

# SUPPLEMENT

for

## Pairwise Tests of Purchasing Power Parity Using Aggregate and Disaggregate Price Measures

by

M. Hashem Pesaran, Ron P. Smith, Takashi Yamagata,  
Liudmyla Hvozdyk

March 30, 2006

- 1: Real Effective Exchange Rates: Procedures and Results
- 2: Unit Root Tests With Just Intercept
- 3: Seasonal Adjustment: Procedures and Results
- 4: Critical Values
- 5: Average Lags
- 6: Estimates of the Commodity Factor Models
- 7: Countries With Similar Factor Loadings

# 1 Real Effective Exchange Rates: Procedures and Results

Define  $M_{ijt}$  and  $X_{ijt}$  to be import of  $i$  from  $j$  and export from  $i$  to  $j$  at time  $t$ , respectively. The total trade between  $i$  and  $j$  is defined as

$$D_{ijt} = M_{ijt} + X_{ijt}.$$

We compute fixed trade weights by averaging trade flows over a three year time interval 1987–1989, using the data on imports and exports from the IMF Direction of Trade Statistics (DOTS) as

$$w_{i,j} = \frac{\bar{D}_{i,j}}{\sum_{j=0}^N \bar{D}_{i,j}}$$

where  $\bar{D}_{i,j} = \sum_{t=1987}^{1989} D_{i,j}/3$ . Observe that  $w_{ii} = 0$  and  $\sum_{j=0}^N w_{ij} = 1$ . The weights are shown as below:

**Table S1.1: Trade Weights**

(i,j)	BE	DK	FI	FR	DE	GR	IT	NL	PT	ES	UK	US
<b>BE</b>	0	0.0100	0.0067	0.2287	0.2788	0.0049	0.0680	0.2046	0.0061	0.0231	0.1083	0.0608
<b>DK</b>	0.0456	0	0.0473	0.0926	0.3431	0.0094	0.0740	0.0838	0.0133	0.0239	0.1636	0.1034
<b>FI</b>	0.0463	0.0662	0	0.0971	0.2853	0.0087	0.0740	0.0704	0.0131	0.0272	0.1924	0.1193
<b>FR</b>	0.1343	0.0123	0.0093	0	0.2650	0.0085	0.1743	0.0794	0.0163	0.0723	0.1210	0.1072
<b>DE</b>	0.1175	0.0317	0.0179	0.2009	0	0.0142	0.1491	0.1544	0.0125	0.0439	0.1335	0.1243
<b>GR</b>	0.0483	0.0175	0.0120	0.1156	0.3163	0	0.2222	0.0870	0.0042	0.0246	0.0894	0.0630
<b>IT</b>	0.0651	0.0139	0.0090	0.2423	0.3066	0.0205	0	0.0685	0.0116	0.0515	0.0994	0.1116
<b>NL</b>	0.1920	0.0187	0.0114	0.1214	0.3478	0.0079	0.0672	0	0.0080	0.0231	0.1225	0.0800
<b>PT</b>	0.0472	0.0192	0.0126	0.1766	0.1989	0.0035	0.0958	0.0712	0	0.1702	0.1355	0.0692
<b>ES</b>	0.0470	0.0104	0.0081	0.2283	0.2125	0.0064	0.1385	0.0579	0.0503	0	0.1174	0.1231
<b>UK</b>	0.0804	0.0280	0.0224	0.1555	0.2397	0.0074	0.0878	0.1237	0.0151	0.0457	0	0.1944
<b>US</b>	0.0751	0.0162	0.0127	0.1398	0.2582	0.0071	0.1169	0.0914	0.0095	0.0465	0.2267	0

Notes: The source of the data is [http://www.esds.ac.uk/international/support/user\\_guides/imf/dots.asp](http://www.esds.ac.uk/international/support/user_guides/imf/dots.asp)

Table S1.2: ADF Test Results of Real Effective Exchange Rate,  $\bar{q}_{it} = \sum_{j=0}^N w_{ij}q_{ijt}$ , With an Intercept

