A Panel Unit Root Test in the Presence of a Multifactor Error Structure*

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Abstract

This paper extends the cross sectionally augmented panel unit root test proposed by Pesaran (2007) to the case of a multifactor error structure. The basic idea is to exploit information regarding the m unobserved factors that are shared by k other time series in addition to the variable under consideration. Initially we develop a test assuming that m^0, the true number of factors is known, and show that the limit distribution of the test does not depend on any nuisance parameters, so long as $k \geq m^0 - 1$. Small sample properties of the test are investigated by Monte Carlo experiments and shown to be satisfactory. Particularly, in contrast to other existing panel unit root tests, our test has correct size and reasonable power for the case with an intercept and a linear trend as well as with an intercept only, for all combinations of cross section and time series dimensions. An illustrative application is also provided where the proposed panel unit root test is applied to Fisher's inflation parity and real equity prices.

JEL-Classification: C12, C15, C22, C23

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1 Introduction

There is now a sizeable literature on testing for unit roots in panels where both cross section (N) and time (T) dimensions are relatively large. Reviews of this literature are provided in Banerjee (1999), Baltagi and Kao (2000), Choi (2004), and more recently in Breitung and Pesaran (2008). The so called first generation panel unit root tests pioneered by Levin, Lin and Chu (2002) and Im, Pesaran and Shin (2003) focussed on panels where the idiosyncratic errors were cross sectionally uncorrelated. More recently, to deal with a number of applications such as testing for purchasing power parity or output convergence, the interest has shifted to the case where the errors are allowed to be cross sectionally correlated using a residual factor structure. These second generation tests include the contributions of Moon and Perron (2004), Bai and Ng (2004) and Pesaran (2007).[1] The tests proposed by Moon and Perron (2004) and Pesaran (2007) assume that under the null of unit roots the common factor components have the same order of integration as the idiosyncratic components, whilst the test procedures of Bai and Ng (2004) allow the order of integration of the factors to differ from that of the idiosyncratic components, by assuming different processes generating the two. A small sample comparison of some of these tests is provided in Gengenbach, Palm and Urbain (2009).

In the case of the panel unit root test proposed by Pesaran (2007), the cross section dependence is accounted for by augmenting the individual ADF regressions of y_{it} with cross section averages of the dependent variable (current and lagged values, $\Delta\bar{y}_t$, $\bar{y}_{t-1} = N^{-1}\Sigma_{j=1}^{N} y_{j,t-1}$). These cross section averages are used as proxies for the assumed single unobserved common factor. The panel test statistic is then based on the average of the individual t-statistics over the cross section units and is shown to be free of nuisance parameters, although it has a non-normal limit distribution as N and $T \to \infty$. Monte Carlo experiments show that Pesaran's test has desirable small sample properties in the presence of a single unobserved common factor but show serious size distortions if the number of common factors exceeds unity. Bai and Ng (2004) consider whether the source of non-stationarity is due to the common factor and/or idiosyncratic component. Their method involves applying unit root tests to the common factors and the idiosyncratic component separately, where the unobserved factors are replaced with consistent estimates obtained by use of principal components (PC). The pooled tests they propose require an estimate of the true number of factors and the factors themselves. Moon and Perron (2004) follow a similar approach in that they base their test on a principal components estimator of common factors. In particular, their test is based on de-factored observations obtained by projecting the panel data onto the space orthogonal to the (estimated) factor loadings.

This paper extends Pesaran's test and proposes a simple panel unit root test that is valid in the more general case of multiple common factors. In so doing we utilise the information contained in a number of k additional variables, $\mathbf{x}_{it}$, that are assumed to share the same common factors as the original series of interest, y_{it}. The ADF regression for y_{it} is then augmented by the cross section averages of the dependent variable as well as the additional regressors.[2] The test assumes that there exists a number of variables that are simultaneously affected by

[1] Other panel unit root tests include that of Chang (2002) that employs a non-linear IV method to account for cross-section correlation and Phillips and Sul (2003) who use an orthogonalisation procedure to deal with dependence arising from a single common factor. The former is valid for a fixed N and large T.

[2] The idea of augmenting ADF regressions with other covariates has been investigated in the unit root literature by Hansen (1995) and Elliott and Jansson (2003). These authors consider the additional covariates in order to gain power when testing the unit root hypothesis in the case of a single time series. In this paper we augment ADF regressions with cross section averages to eliminate the effects of unobserved common factors in the case of panel unit root tests.

a given set of unobserved common factors. This requirement seems quite plausible in the case of panel data sets from economics and finance where economic agents often face common economic environments. For example, in testing for unit roots in a panel of real outputs one would expect the unobserved common shocks to output (that originate from technology) to also manifest themselves in employment, consumption and investment. In the case of testing for unit roots in inflation across countries, one would expect the unobserved common factors that cross correlate inflation rates to also affect short-term and long-term interest rates across markets and economies. The fundamental issue is to ascertain the nature of dependence and persistence that is observed across markets and over time. The present paper can, therefore, be viewed as a first step in the process of developing a coherent framework for the analysis of unit roots and multiple cointegration in large panels.

Initially we develop a test supposing that m^0, the true number of factors, is known and that all additional variables are $I(1)$ and not cointegrated among themselves. We show that the limit distribution of the test does not depend on the factor loadings or other nuisance parameters so long as $k \geq m^0 - 1$. But, in practice m^0 is rarely known. Given an assumed maximum number of factors, $m_{\max}$, we suggest two strategies for dealing with uncertainty that surrounds the value of m^0. One is to choose the number of additional regressors as $k = m_{\max} - 1$. In this case, the true number of factors are allowed to be any integer value between zero and $m_{\max}$. However, when $m_{\max}$ is assumed to be large, in some situations it can be difficult to find a sufficient number of suitable additional regressors. Another possibility is to estimate m^0 consistently using suitable selection criteria, as is followed in the literature, for example, by Bai and Ng (2004) and Moon and Perron (2004), amongst others.

The small sample properties of the proposed test are investigated by Monte Carlo experiments. The test is shown to have the correct size in a number of different experiments and for relatively small samples. This contrasts the results obtained for some of the prominent existing tests in the literature such as the pooled tests of Bai and Ng (2004) and Moon and Perron (2004) that tend to be over-sized.[3] In terms of power, when the model contains an intercept term only, the pooled tests tend to display higher power in smaller samples as compared to the proposed test, although this could partly reflect the over-sized nature of the pooled tests in small samples.[4] In the case of models with linear trends, our experimental results show that the proposed test can perform better than the pooled tests, both in terms of size and power. Empirical applications to Fisher's inflation parity and real equity prices across different economies illustrate how the proposed test performs in practice.

The plan of the paper is as follows. Section 2 presents the panel data model and the testing procedure and derives the asymptotic distribution of the proposed cross sectionally augmented panel unit root test. Section 3 describes the Monte Carlo experiments and reports the small sample results. Section 4 presents the empirical applications, and Section 5 provides some concluding remarks.

Notation: L denotes a lag operator such that $L^{\ell}\mathbf{x}_t = \mathbf{x}_{t-\ell}$, K denotes a finite positive constant such that $K < \infty$, $||\mathbf{A}|| = [tr(\mathbf{A}\mathbf{A}')]^{1/2}$, $\mathbf{A}^{-}$ denotes the generalised inverse of $\mathbf{A}$, $\mathbf{I}_q$ is a $q \times q$ identity matrix, $\boldsymbol{\tau}_q$ and $\mathbf{0}_q$ are $q \times 1$ vectors of ones and zeros, respectively, $\mathbf{0}_{q\times r}$ is a $q \times r$ null matrix, $\overset{N}{\Longrightarrow}$ ($\overset{N}{\to}$) denotes convergence in distribution (quadratic mean (q.m.) or mean square errors) with T fixed as $N \to \infty$, $\overset{T}{\Longrightarrow}$ ($\overset{T}{\to}$) denotes convergence in distribution (q.m.) with

[3] Westerlund and Larsson (2009) provide further theoretical results on the asymptotic validity of the pooled versions of the PANIC procedure.

[4] We do not present size-corrected power comparisons, since such results are likely to have limited value in empirical applications where such size corrections are not possible.

