditional dependence with autoregressive coefficients summing close to unity. For example, Bera and Higgins (1997) using stock return data estimate $\phi_1 + \phi_2$ close to one (near-integration). Kim and Schmidt (1993) are mainly interested in the size of the DF statistic under GARCH. The potential consequences for power were not explored, particularly when White-correcting.[8] But these are clearly pertinent, as WS-based unit root tests can be more powerful than their ADF counterparts.

In this Section we provide Monte Carlo evidence on the size and power properties of the WS-based tests and, for the recursive statistics, the accuracy of the break point estimator for nearly-integrated GARCH errors. In doing so, we also evaluate the usefulness of White-correcting in these circumstances. The issue of skewness and excess kurtosis is also investigated as both are common features of financial series.[9]

[8] Seo (1999) provides some results demonstrating that DF t-tests haver lower power under conditional heteroskedasticity, but is not concerned with the issue of White-correction.

[9] The unconditional distribution for ϵ_t in the GARCH model with conditionally normal errors has heavier tails than the normal, but these do not adequately capture the excess kurtosis observed in many financial time series.

It is assumed that y_t follows the AR-GARCH process specified in Section 3, with the constant c set to zero. The errors v_t are either $t(5)$ or $\chi^2(3)$-distributed, and the standard normal is also included for comparison purposes. We consider the GARCH parameter combinations $(\phi_1, \phi_2) = \{(0.05, 0.9), (0.1, 0.8), (0.3, 0.6)\}$. The rejection frequencies at the nominal 0.05 level are given in Table 4. Size and power calculations were based on 5,000 and 3,000 replications, respectively.

TABLE 4 HERE

We first report on the WS statistic. This is found to be modestly oversized in finite samples for greater short-run volatility ($\phi_1 = 0.3$), corroborating the results in Kim and Schmidt (1993) and Seo (1999) for the DF t-statistic. When $T = 500$, size distortions are apparent for χ^2 errors only. The WS_w statistic effectively corrects these overrejections but can be somewhat undersized.

Regarding WS test power, as α declines this increases across GARCH parameterizations for given sample size. When T increases, a similar finding emerges for fixed α, reflecting consistency of the test. The ability of the WS_w test to control for size comes at some loss in power which can be substantial, particularly for smaller sample sizes, large α and non-normal errors. Correcting for GARCH when not required ($\phi_1 < 0.3$) yields negligible power losses for large T with the exception of the case $\phi_1 = 0.3$, $\alpha = 0.9$ under t-distributed errors. The same applies for smaller sample sizes provided α is not too large. Otherwise, the decline in power can be pronounced, especially for non-normal errors and larger ϕ_1.

Table 5 reports on the recursive statistic $WS^{f\inf}$.

TABLE 5 HERE

For $T = 200$, the $WS^{f\inf}$ test overrejects when $\phi_1 = 0.3$. Compared to the results for the WS statistic and the corresponding results for the DF-t statistic in Kim and Schmidt (1993), these size distortions are larger and appear to persist for $T = 500$ under all GARCH errors. Moderate overrejections are also apparent for t-GARCH errors and smaller ϕ_1, though mainly for $T = 200$, but these disortions are effectively corrected by the $WS_w^{f\inf}$ test statistic. The usual asympotic theory for White-correction does not extend to dynamic models with a unit root, as can be derived from the simulation results in Nichols and Pagan (1983). However, the necessary condition for existence of the unconditional fourth moment of the errors under White-correction is $3\phi_1^2 + 2\phi_1\phi_2 + \phi_2^2 < 1$, see Bollerslev (1986). Note that for larger ϕ_1 this inequality is closer to being violated than for larger ϕ_2.[10]

Turning to the recursive test, $WS^{f\inf}$ gains power for larger τ^*. Thus, consistency is more apparent for $\tau^* = 0.7$ and smaller ϕ_1, as size distortions are then minimal. This implies that the probability limit function is monotonically decreasing in τ^* under the alternative.[11] For $T = 500$, White-correcting across τ^* results in minor power losses provided α is not too large, and these are somewhat greater for $\tau^* = 0.5$. In contrast, power losses are significant across τ^* for $T = 200$. As before, White-correcting when

[10] He and Teräsvirta (1999) show that the unconditional fourth moment of ϵ_t, where $\epsilon_t = h_t^{1/2} v_t$, v_t is an iid sequence and $h_t = \phi_0 + \phi_1 \epsilon_{t-1}^2 + \phi_2 h_{t-1}$, with $\phi_1 + \phi_2 < 1$, exists in GARCH(1,1) models iff $\phi_2^2 + 2\phi_1\phi_2 m_2 + \phi_1 m_4 < 1$ and $m_i = E\left|v_t\right|^i$.

