# Online Supplement to <br> "An Augmented Anderson-Hsiao Estimator for Dynamic Short-T Panels" 

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This online supplement has two sections. Section S-1 provides plots of rejection frequencies for selected estimators in Monte Carlo experiments described in the paper. Section S-2 provides additional Monte Carlo experiments featuring ARX(1) data generating process.

## S-1 Rejection frequencies for selected estimators in Monte Carlo experiments

This section presents rejection frequencies for selected estimators considered in the Monte Carlo experiments in Section 7 of the paper, and selected sample combinations. Figure S1 compares rejection frequencies of AH and AAH estimators in experiments where both AB and BB restrictions are met ( $\rho=0 \& \kappa=0$ ), for the sample combinations, $n=1000, T=4$ and 6 . Figure S 2 compares rejection frequencies of AAH and AAH-O estimators using the same data generating process ( $\rho=0$ $\& \kappa=0$ ), but plotting rejection frequencies for sample sizes with smaller value of $n=200$ and larger values of $T=10$ and 20 , where the number of moments is a very important small sample issue. Figure S 3 shows the rejection frequencies for $\mathrm{AAH}, \mathrm{AB}$, and BB estimators using the same data generating process ( $\rho=0 \& \kappa=0$ ), using sample combinations $n=1000, T=4$ and 6 . The last figure (Figure S4) compares rejection frequencies of AAH and AB estimators in the experiments where AB restrictions are met but BB restrictions are not met ( $\rho=0$, and $\kappa=1$ ), and using the sample combinations, $n=1000, T=4$ and 6 .

Figure S1: Rejection frequencies (at $5 \%$ nominal level) for AH and AAH estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met



See the notes to Table 2.

Figure S2: Rejection frequencies (at $5 \%$ nominal level) for AAH and AAH-O estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

$$
n=200, T=10
$$

$\phi_{0}=0.4$

$\phi_{0}=0.8$



See the notes to Tables 2 and 4 .

Figure S3: Rejection frequencies (at $5 \%$ nominal level) for $A A H, A B$, and $B B$ estimators when $A B$ and $B B$ restrictions are met
$\phi_{0}=0.4$

$$
n=1000, T=4
$$



$$
n=1000, T=6
$$

$$
\phi_{0}=0.4
$$




See notes to Tables 2 and 6 .

Figure S4: Rejection frequencies (at $5 \%$ nominal level) for AAH and AB estimators when $A B$ restrictions are met and $B B$ restrictions are not met

$$
n=1000, T=4
$$



$$
\phi_{0}=0.8
$$




See the notes to Tables 2 and 6.

## S-2 Monte Carlo experiments for panel ARX(1) model

This section presents Monte Carlo evidence on the relative performance of AAH, AB and BB estimators for the panel $\operatorname{ARX}(1)$ model. ${ }^{24}$

## S-2.1 ARX Monte Carlo Design

We augment the $\operatorname{AR}(1)$ DGP in Section 7 with a strictly exogenous regressor:

$$
\begin{equation*}
y_{i t}=\alpha_{i}+\phi y_{i, t-1}+\beta x_{i t}+u_{i t}, \tag{S.1}
\end{equation*}
$$

for $i=1,2, \ldots, n$, and $t=1,2, \ldots, T$. Individual effects, $\alpha_{i}$, are generated in the same way as in Section 7, see (43). Similarly, the starting values, $y_{i, 0}$, are generated as in (44), namely

$$
\begin{equation*}
y_{i, 0}=\mu_{i}+\kappa \varepsilon_{i}+v_{i}, v_{i} \sim \operatorname{IIDN}(0,1), \tag{S.2}
\end{equation*}
$$

but unlike in Section 7, where $\mu_{i}=\alpha_{i} /(1-\phi)$, the long-run means are generated as $\mu_{i}=$ $\left(\alpha_{i}+\mu_{x i}\right) /(1-\phi)$. The idiosyncratic errors, $u_{i t}$, are generated in the same way as in Section 7. Regressors, $x_{i t}$, are generated as

$$
\begin{equation*}
x_{i t}=\mu_{x, i}(1-\theta)+\theta x_{i, t-1}+\epsilon_{i t}, \tag{S.3}
\end{equation*}
$$

for $i=1,2, \ldots, n$, and $t=1,2, \ldots, T$, with starting values $x_{i, 0}=\mu_{x, i}+\operatorname{IIDN}(0,1)$, where $\mu_{x, i} \sim$ $\operatorname{IIDN}(1,1)$, and $\epsilon_{i t} \sim \operatorname{IIDN}\left(0,1-\theta^{2}\right)$.

We set $\phi=0.8, \beta=0.5$ and $\theta=0.6$, and consider the same two values for $\rho$, namely $\rho=0$ and 0.8 , and the same two values for $\kappa$, namely $\kappa=0$ and 1 , as in Section 7. Under this design, the covariates $x_{i t}$ are strictly exogenous.

