

Panel Unit Root Tests in the Presence of a Multifactor Error Structure*

M. Hashem Pesaran^a L. Vanessa Smith^b
Takashi Yamagata^c

^aUniversity of Southern California and Trinity College, Cambridge

^bCFAP & CIMF, University of Cambridge

^cDepartment of Economics and Related Studies, University of York

1 February 2013

Abstract

This paper extends the cross-sectionally augmented panel unit root test (*CIPS*) proposed by Pesaran (2007) to the case of a multifactor error structure, and proposes a new panel unit root test based on a simple average of cross-sectionally augmented Sargan-Bhargava statistics (*CSB*). The basic idea is to exploit information regarding the m unobserved factors that are shared by k observed time series in addition to the series under consideration. Initially, we develop the tests assuming that m^0 , the true number of factors is known, and show that the limit distribution of the tests does not depend on any nuisance parameters, so long as $k \geq m^0 - 1$. Small sample properties of the tests are investigated by Monte Carlo experiments and are shown to be satisfactory. Particularly, the proposed *CIPS* and *CSB* tests have the correct size for *all* combinations of the cross section (N) and time series (T) dimensions considered. The power of both tests rise with N and T , although the *CSB* test performs better than the *CIPS* test for smaller sample sizes. The various testing procedures are illustrated with empirical applications to real interest rates and real equity prices across countries.

JEL Classifications: C12, C15, C22, C23

Keywords: Panel Unit Root Tests, Cross Section Dependence, Multifactor Error Structure, Fisher Inflation Parity, Real Equity Prices.

*We would like to thank Anindya Banerjee, Soren Johansen and Joachim Westerlund for useful comments. This version has also benefited greatly from constructive comments and suggestions by the Editor (Cheng Hsiao), an associate editor and three anonymous referees.

1 Introduction

There is now a sizeable literature on testing for unit roots in panels where both cross section (N) and time series (T) dimensions are relatively large. Reviews of this literature are provided in Banerjee (1999), Baltagi and Kao (2000), Choi (2004), and in Breitung and Pesaran (2008). The so called first generation panel unit root tests pioneered by Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003) focussed on panels where the errors were cross-sectionally uncorrelated. More recently, to deal with a number of applications such as testing for purchasing power parity or cross country output convergence, the second generation panel unit root tests have focussed on the case where the errors are allowed to be cross-sectionally correlated.

Three main approaches have been proposed. The first, pioneered by Maddala and Wu (1999), and developed further by Chang (2004), Smith et al. (2004), Cerrato and Sarantis (2007), and Palm et al. (2011), applies bootstrap methods to panel unit root tests. The main idea of this approach is to approximate the distribution of the test statistic under cross section dependence by block bootstrap resampling to preserve the pattern of cross section dependence in the panel. This approach allows for general cross section dependence structures, however, it is mainly suited to panels with large T and relatively small N .

The second approach is due to Bai and Ng (2004, 2010) and proposes tests based on a decomposition of the observed series, y_{it} ; $i = 1, 2, \dots, N$, $t = 1, 2, \dots, T$, into two unobserved components, common factors and idiosyncratic errors, and tests for unit roots in both of these components. It is also tested if the unobserved common factors are cointegrated. This is known as the PANIC (panel analysis of nonstationarity in idiosyncratic and common components) approach, and provides indirect tests of unit roots in the observed series. The factors are estimated from m^0 principal components (PC) of Δy_{it} . It is assumed that m^0 , the true number of factors, is known or estimated from the observations. If it is found that the estimated factors contain unit roots and are not cointegrated it is then concluded that the N series are integrated of order 1. If the presence of a unit root in the factors is rejected, in the second stage the PANIC procedure applies panel unit root tests to the N idiosyncratic errors. Estimates of idiosyncratic errors are obtained as defactored observations, also known as PANIC residuals. Moon and Perron (2004) follow a similar approach in that they base their test on a principal components estimator of common factors. In particular, their test is based on defactored observations obtained by projecting the panel data onto the space orthogonal to the estimated factor loadings. Bai and Ng (2010) propose two panel unit root tests which are applied to the PANIC residuals. The first one is based on a pooled estimate of the autoregressive root fitted to the PANIC residuals, as in Moon and Perron (2004), and the second one employs a panel version of the modified Sargan-Bhargava test (*PMSB*).¹

The third approach, proposed in Pesaran (2007), augments the individual Dickey-Fuller (DF) regressions of y_{it} with cross section averages, $\bar{y}_{t-1} = N^{-1}\sum_{j=1}^N y_{j,t-1}$ and $\Delta\bar{y}_t$, to take account of error cross section dependence. These cross-sectionally augmented DF regressions can be further augmented with lagged changes $\Delta y_{i,t-s}$, $\Delta\bar{y}_{t-s}$, for $s = 1, 2, \dots$, to deal with possible serial correlation in the residuals. These doubly augmented DF regressions are referred to as CADF regressions. The panel unit root test statistic is then computed as the average of the CADF statistics. It is shown that the average statistic is free of nuisance parameters but, due to non-zero cross correlation of the individual, $CADF_i$, statistics, the average statistic has a non-normal limit distribution as N and $T \rightarrow \infty$. Monte Carlo experiments show that Pesaran's

¹Westerlund and Larsson (2009) provide further theoretical results on the asymptotic validity of the pooled versions of the PANIC procedure.

test has desirable small sample properties in the presence of a single unobserved common factor but show size distortions if the number of common factors exceeds unity.² A small sample comparison of some of these tests is provided in Gengenbach, Palm and Urbain (2009).³

The data generating mechanisms underlying the PANIC approach differ in one important respect from the ones considered by Moon and Perron (2004) and Pesaran (2007). The latter studies assume that under the null of unit roots the common factor components have the same order of integration as the idiosyncratic components, whilst the PANIC approach allows the order of integration of the factors to differ from that of the idiosyncratic components. However, if the primary objective of the exercise is to test for unit roots in the observed series, y_{it} , the distinction between the common and idiosyncratic components of y_{it} is not essential. The distinction will become relevant if the unit root null hypothesis of y_{it} is not rejected. In that case it would indeed be of interest to investigate further whether the source of the non-stationarity lies with the common factors, the idiosyncratic components, or both.

The present paper extends Pesaran's *CIPS* test to the case of a multifactor error structure. This is a non-trivial yet important extension which is much more broadly applicable. It has also the advantage of being intuitive and simple to implement. Following Bai and Ng (2010) we also consider a panel unit root test based on simple averages of cross-sectionally augmented Sargan-Bhargava type statistics, which we denote by *CSB*. The presence of multiple unobserved factors poses a number of additional challenges. In order to deal with a multifactor structure, we propose to utilize the information contained in a number of k additional variables, \mathbf{x}_{it} , that together are assumed to share the common factors of the series of interest, y_{it} . The ADF regression for y_{it} is then augmented with cross-sectional averages of y_{it} and \mathbf{x}_{it} .⁴

The requirement of finding such additional variables seems quite plausible in the case of panel data sets from economics and finance where economic agents often face common economic environments. Most macroeconomic theories postulate the presence of the same unobserved common factors (such as shocks to technology, tastes and fiscal policy), and it is therefore natural to expect that many macroeconomic variables, such as interest rates, inflation and output share the same factors. If anything, it would be difficult to find macroeconomic time series that do not share one or more common factors. For example, in testing for unit roots in a panel of real outputs one would expect the unobserved common shocks to output (that originate from technology) to also manifest themselves in employment, consumption and investment. In the case of testing for unit roots in inflation across countries, one would expect the unobserved common factors that correlate inflation rates across countries to also affect short-term and long-term interest rates across markets and economies. The fundamental issue is to ascertain the nature of dependence and persistence that is observed across markets and over time. The present paper can, therefore, be viewed as a first step in the process of developing a coherent framework for the analysis of unit roots and multiple cointegration in large panels.

²The cross section augmentation procedure is also employed by Hadri and Kurozumi (2011) in their work on testing the null of stationarity in panels.

³Other panel unit root tests have also been proposed by Chang (2002), who employs a non-linear IV method, Choi and Chue (2007) who use a subsampling method to account for cross-section correlation, and Phillips and Sul (2003) who use an orthogonalisation procedure to deal with error cross-dependence in the case of a single common factor.

⁴The idea of augmenting ADF regressions with other covariates has been investigated in the unit root literature by Hansen (1995) and Elliott and Jansson (2003). These authors consider the additional covariates in order to gain power when testing the unit root hypothesis in the case of a single time series. In this paper we augment ADF regressions with cross section averages to eliminate the effects of unobserved common factors in the case of panel unit root tests.

Initially we develop the tests assuming that m^0 , the true number of factors is known, and show that the limit distribution of *CIPS* and *CSB* tests does not depend on any nuisance parameters, so long as $k \geq m^0 - 1$. But, in practice m^0 is rarely known. Most existing methods of estimating m^0 , such as the information criteria of Bai and Ng (2002), assume that the unobserved factors are strong, in the sense discussed in Chudik, Pesaran and Tosetti (2011). However, in many empirical applications we may not be sure that all the factors are strong. Bailey, Kapetanios and Pesaran (2012, BKP) show that the strength of the factors is determined by the nature of the factor loadings, and depends on the exponent of the cross-sectional dependence, α , defined as $\ln(n)/\ln(N)$, where n is the number of non-zero factor loadings. The value $\alpha = 1$ corresponds to the case of a strong factor, while $\alpha < 1$ gives rise to a large set of practically plausible values ranging from semi-strong to weaker factors. BKP find that for many macroeconomic and financial series of interest, the value of the exponent is less than one. This result casts some doubt on the practical justification of panel unit root tests based on estimated factors by principal components, which are discussed above. The solution offered in this paper deals with the uncertainty surrounding the true number of factors by assuming that there exists a sufficient number of k additional regressors that *together* share at least $m^0 - 1$ of the factors in the model that influence the variable under consideration. This approach does not require all the factors to be strong. This way, by selecting $k = m_{\max} - 1$, where m_{\max} is the assumed maximum number of factors, the estimation of m^0 will not be needed.

For our tests, following the discussions in Im et al. (2003), we propose to use critical values which depend on the values of k , N , T , and lag-augmentation order, p , as they are expected to provide much better finite sample approximations. In empirical applications it is important that the tests being considered have the correct size, otherwise their use could result in misleading conclusions.

Small sample properties of the *CIPS* and *CSB* tests are investigated by Monte Carlo experiments. These tests are shown to have the correct size for *all* combinations of N and T considered in a number of different experiments. The experimental results also show that the proposed *CSB* test has satisfactory power, which for some combinations of N and T tends to be higher than that of the *CIPS* test. Empirical applications to Fisher's inflation parity and real equity prices across different economies illustrate how the proposed tests perform in practice.

The plan of the paper is as follows. Section 2 sets out the panel data model, formulates the *CIPS* test and derives its asymptotic distribution. Section 3 presents the *CSB* test. Section 4 discusses the proposed tests in the presence of residual serial correlation. Section 5 describes the Monte Carlo experiments and reports the small sample results. Section 6 presents the empirical applications, and Section 7 provides some concluding remarks.

Notation: L denotes a lag operator such that $L^\ell \mathbf{x}_t = \mathbf{x}_{t-\ell}$, K denotes a finite positive constant such that $K < \infty$, $\|\mathbf{A}\| = [\text{tr}(\mathbf{A}\mathbf{A}')]^{1/2}$, \mathbf{A}^+ denotes the Moore-Penrose inverse of \mathbf{A} , \mathbf{I}_q is a $q \times q$ identity matrix, $\boldsymbol{\tau}_q$ and $\mathbf{0}_q$ are $q \times 1$ vectors of ones and zeros, respectively, $\mathbf{0}_{q \times r}$ is a $q \times r$ null matrix, $\xrightarrow[N]{N}$ ($\xrightarrow[N]{N}$) denotes convergence in distribution (quadratic mean (q.m.) or mean square errors) with T fixed as $N \rightarrow \infty$, $\xrightarrow[T]{T}$ ($\xrightarrow[T]{T}$) denotes convergence in distribution (q.m.) with N fixed (or when there is no N -dependence) as $T \rightarrow \infty$, $\xrightarrow[N,T]{N,T}$ denotes sequential convergence in distribution with $N \rightarrow \infty$ first followed by $T \rightarrow \infty$, $\xrightarrow[(N,T)j]{N,T}$ denotes joint convergence in distribution with $N, T \rightarrow \infty$ jointly with certain restrictions on the expansion rates of T and N to be specified, if any.

2 Panel Data Model and the *CIPS* Test

Let y_{it} be the observation on the i^{th} cross section unit at time t , and suppose that it is generated as

$$\Delta y_{it} = \beta_i(y_{i,t-1} - \boldsymbol{\alpha}'_{iy} \mathbf{d}_{t-1}) + \boldsymbol{\alpha}'_{iy} \Delta \mathbf{d}_t + u_{it}, \quad i = 1, 2, \dots, N; \quad t = 1, 2, \dots, T, \quad (1)$$

where $\beta_i \leq 0$, \mathbf{d}_t is 2×1 vector consisting of an intercept and a linear trend so that $\mathbf{d}_t = (1, t)'$. Without loss of generality, it is assumed that $\mathbf{d}_0 \equiv \mathbf{0}$, and $\Delta \mathbf{d}_1 \equiv (0, 1)'$. Consider the following multifactor error structure

$$u_{it} = \boldsymbol{\gamma}'_{iy} \mathbf{f}_t + \varepsilon_{iyt}, \quad (2)$$

where \mathbf{f}_t is an $m^0 \times 1$ vector of unobserved common effects, $\boldsymbol{\gamma}_{iy}$ is the associated vector of factor loadings, and ε_{iyt} is the idiosyncratic component. This set up generalises Pesaran's (2007) one factor error specification. We assume that these error processes satisfy the following assumptions:

Assumption 1 (idiosyncratic errors): The idiosyncratic shocks, ε_{iyt} , $i = 1, 2, \dots, N$; $t = 1, 2, \dots, T$, are independently distributed both across i and t , with zero means, variances, σ_i^2 , ($0 < \sigma_i^2 \leq K$), and finite fourth-order moments.

Remark 1 *This assumption, which implies that the idiosyncratic shocks are serially uncorrelated, will be relaxed in Section 4. It is also possible to relax the assumption that the idiosyncratic errors are cross-sectionally independent, and replace it by assuming that ε'_{iyt} s are cross-sectionally weakly dependent in the sense of Chudik, Pesaran, and Tosetti (2011). However, such an extension will not be considered in this paper.*

Assumption 2 (factors): The $m^0 \times 1$ vector \mathbf{f}_t follows a covariance stationary process, with absolute summable autocovariances, distributed independently of $\varepsilon_{iyt'}$ for all i, t and t' . Specifically, we assume that $\mathbf{f}_t = \boldsymbol{\Psi}(L)\mathbf{v}_t$ where $\mathbf{v}_t \sim IID(\mathbf{0}, \boldsymbol{\Omega}_m)$, which has finite fourth-order moments, $\boldsymbol{\Psi}(L) = \sum_{\ell=0}^{\infty} \boldsymbol{\Psi}_\ell L^\ell$, where $\{\ell \boldsymbol{\Psi}_\ell\}_{\ell=0}^{\infty}$ is absolute summable such that $\sum_{\ell=0}^{\infty} \ell |\psi_{rs}^{(\ell)}| < \infty$ for all r, s , with $\psi_{rs}^{(\ell)}$ being the $(r, s)^{\text{th}}$ element of $\boldsymbol{\Psi}_\ell$. Specifically, it is assumed that the inverse of $\boldsymbol{\Lambda}_f$ defined by

$$\boldsymbol{\Lambda}_f = \boldsymbol{\Psi}(1), \quad (3)$$

exists.

Remark 2 *Assumption 2 is quite general but rules out the possibility of the factors having unit roots. In our set up this makes sense since otherwise all series in the panel could be $I(1)$ irrespective of whether $\beta_i = 0$ or not. Also if $\boldsymbol{\gamma}'_{iy} \mathbf{f}_t$ is assumed to be $I(1)$ and cointegrated with y_{it} , then y_{it} will be $I(1)$ even if $\beta_i = 0$, and a test of $\beta_i = 0$ as a unit root test will not be meaningful, which is also noted by Hansen (1995, p. 1159) in a similar context.*

Combining (1) and (2) it follows that

$$\Delta y_{it} = \beta_i(y_{i,t-1} - \boldsymbol{\alpha}'_{iy} \mathbf{d}_{t-1}) + \boldsymbol{\alpha}'_{iy} \Delta \mathbf{d}_t + \boldsymbol{\gamma}'_{iy} \mathbf{f}_t + \varepsilon_{iyt}. \quad (4)$$

The hypothesis that all observed series, y_{it} , have *unit roots and are not cross unit cointegrated* can be expressed as

$$H_0 : \beta_i = 0 \text{ for all } i, \quad (5)$$

against the alternative

$$H_1 : \beta_i < 0 \text{ for } i = 1, 2, \dots, N_1, \beta_i = 0 \text{ for } i = N_1 + 1, N_1 + 2, \dots, N,$$

where $N_1/N \rightarrow \kappa$ and $0 < \kappa \leq 1$ as $N \rightarrow \infty$.

Under the null hypothesis, (4) can be solved for y_{it} to yield

$$y_{it} = y_{i0} + \boldsymbol{\alpha}'_{iy} \mathbf{d}_t + \boldsymbol{\gamma}'_{iy} \mathbf{s}_{ft} + s_{iyt}, \quad i = 1, 2, \dots, N; \quad t = 1, 2, \dots, T, \quad (6)$$

where

$$\mathbf{s}_{ft} = \mathbf{f}_1 + \mathbf{f}_2 + \dots + \mathbf{f}_t, \quad \text{and} \quad s_{iyt} = \varepsilon_{iy1} + \varepsilon_{iy2} + \dots + \varepsilon_{iyt},$$

with y_{i0} being a given initial value. Therefore, under H_0 and Assumptions 1 and 2, y_{it} is composed of the initial value, y_{i0} , a common stochastic component, $\mathbf{s}_{ft} \sim I(1)$, and an idiosyncratic component, $s_{iyt} \sim I(1)$, so that while all units of the panel share the common stochastic trends, \mathbf{s}_{ft} , there is no cointegration among them. Under the alternative stationarity hypothesis, $\beta_i < 0$, we must have $y_{it} \sim I(0)$, and it is therefore *essential* that \mathbf{f}_t is at most an $I(0)$ process.⁵

Remark 3 *Our primary objective is to test for the presence of a unit root in the y_{it} process, which is observed. In contrast, Bai and Ng (2004) consider whether the source of non-stationarity is due to the common factors and/or the idiosyncratic components, neither of which are observed directly. To see how our approach is related to the Bai and Ng (2004, p.1130-1) PANIC framework, consider their specification*

$$\begin{aligned} y_{it} &= \mu_i + \boldsymbol{\gamma}'_{iy} \mathbf{F}_t + e_{iyt}, \\ \Delta \mathbf{F}_t &= \mathbf{C}(L) \mathbf{v}_t, \\ (1 - \rho_i L) e_{iyt} &= \varepsilon_{iyt}, \end{aligned} \quad (7)$$

where $\text{rank}(\mathbf{C}(1)) = r$, with $0 \leq r \leq m^0$ and r is the number of factors that are $I(1)$, and for simplicity let $\varepsilon_{iyt} \sim \text{iid}(0, \sigma_\varepsilon^2)$. Bai and Ng's objective is "to determine r and test if $\rho_i = 1$ when neither \mathbf{F}_t nor e_{iyt} , is observed." (Bai and Ng, 2004;p.1130). From (7) it readily follows that

$$\Delta y_{it} = \beta_i (y_{i,t-1} - \mu_i - \boldsymbol{\gamma}'_{iy} \mathbf{F}_{t-1}) + \boldsymbol{\gamma}'_{iy} \Delta \mathbf{F}_t + \varepsilon_{iyt}, \quad (8)$$

where $\beta_i = -(1 - \rho_i)$. Under $H_0 : \beta_i = 0$, (8) becomes

$$\Delta y_{it} = \boldsymbol{\gamma}'_{iy} \Delta \mathbf{F}_t + \varepsilon_{iyt}, \quad \text{or} \quad y_{it} = y_{i0} + \boldsymbol{\gamma}'_{iy} \mathbf{F}_t + s_{iyt},$$

and since within the Bai and Ng framework \mathbf{F}_t and s_{iyt} are both $I(1)$ processes, then y_{it} must also be $I(1)$. Under the alternative hypothesis $H_1 : \beta_i < 0$, it follows from (8) that if \mathbf{F}_t is $I(1)$ (and possibly cointegrated with y_{it}), y_{it} will be $I(1)$, unless $r = 0$ and there are no common stochastic trends. Therefore, it is meaningful to interpret a test of $\beta_i = 0$ as a panel unit root test only if \mathbf{F}_t is assumed to be $I(0)$. See also Remark 2.

⁵One can test whether f_t is $I(0)$ by applying time series unit root tests to cross section averages, $\bar{y}_t = N^{-1} \sum_{i=1}^N y_{it}$, for $t = 1, 2, \dots, T$. It can be shown that such tests are asymptotically valid as T and $N \rightarrow \infty$, so long as $T/N \rightarrow 0$. However, the power of such tests will depend on T and the cross section dimension is only relevant in ensuring that \bar{y}_t is a good proxy for f_t .

In the case where $m^0 = 1$, Pesaran (2007) proposes a test of $\beta_i = 0$ jointly with $f_t \sim I(0)$, based on DF (or ADF) regressions augmented by current and lagged cross-sectional averages of y_{it} as proxies for the unobserved \mathbf{f}_t . He shows that the resultant test is asymptotically invariant to the factor loadings, γ_{iy} . To deal with the case where $m^0 > 1$ we assume that in addition to y_{it} , there exist k additional observables, say \mathbf{x}_{it} , which depend on at least the same set of common factors, \mathbf{s}_{ft} , although with different factor loadings. For example, in the analysis of output convergence it is reasonable to argue that output, investment, consumption, real equity prices, and oil prices have the same set of factors in common. Similarly, short term and long term interest rates and inflation across countries are likely to have a number of factors in common.

More specifically, suppose the $k \times 1$ vector of additional regressors follow the general linear process

$$\Delta \mathbf{x}_{it} = \mathbf{A}_{ix} \Delta \mathbf{d}_t + \mathbf{\Gamma}_{ix} \mathbf{f}_t + \boldsymbol{\varepsilon}_{ixt}, \quad i = 1, 2, \dots, N; \quad t = 1, 2, \dots, T, \quad (9)$$

where $\mathbf{x}_{it} = (x_{i1t}, x_{i2t}, \dots, x_{ikt})'$, $\mathbf{\Gamma}_{ix} = (\gamma_{ix1}, \gamma_{ix2}, \dots, \gamma_{ixk})'$, $\mathbf{A}_{ix} = (\mathbf{a}_{ix1}, \mathbf{a}_{ix2}, \dots, \mathbf{a}_{ixk})'$, and $\boldsymbol{\varepsilon}_{ixt}$ is the idiosyncratic component of \mathbf{x}_{it} which is $I(0)$ and distributed independently of $\boldsymbol{\varepsilon}_{iyt'}$ for all i, t and t' . Solving for \mathbf{x}_{it} we have

$$\mathbf{x}_{it} = \mathbf{x}_{i0} + \mathbf{A}_{ix} \mathbf{d}_t + \mathbf{\Gamma}_{ix} \mathbf{s}_{ft} + \mathbf{s}_{ixt}, \quad i = 1, 2, \dots, N; \quad t = 1, 2, \dots, T, \quad (10)$$

where $\mathbf{s}_{ixt} = \sum_{s=1}^t \boldsymbol{\varepsilon}_{ixs}$. Combining (6) and (10) we obtain

$$\mathbf{z}_{it} = \mathbf{z}_{i0} + \mathbf{\Gamma}_i \mathbf{s}_{ft} + \mathbf{A}_i \mathbf{d}_t + \mathbf{s}_{it}, \quad (11)$$

where $\mathbf{z}_{it} = (y_{it}, \mathbf{x}'_{it})'$, $\mathbf{\Gamma}_i = (\gamma_{iy}, \mathbf{\Gamma}'_{ix})'$, $\mathbf{A}_i = (\boldsymbol{\alpha}_{iy}, \mathbf{A}'_{ix})'$, and $\mathbf{s}_{it} = (s_{iyt}, \mathbf{s}'_{ixt})'$.

Assumption 3 (factor loadings): $\|\mathbf{A}_i\| \leq K$ and $\|\mathbf{\Gamma}_i\| \leq K$, for all i , with the factors normalized such that $E(\mathbf{f}_t \mathbf{f}'_t) \equiv \mathbf{I}_m$.

Assumption 4 (initial conditions): $E\|\mathbf{s}_{f1}\| \leq K$, $E\|\mathbf{z}_{i0}\| \leq K$, and $E\|\mathbf{s}_{i1}\| \leq K$, for all i .

Remark 4 *Assumption 3 imposes minimal conditions on the factor loadings. For example, it does not rule out possible dependence between the factor loadings and idiosyncratic errors. Also the normalisation of \mathbf{f}_t under Assumption 3 can be achieved by suitable transformations of $\mathbf{\Gamma}_i$ and \mathbf{f}_t (also note that $\boldsymbol{\Psi}_0$ in Assumption 2 is unrestricted). Assumption 4 is also routine in the literature on unit roots.*

Averaging (11) across i we obtain

$$\bar{\mathbf{z}}_t = \bar{\mathbf{z}}_0 + \bar{\mathbf{\Gamma}} \mathbf{s}_{ft} + \bar{\mathbf{A}} \mathbf{d}_t + \bar{\mathbf{s}}_t, \quad (12)$$

where $\bar{\mathbf{z}}_t = N^{-1} \sum_{i=1}^N \mathbf{z}_{it}$, $\bar{\mathbf{A}} = N^{-1} \sum_{i=1}^N \mathbf{A}_i$, and $\bar{\mathbf{s}}_t = N^{-1} \sum_{i=1}^N \mathbf{s}_{it}$.⁶ Writing (4), (11) and (12) in matrix notation, under the null for each i we have

$$\Delta \mathbf{y}_i = \mathbf{F} \gamma_{iy} + \Delta \mathbf{D} \boldsymbol{\alpha}_{iy} + \boldsymbol{\varepsilon}_{iy}, \quad (13)$$

$$\Delta \mathbf{Z}_i = \mathbf{F} \mathbf{\Gamma}'_i + \Delta \mathbf{D} \mathbf{A}'_i + \mathbf{E}_i, \quad (14)$$

$$\Delta \bar{\mathbf{Z}} = \mathbf{F} \bar{\mathbf{\Gamma}}' + \Delta \mathbf{D} \bar{\mathbf{A}}' + \bar{\mathbf{E}}, \quad (15)$$

⁶Weighted cross section averages could also be used with appropriate granularity restrictions on the weights.

where $\mathbf{F} = (\mathbf{f}_1, \mathbf{f}_2, \dots, \mathbf{f}_T)'$, $\Delta \mathbf{D} = (\Delta \mathbf{d}_1, \Delta \mathbf{d}_2, \dots, \Delta \mathbf{d}_T)'$, with $\Delta \mathbf{d}_1 = (0, 1)'$, $\boldsymbol{\varepsilon}_{iy} = (\varepsilon_{iy1}, \varepsilon_{iy2}, \dots, \varepsilon_{iyT})'$, $\Delta \mathbf{Z}_i = (\Delta \mathbf{z}_{i1}, \Delta \mathbf{z}_{i2}, \dots, \Delta \mathbf{z}_{iT})'$, $\mathbf{E}_i = (\boldsymbol{\varepsilon}_{i1}, \boldsymbol{\varepsilon}_{i2}, \dots, \boldsymbol{\varepsilon}_{iT})'$ with $\boldsymbol{\varepsilon}_{it} = (\varepsilon_{iyt}, \boldsymbol{\varepsilon}'_{ixt})'$, $\Delta \bar{\mathbf{Z}} = (\Delta \bar{\mathbf{z}}_1, \Delta \bar{\mathbf{z}}_2, \dots, \Delta \bar{\mathbf{z}}_T)'$ and $\bar{\mathbf{E}} = N^{-1} \sum_{i=1}^N \mathbf{E}_i$. From (15), if $\bar{\Gamma}$ has full column rank m^0 , it follows that

$$\mathbf{F} = \left(\Delta \bar{\mathbf{Z}} - \Delta \mathbf{D} \bar{\mathbf{A}}' - \bar{\mathbf{E}} \right) \bar{\Gamma} (\bar{\Gamma}' \bar{\Gamma})^{-1}. \quad (16)$$

However, as shown in Appendix A, $\bar{\mathbf{E}} \xrightarrow{N} \mathbf{0}$, and hence we obtain that

$$\mathbf{F} - \left(\Delta \bar{\mathbf{Z}} - \Delta \mathbf{D} \bar{\mathbf{A}}' \right) \bar{\Gamma} (\bar{\Gamma}' \bar{\Gamma})^{-1} \xrightarrow{N} \mathbf{0}. \quad (17)$$

This implies that under the null hypothesis linear combinations of $\Delta \bar{\mathbf{Z}}$ and $\Delta \mathbf{D}$ provide valid approximations of \mathbf{F} for large N . This condition on the rank of the cross section average of the factor loadings is stated as an assumption below:

Assumption 5 (rank condition): The $(k+1) \times m^0$ matrix of factor loadings Γ_i is such that

$$\text{rank}(\bar{\Gamma}) = m^0 \leq k+1, \text{ for any } N \text{ and as } N \rightarrow \infty, \quad (18)$$

where $\bar{\Gamma} = N^{-1} \sum_{i=1}^N \Gamma_i$, and $\bar{\Gamma} \xrightarrow{N} \Gamma$, where Γ is a fixed bounded matrix with rank m^0 .