[11] LKSN also found this to be the case for the GLS-detrended DF tests.

not appropriate—for example, due to misspecifying the volatility dynamics—leads to smaller power losses for $T = 500$ when $\tau^* = 0.7$ as long as α is not too large and GARCH errors are not skewed. When $\tau^* = 0.5$ there is some further power reduction.

Table 6 reports on the accuracy of the estimated break fraction.

TABLE 6 HERE

Estimates of the true τ^* based on the $WS^{f\inf}$ test are consistent, and this is more evident for $\tau^* = 0.7$. Convergence is rather slower for $\tau^* = 0.5$, particularly when $T = 200$. In contrast, estimates of τ^* associated with the $WS_w^{f\inf}$ statistic are more accurate for $\tau^* = 0.5$. Convergence is then relatively slower for $\tau^* = 0.7$ and large α. The slow convergence is more pronounced for non-normal errors and greater ϕ_1. The effect of non-normality—skewness more than excess kurtosis—on the accuracy of τ^* is discernible only for estimates implied by the $WS_w^{f\inf}$test. For both tests, the estimated break fraction is more accurate for larger ϕ_1 across T and τ^*.

Finally, we briefly discuss the performance of the reverse and "two-sided" tests, $WS^{r\inf}$ and $\min(WS^{f\inf}, WS^{r\inf})$. The $WS^{r\inf}$ test as well as related estimates of τ^* were found to be inconsistent under the H^{01} alternative, as postulated in Section 2. The same was true for the White-corrected version of this test. Results for the $\min(WS^{f,\inf}, WS^{r,\inf})$ statistic were qualitatively similar to those of $WS^{f\inf}$, while power was higher and τ^* estimates were slightly more accurate using $WS^{f\inf}$.[12]

When relating these findings to the empirical evidence presented in Section 3, it appears that the less significant rejections observed for the 5-year on/off spread under

[12]Detailed results for these tests are available upon request.

White-correction can be attributed to its lower test power, associated with the break point for this series being detected earlier. This finding does not apply to the 1-year spread, possibly due to its corresponding break point occurring later in the sample.

5. CONCLUDING REMARKS

This paper extended the recursive test procedure of Leybourne et al. (2003) by adopting weighted-symmetric estimation to detect a single change in time series persistence. The dynamics of the U.S. yield curve offer a good case in point. An application to U.S. Treasury bond on/off spreads acting as a common empirical proxy of market liquidity found a significant break from $I(0)$ to $I(1)$ in the late 1990s. This finding suggests that the persistence properties of financial time series can be affected by exogenous shocks of systemic origin, as well as by unfolding uncertainty over macroeconomic policy. Thus, although "[...] in the U.S government bond market the announced debt repayment programme has been a major contributor to the persistence of the reduction in market liquidity" (Borio (2000)), over our whole sample period the change in persistence is driven by the debt management policy changes and the subsequent fiscal policy reversal—with their implications for monetary policy and the yield curve. The main result also serves to caution analysts employing on/off spreads on government bond yields as stationary variables in structural models of credit spreads. More generally, our contribution falls in line with the recent literature suggesting liquidity risk is strongly time-varying even in very liquid financial markets. Related theoretical models include Acharya and Pedersen (2005) and Brunnermeier and Pedersen (2005); for empirical evidence see Amihud (2002), Chordia, Roll and Subrahmanyam (2001) and Pastor and Stambaugh (2003) .

The empirical application was supported by implementing non-recursive WS tests to the pre- and post-break subsamples, known to be more powerful than standard and GLS-detrended ADF tests in finite samples. The size and power performance of the WS-based tests, as well as the accuracy of the estimated break point for their recursive counterparts were investigated in a Monte Carlo study under persistent GARCH errors. Conditional heteroskedasticity in time series is known to yield modest overrejections for standard Dickey-Fuller tests. This was also demonstrated by the non-recursive WS tests. For the recursive tests the overrejections were more pronounced. Confirming earlier studies, it was shown that White-correcting can generally eliminate these size distortions. However, we found that this comes at some loss in power, particularly for small sample sizes and greater short-run variation in volatity. We conclude that, when short-run volatity is persistent, employing the White-corrected version of the recursive unit root tests is advisable only in large samples, provided the AR(1) coefficient is not too large. When short-run variation in volatility is smaller, White-correction does not appear necessary. Misspecifying the underlying volatility dynamics would then lead to significant reduction in power in small samples.