Available observations for estimation are $\left(x_{i t}, y_{i t}\right)$ for $t=0,1,2, \ldots, T$. We consider $T=4,6,10$ and $n=100,200,500,1000,2000,8000 . R=2000$ replications were carried out for each experiment.

## S-2.2 AB, BB and AAH Estimators for ARX panel

The AB estimator is implemented as a two-step GMM estimator based on "DIF1" set of moment conditions outlined in Hayakawa and Pesaran (2015), comprising the following $T(T-1) / 2+$

[^0]$(T+1) T / 2-1$ moment conditions:
\[

$$
\begin{equation*}
E\left(y_{i s} \Delta u_{i t}\right)=0, \text { for } s=0,1, \ldots, t-2, t=2,3, \ldots, T \tag{S.4}
\end{equation*}
$$

\]

and

$$
\begin{equation*}
E\left(x_{i s} \Delta u_{i t}\right)=0, \text { for } s=1,2, \ldots, t, t=2,3, \ldots, T \tag{S.5}
\end{equation*}
$$

The BB estimator is a two-step GMM estimator based on "SYS1" set of moment conditions outlined in Hayakawa and Pesaran (2015), comprising the moment conditions in (S.4)-(S.5) plus the following additional $2(T-1)$ moment conditions:

$$
\begin{equation*}
E\left[\Delta y_{i, t-1}\left(\alpha_{i}+u_{i t}\right)\right]=0, \text { for } t=2,3, \ldots, T \tag{S.6}
\end{equation*}
$$

and

$$
\begin{equation*}
E\left[\Delta x_{i t}\left(\alpha_{i}+u_{i t}\right)\right]=0, \text { for } t=2,3, \ldots, T \tag{S.7}
\end{equation*}
$$

Detailed descriptions of these AB and BB estimators are provided in Sections 4 and 5 of Hayakawa and Pesaran (2015).

In addition to AB and BB estimators, we also implement a two-step AAH estimator. This estimator is based on the moment conditions (7) and (13) augmented with additional moment conditions for instrumenting the regressor, $x_{i t}$ for $i=1,2, \ldots, n$. As we noted in Section 5 , this paper has nothing new to add regarding the moment conditions for the exogenous regressors, and standard moment conditions used in the literature can be considered. In the experiments presented here we consider the same subset of available moment conditions for the regressor $x_{i t}$ as chosen for the AB estimator described above. This will make the comparisons between AH and AB methods straightforward and fair. The set of moment conditions for the AAH estimator implemented below is given by the $(T-2)(T-1) / 2+T+(T+1) T / 2-3$ moment conditions in (7), (13) and (S.5).

For the choices of $T=4,6,10$, we respectively have $15,35,99$ moment conditions for the AB estimator, $21,45,117$ moment conditions for the BB estimator, and $14,34,98$ moment conditions for the AAH estimator.

## S-2.3 Results

We consider three sets of experiments, for different values of $\rho$ and $\kappa$. Table S1 reports Bias and RMSE (both $\times 100$ ) of the three estimators in the baseline case where both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met, namely $\rho=\kappa=0$. The reported values for bias are overall relatively small for estimation of both parameters, $\phi$ and $\beta$, and the bias is reduced quite rapidly with an increase in $n$. For most experiments RMSEs of estimating $\beta$ are larger compared to the RMSE obtained for estimating $\phi$. This could be due to relatively low variability of $\Delta x_{i t}$ over $i$ and $t$. For all sample sizes considered, BB estimator has the lowest RMSE values, reflecting that the set of moment conditions underpinning the BB estimator encompasses the moment conditions that underlie the other two estimators. Comparison of RMSE values reported for AAH and AB estimator reveals that AAH has lower RMSE for majority of sample sizes when estimating $\phi$, but somewhat larger RMSE when estimating $\beta$. Size and power of AAH, AB and BB estimators for the baseline experiments (with $\rho=\kappa=0$ ) are reported in Table S2. As in Section 5, we observe large size distortions when the number of moment conditions is relatively large compared with the sample sizes, which can be seen most clearly when $n=100$ or 200 , with the size distortions quickly worsening with as $T$ is increased. For $T=10$, we need at least $n=2000$ for the size distortion to be relatively small.

Findings for experiments when AB restrictions are met and but some of the BB restrictions are not met (namely when $\rho=0$, and $\kappa=1$ ) are summarized in Table S3 (for the bias and RMSE) and in Table S 4 (for the size and power). We see that BB estimator is subject to bias, which does not decrease with an increase in $n$, in line with expectation that BB estimator is no longer consistent in these experiments, since some of the BB moment conditions are no longer valid when $\kappa \neq 1$. Consequently, the reported size distortions of the BB estimator in Table S4 are very large and deteriorate rapidly with an increase in $n$. Comparison of AAH and AB estimators in terms of RMSE (reported in Table S3) reveals that AAH has lower RMSE for all sample sizes with the exception of one sample size ( $n=100, T=4$ ) when estimating $\phi$, whilst the comparisons are more mixed for the estimation of $\beta$, where the AAH estimator still outperforms in majority of sample sizes considered.