Remark 5 *It is not necessary that y_{it} and $(x_{i1t}, x_{i2t}, \dots, x_{ikt})$ have the same cross-sectional dimensions. This is illustrated in Section 6. Also it is not necessary for the rank condition to hold for all cross section units individually, but it must hold on average. For example, the rank condition holds so long as a non-zero fraction of factor loadings, Γ_i , are full rank as $N \rightarrow \infty$. Also, so long as Assumption 5 is satisfied, we do not necessarily require that $\lim_{N \rightarrow \infty} N^{-1} \sum_{i=1}^N \Gamma_i' \Gamma_i$ exists and is positive definite, which is typically assumed for the identification of factors. See, for example, Assumption A(ii) of Bai and Ng (2004) and Assumption 6 of Moon and Perron (2004). Under our framework, a factor can be weak in the equation for y_{it} and strong in the equations for \mathbf{x}_{it} , and vice versa. Such cases do not invalidate the rank condition.*

In view of the above we shall base our test of the panel unit root hypothesis on the t -ratio of the Ordinary Least Squares (OLS) estimator of b_i (\hat{b}_i) in the following cross-sectionally augmented regression

$$\Delta y_{it} = b_i y_{it-1} + \mathbf{c}'_i \bar{\mathbf{z}}_{t-1} + \mathbf{h}'_i \Delta \bar{\mathbf{z}}_t + \mathbf{g}'_i \mathbf{d}_t + \epsilon_{it}. \quad (19)$$

The t -ratio of \hat{b}_i is given by

$$t_i(N, T) = \frac{\Delta \mathbf{y}'_i \bar{\mathbf{M}} \mathbf{y}_{i,-1}}{\hat{\sigma}_i \left(\mathbf{y}'_{i,-1} \bar{\mathbf{M}} \mathbf{y}_{i,-1} \right)^{1/2}} = \frac{\sqrt{T-2k-5} \Delta \mathbf{y}'_i \bar{\mathbf{M}} \mathbf{y}_{i,-1}}{\left(\Delta \mathbf{y}'_i \bar{\mathbf{M}}_i \Delta \mathbf{y}_i \right)^{1/2} \left(\mathbf{y}'_{i,-1} \bar{\mathbf{M}} \mathbf{y}_{i,-1} \right)^{1/2}},$$

where $\Delta \mathbf{y}_i = (\Delta y_{i1}, \Delta y_{i2}, \dots, \Delta y_{iT})'$, $\mathbf{y}_{i,-1} = (y_{i0}, y_{i1}, \dots, y_{i,T-1})'$, $\bar{\mathbf{M}} = \mathbf{I}_T - \bar{\mathbf{W}} (\bar{\mathbf{W}}' \bar{\mathbf{W}})^{-1} \bar{\mathbf{W}}'$, $\bar{\mathbf{W}} = (\bar{\mathbf{w}}_1, \bar{\mathbf{w}}_2, \dots, \bar{\mathbf{w}}_T)'$, $\bar{\mathbf{w}}_t = (\Delta \bar{\mathbf{z}}'_t, \mathbf{d}'_t, \bar{\mathbf{z}}'_{t-1})'$, $\hat{\sigma}_i^2 = \Delta \mathbf{y}'_i \bar{\mathbf{M}}_i \Delta \mathbf{y}_i / (T-2k-5)$, and $\bar{\mathbf{M}}_i = \mathbf{I}_T - \bar{\mathbf{W}}_i (\bar{\mathbf{W}}_i' \bar{\mathbf{W}}_i)^{-1} \bar{\mathbf{W}}_i'$, with $\bar{\mathbf{W}}_i = (\bar{\mathbf{W}}, \mathbf{y}_{i,-1})$. For the intercept only case the degrees of freedom adjustment for $\hat{\sigma}_i^2$ is $T-2k-4$. Using (16) in (13)

$$\Delta \mathbf{y}_i = \Delta \bar{\mathbf{Z}} \boldsymbol{\delta}_i + \Delta \mathbf{D} \boldsymbol{\alpha}_i + \sigma_i \mathbf{v}_i, \quad (20)$$

where $\boldsymbol{\delta}_i = \bar{\boldsymbol{\Gamma}} (\bar{\boldsymbol{\Gamma}} \bar{\boldsymbol{\Gamma}})^{-1} \boldsymbol{\gamma}_{iy}$, $\boldsymbol{\alpha}_i = \boldsymbol{\alpha}_{iy} - \bar{\mathbf{A}}' \boldsymbol{\delta}_i$, $\boldsymbol{v}_i = (\boldsymbol{\varepsilon}_{iy} - \bar{\mathbf{E}} \boldsymbol{\delta}_i) / \sigma_i$, it is also easily seen that $E(\boldsymbol{v}_i \boldsymbol{v}_i') = \mathbf{I}_T + O(N^{-1})$. Therefore,

$$\bar{\mathbf{M}} \Delta \mathbf{y}_i = \sigma_i \bar{\mathbf{M}} \boldsymbol{v}_i. \quad (21)$$

From (14) we also have

$$\mathbf{Z}_{i,-1} = \boldsymbol{\tau}_T \mathbf{z}'_{i0} + \mathbf{S}_{f,-1} \boldsymbol{\Gamma}'_i + \mathbf{D}_{-1} \mathbf{A}'_i + \mathbf{S}_{i,-1}.$$

Taking cross-sectional averages gives

$$\bar{\mathbf{Z}}_{-1} = \boldsymbol{\tau}_T \bar{\mathbf{z}}'_0 + \mathbf{S}_{f,-1} \bar{\boldsymbol{\Gamma}}' + \mathbf{D}_{-1} \bar{\mathbf{A}}' + \bar{\mathbf{S}}_{-1}, \quad (22)$$

where $\mathbf{S}_{f,-1} = (\mathbf{0}_{m^0}, \mathbf{s}_{f1}, \dots, \mathbf{s}_{f,T-1})'$, $\mathbf{D}_{-1} = (\mathbf{0}_2, \mathbf{d}_1, \dots, \mathbf{d}_{T-1})'$, $\mathbf{Z}_{i,-1} = (\mathbf{z}_{i0}, \mathbf{z}_{i1}, \dots, \mathbf{z}_{iT-1})'$, $\mathbf{S}_{i,-1} = (\mathbf{0}_{k+1}, \mathbf{s}_{i1}, \dots, \mathbf{s}_{iT-1})'$, $\bar{\mathbf{Z}}_{-1} = (\bar{\mathbf{z}}_0, \bar{\mathbf{z}}_1, \dots, \bar{\mathbf{z}}_{T-1})'$ and $\bar{\mathbf{S}}_{-1} = N^{-1} \sum_{i=1}^N \mathbf{S}_{i,-1}$.

Similarly from (20)

$$\mathbf{y}_{i,-1} = \hat{y}_{i0} \boldsymbol{\tau}_T + \bar{\mathbf{Z}}_{-1} \boldsymbol{\delta}_i + \mathbf{D}_{-1} \boldsymbol{\alpha}_i + \sigma_i \hat{\mathbf{s}}_{i,-1}, \quad (23)$$

where

$$\hat{\mathbf{s}}_{i,-1} = (\mathbf{s}_{iy,-1} - \bar{\mathbf{S}}_{-1} \boldsymbol{\delta}_i) / \sigma_i, \quad (24)$$

$\mathbf{s}_{iy,-1} = (0, s_{iy1}, \dots, s_{iy,T-1})'$, and $\hat{y}_{i0} = y_{i0} - \bar{\mathbf{z}}'_0 \boldsymbol{\delta}_i$. Therefore,

$$\bar{\mathbf{M}} \mathbf{y}_{i,-1} = \sigma_i \bar{\mathbf{M}} \hat{\mathbf{s}}_{i,-1}. \quad (25)$$

Using (21) and (25), $t_i(N, T)$ can be re-written as

$$t_i(N, T) = \frac{\boldsymbol{v}'_i \bar{\mathbf{M}} \hat{\mathbf{s}}_{i,-1}}{\left(\frac{\boldsymbol{v}'_i \bar{\mathbf{M}}_i \boldsymbol{v}_i}{T-2k-5} \right)^{1/2} \left(\hat{\mathbf{s}}'_{i,-1} \bar{\mathbf{M}} \hat{\mathbf{s}}_{i,-1} \right)^{1/2}}. \quad (26)$$

For fixed N and T , the distribution of $t_i(N, T)$ will depend on the nuisance parameters through their effects on $\bar{\mathbf{M}}_i$ and $\bar{\mathbf{M}}$. However, this dependence vanishes either as $N \rightarrow \infty$, for a fixed T , or as N and $T \rightarrow \infty$, jointly. In addition, under Assumption 4 the effect of the initial cross section mean, $\bar{\mathbf{z}}_0$, also vanishes asymptotically, either as $N \rightarrow \infty$ for a fixed T , or as N and $T \rightarrow \infty$, jointly.⁷

The main results concerning the asymptotic distribution of $t_i(N, T)$ are summarised in the theorem below. The proof is given in the Appendix for the case where $\mathbf{d}_t = (1, 0)'$. The results for the case where $\mathbf{d}_t = (1, t)'$ can be derived in a similar manner and are provided in a supplement available from the authors on request.

Theorem 2.1 *Suppose the series \mathbf{z}_{it} , for $i = 1, 2, \dots, N$, $t = 1, 2, \dots, T$, is generated under (5) according to (11) and $d_t = 1$. Then under Assumptions 1-5 and as N and $T \rightarrow \infty$, such that $\sqrt{T}/N \rightarrow 0$, $t_i(N, T)$ given by (26) has the same sequential ($N \rightarrow \infty, T \rightarrow \infty$) and joint $[(N, T)_j \rightarrow \infty]$ limit distribution, is free of nuisance parameters, and is given by*

$$CADF_i = \frac{\int_0^1 W_i(r) dW_i(r) - \boldsymbol{\omega}'_{i\mathbf{v}} \mathbf{G}_{\mathbf{v}}^{-1} \boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r) dr - \boldsymbol{\pi}'_{i\mathbf{v}} \mathbf{G}_{\mathbf{v}}^{-1} \boldsymbol{\pi}_{i\mathbf{v}} \right)^{1/2}}, \quad (27)$$

⁷The importance of initial values for power properties of panel unit root tests is discussed in Moon et al. (2007), Breitung and Westerlund (2009), and Harris et al. (2010). A further investigation of this issue for the case where the errors are cross sectionally dependent is clearly worthwhile, but will not be pursued in this paper.

where

$$\boldsymbol{\omega}_{iv} = \begin{pmatrix} W_i(1) \\ \int_0^1 [\mathbf{W}_v(r)] dW_i(r) \end{pmatrix}, \quad \boldsymbol{\pi}_{iv} = \begin{pmatrix} \int_0^1 W_i(r) dr \\ \int_0^1 [\mathbf{W}_v(r)] W_i(r) dr \end{pmatrix},$$

$$\mathbf{G}_v = \begin{pmatrix} 1 & \int_0^1 [\mathbf{W}_v(r)]' dr \\ \int_0^1 [\mathbf{W}_v(r)] dr & \int_0^1 [\mathbf{W}_v(r)] [\mathbf{W}_v(r)]' dr \end{pmatrix},$$

$W_i(r)$ is a scalar standard Brownian motion and $\mathbf{W}_v(r)$ is m^0 -dimensional standard Brownian motion defined on $[0,1]$, associated with ε_{iyt} and \mathbf{v}_t , respectively. $W_i(r)$ and $\mathbf{W}_v(r)$ are mutually independent.

See Appendix A for a proof.

Remark 6 Since the random variables $CADF_i$ form an exchangeable sequence, conditional on $\mathbf{W}_v(r)$, $CADF_i$ and $CADF_j$ are independently distributed; see, for example, Theorem 1.2.2 in Taylor et al. (1985, p.13). Unconditionally, however, they are correlated with the same degree of dependence for all $i \neq j$.

Remark 7 When the factors are serially uncorrelated, namely $\mathbf{f}_t \equiv \mathbf{v}_t \sim IID(\mathbf{0}, \mathbf{I}_m)$, (see Assumptions 2 and 3), even for a finite T the limit distribution of $t_i(N, T)$ as $N \rightarrow \infty$, does not depend on the factor loadings and σ_i . In the case where the factors are serially correlated the limit distribution of $t_i(N, T)$ does depend on the serial correlation patterns of \mathbf{f}_t when T is finite. However, as Theorem 2.1 states, the dependence of $t_i(N, T)$ on the autocovariances of \mathbf{f}_t vanishes in the limit when $T \rightarrow \infty$ and $N \rightarrow \infty$, jointly.

Remark 8 When the y_{it} process does not contain a linear time trend but the additional regressors \mathbf{x}_{it} (or some subset thereof) do, the augmented regression (19) must include a linear trend term in order to eliminate the effects of such a trend in $\bar{\mathbf{x}}_t$. Alternatively, in such a case, it can be shown that Theorem 2.1 holds when the additional regressors are replaced by a detrended version of $\bar{\mathbf{x}}_t$. See the empirical Section 6 for more details.

The panel unit root test can now be based on the average of the t-ratios

$$CIPS_{NT} = N^{-1} \sum_{i=1}^N t_i(N, T), \quad (28)$$

which can be viewed as the cross-sectionally augmented version of the IPS test advanced in Im et al. (2003). As in Pesaran (2007), it is theoretically more convenient to work with a suitably truncated version of the $CIPS_{NT}$ test statistic defined by

$$CIPS_{NT}^* = N^{-1} \sum_{i=1}^N t_i^*(N, T), \quad (29)$$

where

$$t_i^*(N, T) = \begin{cases} t_i(N, T), & \text{if } -K_1 < t_i(N, T) < K_2, \\ -K_1, & \text{if } t_i(N, T) \leq -K_1, \\ K_2, & \text{if } t_i(N, T) \geq K_2, \end{cases} \quad (30)$$

and the truncation points K_1 and K_2 are chosen using a normal approximation for $t_i(N, T)$. Specifically, they are set as $K_1 = -E(CADF_i) - \Phi^{-1}(\epsilon/2)\sqrt{Var(CADF_i)}$, and $K_2 = E(CADF_i) + \Phi^{-1}(1 - \epsilon/2)\sqrt{Var(CADF_i)}$, where $\Phi^{-1}(\cdot)$ is the inverse of the cumulative standard normal distribution function. K_1 and K_2 can be obtained using simulated values of $E(CADF_i)$ and $Var(CADF_i)$ with $\epsilon = 1 \times 10^{-6}$ for $N = 200$, and $T = 200$. As with $t_i(N, T)$, the limiting distribution of $t_i^*(N, T)$, denoted by $CADF_i^*$, exists and will be free of nuisance parameters. We have

$$CADF_i^* = \begin{cases} CADF_i, & \text{if } -K_1 < CADF_i < K_2, \\ -K_1, & \text{if } CADF_i \leq -K_1, \\ K_2, & \text{if } CADF_i \geq K_2. \end{cases} \quad (31)$$

It is now straightforward to show that under the null hypothesis the asymptotic distribution of $CIPS^*$ exists and is free from nuisance parameters. To see this, let

$$\Delta_{NT}^* = N^{-1} \sum_{i=1}^N [t_i^*(N, T) - CADF_i^*], \quad (32)$$

and note that $CIPS_{NT}^* = \overline{CADF}^* + \Delta_{NT}^*$, where $\overline{CADF}^* = N^{-1} \sum_{i=1}^N CADF_i^*$. Also, by Theorem 2.1 and the relationships (30) and (31), for each i , $\Delta_i^* = t_i^*(N, T) - CADF_i^* \rightarrow_p 0$, as $(N, T)_j \rightarrow \infty$. Therefore, $\Delta_{NT}^* = CIPS_{NT}^* - \overline{CADF}^* \rightarrow_p 0$, as $(N, T)_j \rightarrow \infty$, since $E|t_i^*(N, T) - CADF_i^*| \leq E|t_i^*(N, T)| + E|CADF_i^*| < K < \infty$, given the truncated nature of the underlying random variables. Furthermore, since by construction, $E|CADF_i^*| < K < \infty$ for each i , then conditional on \mathbf{W}_v , (using Theorem 1.2.2 in Taylor et al. (1985, p.13)), we have

$$\begin{aligned} \overline{CADF}^* &= N^{-1} \sum_{i=1}^N CADF_i^* \xrightarrow{a.s.} E[CADF_1^* | \mathbf{W}_v, -K_1 < CADF_1 < K_2] \\ &+ \pi_2 K_2 - \pi_1 K_1, \end{aligned} \quad (33)$$

where $\pi_1 = \Pr[CADF_i^* \leq -K_1 | \mathbf{W}_v]$ and $\pi_2 = \Pr[CADF_i^* \geq K_2 | \mathbf{W}_v]$. From (33) we have that \overline{CADF}^* converges in distribution as $N \rightarrow \infty$. Hence, it also follows that conditional on \mathbf{W}_v , the truncated statistic, $CIPS_{NT}^*$, will converge to the same distribution as the limiting distribution of \overline{CADF}^* . But due to the dependence of $CADF_i^*$ over i , the limiting distribution is not normal and its critical values need to be computed by stochastic simulation. Further for suitably small choice of ϵ , the simulated critical values of the untruncated statistic, \overline{CADF} , is very close to those of \overline{CADF}^* . The reason for introducing the truncated version of $CIPS$ statistic is purely technical and is aimed at circumventing the difficult problem of establishing that the untruncated statistics, $t_i(N, T)$, have moments. The computation of the critical values of \overline{CADF} is discussed in Section 4.2.

3 The CSB Test

The cross-sectional augmentation approach can also be exploited in the case of other unit root tests, such as the test proposed by Sargan and Bhargava (1983). In the single time series case, the Sargan-Bhargava statistic was modified by Stock (1999) to allow for serial correlation. This test has also been recently adopted by Bai and Ng (2010) in the panel context with good effects.

Recall that the data generating process for y_{it} under the null is given by

$$\Delta y_{it} = \boldsymbol{\alpha}'_{iy} \Delta \mathbf{d}_t + \boldsymbol{\gamma}'_{iy} \mathbf{f}_t + \varepsilon_{iyt}.$$

For each i , the cross-sectionally augmented Sargan-Bhargava statistic, is given by

$$CSB_i(N, T) = T^{-2} \sum_{t=1}^T \hat{u}_{it}^2 / \hat{\sigma}_i^2,$$

where

$$\hat{u}_{it} = \sum_{j=1}^t \hat{\varepsilon}_{ij}, \text{ and } \hat{\sigma}_i^2 = \sum_{t=1}^T \hat{\varepsilon}_{it}^2 / [T - (k + 1)],$$

and $\hat{\varepsilon}_{it}$ are the OLS residuals from the regressions of Δy_{it} on $\Delta \bar{\mathbf{z}}_t$, in the case of models with an intercept only. If the underlying series are trended, $\hat{\varepsilon}_{it}$ must be calculated from a regression of Δy_{it} on an intercept and $\Delta \bar{\mathbf{z}}_t$, with $\hat{\sigma}_i^2$ computed as $\hat{\sigma}_i^2 = \sum_{t=1}^T \hat{\varepsilon}_{it}^2 / [T - (k + 2)]$. The use of cross-sectional augmentation as a way of dealing with the unobserved factors is justified using (17), which renders $\hat{\varepsilon}_{it}$ free of the nuisance parameters (namely the factor loadings). It is now easy to prove that for each i , $CSB_i(N, T)$ statistic converges to a functional of Brownian motions, which is independent of the factors as well as their loadings.⁸ The CSB test is then based on the cross-sectional average of the $CSB_i(N, T)$ statistics, given by

$$CSB_{NT} = N^{-1} \sum_{i=1}^N CSB_i(N, T). \quad (34)$$

Computation of the critical values for the CSB_{NT} statistic using stochastic simulations is described in Section 4.2.

4 The Case of Residual Serial Correlation

In this section we relax Assumption 1, and consider the implications of residual serial correlation for our proposed tests. In error factor models, residual serial correlation can be modelled in a number of different ways, directly *via* the idiosyncratic components, through the factor(s), or a mixture of the two. We focus on the serial correlation in the idiosyncratic errors and model the residual serial correlation as

$$\zeta_{iyt} = \theta_i \zeta_{iy,t-1} + \eta_{iyt}, \quad |\theta_i| < 1, \text{ for } i = 1, 2, \dots, N; \quad t = 1, 2, \dots, T, \quad (35)$$

where ζ_{iyt} is the idiosyncratic component of $u_{it} = \boldsymbol{\gamma}'_{iy} \mathbf{f}_t + \zeta_{iyt}$, and η_{iyt} is independently distributed across both i and t , with zero means and variances, $0 < \sigma_{i\eta}^2 < K < \infty$.

To keep the exposition simple we confine our analysis to the first order stationary processes, though the analysis readily extends to higher order processes. Under (35) we have

$$\Delta y_{it} = \beta_i (y_{i,t-1} - \boldsymbol{\alpha}'_{iy} \mathbf{d}_{t-1}) + \boldsymbol{\alpha}'_{iy} \Delta \mathbf{d}_t + \boldsymbol{\gamma}'_{iy} \mathbf{f}_t + \zeta_{iyt}(\theta_i), \quad (36)$$

where $\zeta_{iyt}(\theta_i) = (1 - \theta_i L)^{-1} \eta_{iyt}$. We also assume the coefficients of the autoregressive process to be homogeneous across i , although this could be relaxed at the cost of more complex mathematical details. Under the null that $\beta_i = 0$, with $\theta_i = \theta$ and $\mathbf{d}_t = (1, 0)'$, (36) reduces to

$$\Delta y_{it} = \boldsymbol{\gamma}'_{iy} \mathbf{f}_t + \zeta_{iyt}(\theta), \quad (37)$$

⁸A proof of this is provided in a supplement, which is available from the authors on request.

and upon using (35) under the null hypothesis we have

$$\Delta y_{it} = \theta \Delta y_{i,t-1} + \gamma'_{iy}(\mathbf{f}_t - \theta \mathbf{f}_{t-1}) + \eta_{iyt}. \quad (38)$$

The individual CADF regressions can be written as

$$\Delta \mathbf{y}_i = b_i \mathbf{y}_{i,-1} + \bar{\mathbf{W}}_{i1} \mathbf{h}_i + \epsilon_i, \text{ for } i = 1, 2, \dots, N, \quad (39)$$

where $\bar{\mathbf{W}}_{i1} = (\Delta \mathbf{y}_{i,-1}, \Delta \bar{\mathbf{Z}}, \Delta \bar{\mathbf{Z}}_{-1}, \boldsymbol{\tau}_T, \bar{\mathbf{Z}}_{-1})$, which is a $T \times (3k + 5)$ matrix. The t -ratio of \hat{b}_i in regression (39) is given by

$$t_i(N, T) = \frac{\Delta \mathbf{y}'_i \bar{\mathbf{M}}_{i1} \mathbf{y}_{i,-1}}{\hat{\sigma}_i \left(\mathbf{y}'_{i,-1} \bar{\mathbf{M}}_{i1} \mathbf{y}_{i,-1} \right)^{1/2}} = \frac{\sqrt{T - (3k + 6)} \Delta \mathbf{y}'_i \bar{\mathbf{M}}_{i1} \mathbf{y}_{i,-1}}{\left(\Delta \mathbf{y}'_i \bar{\mathbf{M}}_{i1,p} \Delta \mathbf{y}_i \right)^{1/2} \left(\mathbf{y}'_{i,-1} \bar{\mathbf{M}}_{i1} \mathbf{y}_{i,-1} \right)^{1/2}}, \quad (40)$$

where $\bar{\mathbf{M}}_{i1} = \mathbf{I}_T - \bar{\mathbf{W}}_{i1} (\bar{\mathbf{W}}'_{i1} \bar{\mathbf{W}}_{i1})^{-1} \bar{\mathbf{W}}'_{i1}$, $\hat{\sigma}_i^2 = [T - (3k + 6)]^{-1} \Delta \mathbf{y}'_i \bar{\mathbf{M}}_{i1,p} \Delta \mathbf{y}_i$ and $\bar{\mathbf{M}}_{i1,p} = \mathbf{I}_T - \mathbf{P}_{i1} (\mathbf{P}'_{i1} \mathbf{P}_{i1})^{-1} \mathbf{P}'_{i1}$, $\mathbf{P}_{i1} = (\bar{\mathbf{W}}_{i1}, \mathbf{y}_{i,-1})$.

Combining (9) with (37), similarly to (14) we obtain

$$\Delta \mathbf{Z}_i = \mathbf{F} \boldsymbol{\Gamma}'_i + \mathbf{E}_i, \quad (41)$$

where $\mathbf{E}_i = (\zeta'_{iy}(\theta), \mathbf{E}'_{ix})'$, with $\mathbf{E}_{ix} = (\epsilon_{ix1}, \epsilon_{ix2}, \dots, \epsilon_{ixT})'$, and $\zeta_{iy}(\theta) = (\zeta_{iy1}(\theta), \zeta_{iy2}(\theta), \dots, \zeta_{iyT}(\theta))'$, with the common factors \mathbf{F} , and factor loadings $\boldsymbol{\Gamma}_i$ defined as in the previous section. Taking cross section averages of (41) we obtain $\Delta \bar{\mathbf{Z}} = \mathbf{F} \bar{\boldsymbol{\Gamma}}' + \bar{\mathbf{E}}$, where as before $\bar{\mathbf{E}} = N^{-1} \sum_{i=1}^N \mathbf{E}_i$. Therefore, assuming that the rank condition, (18), holds

$$\mathbf{F} = (\Delta \bar{\mathbf{Z}} - \bar{\mathbf{E}}) \bar{\boldsymbol{\Gamma}} (\bar{\boldsymbol{\Gamma}}' \bar{\boldsymbol{\Gamma}})^{-1}. \quad (42)$$

Writing (38) in matrix notation and using (42) we have

$$\Delta \mathbf{y}_i = \theta \Delta \mathbf{y}_{i,-1} + (\Delta \bar{\mathbf{Z}} - \theta \Delta \bar{\mathbf{Z}}_{-1}) \boldsymbol{\delta}_i + \sigma_{i\eta} \mathbf{v}_i, \quad (43)$$

with

$$\mathbf{v}_i = [\boldsymbol{\eta}_{iy} - (\bar{\mathbf{E}} - \theta \bar{\mathbf{E}}_{-1}) \boldsymbol{\delta}_i] / \sigma_{i\eta},$$

and $E(\mathbf{v}_i \mathbf{v}'_i) = \mathbf{I}_T + O(N^{-1})$. Furthermore, from (37) using (42) it follows that

$$\mathbf{y}_{i,-1} = \alpha_{iy} \boldsymbol{\tau}_T + \hat{y}_{i0} \boldsymbol{\tau}_T + \bar{\mathbf{Z}}_{-1} \boldsymbol{\delta}_i + \sigma_{i\eta} \hat{\mathbf{s}}_{i\zeta,-1},$$

where

$$\hat{\mathbf{s}}_{i\zeta,-1} = (\mathbf{s}_{i\zeta,-1} - \bar{\mathbf{S}}_{-1} \boldsymbol{\delta}_i) / \sigma_{i\eta},$$

$\mathbf{s}_{i\zeta,-1} = (0, s_{i\zeta1}, \dots, s_{i\zeta,T-1})'$ with $s_{i\zeta t} = \sum_{s=1}^t \zeta_{iys}(\theta)$, $\bar{\mathbf{S}}_{-1} = (\bar{\mathbf{s}}_{\zeta,-1}, \bar{\mathbf{S}}_{x,-1})$ with $\bar{\mathbf{s}}_{\zeta,-1} = N^{-1} \sum_{i=1}^N \mathbf{s}_{i\zeta,-1}$ and $\hat{y}_{i0} = y_{i0} - \bar{\mathbf{z}}'_0 \boldsymbol{\delta}_i$.

