Efficiency Benefits of Transport Cost Reductions

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22 December 1997
(revised March 1998)

Executive Summary

Transport cost benefit analysis, when properly done, takes account of the benefits arising from increased transport demand induced by lower transport costs, and these benefits are typically evaluated by the rule-of-one-half or the trapezium rule. This states that the transport gain is the average level of transport services times the fall in transport costs, $\Sigma\frac{1}{2}(q_i' + q_i).(-\Delta t_i)$, where q_i is the number of trips on route i, $-\Delta t_i$ is the fall in transport costs per trip on that route, and a dash indicates the post-improvement situation. One very important set of issues in undertaking such transport evaluations is to pick up the total impact of any scheme, which in a network may affect traffic flows and congestion delays on a whole range of routes apart from the one subject to investment.

Transport cost reductions may also yield indirect benefits that are not captured by simple cost-benefit analysis, and that is the subject of current SACTRA interest in the links between transport investment and economic growth. One part of that is the implications of imperfect competition for transport appraisal. These indirect benefits arise from two sources - if prices fall and as a result output increases, then there will be an increase in profits if firms are pricing above (long-run) cost because of imperfect competition. If firms are located at different distances from a given market, then lowering the transport cost will lower the effective delivered price, and increase the intensity of competition between firms, particularly as more firms may find it worthwhile competing for access to a given market.

The paper constructs a model of imperfect competition between suppliers of a homogenous good (which could either be an input or a final consumption good) from geographically dispersed firms who compete imperfectly in a central market place. The impact of improvements which lower the costs of transporting the good to market are examine for the case in which firms price to market and compete in that market, taking the supplies of their rivals as given (though the paper discusses different formulations of competitive behaviour, which may be either more or less intense). Different firms face different delivery costs, and these affect their market share and their impact on prices. The first model examines the effect of a general reduction in transport costs facing all firms, of the kind associated with a fall in fuel excise duty, so the percentage change in transport costs is the same for all, but the relative impact on total costs is larger for more distant firms. The second model examines the effect of a change in the transport cost of

just one firm, of the kind that may arise if just one route is improved.

The question asked is how much larger the total social benefits are compared with the apparent transport benefits, or, more specifically, what is the ratio of the non-transport to the transport benefits, r. In that respect it follows Venables and Gasiorek (1997) in their report for SACTRA. They estimate a multiplier by which direct transport benefits should be multiplied to give the total benefits (equivalent to $1+r$). This paper finds rather lower values for the ratio than they suggest. If ε is the elasticity of demand for the product at the central market place and μ is the ratio of the super-normal profit (arising from imperfect competition) to the price, then a simple analysis which does not model the heterogeneity of transport costs suggests that $r = \varepsilon\mu$, so if $\varepsilon = 2$ and $\mu = 0.1$, then $r = 20\%$ and the multiplier is 1.2, which is quite large.

The first model shows that for the same parameters, but in addition if transport costs are on average 10% of total costs (including transport costs), and if there are 5 competing firms at different distances, with a coefficient of variation of transport costs of 25%, then r is only 3% for the case of linear demand, and 9% if demand has constant elasticity, in both cases considerably below the simple estimate of 20%. The reason is that a generalised fall in transport costs increases the market share of more distant firms, but these firms have higher total costs (production plus delivery) and so the gains from increased output are reduced by the increase in these costs. The lesson of the model is that it is important to model the nature of imperfect competition carefully, because it does not necessarily follow that reducing some firms' costs will be welfare improving - although prices may fall and benefit consumers, total costs may rise, and the fall in profits can outweigh the gain in consumer benefits.

The second model looks at the effect of a localised transport improvement that lowers delivery costs for just firm i. In the linear case, if s_i is the market share of firm i, the indirect benefit ratio $r = 1 - 1/[s_i(n+1)]$. This equation has a quite remarkable simplicity and surprising implication. If transport costs are lowered for firms whose market share is *less* than $1/(n+1)$, r is negative and production efficiency decrease. These are precisely the more distant firms that benefit more from transport cost reductions. The reason is that the least socially costly way of producing goods in this simple model is to use inputs from the closest suppliers. If the transport costs of more distant firms fall, then their market share rises, but this increases total production costs.