N fixed (or when there is no N-dependence) as $T \to \infty$, $\overset{N,T}{\Longrightarrow}$ denotes sequential convergence in distribution with $N \to \infty$ first followed by $T \to \infty$, $\overset{(N,T)_j}{\Longrightarrow}$ denotes joint convergence in distribution with N,$T \to \infty$ jointly with certain restrictions on the expansion rates of T and N to be specified, if any.

2 Panel Data Model and Tests

Let y_{it} be the observation on the i^{th} cross section unit at time t generated as

$$\Delta y_{it} = \beta_i(y_{i,t-1} - \boldsymbol{\alpha}'_{iy}\mathbf{d}_{t-1}) + \boldsymbol{\alpha}'_{iy}\Delta\mathbf{d}_t + u_{it},\ i = 1, 2, ..., N;\ t = 1, 2, ..., T \tag{1}$$

where $\beta_i = -(1 - \rho_i)$, $\mathbf{d}_t$ is 2×1 vector consisting of an intercept and a linear trend so that $\mathbf{d}_t = (1, t)'$. Without loss of generality, it is assumed that $\mathbf{d}_0 \equiv \mathbf{0}$. Consider the following multifactor error structure

$$u_{it} = \boldsymbol{\gamma}'_{iy}\mathbf{f}_t + \varepsilon_{iyt} \tag{2}$$

where $\mathbf{f}_t$ is an $m^0 \times 1$ vector of unobserved common effects, $\boldsymbol{\gamma}_{iy}$ is the associated vector of factor loadings, and ε_{iyt} is the idiosyncratic component. This set up generalises Pesaran's (2007) one factor error specification. We assume that these error processes satisfy the following assumptions:

Assumption 1 (idiosyncratic errors): The idiosyncratic shocks, ε_{iyt}, $i = 1, 2, ..., N$; $t = 1, 2, ..., T$, are independently distributed both across i and t, have zero means, variances $0 < \sigma_i^2 \leq K$ and finite fourth-order moments.

Remark 1 *This assumption, which implies that the idiosyncratic shocks are serially uncorrelated, will be relaxed in Section 2.1. It is also possible to relax the assumption that the idiosyncratic errors are cross sectionally independent, and replace it by assuming that $\varepsilon'_{iyt}s$ are cross sectionally weakly dependent in the sense of Chudik, Pesaran, and Tosetti (2009). However, such an extension will not be considered in this paper.*

Assumption 2 (factors): The $m^0 \times 1$ vector $\mathbf{f}_t$ follows a covariance stationary process, with absolute summable autocovariances, distributed independently of $\varepsilon_{iyt'}$ for all i, t and t'. Specifically, we assume that $\mathbf{f}_t = \boldsymbol{\Psi}(L)\mathbf{v}_t$, where $\mathbf{v}_t \sim IID(\mathbf{0}, \mathbf{I}_m)$, which have finite fourth-order moments, $\boldsymbol{\Psi}(L) = \sum_{\ell=0}^{\infty} \boldsymbol{\Psi}_\ell L^\ell$ with $\{\ell\boldsymbol{\Psi}_\ell\}_{\ell=0}^{\infty}$ being absolute summable such that $\sum_{\ell=0}^{\infty} \ell|\psi_{rs}^{(\ell)}|$ with $\psi_{rs}^{(\ell)}$ being the $(r, s)^{th}$ element of $\boldsymbol{\Psi}_\ell$, and specifically the inverse of $\boldsymbol{\Lambda}_f$ defined by

$$\boldsymbol{\Lambda}_f = \boldsymbol{\Psi}(1) \tag{3}$$

exists.

Remark 2 *Since $\boldsymbol{\Psi}_0$ is not restricted it can always be chosen such that $E(\mathbf{v}_t\mathbf{v}'_t) = \mathbf{I}_m$, without loss of generality. Assumption 2 is quite general but rules out the possibility of the factors having unit roots. This seems reasonable since otherwise all series in the panel could be $I(1)$ irrespective of whether $\beta_i = 0$ or not. Also if $\boldsymbol{\gamma}'_{iy}\mathbf{f}_t$ is assumed to be $I(1)$ and cointegarted with y_{it}, then y_{it} will be $I(1)$ even if $\beta_i = 0$, and as noted by Hansen (1995, p. 1159) in a similar context, a test of $\beta_i = 0$ as a unit root test will not be meaningful.*

Combining (1) and (2) it follows that

$$\Delta y_{it} = \beta_i(y_{i,t-1} - \boldsymbol{\alpha}'_{iy}\mathbf{d}_{t-1}) + \boldsymbol{\alpha}'_{iy}\Delta\mathbf{d}_t + \boldsymbol{\gamma}'_{iy}\mathbf{f}_t + \varepsilon_{iyt}. \tag{4}$$

The hypothesis that all the series, y_{it}, have *a unit root and are not cross unit cointegrated* can be expressed as

$$H_0 : \beta_i = 0 \text{ for all } i, \tag{5}$$

against the alternative

$$H_1 : \beta_i < 0 \text{ for } i = 1, 2, ..., N_1, \beta_i = 0 \text{ for } i = N_1 + 1, N_1 + 2, .., N$$

where $N_1/N \to \kappa$ and $0 < \kappa \leq 1$ as $N \to \infty$.

Note that under the null hypothesis, (4) can be solved for y_{it} to yield

$$y_{it} = y_{i0} + \boldsymbol{\alpha}'_{iy}\mathbf{d}_t + \boldsymbol{\gamma}'_{iy}\mathbf{s}_{ft} + s_{iyt},\ i = 1, 2, ..., N;\ t = 1, 2, ..., T \tag{6}$$

where

$$\begin{aligned} \mathbf{s}_{ft} &= \mathbf{f}_1 + \mathbf{f}_2 + ... + \mathbf{f}_t, \\ s_{iyt} &= \varepsilon_{1yt} + \varepsilon_{2yt} + ... + \varepsilon_{iyt}, \end{aligned}$$

with y_{i0} being a given initial value. Therefore, under H_0 and Assumptions 1 and 3, y_{it} is composed of the initial value, y_{i0}, a common stochastic component, $\mathbf{s}_{ft} \sim I(1)$, and an idiosyncratic component, $s_{iyt} \sim I(1)$, so that while all units of the panel share the common stochastic trends, $\mathbf{s}_{ft}$, there is no cointegration among them. Under the alternative hypothesis, $\beta_i < 0$, we have $y_{it} \sim I(0)$, and it is *essential* that $\mathbf{f}_t$ is at most an $I(0)$ process.

In the case where $m^0 = 1$, Pesaran (2007) proposes a test of $\beta_i = 0$ jointly with $f_t \sim I(0)$, based on DF (or ADF) regressions augmented by the current and lagged cross section averages of y_{it} as proxies for the unobserved $\mathbf{f}_t$. He shows that the resultant test is asymptotically invariant to the factor loadings, $\boldsymbol{\gamma}_{iy}$. To deal with the case where $m^0 > 1$ we assume that in addition to y_{it}, there exists k additional observables, say $\mathbf{x}_{it}$, which depend on at least the same set of common factors, $\mathbf{s}_{ft}$, although with different factor loadings. For example, in the analysis of output convergence it is reasonable to argue that output, investment, consumption, real equity prices, and oil prices have the same set of factors in common. Similarly, short term and long term interest rates and inflation across countries are likely to have a number of factors in common.

More specifically, suppose the $k \times 1$ vector of additional regressors follow the general linear process

$$\Delta\mathbf{x}_{it} = \mathbf{A}_{ix}\Delta\mathbf{d}_t + \boldsymbol{\Gamma}_{ix}\mathbf{f}_t + \boldsymbol{\varepsilon}_{ixt},\ i = 1, 2, ..., N;\ t = 1, 2, ..., T \tag{7}$$

where $\mathbf{x}_{it} = (x_{i1t}, x_{i2t}, ..., x_{ikt})'$, $\boldsymbol{\Gamma}_{ix} = (\boldsymbol{\gamma}_{ix1}, \boldsymbol{\gamma}_{ix2}, ..., \boldsymbol{\gamma}_{ixk})'$, $\mathbf{A}_{ix} = (\mathbf{a}_{ix1}, \mathbf{a}_{ix2}, ..., \mathbf{a}_{ixk})'$, and $\boldsymbol{\varepsilon}_{ixt}$ is the idiosyncratic component of $\mathbf{x}_{it}$ which is $I(0)$ and distributed independently of $\varepsilon_{iyt'}$ for all i, t and t'. The level equation can be written as

$$\mathbf{x}_{it} = \mathbf{x}_{i0} + \mathbf{A}_{ix}\mathbf{d}_t + \boldsymbol{\Gamma}_{ix}\mathbf{s}_{ft} + \mathbf{s}_{ixt},\ i = 1, 2, ..., N;\ t = 1, 2, ..., T \tag{8}$$

where $\mathbf{s}_{ixt} = \sum_{s=1}^{t} \boldsymbol{\varepsilon}_{ixs}$.