The test statistic (40) then becomes

$$t_i(N, T) = \frac{\mathbf{v}'_i \bar{\mathbf{M}}_{i1} \hat{\mathbf{s}}_{i\zeta,-1}}{\left(\frac{\mathbf{v}'_i \bar{\mathbf{M}}_{i1,p} \mathbf{v}_i}{T - 3k - 6} \right)^{1/2} \left(\hat{\mathbf{s}}'_{i\zeta,-1} \bar{\mathbf{M}}_{i1} \hat{\mathbf{s}}_{i\zeta,-1} \right)^{1/2}}. \quad (44)$$

Theorem 4.1 Suppose the series \mathbf{z}_{it} , for $i = 1, 2, \dots, N$, $t = 1, 2, \dots, T$, is generated under (5) according to (41) and $|\theta| < 1$. Then under Assumptions 1-5 and as N and $T \rightarrow \infty$, $t_i(N, T)$ in (44) has the same sequential ($N \rightarrow \infty, T \rightarrow \infty$) and joint $[(N, T)_j \rightarrow \infty]$ limit distribution given by (27) obtained for $\theta = 0$.

The proof is provided in a supplement available from the authors on request.

For a general AR(p) error specification, the CADF regressions in (39) must also be augmented by further lagged changes. More specifically, in this case the $t_i(N, T)$ statistic should be computed as the OLS t -ratio of b_i in the following p^{th} order augmented regression:

$$\Delta \mathbf{y}_i = b_i \mathbf{y}_{i,-1} + \bar{\mathbf{W}}_{ip} \mathbf{h}_{ip} + \epsilon_i, \quad (45)$$

where $\bar{\mathbf{W}}_{ip} = (\Delta \mathbf{y}_{i,-1}, \Delta \mathbf{y}_{i,-2}, \dots, \Delta \mathbf{y}_{i,-p}; \Delta \bar{\mathbf{z}}, \Delta \bar{\mathbf{z}}_{-1}, \dots, \Delta \bar{\mathbf{z}}_{-p}; \boldsymbol{\tau}_T; \bar{\mathbf{z}}_{-1})$, which is a $T \times (k + 2)(p + 1)$ data matrix. In the case where $\mathbf{d}_t = (1, t)'$, (45) should include a linear trend term, with the degrees of freedom term associated with the error variance adjusted accordingly.

Similarly it can be shown that the $CSB_i(N, T)$ statistics have the same limiting distribution as for $\theta = 0$,⁹ and from the above it follows that in the case of first order residual serial correlation, the cross section augmented regression should be augmented further with the term $\Delta \bar{\mathbf{z}}_{t-1}$, so that

$$\Delta y_{it} = b_i \Delta y_{i,t-1} + \mathbf{c}'_{i0} \Delta \bar{\mathbf{z}}_t + \mathbf{c}'_{i1} \Delta \bar{\mathbf{z}}_{t-1} + \epsilon_{it},$$

which for higher order serial correlation generalises to

$$\Delta y_{it} = \sum_{\ell=1}^p b_{i\ell} \Delta y_{i,t-\ell} + \sum_{\ell=0}^p \mathbf{c}'_{i\ell} \Delta \bar{\mathbf{z}}_{t-\ell} + \epsilon_{it}, \quad (46)$$

with

$$CSB_i(N, T) = T^{-2} \sum_{t=1}^T \hat{u}_{it}^2 / \hat{\sigma}_i^2,$$

where $\hat{u}_{it} = \sum_{j=1}^t \hat{\epsilon}_{ij}$, $\hat{\sigma}_i^2 = \sum_{t=1}^T \hat{\epsilon}_{it}^2 / [T - p - (p + 1)(k + 1)]$, and

$$\hat{\epsilon}_{it} = \Delta y_{it} - \sum_{\ell=1}^p \hat{b}_{i\ell} \Delta y_{i,t-\ell} - \sum_{\ell=0}^p \hat{\mathbf{c}}'_{i\ell} \Delta \bar{\mathbf{z}}_{t-\ell}.$$

In the case where $\mathbf{d}_t = (1, t)'$, (46) should include an intercept term, with the degrees of freedom term associated with the error variance adjusted accordingly.

4.1 Uncertainty Surrounding the Number of Factors

So far we have considered the case in which the true number of unobserved factors, m^0 , is given. In practice m^0 is rarely known, although it is reasonable to assume that it is bounded by a finite integer value, m_{\max} . In the case of the proposed test there are two possible ways that one could proceed when m^0 is not known.

One approach would be to estimate m^0 using a suitable statistical technique such as the information criteria proposed by Bai and Ng (2002). Most existing methods of estimating m^0 assume that the unobserved factors are strong, in the sense discussed in Chudik, Pesaran and

⁹ A proof is included in a supplement available upon request from the authors

Tosetti (2011). However, in many empirical applications we may not be sure that all unobserved factors are strong. Bailey, Kapetanios and Pesaran (2012) propose measuring the strength of the factors by the exponent of the cross-section dependence, α , defined as $\ln(n)/\ln(N)$, where n is the number of non-zero factor loadings. The value $\alpha = 1$ corresponds to the case of a strong factor, while values of α in the range $(1/2, 1)$ correspond to factors that are semi-strong. Bailey et al. (2012) estimate the exponent of the cross-sectional dependence for macroeconomic and financial series of interest, and find that mostly it is less than one. This raises interesting technical issues concerning the determination of the number of factors and if they are strong, as assumed by the standard theory.

Alternatively, assuming that there exists a sufficient number of additional regressors that share at least $m^0 - 1$ of the factors included in the model for y_{it} , one could set $k = m_{\max} - 1$ (where $m^0 \leq m_{\max}$), and use the k additional regressors for augmenting the regressions when computing *CIPS* and *CSB* statistics. This approach is likely to work in practice when m_{\max} is relatively small (2 or 3), and does not require all the factors to be strong. However, when m_{\max} is believed to be large, the *CIPS* and *CSB* tests are likely to lose power due to loss of degrees of freedom. More importantly, it might be difficult to find a sufficient number of additional regressors to deal with the adverse effects of the unobserved factors on our proposed test statistics.¹⁰

4.2 Computation of Critical Values of *CIPS* and *CSB* Tests

Critical values for the *CIPS* and *CSB* tests are obtained by stochastic simulation. We propose to use critical values which depend on the values of k , N , T , and lag-augmentation order, p , since they are expected to provide much better finite sample approximations as discussed in Im et al. (2003). To compute the critical values, y_{it} is generated as

$$y_{it} = y_{it-1} + \varepsilon_{iyt}, \quad i = 1, 2, \dots, N; \quad t = 1, 2, \dots, T, \quad (47)$$

where $\varepsilon_{iyt} \sim iidN(0, 1)$ with $y_{i0} = 0$. The j^{th} element of the $k \times 1$ vector of the additional regressors x_{it} , is generated as

$$x_{ijt} = x_{ij,t-1} + \varepsilon_{ixjt}, \quad i = 1, 2, \dots, N; \quad j = 1, 2, \dots, k; \quad t = 1, 2, \dots, T, \quad (48)$$

with $\varepsilon_{ixjt} \sim iidN(0, 1)$ and $x_{ij0} = 0$.

For the *CIPS* test the individual $t_i(N, T)$ statistic is calculated as the t -ratio of the coefficient on $y_{i,t-1}$ of the CADF regression of Δy_{it} on an intercept, $y_{i,t-1}$, \bar{z}'_{t-1} , $\Delta \bar{z}'_t$, $\Delta \bar{z}'_{t-1}$, ..., $\Delta \bar{z}'_{t-p}$, and $\Delta y'_{i,t-1}$, ..., $\Delta y'_{i,t-p}$ under Case I where the model only contains an intercept, and Case II where the CADF regressions also include a linear time trend. The *CIPS* statistic is then computed as $CIPS_{NT} = N^{-1} \sum_{i=1}^N t_i(N, T)$.

For the *CSB* test, the individual CSB_i statistic is computed as $CSB_i = T^{-2} \sum_{t=1}^T \hat{e}_{it}^2 / \hat{\sigma}_i^2$, with $\hat{u}_{it} = \sum_{j=1}^t \hat{e}_{ij}$ and $\hat{\sigma}_i^2 = \sum_{t=1}^T \hat{e}_{it}^2 / [T - p - (p+1)(k+1)]$, where \hat{e}_{it} are the estimated residuals from the regression of Δy_{it} on $\Delta y_{i,t-1}$, ..., $\Delta y_{i,t-p}$ and $\Delta \bar{z}'_t$, $\Delta \bar{z}'_{t-1}$, ..., $\Delta \bar{z}'_{t-p}$, under Case I. Under Case II, $\hat{\sigma}_i^2 = \sum_{t=1}^T \hat{e}_{it}^2 / [T - (p+1)(k+2)]$, where \hat{e}_{it} are the estimated residuals from the regression of Δy_{it} on an intercept, $\Delta y_{i,t-1}$, ..., $\Delta y_{i,t-p}$ and $\Delta \bar{z}'_t$, $\Delta \bar{z}'_{t-1}$, ..., $\Delta \bar{z}'_{t-p}$. The *CSB* statistic is computed as $CSB_{NT} = N^{-1} \sum_{i=1}^N CSB_i$.

¹⁰In the presence of uncertainty regarding the integration and/or the cointegration properties of the additional regressors, one could employ the bounds testing approach proposed by Pesaran et al. (2001). A detailed discussion of such an approach is outside the scope of the present paper.

The $100\alpha\%$ critical values of the *CIPS* and *CSB* statistics are computed for $N, T = 20, 30, 50, 70, 100, 200$, $k = 0, 1, 2, 3$ and $p = 0, 1, \dots, 4$, as their α quantiles for $\alpha = 0.01, 0.05, 0.1$ based on 10,000 and 50,000 replications, respectively.^{11, 12, 13} The (N, T, p, k) specific critical values for both tests are reported in Appendix B (*CIPS* and *CSB* Critical Value Tables).

It is worth noting that the reported critical values of the *CIPS* test statistics depend on k , and not on m^0 . When $m^0 \leq k+1$ the asymptotic distribution of the *CIPS* test depends on m^0 , but the critical values depend on the number of additional regressors actually included when simulating the critical values. A similar situation also arises when critical values are computed for standard DF statistics with deterministics, using stochastic simulations. When carrying out the simulations what matters is the nature of the deterministic variables and the number of additional $I(1)$ regressors that are added to the augmented DF and SB regressions.

5 Small Sample Performance: Monte Carlo Evidence

In what follows we investigate by means of Monte Carlo simulations the small sample properties of the *CIPS* and *CSB* tests defined by (28) and (34), using the (N, T, p, k) specific critical values which are reported in Appendix B.¹⁴ Both the *CIPS* and *CSB* tests reject the null when the value of the statistic is smaller than the relevant critical value, at the chosen level of significance.

5.1 Monte Carlo Design

In their Monte Carlo experiments Bai and Ng (2010, Section 5) set $m^0 = 1$ and do not allow for serial correlation in the idiosyncratic errors. Here we consider a more general set up and allow for two factors ($m^0 = 2$), and also consider experiments where the idiosyncratic errors are serially correlated. Following Bailey, Kapetanios and Pesaran (2012) we generate one of the factors in the y_{it} equations as strong and the second factor as semi-strong. Accordingly, the data generating process (DGP) for the $\{y_{it}\}$ is given by

$$y_{it} = d_{iyt} + \rho_i y_{i,t-1} + \gamma_{iy1} f_{1t} + \gamma_{iy2} f_{2t} + \varepsilon_{iyt}, i = 1, 2, \dots, N; t = -49, \dots, T, \quad (49)$$

with $y_{i,-50} = 0$, where $\gamma_{iy1} \sim iidU[0, 2]$, for $i = 1, 2, \dots, N$; $\gamma_{iy2} \sim iidU[0, 1]$ for $i = 1, \dots, [N^\alpha]$, and $\gamma_{iy2} = 0$ for $i = [N^\alpha] + 1, [N^\alpha] + 2, \dots, N$ (where $[\cdot]$ denotes the integer part); $f_{\ell t} \sim iidN(0, 1)$ for $\ell = 1, 2$, $\varepsilon_{iyt} \sim iidN(0, \sigma_i^2)$ with $\sigma_i^2 \sim iidU[0.5, 1.5]$. The exponent of cross-sectional dependence of the first (strong) factor is 1, and for the second (semi-strong) factor, it is set to 0.75, guided by the empirical results reported in Bailey et al. (2012). See, also Chudik et al. (2011).

At the stage of implementing the tests, we assume that $m_{\max} = 2$, and hence set $k = m_{\max} - 1 = 1$. The additional regressor, x_{it} , is generated as

$$\Delta x_{it} = d_{ix} + \gamma_{ix1} f_{1t} + \varepsilon_{ixt}, \quad (50)$$

¹¹We used 50,000 replications to obtain the critical values of the *CSB* test statistic, since we found that its critical values based on 10,000 replications were in certain instances not precise enough especially for small sample sizes.

¹²It is also possible to simulate the critical values directly using (27) by replacing the integrals of the Brownian motions with their simulated counterparts. Our analysis suggests that the critical values obtained from this procedure closely match the ones tabulated in the Appendix of the paper.

¹³In principle it might be possible that N and T specific critical values could also be used for the other tests proposed in the literature but such an exercise is beyond the scope of the present paper.

¹⁴Monte Carlo comparison results with several panel unit root tests proposed in the literature are included in a supplement available from the authors upon request.

where

$$\varepsilon_{ixt} = \rho_{ix}\varepsilon_{ixt-1} + \varpi_{ixt}, \varpi_{ixt} \sim iidN(0, 1 - \rho_{ix}^2), \quad (51)$$

$i = 1, 2, \dots, N; t = -49, \dots, T$, with $\varepsilon_{ix, -50} = 0$, and $\rho_{ix} \sim iidU[0.2, 0.4]$. The factor loadings in (50) are generated as $\gamma_{ix1} \sim iidU[0, 2]$, so that

$$E(\mathbf{\Gamma}_i) = \begin{pmatrix} 1 & \frac{1}{2}N^{-0.25} \\ 1 & 0 \end{pmatrix}, \quad (52)$$

and hence the rank condition (18) is satisfied when N is finite, but fails when $N \rightarrow \infty$. In this way we also check the robustness of the *CIPS* and *CSB* tests to failure of the rank condition for sufficiently large N .

We considered two specifications for the deterministic in y_{it} and x_{it} . For the case of an intercept only, $d_{iyt} = (1 - \rho_i)\alpha_{iy}$ with $\alpha_{iy} \sim iidN(1, 1)$ and $d_{ix} = 0$; for the case of an intercept and a linear trend, $d_{iyt} = \mu_{iy} + (1 - \rho_i)\delta_i t$ with $\mu_{iy} \sim iidU[0.0, 0.02]$ and $\delta_i \sim iidU[0.0, 0.02]$, and $d_{ix} = \delta_{ix}$ with $\delta_{ix} \sim iidU[0.0, 0.02]$.

To examine the impact of the residual serial correlation on the proposed tests we consider the DGPs in which the idiosyncratic errors ε_{iyt} are generated as

$$\varepsilon_{iyt} = \rho_{iy\varepsilon}\varepsilon_{iyt-1} + (1 - \rho_{iy\varepsilon}^2)^{1/2}\eta_{iyt}, \text{ for } t = -49, -48, \dots, 0, 1, \dots, T, \quad (53)$$

with $\varepsilon_{iy, -50} = 0$, where $\eta_{iyt} \sim iidN(0, \sigma_i^2)$, and $\sigma_i^2 \sim iidU[0.5, 1.5]$. We considered a positively serially correlated case, $\rho_{iy\varepsilon} \sim iidU[0.2, 0.4]$, as well as a negatively serially correlated case, $\rho_{iy\varepsilon} \sim iidU[-0.4, -0.2]$. The first 50 observations are discarded.

In the case where the errors of y_{it} are serially correlated, lag augmentation is required for the asymptotic validity of the *CIPS* and *CSB* tests. In all Monte Carlo results that follow, lag augmentation is performed and selected according to $\hat{p} = \lceil 4(T/100)^{1/4} \rceil$ (where $\lceil \cdot \rceil$ denotes the integer part).

The parameters $\alpha_{iy}, \delta_i, \mu_{iy}, \delta_{ix}, \rho_{iy\varepsilon}, \gamma_{iy1}, \gamma_{iy2}, \rho_i, \gamma_{ix1}, \rho_{ix}$, and σ_i are redrawn over each replication. The DGP is given by (49) with $\rho_i = \rho = 1$ for size, and $\rho_i \sim iidU[0.90, 0.99]$ for power. All tests are conducted at the 5% significance level. All combinations of $N, T = 20, 30, 50, 70, 100, 200$ are considered, and all experiments are based on 2,000 replications each.

5.2 Results

Size and power of the tests are summarised in Tables 1 to 3. Table 1 provides the results for the panel with serially uncorrelated idiosyncratic errors for the intercept only and linear trend cases. For the case with an intercept, the *CIPS* and *CSB* tests have the correct size for all combinations of sample sizes, even when T is small relative to N . In terms of power, the *CSB* test has satisfactory power which is almost consistently higher than that of *CIPS*. For the case with a linear trend, again the *CIPS* and *CSB* tests have the correct size for all combinations of sample sizes and their power rises in N and T , as to be expected. Power discrepancies between the *CSB* and *CIPS* tests are less pronounced in this case, with the former still showing higher power than the latter.

Table 2 presents the results for the case where ε_{iyt} are positively serially correlated for the intercept only and linear trend cases, respectively. The results for the case where ε_{iyt} are negatively serially correlated are summarised in Table 3. The size and power of the *CIPS* and *CSB* tests are not much affected by residual serial correlation once the underlying regressions are augmented with lagged changes as in (45) and (46). The proposed rule of choosing the lag

augmentation order $\hat{p} = \lceil 4(T/100)^{1/4} \rceil$ seems to work well in our experimental designs, whether serial correlation is present in the data generating process or not.

Overall, the *CIPS* and *CSB* tests do not display any size distortions for all values of N and T , irrespective of whether the idiosyncratic errors are serially correlated or not, with the *CSB* test displaying higher power than the *CIPS* test for smaller T .

6 Empirical Applications

As an illustration of the proposed tests we consider two applications. One to the real interest rates across $N = 32$ economies, and another to the real equity prices across $N = 26$ markets. For both applications we employ quarterly observations over the period 1979Q2 – 2009Q4 (i.e. 123 data points). Under the Fisher parity hypothesis, real interest rates, defined as the difference between the nominal short-term interest rate and the inflation rate, are stationary. The second application is chosen as it is generally believed that real equity prices are $I(1)$, and it would be interesting to see if the outcomes of the tests are in line with this belief.

As noted in Section 4.1, and as with other panel unit root tests that are based on principal components, we need to decide on m_{\max} . In the present application we set $m_{\max} = 4$. This choice is based on the recent literature that argues that 2 to 6 unobserved common factors are sufficient to explain variations in most macroeconomic variables. See, for example, Stock and Watson (2002) and Eickmeier (2009), among others. This suggests that at most three additional $I(1)$ regressors ($k = m_{\max} - 1 = 3$) are needed for the implementation of *CIPS* and *CSB* tests. The set of regressors that are likely to share common factors with real interest rates, $r_{it}^S - \pi_{it}$, and real equity prices, eq_{it} , are as follows:

	y_{it}	Additional regressors (\mathbf{x}_{it})
Real Interest Rates ($N = 32$)	$r_{it}^S - \pi_{it}$	$poil_t, r_{it}^L, eq_{it}, ep_{it}, gdp_{it}$
Real Equity Prices ($N = 26$)	eq_{it}	$poil_t, r_{it}^L, \pi_{it}, ep_{it}, gdp_{it}$

where

$$\begin{aligned} r_{it}^S &= 0.25 * \ln(1 + R_{it}^S/100), \quad \pi_{it} = p_{it} - p_{it-1} \text{ with } p_{it} = \ln(CPI_{it}), \quad poil_t = \ln(POIL_t), \\ r_{it}^L &= 0.25 * \ln(1 + R_{it}^L/100), \quad ep_{it} = e_{it} - p_{it} \text{ with } e_{it} = \ln(E_{it}), \quad eq_{it} = \ln(EQ_{it}/CPI_{it}), \\ gdp_{it} &= \ln(GDP_{it}/CPI_{it}), \end{aligned}$$

R_{it}^S is the short-term (three month) rate of interest, measured in per annum in per cent in country i at time t , CPI_{it} the consumer price index, $POIL_t$ the price of Brent Crude oil, R_{it}^L the long-term rate of interest per annum in per cent (typically the yield on ten year government bonds), E_{it} the nominal exchange rate of country i in terms of US dollars, EQ_{it} the nominal equity price index, and GDP_{it} the nominal Gross Domestic Product of country i during period t in domestic currency.¹⁵

When testing for unit roots in real interest rates, $\rho_{it}^S = r_{it}^S - \pi_{it}$, we consider ADF regressions without linear trends, and detrend the possibly trended variables $poil_t$, eq_{it} , ep_{it} and gdp_{it} , before the ADF regressions are augmented with their cross-sectional averages. See, also Remark 8. These detrended components are computed as residuals from the regressions of $poil_t$, eq_{it} , ep_{it}

¹⁵The data are publicly available at: <http://www-cfap.jbs.cam.ac.uk/research/gvartoolbox/download.html>. A detailed description of the data and sources can be found in the Appendix of the user guide of the gvartoolbox by Smith, L.V. and A. Galesi (2011), available at the same web address.

and gdp_{it} on a linear deterministic trend which does not affect the $I(1)$ properties of these variables.

The 32 countries considered are: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, France, Finland, Germany, Indonesia, India, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, Spain, Sweden, Switzerland, Singapore, South Africa, Thailand, Turkey, UK, and the US. Note that not all components of \mathbf{x}_{it} are available for all countries due to data limitations. In particular, there are 26 series for eq_{it} , 31 series for ep_{it} , and 18 series for r_{it}^L .

For $m_{\max} = 4$, we consider the application of the *CIPS* and *CSB* tests allowing the number of factors, m^0 , to take any value between 1 and 4, as a way of dealing with the sampling uncertainty associated with basing our tests on a particular choice of m^0 .¹⁶ To check the robustness of the test outcomes to the choice of the additional regressors used in augmentation, we present the results of these tests for all possible combinations of candidate regressors. For $m^0 = 1$ no additional regressors are required for augmentation apart from \bar{y}_t , for $m^0 = 2$ one additional regressor is required, and so on. We set the lag order to $\hat{p} = \lceil 4(T/100)^{1/4} \rceil$, as discussed in the previous section.

The test results for the real interest rates are reported in Table 4. As can be seen, the *CIPS* test strongly rejects the null hypothesis of the panel unit root at the 1% level, for all values of m^0 , and for all combinations of candidate regressors. The test results based on the *CSB* test are very similar, although there are some exceptions. The *CSB* test does not reject when $m^0 = 3$ with $\mathbf{x}_{it} = (poil_t, \bar{r}_t^L)$, and when $m^0 = 4$ with $\mathbf{x}_{it} = (poil_t, \bar{r}_t^L, \bar{eq}_t)$, $\mathbf{x}_{it} = (poil_t, \bar{r}_t^L, \bar{ep}_t)$ and $\mathbf{x}_{it} = (poil_t, \bar{r}_t^L, \bar{gdp}_t)$, out of the twenty possible combinations. These results suggest that for a significant number of countries the Fisher parity holds. This is in line with recent findings reported in Westerlund (2008) using panel cointegration tests.

The results of panel unit root tests applied to real equity prices are summarised in Table 5. The test outcomes are generally as to be expected. The null hypothesis of a panel unit root in real equity prices cannot be rejected in most cases. When the *CIPS* test is used, the null of the panel unit root is rejected once at the 1% level (out of 26 cases), and 6 times at the 5% level. There are fewer rejections when the *CSB* test is used, namely 2 out of 26 cases. Overall, the test results are in line with the generally accepted view that real equity prices approximately follow random walks with a drift.

We also applied other panel unit root tests proposed in the literature. Specifically, we consider the pooled test statistic $P_{\hat{\epsilon}}$ of Bai and Ng (2004) based on the PANIC residuals, a panel version of the modified Sargan–Bhargava test (denoted by *PMSB*) and a PANIC residual-based Moon and Perron (2004) type test (denoted by P_b), both of which are proposed by Bai and Ng (2010), the t_b^* statistic of Moon and Perron (2004) for the case of an intercept only,¹⁷ a defactored version of the optimal invariant test of Ploberger and Phillips (2002), denoted by *PP*, for the case of an intercept and a linear trend, and the defactored version of the common

¹⁶Setting $m_{\max} = 4$, and using the information criterion IC_1 proposed by Bai and Ng (2004), the number of factors selected was $\hat{m}^0 = 3$ for the real interest rates, and $\hat{m}^0 = 4$ for the real equity prices. The last result suggests that the number of factors in real equity prices could be even higher than 4, but we did not consider $m_{\max} > 4$ given the number of observations available and the tendency of IC_1 selection criteria to over-estimate the number of factors, particularly if some of the factors are not strong.

¹⁷The t_a^* test of Moon and Perron (2004) is not included since they summarise the experimental results saying “in almost all cases, the test based on the t_b^* statistic has better size properties.” Similarly, the P_a test of Bai and Ng (2010) is not included.

point optimal test of Moon, Perron and Phillips (2011), denoted by \widetilde{CPO} .^{18 19 20} Details of the computation of the statistics $P_{\hat{\epsilon}}$, P_b , t_b^* , $PMSB$, PP , and \widetilde{CPO} are provided in a supplement, which is available from the authors on request.

The results are summarised in Table 6. For computation of the $P_{\hat{\epsilon}}$ statistic, the lag order $\hat{p} = \lceil 4(T/100)^{1/4} \rceil$ was used. Application of these tests to real interest rates yield mixed results. The P_b , t_b^* and \widetilde{CPO} tests strongly reject the null hypothesis at the 1%, for all values of m^0 , which is in accordance with the results of the $CIPS$ and CSB tests. In contrast, the $P_{\hat{\epsilon}}$ test does not reject the null hypothesis, irrespective of the number of factors considered. Results for the $PMSB$ test are mixed. It rejects the null at the 5% level for $m^0 = 1$ and 4, but fails to reject the null when $m^0 = 2$ and 3, illustrating the sensitivity of this test to the assumed number of factors. For real equity prices almost all the tests considered show strong rejections of the null hypothesis for all values of m^0 , which conflicts with the generally accepted view that real equity prices approximately follow random walks with a drift, the exceptions being the $P_{\hat{\epsilon}}$, the PP and \widetilde{CPO} tests (the latter two only when $m^0 = 4$), and the $PMSB$ test only when $m^0 = 1$.

7 Concluding Remarks

This paper considers two simple panel unit root tests that are valid in the presence of cross-sectional dependence induced by m^0 stationary common factors. The first test, $CIPS$, is an extension of the test proposed in Pesaran (2007) and is based on the average of t-ratios from ADF regressions augmented by the cross section averages of the dependent variable as well as k additional regressors with similar common factor features. The second test, CSB , is based on averages of cross-sectionally augmented Sargan-Bhargava statistics. Initially we develop the tests assuming that m^0 , the true number of factors is known, and show that the limit distributions of the tests do not depend on any nuisance parameters, so long as $k \geq m^0 - 1$. To deal with the uncertainty that surrounds the value of m^0 in practice, we propose to either choose the number of additional regressors as $k = m_{\max} - 1$, where $m_{\max} \leq m^0$, which avoids

¹⁸The $P_{\hat{\epsilon}}$ test is defined in Section 2.4 of Bai and Ng (2004, p.1140), the t_b^* test in Section 2.2.2 of Moon and Perron (2004, p.91), the P_b and $PMSB$ tests in Section 3, p.1094, eq. (9) and Section 3.1, p.1095, eq.(11), respectively of Bai and Ng (2010), the PP test in Section 5.3.1, p.429, eq. (20) in Moon et al. (2007), and the \widetilde{CPO} test in Section 2.2, p.4; Section 2.3, p.5, of Moon et al. (2011). In computing the \widetilde{CPO} test statistic we set the constant term (the ‘c’ term in Moon et al.) to unity. Also, following Moon and Perron (2004), the long-run variances for the $PMSB$, P_b , t_b^* , PP and \widetilde{CPO} test statistics are estimated by means of the Andrews and Monahan (1992) method using the quadratic spectral kernel and prewhitening. See Moon and Perron (2004) for further details.