The steady decrease in transport costs has had a dramatic effect on the average distance over which firms source their requirements, and the resulting increase in market area can be expected to generate additional benefits of increased choice and competition. The simple models considered here allow the effect of lower transport costs to affect the efficiency of the economy, but the resulting impacts appear small, and can be counterintuitive, with lowered transport costs reducing production efficiency. The moral of this paper is that careful modelling of the impact of transport improvements on production costs will be required to capture the indirect efficiency benefits, as these can go either way.

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Transport cost reductions are typically evaluated by the rule-of-one-half or the usual trapezium rule that the transport gain is the average level of transport services times the *fall* in transport costs, $\Sigma\frac{1}{2}(q_i' + q_i).(-\Delta t_i)$, where q_i is the number of trips on route i, $-\Delta t_i$ is the fall in transport costs per trip on that route, and a dash indicates the post-improvement situation.

Transport cost reductions yield indirect benefits from two sources - if prices fall and as a result output increases, then there will be an increase in profits if firms are pricing above (long-run) cost because of imperfect competition, and there may be further effects on the intensity of competition between firms. If firms are located at different distances from a given market, then lowering the transport cost will lower the effective delivered price, and increase the intensity of competition between firms, particularly as more firms may find it worthwhile competing for access to a given market. This paper attempts to model these indirect benefits in a simple framework, which can readily be elaborated.

1. Nash-Cournot competition at a central market place

Suppose n firms compete to deliver a homogenous good to a central market. If all firms have the same unit production costs c, and the transport cost from firm i located at distance d_i is td_i (where t is the transport cost per unit per km), then the cost of delivering one unit to the market place will be $c_i \equiv c + td_i$, and the delivered price will be p. Suppose that each firm chooses its output, q_i, taking the output of other firms as given (that is firms compete in quantities - the Nash-Cournot assumption). Total demand, $Q(p)$, will depend on the market price, p, which will be set by total supply (and firms will accept the same delivered price by bearing the transport costs themselves). Profit of firm i is $(p(Q)-c_i)q_i - F$, where F is fixed cost. The first order condition for maximum profit is

$$p - c_i - p\frac{q_i}{Q}\left(-\frac{Q}{p}\frac{dp}{dQ}\right)\frac{\partial Q}{\partial q_i} = 0. \tag{1}$$

The Nash-Cournot assumption that other firms are assumed not to vary their output in response to changes in q_i implies that $\partial Q/\partial q_i = 1$. If the market demand schedule has constant elasticity $\varepsilon \equiv -\, d\ lnQ/d\ lnp$ (defined to be a positive number) then (1) can be rewritten in terms of the market share of firm i, $s_i = q_i/Q$, and the demand elasticity ε:

$$s_i = \left(1 - \frac{c_i}{p}\right)\varepsilon, \qquad i = 1,2,..,n. \tag{2}$$

Summing over the *n* firms gives an equation for the equilibrium price, *p*, which depends on the *unweighted* average cost, $\bar{c}$, the number of firms supplying the market, *n*, and the elasticity of demand, ε. From the price, the levels of firm output can be recovered:

$$\begin{aligned} p &= \left(\frac{\bar{c}}{1-1/n\varepsilon}\right), \quad \bar{c} = \frac{1}{n}\sum_i^n c_i, \\ q_i &= \left(\varepsilon\left(1 - \frac{c_i}{\bar{c}}\right) + \frac{c_i}{n\bar{c}}\right)Q = Q\left(\frac{1}{n} - \frac{\varepsilon\delta_i t}{p}\right), \quad \delta_i = \frac{1}{n}\sum_i d_i, \quad \delta_i \equiv d_i - \bar{d}. \end{aligned} \tag{3}$$

Output of firm *i*, q_i is made up of the average output per firm and the effect of differential transport access (measured by δ_i),showing that more distant firms have smaller market shares. Competitive pressure increases with *n* and ε, and helps drive the price down closer to cost, $\bar{c}$. Note that the number of firms may be endogenous if *p* falls below the unit cost of the most expensive firm(s), for then both $\bar{c}$ and *n* will fall.