Combining (6) and (8) we have

$$\mathbf{z}_{it} = \mathbf{z}_{i0} + \boldsymbol{\Gamma}_i\mathbf{s}_{ft} + \mathbf{A}_i\mathbf{d}_t + \mathbf{s}_{it}, \tag{9}$$

where $\mathbf{z}_{it} = (y_{it}, \mathbf{x}'_{it})'$, $\mathbf{\Gamma}_i = \left(\boldsymbol{\gamma}_{iy}, \mathbf{\Gamma}'_{ix}\right)'$, $\mathbf{A}_i = (\boldsymbol{\alpha}_{iy}, \mathbf{A}'_{ix})'$, and $\mathbf{s}_{it} = (s_{iyt}, \mathbf{s}'_{ixt})'$. Without loss of generality we set $\mathbf{s}_{f0} = \mathbf{0}_{m^0}$ and $\mathbf{s}_{i0} = \mathbf{0}_{k+1}$.

Assumption 3 (factor loadings): $\|\mathbf{A}_i\| \leq K$ and $\|\mathbf{\Gamma}_i\| \leq K$, for all i, and $\mathbf{\Gamma}_i$ are set such that $E(\mathbf{f}_t\mathbf{f}'_t) \equiv \mathbf{I}_m$.

Assumption 4 (initial conditions): $E\|\mathbf{s}_{f1}\| \leq K$, and $E\|\mathbf{z}_{i0}\| \leq K$, $E\|\mathbf{s}_{i1}\| \leq K$, for all i.

Remark 3 *Assumption 3 imposes minimal conditions on the factor loadings. For example, it does not rule out possible dependence between the factor loadings and idiosyncratic errors. Also the normalization of* $\mathbf{f}_t$ *so that its variance covariance matrix is an identity matrix is innocuous since otherwise* $\mathbf{\Gamma}_i$ *and* $\mathbf{f}_t$ *can be suitably transformed so that Assumption 3 holds. Assumption 4 is also routine in the literature on unit roots.*

Averaging (9) across i we obtain

$$\bar{\mathbf{z}}_t = \bar{\mathbf{z}}_0 + \bar{\mathbf{\Gamma}}\mathbf{s}_{ft} + \bar{\mathbf{A}}\mathbf{d}_t + \bar{\mathbf{s}}_t, \tag{10}$$

where $\bar{\mathbf{z}}_t = N^{-1}\sum_{i=1}^N \mathbf{z}_{it}$, $\bar{\mathbf{A}} = N^{-1}\sum_{i=1}^N \mathbf{A}_i$, and $\bar{\mathbf{s}}_t = N^{-1}\sum_{i=1}^N \mathbf{s}_{it}$.[5] Writing (4), (9) and (10) in matrix notation, under the null for each i we have

$$\Delta\mathbf{y}_i = \mathbf{F}\boldsymbol{\gamma}_{iy} + \Delta\mathbf{D}\boldsymbol{\alpha}_{iy} + \boldsymbol{\varepsilon}_{iy}, \tag{11}$$

$$\Delta\mathbf{Z}_i = \mathbf{F}\mathbf{\Gamma}'_i + \Delta\mathbf{D}\mathbf{A}'_i + \mathbf{E}_i, \tag{12}$$

$$\Delta\bar{\mathbf{Z}} = \mathbf{F}\bar{\mathbf{\Gamma}}' + \Delta\mathbf{D}\bar{\mathbf{A}}' + \bar{\mathbf{E}}, \tag{13}$$

where $\mathbf{F} = (\mathbf{f}_1, \mathbf{f}_2, ..., \mathbf{f}_T)'$, $\Delta\mathbf{D} = (\Delta\mathbf{d}_1, \Delta\mathbf{d}_2, ..., \Delta\mathbf{d}_T)'$, $\boldsymbol{\varepsilon}_{iy} = (\varepsilon_{iy1}, \varepsilon_{iy2}, ..., \varepsilon_{iyT})'$, $\Delta\mathbf{Z}_i = (\Delta\mathbf{z}_{i1}, \Delta\mathbf{z}_{i2}, ..., \Delta\mathbf{z}_{iT})'$, $\mathbf{E}_i = (\boldsymbol{\varepsilon}_{i1}, \boldsymbol{\varepsilon}_{i2}, ..., \boldsymbol{\varepsilon}_{iT})'$ with $\boldsymbol{\varepsilon}_{it} = (\varepsilon_{iyt}, \boldsymbol{\varepsilon}'_{ixt})'$ $\Delta\bar{\mathbf{Z}} = (\Delta\bar{\mathbf{z}}_1, \Delta\bar{\mathbf{z}}_2, ..., \Delta\bar{\mathbf{z}}_T)'$ and $\bar{\mathbf{E}} = N^{-1}\sum_{i=1}^N \mathbf{E}_i$. From (13), if $\bar{\mathbf{\Gamma}}$ has full column rank m^0, it follows that

$$\mathbf{F} = \left(\Delta\bar{\mathbf{Z}} - \Delta\mathbf{D}\bar{\mathbf{A}}' - \bar{\mathbf{E}}\right)\bar{\mathbf{\Gamma}}\left(\bar{\mathbf{\Gamma}}'\bar{\mathbf{\Gamma}}\right)^{-1}. \tag{14}$$

However, from Appendix A.2.1 we have that $\bar{\mathbf{E}} \xrightarrow{N} \mathbf{0}$ for each t and hence we obtain that

$$\mathbf{F} - \left(\Delta\bar{\mathbf{Z}} - \Delta\mathbf{D}\bar{\mathbf{A}}'\right)\bar{\mathbf{\Gamma}}\left(\bar{\mathbf{\Gamma}}'\bar{\mathbf{\Gamma}}\right)^{-1} \xrightarrow{N} \mathbf{0}. \tag{15}$$

This implies that the linear combinations of $(\Delta\bar{\mathbf{Z}}, \Delta\mathbf{D})$ would be a valid approximation of $\mathbf{F}$ for large N. This condition on the rank of the cross section average of factor loadings is stated as an assumption below:

Assumption 5 (rank condition): The $(k+1) \times m^0$ matrix of factor loadings $\mathbf{\Gamma}_i$ is such that

$$rank(\bar{\mathbf{\Gamma}}) = m^0 \leq k+1, \text{ for any } N \text{ and as } N \to \infty, \tag{16}$$

where $\bar{\mathbf{\Gamma}} = N^{-1}\sum_{i=1}^N \mathbf{\Gamma}_i$, and $\bar{\mathbf{\Gamma}} \xrightarrow{N} \mathbf{\Gamma}$, where $\mathbf{\Gamma}$ is a fixed bounded matrix with rank m^0.

[5] Weighted cross section averages could also be used with appropriate granularity restrictions on the weights.