¹⁹We are grateful to Roger Moon and Benoit Perron for helpful email correspondence with regard to the implementation of the tests in Moon et al. (2007). We would also like to thank Serena Ng for generously providing her matlab codes for computation of the test statistics proposed by Bai and Ng (2010).

²⁰The theory of the CPO test is developed by Moon et al. (2007) for the serially uncorrelated case, but it is claimed (see Section 6.4 in Moon et al. (2007, p. 436)), that replacing variances in their CPO statistic with long-run variances should result in a test with a correct size under quite general short memory error autocorrelations. However, our preliminary Monte Carlo experiments suggested that this claim might not be valid. Upon communicating these results to the authors, Moon, Perron and Phillips provided us with another modification of the CPO test that appropriately allows for residual serial correlation (see Moon, Perron and Phillips, 2011). In addition to replacing the variance of the errors by the long run variance, in this recent paper Moon et al. also adjust the centering of the statistic to accommodate for the second-order bias induced by the correlation between the error and lagged values of the dependent variable. We only consider this modified CPO test, denoted by \widetilde{CPO} .

having to estimate m^0 , or to estimate m^0 consistently using suitable selection criteria.

Small sample properties of the proposed tests are investigated by Monte Carlo experiments, which suggest that the proposed *CIPS* and *CSB* tests have the correct size across all combinations of N and T considered. The power of the *CSB* and *CIPS* tests rises in N and T and reaches quite acceptable levels when N and T are sufficiently large. For smaller T , the *CSB* test has higher power than that of *CIPS*, and should thus be preferred in such cases. In empirical applications it is important that the tests being considered have the correct size, otherwise their use could result in misleading conclusions.

The paper also applies the various panel unit root tests to real interest rates and real equity prices across countries. All tests, except the $P_{\hat{\epsilon}}$ test of Bai and Ng (2004) and the *PMSB* test of Bai and Ng (2010) reject the null of a unit root in real interest rates, which is in line with panel cointegration tests of the Fisher parity equation. For real equity prices, only our proposed tests, *CIPS* and *CSB*, and the $P_{\hat{\epsilon}}$ test do not reject the null of panel unit roots in real equity prices across, which is in accordance with the generally accepted view that real equity prices approximately follow random walks with a drift.

For our tests, following the discussions in Im et al. (2003), we propose to use critical values which depend on the values of k , N , T , and lag-augmentation order, p , as they are expected to provide much better finite sample approximations.

The good small sample results reported for the *CIPS* and *CSB* tests comes at a cost, as the tests require the existence of additional $I(1)$ regressors that share the same common factors with y_{it} . We have argued that this might not be a problem when m^0 , the true number of factors in y_{it} , is not too large. For example, if $m^0 \leq 2$, only one additional regressor is needed at most to apply the test, and this is unlikely to be a problem in practice, where most macro and finance series are often driven by a small number of common factors. For larger values of m^0 a more careful consideration of the testing problem is required. In such cases it seems more appropriate if the problem of panel unit root testing is considered as part of a more general problem, where robustness of the panel unit root test outcomes to alternative assumptions regarding the integration and cointegration properties of the additional regressors is considered and evaluated.

Appendix A Mathematical Proofs

Lemmas

Lemma A.1 *Under Assumptions 1-5*

$$\begin{aligned}
\boldsymbol{\epsilon}'_{iy} \bar{\mathbf{E}}/T &= O_p\left(T^{-1/2}N^{-1/2}\right), \mathbf{s}'_{iy,-1} \bar{\mathbf{E}}/T = O_p\left(N^{-1/2}\right), \text{ uniformly over } i \\
\bar{\mathbf{S}}'_{-1} \boldsymbol{\epsilon}_{iy}/T &= O_p\left(N^{-1/2}\right), \mathbf{s}'_{iy,-1} \bar{\mathbf{S}}_{-1}/T^2 = O_p\left(N^{-1/2}\right), \text{ uniformly over } i \\
\bar{\mathbf{E}}' \bar{\mathbf{E}}/T &= O_p\left(N^{-1}\right), \bar{\mathbf{S}}'_{-1} \bar{\mathbf{E}}/T = O_p\left(N^{-1}\right), \bar{\mathbf{S}}'_{-1} \bar{\mathbf{S}}_{-1}/T^2 = O_p\left(N^{-1}\right) \\
\mathbf{F}' \bar{\mathbf{E}}/T &= O_p\left(T^{-1/2}N^{-1/2}\right), \bar{\mathbf{S}}'_{-1} \mathbf{F}/T = O_p\left(N^{-1/2}\right) \\
\boldsymbol{\tau}'_T \bar{\mathbf{E}}/T &= O_p\left(T^{-1/2}N^{-1/2}\right), \bar{\mathbf{S}}'_{-1} \boldsymbol{\tau}_T/T = O_p\left(T/N\right) \\
\mathbf{S}'_{f,-1} \bar{\mathbf{E}}/T &= O_p\left(N^{-1/2}\right), \frac{\mathbf{S}'_{f,-1} \bar{\mathbf{S}}_{-1}}{T^2} = O_p\left(N^{-1/2}\right).
\end{aligned}$$

Similar order results hold for the case of serially correlated errors.

Proof. See Appendix A.1 of Pesaran (2007). ■

Lemma A.2 Consider a full column rank $m \times n$ matrix \mathbf{A} ($m > n$) and an $n \times n$ non-singular symmetric matrix $\mathbf{\Omega}$. Then $\mathbf{A}'(\mathbf{A}\mathbf{\Omega}\mathbf{A}')^+ \mathbf{A} = \mathbf{\Omega}^{-1}$, where $(\mathbf{A}\mathbf{\Omega}\mathbf{A}')^+$ is the Moore-Penrose inverse of $\mathbf{A}\mathbf{\Omega}\mathbf{A}'$.

Proof. Using the fact that $\mathbf{A}^+ \mathbf{A} = \mathbf{I}_n$ and $\mathbf{A}' \mathbf{A}'^+ = \mathbf{I}_n$ and that $(\mathbf{A}\mathbf{\Omega}\mathbf{A}')^+ = \mathbf{A}'^+ \mathbf{\Omega}^{-1} \mathbf{A}^+$ (see Magnus and Neudecker 1999, p.34), $\mathbf{A}'(\mathbf{A}\mathbf{\Omega}\mathbf{A}')^+ \mathbf{A} = \mathbf{A}' \mathbf{A}'^+ \mathbf{\Omega}^{-1} \mathbf{A}^+ \mathbf{A} = \mathbf{\Omega}^{-1}$, as required. ■

Proof of Theorem 2.1:

In the case where $\mathbf{d}_t = 1$, using (26) we have

$$t_i(N, T) = \frac{\frac{\mathbf{v}'_i \bar{\mathbf{M}} \hat{\mathbf{s}}_{i,-1}}{T}}{\left(\frac{\mathbf{v}'_i \bar{\mathbf{M}}_i \mathbf{v}_i}{T-2k-4} \right)^{1/2} \left(\frac{\hat{\mathbf{s}}'_{i,-1} \bar{\mathbf{M}} \hat{\mathbf{s}}_{i,-1}}{T^2} \right)^{1/2}}, \quad (\text{A.1})$$

where $\mathbf{v}_i = (\boldsymbol{\varepsilon}_{iy} - \bar{\mathbf{E}} \boldsymbol{\delta}_i) / \sigma_i$, and $\hat{\mathbf{s}}_{i,-1} = (\mathbf{s}'_{iy,-1} - \bar{\mathbf{S}}_{-1} \boldsymbol{\delta}_i) / \sigma_i$, $\bar{\mathbf{M}} = \mathbf{I}_T - \bar{\mathbf{W}} (\bar{\mathbf{W}}' \bar{\mathbf{W}})^{-1} \bar{\mathbf{W}}'$, and $\bar{\mathbf{W}} = (\Delta \bar{\mathbf{Z}}, \boldsymbol{\tau}_T, \bar{\mathbf{Z}}_{-1})$. Also, since $d_t = 1$, from (15) and (22) we have $\Delta \bar{\mathbf{Z}} = \mathbf{F} \bar{\boldsymbol{\Gamma}}' + \bar{\mathbf{E}}$ and $\bar{\mathbf{Z}}_{-1} = \boldsymbol{\tau}_T \bar{\mathbf{z}}'_0 + \mathbf{S}_{f,-1} \bar{\boldsymbol{\Gamma}}' + \bar{\mathbf{S}}_{-1}$. Let $\mathbf{W}_f = (\mathbf{F}, \boldsymbol{\tau}_T, \mathbf{S}_{f,-1})$ and $\bar{\boldsymbol{\Xi}} = (\bar{\mathbf{E}}, \mathbf{0}_T, \bar{\mathbf{S}}_{-1})$ so that

$$\bar{\mathbf{W}}' = \mathbf{Q}_N \mathbf{W}'_f + \bar{\boldsymbol{\Xi}}', \quad \text{where} \quad \mathbf{Q}_N = \begin{pmatrix} \bar{\boldsymbol{\Gamma}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \bar{\mathbf{z}}_0 & \bar{\boldsymbol{\Gamma}} \end{pmatrix}. \quad (\text{A.2})$$

Consider the numerator of (A.1), and note that

$$\frac{\mathbf{v}'_i \bar{\mathbf{M}} \hat{\mathbf{s}}_{i,-1}}{T} = \frac{\mathbf{v}'_i \hat{\mathbf{s}}_{i,-1}}{T} - (\mathbf{v}'_i \bar{\mathbf{W}} \mathbf{B}) (\mathbf{B} \bar{\mathbf{W}}' \bar{\mathbf{W}} \mathbf{B})^{-1} \left(\frac{\mathbf{B} \bar{\mathbf{W}}' \hat{\mathbf{s}}_{i,-1}}{T} \right), \quad (\text{A.3})$$

where $\mathbf{B} = \begin{pmatrix} T^{-1/2} \mathbf{I}_{k+2} & \mathbf{0} \\ \mathbf{0} & T^{-1} \mathbf{I}_{k+1} \end{pmatrix}$. Using Lemma A.1 together with the results in Proposition 17.1 of Hamilton (1994; p.486) we have

$$\frac{\hat{\mathbf{s}}'_{i,-1} \mathbf{v}_i}{T^{3/2}} = \frac{\mathbf{s}'_{iy,-1} \boldsymbol{\varepsilon}_{iy}}{\sigma_i^2 T^{3/2}} + O_p \left(\frac{1}{\sqrt{NT}} \right) \xrightarrow{(N,T)j} \int_0^1 W_i(r) dW_i(r). \quad (\text{A.4})$$

where $W_i(r)$ is a standard Brownian motion defined on $[0,1]$, associated with $\boldsymbol{\varepsilon}_{iyt}$. Using (A.2) it follows that

$$\mathbf{B} \bar{\mathbf{W}}' \mathbf{v}_i = \mathbf{B} \mathbf{Q}_N \mathbf{W}'_f \mathbf{v}_i + \mathbf{B} \bar{\boldsymbol{\Xi}}' \mathbf{v}_i, \quad (\text{A.5})$$

$$\mathbf{B} \bar{\mathbf{W}}' \hat{\mathbf{s}}_{i,-1} / T = \mathbf{B} \mathbf{Q}_N \mathbf{W}'_f \hat{\mathbf{s}}_{i,-1} / T + \mathbf{B} \bar{\boldsymbol{\Xi}}' \hat{\mathbf{s}}_{i,-1} / T, \quad (\text{A.6})$$

$$\mathbf{B} \bar{\mathbf{W}}' \bar{\mathbf{W}} \mathbf{B} = \mathbf{B} \mathbf{Q}_N \mathbf{W}'_f \mathbf{W}_f \mathbf{Q}'_N \mathbf{B} + \mathbf{B} \mathbf{Q}_N \mathbf{W}'_f \bar{\boldsymbol{\Xi}} \mathbf{B} + \mathbf{B} \bar{\boldsymbol{\Xi}}' \mathbf{W}_f \mathbf{Q}'_N \mathbf{B} + \mathbf{B} \bar{\boldsymbol{\Xi}}' \bar{\boldsymbol{\Xi}} \mathbf{B}. \quad (\text{A.7})$$

From Lemma A.1, it is easily seen that, as $(T, N) \xrightarrow{j} \infty$ with $\sqrt{T}/N \rightarrow 0$,

$$\mathbf{B} \bar{\boldsymbol{\Xi}}' \mathbf{v}_i \xrightarrow{(N,T)j} \mathbf{0}, \quad \mathbf{B} \bar{\boldsymbol{\Xi}}' \hat{\mathbf{s}}_{i,-1} / T \xrightarrow{(N,T)j} \mathbf{0}, \quad \mathbf{B} \bar{\boldsymbol{\Xi}}' \bar{\boldsymbol{\Xi}} \mathbf{B} \xrightarrow{(N,T)j} \mathbf{0}, \quad \text{and} \quad \mathbf{B} \mathbf{Q}_N \mathbf{W}'_f \bar{\boldsymbol{\Xi}} \mathbf{B} \xrightarrow{(N,T)j} \mathbf{0}. \quad (\text{A.8})$$

Define $\mathbf{C} = \begin{pmatrix} T^{-1/2} \mathbf{I}_{m^0+1} & \mathbf{0} \\ \mathbf{0} & T^{-1} \mathbf{I}_{m^0} \end{pmatrix}$, so that, using Lemma A.1 and the results in Proposition 17.1 and 18.1 of Hamilton (1994; p.486, p.547-8), as $(T, N) \xrightarrow{j} \infty$ with $\sqrt{T}/N \rightarrow 0$ we have

$$\mathbf{B} \mathbf{Q}_N \mathbf{W}'_f \mathbf{v}_i = \mathbf{Q}_N \mathbf{C} \mathbf{W}'_f \mathbf{v}_i \xrightarrow{(N,T)j} \mathbf{Q} \boldsymbol{\vartheta}_{if}, \quad (\text{A.9})$$

$$\mathbf{B} \mathbf{Q}_N \mathbf{W}'_f \hat{\mathbf{s}}_{i,-1} / T = \mathbf{Q}_N \mathbf{C} \mathbf{W}'_f \hat{\mathbf{s}}_{i,-1} / T \xrightarrow{(N,T)j} \mathbf{Q} \boldsymbol{\kappa}_{if}, \quad (\text{A.10})$$

$$\mathbf{B} \mathbf{Q}_N \mathbf{W}'_f \mathbf{W}_f \mathbf{Q}'_N \mathbf{B} = \mathbf{Q}_N \mathbf{C} \mathbf{W}'_f \mathbf{W}_f \mathbf{C} \mathbf{Q}'_N \xrightarrow{(N,T)j} \mathbf{Q} \boldsymbol{\Upsilon}_f \mathbf{Q}', \quad (\text{A.11})$$

where

$$\begin{aligned} \mathbf{Q} &= \text{plim}_{N \rightarrow \infty} \mathbf{Q}_N, \quad \boldsymbol{\vartheta}_{if} = \begin{pmatrix} \boldsymbol{\Lambda}_f \mathbf{W}_{\mathbf{v},i}(1) \\ \boldsymbol{\Lambda}_f^* \boldsymbol{\omega}_{i\mathbf{v}} \end{pmatrix}, \quad \boldsymbol{\kappa}_{if} = \begin{pmatrix} \mathbf{0} \\ \boldsymbol{\Lambda}_f^* \boldsymbol{\pi}_{i\mathbf{v}} \end{pmatrix}, \quad \boldsymbol{\Upsilon}_f = \begin{pmatrix} \mathbf{I}_{m^0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_f^* \mathbf{G}_{\mathbf{v}} \boldsymbol{\Lambda}_f^{*'} \end{pmatrix}, \quad \boldsymbol{\Lambda}_f^* = \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_f \end{pmatrix}, \\ \boldsymbol{\omega}_{i\mathbf{v}} &= \begin{pmatrix} W_i(1) \\ \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)] dW_i(r) \end{pmatrix}, \quad \boldsymbol{\pi}_{i\mathbf{v}} = \begin{pmatrix} \int_0^1 W_i(r) dr \\ \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)] W_i(r) dr \end{pmatrix}, \quad \mathbf{G}_{\mathbf{v}} = \begin{pmatrix} 1 & \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)]' dr \\ \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)] dr & \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)] [\mathbf{W}_{\mathbf{v}}(r)]' dr \end{pmatrix}, \end{aligned} \quad (\text{A.12})$$

Λ_f is defined by (3), $\mathbf{W}_{\mathbf{v},i}(1)$ is defined such that $T^{-1/2} \sum_{t=1}^T \mathbf{v}_t \varepsilon_{iyt} / \sigma_i \xrightarrow{T} \mathbf{W}_{\mathbf{v},i}(1)$ with \mathbf{v}_t defined as in Assumption 2, $\mathbf{W}_{\mathbf{v}}(r)$ is an m^0 -dimensional standard Brownian motion associated with \mathbf{v}_t defined on $[0,1]$, and $W_i(r)$ is defined as above. These two groups of Brownian motions ($\mathbf{W}_{\mathbf{v}}(r), W_i(r)$) are independent of each other. Collecting the results from (A.5) to (A.11), as well as using Lemma A.2 (since \mathbf{Q} has full column rank) we have

$$\begin{aligned} & (\mathbf{v}'_i \bar{\mathbf{W}} \mathbf{B}) \left(\mathbf{B} \bar{\mathbf{W}}' \bar{\mathbf{W}} \mathbf{B} \right)^{-1} \left(T^{-1} \mathbf{B} \bar{\mathbf{W}}' \hat{\mathbf{s}}_{i,-1} \right) \xrightarrow{(N,T)j} \vartheta'_{if} \mathbf{Q}' (\mathbf{Q} \Upsilon_f \mathbf{Q}')^+ \mathbf{Q} \kappa_{if} \\ & = \vartheta'_{if} \Upsilon_f^{-1} \kappa_{if} = \omega'_{iv} \Lambda_f' \left(\Lambda_f^* \mathbf{G}_{\mathbf{v}} \Lambda_f^* \right)^{-1} \Lambda_f^* \pi_{iv} = \omega'_{iv} \mathbf{G}_{\mathbf{v}}^{-1} \pi_{iv}. \end{aligned} \quad (\text{A.13})$$

Therefore, together with (A.3), (A.4) and (A.13), as $(T, N) \xrightarrow{j} \infty$ with $\sqrt{T}/N \rightarrow 0$ we have

$$\frac{\mathbf{v}'_i \bar{\mathbf{M}} \hat{\mathbf{s}}_{i,-1}}{T} \xrightarrow{(N,T)j} \int_0^1 W_i(r) dW_i(r) - \omega'_{iv} \mathbf{G}_{\mathbf{v}}^{-1} \pi_{iv}. \quad (\text{A.14})$$

In a similar manner, noting that as $(T, N) \xrightarrow{j} \infty$ with $\sqrt{T}/N \rightarrow 0$

$$\frac{\hat{\mathbf{s}}'_{i,-1} \hat{\mathbf{s}}_{i,-1}}{T^2} = \frac{\mathbf{s}'_{iy,-1} \mathbf{s}_{iy,-1}}{\sigma_i^2 T^2} + O_p \left(\frac{1}{\sqrt{N}} \right) \xrightarrow{(N,T)j} \int_0^1 W_i^2(r) dr, \quad (\text{A.15})$$

it follows that

$$\frac{\hat{\mathbf{s}}'_{i,-1} \bar{\mathbf{M}} \hat{\mathbf{s}}_{i,-1}}{T} \xrightarrow{(N,T)j} \int_0^1 W_i^2(r) dr - \pi'_{iv} \mathbf{G}_{\mathbf{v}}^{-1} \pi_{iv}. \quad (\text{A.16})$$

Next, consider $\mathbf{v}'_i \bar{\mathbf{M}}_i \mathbf{v}_i / (T - 2k - 4)$. Note that $\bar{\mathbf{M}}_i \mathbf{v}_i$ are the residuals from the regression of \mathbf{v}_i on $\bar{\mathbf{W}}_i = (\bar{\mathbf{W}}, \mathbf{y}_{i,-1})$, but from equation (23) $\mathbf{y}_{i,-1}$ has components $(\bar{\mathbf{Z}}_{-1}, \boldsymbol{\tau}_T, \hat{\mathbf{s}}_{i,-1})$. As $(\bar{\mathbf{Z}}_{-1}, \boldsymbol{\tau}_T) \subset \bar{\mathbf{W}}$, but $\hat{\mathbf{s}}_{i,-1}$ is not contained in $\bar{\mathbf{W}}$, we have $\bar{\mathbf{M}}_i \mathbf{v}_i = \bar{\mathbf{M}}_i^* \mathbf{v}_i$, where $\bar{\mathbf{M}}_i^* = \mathbf{I}_T - \bar{\mathbf{H}}_i (\bar{\mathbf{H}}_i' \bar{\mathbf{H}}_i)^{-1} \bar{\mathbf{H}}_i'$ with $\bar{\mathbf{H}}_i = (\bar{\mathbf{W}}, \hat{\mathbf{s}}_{i,-1})$. Thus

$$\frac{\mathbf{v}'_i \bar{\mathbf{M}}_i^* \mathbf{v}_i}{T - 2k - 4} = \frac{\mathbf{v}'_i \mathbf{v}_i}{T - 2k - 4} - \frac{(\mathbf{v}'_i \bar{\mathbf{H}}_i \mathbf{B}_*) (\mathbf{B}_* \bar{\mathbf{H}}_i' \bar{\mathbf{H}}_i \mathbf{B}_*)^{-1} (\mathbf{B}_* \bar{\mathbf{H}}_i' \mathbf{v}_i)}{T - 2k - 4}, \quad (\text{A.17})$$

where $\mathbf{B}_* = \begin{pmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & T^{-1} \end{pmatrix}_{(2k+4) \times (2k+4)}$. First note that using Lemma A.1 we have

$$\mathbf{v}'_i \mathbf{v}_i / (T - 2k - 4) \xrightarrow{(N,T)j} 1. \quad (\text{A.18})$$

We also have that

$$\mathbf{B}_* \bar{\mathbf{H}}_i' \mathbf{v}_i = \begin{pmatrix} \mathbf{B} \bar{\mathbf{W}}' \mathbf{v}_i \\ \hat{\mathbf{s}}'_{i,-1} \mathbf{v}_i / T \end{pmatrix}, \quad \mathbf{B}_* \bar{\mathbf{H}}_i' \bar{\mathbf{H}}_i \mathbf{B}_* = \begin{pmatrix} \mathbf{B} \bar{\mathbf{W}}' \bar{\mathbf{W}} \mathbf{B} & \mathbf{B} \bar{\mathbf{W}}' \hat{\mathbf{s}}_{i,-1} / T \\ \hat{\mathbf{s}}'_{i,-1} \bar{\mathbf{W}} \mathbf{B} / T & \hat{\mathbf{s}}'_{i,-1} \hat{\mathbf{s}}_{i,-1} / T^2 \end{pmatrix},$$

so then using (A.4), (A.15), and following the same line of analysis as for the results in (A.13), it can be seen that $(\mathbf{v}'_i \bar{\mathbf{H}}_i \mathbf{B}_*) (\mathbf{B}_* \bar{\mathbf{H}}_i' \bar{\mathbf{H}}_i \mathbf{B}_*)^{-1} (\mathbf{B}_* \bar{\mathbf{H}}_i' \mathbf{v}_i)$ in (A.17) will tend to a function of standard Brownian motions as $(T, N) \xrightarrow{j} \infty$ with $\sqrt{T}/N \rightarrow 0$. Thus, dividing by $T - 2k - 4$ makes the second term of (A.17) asymptotically negligible, and together with the results in (A.17) and (A.18) we have that $\frac{\mathbf{v}'_i \bar{\mathbf{M}}_i^* \mathbf{v}_i}{T - 2k - 4} \xrightarrow{(N,T)j} 1$. Thus, as $(T, N) \xrightarrow{j} \infty$ with $\sqrt{T}/N \rightarrow 0$,

$$\mathbf{v}'_i \bar{\mathbf{M}}_i \mathbf{v}_i / (T - 2k - 4) \xrightarrow{(N,T)j} 1. \quad (\text{A.19})$$

Finally, from the results in (A.1), (A.14), (A.16) and (A.19), we have, as $\sqrt{T}/N \rightarrow 0$,

$$t_i(N, T) \xrightarrow{(N,T)j} \frac{\int_0^1 W_i(r) dW_i(r) - \omega'_{iv} \mathbf{G}_{\mathbf{v}}^{-1} \pi_{iv}}{\left(\int_0^1 W_i^2(r) dr - \pi'_{iv} \mathbf{G}_{\mathbf{v}}^{-1} \pi_{iv} \right)^{1/2}}, \quad (\text{A.20})$$

as required. Condition $\sqrt{T}/N \rightarrow 0$ is satisfied so long as $T/N \rightarrow \delta$, as N and $T \rightarrow \infty$, where δ is a fixed finite non-zero positive constant. For sequential asymptotics, with $N \rightarrow \infty$, first, we note that for a fixed T and as $N \rightarrow \infty$, $\mathbf{Q} = \text{plim}_{N \rightarrow \infty} \mathbf{Q}_N$, and by Lemma A.1, (A.8) continues to hold (replacing $\xrightarrow{(N,T)j}$ by \xrightarrow{N}). Then, letting $T \rightarrow \infty$ yields (A.20).