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Figure 1. U.S. Treasury Bond On/Off Spread: 1991-2002

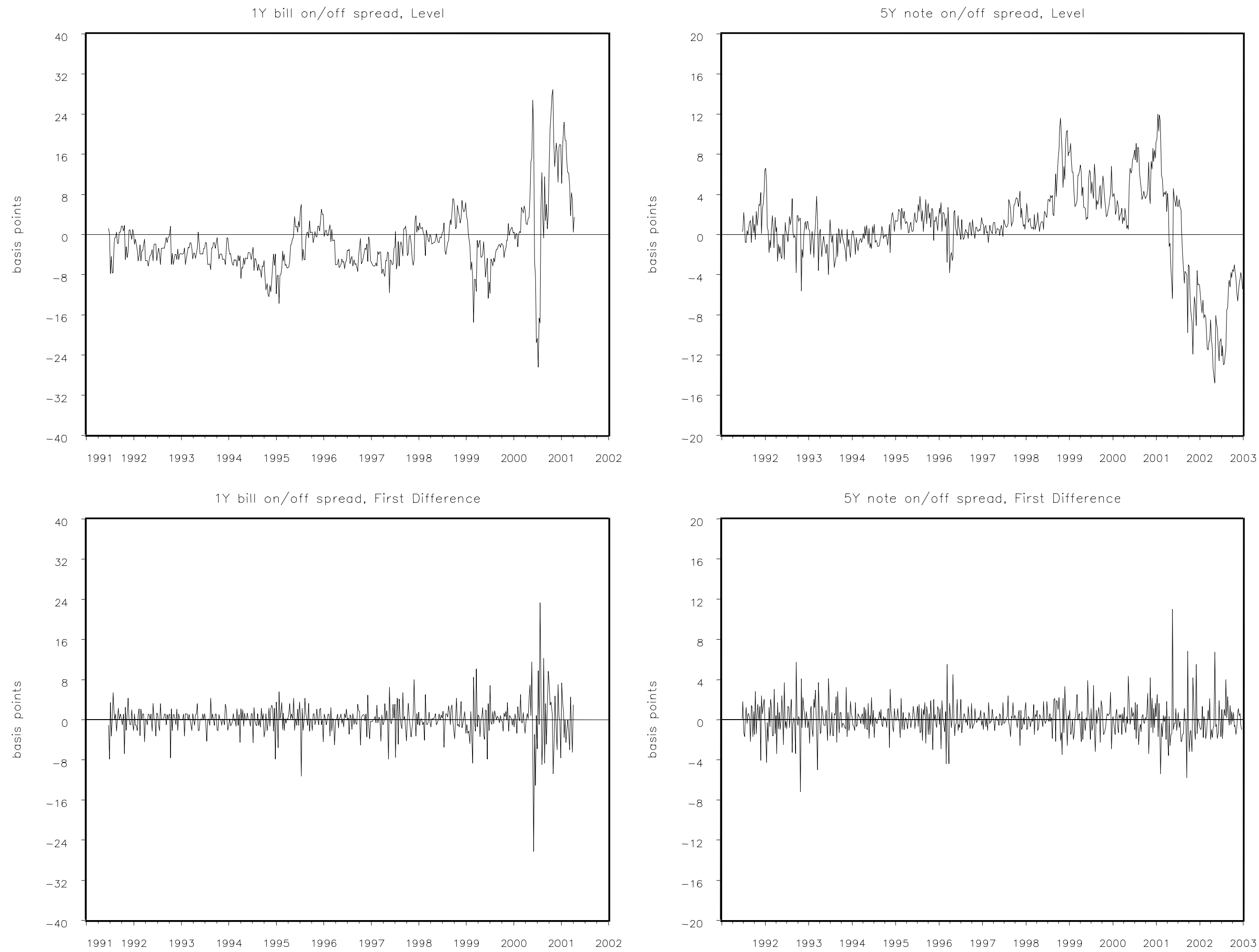

TABLE 1

Panel A	On/Off Bond Spread Statistics: 1991-2002						
Series	Mean	Std. Dev.	Max	Min	Skewness	Kurtosis	Jarque-Bera
1Y	-1.632	6.311	28.90	-26.50	1.518	8.437	820.76
5Y	0.684	4.202	12.00	-14.80	-0.771	4.947	152.18