Remark 4 *From the equations (9) and (14), it is clear that our approach approximates* $\mathbf{s}_{ft}$ *of* $m^0 \times 1$ *dimension by linear combinations of the cross section average* $\bar{\mathbf{z}}_t = (\bar{y}_t, \bar{\mathbf{x}}_t')'$ *of* $k+1$ *dimension for large* N. *Thus, the rank condition (16),* $rank(\bar{\mathbf{\Gamma}}) = m^0 \leq k+1$, *which implies* $k \geq m^0 - 1$, *is of importance.*

Remark 5 *It is not necessary that* y_{it} *and* $(x_{i1t}, x_{i2t}, \ldots, x_{ikt})$ *have the same cross section dimensions. This is illustrated in Section 4.*

Remark 6 *Note that it is not necessary for the rank condition to hold for all cross section units individually, but that it must hold on average. For example, the rank condition holds so long as a non-zero fraction of factor loadings,* $\mathbf{\Gamma}_i$, *are full rank as* $N \to \infty$. *Also, so long as Assumption 5 is satisfied, we do not necessarily require that* $\lim_{N\to\infty} N^{-1}\sum_{i=1}^{N} \mathbf{\Gamma}_i'\mathbf{\Gamma}_i$ *exists and is positive definite, which is typically assumed for the identification of factors. See, for example, Assumption A(ii) of Bai and Ng (2004) and Assumption 6 of Moon and Perron (2004).*

In view of the above we shall base our test of the panel unit root hypothesis on the t-ratio of the ordinary least square (OLS) estimate of b_i ($\hat{b}_i$) in the following cross sectionally augmented regression

$$\Delta y_{it} = b_i y_{it-1} + \mathbf{c}_i'\bar{\mathbf{z}}_{t-1} + \mathbf{h}_i'\Delta\bar{\mathbf{z}}_t + \mathbf{g}_i'\Delta\mathbf{d}_t + \epsilon_{it}.$$

The t-ratio of $\hat{b}_i$ in this regression is given by

$$t_i(N,T) = \frac{\Delta\mathbf{y}_i'\bar{\mathbf{M}}\mathbf{y}_{i,-1}}{\hat{\sigma}_i\left(\mathbf{y}_{i,-1}'\bar{\mathbf{M}}\mathbf{y}_{i,-1}\right)^{1/2}} = \frac{\sqrt{T-(2k+5)}\Delta\mathbf{y}_i'\bar{\mathbf{M}}\mathbf{y}_{i,-1}}{\left(\Delta\mathbf{y}_i'\bar{\mathbf{M}}_i\Delta\mathbf{y}_i\right)^{1/2}\left(\mathbf{y}_{i,-1}'\bar{\mathbf{M}}\mathbf{y}_{i,-1}\right)^{1/2}},$$

where $\Delta\mathbf{y}_i = (\Delta y_{i1}, \Delta y_{i2}, \ldots, \Delta y_{iT})'$, $\mathbf{y}_{i,-1} = (y_{i0}, y_{i1}, \ldots, y_{i,T-1})'$, $\bar{\mathbf{M}} = \mathbf{I}_T - \bar{\mathbf{W}}\left(\bar{\mathbf{W}}'\bar{\mathbf{W}}\right)^{-1}\bar{\mathbf{W}}'$, $\bar{\mathbf{W}} = (\bar{\mathbf{w}}_1, \bar{\mathbf{w}}_2, \ldots, \bar{\mathbf{w}}_T)'$, $\bar{\mathbf{w}}_t = \left(\Delta\bar{\mathbf{z}}_t', \mathbf{d}_t', \bar{\mathbf{z}}_{t-1}'\right)'$,

$$\hat{\sigma}_i^2 = \frac{\Delta\mathbf{y}_i'\bar{\mathbf{M}}_i\Delta\mathbf{y}_i}{T-(2k+5)},$$

and $\bar{\mathbf{M}}_i = \mathbf{I}_T - \bar{\mathbf{W}}_i\left(\bar{\mathbf{W}}_i'\bar{\mathbf{W}}_i\right)^{-1}\bar{\mathbf{W}}_i'$, with $\bar{\mathbf{W}}_i = \left(\bar{\mathbf{W}}, \mathbf{y}_{i,-1}\right)$.

Using (14) in (11)

$$\Delta\mathbf{y}_i = \Delta\bar{\mathbf{Z}}\boldsymbol{\delta}_i + \Delta\mathbf{D}\boldsymbol{\alpha}_i + \sigma_i\boldsymbol{\upsilon}_i, \tag{17}$$

where

$$\boldsymbol{\delta}_i = \bar{\mathbf{\Gamma}}\left(\bar{\mathbf{\Gamma}}'\bar{\mathbf{\Gamma}}\right)^{-1}\boldsymbol{\gamma}_{iy},\ \boldsymbol{\alpha}_i = \boldsymbol{\alpha}_{iy} - \bar{\mathbf{A}}'\boldsymbol{\delta}_i,\ \boldsymbol{\upsilon}_i = \left(\boldsymbol{\varepsilon}_{iy} - \bar{\mathbf{E}}\boldsymbol{\delta}_i\right)/\sigma_i.$$

It is also easily seen that $E(\boldsymbol{\upsilon}_i\boldsymbol{\upsilon}_i') = \mathbf{I}_T + O(N^{-1})$. Therefore, we have

$$\bar{\mathbf{M}}\Delta\mathbf{y}_i = \sigma_i\bar{\mathbf{M}}\boldsymbol{\upsilon}_i. \tag{18}$$

From (12) and (13) we obtain

$$\mathbf{Z}_{i,-1} = \boldsymbol{\tau}_T\mathbf{z}_{i0}' + \mathbf{S}_{f,-1}\mathbf{\Gamma}_i' + \mathbf{D}_{-1}\mathbf{A}_i' + \mathbf{S}_{i,-1}.$$

Also

$$\bar{\mathbf{Z}}_{-1} = \boldsymbol{\tau}_T\bar{\mathbf{z}}_0' + \mathbf{S}_{f,-1}\bar{\mathbf{\Gamma}}' + \mathbf{D}_{-1}\bar{\mathbf{A}}' + \bar{\mathbf{S}}_{i,-1} \tag{19}$$

where $\mathbf{S}_{f,-1} = (\mathbf{0}_{m^0}, \mathbf{s}_{f1}, \ldots, \mathbf{s}_{f,T-1})'$, $\mathbf{D}_{-1} = (\mathbf{0}, \mathbf{d}_1, \ldots, \mathbf{d}_{T-1})'$, $\mathbf{Z}_{i,-1} = (\mathbf{z}_{i0}, \mathbf{z}_{i1}, \ldots, \mathbf{z}_{iT-1})'$, $\mathbf{S}_{i,-1} = (\mathbf{0}_{k+1}, \mathbf{s}_{i1}, \ldots, \mathbf{s}_{i,T-1})'$, $\bar{\mathbf{Z}}_{-1} = (\bar{\mathbf{z}}_0, \bar{\mathbf{z}}_1, \ldots, \bar{\mathbf{z}}_{T-1})'$ and $\bar{\mathbf{S}}_{-1} = N^{-1}\sum_{i=1}^{N}\mathbf{S}_{i,-1}$.

Similarly from (17)

$$\mathbf{y}_{i,-1} = \mathring{y}_{i0}\boldsymbol{\tau}_T + \bar{\mathbf{Z}}_{-1}\boldsymbol{\delta}_i + \mathbf{D}_{-1}\boldsymbol{\alpha}_i + \sigma_i\mathring{\mathfrak{s}}_{i,-1}, \tag{20}$$

where

$$\mathring{\mathfrak{s}}_{i,-1} = (\mathbf{s}_{iy,-1} - \bar{\mathbf{S}}_{-1}\boldsymbol{\delta}_i)/\sigma_i, \tag{21}$$

$\mathbf{s}_{iy,-1} = (0, s_{iy1}, ..., s_{iy,T-1})'$ and $\mathring{y}_{i0} = y_{i0} - \bar{\mathbf{z}}_0'\boldsymbol{\delta}_i$.

Therefore,

$$\bar{\mathbf{M}}\mathbf{y}_{i,-1} = \sigma_i\bar{\mathbf{M}}\mathring{\mathfrak{s}}_{i,-1}. \tag{22}$$

Using (18) and (22), $t_i(N,T)$ can be re-written as

$$t_i(N,T) = \frac{\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}\mathring{\mathfrak{s}}_{i,-1}}{\left(\frac{\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}_i\boldsymbol{\upsilon}_i}{T-2k-5}\right)^{1/2}\left(\mathring{\mathfrak{s}}_{i,-1}'\bar{\mathbf{M}}\mathring{\mathfrak{s}}_{i,-1}\right)^{1/2}}. \tag{23}$$

For fixed N and T, the distribution of $t_i(N,T)$ will depend on the nuisance parameters through their effects on $\bar{\mathbf{M}}_i$ and $\bar{\mathbf{M}}$. However, this dependence vanishes as $N \to \infty$, for fixed T. In the case of fixed T however, the effect of the initial cross section mean, $\bar{\mathbf{z}}_0$, must be eliminated in order to ensure that $t_i(N,T)$ does not depend on nuisance parameters. This can be achieved by working with the deviations, $\mathbf{z}_{it} - \bar{\mathbf{z}}_0$.