Appendix B: Critical Value Tables for CIPS and CSB tests

Table B1: Critical Values of the Average of Individual Cross-Sectionally Augmented Dickey-Fuller Distributions with k Additional Regressors in the Case of Models with an Intercept Only

		$k = 1$																	
p	(T,N)	1% (CADF)						5% (CADF)						10% (CADF)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	-2.64	-2.56	-2.51	-2.45	-2.42	-2.39	-2.44	-2.38	-2.34	-2.30	-2.29	-2.26	-2.33	-2.28	-2.25	-2.22	-2.21	-2.19
	30	-2.64	-2.55	-2.48	-2.46	-2.44	-2.40	-2.45	-2.39	-2.35	-2.33	-2.31	-2.29	-2.34	-2.30	-2.27	-2.25	-2.24	-2.22
	50	-2.62	-2.56	-2.49	-2.46	-2.44	-2.42	-2.45	-2.40	-2.36	-2.34	-2.32	-2.30	-2.35	-2.32	-2.28	-2.27	-2.26	-2.25
	70	-2.64	-2.55	-2.49	-2.46	-2.45	-2.43	-2.46	-2.40	-2.37	-2.35	-2.33	-2.32	-2.36	-2.32	-2.30	-2.28	-2.27	-2.26
	100	-2.63	-2.56	-2.51	-2.47	-2.45	-2.43	-2.47	-2.41	-2.38	-2.35	-2.34	-2.32	-2.37	-2.33	-2.31	-2.28	-2.28	-2.26
	200	-2.63	-2.56	-2.50	-2.48	-2.46	-2.43	-2.46	-2.42	-2.38	-2.36	-2.35	-2.33	-2.37	-2.34	-2.31	-2.29	-2.29	-2.27
1	20	-2.63	-2.54	-2.45	-2.41	-2.40	-2.35	-2.38	-2.32	-2.27	-2.23	-2.22	-2.19	-2.26	-2.20	-2.17	-2.14	-2.12	-2.10
	30	-2.61	-2.52	-2.46	-2.43	-2.41	-2.36	-2.41	-2.34	-2.30	-2.28	-2.26	-2.24	-2.29	-2.24	-2.22	-2.19	-2.18	-2.17
	50	-2.62	-2.55	-2.48	-2.44	-2.43	-2.40	-2.43	-2.37	-2.33	-2.31	-2.30	-2.28	-2.33	-2.29	-2.25	-2.24	-2.23	-2.21
	70	-2.62	-2.54	-2.47	-2.45	-2.43	-2.41	-2.45	-2.39	-2.35	-2.33	-2.32	-2.29	-2.35	-2.30	-2.27	-2.26	-2.24	-2.23
	100	-2.62	-2.54	-2.50	-2.46	-2.44	-2.41	-2.45	-2.40	-2.37	-2.34	-2.32	-2.31	-2.36	-2.32	-2.30	-2.27	-2.26	-2.24
	200	-2.62	-2.55	-2.50	-2.47	-2.46	-2.43	-2.46	-2.41	-2.37	-2.36	-2.34	-2.32	-2.37	-2.33	-2.30	-2.28	-2.28	-2.26
2	20	-2.44	-2.34	-2.25	-2.21	-2.17	-2.12	-2.18	-2.11	-2.05	-2.01	-1.99	-1.96	-2.03	-1.98	-1.94	-1.91	-1.89	-1.88
	30	-2.50	-2.41	-2.33	-2.30	-2.28	-2.24	-2.27	-2.21	-2.17	-2.14	-2.12	-2.10	-2.15	-2.11	-2.07	-2.05	-2.04	-2.03
	50	-2.55	-2.49	-2.40	-2.37	-2.35	-2.32	-2.35	-2.31	-2.25	-2.24	-2.22	-2.20	-2.25	-2.22	-2.17	-2.16	-2.15	-2.13
	70	-2.57	-2.49	-2.43	-2.41	-2.39	-2.36	-2.39	-2.34	-2.29	-2.27	-2.26	-2.24	-2.30	-2.25	-2.22	-2.20	-2.19	-2.18
	100	-2.59	-2.51	-2.46	-2.42	-2.40	-2.38	-2.41	-2.37	-2.33	-2.30	-2.29	-2.27	-2.32	-2.28	-2.26	-2.23	-2.22	-2.20
	200	-2.61	-2.54	-2.48	-2.46	-2.43	-2.40	-2.43	-2.39	-2.35	-2.34	-2.33	-2.30	-2.35	-2.31	-2.28	-2.26	-2.27	-2.24
3	20	-2.45	-2.32	-2.21	-2.13	-2.09	-2.02	-2.12	-2.05	-1.97	-1.90	-1.89	-1.85	-1.96	-1.90	-1.83	-1.80	-1.77	-1.74
	30	-2.43	-2.34	-2.28	-2.24	-2.21	-2.18	-2.21	-2.13	-2.09	-2.07	-2.05	-2.03	-2.09	-2.02	-1.99	-1.97	-1.96	-1.94
	50	-2.53	-2.45	-2.38	-2.36	-2.33	-2.30	-2.32	-2.27	-2.22	-2.20	-2.19	-2.17	-2.21	-2.18	-2.13	-2.12	-2.11	-2.09
	70	-2.56	-2.47	-2.42	-2.39	-2.37	-2.35	-2.37	-2.31	-2.27	-2.25	-2.24	-2.22	-2.27	-2.23	-2.19	-2.18	-2.16	-2.15
	100	-2.59	-2.50	-2.46	-2.42	-2.39	-2.37	-2.39	-2.35	-2.32	-2.29	-2.27	-2.25	-2.30	-2.26	-2.24	-2.21	-2.20	-2.19
	200	-2.61	-2.54	-2.47	-2.45	-2.43	-2.40	-2.43	-2.39	-2.34	-2.33	-2.32	-2.30	-2.34	-2.31	-2.27	-2.26	-2.26	-2.24
4	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.29	-2.20	-2.12	-2.06	-2.05	-2.03	-2.05	-1.99	-1.94	-1.91	-1.89	-1.86	-1.93	-1.87	-1.84	-1.81	-1.79	-1.78
	50	-2.44	-2.37	-2.30	-2.28	-2.23	-2.21	-2.24	-2.19	-2.14	-2.12	-2.11	-2.08	-2.13	-2.09	-2.05	-2.04	-2.02	-2.01
	70	-2.52	-2.43	-2.37	-2.34	-2.32	-2.30	-2.32	-2.26	-2.22	-2.20	-2.19	-2.17	-2.22	-2.17	-2.14	-2.12	-2.11	-2.10
	100	-2.54	-2.48	-2.42	-2.38	-2.35	-2.33	-2.36	-2.31	-2.28	-2.25	-2.23	-2.22	-2.26	-2.23	-2.20	-2.17	-2.16	-2.15
	200	-2.60	-2.52	-2.46	-2.44	-2.42	-2.39	-2.41	-2.37	-2.33	-2.30	-2.30	-2.28	-2.32	-2.29	-2.26	-2.24	-2.24	-2.22
$k = 2$																			
p	(T,N)	1% (CADF)						5% (CADF)						10% (CADF)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	-2.84	-2.78	-2.68	-2.65	-2.59	-2.57	-2.60	-2.56	-2.49	-2.48	-2.45	-2.43	-2.48	-2.45	-2.40	-2.39	-2.37	-2.35
	30	-2.84	-2.76	-2.68	-2.65	-2.63	-2.59	-2.64	-2.58	-2.53	-2.51	-2.50	-2.47	-2.53	-2.49	-2.45	-2.43	-2.42	-2.40
	50	-2.84	-2.77	-2.70	-2.67	-2.66	-2.62	-2.66	-2.61	-2.56	-2.55	-2.53	-2.51	-2.57	-2.52	-2.48	-2.48	-2.46	-2.45
	70	-2.86	-2.78	-2.72	-2.68	-2.67	-2.64	-2.67	-2.62	-2.58	-2.56	-2.55	-2.53	-2.58	-2.54	-2.50	-2.49	-2.48	-2.47
	100	-2.85	-2.79	-2.72	-2.69	-2.67	-2.64	-2.68	-2.64	-2.59	-2.57	-2.56	-2.54	-2.59	-2.56	-2.52	-2.50	-2.49	-2.48
	200	-2.87	-2.80	-2.74	-2.70	-2.68	-2.66	-2.69	-2.65	-2.61	-2.58	-2.56	-2.55	-2.60	-2.57	-2.53	-2.52	-2.50	-2.49
1	20	-2.78	-2.68	-2.55	-2.53	-2.48	-2.46	-2.47	-2.40	-2.33	-2.32	-2.30	-2.27	-2.33	-2.27	-2.23	-2.21	-2.19	-2.17
	30	-2.76	-2.69	-2.61	-2.57	-2.56	-2.51	-2.54	-2.49	-2.43	-2.41	-2.39	-2.36	-2.42	-2.38	-2.34	-2.32	-2.31	-2.29
	50	-2.80	-2.74	-2.67	-2.64	-2.61	-2.58	-2.61	-2.56	-2.51	-2.50	-2.47	-2.45	-2.50	-2.47	-2.43	-2.42	-2.40	-2.38
	70	-2.83	-2.75	-2.68	-2.65	-2.64	-2.60	-2.65	-2.59	-2.53	-2.52	-2.51	-2.49	-2.54	-2.49	-2.46	-2.44	-2.44	-2.42
	100	-2.84	-2.78	-2.70	-2.67	-2.65	-2.62	-2.65	-2.61	-2.57	-2.54	-2.53	-2.51	-2.56	-2.52	-2.49	-2.47	-2.46	-2.45
	200	-2.85	-2.80	-2.72	-2.69	-2.67	-2.65	-2.69	-2.65	-2.61	-2.57	-2.54	-2.53	-2.51	-2.56	-2.52	-2.49	-2.49	-2.48
2	20	-2.71	-2.51	-2.33	-2.27	-2.21	-2.16	-2.29	-2.17	-2.08	-2.03	-1.99	-1.95	-2.08	-2.01	-1.93	-1.90	-1.87	-1.84
	30	-2.58	-2.50	-2.41	-2.36	-2.34	-2.30	-2.35	-2.30	-2.22	-2.20	-2.18	-2.15	-2.21	-2.17	-2.12	-2.10	-2.10	-2.07
	50	-2.70	-2.63	-2.55	-2.53	-2.50	-2.47	-2.50	-2.45	-2.39	-2.39	-2.37	-2.34	-2.38	-2.35	-2.30	-2.30	-2.28	-2.26
	70	-2.75	-2.68	-2.61	-2.58	-2.57	-2.53	-2.57	-2.51	-2.46	-2.44	-2.44	-2.41	-2.45	-2.42	-2.38	-2.37	-2.36	-2.34
	100	-2.79	-2.72	-2.65	-2.62	-2.60	-2.57	-2.60	-2.56	-2.51	-2.49	-2.47	-2.46	-2.51	-2.46	-2.44	-2.42	-2.40	-2.39
	200	-2.84	-2.77	-2.69	-2.66	-2.65	-2.62	-2.67	-2.62	-2.57	-2.54	-2.53	-2.51	-2.56	-2.53	-2.49	-2.48	-2.46	-2.45
3	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.47	-2.37	-2.27	-2.21	-2.19	-2.16	-2.20	-2.14	-2.07	-2.04	-2.02	-2.00	-2.07	-2.02	-1.96	-1.94	-1.93	-1.90
	50	-2.64	-2.57	-2.49	-2.48	-2.44	-2.41	-2.43	-2.38	-2.32	-2.31	-2.29	-2.27	-2.31	-2.28	-2.23	-2.23	-2.21	-2.19
	70	-2.72	-2.65	-2.56	-2.54	-2.53	-2.49	-2.51	-2.47	-2.42	-2.40	-2.39	-2.36	-2.41	-2.37	-2.33	-2.32	-2.31	-2.30
	100	-2.77	-2.69	-2.63	-2.60	-2.58	-2.54	-2.58	-2.53	-2.49	-2.46	-2.45	-2.43	-2.48	-2.44	-2.41	-2.39	-2.37	-2.36
	200	-2.82	-2.76	-2.68	-2.65	-2.63	-2.61	-2.65	-2.61	-2.55	-2.53	-2.52	-2.50	-2.55	-2.52	-2.48	-2.46	-2.44	-2.44
4	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.28	-2.15	-2.04	-2.01	-1.97	-1.89	-2.00	-1.90	-1.83	-1.81	-1.78	-1.72	-1.85	-1.77	-1.72	-1.70	-1.68	-1.64
	50	-2.53	-2.45	-2.38	-2.35	-2.31	-2.28	-2.31	-2.25	-2.20	-2.19	-2.16	-2.14	-2.19	-2.15	-2.10	-2.10	-2.08	-2.06
	70	-2.64	-2.59	-2.50	-2.48	-2.45	-2.41	-2.45	-2.38	-2.34	-2.32	-2.31	-2.29	-2.33	-2.29	-2.25	-2.24	-2.23	-2.21
	100	-2.71	-2.65	-2.58	-2.55	-2.53	-2.49	-2.53	-2.47	-2.44	-2.41	-2.39	-2.37	-2.43	-2.38	-2.35	-2.33	-2.32	-2.30
	200	-2.80	-2.74	-2.66	-2.63	-2.60	-2.58	-2.62	-2.58	-2.53	-2.51	-2.49	-2.47	-2.52	-2.49	-2.45	-2.43	-2.42	-2.41

Table B1 Continued

		$k = 3$																	
p	(T,N)	1% (CADF)						5% (CADF)						10% (CADF)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	-2.99	-2.91	-2.82	-2.76	-2.75	-2.71	-2.73	-2.68	-2.63	-2.58	-2.57	-2.54	-2.60	-2.56	-2.52	-2.49	-2.48	-2.45
	30	-3.00	-2.89	-2.85	-2.81	-2.78	-2.73	-2.79	-2.73	-2.69	-2.66	-2.64	-2.61	-2.68	-2.63	-2.60	-2.57	-2.56	-2.54
	50	-3.01	-2.95	-2.88	-2.86	-2.82	-2.80	-2.83	-2.79	-2.74	-2.72	-2.70	-2.69	-2.73	-2.70	-2.65	-2.64	-2.63	-2.62
	70	-3.04	-2.97	-2.89	-2.86	-2.84	-2.82	-2.86	-2.81	-2.77	-2.74	-2.72	-2.71	-2.76	-2.73	-2.69	-2.67	-2.66	-2.65
	100	-3.05	-2.97	-2.92	-2.88	-2.86	-2.83	-2.87	-2.82	-2.78	-2.76	-2.74	-2.73	-2.78	-2.74	-2.71	-2.69	-2.68	-2.66
	200	-3.07	-2.99	-2.92	-2.90	-2.87	-2.85	-2.89	-2.84	-2.80	-2.79	-2.76	-2.75	-2.80	-2.76	-2.72	-2.71	-2.70	-2.68
1	20	-2.91	-2.77	-2.66	-2.58	-2.52	-2.47	-2.55	-2.47	-2.39	-2.34	-2.30	-2.26	-2.37	-2.30	-2.25	-2.20	-2.19	-2.15
	30	-2.86	-2.77	-2.69	-2.67	-2.62	-2.59	-2.63	-2.56	-2.52	-2.49	-2.46	-2.42	-2.50	-2.45	-2.41	-2.39	-2.38	-2.35
	50	-2.94	-2.88	-2.80	-2.78	-2.74	-2.72	-2.75	-2.69	-2.65	-2.62	-2.61	-2.59	-2.65	-2.60	-2.56	-2.54	-2.53	-2.52
	70	-2.99	-2.90	-2.85	-2.82	-2.79	-2.77	-2.80	-2.75	-2.70	-2.67	-2.67	-2.65	-2.70	-2.66	-2.62	-2.60	-2.59	-2.58
	100	-3.01	-2.93	-2.87	-2.84	-2.83	-2.80	-2.83	-2.78	-2.74	-2.72	-2.70	-2.68	-2.73	-2.70	-2.66	-2.64	-2.63	-2.61
	200	-3.05	-2.96	-2.90	-2.89	-2.86	-2.82	-2.87	-2.82	-2.77	-2.76	-2.74	-2.72	-2.78	-2.74	-2.70	-2.69	-2.68	-2.66
2	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.59	-2.47	-2.39	-2.35	-2.31	-2.27	-2.34	-2.27	-2.20	-2.17	-2.15	-2.11	-2.20	-2.15	-2.10	-2.07	-2.05	-2.02
	50	-2.81	-2.74	-2.65	-2.63	-2.58	-2.58	-2.60	-2.55	-2.50	-2.46	-2.45	-2.44	-2.48	-2.45	-2.40	-2.38	-2.37	-2.35
	70	-2.90	-2.81	-2.76	-2.72	-2.69	-2.67	-2.70	-2.65	-2.60	-2.57	-2.57	-2.54	-2.59	-2.55	-2.52	-2.49	-2.49	-2.47
	100	-2.96	-2.88	-2.81	-2.78	-2.75	-2.72	-2.76	-2.71	-2.67	-2.64	-2.63	-2.61	-2.67	-2.62	-2.59	-2.57	-2.55	-2.54
	200	-3.01	-2.94	-2.87	-2.85	-2.83	-2.79	-2.84	-2.79	-2.74	-2.73	-2.71	-2.69	-2.75	-2.70	-2.66	-2.66	-2.65	-2.63
3	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.51	-2.35	-2.19	-2.14	-2.09	-2.02	-2.15	-2.04	-1.96	-1.92	-1.89	-1.84	-1.98	-1.91	-1.85	-1.81	-1.78	-1.75
	50	-2.72	-2.62	-2.54	-2.52	-2.48	-2.46	-2.49	-2.43	-2.37	-2.34	-2.34	-2.31	-2.37	-2.33	-2.28	-2.26	-2.25	-2.23
	70	-2.82	-2.77	-2.71	-2.66	-2.63	-2.60	-2.63	-2.57	-2.53	-2.50	-2.49	-2.47	-2.51	-2.48	-2.44	-2.41	-2.41	-2.39
	100	-2.92	-2.83	-2.76	-2.73	-2.71	-2.68	-2.72	-2.67	-2.62	-2.59	-2.58	-2.56	-2.62	-2.58	-2.54	-2.52	-2.51	-2.49
	200	-3.00	-2.94	-2.85	-2.84	-2.82	-2.78	-2.82	-2.77	-2.72	-2.71	-2.69	-2.67	-2.72	-2.68	-2.64	-2.64	-2.62	-2.61
4	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	50	-2.52	-2.44	-2.36	-2.33	-2.30	-2.28	-2.32	-2.24	-2.19	-2.16	-2.14	-2.12	-2.19	-2.13	-2.09	-2.07	-2.06	-2.03
	70	-2.73	-2.65	-2.58	-2.55	-2.52	-2.50	-2.53	-2.46	-2.41	-2.39	-2.37	-2.35	-2.40	-2.36	-2.32	-2.29	-2.29	-2.27
	100	-2.84	-2.78	-2.71	-2.67	-2.65	-2.61	-2.65	-2.60	-2.54	-2.52	-2.51	-2.48	-2.54	-2.50	-2.46	-2.44	-2.43	-2.41
	200	-2.98	-2.90	-2.82	-2.81	-2.78	-2.74	-2.78	-2.73	-2.68	-2.68	-2.66	-2.63	-2.69	-2.64	-2.61	-2.60	-2.59	-2.57

Notes: The critical values are obtained by stochastic simulation. The data generating process is $y_{it} = y_{i,t-1} + \varepsilon_{iyt}$, where $\varepsilon_{iyt} \sim iidN(0, 1)$, with $y_{i,-p} = 0$, and the j^{th} element of the $k \times 1$ vector of additional regressors, x_{it} , is generated as $x_{ijt} = x_{ij,t-1} + \varepsilon_{ixjt}$, where $\varepsilon_{ixjt} \sim iidN(0, 1)$, and $x_{ij,-p} = 0$, $i = 1, 2, \dots, N$; $j = 1, 2, \dots, k$; $t = -p, \dots, T$. Critical values for the case where $k = 0$ are provided in Pesaran (2007). The $CADF_i$ statistic is computed as the t -ratio of the coefficient on $y_{i,t-1}$ of the regression of Δy_{it} on $y_{i,t-1}$, $\mathbf{w}'_{it,p} = (\bar{\mathbf{z}}'_{t-1}; \Delta \bar{\mathbf{z}}'_t, \Delta \bar{\mathbf{z}}'_{t-1}, \dots, \Delta \bar{\mathbf{z}}'_{t-p}; \Delta y'_{t-1}, \dots, \Delta y'_{t-p})$, including an intercept, with $\bar{\mathbf{z}}_t = N^{-1} \sum_{i=1}^N (y_{it}, \mathbf{x}'_{it})'$, and the average of the individual $CADF_i$ is computed as $\bar{CADF} = N^{-1} \sum_{i=1}^N CADF_i$. $(100 \times \alpha)\%$ critical values are obtained as the α quantiles of \bar{CADF} for $\alpha = 0.01, 0.05, 0.1$. Computations are based on 10,000 replications. Where values are not reported, this is due to insufficient degrees of freedom.

Table B2: Critical Values of the Average of Individual Cross-Sectionally Augmented Dickey-Fuller Distributions with k Additional Regressors in the Case of Models with an Intercept and a Linear Trend

		$k = 1$																	
p	(T,N)	1% (CADF)						5% (CADF)						10% (CADF)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	-3.12	-3.02	-2.94	-2.91	-2.86	-2.82	-2.90	-2.85	-2.78	-2.75	-2.73	-2.70	-2.78	-2.74	-2.70	-2.67	-2.65	-2.63
	30	-3.07	-2.99	-2.92	-2.89	-2.86	-2.82	-2.89	-2.84	-2.78	-2.76	-2.74	-2.72	-2.79	-2.75	-2.71	-2.69	-2.68	-2.66
	50	-3.06	-2.99	-2.93	-2.88	-2.88	-2.83	-2.89	-2.85	-2.81	-2.77	-2.76	-2.73	-2.80	-2.77	-2.74	-2.71	-2.70	-2.68
	70	-3.07	-3.00	-2.93	-2.89	-2.86	-2.83	-2.91	-2.85	-2.81	-2.79	-2.77	-2.74	-2.82	-2.77	-2.75	-2.72	-2.71	-2.69
	100	-3.06	-3.00	-2.92	-2.90	-2.88	-2.83	-2.91	-2.86	-2.81	-2.79	-2.77	-2.75	-2.83	-2.79	-2.75	-2.73	-2.71	-2.70
	200	-3.06	-2.99	-2.94	-2.91	-2.88	-2.84	-2.91	-2.86	-2.82	-2.79	-2.78	-2.75	-2.83	-2.79	-2.75	-2.74	-2.72	-2.70
1	20	-3.09	-2.99	-2.90	-2.88	-2.85	-2.81	-2.83	-2.76	-2.72	-2.67	-2.65	-2.63	-2.69	-2.64	-2.60	-2.57	-2.56	-2.53
	30	-3.06	-2.96	-2.89	-2.88	-2.83	-2.80	-2.85	-2.79	-2.74	-2.71	-2.70	-2.67	-2.73	-2.69	-2.65	-2.63	-2.62	-2.60
	50	-3.06	-2.96	-2.90	-2.87	-2.86	-2.82	-2.87	-2.82	-2.78	-2.74	-2.73	-2.71	-2.77	-2.73	-2.70	-2.67	-2.67	-2.65
	70	-3.05	-2.99	-2.90	-2.88	-2.84	-2.82	-2.89	-2.83	-2.78	-2.76	-2.75	-2.73	-2.80	-2.75	-2.72	-2.70	-2.69	-2.67
	100	-3.06	-2.99	-2.91	-2.89	-2.87	-2.82	-2.90	-2.85	-2.80	-2.77	-2.76	-2.73	-2.81	-2.77	-2.73	-2.71	-2.70	-2.68
	200	-3.06	-2.99	-2.93	-2.90	-2.87	-2.83	-2.90	-2.86	-2.81	-2.78	-2.77	-2.75	-2.82	-2.78	-2.75	-2.73	-2.71	-2.70
2	20	-2.88	-2.74	-2.64	-2.60	-2.56	-2.52	-2.57	-2.49	-2.42	-2.38	-2.37	-2.34	-2.42	-2.36	-2.31	-2.27	-2.26	-2.24
	30	-2.93	-2.81	-2.75	-2.72	-2.68	-2.64	-2.68	-2.62	-2.57	-2.55	-2.53	-2.50	-2.57	-2.51	-2.47	-2.46	-2.45	-2.43
	50	-2.96	-2.89	-2.83	-2.79	-2.78	-2.74	-2.78	-2.73	-2.70	-2.66	-2.65	-2.62	-2.68	-2.64	-2.61	-2.58	-2.57	-2.55
	70	-3.00	-2.94	-2.84	-2.83	-2.80	-2.77	-2.83	-2.77	-2.72	-2.71	-2.69	-2.67	-2.73	-2.69	-2.66	-2.64	-2.62	-2.61
	100	-3.03	-2.96	-2.88	-2.85	-2.83	-2.79	-2.85	-2.81	-2.76	-2.74	-2.71	-2.70	-2.76	-2.72	-2.69	-2.67	-2.65	-2.64
	200	-3.03	-2.97	-2.91	-2.88	-2.85	-2.81	-2.88	-2.83	-2.79	-2.77	-2.75	-2.72	-2.79	-2.76	-2.73	-2.71	-2.70	-2.68
3	20	-2.96	-2.80	-2.65	-2.58	-2.50	-2.42	-2.57	-2.45	-2.36	-2.31	-2.27	-2.22	-2.37	-2.29	-2.21	-2.18	-2.15	-2.11
	30	-2.84	-2.76	-2.66	-2.63	-2.62	-2.56	-2.60	-2.52	-2.48	-2.46	-2.43	-2.41	-2.48	-2.41	-2.38	-2.37	-2.34	-2.33
	50	-2.94	-2.86	-2.81	-2.75	-2.74	-2.71	-2.73	-2.70	-2.65	-2.61	-2.61	-2.58	-2.63	-2.60	-2.57	-2.54	-2.53	-2.51
	70	-2.98	-2.94	-2.83	-2.81	-2.78	-2.76	-2.81	-2.74	-2.71	-2.68	-2.66	-2.64	-2.71	-2.66	-2.63	-2.61	-2.60	-2.58
	100	-3.00	-2.94	-2.86	-2.84	-2.82	-2.78	-2.84	-2.79	-2.74	-2.72	-2.70	-2.68	-2.74	-2.71	-2.67	-2.65	-2.64	-2.62
	200	-3.03	-2.96	-2.90	-2.87	-2.85	-2.81	-2.88	-2.82	-2.78	-2.76	-2.75	-2.72	-2.78	-2.75	-2.72	-2.70	-2.69	-2.67
4	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.68	-2.56	-2.47	-2.43	-2.40	-2.35	-2.41	-2.34	-2.29	-2.26	-2.24	-2.20	-2.28	-2.23	-2.18	-2.16	-2.14	-2.12
	50	-2.83	-2.76	-2.70	-2.67	-2.65	-2.62	-2.66	-2.59	-2.56	-2.52	-2.51	-2.48	-2.54	-2.50	-2.46	-2.44	-2.43	-2.40
	70	-2.93	-2.86	-2.77	-2.75	-2.72	-2.70	-2.75	-2.68	-2.64	-2.62	-2.60	-2.58	-2.65	-2.59	-2.56	-2.55	-2.53	-2.52
	100	-2.96	-2.91	-2.83	-2.81	-2.78	-2.74	-2.79	-2.75	-2.70	-2.68	-2.66	-2.64	-2.70	-2.66	-2.62	-2.61	-2.59	-2.58
	200	-3.02	-2.94	-2.88	-2.85	-2.83	-2.79	-2.86	-2.81	-2.76	-2.74	-2.73	-2.70	-2.77	-2.73	-2.70	-2.68	-2.67	-2.65
		$k = 2$																	
p	(T,N)	1% (CADF)						5% (CADF)						10% (CADF)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	-3.26	-3.15	-3.09	-3.04	-3.00	-2.97	-3.01	-2.95	-2.89	-2.86	-2.84	-2.82	-2.89	-2.84	-2.79	-2.77	-2.75	-2.73
	30	-3.25	-3.15	-3.07	-3.03	-3.02	-2.97	-3.04	-2.98	-2.93	-2.90	-2.88	-2.86	-2.93	-2.89	-2.84	-2.82	-2.81	-2.79
	50	-3.23	-3.16	-3.09	-3.06	-3.03	-3.01	-3.05	-3.01	-2.96	-2.94	-2.92	-2.90	-2.97	-2.93	-2.89	-2.87	-2.86	-2.84
	70	-3.25	-3.17	-3.10	-3.06	-3.04	-3.01	-3.08	-3.03	-2.98	-2.95	-2.93	-2.92	-2.99	-2.94	-2.91	-2.89	-2.88	-2.86
	100	-3.25	-3.17	-3.11	-3.08	-3.06	-3.02	-3.09	-3.04	-2.99	-2.96	-2.95	-2.93	-3.00	-2.96	-2.92	-2.90	-2.89	-2.88
	200	-3.25	-3.18	-3.12	-3.08	-3.06	-3.03	-3.09	-3.04	-3.00	-2.97	-2.96	-2.94	-3.01	-2.97	-2.94	-2.91	-2.91	-2.89
1	20	-3.18	-3.06	-2.97	-2.89	-2.87	-2.84	-2.88	-2.78	-2.71	-2.68	-2.66	-2.64	-2.71	-2.64	-2.58	-2.57	-2.55	-2.52
	30	-3.16	-3.08	-3.00	-2.95	-2.93	-2.89	-2.92	-2.87	-2.82	-2.78	-2.77	-2.75	-2.81	-2.76	-2.72	-2.69	-2.68	-2.67
	50	-3.21	-3.11	-3.04	-3.01	-2.99	-2.96	-3.00	-2.95	-2.90	-2.87	-2.86	-2.84	-2.90	-2.86	-2.82	-2.80	-2.79	-2.77
	70	-3.21	-3.14	-3.07	-3.03	-3.01	-2.98	-3.06	-2.99	-2.94	-2.91	-2.89	-2.87	-2.95	-2.90	-2.86	-2.84	-2.83	-2.82
	100	-3.24	-3.15	-3.09	-3.05	-3.03	-3.00	-3.06	-3.01	-2.96	-2.93	-2.92	-2.90	-2.96	-2.93	-2.89	-2.87	-2.86	-2.85
	200	-3.25	-3.17	-3.12	-3.08	-3.05	-3.02	-3.09	-3.03	-2.99	-2.96	-2.95	-2.93	-3.00	-2.95	-2.92	-2.90	-2.89	-2.87
2	20	-3.43	-3.17	-2.92	-2.82	-2.75	-2.61	-2.77	-2.64	-2.51	-2.45	-2.41	-2.34	-2.52	-2.42	-2.33	-2.28	-2.25	-2.21
	30	-2.92	-2.86	-2.76	-2.71	-2.69	-2.64	-2.68	-2.63	-2.55	-2.53	-2.51	-2.49	-2.56	-2.51	-2.46	-2.43	-2.42	-2.40
	50	-3.09	-3.00	-2.93	-2.90	-2.87	-2.84	-2.88	-2.82	-2.77	-2.75	-2.73	-2.71	-2.77	-2.72	-2.68	-2.67	-2.65	-2.64
	70	-3.14	-3.06	-3.00	-2.95	-2.93	-2.90	-2.96	-2.90	-2.85	-2.82	-2.81	-2.79	-2.85	-2.81	-2.77	-2.75	-2.74	-2.72
	100	-3.18	-3.10	-3.04	-3.00	-2.98	-2.95	-3.01	-2.95	-2.90	-2.88	-2.86	-2.84	-2.91	-2.87	-2.83	-2.81	-2.80	-2.78
	200	-3.23	-3.14	-3.08	-3.04	-3.02	-2.99	-3.05	-3.00	-2.96	-2.93	-2.92	-2.90	-2.96	-2.93	-2.89	-2.87	-2.86	-2.85
3	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.81	-2.70	-2.59	-2.54	-2.50	-2.48	-2.53	-2.45	-2.37	-2.34	-2.31	-2.30	-2.38	-2.32	-2.26	-2.24	-2.21	-2.20
	50	-3.00	-2.92	-2.85	-2.83	-2.80	-2.76	-2.80	-2.74	-2.69	-2.66	-2.64	-2.63	-2.68	-2.64	-2.59	-2.57	-2.56	-2.55
	70	-3.09	-3.03	-2.95	-2.91	-2.88	-2.86	-2.91	-2.85	-2.80	-2.77	-2.76	-2.74	-2.81	-2.75	-2.71	-2.70	-2.68	-2.67
	100	-3.16	-3.07	-3.02	-2.97	-2.96	-2.93	-2.97	-2.92	-2.87	-2.84	-2.83	-2.81	-2.87	-2.83	-2.80	-2.78	-2.76	-2.75
	200	-3.21	-3.14	-3.07	-3.03	-3.01	-2.99	-3.04	-2.99	-2.95	-2.92	-2.90	-2.89	-2.95	-2.91	-2.88	-2.85	-2.85	-2.83
4	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.63	-2.47	-2.34	-2.28	-2.21	-2.18	-2.31	-2.18	-2.10	-2.06	-2.02	-1.98	-2.13	-2.04	-1.98	-1.94	-1.92	-1.89
	50	-2.86	-2.79	-2.71	-2.69	-2.66	-2.61	-2.65	-2.59	-2.54	-2.52	-2.49	-2.47	-2.54	-2.49	-2.45	-2.42	-2.41	-2.39
	70	-2.99	-2.93	-2.87	-2.83	-2.79	-2.77	-2.81	-2.75	-2.70	-2.67	-2.66	-2.64	-2.71	-2.66	-2.62	-2.60	-2.58	-2.57
	100	-3.10	-3.02	-2.95	-2.92	-2.91	-2.87	-2.91	-2.85	-2.81	-2.79	-2.77	-2.75	-2.80	-2.76	-2.73	-2.71	-2.70	-2.68
	200	-3.19	-3.11	-3.05	-3.01	-2.98	-2.96	-3.01	-2.96	-2.92	-2.89	-2.88	-2.86	-2.92	-2.88	-2.85	-2.83	-2.81	-2.80