	Test	DF	ADF(1)	ADF(2)	ADF(3)	ADF(4)	ADF(5)	ADF(6)	ADF(7)	ADF(8)	ADF(9)	ADF(10)	ADF(11)	ADF(12)
BE	Statistic	-1.013	<b>-1.602</b>	-1.548	-1.825	-1.705	-1.679	-1.323	-1.489	-1.479	-1.616	-1.497	-1.649	-1.399
	AIC	570.013	<b>575.964</b>	574.977	575.672	574.781	573.784	574.205	573.732	572.740	572.152	571.350	570.971	571.254
DE	Statistic	-0.419	<b>-0.777</b>	-0.863	-0.840	-0.676	-0.697	-0.904	-1.040	-0.966	-1.097	-1.390	-1.519	-1.728
	AIC	555.388	<b>555.388</b>	554.539	553.540	553.092	552.110	551.790	551.105	550.118	549.351	549.717	549.077	548.856
DK	Statistic	-0.823	-1.232	-1.100	<b>-1.325</b>	-1.229	-1.218	-1.086	-0.953	-1.093	-1.066	-0.994	-0.916	-0.940
	AIC	568.716	574.972	574.496	<b>575.657</b>	574.857	573.857	573.340	572.936	572.762	571.781	571.050	570.492	569.556
ES	Statistic	-0.910	<b>-1.257</b>	-1.254	-1.290	-1.153	-1.183	-1.358	-1.489	-1.392	-1.485	-1.403	-1.289	-1.241
	AIC	490.756	<b>499.201</b>	498.204	497.318	497.225	496.313	497.108	496.976	496.259	495.735	494.934	494.468	493.546
IT	Statistic	-1.072	-1.524	-1.208	-1.566	<b>-1.266</b>	-1.416	-1.397	-1.293	-1.298	-1.386	-1.468	-1.459	-1.588
	AIC	464.282	471.204	473.074	477.193	<b>478.203</b>	477.875	476.875	476.001	475.020	474.248	473.557	472.559	472.175
FR	Statistic	-1.780	-2.528	-2.221	-2.487	-2.127	-1.988	-1.681	-1.990	<b>-2.270</b>	-2.317	-2.329	-2.187	-1.960
	AIC	552.808	558.853	558.590	558.835	559.262	558.361	558.790	559.774	<b>559.911</b>	559.043	558.095	557.246	557.133
GR	Statistic	-1.626	-1.910	<b>-1.600</b>	-1.444	-1.287	-1.242	-1.100	-1.320	-1.313	-1.074	-0.877	-0.807	-0.925
	AIC	435.685	436.879	<b>437.622</b>	437.081	436.625	435.647	435.105	435.565	434.568	434.899	435.037	434.181	433.773
NL	Statistic	-1.112	<b>-1.520</b>	-1.598	-1.392	-1.487	-1.564	-1.933	-1.877	-1.808	-2.257	-2.187	-2.152	-2.784
	AIC	602.563	<b>604.781</b>	603.953	603.530	602.792	601.965	603.001	602.001	601.003	602.179	601.179	600.185	604.073
PT	Statistic	-0.358	-0.774	<b>-0.549</b>	-0.676	-0.580	-0.418	-0.298	-0.260	-0.314	-0.444	-0.456	-0.454	-0.612
	AIC	480.016	484.189	<b>484.593</b>	484.149	483.373	483.105	482.526	481.560	480.743	480.324	479.335	478.335	478.324
FI	Statistic	-0.981	<b>-1.320</b>	-1.295	-1.278	-1.365	-1.584	-1.718	-1.494	-1.524	-1.667	-1.937	-1.804	-2.120
	AIC	455.064	<b>460.633</b>	459.640	458.641	458.051	458.837	458.461	458.606	457.678	457.417	458.266	457.461	458.689
UK	Statistic	-1.759	<b>-2.641</b>	-2.515	-2.667	-2.657	-2.151	-2.151	-2.220	-2.162	-2.172	-2.005	-2.074	-2.039
	AIC	429.512	<b>441.896</b>	440.939	440.448	439.490	441.741	440.764	440.014	439.040	438.078	437.824	437.303	436.319
US	Statistic	-0.809	<b>-1.245</b>	-1.139	-1.308	-1.247	-1.215	-1.164	-1.201	-1.317	-1.424	-1.498	-1.435	-1.554
	AIC	368.561	<b>375.657</b>	374.962	375.153	374.227	373.239	372.287	371.371	370.888	370.334	369.585	368.645	368.243
CV	10%	-2.586	-2.584	-2.574	-2.572	-2.579	-2.562	-2.549	-2.541	-2.535	-2.537	-2.525	-2.547	-2.533
CV	5%	-2.882	-2.880	-2.872	-2.884	-2.872	-2.872	-2.864	-2.884	-2.864	-2.872	-2.840	-2.867	-2.854

Notes: The real effective exchange rate,  $\bar{q}_{it} = \sum_{j=0}^N w_{ij}q_{ijt}$ , is based on the CPI of the countries. The lag augmentation is chosen by AIC from  $\ell = 1, 2, \dots, 12$ . In bold is the maximum AIC value and the corresponding test statistic. The last two rows represent critical values which depend upon  $T$  and are obtained by simulation with 10000 replications.

Table S1.3: ADF Test Results of Real Effective Exchange Rate,  $\bar{q}_{it} = \sum_{j=0}^N w_{ij}q_{ijt}$ , With an Intercept and a Trend

	Test	DF	ADF(1)	ADF(2)	ADF(3)	ADF(4)	ADF(5)	ADF(6)	ADF(7)	ADF(8)	ADF(9)	ADF(10)	ADF(11)	ADF(12)
BE	Statistic	-4.579	-5.090	-5.046	<b>-5.382</b>	-5.279	-5.306	-4.969	-5.137	-5.127	-5.238	-5.139	-5.245	-5.101
	AIC	579.588	586.541	585.559	<b>587.255</b>	586.255	585.539	585.321	585.338	584.436	584.136	583.280	583.106	583.532
DE	Statistic	-1.378	<b>-1.716</b>	-1.794	-1.773	-1.594	-1.621	-1.838	-1.989	-1.925	-2.060	-2.390	-2.574	-2.858
	AIC	554.003	<b>555.826</b>	555.027	554.028	553.440	552.493	552.345	551.803	550.803	550.160	550.865	550.506	550.723
DK	Statistic	-1.707	-2.275	-2.106	<b>-2.465</b>	-2.353	-2.361	-2.193	-2.018	-2.229	-2.203	-2.090	-1.950	-2.006
	AIC	568.860	575.814	575.140	<b>576.852</b>	575.921	574.966	574.235	573.609	573.754	572.758	571.863	571.094	570.263
ES	Statistic	-1.114	<b>-1.602</b>	-1.606	-1.668	-1.494	-1.536	-1.778	-1.971	-1.843	-2.018	-1.905	-1.718	-1.655
	AIC	489.971	<b>498.706</b>	497.721	496.894	496.695	495.817	496.800	496.852	496.033	495.728	494.826	494.170	493.204
IT	Statistic	-1.161	-1.557	-1.209	-1.535	<b>-1.193</b>	-1.297	-1.214	-1.016	-0.836	-0.837	-0.897	-0.837	-0.963
	AIC	465.902	471.609	474.283	477.531	<b>479.089</b>	478.407	477.462	476.899	476.022	475.029	474.128	473.161	472.481
FR	Statistic	-2.600	-3.267	-2.972	-3.195	-2.880	-2.739	-2.473	-2.718	<b>-2.936</b>	-2.952	-2.943	-2.798	-2.594
	AIC	554.331	560.373	560.083	560.289	560.891	560.016	560.683	561.508	<b>561.518</b>	560.598	559.627	558.829	558.949
GR	Statistic	-2.189	-2.402	<b>-2.125</b>	-1.979	-1.834	-1.774	-1.636	-1.785	-1.753	-1.525	-1.335	-1.247	-1.309
	AIC	436.746	437.686	<b>438.662</b>	438.298	438.076	437.156	436.882	436.988	435.993	436.693	437.253	436.633	435.921
NL	Statistic	-1.253	<b>-1.628</b>	-1.692	-1.486	-1.578	-1.652	-2.013	-1.953	-1.877	-2.307	-2.232	-2.193	-2.811
	AIC	602.203	<b>604.275</b>	603.417	603.028	602.290	601.462	602.473	601.474	600.476	601.570	600.570	599.575	603.380
PT	Statistic	-2.600	-2.843	<b>-2.657</b>	-2.742	-2.663	-2.558	-2.475	-2.440	-2.483	-2.592	-2.607	-2.602	-2.818
	AIC	483.214	487.432	<b>487.684</b>	487.313	486.492	486.235	485.662	484.696	483.948	483.677	482.741	481.759	482.223
FI	Statistic	-1.188	<b>-1.501</b>	-1.476	-1.458	-1.541	-1.750	-1.878	-1.661	-1.687	-1.816	-2.063	-1.936	-2.227
	AIC	454.438	<b>459.982</b>	458.990	457.991	457.402	458.193	457.820	457.962	457.032	456.753	457.567	456.774	457.966
UK	Statistic	-1.943	<b>-2.914</b>	-2.789	-2.966	-2.970	-2.404	-2.414	-2.510	-2.448	-2.472	-2.264	-2.380	-2.340
	AIC	428.860	<b>441.659</b>	440.675	440.302	439.389	441.342	440.393	439.733	438.738	437.819	437.423	437.038	436.038
US	Statistic	-1.988	<b>-2.414</b>	-2.306	-2.518	-2.453	-2.425	-2.374	-2.420	-2.536	-2.636	-2.720	-2.658	-2.765
	AIC	369.442	<b>376.910</b>	376.125	376.606	375.620	374.622	373.631	372.789	372.438	371.992	371.365	370.381	370.085
CV	10%	-3.138	-3.124	-3.124	-3.131	-3.127	-3.126	-3.110	-3.100	-3.101	-3.080	-3.066	-3.080	-3.058
CV	5%	-3.447	-3.445	-3.427	-3.424	-3.414	-3.419	-3.404	-3.413	-3.412	-3.429	-3.393	-3.388	-3.370