Panel B	Maximum Likelihood Parameter Estimates AR(1)-GARCH(1,1) with Student-t Innovations				
Series	c	a	ϕ_0	ϕ_1	ϕ_2
1Y	-0.227	0.866	0.394	0.129	0.861
	(0.096)	(0.019)	(0.231)	(0.057)	(0.046)
5Y	0.038	0.931	0.096	0.122	0.853
	(0.051)	(0.013)	(0.051)	(0.039)	(0.041)

Panel C	ADF and WS Tests for Whole Sample			
Series	ADF	ADF_w	WS	WS_w
1Y	-3.009^b	-2.102	-3.168^a	-1.680
5Y	-1.409	-1.299	-1.711	-1.209

Note: Superscript a, b, c denotes significance at the 0.01, 0.05 and 0.10 level, respectively. Standard errors are in parentheses. ADF_w and WS_w refer to White's heteroskedastic robust version of the ADF and WS tests.

TABLE 2
Recursive WS tests for a change in persistence

Panel A Series	$WS^{f\,\inf}$	Break date	$WS^{r\,\inf}$	Break date	$\min(.,.)$	Break date
1Y	-3.602^b	03/03/99	-2.808	n/a	-3.602^b	03/03/99
5Y	-3.603^b	30/07/97	-2.371	n/a	-3.603^b	30/07/97
Panel B Series	$WS_w^{f\,\inf}$	Break date	$WS_w^{r\,\inf}$	Break date	$\min_w(.,.)$	Break date
1Y	-3.282^b	24/03/99	-1.692	n/a	-3.282^b	24/03/99
5Y	-2.927^c	27/05/98	-1.555	n/a	-2.927	n/a

Note: Statistics $\min(WS^{f\,\inf}, WS^{r\,\inf})$ and $\min(WS_w^{f\,\inf}, WS_w^{r\,\inf})$ are respectively denoted min and $\min_w$. Break dates are reported only when the null is rejected. The significant break points are 395 (03/03/99), 314 (30/07/97), 398 (24/03/99), 357 (27/05/98).

TABLE 3
WS tests for subsamples

Panel A	$WS^{f\,\inf}$	
Series	Pre-break	Post-break
1Y	-3.602^a	-1.847
5Y	-3.603^a	-1.736
Panel B	$WS_w^{f\,\inf}$	
Series	Pre-break	Post-break
1Y	-3.282^a	-1.651
5Y	-2.927^b	-1.261

TABLE 4

Empirical size and power of WS test at nominal 0.05-level under GARCH(1,1)