Table B2 Continued

		$k = 3$																	
		1% (\overline{CADF})						5% (\overline{CADF})						10% (\overline{CADF})					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	-3.39	-3.29	-3.19	-3.13	-3.09	-3.08	-3.10	-3.03	-2.97	-2.94	-2.92	-2.90	-2.96	-2.90	-2.87	-2.84	-2.82	-2.80
	30	-3.38	-3.26	-3.20	-3.16	-3.14	-3.10	-3.16	-3.09	-3.04	-3.01	-3.00	-2.96	-3.05	-2.99	-2.95	-2.93	-2.91	-2.89
	50	-3.39	-3.31	-3.25	-3.20	-3.18	-3.15	-3.20	-3.15	-3.11	-3.07	-3.06	-3.04	-3.11	-3.07	-3.03	-3.01	-2.99	-2.98
	70	-3.41	-3.32	-3.25	-3.24	-3.19	-3.17	-3.23	-3.17	-3.13	-3.10	-3.09	-3.07	-3.13	-3.09	-3.06	-3.04	-3.02	-3.01
	100	-3.41	-3.33	-3.28	-3.24	-3.22	-3.18	-3.25	-3.20	-3.15	-3.12	-3.10	-3.09	-3.16	-3.12	-3.08	-3.06	-3.04	-3.03
1	20	-3.43	-3.34	-3.30	-3.26	-3.23	-3.20	-3.27	-3.21	-3.17	-3.14	-3.13	-3.11	-3.18	-3.13	-3.10	-3.09	-3.07	-3.05
	30	-3.43	-3.26	-3.05	-2.99	-2.93	-2.86	-2.97	-2.85	-2.74	-2.68	-2.65	-2.61	-2.74	-2.66	-2.59	-2.53	-2.51	-2.48
	50	-3.23	-3.13	-3.03	-3.00	-2.97	-2.93	-2.97	-2.90	-2.84	-2.81	-2.79	-2.77	-2.83	-2.79	-2.74	-2.72	-2.70	-2.68
	70	-3.31	-3.22	-3.17	-3.12	-3.09	-3.07	-3.11	-3.05	-3.00	-2.98	-2.96	-2.94	-3.00	-2.96	-2.92	-2.90	-2.89	-2.87
	100	-3.34	-3.26	-3.21	-3.18	-3.14	-3.11	-3.16	-3.11	-3.06	-3.03	-3.02	-3.00	-3.06	-3.02	-2.99	-2.96	-2.95	-2.94
2	20	-3.37	-3.29	-3.23	-3.20	-3.18	-3.15	-3.20	-3.15	-3.10	-3.08	-3.05	-3.04	-3.11	-3.07	-3.03	-3.01	-2.99	-2.98
	30	-3.42	-3.34	-3.27	-3.24	-3.22	-3.19	-3.25	-3.19	-3.15	-3.13	-3.10	-3.08	-3.16	-3.11	-3.08	-3.06	-3.05	-3.03
	50	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	70	-2.92	-2.77	-2.67	-2.62	-2.60	-2.55	-2.62	-2.54	-2.45	-2.43	-2.41	-2.38	-2.47	-2.41	-2.34	-2.32	-2.31	-2.29
	100	-3.17	-3.06	-2.99	-2.96	-2.92	-2.90	-2.94	-2.87	-2.83	-2.80	-2.78	-2.76	-2.82	-2.78	-2.73	-2.71	-2.70	-2.68
3	20	-3.24	-3.16	-3.09	-3.07	-3.03	-3.00	-3.04	-2.99	-2.94	-2.91	-2.90	-2.88	-2.95	-2.90	-2.86	-2.84	-2.83	-2.81
	30	-3.30	-3.23	-3.16	-3.13	-3.10	-3.07	-3.12	-3.07	-3.02	-3.00	-2.97	-2.96	-3.02	-2.99	-2.95	-2.93	-2.91	-2.90
	50	-3.37	-3.29	-3.24	-3.20	-3.18	-3.15	-3.21	-3.15	-3.11	-3.09	-3.07	-3.05	-3.12	-3.07	-3.04	-3.03	-3.01	-2.99
	70	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	100	-2.85	-2.75	-2.53	-2.45	-2.36	-2.31	-2.46	-2.36	-2.22	-2.18	-2.14	-2.10	-2.27	-2.17	-2.09	-2.05	-2.02	-1.99
4	20	-3.04	-2.94	-2.86	-2.83	-2.79	-2.76	-2.81	-2.74	-2.68	-2.66	-2.64	-2.61	-2.69	-2.63	-2.59	-2.57	-2.55	-2.53
	30	-3.16	-3.11	-3.02	-2.99	-2.95	-2.93	-2.96	-2.91	-2.86	-2.84	-2.81	-2.79	-2.85	-2.81	-2.78	-2.75	-2.74	-2.72
	50	-3.26	-3.19	-3.12	-3.09	-3.06	-3.03	-3.08	-3.02	-2.97	-2.95	-2.93	-2.91	-2.97	-2.93	-2.89	-2.87	-2.86	-2.85
	70	-3.34	-3.28	-3.22	-3.19	-3.16	-3.13	-3.18	-3.13	-3.09	-3.06	-3.05	-3.03	-3.09	-3.05	-3.02	-3.00	-2.98	-2.97
	100	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
5	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	50	-2.83	-2.72	-2.66	-2.62	-2.58	-2.54	-2.59	-2.53	-2.47	-2.44	-2.42	-2.38	-2.47	-2.42	-2.37	-2.35	-2.33	-2.30
	70	-3.06	-2.97	-2.89	-2.85	-2.82	-2.80	-2.84	-2.78	-2.73	-2.70	-2.68	-2.66	-2.73	-2.67	-2.64	-2.61	-2.60	-2.58
	100	-3.18	-3.11	-3.05	-3.00	-2.99	-2.95	-2.98	-2.94	-2.89	-2.86	-2.84	-2.83	-2.88	-2.85	-2.81	-2.78	-2.77	-2.76
200	-3.32	-3.24	-3.18	-3.15	-3.12	-3.10	-3.15	-3.09	-3.05	-3.03	-3.01	-2.99	-3.05	-3.01	-2.98	-2.96	-2.94	-2.93	

Notes: The critical values are obtained by stochastic simulation. The data generating process is $y_{it} = y_{i,t-1} + \varepsilon_{iyt}$, where $\varepsilon_{iyt} \sim iidN(0, 1)$, with $y_{i,-p} = 0$, and the j^{th} element of the $k \times 1$ vector of additional regressors, x_{it} , is generated as $x_{ijt} = x_{ij,t-1} + \varepsilon_{ixjt}$, where $\varepsilon_{ixjt} \sim iidN(0, 1)$ and $x_{ij,-p} = 0$, $i = 1, 2, \dots, N$; $j = 1, 2, \dots, k$; $t = -p, \dots, T$. The relevant critical values for the case where $k = 0$ is provided in Pesaran (2007). The \overline{CADF}_i statistic is computed as the t -ratio of the coefficient on $y_{i,t-1}$ of the regression of Δy_{it} on $y_{i,t-1}$, $\mathbf{w}'_{it,p} = (\bar{\mathbf{z}}'_{t-1}; \Delta \bar{\mathbf{z}}'_t, \Delta \bar{\mathbf{z}}'_{t-1}, \dots, \Delta \bar{\mathbf{z}}'_{t-p}; \Delta y'_{i,t-1}, \dots, \Delta y'_{i,t-p})$, including an intercept and a linear trend, with $\bar{\mathbf{z}}_t = N^{-1} \sum_{i=1}^N (y_{it}, \mathbf{x}'_{it})'$, and the average of the individual statistics is computed as $\overline{CADF} = N^{-1} \sum_{i=1}^N \overline{CADF}_i$. $(100 \times \alpha)\%$ critical values are obtained as the α quantiles of \overline{CADF} for $\alpha = 0.01, 0.05, 0.1$. Computations are based on 10,000 replications. Where values are not reported, this is due to insufficient degrees of freedom.

Table B3: Critical Values of the Average of Individual Cross-Sectionally Augmented Sargan-Barghava Distributions with k Additional Regressors in the Case of Models with an Intercept Only

		$k = 0$																	
p	(T,N)	1% (CSB)						5% (CSB)						10% (CSB)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	0.259	0.292	0.330	0.347	0.360	0.377	0.315	0.343	0.373	0.389	0.401	0.420	0.348	0.374	0.398	0.412	0.423	0.440
	30	0.260	0.298	0.335	0.355	0.372	0.398	0.315	0.344	0.377	0.393	0.408	0.429	0.348	0.373	0.401	0.414	0.427	0.445
	50	0.259	0.295	0.334	0.358	0.378	0.409	0.313	0.343	0.376	0.394	0.410	0.434	0.345	0.372	0.400	0.415	0.428	0.448
	70	0.257	0.293	0.333	0.358	0.377	0.411	0.311	0.344	0.376	0.395	0.410	0.434	0.343	0.372	0.400	0.416	0.429	0.448
	100	0.257	0.293	0.335	0.358	0.377	0.412	0.311	0.343	0.375	0.395	0.410	0.435	0.344	0.371	0.400	0.415	0.428	0.448
	200	0.257	0.293	0.333	0.356	0.378	0.411	0.310	0.342	0.375	0.394	0.410	0.435	0.343	0.371	0.399	0.415	0.428	0.449
1	20	0.220	0.238	0.258	0.266	0.271	0.274	0.268	0.287	0.305	0.315	0.321	0.330	0.296	0.314	0.331	0.340	0.346	0.355
	30	0.236	0.264	0.291	0.303	0.312	0.323	0.284	0.308	0.332	0.344	0.354	0.368	0.313	0.334	0.355	0.365	0.375	0.387
	50	0.247	0.279	0.312	0.330	0.346	0.368	0.295	0.323	0.351	0.366	0.379	0.398	0.325	0.349	0.373	0.385	0.397	0.413
	70	0.248	0.282	0.319	0.340	0.358	0.385	0.299	0.328	0.358	0.375	0.389	0.410	0.330	0.355	0.381	0.395	0.406	0.424
	100	0.251	0.285	0.325	0.346	0.365	0.395	0.302	0.332	0.363	0.381	0.395	0.418	0.334	0.360	0.387	0.401	0.413	0.431
	200	0.253	0.290	0.328	0.352	0.372	0.404	0.306	0.337	0.369	0.387	0.403	0.427	0.337	0.365	0.393	0.408	0.420	0.440
2	20	0.186	0.197	0.205	0.208	0.210	0.211	0.235	0.248	0.257	0.263	0.265	0.269	0.264	0.276	0.287	0.294	0.297	0.302
	30	0.219	0.239	0.255	0.262	0.266	0.272	0.266	0.285	0.303	0.312	0.318	0.325	0.294	0.312	0.328	0.337	0.344	0.352
	50	0.241	0.268	0.296	0.309	0.320	0.334	0.288	0.311	0.337	0.349	0.361	0.375	0.317	0.337	0.359	0.370	0.380	0.393
	70	0.245	0.276	0.311	0.328	0.344	0.363	0.294	0.322	0.349	0.364	0.377	0.395	0.325	0.349	0.371	0.384	0.395	0.410
	100	0.249	0.283	0.321	0.340	0.356	0.383	0.300	0.329	0.358	0.375	0.388	0.409	0.331	0.355	0.380	0.394	0.405	0.422
	200	0.252	0.289	0.328	0.349	0.368	0.399	0.304	0.336	0.367	0.384	0.399	0.423	0.336	0.363	0.390	0.404	0.417	0.436
3	20	0.146	0.151	0.155	0.158	0.158	0.160	0.190	0.198	0.203	0.208	0.207	0.210	0.217	0.226	0.233	0.237	0.238	0.241
	30	0.191	0.203	0.214	0.218	0.220	0.224	0.235	0.250	0.264	0.268	0.273	0.277	0.261	0.276	0.289	0.296	0.300	0.305
	50	0.225	0.246	0.270	0.279	0.288	0.293	0.270	0.291	0.311	0.322	0.330	0.341	0.297	0.315	0.333	0.342	0.351	0.362
	70	0.237	0.263	0.292	0.307	0.319	0.332	0.283	0.306	0.332	0.344	0.355	0.369	0.311	0.331	0.352	0.364	0.373	0.386
	100	0.243	0.274	0.309	0.327	0.340	0.362	0.292	0.318	0.346	0.360	0.373	0.391	0.322	0.343	0.367	0.380	0.390	0.405
	200	0.250	0.286	0.322	0.344	0.362	0.391	0.301	0.330	0.361	0.378	0.392	0.415	0.332	0.357	0.383	0.398	0.409	0.428
4	20	0.114	0.117	0.119	0.121	0.121	0.122	0.153	0.157	0.160	0.163	0.162	0.164	0.178	0.183	0.189	0.191	0.191	0.194
	30	0.167	0.173	0.180	0.184	0.185	0.187	0.212	0.222	0.232	0.236	0.237	0.240	0.238	0.249	0.260	0.264	0.267	0.271
	50	0.214	0.232	0.250	0.255	0.261	0.264	0.259	0.276	0.293	0.302	0.308	0.316	0.285	0.301	0.318	0.325	0.332	0.341
	70	0.230	0.254	0.279	0.290	0.300	0.310	0.276	0.298	0.321	0.331	0.341	0.352	0.303	0.322	0.342	0.352	0.361	0.372
	100	0.241	0.269	0.300	0.318	0.329	0.346	0.288	0.313	0.340	0.352	0.363	0.380	0.316	0.337	0.360	0.372	0.381	0.396
	200	0.249	0.283	0.319	0.340	0.358	0.385	0.299	0.328	0.358	0.374	0.389	0.410	0.330	0.355	0.381	0.394	0.405	0.423
		$k = 1$																	
p	(T,N)	1% (CSB)						5% (CSB)						10% (CSB)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	0.241	0.270	0.300	0.311	0.323	0.335	0.294	0.319	0.345	0.358	0.368	0.382	0.326	0.349	0.372	0.382	0.392	0.406
	30	0.248	0.284	0.319	0.338	0.353	0.373	0.303	0.329	0.361	0.376	0.388	0.407	0.334	0.358	0.385	0.397	0.409	0.424
	50	0.251	0.288	0.327	0.348	0.367	0.395	0.305	0.336	0.368	0.384	0.400	0.422	0.338	0.365	0.391	0.405	0.418	0.437
	70	0.253	0.288	0.328	0.352	0.370	0.403	0.307	0.339	0.371	0.389	0.403	0.427	0.338	0.368	0.394	0.409	0.421	0.441
	100	0.254	0.290	0.329	0.353	0.374	0.406	0.307	0.340	0.371	0.389	0.405	0.430	0.340	0.368	0.395	0.411	0.424	0.443
	200	0.254	0.291	0.332	0.354	0.376	0.410	0.307	0.340	0.373	0.392	0.408	0.433	0.339	0.369	0.397	0.413	0.426	0.446
1	20	0.182	0.195	0.205	0.208	0.214	0.217	0.225	0.239	0.252	0.257	0.261	0.267	0.253	0.266	0.278	0.284	0.288	0.293
	30	0.211	0.236	0.255	0.265	0.273	0.277	0.257	0.279	0.298	0.307	0.315	0.324	0.284	0.303	0.321	0.330	0.337	0.346
	50	0.233	0.264	0.293	0.310	0.322	0.338	0.281	0.307	0.331	0.346	0.357	0.373	0.310	0.332	0.354	0.366	0.376	0.390
	70	0.240	0.273	0.308	0.325	0.341	0.365	0.289	0.318	0.346	0.361	0.374	0.393	0.320	0.345	0.368	0.381	0.392	0.408
	100	0.245	0.279	0.315	0.337	0.356	0.383	0.296	0.325	0.354	0.372	0.386	0.408	0.327	0.352	0.377	0.391	0.403	0.421
	200	0.249	0.286	0.324	0.346	0.367	0.399	0.301	0.333	0.366	0.383	0.398	0.422	0.333	0.362	0.389	0.404	0.416	0.436
2	20	0.130	0.135	0.139	0.140	0.143	0.144	0.170	0.176	0.180	0.182	0.185	0.187	0.194	0.201	0.207	0.210	0.211	0.214
	30	0.180	0.192	0.203	0.209	0.211	0.213	0.223	0.237	0.248	0.254	0.258	0.262	0.249	0.263	0.275	0.280	0.284	0.289
	50	0.219	0.242	0.265	0.276	0.283	0.291	0.265	0.285	0.306	0.315	0.324	0.334	0.292	0.311	0.329	0.338	0.345	0.355
	70	0.230	0.260	0.291	0.305	0.316	0.332	0.279	0.305	0.329	0.342	0.353	0.366	0.308	0.330	0.351	0.362	0.371	0.384
	100	0.239	0.273	0.305	0.324	0.341	0.363	0.290	0.316	0.343	0.360	0.372	0.391	0.319	0.343	0.366	0.379	0.389	0.406
	200	0.248	0.282	0.322	0.343	0.361	0.392	0.298	0.330	0.361	0.378	0.392	0.416	0.329	0.357	0.384	0.398	0.410	0.429
3	20	0.085	0.087	0.088	0.089	0.091	0.090	0.112	0.116	0.118	0.118	0.119	0.121	0.132	0.136	0.139	0.140	0.141	0.142
	30	0.141	0.148	0.154	0.157	0.159	0.161	0.180	0.188	0.197	0.200	0.202	0.203	0.204	0.213	0.222	0.225	0.227	0.229
	50	0.196	0.212	0.229	0.236	0.241	0.244	0.238	0.254	0.271	0.278	0.283	0.290	0.264	0.278	0.293	0.300	0.306	0.313
	70	0.215	0.240	0.264	0.275	0.285	0.293	0.261	0.282	0.304	0.314	0.322	0.332	0.289	0.307	0.325	0.335	0.342	0.352
	100	0.231	0.260	0.289	0.306	0.317	0.335	0.278	0.302	0.327	0.340	0.351	0.367	0.305	0.327	0.348	0.360	0.369	0.383
	200	0.243	0.278	0.313	0.335	0.351	0.380	0.293	0.323	0.352	0.368	0.382	0.404	0.323	0.349	0.375	0.389	0.400	0.417
4	20	0.048	0.050	0.051	0.051	0.051	0.052	0.066	0.068	0.068	0.069	0.069	0.070	0.079	0.081	0.082	0.083	0.084	0.084
	30	0.109	0.113	0.118	0.119	0.119	0.121	0.142	0.147	0.153	0.155	0.156	0.157	0.165	0.170	0.176	0.179	0.180	0.181
	50	0.176	0.187	0.199	0.204	0.207	0.208	0.217	0.230	0.241	0.247	0.250	0.255	0.242	0.254	0.266	0.270	0.275	0.280
	70	0.203	0.224	0.243	0.250	0.257	0.262	0.248	0.266	0.284	0.291	0.298	0.305	0.274	0.290	0.306	0.314	0.320	0.327
	100	0.224	0.250	0.275	0.290	0.301	0.312	0.269	0.292	0.315	0.327	0.335	0.349	0.296	0.316	0.336	0.346	0.354	0.366
	200	0.242	0.274	0.310	0.329	0.344	0.370	0.289	0.319	0.347	0.363	0.375	0.396	0.319	0.345	0.369	0.383	0.393	0.410

Table B3 Continued

		$k = 2$																	
p	(T,N)	1% (CSB)						5% (CSB)						10% (CSB)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	0.224	0.250	0.272	0.281	0.291	0.300	0.275	0.298	0.319	0.329	0.336	0.347	0.304	0.326	0.345	0.354	0.362	0.372
	30	0.235	0.270	0.303	0.320	0.332	0.350	0.289	0.316	0.344	0.358	0.370	0.386	0.320	0.344	0.367	0.380	0.390	0.404
	50	0.247	0.281	0.319	0.339	0.357	0.383	0.299	0.328	0.359	0.376	0.390	0.411	0.329	0.356	0.382	0.396	0.408	0.425
	70	0.249	0.284	0.322	0.344	0.363	0.394	0.301	0.332	0.364	0.381	0.396	0.419	0.334	0.360	0.387	0.403	0.414	0.433
	100	0.251	0.286	0.328	0.350	0.370	0.401	0.304	0.335	0.368	0.385	0.402	0.426	0.336	0.363	0.391	0.406	0.420	0.439
	200	0.250	0.289	0.330	0.352	0.374	0.407	0.306	0.338	0.372	0.389	0.406	0.431	0.338	0.367	0.395	0.411	0.424	0.444
1	20	0.147	0.157	0.164	0.166	0.168	0.171	0.186	0.197	0.205	0.208	0.212	0.214	0.211	0.221	0.230	0.234	0.237	0.240
	30	0.189	0.207	0.223	0.232	0.237	0.243	0.232	0.249	0.265	0.273	0.279	0.286	0.258	0.274	0.289	0.296	0.301	0.308
	50	0.221	0.249	0.277	0.290	0.301	0.313	0.266	0.292	0.314	0.326	0.335	0.349	0.294	0.317	0.336	0.347	0.355	0.367
	70	0.233	0.262	0.295	0.313	0.326	0.346	0.280	0.305	0.333	0.348	0.359	0.377	0.309	0.331	0.355	0.368	0.377	0.392
	100	0.240	0.272	0.309	0.329	0.346	0.372	0.289	0.318	0.348	0.363	0.377	0.397	0.320	0.344	0.369	0.382	0.394	0.411
	200	0.244	0.282	0.322	0.343	0.362	0.394	0.298	0.330	0.362	0.378	0.394	0.418	0.330	0.357	0.384	0.399	0.411	0.431
2	20	0.086	0.088	0.091	0.092	0.093	0.093	0.114	0.118	0.120	0.121	0.123	0.123	0.133	0.137	0.140	0.142	0.143	0.144
	30	0.144	0.152	0.160	0.164	0.168	0.170	0.183	0.191	0.201	0.205	0.207	0.209	0.207	0.216	0.226	0.229	0.232	0.234
	50	0.198	0.217	0.237	0.246	0.250	0.259	0.241	0.259	0.277	0.284	0.291	0.299	0.267	0.284	0.299	0.307	0.313	0.320
	70	0.216	0.243	0.270	0.283	0.292	0.306	0.263	0.286	0.308	0.320	0.329	0.341	0.291	0.311	0.330	0.341	0.349	0.360
	100	0.231	0.261	0.293	0.311	0.326	0.344	0.279	0.304	0.331	0.345	0.357	0.374	0.308	0.330	0.353	0.365	0.375	0.389
	200	0.243	0.279	0.316	0.337	0.354	0.384	0.294	0.324	0.355	0.371	0.385	0.408	0.324	0.351	0.377	0.391	0.403	0.421
3	20	0.038	0.039	0.040	0.041	0.040	0.041	0.052	0.054	0.054	0.055	0.055	0.055	0.063	0.064	0.065	0.066	0.066	0.067
	30	0.101	0.105	0.109	0.111	0.112	0.113	0.131	0.136	0.141	0.142	0.144	0.145	0.151	0.156	0.162	0.163	0.165	0.166
	50	0.168	0.182	0.195	0.200	0.201	0.205	0.207	0.221	0.232	0.238	0.243	0.247	0.231	0.244	0.255	0.261	0.265	0.269
	70	0.198	0.218	0.238	0.248	0.253	0.263	0.240	0.259	0.276	0.286	0.292	0.300	0.266	0.282	0.298	0.306	0.312	0.320
	100	0.219	0.245	0.273	0.286	0.297	0.310	0.263	0.285	0.309	0.321	0.331	0.344	0.290	0.311	0.330	0.340	0.349	0.361
	200	0.238	0.273	0.307	0.326	0.342	0.368	0.286	0.316	0.345	0.359	0.373	0.393	0.316	0.342	0.366	0.379	0.390	0.407
4	20	0.005	0.005	0.006	0.006	0.006	0.006	0.008	0.008	0.008	0.008	0.008	0.008	0.010	0.010	0.010	0.010	0.010	0.010
	30	0.065	0.068	0.069	0.070	0.070	0.071	0.087	0.089	0.092	0.093	0.093	0.093	0.102	0.105	0.108	0.109	0.109	0.110
	50	0.142	0.151	0.158	0.161	0.163	0.165	0.178	0.187	0.195	0.199	0.202	0.204	0.200	0.210	0.218	0.221	0.225	0.227
	70	0.179	0.195	0.211	0.216	0.221	0.227	0.220	0.235	0.249	0.256	0.260	0.266	0.245	0.258	0.272	0.278	0.282	0.288
	100	0.208	0.229	0.253	0.263	0.273	0.282	0.250	0.270	0.292	0.301	0.309	0.319	0.276	0.295	0.313	0.321	0.328	0.338
	200	0.234	0.267	0.299	0.317	0.332	0.355	0.281	0.309	0.337	0.351	0.363	0.382	0.311	0.335	0.358	0.371	0.381	0.396

		$k = 3$																	
p	(T,N)	1% (CSB)						5% (CSB)						10% (CSB)					
		20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	0.205	0.228	0.245	0.253	0.260	0.268	0.254	0.274	0.292	0.301	0.307	0.316	0.284	0.301	0.318	0.326	0.331	0.341
	30	0.227	0.257	0.286	0.301	0.313	0.327	0.277	0.302	0.328	0.340	0.350	0.365	0.307	0.329	0.351	0.362	0.371	0.384
	50	0.241	0.275	0.310	0.330	0.347	0.371	0.292	0.320	0.350	0.365	0.380	0.399	0.323	0.348	0.374	0.386	0.398	0.415
	70	0.246	0.280	0.317	0.341	0.356	0.386	0.297	0.329	0.358	0.375	0.389	0.412	0.328	0.355	0.382	0.395	0.407	0.426
	100	0.249	0.283	0.322	0.346	0.365	0.396	0.299	0.332	0.364	0.381	0.397	0.420	0.332	0.360	0.386	0.402	0.415	0.433
	200	0.250	0.288	0.330	0.350	0.371	0.405	0.303	0.337	0.369	0.388	0.404	0.428	0.336	0.366	0.394	0.408	0.421	0.442
1	20	0.116	0.121	0.128	0.129	0.131	0.131	0.150	0.156	0.163	0.165	0.165	0.168	0.172	0.178	0.184	0.187	0.188	0.191
	30	0.169	0.183	0.195	0.201	0.206	0.209	0.207	0.222	0.234	0.241	0.246	0.250	0.232	0.245	0.257	0.264	0.268	0.272
	50	0.209	0.233	0.259	0.271	0.279	0.291	0.253	0.274	0.296	0.307	0.315	0.326	0.280	0.298	0.318	0.327	0.334	0.345
	70	0.222	0.254	0.283	0.300	0.311	0.330	0.270	0.296	0.321	0.334	0.344	0.361	0.299	0.321	0.343	0.353	0.363	0.377
	100	0.234	0.264	0.299	0.320	0.336	0.359	0.282	0.309	0.339	0.354	0.367	0.386	0.312	0.335	0.360	0.374	0.384	0.400
	200	0.243	0.280	0.317	0.340	0.359	0.390	0.295	0.326	0.358	0.375	0.389	0.413	0.326	0.355	0.380	0.395	0.407	0.426
2	20	0.049	0.050	0.052	0.052	0.052	0.052	0.066	0.068	0.070	0.070	0.070	0.071	0.079	0.081	0.083	0.083	0.083	0.084
	30	0.113	0.119	0.127	0.127	0.131	0.132	0.146	0.153	0.159	0.162	0.163	0.165	0.167	0.174	0.180	0.182	0.184	0.186
	50	0.178	0.195	0.211	0.218	0.223	0.228	0.218	0.233	0.248	0.255	0.260	0.266	0.243	0.257	0.271	0.277	0.281	0.288
	70	0.205	0.229	0.251	0.262	0.269	0.281	0.248	0.269	0.289	0.298	0.306	0.317	0.276	0.293	0.310	0.318	0.326	0.336
	100	0.223	0.249	0.280	0.297	0.309	0.326	0.268	0.292	0.317	0.331	0.342	0.357	0.296	0.317	0.339	0.350	0.360	0.373
	200	0.239	0.273	0.310	0.332	0.347	0.375	0.289	0.319	0.348	0.365	0.378	0.400	0.319	0.346	0.370	0.385	0.396	0.413
3	20	0.005	0.005	0.005	0.005	0.005	0.005	0.007	0.007	0.008	0.008	0.008	0.008	0.009	0.010	0.010	0.010	0.010	0.010
	30	0.067	0.069	0.072	0.071	0.074	0.074	0.088	0.091	0.093	0.094	0.095	0.096	0.102	0.106	0.109	0.110	0.110	0.111
	50	0.144	0.153	0.162	0.168	0.170	0.172	0.178	0.189	0.198	0.202	0.205	0.209	0.200	0.210	0.219	0.222	0.226	0.229
	70	0.180	0.198	0.213	0.222	0.224	0.232	0.220	0.236	0.251	0.257	0.263	0.269	0.244	0.258	0.272	0.278	0.284	0.290
	100	0.206	0.230	0.254	0.267	0.276	0.288	0.249	0.270	0.291	0.301	0.310	0.321	0.275	0.294	0.312	0.321	0.329	0.339
	200	0.232	0.264	0.298	0.318	0.332	0.357	0.280	0.309	0.336	0.351	0.364	0.383	0.308	0.334	0.358	0.370	0.380	0.397
4	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	0.030	0.031	0.031	0.032	0.032	0.032	0.041	0.042	0.043	0.043	0.044	0.044	0.049	0.050	0.051	0.051	0.052	0.052
	50	0.111	0.118	0.122	0.126	0.127	0.128	0.141	0.149	0.154	0.157	0.158	0.160	0.161	0.168	0.174	0.176	0.178	0.180
	70	0.156	0.170	0.181	0.186	0.188	0.192	0.193	0.206	0.217	0.222	0.225	0.230	0.216	0.228	0.238	0.243	0.247	0.251
	100	0.192	0.211	0.232	0.240	0.248	0.256	0.233	0.250	0.268	0.277	0.284	0.290	0.257	0.274	0.289	0.297	0.303	0.310
	200	0.226	0.257	0.289	0.306	0.319	0.341	0.271	0.300	0.325	0.340	0.351	0.368	0.301	0.324	0.347	0.359	0.368	0.383

Notes: The

Where values are not reported, this is due to insufficient degrees of freedom.