Notes: The real effective exchange rate,  $\bar{q}_{it} = \sum_{j=0}^N w_{ij}q_{ijt}$ , is based on the CPI of the countries. The lag augmentation is chosen by AIC from  $\ell = 1, 2, \dots, 12$ . In bold is the maximum AIC value and the corresponding test statistic. The last two rows represent critical values which depend upon  $T$  and are obtained by simulation with 10000 replications.

## 2 Unit Root Tests With Just Intercept

Table S2.1: Fractions of Pairs  $q_{ijct}$  for Which the Unit Root Hypothesis is Rejected at 10% Significance Level in the Case With an Intercept, for All 12 Countries

	Lag Order Chosen by AIC			Lag Order Chosen by SBC			KPSS		
	ADF	ADF-GLS	ADF-WS	ADF	ADF-GLS	ADF-WS	$0.75T^{1/3}$	$2T^{1/2}$	$av$
<b>CPI</b>	7.58	6.06	7.58	9.09	10.61	10.61	96.97	31.82	53.03
<b>Subcategories</b>									
<b>Bread</b>	6.06	4.55	6.06	9.09	7.58	10.61	96.97	50.00	72.73
<b>Meat</b>	12.12	19.70	19.70	10.61	18.18	19.70	95.45	25.76	50.00
<b>Dairy</b>	22.73	13.64	21.21	24.24	12.12	19.70	89.39	28.79	51.52
<b>Fruits</b>	28.79	18.18	30.30	53.03	46.97	65.15	77.27	37.88	50.00
<b>Tobacco</b>	21.21	19.70	21.21	21.21	16.67	22.73	81.82	28.79	46.97
<b>Alcohol</b>	9.09	18.18	19.70	6.06	18.18	16.67	93.94	33.33	59.09
<b>Clothing</b>	6.06	10.61	10.61	6.06	9.09	9.09	92.42	60.61	77.27
<b>Footwear</b>	18.18	6.06	13.64	15.15	6.06	7.58	86.36	37.88	46.97
<b>Rents</b>	18.18	9.09	12.73	14.55	5.45	7.27	94.55	30.91	49.09
<b>Fuel</b>	21.82	18.18	27.27	38.18	23.64	34.55	90.91	23.64	49.09
<b>Furnit.</b>	12.73	5.45	5.45	10.91	3.64	3.64	90.91	50.91	67.27
<b>Dom. Appl.</b>	9.09	9.09	5.45	5.45	5.45	5.45	94.55	56.36	67.27
<b>Vehicles</b>	7.58	3.03	6.06	10.61	3.03	4.55	95.45	45.45	63.64
<b>Pub. Transp</b>	13.64	9.09	12.12	19.70	7.58	12.12	90.91	24.24	46.97
<b>Comm.</b>	6.06	15.15	15.15	6.06	16.67	18.18	90.91	33.33	51.52
<b>Sound</b>	9.09	12.73	14.55	18.18	14.55	20.00	87.27	36.36	47.27
<b>Leisure</b>	16.67	9.09	13.64	15.15	7.58	9.09	98.48	22.73	57.58
<b>Books</b>	10.61	10.61	7.58	9.09	9.09	9.09	93.94	39.39	69.70
<b>Hotels</b>	7.58	10.61	16.67	4.55	7.58	13.64	96.97	33.33	60.61
<b>Average</b>	13.54	11.72	14.69	15.68	12.58	16.25	91.50	36.83	57.08

Notes: ADF is a standard Dicky-Fuller unit root test, ADF-GLS is Elliot et al. (1996) test, ADF-WS is Park and Fuller's (1995) weighted symmetric test. Unit root tests are conducted at 10% significance level for  $N_c(N_c + 1)/2$  distinct pairs of  $q_{ijct}$ ,  $i \neq j$ , for each commodity  $c$ , and augmentation orders are chosen by AIC and SBC of ADF regression from augmentation orders  $p = 0, 1, \dots, 12$ , then the fraction of the rejected pairs over  $N_c(N_c + 1)/2$  are computed. Critical values are tabulated in Section 4, which are different for  $T_{ijc}$  and augmentation orders. Lag window to compute KPSS statistics is shown in the second row. The category 'Average' located at the bottom row of the table is a simple average of fractions of all the commodities except for CPI.