The main asymptotic results concerning the distribution of $t_i(N,T)$ are summarised in the theorems below. The proofs are given in the Appendix for the case where $\mathbf{d}_t = (1,0)'$, $t = 0, 1, ..., T$, which implies $\Delta\mathbf{D} = \mathbf{0}$. The asymptotic results for the case where $\mathbf{d}_t = (1,t)'$ can be derived in a similar manner.

Theorem 2.1 *Suppose the series $\mathbf{z}_{it}$, for $i = 1, 2, ..., N$, and $t = 1, 2, ..., T$, is generated under (5) according to (9), $d_t = 1$ with $\bar{\mathbf{z}}_0$ set to a zero vector. Then under Assumptions 1-5, the distribution of $t_i(N,T)$ given by (23), will be free of nuisance parameters as $N \to \infty$ for any fixed $T > 2k+4$. In particular, we have (in quadratic mean)*

$$t_i(N,T) \overset{N}{\to} \frac{\frac{\boldsymbol{\varepsilon}_{iy}'\mathbf{s}_{iy,-1}}{\sigma_i^2 T} - \mathbf{q}_{iT}'\boldsymbol{\Upsilon}_{fT}^{-1}\mathbf{h}_{iT}}{\left(\frac{\boldsymbol{\varepsilon}_{iy}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i^2(T-2k-4)} - \frac{\mathbf{g}_{iT}'\mathbf{Q}_{iT}^{-1}\mathbf{g}_{iT}}{(T-2k-4)}\right)^{1/2}\left(\frac{\mathbf{s}_{iy,-1}'\mathbf{s}_{iy,-1}}{\sigma_i^2T^2} - \mathbf{h}_{iT}'\boldsymbol{\Upsilon}_{fT}^{-1}\mathbf{h}_{iT}\right)^{1/2}},$$

where

$$\underset{(2m+1\times1)}{\mathbf{q}_{iT}} = \begin{pmatrix} \frac{\mathbf{F}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i\sqrt{T}} \\ \frac{\boldsymbol{\tau}_T'\boldsymbol{\varepsilon}_{iy}}{\sigma_i\sqrt{T}} \\ \frac{\mathbf{S}_{f,-1}'\boldsymbol{\varepsilon}_{iy}}{\sigma_iT} \end{pmatrix}, \quad \underset{(2m+1\times1)}{\mathbf{h}_{iT}} = \begin{pmatrix} \frac{\mathbf{F}'\mathbf{s}_{iy,-1}}{\sigma_iT^{3/2}} \\ \frac{\boldsymbol{\tau}_T'\mathbf{s}_{iy,-1}}{\sigma_iT^{3/2}} \\ \frac{\mathbf{S}_{f,-1}'\mathbf{s}_{iy,-1}}{\sigma_iT^2} \end{pmatrix}, \; \mathbf{g}_{iT} = \begin{pmatrix} \mathbf{q}_{iT} \\ \frac{\mathbf{s}_{iy,-1}'\boldsymbol{\varepsilon}_i}{\sigma_i^2T} \end{pmatrix}$$

$$\boldsymbol{\Upsilon}_{fT} = \begin{pmatrix} \frac{\mathbf{F}'\mathbf{F}}{T} & \frac{\mathbf{F}'\boldsymbol{\tau}_T}{T} & \frac{\mathbf{F}'\mathbf{S}_{f,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\tau}_T'\mathbf{F}}{T} & 1 & \frac{\boldsymbol{\tau}_T'\mathbf{S}_{f,-1}}{T^{3/2}} \\ \frac{\mathbf{S}_{f,-1}'\mathbf{F}}{T^{3/2}} & \frac{\mathbf{S}_{f,-1}'\boldsymbol{\tau}_T}{T^{3/2}} & \frac{\mathbf{S}_{f,-1}'\mathbf{S}_{f,-1}}{T^2} \end{pmatrix}, \; \mathbf{Q}_{iT} = \begin{pmatrix} \boldsymbol{\Upsilon}_{fT} & \mathbf{h}_{iT} \\ \mathbf{h}_{iT}' & \frac{\mathbf{s}_{iy,-1}'\mathbf{s}_{iy,-1}}{\sigma_i^2T^2} \end{pmatrix}.$$

See Appendix A.3 for a proof.

Remark 7 *When the factors are serially uncorrelated, namely* $\mathbf{f}_t \equiv \mathbf{v}_t \sim IID(\mathbf{0}, \mathbf{I}_m)$, *(see Assumption 2), even for a finite* T *the limit distribution of* $t_i(N,T)$ *as* $N \to \infty$, *does not depend on the factor loadings and* σ_i. *In the case where the factors are serially correlated the limit distribution of* $t_i(N,T)$ *does depend on the serial correlation patterns of* $\mathbf{f}_t$ *when* T *is finite. However, as stated in the next theorem, the dependence of* $t_i(N,T)$ *on the autocovariances of* $\mathbf{f}_t$ *disappears in the limit when* $T \to \infty$ *and* $N \to \infty$, *jointly.*

Theorem 2.2 *Suppose the series* $\mathbf{z}_{it}$, *for* $i = 1, 2, ..., N$, $t = 1, 2, ..., T$, *is generated under (5) according to (9) and* $d_t = 1$. *Then under Assumptions 1-5 and as* N *and* $T \to \infty$, *such that* $\sqrt{T}/N \to 0$, $t_i(N,T)$ *given by (23) has the same sequential* $(N \to \infty, T \to \infty)$ *and joint* $[(N,T)_j \to \infty]$ *limit distribution, is free of nuisance parameters, and is given by*

$$CADF_i = \frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}'_{i\mathbf{v}}\mathbf{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r)dr - \boldsymbol{\pi}'_{i\mathbf{v}}\mathbf{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}}, \tag{24}$$

where

$$\boldsymbol{\omega}_{i\mathbf{v}} = \begin{pmatrix} W_i(1) \\ \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)]\, dW_i(r) \end{pmatrix}, \quad \boldsymbol{\pi}_{i\mathbf{v}} = \begin{pmatrix} \int_0^1 W_i(r)dr \\ \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)]\, W_i(r)dr \end{pmatrix},$$

$$\mathbf{G}_{\mathbf{v}} = \begin{pmatrix} 1 & \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)]'\, dr \\ \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)]\, dr & \int_0^1 [\mathbf{W}_{\mathbf{v}}(r)]\,[\mathbf{W}_{\mathbf{v}}(r)]'\, dr \end{pmatrix},$$

$W_i(r)$ *is a scaler standard Brownian motion and* $\mathbf{W}_{\mathbf{v}}(r)$ *is* m^0*-dimensional standard Brownian motion defined on [0,1], associated with* ε_{iyt} *and* $\mathbf{v}_t$, *respectively.* $W_i(r)$ *and* $\mathbf{W}_{\mathbf{v}}(r)$ *are mutually independent.*

See Appendix A.4 for a proof.

Remark 8 *Conditional on* $\mathbf{W}_{\mathbf{v}}(r)$, $CADF_i$ *and* $CADF_j$ *are independently distributed, but unconditionally they are correlated with the same degree of dependence for all* $i \neq j$.

Having established that the limit distribution of the individual $t_i(N,T)$ statistic is free of nuisance parameters, we now focus on panel unit root tests based on the average of a suitably truncated version of $t_i(N,T)$ which we denote by $t_i^*(N,T)$. The truncation is carried out as in Pesaran (2007) to avoid certain technical difficulties concerning the existence of the moments of the non-truncated version of the individual statistics when T is finite. The truncated statistics are defined by

$$t_i^*(N,T) = \begin{cases} t_i(N,T), & \text{if } -K_1 < t_i(N,T) < K_2, \\ -K_1, & \text{if } t_i(N,T) \leq -K_1, \\ K_2, & \text{if } t_i(N,T) \geq K_2, \end{cases}$$

where K_1 and K_2 are positive constants that are sufficiently large so that $\Pr[-K_1 < t_i(N,T) < K_2]$ is sufficiently large. Using the normal approximation of $t_i(N,T)$, we would have $K_1 = -E(CADF_i) - \Phi^{-1}(\varepsilon/2)\sqrt{Var(CADF_i)}$, and $K_2 = E(CADF_i) + \Phi^{-1}(\varepsilon/2)\sqrt{Var(CADF_i)}$, where $\Phi^{-1}(.)$ is the inverse of the cumulative standard normal distribution function, and ε is a sufficiently small positive constant. K_1 and K_2 can now be obtained using simulated values of $E(CADF_i)$ and $Var(CADF_i)$ with $\varepsilon = 1 \times 10^{-6}$ for $N = 200$, and $T = 200$. The truncation does not affect the limit distribution and Theorem 2.1 continues to apply to $t_i^*(N,T)$ so that

$$t_i^*(N,T) - CADF_i^* = o_p(1), \tag{25}$$

where

$$CADF_i^* = \begin{cases} CADF_i, & \text{if } -K_1 < CADF_i < K_2, \\ -K_1, & \text{if } CADF_i \leq -K_1, \\ K_2, & \text{if } CADF_i \geq K_2. \end{cases}.$$