Table B4: Critical Values of the Average of Individual Cross-Sectionally Augmented Sargan-Barghava Distributions with k Additional Regressors in the Case of Models with an Intercept and a Linear Trend

		$k = 0$																	
		1% (CSB)						5% (CSB)						10% (CSB)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	0.100	0.108	0.116	0.121	0.124	0.128	0.113	0.120	0.127	0.131	0.134	0.138	0.121	0.128	0.134	0.137	0.139	0.143
	30	0.101	0.110	0.120	0.125	0.129	0.136	0.115	0.123	0.131	0.134	0.138	0.144	0.123	0.130	0.136	0.140	0.143	0.147
	50	0.101	0.111	0.121	0.127	0.132	0.140	0.116	0.124	0.132	0.137	0.141	0.147	0.124	0.131	0.138	0.142	0.145	0.150
	70	0.102	0.111	0.122	0.127	0.133	0.142	0.116	0.124	0.133	0.137	0.142	0.148	0.125	0.132	0.139	0.142	0.146	0.151
	100	0.102	0.112	0.123	0.128	0.134	0.142	0.116	0.125	0.133	0.138	0.142	0.148	0.125	0.132	0.140	0.144	0.147	0.152
	200	0.101	0.112	0.123	0.129	0.134	0.143	0.117	0.125	0.134	0.138	0.143	0.149	0.125	0.133	0.140	0.144	0.147	0.153
1	20	0.091	0.096	0.100	0.101	0.102	0.104	0.104	0.109	0.113	0.115	0.116	0.118	0.111	0.116	0.120	0.122	0.123	0.125
	30	0.098	0.105	0.112	0.114	0.116	0.120	0.110	0.116	0.122	0.125	0.128	0.131	0.118	0.123	0.128	0.131	0.133	0.136
	50	0.100	0.109	0.118	0.122	0.127	0.132	0.114	0.121	0.128	0.132	0.135	0.140	0.122	0.128	0.134	0.137	0.140	0.144
	70	0.101	0.110	0.120	0.125	0.130	0.137	0.115	0.122	0.130	0.134	0.138	0.143	0.123	0.129	0.136	0.139	0.142	0.147
	100	0.101	0.111	0.121	0.127	0.131	0.139	0.115	0.124	0.132	0.136	0.140	0.146	0.124	0.131	0.137	0.141	0.144	0.149
	200	0.101	0.112	0.122	0.128	0.133	0.142	0.116	0.124	0.133	0.138	0.141	0.148	0.125	0.132	0.139	0.143	0.146	0.151
2	20	0.081	0.084	0.085	0.085	0.086	0.087	0.094	0.097	0.099	0.100	0.100	0.102	0.102	0.105	0.108	0.109	0.109	0.110
	30	0.094	0.099	0.103	0.105	0.106	0.108	0.107	0.112	0.117	0.118	0.120	0.121	0.115	0.119	0.123	0.125	0.127	0.129
	50	0.101	0.109	0.116	0.119	0.123	0.125	0.114	0.120	0.127	0.130	0.133	0.136	0.122	0.127	0.133	0.136	0.138	0.141
	70	0.102	0.110	0.120	0.124	0.128	0.133	0.116	0.123	0.130	0.134	0.137	0.142	0.124	0.130	0.136	0.139	0.142	0.146
	100	0.102	0.111	0.122	0.127	0.131	0.138	0.116	0.124	0.132	0.136	0.140	0.145	0.125	0.131	0.138	0.141	0.144	0.149
	200	0.102	0.112	0.122	0.129	0.134	0.142	0.117	0.125	0.133	0.138	0.142	0.148	0.126	0.132	0.139	0.143	0.146	0.151
3	20	0.066	0.067	0.068	0.069	0.069	0.070	0.077	0.079	0.081	0.081	0.081	0.082	0.085	0.087	0.088	0.089	0.089	0.090
	30	0.085	0.088	0.091	0.092	0.093	0.093	0.098	0.102	0.105	0.106	0.107	0.107	0.105	0.109	0.112	0.114	0.115	0.116
	50	0.097	0.104	0.110	0.112	0.114	0.116	0.110	0.116	0.121	0.124	0.126	0.128	0.117	0.123	0.127	0.130	0.132	0.134
	70	0.101	0.108	0.116	0.119	0.123	0.126	0.113	0.120	0.127	0.129	0.132	0.136	0.121	0.127	0.132	0.135	0.138	0.141
	100	0.102	0.110	0.119	0.125	0.128	0.134	0.115	0.122	0.129	0.134	0.137	0.142	0.123	0.129	0.135	0.139	0.141	0.145
	200	0.102	0.112	0.122	0.128	0.132	0.140	0.117	0.124	0.132	0.137	0.141	0.146	0.125	0.132	0.138	0.142	0.145	0.150
4	20	0.052	0.052	0.053	0.053	0.054	0.054	0.061	0.062	0.063	0.063	0.063	0.063	0.067	0.069	0.070	0.070	0.070	0.070
	30	0.076	0.078	0.080	0.081	0.081	0.081	0.089	0.091	0.094	0.094	0.095	0.095	0.097	0.099	0.102	0.103	0.104	0.104
	50	0.094	0.100	0.104	0.106	0.107	0.108	0.107	0.113	0.117	0.119	0.120	0.122	0.115	0.120	0.124	0.126	0.127	0.129
	70	0.100	0.106	0.114	0.116	0.118	0.121	0.112	0.119	0.125	0.127	0.130	0.133	0.120	0.125	0.131	0.133	0.135	0.138
	100	0.102	0.110	0.119	0.123	0.126	0.131	0.115	0.122	0.129	0.133	0.136	0.140	0.123	0.129	0.135	0.138	0.141	0.144
	200	0.103	0.112	0.122	0.128	0.133	0.140	0.117	0.125	0.133	0.137	0.141	0.146	0.125	0.132	0.139	0.142	0.145	0.150
		$k = 1$																	
		1% (CSB)						5% (CSB)						10% (CSB)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	0.092	0.100	0.107	0.110	0.112	0.115	0.106	0.112	0.118	0.121	0.124	0.127	0.114	0.119	0.125	0.127	0.129	0.132
	30	0.097	0.105	0.114	0.119	0.123	0.128	0.111	0.118	0.125	0.128	0.132	0.136	0.119	0.125	0.131	0.134	0.137	0.141
	50	0.099	0.108	0.119	0.124	0.129	0.136	0.113	0.121	0.129	0.133	0.137	0.143	0.121	0.128	0.135	0.139	0.142	0.146
	70	0.100	0.109	0.120	0.126	0.131	0.139	0.114	0.122	0.131	0.135	0.139	0.145	0.123	0.130	0.137	0.141	0.144	0.149
	100	0.100	0.110	0.121	0.127	0.132	0.141	0.115	0.123	0.132	0.136	0.141	0.147	0.124	0.131	0.138	0.142	0.145	0.150
	200	0.101	0.111	0.122	0.128	0.134	0.142	0.116	0.124	0.133	0.138	0.142	0.149	0.125	0.132	0.139	0.143	0.147	0.152
1	20	0.076	0.079	0.081	0.082	0.083	0.084	0.087	0.090	0.093	0.094	0.095	0.096	0.094	0.097	0.100	0.101	0.102	0.103
	30	0.088	0.094	0.099	0.101	0.102	0.105	0.100	0.105	0.110	0.112	0.114	0.116	0.107	0.112	0.116	0.118	0.120	0.122
	50	0.095	0.103	0.111	0.115	0.118	0.123	0.108	0.115	0.121	0.125	0.128	0.132	0.116	0.121	0.127	0.130	0.132	0.136
	70	0.098	0.106	0.116	0.120	0.124	0.131	0.111	0.118	0.126	0.130	0.133	0.138	0.119	0.125	0.132	0.135	0.138	0.141
	100	0.099	0.108	0.118	0.124	0.128	0.135	0.113	0.121	0.128	0.133	0.136	0.142	0.121	0.128	0.134	0.138	0.141	0.146
	200	0.100	0.110	0.121	0.126	0.132	0.140	0.115	0.123	0.132	0.136	0.140	0.146	0.124	0.130	0.138	0.141	0.145	0.150
2	20	0.056	0.057	0.059	0.059	0.060	0.060	0.066	0.068	0.069	0.070	0.070	0.071	0.073	0.074	0.076	0.076	0.077	0.077
	30	0.078	0.081	0.083	0.084	0.085	0.086	0.089	0.093	0.096	0.097	0.098	0.099	0.096	0.100	0.102	0.104	0.105	0.106
	50	0.092	0.099	0.104	0.106	0.108	0.111	0.104	0.110	0.115	0.118	0.120	0.122	0.112	0.117	0.121	0.124	0.126	0.128
	70	0.096	0.104	0.112	0.116	0.119	0.123	0.109	0.115	0.123	0.125	0.129	0.132	0.117	0.122	0.128	0.131	0.134	0.137
	100	0.099	0.108	0.117	0.122	0.126	0.132	0.112	0.119	0.127	0.131	0.134	0.139	0.120	0.127	0.133	0.136	0.139	0.143
	200	0.101	0.110	0.121	0.126	0.131	0.139	0.115	0.123	0.131	0.136	0.139	0.145	0.124	0.130	0.137	0.141	0.144	0.149
3	20	0.036	0.037	0.038	0.038	0.038	0.038	0.043	0.044	0.045	0.045	0.045	0.045	0.048	0.049	0.050	0.050	0.050	0.050
	30	0.063	0.065	0.067	0.067	0.068	0.068	0.074	0.076	0.078	0.078	0.079	0.079	0.080	0.083	0.084	0.085	0.086	0.086
	50	0.085	0.089	0.094	0.095	0.096	0.098	0.097	0.101	0.105	0.107	0.108	0.110	0.104	0.108	0.112	0.113	0.114	0.116
	70	0.092	0.099	0.105	0.108	0.110	0.113	0.104	0.110	0.116	0.118	0.121	0.123	0.112	0.117	0.122	0.124	0.126	0.129
	100	0.097	0.104	0.113	0.116	0.120	0.124	0.110	0.116	0.123	0.126	0.129	0.133	0.117	0.123	0.128	0.131	0.134	0.137
	200	0.100	0.109	0.119	0.124	0.129	0.136	0.114	0.122	0.130	0.134	0.137	0.143	0.122	0.128	0.135	0.139	0.142	0.146
4	20	0.019	0.019	0.020	0.020	0.020	0.020	0.023	0.023	0.024	0.024	0.024	0.024	0.026	0.026	0.027	0.027	0.027	0.027
	30	0.049	0.050	0.052	0.052	0.052	0.053	0.059	0.060	0.061	0.061	0.062	0.062	0.065	0.066	0.067	0.068	0.068	0.068
	50	0.078	0.081	0.085	0.085	0.086	0.087	0.090	0.093	0.096	0.097	0.098	0.099	0.097	0.100	0.103	0.104	0.105	0.106
	70	0.088	0.094	0.099	0.101	0.103	0.105	0.101	0.106	0.110	0.112	0.114	0.116	0.108	0.113	0.117	0.119	0.120	0.122
	100	0.095	0.102	0.109	0.112	0.116	0.119	0.108	0.114	0.120	0.123	0.126	0.129	0.115	0.121	0.126	0.128	0.131	0.134
	200	0.100	0.109	0.118	0.123	0.128	0.134	0.113	0.121	0.129	0.133	0.136	0.141	0.122	0.128	0.135	0.138	0.141	0.145

Table B4 Continued
k = 2

		1% (CSB)						5% (CSB)						10% (CSB)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	0.086	0.092	0.097	0.100	0.102	0.104	0.098	0.104	0.109	0.112	0.113	0.116	0.106	0.111	0.116	0.118	0.120	0.122
	30	0.093	0.101	0.109	0.113	0.116	0.120	0.106	0.113	0.119	0.123	0.125	0.130	0.114	0.119	0.125	0.128	0.131	0.134
	50	0.096	0.106	0.116	0.121	0.126	0.132	0.110	0.119	0.126	0.130	0.134	0.139	0.119	0.126	0.132	0.136	0.139	0.143
	70	0.098	0.108	0.118	0.123	0.129	0.137	0.112	0.120	0.129	0.133	0.137	0.143	0.121	0.127	0.135	0.138	0.142	0.146
	100	0.100	0.109	0.120	0.126	0.131	0.139	0.114	0.122	0.130	0.135	0.139	0.145	0.122	0.130	0.136	0.140	0.144	0.148
	200	0.100	0.111	0.122	0.128	0.133	0.141	0.115	0.124	0.133	0.137	0.141	0.148	0.124	0.132	0.139	0.142	0.146	0.151
1	20	0.062	0.063	0.065	0.066	0.066	0.067	0.071	0.074	0.076	0.076	0.077	0.078	0.078	0.080	0.082	0.083	0.083	0.084
	30	0.079	0.083	0.087	0.089	0.090	0.092	0.090	0.094	0.098	0.099	0.101	0.103	0.097	0.100	0.104	0.106	0.107	0.109
	50	0.090	0.097	0.105	0.108	0.111	0.115	0.102	0.109	0.115	0.118	0.120	0.124	0.110	0.115	0.121	0.123	0.125	0.128
	70	0.095	0.102	0.112	0.116	0.120	0.125	0.107	0.114	0.121	0.125	0.128	0.132	0.115	0.121	0.127	0.130	0.132	0.136
	100	0.097	0.106	0.115	0.121	0.125	0.131	0.110	0.118	0.126	0.130	0.133	0.138	0.118	0.125	0.131	0.135	0.138	0.142
	200	0.100	0.110	0.120	0.125	0.131	0.138	0.114	0.122	0.131	0.135	0.139	0.145	0.122	0.129	0.136	0.140	0.143	0.148
2	20	0.036	0.036	0.037	0.038	0.038	0.038	0.043	0.044	0.045	0.045	0.045	0.045	0.048	0.049	0.049	0.050	0.050	0.050
	30	0.063	0.065	0.067	0.068	0.068	0.069	0.073	0.075	0.077	0.078	0.078	0.079	0.079	0.082	0.084	0.084	0.085	0.086
	50	0.083	0.089	0.094	0.096	0.097	0.099	0.095	0.100	0.105	0.106	0.108	0.110	0.102	0.107	0.111	0.112	0.114	0.116
	70	0.091	0.097	0.105	0.108	0.111	0.114	0.103	0.109	0.115	0.118	0.120	0.123	0.110	0.116	0.121	0.123	0.125	0.128
	100	0.095	0.104	0.112	0.116	0.120	0.125	0.108	0.115	0.122	0.125	0.129	0.133	0.116	0.122	0.128	0.131	0.133	0.137
	200	0.099	0.109	0.119	0.124	0.129	0.136	0.113	0.121	0.129	0.133	0.137	0.143	0.121	0.128	0.135	0.138	0.141	0.146
3	20	0.014	0.014	0.014	0.014	0.014	0.015	0.017	0.017	0.018	0.018	0.018	0.018	0.020	0.020	0.020	0.020	0.020	0.020
	30	0.045	0.045	0.047	0.047	0.047	0.048	0.053	0.054	0.055	0.055	0.056	0.056	0.058	0.059	0.060	0.061	0.061	0.061
	50	0.073	0.077	0.080	0.081	0.082	0.083	0.084	0.088	0.091	0.092	0.093	0.094	0.090	0.094	0.097	0.098	0.099	0.100
	70	0.084	0.090	0.095	0.097	0.100	0.101	0.096	0.101	0.106	0.108	0.109	0.112	0.103	0.107	0.112	0.113	0.115	0.117
	100	0.091	0.099	0.106	0.109	0.112	0.116	0.104	0.110	0.116	0.119	0.122	0.125	0.111	0.116	0.122	0.124	0.127	0.129
	200	0.098	0.107	0.116	0.121	0.126	0.132	0.111	0.119	0.127	0.130	0.134	0.139	0.119	0.126	0.132	0.135	0.138	0.143
4	20	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
	30	0.028	0.029	0.029	0.029	0.029	0.030	0.034	0.034	0.035	0.035	0.035	0.036	0.038	0.038	0.039	0.039	0.039	0.039
	50	0.063	0.065	0.067	0.068	0.068	0.070	0.073	0.076	0.078	0.078	0.079	0.080	0.079	0.082	0.084	0.085	0.085	0.086
	70	0.078	0.083	0.087	0.088	0.089	0.090	0.089	0.093	0.097	0.099	0.100	0.101	0.096	0.100	0.103	0.105	0.106	0.108
	100	0.088	0.095	0.100	0.103	0.105	0.108	0.100	0.106	0.111	0.113	0.115	0.118	0.107	0.112	0.117	0.119	0.121	0.123
	200	0.097	0.105	0.115	0.119	0.124	0.129	0.110	0.118	0.125	0.129	0.132	0.137	0.118	0.124	0.131	0.134	0.136	0.140

		1% (CSB)						5% (CSB)						10% (CSB)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
0	20	0.080	0.085	0.089	0.091	0.092	0.094	0.092	0.096	0.100	0.102	0.104	0.106	0.099	0.103	0.107	0.109	0.110	0.112
	30	0.089	0.097	0.103	0.107	0.110	0.114	0.101	0.108	0.114	0.117	0.119	0.123	0.109	0.115	0.120	0.122	0.125	0.128
	50	0.094	0.104	0.112	0.118	0.122	0.128	0.108	0.116	0.123	0.127	0.131	0.135	0.116	0.123	0.129	0.132	0.135	0.139
	70	0.097	0.106	0.116	0.121	0.126	0.134	0.111	0.118	0.127	0.131	0.135	0.140	0.119	0.126	0.133	0.136	0.139	0.144
	100	0.099	0.109	0.119	0.124	0.129	0.137	0.113	0.121	0.129	0.133	0.137	0.143	0.121	0.128	0.135	0.139	0.142	0.147
	200	0.100	0.110	0.120	0.127	0.132	0.141	0.115	0.123	0.132	0.136	0.140	0.147	0.124	0.130	0.138	0.142	0.145	0.150
1	20	0.048	0.049	0.050	0.051	0.051	0.052	0.057	0.058	0.059	0.060	0.060	0.060	0.062	0.063	0.065	0.065	0.066	0.066
	30	0.069	0.073	0.076	0.077	0.079	0.080	0.080	0.084	0.086	0.088	0.089	0.090	0.087	0.090	0.092	0.094	0.095	0.096
	50	0.085	0.092	0.098	0.101	0.104	0.107	0.097	0.103	0.108	0.111	0.113	0.116	0.104	0.109	0.114	0.116	0.118	0.121
	70	0.090	0.099	0.107	0.111	0.115	0.119	0.103	0.110	0.117	0.120	0.123	0.127	0.111	0.117	0.122	0.125	0.127	0.131
	100	0.095	0.104	0.113	0.118	0.121	0.128	0.108	0.115	0.123	0.127	0.130	0.135	0.116	0.122	0.128	0.132	0.134	0.139
	200	0.098	0.108	0.118	0.124	0.129	0.137	0.113	0.120	0.129	0.133	0.137	0.143	0.121	0.128	0.135	0.138	0.141	0.146
2	20	0.019	0.019	0.019	0.020	0.020	0.020	0.023	0.023	0.024	0.024	0.024	0.024	0.026	0.026	0.027	0.027	0.027	0.027
	30	0.049	0.051	0.052	0.052	0.052	0.053	0.057	0.059	0.060	0.061	0.061	0.062	0.063	0.065	0.066	0.066	0.067	0.067
	50	0.075	0.080	0.084	0.085	0.087	0.088	0.086	0.090	0.094	0.095	0.097	0.099	0.093	0.097	0.100	0.101	0.103	0.104
	70	0.085	0.091	0.098	0.100	0.103	0.106	0.097	0.102	0.108	0.110	0.112	0.115	0.104	0.109	0.114	0.116	0.118	0.120
	100	0.092	0.099	0.108	0.112	0.115	0.119	0.104	0.111	0.117	0.121	0.123	0.127	0.112	0.117	0.123	0.126	0.128	0.131
	200	0.097	0.106	0.116	0.121	0.126	0.134	0.111	0.119	0.127	0.131	0.134	0.140	0.120	0.126	0.133	0.136	0.139	0.144
3	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	0.028	0.029	0.029	0.030	0.030	0.030	0.034	0.035	0.035	0.036	0.036	0.036	0.038	0.039	0.039	0.039	0.039	0.040
	50	0.062	0.065	0.067	0.068	0.069	0.070	0.072	0.074	0.077	0.078	0.079	0.080	0.078	0.080	0.083	0.084	0.084	0.085
	70	0.076	0.081	0.086	0.087	0.089	0.091	0.087	0.092	0.096	0.097	0.099	0.101	0.094	0.098	0.102	0.103	0.104	0.106
	100	0.086	0.093	0.100	0.103	0.105	0.108	0.098	0.104	0.110	0.112	0.114	0.117	0.105	0.110	0.115	0.117	0.119	0.122
	200	0.095	0.104	0.113	0.118	0.122	0.129	0.109	0.116	0.123	0.127	0.130	0.135	0.117	0.122	0.129	0.132	0.135	0.139
4	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	0.011	0.011	0.011	0.012	0.011	0.012	0.014	0.014	0.014	0.014	0.014	0.014	0.016					

replications. Where values are not reported, this is due to insufficient degrees of freedom.

References

- Andrews, D. and C. Monahan (1992). Heteroskedasticity and autocorrelation consistent covariance matrix estimation. *Econometrica* 60, 953-966.
- Bai, J. and S. Ng (2002). Determining the number of factors in approximate factor models. *Econometrica* 70, 191-221.
- Bai, J. and S. Ng (2004). A panic on unit root tests and cointegration. *Econometrica* 72, 1127-1177.
- Bai, J. and S. Ng (2010). Panel unit root tests with cross section dependence: A further investigation. *Econometric Theory* 26, 1088-1114.
- Bailey, N., Kapetanios, G. and M.H. Pesaran (2012). Exponent of cross-sectional dependence: Estimation and inference. IZA Discussion Paper No. 6318 (<http://ftp.iza.org/dp6318.pdf>).
- Baltagi, B.H. and C. Kao (2000). Nonstationary panels, cointegration in panels and dynamic panels. A Survey, in: B. Baltagi (ed.), Nonstationary panels, panel cointegration, and dynamic panels, *Advances in Econometrics* 15, 7-52, Amsterdam: JAI Press.
- Banerjee, A. (1999). Panel data unit roots and cointegration: An overview. *Oxford Bulletin of Economics and Statistics* 61, 607-629.
- Breitung, J. and M.H. Pesaran (2008). Unit roots and cointegration in panels. L. Matyas, and P. Sevestre (eds), *The Econometrics of Panel Data (Third Edition)*, Ch 9, 279-322, Kluwer Academic Publishers.
- Breitung, J. and J. Westerlund (2009). Myths and facts about panel unit root tests. Mimeo.
- Cerrato, M. and N. Sarantis (2007). A bootstrap panel unit root test under cross-sectional dependence, with an application to PPP. *Computational Statistics & Data Analysis* 51, 4028 – 4037.
- Chang, Y. (2002). Nonlinear IV unit root tests in panels with cross sectional dependency. *Journal of Econometrics* 110, 261-292.
- Chang, Y. (2004). Bootstrap Unit root Tests in Panels with Cross-sectional Dependency, *Journal of Econometrics* 120, 263-293.
- Choi, I. (2004). Nonstationary panels. *Palgrave Handbooks of Econometrics*, Vol. 1.
- Choi, I. and T.K. Chue (2007). Subsampling Hypothesis Tests for Nonstationary Panels with Applications to Exchange Rates and Stock Prices. *Journal of Applied Econometrics* 22, 233–264.
- Chudik, A., Pesaran, M.H. and E. Tosetti (2011). Weak and strong cross section dependence and estimation of large panels. *Econometrics Journal* 14, C45-C90.
- Eickmeier, S. (2009). Comovements and heterogeneity in the Euro area analyzed in a non-stationary dynamic factor model. *Journal of Applied Econometrics* 24, 933-959.
- Elliott, G. and M. Jansson (2003). Testing for unit roots with stationary covariates, *Journal of Econometrics* 115, 75 – 89.
- Gengenbach, C., Palm, F.C. and J-P. Urbain (2009). Panel unit root tests in the presence of cross-sectional dependencies: Comparison and implications for modelling. *Econometric Reviews* 29, 111-145.
- Hadri, K. and E. Kurozumi, (2011). A locally optimal test for no unit root in cross-sectionally dependent panel data. *Hitotsubashi Journal of Economics*, 52, 165-184.
- Hamilton, J.D. (1994). *Time Series Analysis*. Princeton N.J.: Princeton University Press.
- Hansen, B.E. (1995). Rethinking the univariate approach to unit root testing: using covariates to increase power. *Econometric Theory* 11, 1148-1171.
- Harris, D., Harvey, D.I., Leybourne, S. J. and N.D. Sakkas (2010). Local asymptotic power of the Im-Pesaran-Shin panel unit root test and the impact of initial observations. *Econometric Theory* 26, 311-324.
- Im, K.S., Pesaran, M.H. and Y. Shin (2003). Testing for unit roots in heterogeneous panels. *Journal of Econometrics* 115, 53-74.
- Levin, A., Lin, C. and C.J. Chu (2002). Unit root tests in panel data: Asymptotic and finite-sample properties. *Journal of Econometrics* 108, 1–24.
- Maddala, G.S. and S. Wu (1999). A comparative study of unit root tests with panel data and a new simple test. *Oxford Bulletin of Economics and Statistics* 61, 631–652.
- Magnus, J.R. and H. Neudecker (1999). *Matrix differential calculus: with applications in statistics and econometrics*. Revised edition, John Wiley and Sons.

