Supplement to

An Exponential Class of Dynamic Binary Choice Panel Data Models with Fixed Effects

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1 Monte Carlo Evidence on Average Partial Under a Probit Specification

Here we repeat the exercise in subsection 5.4 of Al-Sadoon, Li and Pesaran (2016) using a probit specification rather than a logit. Thus, we take the DGP to be

$$\Pr(y_{it} = 1 \,|y_{i,t-1}, c_{ip}, x_{it}\,) = \Phi(\rho_p y_{i,t-1} + \beta_p x_{it} + c_{ip}),$$

where $\Phi(\cdot)$ is the CDF of the standard normal. Then the marginal effect for x_{it} is

$$\frac{\partial \Pr\left(y_{it} = 1|y_{i,t-1}, c_{ip}, x_{it}\right)}{\partial x_{it}} = \phi(\rho_p y_{i,t-1} + \beta_p x_{it} + c_{ip})\beta_p,$$

where $\phi(z) = \frac{\partial}{\partial z}\Phi(z) = \frac{1}{\sqrt{2\pi}}e^{-z^2/2}$. On the other hand, the marginal effect of $y_{i,t-1}$ is given as

$$\Pr\left(y_{it} = 1\,|y_{i,t-1} = 1, c_{ip}, x_{it}\,\right) - \Pr\left(y_{it} = 1\,|y_{i,t-1} = 0, c_{il}, x_{it}\,\right) = \Phi(\rho_p + \beta_p x_{it} + c_{ip}) - \Phi(\beta_p x_{it} + c_{ip}).$$

The average marginal effects may be calculated as,

$$\begin{aligned}
APEX\left(y_{i,t-1} = 1,\, x_{it} = \bar{x}\right) &= \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \phi(\rho_p + \beta_p \bar{x} + c_{ip})\beta_p, \\
APEX\left(y_{i,t-1} = 0,\, x_{it} = \bar{x}\right) &= \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \phi(\beta_p \bar{x} + c_{ip})\beta_p, \\
APEY\left(x_{it} = \bar{x}\right) &= \lim_{N\to\infty} \frac{1}{N}\sum_{i=1}^{N} \left[\Phi(\rho_p + \beta_p \bar{x} + c_{ip}) - \Phi(\beta_p \bar{x} + c_{ip})\right],
\end{aligned}$$

where the averages over i are obtained by drawing from the distribution of c_{ip}.

Now given ρ_e and β_e under the exponential specification, we can obtain the corresponding exponential fixed effects as

$$1 - e^{-c_{ie} - \boldsymbol{\beta}_e \bar{x}_i} = \Phi(c_{ip} + \beta_p \bar{x}_i),$$

which yields

$$e^{-c_{ie}} = e^{\beta_e \bar{x}_i} \left[1 - \Phi(c_{ip} + \beta_p \bar{x}_i)\right].$$

We may then estimate the average partial effects as

$$\widehat{APEX}\left(y_{i,t-1} = 1,\, x_{it} = \bar{x}\right) = \widehat{\beta_e} e^{-\widehat{\rho_e} - \widehat{\beta}_e \bar{x}} \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} e^{\hat{\beta}_e \bar{x}_i} (1 - \Phi(c_{ip} + \beta_p \bar{x}_i)),$$

$$\widehat{APEX}\left(y_{i,t-1} = 0,\, x_{it} = \bar{x}\right) = \widehat{\beta_e} e^{-\widehat{\beta}_e \bar{x}} \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} e^{\hat{\beta}_e \bar{x}_i} (1 - \Phi(c_{ip} + \beta_p \bar{x}_i)),$$

$$\widehat{APEY}\left(x_{it} = \bar{x}\right) = e^{-\hat{\beta}_e \bar{x}} (1 - e^{-\hat{\rho}_e}) \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} e^{\hat{\beta}_e \bar{x}_i} (1 - \Phi(c_{ip} + \beta_p \bar{x}_i)).$$

We use the same parameters as in Section 5.4 of the paper, namely $\rho_l = 0.5$, $\beta_l = 1$, $x_{it} \sim N(0, \pi^2/3)$, and $c_{il} \sim N(0,1)$. To minimize the impact of the initial values we drop the first 100 observations, while being careful to keep x_{it} fixed across replications. The simulations are based on $N = 1,000$, $T = 3$, and each experiment is repeated $2,000$ times to obtain the mean, variance, bias, and RMSE of the APEs. We vary the DGP and the data sets just as we did in the exercise using the logistic distribution as the DGP.