Last but not least, Tables S5-S6 report Monte Carlo findings for panel ARX(1) experiments when some of AB and BB restrictions are not met ( $\rho=0.8$, and $\kappa=1$ ). In the case, not surprisingly, AB and BB estimators being based on invalid moment conditions are biased (Table S5) for most
of the sample sizes. It is interesting to note that the bias is quite small for the AB estimator and values of $T=10$, when parameter $\beta$ is estimated. Bias distortions of AB and BB estimators manifest in large size distortions that rise in $n$ (Table S8). In contrast, the AAH estimator shows qualitatively similar performance compared with the previous experiments. In particular, there are no serious size distortions when $n$ is sufficiently large relative to $T$. For example, for $T=4$ the AAH estimator does not show any size distortions for values of $n \geq 200$, irrespective of whether we consider estimating $\phi$ and $\beta$. But to avoid size distortions when $T=6$ then we need $n \geq 500$, and so on. The AAH continues to have satisfactory power which rise with $n$.

Table S1: Bias and RMSE of AAH, AB and BB estimators in panel ARX(1) experiments when both Arellano and Bond (AB) and Blundell and Bond (BB)

## restrictions are met

$\rho=0$, and $\kappa=0$

| $T$ | $n$ | Bias ( $\times 100$ ) |  |  |  |  |  | RMSE $(\times 100)$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  |
|  |  | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB |
| 4 | 100 | 1.51 | -0.24 | 0.47 | 0.03 | 0.44 | -1.17 | 7.39 | 8.24 | 2.76 | 8.71 | 8.19 | 8.07 |
| 4 | 200 | 0.79 | -0.16 | 0.21 | -0.20 | 0.05 | -0.75 | 4.71 | 5.69 | 1.88 | 5.74 | 5.48 | 5.36 |
| 4 | 500 | 0.34 | 0.02 | 0.03 | 0.00 | 0.10 | -0.23 | 2.79 | 3.58 | 1.15 | 3.54 | 3.49 | 3.34 |
| 4 | 1000 | 0.24 | 0.03 | 0.05 | -0.05 | -0.01 | -0.20 | 2.01 | 2.56 | 0.81 | 2.57 | 2.54 | 2.40 |
| 4 | 2000 | 0.10 | 0.06 | 0.00 | 0.03 | 0.06 | -0.05 | 1.36 | 1.77 | 0.57 | 1.78 | 1.78 | 1.69 |
| 4 | 8000 | 0.01 | 0.00 | 0.00 | -0.02 | -0.02 | -0.03 | 0.68 | 0.90 | 0.28 | 0.90 | 0.90 | 0.84 |
| 6 | 100 | 1.65 | 0.13 | 0.88 | -0.42 | 0.02 | 0.88 | 5.10 | 5.63 | 2.78 | 7.68 | 6.48 | 7.01 |
| 6 | 200 | 1.03 | 0.32 | 0.48 | -0.02 | 0.17 | 0.48 | 3.07 | 3.67 | 1.71 | 4.51 | 4.19 | 4.21 |
| 6 | 500 | 0.47 | 0.27 | 0.20 | -0.05 | 0.00 | 0.20 | 1.78 | 2.21 | 1.00 | 2.72 | 2.68 | 2.62 |
| 6 | 1000 | 0.25 | 0.06 | 0.07 | 0.06 | 0.08 | 0.07 | 1.19 | 1.50 | 0.70 | 1.89 | 1.87 | 1.76 |
| 6 | 2000 | 0.11 | 0.04 | 0.03 | -0.02 | -0.01 | 0.03 | 0.82 | 1.09 | 0.48 | 1.28 | 1.29 | 1.22 |
| 6 | 8000 | 0.01 | 0.00 | 0.01 | -0.01 | -0.01 | 0.01 | 0.41 | 0.55 | 0.24 | 0.64 | 0.64 | 0.60 |
| 10 | 100 | 3.84 | 0.36 | - | -1.49 | 1.14 | - | 10.98 | 18.90 | - | 16.13 | 26.23 | - |
| 10 | 200 | 1.26 | 0.71 | 0.88 | -0.08 | 0.15 | 0.88 | 2.63 | 2.76 | 1.89 | 4.13 | 3.63 | 3.77 |
| 10 | 500 | 0.58 | 0.35 | 0.37 | 0.09 | 0.17 | 0.37 | 1.27 | 1.39 | 0.89 | 2.08 | 1.99 | 1.94 |
| 10 | 1000 | 0.31 | 0.20 | 0.20 | 0.05 | 0.08 | 0.20 | 0.80 | 0.94 | 0.59 | 1.34 | 1.34 | 1.27 |
| 10 | 2000 | 0.16 | 0.10 | 0.10 | 0.05 | 0.06 | 0.10 | 0.53 | 0.66 | 0.40 | 0.95 | 0.96 | 0.89 |
| 10 | 8000 | 0.03 | 0.02 | 0.02 | 0.00 | 0.00 | 0.02 | 0.25 | 0.32 | 0.19 | 0.47 | 0.47 | 0.44 |