- Moon, H.R. and B. Perron (2004). Testing for a unit root in panels with dynamic factors. *Journal of Econometrics* 122, 81-126.
- Moon, H.R., Perron, B. and P.C.B. Phillips (2007). Incidental trends and the power of panel unit root tests. *Journal of Econometrics* 141, 416-459.
- Moon, H.R., Perron, B. and P.C.B. Phillips (2011). Point optimal panel unit root tests with serially correlated errors. Unpublished Manuscript.
- Palm, F.C., Smeekes, S., and J. Urbain (2011). Cross-sectional dependence robust block bootstrap panel unit root tests. *Journal of Econometrics* 163, 85-104.
- Pesaran, M.H. (2007). A simple panel unit root test in the presence of cross section dependence. *Journal of Applied Econometrics* 22, 265-312.
- Pesaran, M.H., Shin, Y. and R.J. Smith (2001). Bounds testing approach to the analysis of level relationships. *Journal of Applied Econometrics* 16, 289-326.
- Phillips, P.C.B and D. Sul (2003). Dynamic panel estimation and homogeneity testing under cross section dependence. *Econometrics Journal* 6, 217-259.
- Ploberger, W. and P.C.B. Phillips (2002). Optimal testing for unit roots in panel data. Unpublished manuscript.
- Sargan, J.D. and A. Bhargava (1983). Testing for residuals from least squares regression being generated by Gaussian random walk. *Econometrica* 51, 153-174.
- Smith, L.V. and A. Galesi (2011). GVAR Toolbox 1.1, www.cfap.jbs.cam.ac.uk/research/gvartoolbox/download.html.
- Smith, L.V., Leybourne, S., Kim, T.-H. and P. Newbold (2004). More powerful panel data unit root tests with an application to mean reversion in real exchange rates. *Journal of Applied Econometrics* 19, 147-170.
- Stock, J.H. (1999). A class of tests for integration and cointegration, p.135-167 in R.F. Engle and H. White (Eds.) *Cointegration, Causality and Forecasting*. Oxford University Press.
- Stock, J.H. and M.W. Watson (2002). Macroeconomic forecasting using diffusion indexes. *Journal of Business and Economic Statistics*, 20, 147-162.
- Taylor, R.L., Daffer, P.Z. and R.F. Patterson (1985). Limit theorems for sums of exchangeable random variables. Rowman & Allanheld: Totowa, NJ.
- Westerlund, J. (2008). Panel cointegration tests of the Fisher hypothesis. *Journal of Applied Econometrics* 23, 193-233.
- Westerlund, J. and R. Larsson (2009). A note on the pooling of individual PANIC unit root tests. *Econometric Theory* 25, 1851-1868.

Table 1: Size and Power of *CIPS* and *CSB* Panel Unit Root Tests with Two Factors, Factors and Idiosyncratic Errors are Serially Uncorrelated, $m^0 = 2$ Known

	Intercept Only											
	Size: $\rho_i = \rho = 1$						Power: $\rho_i \sim iidU[0.90, 0.99]$					
(T, N)	20	30	50	70	100	200	20	30	50	70	100	200
<i>CIPS</i> ($\hat{p}, k = 1$)												
20	5.75	6.40	5.10	5.50	5.50	6.10	7.80	10.70	10.85	13.15	11.95	14.85
30	5.40	6.60	5.35	5.70	5.85	6.15	11.40	13.65	17.10	17.10	18.55	21.85
50	5.00	5.60	5.90	6.10	4.80	5.90	17.35	22.10	27.10	27.50	32.05	38.40
70	5.45	4.85	4.60	5.70	5.35	5.25	27.95	33.40	40.75	47.45	50.00	56.35
100	5.65	7.05	6.10	4.95	5.75	5.45	44.65	54.45	67.10	68.20	78.60	82.15
200	4.95	4.55	5.60	5.65	4.85	4.80	97.40	99.50	99.95	99.95	100.00	100.00
<i>CSB</i> ($\hat{p}, k = 1$)												
20	6.35	6.10	5.60	4.95	5.80	6.10	14.25	15.80	18.50	23.45	24.80	31.20
30	5.70	5.85	5.20	5.60	5.55	4.10	20.50	24.80	31.70	36.80	40.50	46.95
50	6.35	6.00	5.80	5.85	5.55	5.55	39.20	47.75	62.20	70.30	77.25	87.70
70	5.70	5.80	6.35	6.15	5.75	5.60	61.40	75.40	89.55	94.30	98.00	99.50
100	4.55	5.20	5.95	6.10	5.40	6.60	79.05	89.65	97.95	98.70	99.60	99.95
200	6.50	4.75	6.15	5.15	6.20	5.85	94.85	97.80	99.45	99.90	99.95	100.00
	Intercept and Linear Trend											
<i>CIPS</i> ($\hat{p}, k = 1$)												
20	6.45	5.20	6.30	6.30	5.45	5.50	7.25	6.55	7.85	7.85	5.80	8.05
30	5.30	5.40	5.90	6.80	5.85	5.45	6.85	8.15	9.00	10.45	11.95	11.75
50	6.35	5.45	5.65	6.10	5.85	5.35	10.00	10.40	13.00	14.00	17.90	20.75
70	5.55	5.50	5.60	5.20	4.65	4.65	14.70	17.40	22.15	25.75	26.65	31.35
100	5.20	5.90	6.30	5.25	5.00	5.10	23.45	29.60	37.85	39.40	46.45	52.10
200	5.60	5.70	5.65	5.30	6.15	3.75	83.80	91.25	97.85	99.25	99.80	99.95
<i>CSB</i> ($\hat{p}, k = 1$)												
20	6.35	5.40	5.80	5.15	5.20	5.65	8.60	8.85	11.55	12.10	13.35	19.25
30	6.80	6.15	5.80	5.95	5.85	5.70	10.65	12.10	14.45	18.45	20.65	25.80
50	5.95	5.80	5.20	5.60	4.50	5.80	15.50	19.15	23.50	29.65	33.55	41.75
70	6.05	4.95	5.90	5.70	5.85	5.25	25.50	33.60	46.45	54.70	65.75	80.40
100	4.65	5.55	5.80	6.35	5.45	5.00	44.15	58.25	75.85	84.95	91.95	97.90
200	5.40	5.10	5.10	6.20	6.15	5.75	87.20	94.85	98.75	99.60	99.85	100.00

Notes: In the intercept only case, y_{it} is generated as $y_{it} = d_{iyt} + \rho_i y_{i,t-1} + \gamma_{iy1} f_{1t} + \gamma_{iy2} f_{2t} + \varepsilon_{iyt}$, $i = 1, 2, \dots, N$; $t = -49, 48, \dots, 0, 1, \dots, T$, with $y_{i,-50} = 0$, where $\gamma_{iy1} \sim iidU[0, 2]$, for $i = 1, 2, \dots, N$; $\gamma_{iy2} \sim iidU[0, 1]$ for $i = 1, \dots, [N^\alpha]$ and $\gamma_{iy2} = 0$ for $i = [N^\alpha] + 1, [N^\alpha] + 2, \dots, N$ (where $[\cdot]$ denotes the integer part); $f_{\ell t} \sim iidN(0, 1)$ for $\ell = 1, 2$, $\varepsilon_{iyt} \sim iidN(0, \sigma_i^2)$ with $\sigma_i^2 \sim iidU[0.5, 1.5]$; $\Delta x_{it} = d_{ix} + \gamma_{ix1} f_{1t} + \varepsilon_{ixt}$, where, $d_{ix} = 0$, $\varepsilon_{ixt} = \rho_{ix} \varepsilon_{ix,t-1} + \varpi_{ixt}$, $\varpi_{ixt} \sim iidN(0, 1 - \rho_{ix}^2)$, $i = 1, 2, \dots, N$; $t = -49, 48, \dots, 0, 1, \dots, T$, with $\varepsilon_{ix,-50} = 0$, and $\rho_{ix} \sim iidU[0.2, 0.4]$. The factor loadings in (50) are generated as $\gamma_{ix1} \sim iidU[0, 2]$; $d_{iyt} = (1 - \rho_i) \alpha_{iy}$ with $\alpha_{iy} \sim iidN(1, 1)$. The parameters α_{iy} , ρ_{iy} , γ_{iy1} , γ_{iy2} , ρ_i , γ_{ix1} , ρ_{ix} , and σ_i are redrawn over each replication. The first 50 observations are discarded. The *CIPS*(\hat{p}) and the *CSB*(\hat{p}) tests are the proposed panel unit root tests, defined by (28) and (34), respectively, based on cross section augmentation using y_{it} and x_{it} with lag-augmentation order selected according to $\hat{p} = \lceil 4(T/100)^{1/4} \rceil$. In the intercept and linear trend case, y_{it} is generated as described above, but $d_{iyt} = \mu_{iy} + (1 - \rho_i) \delta_{it}$ with $\mu_{iy} \sim iidU[0.0, 0.02]$ and $\delta_{it} \sim iidU[0.0, 0.02]$, and $d_{ixt} = \delta_{ix}$ with and $\delta_{ix} \sim iidU[0.0, 0.02]$. The tests are conducted at the 5% significance level, based on the critical values for the corresponding N, T, \hat{p} and the number of additional regressors, k . All experiments are based on 2000 replications.

Table 2: Size and Power of *CIPS* and *CSB* Panel Unit Root Tests with Two Factors, Factors are Serially Uncorrelated but Idiosyncratic Errors are Positively Serially Correlated, $m^0 = 2$ Known

		Intercept Only											
		Size: $\rho_i = \rho = 1$					Power: $\rho_i \sim iidU[0.90, 0.99]$						
(T, N)		20	30	50	70	100	200	20	30	50	70	100	200
<i>CIPS</i> ($\hat{\rho}, k = 1$)													
20		5.00	5.65	4.05	4.30	3.80	4.15	7.40	8.65	8.50	10.70	9.25	11.65
30		4.40	5.45	3.85	4.20	4.15	4.55	9.65	11.45	14.55	14.80	16.00	18.75
50		4.30	5.30	5.25	4.70	3.90	5.15	16.55	20.40	24.05	24.65	28.75	34.60
70		4.90	5.00	4.45	5.00	4.35	4.30	26.10	30.55	37.55	44.40	45.15	51.50
100		5.45	6.20	5.60	4.10	5.55	4.95	41.95	51.10	62.85	62.65	74.60	78.25
200		4.75	4.45	5.05	5.55	4.65	4.55	96.45	99.10	99.80	100.00	100.00	100.00
<i>CSB</i> ($\hat{\rho}, k = 1$)													
20		6.85	6.40	6.15	6.30	6.75	6.20	13.40	15.30	17.70	23.25	23.65	30.55
30		5.40	6.25	5.65	5.65	6.00	4.45	18.65	22.95	28.75	33.85	37.70	43.90
50		5.90	5.60	5.95	5.65	5.65	6.10	36.65	43.55	58.60	67.40	74.25	86.10
70		5.15	6.15	5.50	5.85	5.20	5.80	60.45	74.85	90.65	95.70	99.05	99.85
100		4.35	4.80	5.75	5.75	5.15	6.30	80.35	90.90	98.60	99.50	99.90	100.00
200		6.35	4.40	5.40	5.10	5.65	5.45	97.25	99.10	99.75	100.00	100.00	100.00
		Intercept and Linear Trend											
<i>CIPS</i> ($\hat{\rho}, k = 1$)													
20		5.05	3.90	3.85	4.15	3.50	2.75	6.15	4.55	5.50	5.65	4.10	5.05
30		4.15	5.05	4.35	4.95	3.75	3.45	6.00	6.30	6.40	7.95	9.35	9.25
50		5.80	4.50	4.65	5.05	4.95	4.30	8.95	9.55	11.05	12.15	16.30	19.05
70		5.10	4.65	4.45	4.65	3.95	4.00	13.85	15.60	18.95	23.30	24.55	28.70
100		5.25	5.50	5.30	4.70	4.05	4.50	21.70	27.60	33.65	36.60	43.00	47.15
200		5.60	4.85	5.75	4.85	5.75	3.35	79.35	89.95	96.55	98.40	99.35	99.90
<i>CSB</i> ($\hat{\rho}, k = 1$)													
20		6.50	5.70	6.05	4.65	5.50	5.30	8.55	8.65	10.80	11.15	13.00	18.20
30		5.65	5.05	5.20	5.25	5.00	4.85	8.70	9.70	12.90	15.35	18.30	22.40
50		4.80	5.25	4.25	4.45	4.20	4.65	12.40	15.70	19.35	24.45	28.15	36.10
70		4.90	3.65	4.70	4.20	4.35	3.80	21.65	28.75	39.10	47.85	58.50	73.10
100		4.15	4.30	5.10	4.50	4.45	4.35	40.20	55.15	72.60	82.25	90.60	98.10
200		4.45	3.95	4.10	4.70	5.10	4.80	90.40	96.90	99.50	99.95	100.00	100.00

Notes: y_{it} is generated as described in the notes to Table 1, except that $\varepsilon_{iyt} = \rho_{iy\varepsilon}\varepsilon_{iyt-1} + (1 - \rho_{iy\varepsilon}^2)^{1/2}\eta_{iyt}$, $\eta_{iyt} \sim iidN(0, \sigma_\eta^2)$, $\varepsilon_{iy, -50} = 0$, $\sigma_\eta^2 \sim iidU[0.5, 1.5]$, $\rho_{iy\varepsilon} \sim iidU[0.2, 0.4]$. See also the notes to Table 1 for the specification of the rest of the parameters.

Table 3: Size and Power of *CIPS* and *CSB* Panel Unit Root Tests with Two Factors, Factors are Serially Uncorrelated but Idiosyncratic Errors are Negatively Serially Correlated, $m^0 = 2$ Known

		Intercept Only											
		Size: $\rho_i = \rho = 1$					Power: $\rho_i \sim iidU[0.90, 0.99]$						
(T, N)		20	30	50	70	100	200	20	30	50	70	100	200
<i>CIPS</i> ($\hat{\rho}, k = 1$)													
20		6.65	7.20	6.35	6.60	7.40	6.85	8.45	11.55	11.60	13.95	13.20	15.60
30		6.60	7.10	6.45	6.70	7.40	7.45	12.10	14.90	17.05	18.00	19.15	22.95
50		5.10	6.20	6.40	6.90	5.70	6.05	17.60	21.90	26.95	28.30	31.90	38.50
70		5.95	5.50	5.35	6.00	6.30	5.65	28.80	33.45	41.55	47.65	50.95	56.85
100		6.30	7.40	7.00	5.40	6.15	5.95	46.40	56.25	68.20	70.10	79.35	83.35
200		6.20	5.15	6.15	5.65	5.45	5.20	97.75	99.50	99.90	99.90	100.00	100.00
<i>CSB</i> ($\hat{\rho}, k = 1$)													
20		6.60	5.40	5.25	4.65	6.10	5.65	14.45	16.40	19.30	24.60	25.80	31.30
30		6.05	5.85	5.25	5.55	5.80	4.30	21.55	25.65	32.15	36.90	41.45	47.25
50		6.55	5.95	6.35	6.50	5.40	5.80	38.70	46.65	60.95	67.50	75.50	85.15
70		6.35	6.25	6.75	6.25	6.45	5.15	58.65	71.85	84.45	90.25	94.35	97.80
100		5.60	6.00	6.20	6.40	5.60	6.50	74.25	85.80	94.30	95.60	98.30	99.70
200		7.05	5.90	6.75	6.10	7.00	6.80	91.40	94.10	97.90	98.90	99.50	100.00
		Intercept and Linear Trend											
<i>CIPS</i> ($\hat{\rho}, k = 1$)													
20		6.65	7.15	7.95	7.60	7.35	6.15	6.75	7.05	9.70	9.50	8.90	8.00
30		6.00	6.40	6.70	7.35	7.40	7.10	8.00	8.40	9.65	11.05	12.70	13.00
50		6.85	5.85	7.25	6.75	6.85	5.90	11.30	11.65	13.50	15.80	17.85	19.80
70		5.90	6.15	6.40	6.60	5.10	6.00	15.30	17.85	23.55	25.75	28.55	30.80
100		6.15	6.70	5.90	6.30	5.85	5.75	24.75	29.35	36.00	42.05	45.05	55.30
200		7.20	5.80	6.85	4.50	6.00	4.65	84.20	92.35	98.20	98.95	99.80	99.95
<i>CSB</i> ($\hat{\rho}, k = 1$)													
20		6.85	6.35	6.05	5.50	6.20	5.25	8.85	8.70	11.95	11.90	15.25	18.55
30		6.75	5.65	6.90	6.65	5.95	6.60	11.35	14.45	17.45	18.15	22.55	28.70
50		5.75	7.05	6.00	6.75	5.05	6.00	16.25	21.15	28.70	30.55	35.90	44.20
70		6.75	6.40	6.60	5.95	7.45	5.90	26.60	34.95	48.35	55.45	66.35	79.10
100		6.00	6.70	6.30	8.10	7.45	5.95	41.65	54.85	74.30	80.10	89.10	96.90
200		6.75	6.80	6.95	6.95	6.95	8.15	82.50	90.55	95.80	97.95	99.15	99.85

Notes: y_{it} is generated as described in the notes to Table 1, except that $\varepsilon_{iyt} = \rho_{iy\varepsilon}\varepsilon_{iyt-1} + (1 - \rho_{iy\varepsilon}^2)^{1/2}\eta_{iyt}$, $\eta_{iyt} \sim iidN(0, \sigma_\eta^2)$, $\varepsilon_{iy, -50} = 0$, $\sigma_\eta^2 \sim iidU[0.5, 1.5]$, $\rho_{iy\varepsilon} \sim iidU[-0.4, -0.2]$. See also the notes to Table 1 for the specification of the rest of the parameters.

Table 4: Results of the *CIPS* and the *CSB* Panel Unit Root Tests Applied to Real Interest Rates for all $m^0 - 1$ Combinations of the 5 Additional Regressors (1979Q2 – 2009Q4)

Real Interest Rates ($N = 32, T = 118$)									
Intercept Only									
$m^0 = 1$			$m^0 = 3$			$m^0 = 4$			
<i>CIPS</i> (\hat{p})		<i>CSB</i> (\hat{p})	\bar{x}_t	<i>CIPS</i> (\hat{p})	<i>CSB</i> (\hat{p})	\bar{x}_t	<i>CIPS</i> (\hat{p})	<i>CSB</i> (\hat{p})	
-2.983 [†]		0.157 [†]	$poilt_t, \bar{r}_t^L$	-3.603 [†]	0.319	$poilt_t, \bar{r}_t^L, \bar{e}q_t$	-3.545 [†]	0.282	
			$poilt_t, \bar{e}q_t$	-3.319 [†]	0.166 [†]	$poilt_t, \bar{r}_t^L, \bar{e}p_t$	-3.566 [†]	0.321	
$m^0 = 2$			$poilt_t, \bar{e}q_t$	-3.286 [†]	0.158 [†]	$poilt_t, \bar{r}_t^L, \bar{gdp}_t$	-3.530 [†]	0.284	
\bar{x}_t	<i>CIPS</i> (\hat{p})	<i>CSB</i> (\hat{p})	$poilt_t, \bar{gdp}_t$	-3.318 [†]	0.161 [†]	$poilt_t, \bar{e}q_t, \bar{e}p_t$	-3.257 [†]	0.164 [†]	
$poilt_t$	-3.347 [†]	0.163 [†]	$\bar{r}_t^L, \bar{e}q_t$	-3.671 [†]	0.234 [†]	$poilt_t, \bar{e}q_t, \bar{gdp}_t$	-3.325 [†]	0.161 [†]	
\bar{r}_t^L	-3.604 [†]	0.243 [†]	$\bar{r}_t^L, \bar{e}p_t$	-3.457 [†]	0.245*	$poilt_t, \bar{e}p_t, \bar{gdp}_t$	-3.286 [†]	0.160 [†]	
$\bar{e}q_t$	-2.878 [†]	0.169 [†]	\bar{r}_t^L, \bar{gdp}_t	-3.504 [†]	0.243 [†]	$\bar{r}_t^L, \bar{e}q_t, \bar{e}p_t$	-3.619 [†]	0.233*	
$\bar{e}p_t$	-2.866 [†]	0.153 [†]	$\bar{e}q_t, \bar{e}p_t$	-2.805 [†]	0.165 [†]	$\bar{r}_t^L, \bar{e}q_t, \bar{gdp}_t$	-3.684 [†]	0.240*	
\bar{gdp}_t	-3.117 [†]	0.162 [†]	$\bar{e}q_t, \bar{gdp}_t$	-3.354 [†]	0.165 [†]	$\bar{r}_t^L, \bar{e}p_t, \bar{gdp}_t$	-3.365 [†]	0.234*	
			$\bar{e}p_t, \bar{gdp}_t$	-3.129 [†]	0.159 [†]	$\bar{e}q_t, \bar{e}p_t, \bar{gdp}_t$	-3.407 [†]	0.161 [†]	

Notes: † and * denote rejections at 1% and 5% significance levels, respectively. For the selected lag order $\hat{p} = [4(T/100)^{1/4}]$, the critical values for the *CIPS* test in the case where $m^0 = 1$ are -2.238, and -2.106 for the 1%, and 5% significance levels, respectively. For $m^0 = 2$ they are -2.486, and -2.335, for $m^0 = 3$ they are -2.669, and -2.504, and for $m^0 = 4$ they are -2.816, and -2.641. Similarly for the *CSB* test, the critical values for $m^0 = 1$ are 0.279, and 0.322, for $m^0 = 2$ they are 0.261, and 0.304, for $m^0 = 3$ they are 0.245, and 0.287, and for $m^0 = 4$ they are 0.231, and 0.270. The variables under the heading \bar{x}_t indicate the regressors used for cross section augmentation in addition to \bar{y}_t , where $y_{it} = r_{it}^S - \pi_{it}$. In the case where $m^0 = 1$ no additional regressors are used. The variables $poilt_t, \bar{e}q_t, \bar{e}p_t$, and \bar{gdp}_t are detrended.

Table 5: Results of *CIPS* and *CSB* Panel Unit Root Tests Applied to Real Equity Prices for all $m^0 - 1$ Combinations of the Five Additional Regressors (1979Q2 – 2009Q4)

Real Equity Prices ($N = 26, T = 118$)									
Intercept and Trend									
$m^0 = 1$			$m^0 = 3$			$m^0 = 4$			
<i>CIPS</i> (\hat{p})		<i>CSB</i> (\hat{p})	\bar{x}_t	<i>CIPS</i> (\hat{p})	<i>CSB</i> (\hat{p})	\bar{x}_t	<i>CIPS</i> (\hat{p})	<i>CSB</i> (\hat{p})	
-2.594		0.143	$poilt_t, \bar{r}_t^L$	-3.022*	0.113	$poilt_t, \bar{r}_t^L, \bar{\pi}_t$	-2.716	0.116	
			$poilt_t, \bar{\pi}_t$	-2.933*	0.125	$poilt_t, \bar{r}_t^L, \bar{e}p_t$	-3.051*	0.094*	
$m^0 = 2$			$poilt_t, \bar{e}p_t$	-3.098 [†]	0.122	$poilt_t, \bar{r}_t^L, \bar{gdp}_t$	-2.423	0.106	
\bar{x}_t	<i>CIPS</i> (\hat{p})	<i>CSB</i> (\hat{p})	$poilt_t, \bar{gdp}_t$	-2.416	0.118	$poilt_t, \bar{\pi}_t, \bar{e}p_t$	-3.044*	0.119	
$poilt_t$	-2.819*	0.125	$\bar{r}_t^L, \bar{\pi}_t$	-2.567	0.136	$poilt_t, \bar{\pi}_t, \bar{gdp}_t$	-2.455	0.118	
\bar{r}_t^L	-2.791*	0.132	$\bar{r}_t^L, \bar{e}p_t$	-2.711	0.124	$poilt_t, \bar{e}p_t, \bar{gdp}_t$	-2.965	0.116	
$\bar{\pi}_t$	-2.731	0.142	\bar{r}_t^L, \bar{gdp}_t	-2.433	0.116	$\bar{r}_t^L, \bar{\pi}_t, \bar{e}p_t$	-2.527	0.128	
$\bar{e}p_t$	-2.759	0.135	$\bar{\pi}_t, \bar{e}p_t$	-2.664	0.135	$\bar{r}_t^L, \bar{\pi}_t, \bar{gdp}_t$	-2.249	0.116	
\bar{gdp}_t	-2.297	0.132	$\bar{\pi}_t, \bar{gdp}_t$	-2.494	0.129	$\bar{r}_t^L, \bar{e}p_t, \bar{gdp}_t$	-2.624	0.101*	
			$\bar{e}p_t, \bar{gdp}_t$	-2.665	0.125	$\bar{\pi}_t, \bar{e}p_t, \bar{gdp}_t$	-2.580	0.122	

Notes: The critical values for the *CIPS* test in the case where $m^0 = 1$ are -2.757, and -2.619 for the 1%, and 5% significance levels, respectively. For $m^0 = 2$ they are -2.926, and -2.773, for $m^0 = 3$ they are -3.075, and -2.911 and for $m^0 = 4$ they are -3.190, and -3.006. Similarly for the *CSB* test, the critical values for $m^0 = 1$ are 0.108, and 0.121, for $m^0 = 2$ they are 0.102, and 0.114, for $m^0 = 3$ they are 0.096, and 0.108, and for $m^0 = 4$ they are 0.090, and 0.101. See also the notes to Table 4.

Table 6: Results for $P_{\hat{e}}(\hat{p}), P_b, t_b^*, PP, PMSB$ and \widetilde{CPO} Panel Unit Root Tests for Real Interest Rates and Real Equity Prices (1979Q2 – 2009Q4)

m^0	PANEL A					PANEL B				
	Real Interest Rates ($N = 32$)					Real Equity Prices ($N = 26$)				
	With an Intercept					With an Intercept and a Linear Trend				
	$P_{\hat{e}}(\hat{p})$	P_b	t_b^*	$PMSB$	\widetilde{CPO}	$P_{\hat{e}}(\hat{p})$	P_b	PP	$PMSB$	\widetilde{CPO}
1	9.768	-9.140 [†]	-19.187 [†]	-1.804*	-4.114 [†]	4.226	-1.776*	-2.084*	-1.349	-2.181*
2	8.826	-3.405 [†]	-18.230 [†]	-1.206	-4.124 [†]	3.231	-2.232*	-1.916*	-1.689*	-2.019*
3	8.326	-3.263 [†]	-22.527 [†]	-1.547	-4.170 [†]	3.430	-2.788 [†]	-1.655*	-1.989*	-1.765*
4	9.197	-6.371 [†]	-26.613 [†]	-2.124*	-4.157 [†]	3.082	-2.427 [†]	-1.217	-1.778*	-1.347

$P_{\hat{e}}(\hat{p})$ is the test of Bai and Ng (2004) with lag-augmentation order $\hat{p} = [4(T/100)^{1/4}]$ and $PMSB$ and P_b are the pooled tests of Bai and Ng (2010), all of which are based on two extracted factors from y_{it} . The t_b^* test is the Moon and Perron (2004) test, and the \widetilde{CPO} is the defactored point optimal test with serially correlated errors of Moon, Perron and Phillips (2011), based on two extracted factors from y_{it} . The $PMSB, P_b, t_b^*, \widetilde{CPO}$ tests use the automatic lag-order selection for the estimation of the long-run variances following Andrews and Monahan (1992). See also the notes to Table 4.