If market demand is linear, $p = a - bQ$, the elasticity at price *p* is $\varepsilon = p/bQ$, and the first order conditions and market aggregation give

$$p = \frac{n\bar{c} + a}{n+1}, \qquad q_i = \frac{a - c - \bar{d}t}{b(n+1)} - \frac{\delta_i t}{b} = Q\left(\frac{1}{n} - \frac{\varepsilon\delta_i t}{p}\right), \tag{4}$$

exactly as in the constant elastic case.

The inefficiency of the market arises from two sources - the price is above the competitive level, which would be the lowest cost, c_0 (with all output supplied from the least cost firm), and the output is supplied not from the least cost firm, but from a range of firms with average cost $\bar{c}$ higher than the least cost. The natural way to investigate the impact of a change in transport costs on value added is to start from a measure of total value added, made up of the sum of consumer benefits (measured in money terms as a consumer surplus), *V(p)*, plus industry profits, $\Pi = pQ - C - T$, where *C* is total production cost, *cQ*, and *T* is total transport cost, $\Sigma d_i q_i t$. Total value added is then $W = V(p) + \Pi$, and the impact of changes in transport costs, *t*, on value added is given by

$$\begin{aligned} \Delta W &= V(p') - V(p) + \sum_i (p' - c - t'd_i)(q_i + \Delta q_i) - \sum_i (p - c - td_i)q_i, \\ &= -\Delta p\left(Q + \tfrac{1}{2}\Delta Q\right) + \Delta p.Q - \Delta t \sum_i d_i q_i + \sum_i (p' - c - t'd_i)\Delta q_i, \\ &= -\tfrac{1}{2}\Delta p.\Delta Q - \Delta t \sum_i d_i q_i + \sum_i (p' - c - t'd_i)\Delta q_i, \end{aligned} \tag{5}$$

where a dash indicates the value after the change, and $\Delta x \equiv x' - x$ is the difference between the after-change value and the pre-change value. Note that $\Delta V(p) = -\Delta p(Q + \frac{1}{2}\Delta Q)$ is the gain in consumer surplus from the fall in prices, and that the first and third term in the second line of (5) cancel as income is transferred from one side of the market to the other through the change in price. In the competitive case, where $d_i = 1$, $p' = c+t'$, and $\Delta p = \Delta t$, there is no gain in productive efficiency, and $\Delta W = -\frac{1}{2}\Delta p.\Delta Q - Q\Delta t = (Q+\frac{1}{2}\Delta Q).(-\Delta t)$, the usual transport benefit.

These terms can be interpreted using a standard surplus analysis suggested by Venables and Gasiorek (1997) and set out in fig.1 for a commodity produced in a single location at a unit cost c where the transport cost to deliver it to market is initially t and price is p. Area A measures the value of fall in transport cost at the initial output, $-\Delta t.Q$, corresponding to the middle term in (5), to which should be added the gain in consumer surplus resulting from the fall in price, $-\frac{1}{2}\Delta p.\Delta Q$, shown as area B, and given by the first term in (5). These two benefits make up the transport benefits measured by a standard transport cost-benefit analysis (CBA) and captured in the rule of one-half if $\Delta t = \Delta p$. To this should be added the margin between the price and the sum of production and delivery cost, times the increase in output, $(p'-c-t')\Delta Q$, the middle term in (5) and area C in fig. 1. This is the gain arising from taking account of the distortion between the imperfectly competitive price, p', and the competitive price, $c+t'$, which is ignored in standard transport cost-benefit analysis (though it is taken into account in normal CBA whenever such distortions, usually arising from taxes, can be identified).