ϕ_1	ϕ_2		α: 0.70 WS	WS_w	0.80 WS	WS_w	0.90 WS	WS_w	1.00 WS	WS_w
					$T = 100$					
0.05	0.9	$N(0,1)$	1.000	0.992	0.968	0.927	0.531	0.483	0.055	0.056
		$t(5)$	0.999	0.960	0.974	0.846	0.623	0.469	0.055	0.042
		$\chi^2(3)$	0.996	0.909	0.958	0.746	0.546	0.368	0.050	0.046
0.1	0.8	$N(0,1)$	0.999	0.986	0.964	0.898	0.536	0.465	0.058	0.053
		$t(5)$	0.998	0.936	0.971	0.808	0.625	0.438	0.057	0.040
		$\chi^2(3)$	0.994	0.885	0.956	0.706	0.564	0.351	0.059	0.045
0.3	0.6	$N(0,1)$	0.995	0.922	0.944	0.764	0.553	0.387	0.072	0.050
		$t(5)$	0.996	0.856	0.955	0.690	0.626	0.364	0.075	0.036
		$\chi^2(3)$	0.989	0.811	0.948	0.628	0.613	0.312	0.079	0.046
					$T = 200$					
0.05	0.9	$N(0,1)$	1.000	1.000	1.000	0.999	0.960	0.913	0.058	0.060
		$t(5)$	1.000	0.994	1.000	0.981	0.975	0.856	0.058	0.047
		$\chi^2(3)$	1.000	0.990	0.999	0.948	0.960	0.735	0.058	0.048
0.1	0.8	$N(0,1)$	1.000	1.000	1.000	0.995	0.956	0.885	0.060	0.058
		$t(5)$	1.000	0.986	0.999	0.960	0.971	0.809	0.063	0.046
		$\chi^2(3)$	1.000	0.974	0.998	0.916	0.956	0.678	0.061	0.046
0.3	0.6	$N(0,1)$	1.000	0.982	0.998	0.942	0.934	0.716	0.076	0.051
		$t(5)$	0.999	0.942	0.998	0.874	0.955	0.648	0.074	0.043
		$\chi^2(3)$	0.998	0.916	0.995	0.800	0.949	0.535	0.082	0.044
					$T = 500$					
0.05	0.9	$N(0,1)$	1.000	1.000	1.000	1.000	1.000	1.000	0.056	0.053
		$t(5)$	1.000	0.998	1.000	0.995	1.000	0.986	0.054	0.043
		$\chi^2(3)$	1.000	0.999	1.000	0.997	0.999	0.965	0.058	0.050
0.1	0.8	$N(0,1)$	1.000	1.000	1.000	1.000	1.000	1.000	0.056	0.052
		$t(5)$	1.000	0.966	1.000	0.990	1.000	0.971	0.053	0.040
		$\chi^2(3)$	1.000	0.997	1.000	0.988	0.999	0.930	0.061	0.046
0.3	0.6	$N(0,1)$	1.000	0.996	1.000	0.987	0.999	0.941	0.064	0.044
		$t(5)$	1.000	0.977	1.000	0.946	0.999	0.850	0.065	0.035
		$\chi^2(3)$	1.000	0.960	1.000	0.904	0.997	0.755	0.080	0.035

Note: The DGP is $y_t = \alpha y_{t-1} + \epsilon_t$, where $\epsilon_t = h_t^{1/2} v_t$, $h_t = \phi_0 + \phi_1 \epsilon_{t-1}^2 + \phi_2\, h_{t-1}$ and v_t is distributed as in the first column. The t and χ^2 distributions are standardized as $\frac{t(n)}{(\frac{n}{n-2})^{1/2}}$ and $\frac{\chi^2(n)-n}{(2n)^{1/2}}$ with n degrees of freedom. The unconditional variance is 1 by setting $\phi_0 = 1 - \phi_1 - \phi_2$, without loss of generality.

TABLE 5
Empirical size and power (under H^{01}): $WS^{f\inf}$ at 0.05 level under GARCH(1,1) as in Table 6