### 3 Seasonal Adjustment: Procedures and Results

Price data series can be written as (suppressing  $i, c$ )

$$P_{ym}, y = 1, 2, \dots, Y_m; m = 1, 2, \dots, 12,$$

where  $y$  indicates year,  $y = 1, 2, \dots, Y_m$ , and  $m$  stands for month,  $m = 1, 2, \dots, 12$ . We use, as usually, lower case to indicate logarithm:

$$p_{ym} = \ln(P_{ym}).$$

Next, we take the first difference:

$$\Delta p_{ym} = p_{ym} - p_{y,m-1}.$$

Note that for  $p_{ym}$  with  $m = 1$ ,  $p_{y,m-1} \equiv p_{y-1,12}$ . As the initial observation is lost, it is replaced with  $\Delta p_{y+1,1}$ , which is the lagged price of the initial month of the second year. Then, take the monthly average of  $\Delta p_{ym}$ :

$$\Delta \bar{p}_m = \frac{1}{Y_m} \sum_{y=1}^{Y_m} \Delta p_{ym}.$$

Note that the number of years,  $Y_m$ , depends on the month. Also, as  $\sum_{m=1}^M \Delta \bar{p}_m \neq 0$  we compute demeaned monthly average,  $\Delta \bar{p}_m^+ = \Delta \bar{p}_m - a$ ,  $a = M^{-1} \sum_{m=1}^M \Delta \bar{p}_m$ , and define monthly demeaned lagged price as

$$\Delta \hat{p}_{ym} = \Delta p_{ym} - \Delta \bar{p}_m^+.$$

This ensures that

$$\sum_{m=1}^M \Delta \hat{p}_{ym} = \sum_{m=1}^M \Delta p_{ym} \text{ for all } y,$$

which we wanted. Next, we recover level price by taking a cumulative sum. Here, the initial observation is replaced with the original initial observations of  $p_{ym}$ , and we find seasonally adjusted log of price,  $p_{ym}^*$ . Transform double index to the single index, i.e.  $\{p_{11}^*, p_{12}^*, \dots, p_{1m}^*; p_{21}^*, p_{22}^*, \dots, p_{2m}^*; \dots; p_{y1}^*, p_{y2}^*, \dots, p_{ym}^*\}$  to  $p_t^*$ ,  $t = 1, 2, \dots, T$ ,  $T = 12Y_m$ . We define the log of seasonally adjusted price index of country  $i$  for commodity  $c$  at time  $t$  as

$$q_{ict}^* = e_{it} - p_{ct}^* + p_{ict}^*,$$

where  $e_{it}$  is the log of the bilateral exchange rate between country  $i$  and the US,  $p_{ct}^*$  is the log of seasonally adjusted US price index, and  $p_{ict}^*$  is the log of the seasonally adjusted price index of country  $i$ 's commodity  $c$  at time  $t$ . The log of seasonally adjusted real exchange rate for commodity  $c$  between any other pair of countries  $i, j \neq 0$  is computed as

$$q_{ijct}^* = q_{ict}^* - q_{jct}^*.$$

### Criterion for Seasonal Adjustment Application

We apply seasonal adjustment described above if the following criteria are satisfied. First, we run a joint significance test of the regression of  $\Delta p_{ict}$  on seasonal dummies for all commodity categories across countries at 1% level. Next, if the joint tests are significant for a half of the countries or more, we examine whether the time series plot of  $p_{ict}$  exhibits seasonality pattern. Otherwise, no seasonal adjustment is applied. We also do not apply seasonal adjustment if the level price exhibits a step function. The summary of seasonal adjustment is reported below.

**Table S3.1: Summary on Seasonal Adjustment Application**

Commodity	# rej at 1%	Description	Adjust?
Bread	5/12	Step function	No
Meat	8/12	Not step function except DK	Yes except BE, DE,DK,FE,US
Dairy	4/12	Only DE, ES, NL, US's dummies are jointly significant. For DE and ES seasonal adjustment seems effective	No
Fruits	12/12	It may be reasonable to seasonally adjust all	Yes
Tobacco	6/12	Clear step function	No
Alcohol	5/12	DK and FI are step functions, but others are smooth.	No
Clothing	11/12	Except US, seasonality exists (For us, p-value is 0.88)	Yes, except US
Footwear	9/12	Except US, seasonality exists (For us, p-value is 0.999)	Yes, except US
Rents	10/12	Most series are step functions	No
Fuel	4/12	No seasonality	No
Furnit.	10/12	It may be reasonable to seasonally adjust all	Yes
Dom. Appl.	7/12	It may be reasonable to seasonally adjust all	Yes
Vehicles	6/12	It is not very clear. DK is step function.	Yes except DK
Pub. Transp	11/12	Clear step function is only GR, others are kind of mixture.	No
Comm.	5/12	Most series are step functions	No
Sound	8/11	Sudden jump for US after 1995M10. DK is a sort of step function	Yes except DK
Leisure	11/12	Mixture of step type movement and seasonality.	No
Books	4/12	Step function	No
Hotels	11/12	DK and FI are step functions. Higher seasonality for BE after 1992M1, for NL after 1991M1.	Yes, except DK, FI,BE,NL
CPI	11/12	All to be adjusted	Yes

**Table S3.2: Fractions of Pairs  $q_{ijct}^*$  (Seasonally Adjusted) for Which the Unit Root Hypothesis is Rejected at 10% Significance Level in the Case with an Intercept and a Trend, for All 12 Countries**

	Lag Order Chosen by AIC			Lag Order Chosen by SBC			KPSS	
	ADF	ADF-GLS	ADF-WS	ADF	ADF-GLS	ADF-WS	$0.75T^{1/3}$	$2T^{1/2}$
<b>CPI</b>	13.64	4.55	9.09	12.12	10.61	12.12	96.97	31.82
<b>Subcategories</b>								
<b>Bread</b>	13.64	13.64	12.12	13.64	7.58	10.61	98.48	22.73
<b>Meat</b>	21.21	10.61	15.15	13.64	10.61	15.15	98.48	28.79
<b>Dairy</b>	13.64	13.64	12.12	13.64	7.58	10.61	98.48	22.73
<b>Fruits</b>	42.42	39.39	39.39	57.58	54.55	59.09	93.94	24.24
<b>Tobacco</b>	40.91	15.15	22.73	42.42	19.70	25.76	93.94	16.67
<b>Alcohol</b>	19.70	16.67	18.18	15.15	12.12	13.64	96.97	28.79
<b>Clothing</b>	12.12	7.58	7.58	18.18	3.03	4.55	98.48	18.18
<b>Footwear</b>	22.73	1.52	6.06	21.21	3.03	6.06	96.97	28.79
<b>Rents</b>	16.36	7.27	10.91	12.73	1.82	3.64	98.18	34.55
<b>Fuel</b>	27.27	23.64	27.27	32.73	30.91	34.55	96.36	30.91
<b>Furnit.</b>	27.27	9.09	16.67	24.24	6.06	12.12	98.48	21.21
<b>Dom. Appl.</b>	27.27	15.15	21.21	15.15	7.58	12.12	93.94	12.12
<b>Vehicles</b>	18.18	15.15	19.70	15.15	13.64	21.21	95.45	30.30
<b>Pub. Transp</b>	13.64	7.58	9.09	18.18	9.09	9.09	95.45	33.33
<b>Comm.</b>	7.58	12.12	7.58	7.58	9.09	4.55	100.00	22.73
<b>Sound</b>	25.45	16.36	21.82	25.45	20.00	21.82	98.18	27.27
<b>Leisure</b>	13.64	7.58	10.61	12.12	7.58	10.61	98.48	33.33
<b>Books</b>	22.73	15.15	19.70	18.18	16.67	16.67	95.45	18.18
<b>Hotels</b>	10.61	12.12	9.09	12.12	7.58	7.58	100.00	40.91
<b>Average</b>	20.86	13.65	16.16	20.48	13.06	15.76	97.14	26.09

