On the Interpretation of Panel Unit Root Tests*

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Commenting on Bayoumi and MacDonald (1999), Sarno and Taylor (2000) raise a logical issue concerning the interpretation of panel unit root tests and as an alternative suggest the use of system-based cointegration tests a là Johansen. Sarno and Taylor (ST) argue that they have identified a "logical flaw" in panel unit root tests that invalidate them for the analysis of the type undertaken by Bayoumi and MacDonald (BM). Their argument is based on the following articulation of the null and the alternative hypotheses that they claim underlie panel unit root tests:

$$H_0 \quad : \quad \text{All series in the panel are generated by a unit root process.}$$

$$H_1^{ST} \quad : \quad \text{At least one of the series in the panel is generated by a stationary process.}$$

In their reply Bayoumi and MacDonald (2000) agree with the above formulation of the null hypothesis, but introduce the following alternative hypothesis:

$$H_1^{BM} : \text{Each of the series are stationary as a panel.}$$

While the formulation of the null hypothesis is relatively uncontroversial, neither of the above two formulations of the alternative hypothesis is satisfactory. To see

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why this is so consider the following simple dynamic heterogeneous panel on N cross sections observed over T time periods:

$$y_{it} = (1 - \phi_i)\,\mu_i + \phi_i y_{i,t-1} + \varepsilon_{it},\ i = 1, 2, ..., N;\ t = 1, 2, ..., T, \tag{1}$$

where initial values, y_{i0}, are given. (1) can be expressed as

$$\Delta y_{it} = \alpha_i + \beta_i y_{i,t-1} + \varepsilon_{it}, \tag{2}$$

where $\alpha_i = (1 - \phi_i)\,\mu_i,\ \beta_i = -(1 - \phi_i)$ and $\Delta y_{it} = y_{it} - y_{i,t-1}$. The null hypothesis of unit roots can then be written as

$$H_0 : \beta_i = 0 \text{ for all } i. \tag{3}$$

The specification of the alternative hypothesis critically depends on what assumption one makes about the nature of the homogeneity/heterogeneity of the panel. Under a homogeneous alternative we have $\beta_i = \beta \neq 0$ for all i, and BM are justified and H_1^{BM} is meaningful. Harris and Tzavalis (1999) consider this case in the context of a simple dynamic panel. But as ST point out one of the attractive features of heterogeneous panel unit root tests is the fact that they allow for heterogeneous dynamics across i, and the homogeneous alternative H_1^{BM}, as put forward in BM's reply is likely to be unduly restrictive, particularly for cross-country studies involving differing short-run dynamics. This homogeneous alternative seems particularly inappropriate in the case of the purchasing power parity (PPP) hypothesis, where y_{it} is taken to be the real exchange rate. There are no theoretical grounds for the imposition of the homogeneity hypothesis, $\beta_i = \beta$, under PPP.

The alternative hypothesis favoured by ST stands at the other extreme and in terms of the above notations states that

$$H_1^{ST} : \beta_i < 0, \text{ for one or more } i.$$

This alternative is only appropriate when N is finite, namely the multivariate model in a fixed number of variables analyzed in the time series literature. In heterogeneous panel unit root tests both N and T are assumed to be sufficiently large, and in the case of the Levin and Lin (1993) test it is also required that $N/T \to 0$, as both N and $T \to \infty$. It is now easily seen that in the case of large N and T panels the panel unit root test will be inconsistent (will have power equal to the test size) if the alternative, H_1^{ST}, is adopted. Therefore, ST's formulation will not be satisfactory. In this context a more appropriate alternative is given by

$$H_1 : \beta_i < 0,\ i = 1, 2, \ldots, N_1,\ \beta_i = 0,\ i = N_1 + 1, N_1 + 2, \ldots, N, \tag{4}$$

such that

$$\lim_{N \to \infty} \frac{N_1}{N} = \delta,\ 0 < \delta \leq 1. \tag{5}$$

Using the above specification the null and the alternative hypotheses can also be written as

$$\begin{aligned} H_0 &: \delta = 0. \\ H_1 &: \delta > 0. \end{aligned}$$

In other words, rejection of the unit root null hypothesis can be interpreted as providing evidence in favour of rejecting the unit root hypothesis for a non-zero fraction of panel members as $N \to \infty$. Therefore, for large N and T panels it is reasonable to entertain alternatives that lie somewhere between the two extremes of H_1^{BM} and H_1^{ST}.

The second point of contention between ST and BM is over how best to test for unit roots in panels. The problem here is also related to the relative size of N and T. In the analysis of U.S. states, BM have $N = 47$ which is large relative to $T = 30$ (recall that the application of the Levin and Lin (LL) test requires that N/T should be reasonably small!), while in the case of the Canadian data set $N = 9$, which is small both absolutely as well as relative to $T = 23$.[1] It is not clear that the LL testing procedure is appropriate in either of these two cases. Also the LL test is known to have undesirable small sample properties as documented in Im, Pesaran and Shin (1997). On the other hand, BM are correct when they argue that the cointegration type unit-root tests favoured by ST are likely to be relevant only when N is small and T relatively large.[2]

The inter-change between ST and BM highlights the differing nature of the null and the alternative hypotheses in dynamic heterogeneous panels. The heterogeneity of panel data models used in cross-country analysis introduces a new

[1] Note that the validity of the LL test requires N to be sufficiently large in an absolute sense, and at the same time small relative to T.

[2] A number of other approaches to unit root testing in heterogeneous panels have also been proposed in the literature. See, for example, Bowman (1999), Choi (1999), Hadri (1999), and Maddala and Wu (1999). A review of this literature is provided in Baltagi and Kao (2000).

kind of asymmetry in the way the null and the alternative hypotheses are treated, which is not usually present in the univariate time series (or cross-section) models. This is because the same null hypothesis is imposed across all i but the specification of the alternative hypothesis is allowed to vary with i. This asymmetry is assumed away in homogeneous panels. However, as demonstrated in Pesaran and Smith (1995) neglected heterogeneity (even if purely random) can lead to spurious results in dynamic panels. Therefore, in cross-country analysis of the type undertaken by BM and ST where slope heterogeneity is a norm, the asymmetry of the null and the alternative hypotheses has to be taken into account. The appropriate response critically depends on the relative size of N and T. In large N heterogeneous panel data models with T small (say around 15) it is only possible to devise sufficiently powerful unit root tests which are informative in some average sense, namely whether the null of a unit root can be rejected in the case of a significant fraction of the countries in the panel.[3] To identify the exact proportion of the sample for which the null hypothesis is rejected one requires country-specific data sets with T sufficiently large. But if T is large enough for reliable country-specific inferences to be made, then there seems little rationale in pooling countries into a panel.

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[3]Some of these difficulties can be circumvented if slope heterogeneity can be modelled in a sensible and parsimonious manner.

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