The panel unit root tests associated with the non-truncated and truncated versions of the individual unit root tests are given by

$$CIPS(N,T) = N^{-1} \sum_{i=1}^{N} t_i(N,T), \tag{26}$$

and

$$CIPS^*(N,T) = N^{-1} \sum_{i=1}^{N} t_i^*(N,T). \tag{27}$$

Since by construction all moments of $t_i^*(N,T)$ exist, using (25) it now follows (under assumptions of Theorem 2.2) that

$$CIPS^*(N,T) - \overline{CADF^*} = o_p(1), \text{ almost surely,}$$

where $\overline{CADF^*} = N^{-1} \sum_{i=1}^{N} CADF_i^*$. Hence, $CIPS^*(N,T)$ has the same limit distribution as $\overline{CADF^*}$, almost surely. But following Pesaran (2007, Section 4), it can be seen that the limit distribution of $\overline{CADF^*}$ exits and is free from nuisance parameters, although it is not analytically tractable. But the critical values of the distribution of $\overline{CADF^*}$ (or $\overline{CADF} = N^{-1} \sum_{i=1}^{N} CADF_i$) can be obtained by stochastic simulations.[6]

2.1 The Case of Serially Correlated Errors

In this section we relax Assumption 1, and allow for residual serial correlation. The residual serial correlation can be modeled in a number of different ways, directly via the idiosyncratic components, through the common effects or a mixture of the two. We focus on the first specification where cross section dependence is present under the multifactor error structure

$$u_{it} = \boldsymbol{\gamma}_{iy}' \mathbf{f}_t + \zeta_{iyt}$$

and residual serial correlation is modeled as

$$\zeta_{iyt} = \theta_i \zeta_{iy,t-1} + \eta_{iyt}, \;\; |\theta_i| < 1, \text{ for } i = 1, 2, ..., N; \; t = 1, 2, ..., T, \tag{28}$$

[6] We only report results for the non-truncated version of the test statistics. The results for the truncated version are very similar and are available upon request.

where η_{iyt} is independently distributed across time, with zero mean and finite positive variance, $\sigma^2_{i\eta}$.

In what follows we confine our attention to first order stationary processes for expositional convenience, though the analysis readily extends to higher order processes as well as to the alternative specifications of serial correlation mentioned above.

Under the above specification we have

$$\Delta y_{it} = \beta_i(y_{i,t-1} - \boldsymbol{\alpha}'_{iy}\mathbf{d}_{t-1}) + \boldsymbol{\alpha}'_{iy}\Delta\mathbf{d}_t + \boldsymbol{\gamma}'_{iy}\mathbf{f}_t + \zeta_{iyt}(\theta_i) \tag{29}$$

where $\zeta_{iyt}(\theta_i) = (1-\theta_i L)^{-1}\eta_{iyt}$. We also assume the coefficients of the autoregressive process to be homogeneous across i, although this could be relaxed at the cost of more complex mathematical details. Under the null that $\beta_i = 0$, with $\theta_i = \theta$ and $d_t = 1$, (29) becomes

$$\Delta y_{it} = \boldsymbol{\gamma}'_{iy}\mathbf{f}_t + \zeta_{iyt}(\theta), \tag{30}$$

or

$$\Delta y_{it} = \theta\Delta y_{i,t-1} + \boldsymbol{\gamma}'_{iy}(\mathbf{f}_t - \theta\mathbf{f}_{t-1}) + \eta_{iyt}. \tag{31}$$

Combining (7) with (30), similarly to (12) we obtain

$$\Delta\mathbf{Z}_i = \mathbf{F}\boldsymbol{\Gamma}'_i + \mathbf{E}_i, \tag{32}$$

where $\mathbf{E}_i = (\boldsymbol{\zeta}'_{iy}(\theta), \mathbf{E}'_{ix})'$ with $\mathbf{E}_{ix} = (\boldsymbol{\varepsilon}_{ix1}, \boldsymbol{\varepsilon}_{ix2}, ..., \boldsymbol{\varepsilon}_{ixT})'$and $\boldsymbol{\zeta}_{iy}(\theta) = \left(\zeta_{iy1}(\theta), \zeta_{iy2}(\theta), ..., \zeta_{iyT}(\theta)\right)'$, with the common factors $\mathbf{F}$, and factor loadings $\boldsymbol{\Gamma}_i$ defined as in the previous section. Taking cross section averages of (32) we have that

$$\Delta\bar{\mathbf{Z}} = \mathbf{F}\bar{\boldsymbol{\Gamma}}' + \bar{\mathbf{E}},$$

where as before $\bar{\mathbf{E}} = N^{-1}\sum_{i=1}^N \mathbf{E}_i$, from which it follows under rank condition (16) that

$$\mathbf{F} = \left(\Delta\bar{\mathbf{Z}} - \bar{\mathbf{E}}\right)\bar{\boldsymbol{\Gamma}}\left(\bar{\boldsymbol{\Gamma}}'\bar{\boldsymbol{\Gamma}}\right)^{-1}. \tag{33}$$

Thus in testing (5) we use the following cross sectionally augmented regression

$$\Delta\mathbf{y}_i = b_i\mathbf{y}_{i,-1} + \bar{\mathbf{W}}_{i1}\mathbf{h}_i + \epsilon_i, \tag{34}$$

where $\bar{\mathbf{W}}_{i1} = (\Delta\mathbf{y}_{i,-1}, \Delta\bar{\mathbf{Z}}, \Delta\bar{\mathbf{Z}}_{-1}, \boldsymbol{\tau}_T, \bar{\mathbf{Z}}_{-1})$, which is a $T\times(3k+5)$ matrix.

The t-ratio of $\hat{b}_i$ in regression (34) is given by

$$t_i(N,T) = \frac{\Delta\mathbf{y}'_i\bar{\mathbf{M}}_{i1}\mathbf{y}_{i,-1}}{\hat{\sigma}_i\left(\mathbf{y}'_{i,-1}\bar{\mathbf{M}}_{i1}\mathbf{y}_{i,-1}\right)^{1/2}} = \frac{\sqrt{T-(3k+6)}\Delta\mathbf{y}'_i\bar{\mathbf{M}}_{i1}\mathbf{y}_{i,-1}}{\left(\Delta\mathbf{y}'_i\bar{\mathbf{M}}_{i1,p}\Delta\mathbf{y}_i\right)^{1/2}\left(\mathbf{y}'_{i,-1}\bar{\mathbf{M}}_{i1}\mathbf{y}_{i,-1}\right)^{1/2}}, \tag{35}$$

where $\bar{\mathbf{M}}_{i1} = \mathbf{I}_T - \bar{\mathbf{W}}_{i1}(\bar{\mathbf{W}}'_{i1}\bar{\mathbf{W}}_{i1})^{-1}\bar{\mathbf{W}}'_{i1}$, $\hat{\sigma}^2_i = [T-(3k+6)]^{-1}\Delta\mathbf{y}'_i\bar{\mathbf{M}}_{i1,p}\Delta\mathbf{y}_i$ and $\bar{\mathbf{M}}_{i1,p} = \mathbf{I}_T - \mathbf{P}_{i1}(\mathbf{P}'_{i1}\mathbf{P}_{i1})^{-1}\mathbf{P}'_{i1}$, $\mathbf{P}_{i1} = (\bar{\mathbf{W}}_{i1}, \mathbf{y}_{i,-1})$.