Notes: "AAH" is the augmented Anderson and Hsiao 2-step GMM estimator based on the $(T-2)(T-1) / 2+T+(T+1) T / 2-3$ moment conditions in (7), (13), and (S.5), "AB" is 2-step GMM estimator based on the $T(T-1) / 2+(T+1) T / 2-1$ moment conditions in (S.4)-(S.5), and "BB" is 2-step GMM estimator based on the $T(T-1) / 2+(T+1) T / 2-1+2(T-1)$ moment conditions in (S.4)-(S.7). See Subsection S-2.2 in Appendix for further details. The DGP is given by $y_{i t}=\alpha_{i}+\phi y_{i, t-1}+\beta x_{i t}+u_{i t}$, for $i=1,2, \ldots, n$, and $t=1,2, \ldots, T$, with initial values given by $y_{i, 0}=\mu_{i}+\kappa \varepsilon_{i}+v_{i}, x_{i, 0}=\mu_{x i}+I I D N(0,1)$, where $\mu_{i}=\left(\alpha_{i}+\mu_{x i}\right) /(1-\phi), \mu_{x, i} \sim \operatorname{IIDN}(1,1), \alpha_{i}=\sum_{t=1}^{T} \rho^{t} u_{i t}+\varepsilon_{i}, \varepsilon_{i} \sim \operatorname{IIDN}(1,1)$, and $v_{i} \sim \operatorname{IIDN}(0,1)$. This table reports findings for experiments where $\kappa=\rho=0$, namely AB and BB restrictions are met. BB restrictions are not satisfied when $\kappa \neq 0$, and AB restrictions are not satisfied when $\rho \neq 0$. Errors $u_{i t}$ are generated to be cross-sectionally heteroskedastic and non-normal, $u_{i t}=\left(e_{i t}-2\right) \sigma_{i a} / 2$ for $t \leq[T / 2]$, and $u_{i t}=\left(e_{i t}-2\right) \sigma_{i b} / 2$ for $t>[T / 2]$, with $\sigma_{i a}^{2} \sim I I D U(0.25,0.75)$, $\sigma_{i b}^{2} \sim I I D U(1,2), e_{i t} \sim I I D \chi^{2}(2)$, and $[T / 2]$ is the integer part of $T / 2$. Errors $\epsilon_{i t}$ are generated as $\epsilon_{i t} \sim I I D N\left(0,1-\theta^{2}\right)$. See Subsection S-2.1 for a full description of the MC experiments. The number of time periods available for estimation is $T+1$, namely $\left(x_{i 0}, y_{i 0}\right),\left(x_{i 1}, y_{i 1}\right), \ldots,\left(x_{i T} y_{i T}\right)$, is available for $i=1,2, \ldots, n$.

Table S2: Size and Power of AAH, AB and BB estimators in panel ARX(1) experiments when both Arellano and Bond (AB) and Blundell and Bond (BB)
restrictions are met
$\rho=0$, and $\kappa=0$