					$\tau^* = 0.5$					
					$T = 200$					
ϕ_1	ϕ_2	α:	0.70		0.80		0.90		1.00	
			$WS^{f\inf}$	$WS_w^{f\inf}$	$WS^{f\inf}$	$WS_w^{f\inf}$	$WS^{f\inf}$	$WS_w^{f\inf}$	$WS^{f\inf}$	$WS_w^{f\inf}$
0.05	0.9	$N(0,1)$	0.994	0.959	0.867	0.727	0.385	0.262	0.067	0.064
		$t(5)$	0.989	0.792	0.862	0.750	0.378	0.166	0.070	0.042
		$\chi^2(3)$	0.991	0.774	0.874	0.521	0.392	0.199	0.059	0.047
0.1	0.8	$N(0,1)$	0.991	0.935	0.859	0.704	0.387	0.262	0.066	0.059
		$t(5)$	0.984	0.780	0.846	0.509	0.386	0.171	0.071	0.039
		$\chi^2(3)$	0.990	0.766	0.871	0.514	0.415	0.203	0.060	0.044
0.3	0.6	$N(0,1)$	0.984	0.843	0.862	0.592	0.455	0.227	0.093	0.061
		$t(5)$	0.978	0.705	0.846	0.452	0.445	0.161	0.098	0.042
		$\chi^2(3)$	0.984	0.734	0.891	0.499	0.493	0.212	0.088	0.050
					$T = 500$					
0.05	0.9	$N(0,1)$	1.000	1.000	1.000	0.999	0.959	0.881	0.061	0.061
		$t(5)$	1.000	0.981	1.000	0.946	0.957	0.700	0.068	0.049
		$\chi^2(3)$	1.000	0.980	1.000	0.924	0.961	0.670	0.063	0.050
0.1	0.8	$N(0,1)$	1.000	1.000	1.000	0.996	0.952	0.855	0.061	0.056
		$t(5)$	1.000	0.975	1.000	0.928	0.950	0.664	0.070	0.042
		$\chi^2(3)$	1.000	0.971	0.999	0.897	0.958	0.621	0.066	0.045
0.3	0.6	$N(0,1)$	1.000	0.988	1.000	0.947	0.939	0.680	0.091	0.048
		$t(5)$	1.000	0.936	0.999	0.831	0.936	0.508	0.103	0.036
		$\chi^2(3)$	1.000	0.919	0.997	0.810	0.957	0.506	0.097	0.041
					$\tau^* = 0.7$					
					$T = 200$					
ϕ_1	ϕ_2	α:	0.70		0.80		0.90		1.00	
			$WS^{f\inf}$	$WS_w^{f\inf}$	$WS^{f\inf}$	$WS_w^{f\inf}$	$WS^{f\inf}$	$WS_w^{f\inf}$	$WS^{f\inf}$	$WS_w^{f\inf}$
0.05	0.9	$N(0,1)$	1.000	0.995	0.981	0.885	0.503	0.351	0.067	0.064
		$t(5)$	0.998	0.905	0.971	0.691	0.492	0.224	0.070	0.042
		$\chi^2(3)$	0.999	0.874	0.979	0.663	0.538	0.253	0.059	0.047
0.1	0.8	$N(0,1)$	1.000	0.990	0.978	0.865	0.516	0.343	0.066	0.059
		$t(5)$	0.998	0.892	0.967	0.668	0.508	0.225	0.071	0.039
		$\chi^2(3)$	0.999	0.867	0.976	0.646	0.559	0.258	0.060	0.044
0.3	0.6	$N(0,1)$	0.998	0.924	0.964	0.725	0.573	0.284	0.093	0.061
		$t(5)$	0.996	0.806	0.955	0.567	0.551	0.205	0.098	0.042
		$\chi^2(3)$	0.997	0.817	0.973	0.598	0.641	0.254	0.088	0.050
					$T = 500$					
0.05	0.9	$N(0,1)$	1.000	1.000	1.000	1.000	0.999	0.981	0.061	0.061
		$t(5)$	1.000	0.990	1.000	0.976	0.997	0.989	0.068	0.049
		$\chi^2(3)$	1.000	0.993	1.000	0.968	0.997	0.794	0.063	0.050
0.1	0.8	$N(0,1)$	1.000	1.000	1.000	1.000	0.999	0.962	0.061	0.056
		$t(5)$	1.000	0.987	1.000	0.965	0.995	0.806	0.070	0.042
		$\chi^2(3)$	1.000	0.984	1.000	0.943	0.996	0.746	0.066	0.045
0.3	0.6	$N(0,1)$	1.000	0.994	1.000	0.973	0.993	0.813	0.091	0.048
		$t(5)$	1.000	0.963	0.999	0.891	0.989	0.634	0.103	0.036
		$\chi^2(3)$	1.000	0.950	0.999	0.868	0.990	0.606	0.097	0.041

TABLE 6

Break point estimates under H^{01}: $WS^{f\,\inf}$ at 0.05 level under GARCH(1,1) as in Table 6

ϕ_1	ϕ_2		α: 0.70 $WS^{f\,\inf}$	0.70 $WS_w^{f\,\inf}$	0.80 $WS^{f\,\inf}$	0.80 $WS_w^{f\,\inf}$	0.90 $WS^{f\,\inf}$	0.90 $WS_w^{f\,\inf}$
$\tau^* = 0.5$								
$T = 200$								
0.05	0.9	$N(0,1)$	0.583	0.552	0.599	0.562	0.611	0.566
		$t(5)$	0.586	0.510	0.603	0.533	0.617	0.551
		$\chi^2(3)$	0.586	0.492	0.604	0.510	0.615	0.524
0.1	0.8	$N(0,1)$	0.566	0.535	0.582	0.549	0.598	0.553
		$t(5)$	0.568	0.501	0.584	0.521	0.601	0.538
		$\chi^2(3)$	0.569	0.481	0.587	0.497	0.599	0.512
0.3	0.6	$N(0,1)$	0.564	0.519	0.580	0.525	0.590	0.529
		$t(5)$	0.563	0.486	0.579	0.500	0.590	0.517
		$\chi^2(3)$	0.565	0.469	0.580	0.480	0.594	0.486
$T = 500$								
0.05	0.9	$N(0,1)$	0.536	0.524	0.549	0.538	0.579	0.563
		$t(5)$	0.539	0.505	0.554	0.528	0.582	0.558
		$\chi^2(3)$	0.537	0.491	0.552	0.509	0.582	0.536
0.1	0.8	$N(0,1)$	0.528	0.516	0.541	0.528	0.568	0.551
		$t(5)$	0.531	0.499	0.546	0.519	0.570	0.549
		$\chi^2(3)$	0.530	0.481	0.543	0.499	0.571	0.526
0.3	0.6	$N(0,1)$	0.527	0.501	0.539	0.513	0.566	0.538
		$t(5)$	0.529	0.481	0.541	0.499	0.567	0.529
		$\chi^2(3)$	0.527	0.461	0.541	0.475	0.568	0.494