Probit vs. Implied Exponential Average Partial Effects.

Experiment	1	2	3	4	5	6	7	8	9	10	11
$APEX_1$	0.2641	0.2429	0.2784	0.3967	0.1319	0.2127	0.3223	0.2643	0.2640	0.2695	0.2636
Mean $\widehat{APEX}_1$	0.1566	0.1441	0.1697	0.1703	0.1151	0.1469	0.1629	0.1297	0.1941	0.1667	0.1484
Variance $\widehat{APEX}_1$	0.0003	0.0003	0.0003	0.0002	0.0002	0.0002	0.0003	0.0002	0.0005	0.0000	0.0001
Bias $\widehat{APEX}_1$	-0.1075	-0.0988	-0.1086	-0.2264	-0.0169	-0.0658	-0.1594	-0.1346	-0.0698	-0.1028	-0.1152
RMSE $\widehat{APEX}_1$	0.1089	0.1001	0.1101	0.2268	0.0222	0.0676	0.1603	0.1352	0.0732	0.1030	0.1154
$APEX_0$	0.2842	0.2842	0.2842	0.4263	0.1421	0.2237	0.3578	0.2842	0.2842	0.2857	0.2820
Mean $\widehat{APEX}_0$	0.1879	0.1877	0.1877	0.2016	0.1399	0.1638	0.2184	0.1539	0.2361	0.1993	0.1906
Variance $\widehat{APEX}_0$	0.0004	0.0004	0.0003	0.0003	0.0001	0.0002	0.0005	0.0002	0.0004	0.0000	0.0001
Bias $\widehat{APEX}_0$	-0.0963	-0.0965	-0.0965	-0.2247	-0.0021	-0.0599	-0.1393	-0.1303	-0.0480	-0.0864	-0.0915
RMSE $\widehat{APEX}_0$	0.0982	0.0986	0.0981	0.2253	0.0108	0.0612	0.1411	0.1312	0.0519	0.0867	0.0918
$APEY$	0.1385	0.2020	0.0705	0.1386	0.1384	0.1098	0.1727	0.1385	0.1384	0.1403	0.1378
Mean $\widehat{APEY}$	0.0703	0.0987	0.0401	0.0599	0.0831	0.0456	0.1022	0.0628	0.0799	0.0725	0.1030
Variance $\widehat{APEY}$	0.0012	0.0011	0.0013	0.0008	0.0022	0.0014	0.0012	0.0010	0.0016	0.0002	0.0002
Bias $\widehat{APEY}$	-0.0682	-0.1033	-0.0304	-0.0787	-0.0553	-0.0642	-0.0706	-0.0758	-0.0586	-0.0678	-0.0348
RMSE $\widehat{APEY}$	0.0767	0.1085	0.0472	0.0839	0.0727	0.0746	0.0786	0.0824	0.0710	0.0691	0.0374

The average partial effects are $APEX_1 = \int \frac{\partial P[y_{it}=1|y_{i,t-1}=1,c_{ip},x_{it}]}{\partial x_{it}}\Big|_{x_{it}=\bar{x}} dF_c(c_i)$, $APEX_0 = \int \frac{\partial P[y_{it}=1|y_{i,t-1}=0,c_{ip},x_{it}]}{\partial x_{it}}\Big|_{x_{it}=\bar{x}} dF_c(c_i)$, and $APEY = \int \left(P[y_{it}=1\,|y_{i,t-1}=1, c_{ip}, x_{it}=\bar{x}] - P[y_{it}=1\,|y_{i,t-1}=0, c_{ip}, x_{it}=\bar{x}]\right) dF_c(c_i)$, where F_c is the distribution function of the fixed effects. See the discussion above for the calculation and estimation of these quantities.

The simulations are as follows: (1) the benchmark, (2) ρ increased to 0.75, (3) ρ decreased to 0.25, (4) β increased to 1.5, (5) β decreased to 0.5, (6) σ_c increased to 1.5, (7) σ_c decreased to 0.5, (8) σ_x increased by 0.5, (9) σ_x decreased by 0.5, (10) N increased to 10,000, (11) T increased to 8. Parameters are estimated using the full set of linear instruments.

REFERENCE

Al-Sadoon, Majid and Li, Tong and Pesaran, M. Hashem, An Exponential Class of Dynamic Binary Choice Panel Data Models with Fixed Effects (January 18, 2016). USC-INET Research Paper No. 16-03. Available at SSRN: http://ssrn.com/abstract=2720447 or http://dx.doi.org/10.2139/ssrn.2720447.