| $T$ | $n$ | Size (5\% level, $\times 100$ ) |  |  |  |  |  | Power (5\% level, $\times 100$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  | $H_{1}: \phi=\phi_{0}+0.1$ |  |  | $H_{1}: \beta_{0}=\beta_{0}+0.1$ |  |  |
|  |  | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB |
| 4 | 100 | 10.60 | 11.35 | 17.90 | 8.75 | 11.70 | 17.40 | 11.40 | 12.55 | 20.60 | 10.20 | 12.55 | 19.55 |
| 4 | 200 | 6.95 | 7.75 | 10.95 | 5.85 | 7.35 | 10.10 | 8.65 | 10.55 | 23.60 | 7.85 | 9.05 | 12.80 |
| 4 | 500 | 5.40 | 6.45 | 7.60 | 4.80 | 5.85 | 6.20 | 10.65 | 9.10 | 43.40 | 8.10 | 9.85 | 12.65 |
| 4 | 1000 | 6.05 | 6.25 | 6.25 | 5.65 | 6.25 | 6.75 | 17.55 | 13.25 | 71.55 | 13.35 | 13.90 | 16.70 |
| 4 | 2000 | 4.85 | 4.45 | 5.20 | 5.20 | 5.90 | 5.60 | 28.90 | 19.30 | 94.80 | 19.60 | 20.20 | 25.10 |
| 4 | 8000 | 5.10 | 5.00 | 5.00 | 5.30 | 5.40 | 5.50 | 82.50 | 60.35 | 100.00 | 62.55 | 62.40 | 67.35 |
| 6 | 100 | 25.60 | 24.30 | 39.00 | 27.60 | 25.75 | 37.50 | 24.10 | 26.45 | 40.90 | 30.20 | 26.80 | 42.50 |
| 6 | 200 | 13.65 | 13.45 | 19.95 | 11.55 | 11.45 | 16.40 | 14.95 | 16.45 | 33.20 | 15.85 | 15.50 | 25.65 |
| 6 | 500 | 8.50 | 8.40 | 9.95 | 7.40 | 7.90 | 11.00 | 20.80 | 16.40 | 54.90 | 15.40 | 16.55 | 24.60 |
| 6 | 1000 | 6.60 | 5.60 | 6.90 | 6.45 | 5.75 | 6.70 | 35.45 | 25.85 | 83.60 | 20.20 | 20.85 | 28.00 |
| 6 | 2000 | 5.90 | 5.50 | 6.05 | 4.10 | 4.70 | 5.75 | 64.05 | 43.90 | 98.95 | 34.90 | 34.50 | 43.25 |
| 6 | 8000 | 5.10 | 5.65 | 5.80 | 4.80 | 4.80 | 4.45 | 99.60 | 95.60 | 100.00 | 86.45 | 87.05 | 92.30 |
| 10 | 100 | 94.90 | 97.25 | - | 94.40 | 97.35 | - | 95.50 | 96.80 | - | 95.45 | 96.80 | - |
| 10 | 200 | 44.20 | 38.25 | 55.40 | 40.50 | 36.65 | 47.25 | 40.10 | 41.65 | 57.85 | 44.60 | 40.35 | 57.00 |
| 10 | 500 | 18.90 | 13.40 | 18.95 | 14.30 | 13.10 | 17.10 | 42.50 | 37.55 | 73.55 | 30.25 | 28.95 | 43.35 |
| 10 | 1000 | 10.80 | 8.70 | 12.55 | 8.70 | 7.80 | 8.85 | 72.30 | 58.40 | 94.35 | 37.85 | 36.85 | 51.90 |
| 10 | 2000 | 7.65 | 6.70 | 7.80 | 6.45 | 6.60 | 7.00 | 96.45 | 85.50 | 99.85 | 59.95 | 58.35 | 70.65 |
| 10 | 8000 | 5.65 | 5.60 | 5.35 | 5.35 | 5.20 | 5.35 | 100.00 | 100.00 | 100.00 | 99.35 | 99.20 | 99.65 |

See the notes to Table S1

Table S3: Bias and RMSE of AAH, AB and BB estimators in panel ARX(1) experiments when Arellano Bond (AB) restrictions are met and Blundell and Bond (BB) restrictions are not met

$$
\rho=0, \text { and } \kappa=1
$$

|  | $n$ | Bias ( $\times 100$ ) |  |  |  |  |  | RMSE ( $\times 100$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  |
| T |  | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB |
| 4 | 100 | 1.50 | -0.17 | 3.12 | 0.04 | 0.44 | -0.04 | 7.36 | 7.35 | 3.77 | 8.76 | 8.17 | 8.08 |
| 4 | 200 | 0.77 | -0.09 | 3.19 | -0.19 | 0.08 | 0.45 | 4.62 | 5.08 | 3.52 | 5.73 | 5.48 | 5.40 |
| 4 | 500 | 0.32 | 0.02 | 3.25 | 0.00 | 0.10 | 1.10 | 2.73 | 3.19 | 3.38 | 3.54 | 3.49 | 3.58 |
| 4 | 1000 | 0.23 | 0.02 | 3.32 | -0.05 | 0.00 | 1.16 | 1.95 | 2.27 | 3.38 | 2.56 | 2.54 | 2.70 |
| 4 | 2000 | 0.10 | 0.05 | 3.32 | 0.03 | 0.06 | 1.33 | 1.33 | 1.58 | 3.35 | 1.78 | 1.78 | 2.18 |
| 4 | 8000 | 0.01 | 0.01 | 3.35 | -0.02 | -0.01 | 1.37 | 0.66 | 0.80 | 3.36 | 0.90 | 0.89 | 1.62 |
| 6 | 100 | 1.58 | 0.11 | 3.08 | -0.39 | 0.04 | 3.08 | 4.95 | 5.09 | 3.74 | 7.71 | 6.50 | 6.81 |
| 6 | 200 | 1.00 | 0.26 | 3.03 | 0.00 | 0.20 | 3.03 | 3.00 | 3.31 | 3.31 | 4.53 | 4.21 | 4.20 |
| 6 | 500 | 0.47 | 0.24 | 3.05 | -0.04 | 0.01 | 3.05 | 1.74 | 1.99 | 3.16 | 2.73 | 2.68 | 2.84 |
| 6 | 1000 | 0.24 | 0.04 | 3.05 | 0.06 | 0.09 | 3.05 | 1.16 | 1.36 | 3.11 | 1.89 | 1.87 | 2.34 |
| 6 | 2000 | 0.11 | 0.03 | 3.09 | -0.02 | -0.01 | 3.09 | 0.81 | 0.98 | 3.12 | 1.28 | 1.29 | 2.03 |
| 6 | 8000 | 0.01 | 0.00 | 3.11 | -0.01 | -0.01 | 3.11 | 0.40 | 0.50 | 3.12 | 0.64 | 0.65 | 1.84 |
| 10 | 100 | 3.82 | 0.35 | - | -1.69 | 0.78 | - | 10.72 | 18.94 | - | 16.14 | 27.32 | - |
| 10 | 200 | 1.24 | 0.61 | 2.74 | -0.04 | 0.19 | 2.74 | 2.60 | 2.52 | 3.07 | 4.14 | 3.66 | 3.69 |
| 10 | 500 | 0.57 | 0.29 | 2.55 | 0.11 | 0.20 | 2.55 | 1.25 | 1.28 | 2.65 | 2.09 | 2.01 | 2.28 |
| 10 | 1000 | 0.30 | 0.18 | 2.55 | 0.06 | 0.10 | 2.55 | 0.78 | 0.86 | 2.60 | 1.35 | 1.35 | 2.00 |
| 10 | 2000 | 0.16 | 0.09 | 2.55 | 0.06 | 0.07 | 2.55 | 0.52 | 0.60 | 2.57 | 0.95 | 0.96 | 1.97 |
| 10 | 8000 | 0.03 | 0.02 | 2.55 | 0.00 | 0.00 | 2.55 | 0.24 | 0.29 | 2.56 | 0.47 | 0.47 | 1.92 |