Transport benefits

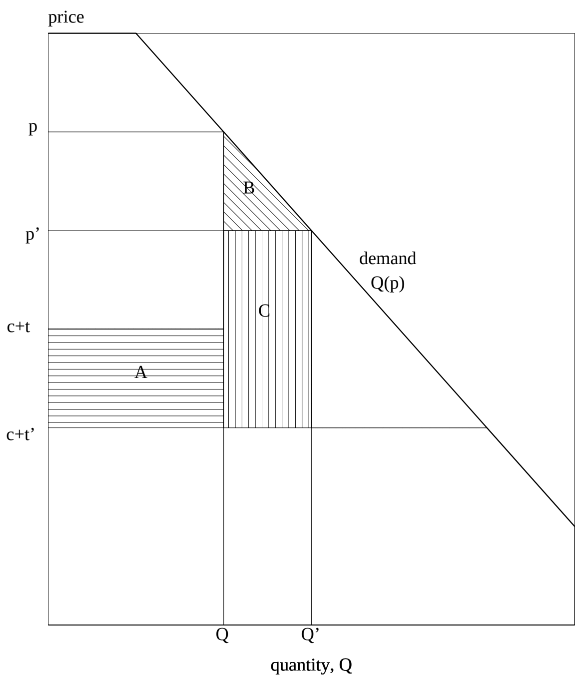

Fig. **1**

Geometrically, if ΔW is total change in value added (A+B+C in fig. 1) while ΔT is the direct transport cost reduction benefit (A+B in fig. 1), then

$$\frac{\Delta W}{\Delta T} = 1 - \frac{(p-c-t)\Delta Q}{\bar{Q}\Delta t} = 1 + \left(-\frac{p}{\bar{Q}}\frac{\Delta Q}{\Delta p}\right)\frac{p-c-t}{p} = 1 + \frac{\varepsilon(p-c-t)}{p} \equiv 1 + r, \qquad (6)$$

where $\bar{Q} = Q+\frac{1}{2}\Delta Q$ is the average level of output pre- and post-improvement. The main simplifying assumptions in (6) are that unit transport cost reductions translate one-for-one into price decreases, so $\Delta p = \Delta t$, and all firms are located at a constant distance taken as

1 unit (unlike the more complex model). Venables and Gasiorek (1997) suggest numbers for ε of about 2 and for the profit markup, $(p-c-t)/p$ of about 0.2, giving a multiplier of 1.4, or a value for r of 0.4, which is large. Later we suggest rather smaller values for the markup of 0.1, which, on this formula would give a value for r of 0.2. We shall see that taking account of the heterogeneity of transport costs considerably reduces this value.

The result of an infinitesimal change in transport costs can be found by differentiating $W = V(p) + (p-c)Q - t\Sigma d_i q_i$:

$$\begin{aligned}\frac{dW}{dt} &= \left(\frac{dV}{dp}+Q\right)\frac{dp}{dt} + \sum_i (p-c-td_i)\frac{dq_i}{dt} - \sum_i d_i q_i,\\ &= (p-c-\bar{d}t)\frac{dQ}{dt} - \sum_i \left((d_i-\bar{d})t\frac{dq_i}{dt} + d_i q_i\right),\\ &= (p-c-\bar{d}t)Q\left(-\frac{p}{Q}\frac{dQ}{dp}\right)\left(-\frac{1}{p}\frac{dp}{dt}\right) - \sum_i (t\delta_i\frac{dq_i}{dt} + d_i q_i).\end{aligned} \tag{7}$$

(Note that in this case from Roy's identity, $dV/dp = -Q$, so again the first two terms on the first line cancel.)

The object of this paper is to estimate the relationship between the indirect efficiency benefits of reducing transport costs, and the direct benefits, but this will depend on how the benefits are classified. The guiding principle in classifying benefits is that in the competitive case there should be no induced production efficiency gains, which suggests that the induced gains should be defined as $dW/dt+\Sigma d_i q_i$, which is equivalent to taking the transport benefits as $\Sigma d_i q_i.(-\Delta t)$, where $-\Delta t$ is the *fall* in unit transport costs. This is equivalent to assuming no change in average trip length induced by changes in firm behaviour, which means that such changes are counted as part of the improvement in production efficiency (better sourcing of inputs), and not of transport efficiency (ie unit transport costs). The efficiency ratio is $r = (dW/dt+\Sigma d_i q_i)/\Sigma d_i q_i$. It is therefore not the same as subtracting the change in transport costs, $dT/dt = d/dt[t\Sigma d_i q_i]$.[1]