Writing (31) in matrix notation and using (33) we have

$$\Delta\mathbf{y}_i = \theta\Delta\mathbf{y}_{i,-1} + (\Delta\bar{\mathbf{Z}} - \theta\Delta\bar{\mathbf{Z}}_{-1})\boldsymbol{\delta}_i + \sigma_{i\eta}\boldsymbol{\upsilon}_i, \tag{36}$$

with

$$\boldsymbol{\upsilon}_i = [\boldsymbol{\eta}_{iy} - (\bar{\mathbf{E}} - \theta\Delta\bar{\mathbf{E}}_{-1})\boldsymbol{\delta}_i]/\sigma_{i\eta},$$

and $E(\boldsymbol{\upsilon}_i\boldsymbol{\upsilon}_i') = \mathbf{I}_T + O(N^{-1})$. From (36) it follows that

$$\mathbf{y}_{i,-1} = \alpha_{iy}\boldsymbol{\tau}_T + \mathring{y}_{i0}\boldsymbol{\tau}_T + \bar{\mathbf{Z}}_{-1}\boldsymbol{\delta}_i + \sigma_{i\eta}\mathring{\mathfrak{s}}_{i\zeta,-1}$$

where

$$\mathring{\mathfrak{s}}_{i\zeta,-1} = \left(\mathbf{s}_{i\zeta,-1} - \bar{\mathbf{S}}_{-1}\boldsymbol{\delta}_i\right)/\sigma_{i\eta},$$

$\mathbf{s}_{i\zeta,-1} = (0, s_{i\zeta 1}, ..., s_{i\zeta,T-1})'$ with $s_{i\zeta t} = \sum_{s=1}^{t}\zeta_{iys}(\theta)$, $\bar{\mathbf{S}}_{-1} = (\bar{\mathbf{s}}_{\zeta,-1}, \bar{\mathbf{S}}_{x,-1})$ with $\bar{\mathbf{s}}_{\zeta,-1} = N^{-1}\sum_{i=1}^{N}\mathbf{s}_{i\zeta,-1}$ and $\mathring{y}_{i0} = y_{i0} - \bar{\mathbf{z}}_0'\boldsymbol{\delta}_i$.

The test statistic (35) then becomes

$$t_i(N,T) = \frac{\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}_{i1}\mathring{\mathfrak{s}}_{i\zeta,-1}}{\left(\frac{\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}_{i1,p}\boldsymbol{\upsilon}_i}{T-3k-6}\right)^{1/2}\left(\mathring{\mathfrak{s}}_{i\zeta,-1}'\bar{\mathbf{M}}_{i1}\mathring{\mathfrak{s}}_{i\zeta,-1}\right)^{1/2}}. \tag{37}$$

Theorem 2.3 *Suppose the series $\mathbf{z}_{it}$, for $i = 1, 2, ..., N$, $t = 1, 2, ..., T$, is generated under (5) according to (32) and $|\theta| < 1$. Then under Assumptions 1-5 and as N and $T \to \infty$, $t_i(N,T)$ in (37) has the same sequential ($N \to \infty, T \to \infty$) and joint $[(N,T)_j \to \infty]$ limit distribution given by (24) obtained for $\theta = 0$.*

Proof: *See Appendix A.5.*

For an AR(p) error specification in (28), the relevant $t_i(N,T)$ statistic will be given by the OLS t-ratio of b_i in the following p^{th} order augmented regression:

$$\Delta\mathbf{y}_i = b_i\mathbf{y}_{i,-1} + \bar{\mathbf{W}}_{ip}\mathbf{h}_{ip} + \epsilon_i,$$

where $\bar{\mathbf{W}}_{ip} = (\Delta\mathbf{y}_{i,-1}, \Delta\mathbf{y}_{i,-2}, ..., \Delta\mathbf{y}_{i,-p}; \Delta\bar{\mathbf{Z}}, \Delta\bar{\mathbf{Z}}_{-1}, ..., \Delta\bar{\mathbf{Z}}_{-p}; \boldsymbol{\tau}_T; \bar{\mathbf{Z}}_{-1})$, which is a $T\times[(k+2)(p+2)-1]$ matrix.

However, it is easily seen that the limit distribution of $t_i(N,T)$ with $N \to \infty$ for a fixed T depends on the augmentation order, p. Thus, we will obtain critical values of $t_i(N,T)$ for different choices of p.

2.2 Uncertainty about the Number of Factors

So far we have considered the case in which the true number of unobserved factors, m^0, is given. In practice m^0 is not known, although it is reasonable to assume that it is bounded by a sufficiently large integer value, $m_{\max}$. In the case of the proposed test there are two possible ways that one could proceed when m^0 is not known. One possibility would be to set $k = m_{\max} - 1$, if there exists $m_{\max} - 1$ $I(1)$ and not cointegrated additional regressors for augmenting the ADF regressions. In this case, the true number of factors are allowed to be any integer value between zero and $m_{\max}$. However, when $m_{\max}$ is assumed to be large, it can be difficult to find $m_{\max} - 1$ such regressors. Alternatively, m^0 can be estimated consistently using suitable selection criteria, as is followed in the literature, for example, by Bai and Ng (2004) and Moon and Perron (2004), amongst others. Since typically m^0 is estimated to be around 2 to 4 in most economic applications, it may not be particularly difficult to identify suitable additional variables for augmentation.[7]

[7] One could follow the bounds test approach proposed by Pesaran et al. (2001) when there is uncertainty in integration and/or cointegration properties of k additional regressors. This route is not pursued in this paper.

2.3 Critical Values

The critical values of $CADF_i$ and $\overline{CADF} = N^{-1}\sum_{i=1}^{N} CADF_i$ for different values of k, N, T and lag-augmentation p, are obtained by stochastic simulation. Based on the results in Section 2 the limit distribution of $\overline{CADF}$ does not depend on the factor loadings $\mathbf{\Gamma}_i$ or σ_i. This implies that the distribution of the test statistic is invariant to the choice of $\mathbf{\Gamma}_i$ and σ_i so long as $m^0 \leq k+1$. Thus, without loss of generality we set $\mathbf{\Gamma}_i = \mathbf{\Gamma} = \mathbf{0}$, and $\sigma_i = \sigma = 1$ in the simulation exercise.

To be more precise, the y_{it} process is generated as

$$y_{it} = y_{it-1} + \varepsilon_{iyt},\ i = 1, 2, ..., N;\ t = 1, 2, ..., T,$$

where $\varepsilon_{iyt} \sim iidN(0,1)$ with $y_{i0} = 0$. The j^{th} element of the $k \times 1$ vector of the additional regressors x_{it}, is generated as

$$x_{ijt} = x_{ij,t-1} + \varepsilon_{ixjt},\ i = 1, 2, ..., N;\ j = 1, 2, ..., k;\ t = 1, 2, ..., T, \tag{38}$$

with $\varepsilon_{ixjt} \sim iidN(0,1)$ and $x_{ij0} = 0$.

The $CADF_i$ test statistic is calculated as the t-ratio of the coefficient on y_{it-1} of the regression of Δy_{it} on y_{it-1}, $\bar{\mathbf{z}}'_{t-1}$, $(\Delta\mathbf{z}'_{i.t-1}, ..., \Delta\mathbf{z}'_{i.t-p})$, $(\Delta\bar{\mathbf{z}}'_{t-1}, ..., \Delta\bar{\mathbf{z}}'_{t-p})$ where the following cases for the deterministics are entertained

Case I: no deterministics,
Case II: intercept only,
Case III: an intercept and a linear trend,

and $E(CADF_i)$ and $Var(CADF_i)$ are obtained as an average over all replications of $CADF_1$ and the square of the standard deviation of $CADF_1$ respectively, for $N, T = 200$. The $\alpha\%$ critical values of the $CADF_1$ and $\overline{CADF}$ statistics are computed for $N, T = 20, 30, 50, 70, 100, 200$, $k = 1, 2, 3$ and $p = 0, 1, ..., 4$, as the α quantiles of $CADF_1$ and $\overline{CADF}$ for $\alpha = 0.01, 0.05, 0.1$.[8] The critical values of the $\overline{CADF}$ statistic for case II and III are reported in Tables 1 and 2, respectively. Critical values for the $\overline{CADF}$ statistic for case I as well as for the individual statistics $CADF_i$ are available upon request. All stochastic simulations are based on 10,000 replications.[9]

3 Small Sample Performance: Monte Carlo Evidence

In what follows we investigate the small sample properties of the CIPS test defined by (26) and compare its performance to the pooled tests by Bai and Ng (2004), and the t_b^* and $t^{\#}$ tests by Moon and Perron (2004), by means of Monte Carlo experiments. The t_b^* test statistic is for the case with an intercept only, and the $t^{\#}$ test statistic is for the case with an intercept and a linear trend.