Notes: ADF is a standard Dicky-Fuller unit root test, ADF-GLS is Elliot et al. (1996) test, ADF-WS is Park and Fuller's (1995) weighted symmetric test. Unit root tests are conducted at 10% significance level for  $N_c(N_c + 1)/2$  distinct pairs of  $q_{ijct}^*$ ,  $i \neq j$ , where  $q_{ijct}^*$  is based on seasonally adjusted prices, for each commodity  $c$ , and augmentation orders are chosen by AIC and SBC of ADF regression from augmentation orders  $p = 0, 1, \dots, 12$ , then the fraction of the rejected pairs over  $N_c(N_c + 1)/2$  are computed. Critical values are tabulated in Section 4, which are different for  $T_{ijc}$  and augmentation orders. Lag window to compute KPSS statistics is shown at the second row. The category 'Average' located at the bottom row of the table is a simple average of fractions of all the commodities except CPI.



**Table S3.3: Fractions of Pairs  $q_{ijct}^*$  (Seasonally Adjusted) for Which the Unit Root Hypothesis is Rejected at 10% Significance Level in the Case With an Intercept, for All 12 Countries**

	Lag Order Chosen by AIC			Lag Order Chosen by SBC			KPSS		
	ADF	ADF-GLS	ADF-WS	ADF	ADF-GLS	ADF-WS	$0.75T^{1/3}$	$2T^{1/2}$	$av$
<b>CPI</b>	7.58	9.09	10.61	7.58	10.61	7.58	96.97	31.82	53.03
<b>Subcategories</b>									
<b>Bread</b>	6.06	4.55	6.06	9.09	7.58	10.61	96.97	50.00	72.73
<b>Meat</b>	13.64	10.61	16.67	10.61	12.12	15.15	95.45	25.76	50.00
<b>Dairy</b>	6.06	4.55	6.06	9.09	7.58	10.61	96.97	50.00	72.73
<b>Fruits</b>	36.36	30.30	39.39	57.58	45.45	62.12	84.85	36.36	51.52
<b>Tobacco</b>	21.21	19.70	21.21	21.21	16.67	22.73	81.82	28.79	46.97
<b>Alcohol</b>	9.09	18.18	19.70	6.06	18.18	16.67	93.94	33.33	59.09
<b>Clothing</b>	6.06	6.06	3.03	4.55	1.52	0.00	92.42	60.61	77.27
<b>Footwear</b>	13.64	4.55	6.06	12.12	4.55	3.03	87.88	37.88	46.97
<b>Rents</b>	18.18	9.09	12.73	14.55	5.45	7.27	94.55	30.91	49.09
<b>Fuel</b>	21.82	18.18	27.27	38.18	23.64	34.55	90.91	23.64	49.09
<b>Furnit.</b>	18.18	6.06	10.61	16.67	7.58	10.61	92.42	57.58	71.21
<b>Dom. Appl.</b>	9.09	12.12	10.61	7.58	13.64	12.12	95.45	62.12	71.21
<b>Vehicles</b>	6.06	3.03	4.55	10.61	3.03	4.55	95.45	45.45	63.64
<b>Pub. Transp</b>	13.64	9.09	12.12	19.70	7.58	12.12	90.91	24.24	46.97
<b>Comm.</b>	6.06	15.15	15.15	6.06	16.67	18.18	90.91	33.33	51.52
<b>Sound</b>	12.73	10.91	14.55	12.73	10.91	14.55	87.27	36.36	47.27
<b>Leisure</b>	16.67	9.09	13.64	15.15	7.58	9.09	98.48	22.73	57.58
<b>Books</b>	10.61	10.61	7.58	9.09	9.09	9.09	93.94	39.39	69.70
<b>Hotels</b>	4.55	10.61	10.61	1.52	4.55	4.55	96.97	33.33	60.61
<b>Average</b>	13.14	11.18	13.56	14.85	11.76	14.61	92.50	38.52	58.69

Notes: ADF is a standard Dicky-Fuller unit root test, ADF-GLS is Elliot et al. (1996) test, ADF-WS is Park and Fuller's (1995) weighted symmetric test. Unit root tests are conducted at 10% significance level for  $N_c(N_c + 1)/2$  distinct pairs of  $q_{ijct}^*$ ,  $i \neq j$ , where  $q_{ijct}^*$  is based on seasonally adjusted prices, for each commodity  $c$ , and augmentation orders are chosen by AIC and SBC of ADF regression from augmentation orders  $p = 0, 1, \dots, 12$ , then the fraction of the rejected pairs over  $N_c(N_c + 1)/2$  are computed. Critical values are tabulated in Section 4, which are different for  $T_{ijc}$  and augmentation orders. Lag window to compute KPSS statistics is shown at the second row. The category 'Average' located at the bottom row of the table is a simple average of fractions of all the commodities except CPI.