ϕ_1	ϕ_2		α: 0.70 $WS^{f\,\inf}$	0.70 $WS_w^{f\,\inf}$	0.80 $WS^{f\,\inf}$	0.80 $WS_w^{f\,\inf}$	0.90 $WS^{f\,\inf}$	0.90 $WS_w^{f\,\inf}$
$\tau^* = 0.7$								
$T = 200$								
0.05	0.9	$N(0,1)$	0.754	0.717	0.750	0.707	0.718	0.659
		$t(5)$	0.755	0.668	0.751	0.662	0.720	0.637
		$\chi^2(3)$	0.758	0.632	0.758	0.625	0.729	0.592
0.1	0.8	$N(0,1)$	0.739	0.702	0.737	0.691	0.709	0.651
		$t(5)$	0.741	0.655	0.738	0.649	0.709	0.622
		$\chi^2(3)$	0.745	0.620	0.745	0.609	0.719	0.586
0.3	0.6	$N(0,1)$	0.729	0.661	0.725	0.650	0.697	0.614
		$t(5)$	0.727	0.618	0.723	0.612	0.693	0.590
		$\chi^2(3)$	0.735	0.587	0.734	0.582	0.712	0.555
$T = 500$								
0.05	0.9	$N(0,1)$	0.733	0.721	0.742	0.726	0.751	0.726
		$t(5)$	0.736	0.686	0.745	0.699	0.751	0.709
		$\chi^2(3)$	0.734	0.675	0.743	0.681	0.753	0.679
0.1	0.8	$N(0,1)$	0.726	0.710	0.733	0.715	0.741	0.713
		$t(5)$	0.727	0.676	0.735	0.687	0.741	0.694
		$\chi^2(3)$	0.726	0.661	0.735	0.665	0.742	0.661
0.3	0.6	$N(0,1)$	0.720	0.675	0.726	0.677	0.730	0.675
		$t(5)$	0.719	0.639	0.724	0.645	0.727	0.649
		$\chi^2(3)$	0.720	0.617	0.727	0.616	0.733	0.604

Appendix A

Simulated critical values

Panel A Statistic	T	0.01	0.05	0.10	*Panel B* Statistic	T	0.01	0.05	0.10
ADF	500	-3.420	-2.875	-2.578	ADF_w	500	-3.453	-2.905	-2.598
WS	100	-3.124	-2.552	-2.235	WS_w	100	-2.857	-2.299	-2.007
	250	-3.160	-2.554	-2.255		250	-2.796	-2.262	-1.982
	350	-3.111	-2.538	-2.255		350	-2.737	-2.232	-1.971
	400	-3.080	-2.543	-2.222		400	-2.733	-2.225	-1.949
	500	-3.109	-2.540	-2.228		500	-2.745	-2.220	-1.942
$WS^{f\,\inf}$	500	-3.909	-3.325	-3.030	$WS_w^{f\,\inf}$	500	-3.529	-3.004	-2.729
$WS^{r\,\inf}$	500	-3.943	-3.323	-3.033	$WS_w^{r\,\inf}$	500	-3.578	-3.003	-2.721
min	500	-4.162	-3.586	-3.309	$\min_w$	500	-3.770	-3.252	-2.993

Note: Statistics $\min(WS^{f\,\inf}, WS^{r\,\inf})$ and $\min(WS_w^{f\,\inf}, WS_w^{r\,\inf})$ are respectively denoted by min and $\min_w$. Beyond $T = 500$ critical values for WS-based tests appeared to converge.