The value of the transport cost saving term, $\Sigma d_i q_i$, is, for both the linear and constant elastic case (discussed in the appendix)

$$\sum_i d_i q_i = Q\left(\bar{d} - \frac{nt\varepsilon}{p}\mathrm{Var}\, d_i\right) = Q\bar{d}(1-\frac{n\varepsilon t\bar{d}}{p}\sigma_d^2), \quad \mathrm{Var}\, d_i = \frac{1}{n}\sum_i \delta_i^2, \quad \sigma_d^2 \equiv \frac{\mathrm{Var} d_i}{\bar{d}^2}. \tag{8}$$

Note that actual transport costs are less than might be predicted from $Q\bar{d}\Delta t$ as closer firms have a larger market share than more distant firms. The efficiency ratio, r, can be found from (7) as

[1] $\Delta T = Q\Delta t + t\Delta Q \neq (Q+\tfrac{1}{2}\Delta Q)\Delta t$, the proper measure of transport benefits.

$$r = \frac{(p - c - \bar{d}t)\left(-\frac{dQ}{dt}\right) - t\sum_i \delta_i\left(-\frac{dq_i}{dt}\right)}{\bar{d}Q + \sum_i \delta_i q_i}. \qquad (9)$$

This reduces to (6) if $d_i = \bar{d} = 1$ (i.e. $\delta_i = d_i - \bar{d} = 0$). The indirect benefits can most readily be computed for the case of linear demand (and the constant elastic case is examined in the appendix). First, note from (4) that $-dq_i/dt = \bar{d}/b(n+1)+\delta_i/b$, so $-dQ/dt = n\bar{d}/b(n+1)$, and the first efficiency ratio, r, is

$$r = -\frac{(p - c - \bar{d}t)\left(\frac{n\bar{d}}{b(n+1)}\right) - \frac{t}{b}\sum_i \delta_i^2}{Q\bar{d}\left(1 - \frac{n\varepsilon t\bar{d}\sigma_d^2}{p}\right)}. \qquad (10)$$

The elasticity of the linear demand schedule at price p is $\varepsilon = p/bQ$ (where $-b$ is the slope of the demand schedule), so the efficiency ratio can be written in dimensionless units as

$$r = \frac{\varepsilon\left(\frac{p - \bar{c}}{p}\right)\left(1 - \frac{1}{n+1}\right) - \theta}{1 - \theta}, \qquad \theta \equiv \frac{n\varepsilon\bar{d}t\sigma_d^2}{p}. \qquad (11)$$

The size of this ratios can be estimated given observations on market shares, prices and costs. For example, if $\varepsilon = 2$, $n = 5$, $a = 1.5$, $b = 0.5$, then $\bar{c}/p = 0.9$ from (4) for the constant elastic case, and if $\overline{dt}/\bar{c}$, the share of average transport costs in total average delivered unit cost is 0.05, $\overline{dt}/p$= 0.056 and $c/p = 0.855$. If the coefficient of variation of transport costs is 0.25, then $\theta = 0.14$, and the ratio $r = 0.03$, which is remarkably small. If the CV of transport costs were much larger, then with these parameters, r would be negative, which might appear paradoxical. The reason why the apparent production efficiency gains are so low (or even negative) is that the transport benefits estimated by $\Sigma q_i d_i(-\Delta t)$ overstates the fall in transport cost, for a proportionate fall in transport costs leads to longer average trip lengths.[2]

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[2] If the full fall in transport costs, $-dT/dt$ is subtracted from the total gains to give the production efficiency gains, the ratio of these gains to the fall in transport costs is $r^* = (dW/dt+dT/dt)/dT/dt$, or

$$r^* = \frac{\varepsilon\left(\frac{p - c}{p}\right)}{(1 + 1/n)(1 - 2\theta) - \varepsilon\bar{d}t/p}. \qquad (1)$$

This does not reduce to (6) even if $d_i = \bar{d} = 1$, but to $\varepsilon(p-c)/(p-\varepsilon t)$. With the same parameters as before its numerical value would be 0.38, or ten times the other value.