## 4 Critical Values

p,T	With Intercept, 10%					With Intercept and Trend, 10%				
	84	125	132	165-166	175-180	84	125	132	165-166	175-180
	<b>ADF</b>					<b>ADF</b>				
0	-2.594	-2.549	-2.590	-2.586	-2.601	-3.148	-3.151	-3.141	-3.138	-3.130
1	-2.604	-2.571	-2.600	-2.584	-2.603	-3.143	-3.141	-3.148	-3.124	-3.144
2	-2.563	-2.548	-2.564	-2.570	-2.578	-3.122	-3.116	-3.132	-3.124	-3.125
3	-2.592	-2.557	-2.584	-2.572	-2.583	-3.127	-3.124	-3.128	-3.131	-3.126
4	-2.557	-2.547	-2.572	-2.579	-2.572	-3.129	-3.117	-3.120	-3.127	-3.122
5	-2.587	-2.560	-2.576	-2.562	-2.576	-3.116	-3.121	-3.115	-3.126	-3.124
6	-2.534	-2.536	-2.569	-2.549	-2.570	-3.083	-3.098	-3.116	-3.110	-3.118
7	-2.531	-2.533	-2.551	-2.541	-2.576	-3.087	-3.089	-3.117	-3.100	-3.109
8	-2.519	-2.523	-2.539	-2.535	-2.569	-3.062	-3.071	-3.111	-3.101	-3.094
9	-2.529	-2.530	-2.540	-2.537	-2.583	-3.068	-3.075	-3.111	-3.080	-3.111
10	-2.504	-2.522	-2.531	-2.525	-2.574	-3.036	-3.053	-3.090	-3.066	-3.077
11	-2.495	-2.529	-2.526	-2.547	-2.561	-3.030	-3.070	-3.090	-3.080	-3.079
12	-2.491	-2.516	-2.522	-2.533	-2.559	-3.007	-3.042	-3.071	-3.058	-3.082
	<b>ADF-GLS</b>					<b>ADF-GLS</b>				
0	-1.876	-1.793	-1.797	-1.783	-1.754	-2.764	-2.691	-2.684	-2.656	-2.650
1	-1.866	-1.786	-1.789	-1.793	-1.744	-2.759	-2.695	-2.681	-2.663	-2.664
2	-1.843	-1.772	-1.772	-1.777	-1.740	-2.715	-2.667	-2.664	-2.642	-2.638
3	-1.843	-1.778	-1.787	-1.763	-1.728	-2.714	-2.675	-2.677	-2.651	-2.636
4	-1.830	-1.767	-1.773	-1.745	-1.736	-2.691	-2.651	-2.659	-2.638	-2.625
5	-1.833	-1.772	-1.781	-1.738	-1.733	-2.698	-2.666	-2.643	-2.643	-2.630
6	-1.799	-1.748	-1.760	-1.727	-1.722	-2.655	-2.635	-2.640	-2.635	-2.620
7	-1.799	-1.749	-1.751	-1.729	-1.727	-2.642	-2.637	-2.630	-2.625	-2.618
8	-1.789	-1.731	-1.743	-1.717	-1.707	-2.594	-2.619	-2.606	-2.606	-2.600
9	-1.785	-1.720	-1.748	-1.713	-1.711	-2.610	-2.622	-2.599	-2.605	-2.615
10	-1.768	-1.708	-1.725	-1.707	-1.697	-2.562	-2.600	-2.585	-2.576	-2.581
11	-1.750	-1.712	-1.728	-1.707	-1.693	-2.541	-2.594	-2.581	-2.582	-2.584
12	-1.728	-1.696	-1.710	-1.696	-1.674	-2.501	-2.555	-2.555	-2.558	-2.581
	<b>ADF-WS</b>					<b>ADF-WS</b>				
0	-2.279	-2.254	-2.272	-2.275	-2.256	-2.963	-2.932	-2.933	-2.928	-2.928
1	-2.285	-2.268	-2.279	-2.279	-2.274	-2.983	-2.954	-2.958	-2.965	-2.937
2	-2.279	-2.257	-2.267	-2.269	-2.260	-2.969	-2.951	-2.937	-2.939	-2.939
3	-2.297	-2.269	-2.282	-2.288	-2.269	-2.993	-2.976	-2.976	-2.952	-2.946
4	-2.278	-2.262	-2.286	-2.258	-2.261	-2.991	-2.964	-2.983	-2.966	-2.947
5	-2.313	-2.288	-2.287	-2.274	-2.277	-3.034	-2.990	-2.998	-2.982	-2.985
6	-2.306	-2.289	-2.276	-2.268	-2.281	-3.021	-2.980	-2.983	-2.968	-2.964
7	-2.327	-2.277	-2.271	-2.283	-2.285	-3.034	-3.006	-3.003	-2.981	-2.985
8	-2.318	-2.271	-2.278	-2.276	-2.268	-3.017	-2.999	-2.999	-2.980	-2.969
9	-2.329	-2.284	-2.286	-2.291	-2.278	-3.055	-3.017	-3.002	-2.998	-2.980
10	-2.315	-2.284	-2.289	-2.270	-2.264	-3.016	-3.006	-2.993	-2.964	-2.973
11	-2.331	-2.298	-2.302	-2.286	-2.279	-3.032	-3.016	-3.008	-2.983	-2.988
12	-2.313	-2.300	-2.280	-2.269	-2.270	-3.005	-3.003	-3.000	-2.981	-2.988

Note: Critical values are obtained based on 10000 simulations.

## 5 Average Lags

Table S5: Average Lag Order  $\bar{p}$  Chosen by AIC and SBC of the ADF Regressions for  $q_{ijct}$  With a Maximum lag of 12, When the Null Hypothesis is Rejected at 10% Significance Level in the Case with an Intercept and a Trend, for all 12 Countries

	$\bar{p}(\text{AIC})$	$\bar{p}(\text{SBC})$
<b>CPI</b>	4.33	1.17
<b>Subcategories</b>		
<b>Bread</b>	3.38	0.92
<b>Meat</b>	2.56	0.77
<b>Dairy</b>	2.23	0.74
<b>Fruits</b>	8.85	2.24
<b>Tobacco</b>	1.98	0.38
<b>Alcohol</b>	2.86	1.02
<b>Clothing</b>	8.77	6.73
<b>Footwear</b>	7.21	4.94
<b>Rents</b>	5.18	2.38
<b>Fuel</b>	2.73	0.65
<b>Furnit.</b>	5.15	2.67
<b>Dom. Appl.</b>	5.55	2.87
<b>Vehicles</b>	3.03	0.52
<b>Pub. Transp</b>	4.08	0.68
<b>Comm.</b>	2.15	0.62
<b>Sound</b>	4.42	0.64
<b>Leisure</b>	3.79	0.59
<b>Books</b>	2.11	0.48
<b>Hotels</b>	6.59	0.97
<b>Average</b>	4.35	1.62

Notes: This table is a supplement for Table 3 of the paper. The ‘Average’ located at the bottom row of the table is a simple average of the average lag orders chosen by AICs and SBCs for all the commodities except CPI.