Notes: See notes to Table S1.

Table S4: Size and Power of AAH, AB and BB estimators in panel ARX(1) experiments when Arellano Bond (AB) restrictions are met and Blundell and Bond (BB) restrictions are not met

$$
\rho=0, \text { and } \kappa=1
$$

| $T$ | $n$ | Size (5\% level, $\times 100$ ) |  |  |  |  |  | Power (5\% level, $\times 100$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  | $H_{1}: \phi=\phi_{0}+0.1$ |  |  | $H_{1}: \beta_{0}=\beta_{0}+0.1$ |  |  |
|  |  | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB |
| 4 | 100 | 10.00 | 11.80 | 56.20 | 8.65 | 11.60 | 16.35 | 11.20 | 13.55 | 25.15 | 10.15 | 12.60 | 18.20 |
| 4 | 200 | 6.50 | 8.40 | 71.75 | 5.45 | 7.50 | 9.40 | 9.20 | 10.95 | 26.10 | 7.75 | 9.15 | 11.05 |
| 4 | 500 | 5.45 | 6.15 | 95.75 | 4.80 | 5.80 | 8.05 | 10.70 | 10.55 | 38.05 | 8.10 | 9.70 | 7.85 |
| 4 | 1000 | 5.95 | 6.00 | 99.95 | 5.70 | 6.30 | 8.60 | 17.55 | 15.50 | 61.45 | 13.10 | 13.80 | 7.60 |
| 4 | 2000 | 5.10 | 5.00 | 100.00 | 5.10 | 5.90 | 13.70 | 30.10 | 23.25 | 87.15 | 19.55 | 19.95 | 7.35 |
| 4 | 8000 | 5.05 | 4.75 | 100.00 | 5.25 | 5.25 | 37.30 | 83.95 | 70.15 | 100.00 | 62.65 | 62.40 | 12.00 |
| 6 | 100 | 24.90 | 24.65 | 74.20 | 27.20 | 25.35 | 34.70 | 24.35 | 26.45 | 43.85 | 29.45 | 26.55 | 37.65 |
| 6 | 200 | 13.75 | 14.10 | 82.10 | 11.50 | 11.30 | 16.25 | 15.10 | 17.50 | 34.35 | 15.20 | 16.00 | 18.85 |
| 6 | 500 | 8.40 | 8.55 | 97.85 | 7.35 | 7.85 | 12.30 | 20.90 | 18.20 | 40.65 | 15.10 | 16.30 | 11.35 |
| 6 | 1000 | 6.85 | 5.65 | 100.00 | 6.55 | 5.55 | 16.35 | 36.40 | 31.70 | 59.20 | 19.95 | 20.85 | 7.30 |
| 6 | 2000 | 5.75 | 5.10 | 100.00 | 4.00 | 4.55 | 26.55 | 65.75 | 52.20 | 85.90 | 34.30 | 34.25 | 6.95 |
| 6 | 8000 | 4.80 | 5.15 | 100.00 | 4.85 | 4.85 | 79.45 | 99.70 | 97.85 | 100.00 | 86.55 | 86.50 | 6.85 |
| 10 | 100 | 95.95 | 97.25 | - | 93.95 | 97.35 | - | 94.70 | 97.00 | - | 94.70 | 97.05 | - |
| 10 | 200 | 45.55 | 38.55 | 89.95 | 40.00 | 35.85 | 44.95 | 40.00 | 43.95 | 58.50 | 44.85 | 40.55 | 49.70 |
| 10 | 500 | 18.65 | 13.85 | 98.70 | 14.75 | 13.25 | 24.25 | 43.75 | 43.85 | 36.35 | 29.80 | 27.95 | 19.30 |
| 10 | 1000 | 11.00 | 9.15 | 100.00 | 8.15 | 7.70 | 32.15 | 73.50 | 67.25 | 40.00 | 37.40 | 36.10 | 10.55 |
| 10 | 2000 | 7.90 | 6.95 | 100.00 | 6.40 | 6.80 | 55.75 | 97.25 | 91.45 | 53.60 | 59.00 | 57.60 | 8.00 |
| 10 | 8000 | 5.65 | 5.55 | 100.00 | 5.10 | 5.20 | 98.95 | 100.00 | 100.00 | 94.20 | 99.20 | 99.10 | 6.85 |