The ratio of the actual fall in transport cost to the apparent fall can be measured by *dlogT/dlogt*. In the linear case is given by

$$\frac{t}{T}\frac{dT}{dt} = 1 - \frac{\theta + \dfrac{\varepsilon \bar{d} t/p}{1+1/n}}{1-\theta}, \tag{12}$$

which, with these parameters, is 0.73. The share of firm *i* in the output expansion can be found from

$$\frac{\Delta q_i}{\Delta Q} = \frac{dq_i/dt}{dQ/dt} = \frac{1}{n} + \frac{\delta_i(1+1/n)}{\bar{d}}, \tag{13}$$

which increases with distance, d_i. In this case the increased average trip length offsets 27% of the fall in unit transport costs.

2. General conjectural oligopoly model

The Nash-Cournot assumption can be viewed as an assumption about the variation in output of other firms that any firm conjectures will occur in response to its own output variation, $\partial(Q-q_i)/\partial q_i$. Define this conjectural variation to be λ_i, then the Nash-Cournot assumption amounts to $\lambda_i = 0$, perfect competition to $\lambda_i = -1$, and perfect collusion to $\lambda_i = 1/s_i - 1 > 0$. To solve for the price-cost ratio, put $\partial Q/\partial q_i = 1 + \lambda_i$ in (1) and rearrange to give

$$s_i = \left(1 - \frac{c_i}{p}\right)\frac{\varepsilon}{1+\lambda_i}, \qquad \text{whence } p = \frac{\bar{c}}{1 - \dfrac{1+\sum s_i\lambda_i}{n\varepsilon}}. \tag{14}$$

Provided $\lambda_i = \lambda$ all *i*, so that *p* does not depend on s_i, the effect is to modify the effective number of firms from *n* to $n/(1+\lambda)$. Thus in $\lambda = -0.5$ (more competitive), the effective number of firms is doubled.

3. The effect of a localised transport improvement

The analysis so far has explored the impacts of general changes in transport costs, proportional to distance, such as those arising from a change in fuel tax, or general technical improvements in vehicles which lower unit transport costs. Most road improvements only change costs on a single route, and may have a more muted impact on overall efficiency gains. They can be explored by replacing $d_i t$ by t_i, the transport cost paid by firm *i* in delivering goods to the central market. The average cost is now less sensitive to any individual transport cost change, for $\bar{c} = c + \bar{t}$, where $\bar{t} = \Sigma t_i/n$. Consider the linear case, where (3) can be rewritten as

$$bq_i = p - c - t_i = \frac{a - c + \sum_k t_k}{(n+1)} - t_i. \tag{15}$$

The social benefit of changing t_j is

$$\frac{dW}{dt_j} = \sum_i (p - c - t_i)\frac{dq_i}{dt_j} - q_j = \sum_i q_i\left(b\frac{dq_i}{dt_j}\right) - q_j,$$

where the second equality comes from the first order condition for maximising the profit of firm *i* (the first equality in (15).)

$$-b\frac{dq_i}{dt_j} = \delta_{ij} - \frac{1}{n+1}, \quad \delta_{ij} = 0, \quad i \neq j; \quad \delta_{ij} = 1, \quad i = j.$$

(here δ_{ij} is the Kronecker delta.) The efficiency ratio is

$$r = \frac{\sum_i q_i(-b\frac{dq_i}{dt_j})}{q_j} = 1 - \frac{Q}{(n+1)q_j}. \tag{18}$$

This equation has a quite remarkable simplicity and surprising implication. If transport costs are lowered for firms whose market share is *less* than $1/(n+1)$, r is negative and production efficiency decrease. These are precisely the more distant firms that benefit more from transport cost reductions. The reason is that the least socially costly way of producing goods in this simple model is to use inputs from the closest suppliers. If the transport costs of more distant firms fall, then their market share rises, but this increases total production costs. Of course, the total social benefit will still be positive provided $r > -1$, ie $q_j > \frac{1}{2}Q/n+1)$, and will equal $(1+r)q_j.(-\Delta t_j)$, but the transport benefits alone will overstate the social benefits.