## 6 Estimates of the Commodity Factor Models

Country	$\hat{\alpha}_{ic}$	$\hat{\gamma}_{ic1}$	$\hat{\gamma}_{ic2}$	$\hat{\alpha}_{ic}$	$\hat{\gamma}_{ic1}$	$\hat{\gamma}_{ic2}$	$\hat{\alpha}_{ic}$	$\hat{\gamma}_{ic1}$	$\hat{\gamma}_{ic2}$	$\hat{\alpha}_{ic}$	$\hat{\gamma}_{ic1}$	$\hat{\gamma}_{ic2}$
	<b>Bread</b>			<b>Alcohol</b>			<b>Furnit.</b>			<b>Sound</b>		
BE	1.43	-0.39	1.28	1.04	0.54	0.47	0.38	0.36	0.87	1.01	0.66	0.41
DE	-1.45	-0.26	1.10	-2.00	-0.17	1.18	-2.27	0.50	0.61	-2.02	0.61	0.46
DK	-0.79	-0.28	1.34	-0.21	0.50	0.35	-1.42	0.42	0.84	-0.87	0.56	0.56
ES	0.96	-0.34	1.76	1.69	2.23	-1.09	1.04	0.75	0.61	2.35	0.70	0.28
IT	5.02	1.73	-0.89	4.91	1.22	-0.34	3.91	1.21	0.00	4.79	1.03	-0.04
FR	-0.22	-0.13	0.94	-0.59	0.29	0.64	-1.37	0.45	0.74	-0.70	0.47	0.52
GR	2.85	-1.50	2.20	2.34	1.25	-0.42	1.79	0.46	0.58	1.95	0.68	0.35
NL	-0.89	-0.22	0.90	-1.55	0.19	0.71	-1.97	0.32	0.72	-1.15	0.58	0.23
PT	1.49	-1.16	2.42	1.47	1.77	-0.56	0.90	1.34	0.05	.	.	.
FI	-1.15	3.77	-2.77	-0.84	1.40	-0.49	.	.	.	-1.80	1.14	0.17
UK	-1.76	0.49	0.00	-2.53	0.50	0.27	-2.43	0.11	0.60	-1.79	0.71	-0.15
	<b>Meat</b>			<b>Clothing</b>			<b>Dom. Appl.</b>			<b>Leisure</b>		
BE	1.37	-0.20	1.09	0.57	1.22	-0.06	0.23	0.17	1.12	1.08	0.85	0.15
DE	-1.41	-0.23	1.04	-1.98	0.23	0.79	-2.42	0.28	0.87	-1.69	1.27	-0.35
DK	-0.26	0.20	0.67	-0.58	1.52	-0.54	-1.23	0.12	1.07	-0.80	0.31	0.73
ES	1.55	1.14	0.07	1.51	1.14	0.07	1.14	-0.16	1.49	1.75	3.12	-1.96
IT	4.87	0.74	0.14	4.89	1.39	-0.51	4.04	1.44	-0.28	3.77	-0.27	1.52
FR	-0.41	-0.25	1.10	-0.62	1.13	-0.19	-1.09	0.02	1.07	-0.38	2.22	-1.35
GR	3.60	-1.14	1.54	2.43	1.79	-0.97	2.29	0.12	0.73	2.78	5.90	-5.11
NL	-1.12	-0.46	1.20	-0.58	0.32	0.26	-2.11	0.12	0.96	-1.18	1.80	-1.00
PT	2.98	0.57	0.15	0.68	1.18	0.30	1.07	0.25	1.13	2.99	5.18	-4.39
FI	-2.47	3.41	-1.90	0.47	2.66	-2.17	.	.	.	-0.90	1.09	-0.17
UK	-1.41	-0.26	0.62	-2.67	-1.66	2.46	-2.30	-0.26	0.92	-2.06	5.46	-4.78
	<b>Dairy</b>			<b>Footwear</b>			<b>Vehicles</b>			<b>Books</b>		
BE	1.21	0.57	0.38	0.93	1.17	-0.13	1.33	0.92	-0.01	0.45	-0.49	1.68
DE	-1.58	-0.02	0.89	-2.02	0.91	0.12	-2.13	1.46	-0.39	-1.94	-0.15	1.13
DK	-0.64	1.43	-0.43	-0.63	1.17	-0.18	-0.80	0.70	0.36	-1.16	0.27	0.89
ES	1.51	1.09	0.13	1.70	1.35	-0.20	1.86	2.24	-1.15	1.75	0.32	0.79
IT	4.45	1.62	-0.58	4.76	1.54	-0.61	5.02	0.73	0.11	5.19	2.31	-1.50
FR	-0.42	0.34	0.52	-0.60	1.31	-0.38	-0.65	1.04	-0.09	-0.47	-0.33	1.19
GR	2.83	-2.37	3.03	2.96	2.08	-1.43	3.77	-2.39	2.77	0.96	0.72	0.61
NL	-1.44	0.01	0.84	-0.97	0.11	0.59	-1.84	1.20	-0.20	-1.36	-0.29	1.11
PT	2.37	1.67	-0.74	1.30	0.54	0.73	1.98	3.59	-2.58	0.51	-1.08	2.64
FI	-1.55	5.27	-4.07	-0.47	1.41	-0.62	-1.22	0.72	0.32	-0.30	4.06	-3.27
UK	-2.49	-0.55	1.28	-3.16	-1.45	2.41	-2.23	0.70	-0.05	-2.49	-0.21	0.94
	<b>Fruits</b>			<b>Rents</b>			<b>Pub. Transp</b>			<b>Hotels</b>		
BE	1.91	1.01	-0.29	1.92	2.02	-1.19	1.38	0.09	0.81	0.99	1.03	0.00
DE	-2.01	1.15	-0.13	-1.49	1.31	-0.37	-1.52	0.10	0.76	-1.38	1.44	-0.64
DK	-0.08	0.82	-0.02	-0.21	2.16	-1.16	-0.81	0.38	0.69	-0.14	1.56	-0.74
ES	1.35	0.79	0.49	3.23	1.76	-1.03	1.50	0.56	0.66	1.40	0.59	0.67
IT	4.55	1.03	-0.02	4.63	0.58	0.45	4.49	2.18	-1.18	4.51	1.35	-0.33
FR	-0.25	0.75	0.08	-0.29	1.86	-0.92	-0.29	0.06	0.77	-0.64	1.13	-0.20
GR	2.94	1.04	-0.40	3.94	4.69	-4.11	2.90	-0.56	1.23	2.59	0.24	0.54
NL	-1.12	0.97	-0.21	-0.95	2.16	-1.32	-1.29	0.05	0.78	-1.11	1.44	-0.70
PT	2.02	1.00	0.01	.	.	.	2.34	0.58	0.40	1.91	0.02	1.06
FI	-1.55	2.30	-1.20	-0.95	-0.55	1.50	-1.29	2.36	-1.31	-1.42	2.72	-1.64
UK	-2.52	1.40	-0.66	-2.67	1.54	-0.62	-2.22	0.96	-0.32	-2.20	1.56	-0.92