See the notes to Table S1

Table S5: Bias and RMSE of AAH, AB and BB estimators in panel ARX(1) experiments when Arellano Bond (AB) restrictions and Blundell and Bond (BB) restrictions are not met

$$
\rho=0.8, \text { and } \kappa=1
$$

| $T$ | $n$ | Bias ( $\times 100$ ) |  |  |  |  |  | RMSE $(\times 100)$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  |
|  |  | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB |
| 4 | 100 | 1.48 | -4.16 | 1.86 | 0.00 | 0.46 | -0.26 | 7.37 | 9.22 | 2.88 | 8.78 | 8.83 | 8.36 |
| 4 | 200 | 0.77 | -3.56 | 1.86 | -0.21 | 0.27 | 0.15 | 4.62 | 6.86 | 2.41 | 5.73 | 5.87 | 5.49 |
| 4 | 500 | 0.32 | -2.80 | 1.93 | 0.00 | 0.43 | 0.59 | 2.73 | 4.66 | 2.15 | 3.54 | 3.76 | 3.55 |
| 4 | 1000 | 0.23 | -2.58 | 2.01 | -0.05 | 0.35 | 0.58 | 1.95 | 3.81 | 2.12 | 2.56 | 2.72 | 2.55 |
| 4 | 2000 | 0.10 | -2.39 | 2.03 | 0.03 | 0.39 | 0.69 | 1.33 | 3.09 | 2.09 | 1.78 | 1.98 | 1.94 |
| 4 | 8000 | 0.01 | -2.31 | 2.09 | -0.02 | 0.36 | 0.76 | 0.66 | 2.53 | 2.10 | 0.90 | 1.06 | 1.20 |
| 6 | 100 | 1.58 | -1.44 | 2.49 | -0.40 | 0.20 | 2.49 | 4.95 | 5.58 | 3.25 | 7.71 | 6.64 | 6.96 |
| 6 | 200 | 1.00 | -1.05 | 2.40 | 0.00 | 0.45 | 2.40 | 3.00 | 3.62 | 2.75 | 4.53 | 4.37 | 4.30 |
| 6 | 500 | 0.47 | -0.97 | 2.42 | -0.04 | 0.30 | 2.42 | 1.74 | 2.34 | 2.56 | 2.73 | 2.76 | 2.73 |
| 6 | 1000 | 0.24 | -1.04 | 2.44 | 0.06 | 0.41 | 2.44 | 1.16 | 1.79 | 2.51 | 1.89 | 1.97 | 2.08 |
| 6 | 2000 | 0.11 | -1.03 | 2.48 | -0.02 | 0.33 | 2.48 | 0.81 | 1.49 | 2.51 | 1.28 | 1.40 | 1.70 |
| 6 | 8000 | 0.01 | -1.03 | 2.51 | -0.01 | 0.33 | 2.51 | 0.40 | 1.17 | 2.52 | 0.64 | 0.75 | 1.35 |
| 10 | 100 | 3.82 | -0.01 | - | -1.69 | 0.69 | - | 10.72 | 18.21 | - | 16.14 | 25.94 | - |
| 10 | 200 | 1.24 | 0.25 | 2.56 | -0.04 | 0.18 | 2.56 | 2.59 | 2.50 | 2.91 | 4.14 | 3.69 | 3.70 |
| 10 | 500 | 0.57 | -0.07 | 2.38 | 0.11 | 0.21 | 2.38 | 1.25 | 1.25 | 2.48 | 2.09 | 2.02 | 2.19 |
| 10 | 1000 | 0.30 | -0.20 | 2.38 | 0.06 | 0.11 | 2.38 | 0.78 | 0.87 | 2.43 | 1.35 | 1.36 | 1.84 |
| 10 | 2000 | 0.16 | -0.29 | 2.38 | 0.06 | 0.10 | 2.38 | 0.52 | 0.66 | 2.41 | 0.95 | 0.97 | 1.76 |
| 10 | 8000 | 0.03 | -0.36 | 2.38 | 0.00 | 0.04 | 2.38 | 0.24 | 0.47 | 2.39 | 0.47 | 0.48 | 1.68 |

Notes: See notes to Table S1.