If transport costs are negatively correlated with production costs (as they might well be, for an increase in distance and transport cost increases the area that can be searched for cheaper suppliers as the *square* of the distance), this result may be reversed. Equation (18) still holds, but if c_i is now interpreted as unit production costs, then

$$\frac{(n+1)q_j}{Q} = 1 + \frac{1}{n} - (n+1)(\frac{c_i - \bar{c} + t_i - \bar{t}}{p}). \tag{19}$$

Provided transport improvements favour firms whose production and transport costs together are less than the average, there will be positive productive efficiency gains.

4. Conclusions

The steady decrease in transport costs has had a dramatic effect on the average distance over which firms source their requirements, and the resulting increase in market area can be expected to generate additional benefits of increased choice and competition. The simple models considered here allow the effect of lower transport costs to affect the efficiency of the economy, but the resulting impacts appear small, and can be counterintuitive, with lowered transport costs reducing production efficiency. The moral of this paper is that careful modelling of the impact of transport improvements on production costs will be required to capture the indirect efficiency benefits, as these can go either way.

Reference

Venables, A.J. and M. Gasiorek (1997) ‘The welfare implications of transport improvements in the presence of market failure’, SACTRA Working Paper A, Department of Environment, Transport and the Regions, HETA Division.

Appendix The case of constant demand elasticity

For the constant elastic case, $-dQ/dt = \varepsilon Q(1/p)dp/dt$, where dp/dt can be found by differentiating (3):

$$\frac{1}{p}\frac{dp}{dt} = \frac{1}{\bar{c}}\frac{d\bar{c}}{dt} = \frac{\bar{d}}{\bar{c}}, \tag{20}$$

and so can ($-dq_i/dt$):

$$-\frac{dq_i}{dt} = \frac{\varepsilon\bar{d}}{\bar{c}}\left(q_i + \frac{cQ}{p}\frac{\delta_i}{\bar{d}}\right), \tag{21}$$

while from (8), $\Sigma\delta_i q_i = \theta\bar{d}Q$, (where $\theta \equiv n\varepsilon\bar{d}t\sigma_d^2/p$), so

$$t\sum_1 \delta_i\left(-\frac{dq_i}{dt}\right) = \frac{\varepsilon\bar{d}t}{\bar{c}}\sum_i\left(\delta_i q_i + \frac{cQ}{p\bar{d}}\delta_i^2\right) = \frac{\bar{d}Q\theta}{\bar{c}}\left(\varepsilon\bar{d}t + c\right). \tag{22}$$

The ratio of the efficiency gain to the transport benefit is, from (9)

$$\begin{aligned} r &= \frac{\varepsilon\bar{d}Q\dfrac{(p-\bar{c})}{\bar{c}} - \bar{d}Q\theta\left(\dfrac{\varepsilon t\bar{d}+c}{\bar{c}}\right)}{\bar{d}Q(1-\theta)}, \\ &= \frac{\varepsilon\left(\dfrac{p-\bar{c}}{\bar{c}}\right) - \theta\left(1 + \dfrac{\bar{d}t}{\bar{c}}(\varepsilon-1)\right)}{1-\theta}. \end{aligned} \tag{23}$$

The formula is fairly similar to the linear case in (11), and with the same numerical values as before, $r = 0.09$, somewhat larger than for the linear case, and again, readily made negative if the CV of transport costs, σ_d is higher (and with it, θ). The ratio of the actual fall in transport cost to the apparent fall, *dlogT/dlogt*, becomes

$$\frac{t}{T}\frac{dT}{dt} = 1 - \frac{\theta\left(1 + \dfrac{\bar{d}t}{\bar{c}}(\varepsilon-1)\right) + \dfrac{\varepsilon\bar{d}t}{\bar{c}}}{1-\theta}, \tag{24}$$

which, with the same parameters as before, is 0.71, compared to 0.73 in the linear case.