The table is continued on the next page.

Estimates of the Commodity Factor Models, Continued

Country	$\hat{\alpha}_{ic}$	$\hat{\gamma}_{ic1}$	$\hat{\gamma}_{ic2}$	$\hat{\alpha}_{ic}$	$\hat{\gamma}_{ic1}$	$\hat{\gamma}_{ic2}$	$\hat{\alpha}_{ic}$	$\hat{\gamma}_{ic1}$	$\hat{\gamma}_{ic2}$
	Tobacco			Fuel			Comm.		
<b>BE</b>	1.88	0.42	0.32	1.36	1.38	-0.49	0.90	1.24	-0.19
<b>DE</b>	0.25	0.03	0.24	-2.19	1.22	-0.15	-2.43	1.33	-0.17
<b>DK</b>	1.43	0.00	0.32	-1.57	0.52	0.77	-1.13	0.72	0.46
<b>ES</b>	3.47	0.28	0.27	1.69	1.36	-0.23	1.44	1.46	-0.24
<b>IT</b>	4.88	0.68	0.22	4.33	0.34	0.73	4.96	0.88	-0.01
<b>FR</b>	0.97	0.62	-0.24	-1.03	0.96	0.11	-0.34	1.36	-0.51
<b>GR</b>	2.90	0.58	0.08	1.26	2.31	-1.15	1.18	1.58	-0.41
<b>NL</b>	-0.01	0.34	0.07	-1.42	0.98	-0.12	-2.25	1.29	-0.15
<b>PT</b>	2.82	0.59	0.19	1.56	0.61	0.60	2.42	1.81	-0.90
<b>FI</b>	-0.51	-0.40	1.24	.	.	.	1.30	0.00	0.24
<b>UK</b>	-0.66	0.16	-0.02	-3.33	1.15	-0.14	-3.19	1.76	-0.78

Notes: This table reports the OLS estimates for the regressions  $q_{ict} = \hat{\alpha}_{ic} + \hat{\gamma}_{ic1}\bar{q}_{ct} + \hat{\gamma}_{ic2}\bar{q}_t + \hat{\varepsilon}_{ict}$ ,  $i = 0, 1, \dots, N_c$ ;  $c = 1, 2, \dots, C$ ,  $t = 1, 2, \dots, T_{ic}$ , where  $\bar{q}_{ct} = (N_c + 1)^{-1} \sum_{i=0}^{N_c} q_{ict}$  and  $\bar{q}_t = C^{-1} \sum_{c=1}^C \bar{q}_{ct}$ , for each combination of  $(i, c)$ .

## 7 Countries With Similar Factor Loadings

**Table S7: Countries Whose Differences of Factor Loadings are  $\pm 0.1$  From Those of a Base Country**

Category	Base country	Countries with $\pm 0.1$ Difference
<b>Bread</b>	FR	NL
	NL	FR
<b>Meat</b>	BE	DE FR
	DE	BE FR
	FR	BE DE
<b>Dairy</b>	DE	NL
	NL	DE
<b>Fruits</b>	BE	NL
	IT	PT
	NL	BE
	PT	IT
<b>Tobacco</b>	DE	DK
	DK	DE
	IT	PT
	PT	IT
<b>Alcohol</b>	DK	UK
	IT	GR
	FR	NL
	GR	IT
	NL	FR
	UK	DK
<b>Footwear</b>	BE	DK
	DK	BE
<b>Fuel</b>	DE	UK
	UK	DE
<b>Furnit.</b>	BE	DK
	DE	GR
	DK	BE
	GR	DE
<b>Dom. Appl.</b>	BE	DK PT
	DK	BE
	PT	BE
<b>Vehicles</b>	DK	FI
	FI	DK
<b>Pub. Transp</b>	BE	DE FR NL
	DE	BE FR NL
	FR	BE DE NL
	NL	BE DE FR
<b>Comm.</b>	BE	DE NL
	DE	BE NL
	NL	BE DE
<b>Sound</b>	BE	DE GR
	DE	BE
	DK	FR
	ES	GR
	FR	DK
	GR	BE ES
<b>Books</b>	FR	NL
	NL	FR
<b>Hotels</b>	DE	NL
	NL	DE

Notes: The third column of the table reports the countries such that  $|\hat{\gamma}_{ic1} - \hat{\gamma}_{jc1}|$  and  $|\hat{\gamma}_{ic2} - \hat{\gamma}_{jc2}|$  are less than 0.1, where  $i$  is the base country and  $j \neq i$  for each commodity  $c$ ,  $\hat{\gamma}_{ic1}$  and  $\hat{\gamma}_{ic2}$  are factor loadings estimated from the regression  $q_{ict} = \hat{\alpha}_{ic} + \hat{\gamma}_{ic1}\bar{q}_{ct} + \hat{\gamma}_{ic2}\bar{q}_t + \hat{\varepsilon}_{ict}$ ,  $\bar{q}_{ct} = (N_c + 1)^{-1} \sum_{i=0}^{N_c} q_{ict}$  and  $\bar{q}_t = C^{-1} \sum_{c=1}^C \bar{q}_{ct}$ , for each combination of  $(i, c)$ .