Table S6: Size and Power of AAH, AB and BB estimators in panel ARX(1) experiments when Arellano Bond (AB) restrictions and Blundell and Bond (BB) restrictions are not met

$$
\rho=0.8, \text { and } \kappa=1
$$

| $T$ | $n$ | Size (5\% level, $\times 100$ ) |  |  |  |  |  | Power (5\% level, $\times 100$ ) |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $\phi_{0}=0.8$ |  |  | $\beta_{0}=0.5$ |  |  | $H_{1}: \phi=\phi_{0}+0.1$ |  |  | $H_{1}: \beta_{0}=\beta_{0}+0.1$ |  |  |
|  |  | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB | AAH | AB | BB |
| 4 | 100 | 10.05 | 17.00 | 35.15 | 8.65 | 15.35 | 17.20 | 11.25 | 22.20 | 19.30 | 10.15 | 16.70 | 18.35 |
| 4 | 200 | 6.50 | 15.20 | 40.10 | 5.45 | 9.90 | 10.25 | 9.25 | 23.85 | 13.15 | 7.75 | 12.05 | 12.10 |
| 4 | 500 | 5.45 | 14.85 | 64.30 | 4.80 | 8.95 | 7.90 | 10.70 | 31.30 | 9.70 | 8.10 | 12.10 | 10.00 |
| 4 | 1000 | 5.95 | 21.15 | 89.10 | 5.70 | 9.70 | 7.60 | 17.55 | 48.25 | 9.35 | 13.10 | 14.35 | 11.40 |
| 4 | 2000 | 5.10 | 30.80 | 99.15 | 5.10 | 9.95 | 9.30 | 30.10 | 69.50 | 8.90 | 19.55 | 18.60 | 14.10 |
| 4 | 8000 | 5.05 | 71.55 | 100.00 | 5.25 | 10.85 | 16.95 | 83.95 | 99.15 | 11.55 | 62.65 | 48.40 | 32.60 |
| 6 | 100 | 24.95 | 25.85 | 63.85 | 27.15 | 26.55 | 36.70 | 24.40 | 33.90 | 39.90 | 29.40 | 26.85 | 39.25 |
| 6 | 200 | 13.75 | 15.00 | 69.80 | 11.50 | 13.50 | 17.00 | 15.10 | 26.30 | 22.85 | 15.20 | 16.30 | 20.35 |
| 6 | 500 | 8.40 | 10.60 | 92.10 | 7.35 | 9.40 | 10.10 | 20.90 | 36.40 | 18.25 | 15.10 | 15.65 | 15.35 |
| 6 | 1000 | 6.85 | 12.65 | 99.55 | 6.55 | 7.80 | 11.35 | 36.40 | 59.35 | 21.50 | 19.95 | 17.50 | 11.50 |
| 6 | 2000 | 5.75 | 19.70 | 100.00 | 4.00 | 7.30 | 16.10 | 65.75 | 83.80 | 32.20 | 34.30 | 28.05 | 12.60 |
| 6 | 8000 | 4.80 | 52.35 | 100.00 | 4.85 | 9.45 | 49.40 | 99.70 | 99.90 | 80.35 | 86.55 | 72.35 | 25.90 |
| 10 | 100 | 95.90 | 97.40 | - | 93.95 | 96.40 | - | 94.70 | 96.90 | - | 94.70 | 97.40 | - |
| 10 | 200 | 45.55 | 37.20 | 88.50 | 39.95 | 35.90 | 45.35 | 40.00 | 47.65 | 55.15 | 44.85 | 40.45 | 50.05 |
| 10 | 500 | 18.65 | 11.70 | 98.25 | 14.75 | 13.50 | 21.75 | 43.75 | 55.45 | 29.95 | 29.80 | 27.45 | 20.55 |
| 10 | 1000 | 11.00 | 8.75 | 100.00 | 8.15 | 8.10 | 26.60 | 73.50 | 80.35 | 30.00 | 37.40 | 35.75 | 13.35 |
| 10 | 2000 | 7.90 | 9.65 | 100.00 | 6.40 | 6.80 | 45.30 | 97.25 | 97.85 | 36.00 | 59.00 | 56.15 | 11.50 |
| 10 | 8000 | 5.65 | 23.55 | 100.00 | 5.10 | 6.15 | 95.65 | 100.00 | 100.00 | 73.60 | 99.20 | 98.85 | 15.30 |

See the notes to Table S1


[^0]:    ${ }^{24}$ We are grateful to an anonymous referee for suggesting these Monte Carlo experiments.