The pooled test statistics proposed by Bai and Ng (2004), using our notation as set out in Section 2, are computed as follows. Firstly we define the transformed Δy_{it},

$$\underline{\Delta y}_{it} = \begin{cases} \Delta y_{it}, & \text{for the case with an intercept} \\ \Delta y_{it} - \overline{\Delta y}_i, & \text{for the case with an intercept and a linear trend} \end{cases} \tag{39}$$

[8] The critical values for $k = 0$ are tabulated in Pesaran (2007).

[9] It is also possible to simulate the critical values directly using (24) by replacing the integrals of the Brownian motions with their simulated counterparts. Our analysis suggests that the critical values obtained from this procedure closely matches the ones tabulated in the paper.

with $\overline{\Delta y}_i = T^{-1}\sum_{t=1}^{T}\Delta y_{it}$. Apply principal components to the transformed series to estimate $\mathbf{F}$, denoted as $\hat{\mathbf{F}}$, which is $\sqrt{T}$ times the m eigenvectors corresponding to the first m largest eigenvalues of the $T\times T$ matrix $\underline{\Delta\mathbf{Y}}\,\underline{\Delta\mathbf{Y}'}$, where $\underline{\Delta\mathbf{Y}} = (\underline{\Delta\mathbf{y}}_1, \underline{\Delta\mathbf{y}}_2, ..., \underline{\Delta\mathbf{y}}_N)$ with $\underline{\Delta\mathbf{y}}_i = (\underline{\Delta y}_{i1}, \underline{\Delta y}_{i2}, ..., \underline{\Delta y}_{iT})'$. Under the normalisation $\hat{\mathbf{F}}'\hat{\mathbf{F}}/T = \mathbf{I}_m$, the estimates of the factor loadings are given by $\hat{\boldsymbol{\gamma}}_{iy} = \hat{\mathbf{F}}_i'\underline{\Delta\mathbf{y}}_i/T$, which yield the residuals $\hat{\varepsilon}_{iyt} = \underline{\Delta y}_{it} - \hat{\boldsymbol{\gamma}}_{iy}'\hat{\mathbf{f}}_t$. Now set $e_{iyt} = \sum_{s=1}^{t}\hat{\varepsilon}_{iys}$, and compute the ADF statistic for the ADF(p) regressions in e_{iyt} without deterministics for each cross section unit. Denoting this statistic by $t_{BN,i}^{c}$ if y_{it} has individual effects, and by $t_{BN,i}^{\tau}$ if y_{it} has a linear trend, the pooled test statistics are then defined as

$$P_{\hat{u}}^{c} = \frac{\left(-2\sum_{i=1}^{N}\ln(\text{pv}_i^c) - 2N\right)}{\sqrt{4N}} \text{ and } P_{\hat{u}}^{\tau} = \frac{\left(-2\sum_{i=1}^{N}\ln(\text{pv}_i^\tau) - 2N\right)}{\sqrt{4N}},$$

where pv_i^c and pv_i^τ are the p-values of the $t_{BN,i}^{c}$ and $t_{BN,i}^{\tau}$ statistics, respectively. These statistics are asymptotically distributed as standard normal so that the null hypothesis is rejected if $P_{\hat{u}}^{c}$ (or $P_{\hat{u}}^{\tau}$) is larger than 1.645 (at the 5% level).[10]

We also consider variants of $P_{\hat{u}}^{c}$ and $P_{\hat{u}}^{\tau}$ that make use of all the available variables, y_{it} and $\mathbf{x}_{it}$, to estimate the common factors. This version is more directly comparable to the test proposed in this paper which makes use of the additional variables, $\mathbf{x}_{it}$. The procedure is similar to that described above with the principal component estimator of $\mathbf{F}$ now computed using $\underline{\Delta\mathbf{z}}_{it} = (\underline{\Delta y}_{it}, \underline{\Delta\mathbf{x}}_{it}')'$, where$\underline{\Delta\mathbf{x}}_{it}$ is constructed from $\Delta\mathbf{x}_{it}$ in a manner similar to that specified by (39) for$\underline{\Delta y}_{it}$. These variants of $P_{\hat{u}}^{c}$ and $P_{\hat{u}}^{\tau}$ are denoted by $P_{\hat{u},z}^{c}$ and $P_{\hat{u},z}^{\tau}$, respectively.

The t_b^* and $t^{\#}$ test statistics are as defined by Moon and Perron (2004). The t_b^* test is for the case with an intercept only, and the $t^{\#}$ test is for the intercept and a linear trend case.[11] The tests are based on de-factored panel data obtained by projecting the panel data onto the space orthogonal to the (estimated) factor loadings. The nuisance parameters are defined on the residuals of the de-factored data where the long-run variance is estimated by employing Andrews and Monahan's (1992) estimator based on the quadratic spectral kernel and pre-whitening. The null hypothesis is rejected if the test statistics are less than -1.645 (5% level test).

For further details on the above statistics see Bai and Ng (2004) and Moon and Perron (2004). We consider experiments where the number of factors is treated as known as well as unknown.

3.1 Monte Carlo Design

Initially we consider dynamic panel models with fixed effects and a two-factor ($m^0 = 2$) error structure. The data generating process (DGP) is given by

$$y_{it} = (1-\rho_i)\alpha_{iy} + \rho_i y_{i,t-1} + \gamma_{iy1}f_{1t} + \gamma_{iy2}f_{2t} + \varepsilon_{iyt}, i = 1,2,...,N; t = -49,...,T \quad (40)$$

with $y_{i,-50} = 0$, where $\alpha_{iy} \sim iidN(1,1)$, $\gamma_{iy1} \sim iidU[0,2]$, $\gamma_{iy2} \sim iidU[0,2]$,

$$f_{\ell t} = \rho_{f\ell}f_{f\ell,t-1} + v_{\ell t}, v_{\ell t} \sim iidN(0, 1-\rho_{f\ell}^2),\ f_{\ell,-50} = 0$$

[10] In our experiments the $P_{\hat{u}}$ statistics are computed by a GAUSS code which is a translation of the Matlab programme provided by Serena Ng. p-values of $t_{BN,i}^{c}$ and $t_{BN,i}^{\tau}$ are obtained using the tables 'adfnc.asc' and 'lm1.asc', respectively, also provided by Serena Ng.

[11] The t_a^* test, which is also proposed by Moon and Perron (2004), is not considered in our simulations since the t_b^* test is preferred in their paper.

for $\ell = 1, 2$, and

$$\varepsilon_{iyt} = \rho_{iy\varepsilon}\varepsilon_{iyt-1} + \eta_{iyt}, \eta_{iyt} \sim iidN(0, (1 - \rho_{iy\varepsilon}^2)\sigma_i^2),\ \varepsilon_{iy,-50} = 0, \tag{41}$$

$\sigma_i^2 \sim iidU[0.5, 1.5]$.

We include at most two additional regressors, x_{i1t} and x_{i2t} in the experiments. The DGPs are

$$x_{ijt} = x_{ijt-1} + \gamma_{ixj1} f_{1t} + \gamma_{ixj2} f_{2t} + \varepsilon_{ixjt} \text{ for } j = 1, 2, \tag{42}$$

$i = 1, 2, ..., N; t = -49, ..., T$ with $x_{ij,-50} = 0$,

$$\varepsilon_{ixjt} = \rho_{ixj}\varepsilon_{ixjt-1} + \varpi_{ixjt}, , \varpi_{ixjt} \sim iidN(0, 1 - \rho_{ixj}^2), \tag{43}$$

with $\varepsilon_{ixj,-50} = 0$, and $\rho_{ixj} \sim iidU[0.2, 0.4]$ for $j = 1, 2$.

The first set of experiments assumes that $m^0 = 2$ is given and, hence, k is equated to $m^0 - 1 = 1$. We use only one additional regressor, $\bar{x}_{1t}$. The factor loadings in (42) are generated as $\gamma_{ix11} \sim iidU[0, 2]$ and $\gamma_{ix12} = 0$, so that

$$E(\mathbf{\Gamma}_i) = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}, \tag{44}$$

of which the rank condition (16) is satisfied.[12] Note that $\bar{x}_{1t}$ contains s_{f1t} only under this design. As discussed in section 2.2, this is enough for augmenting the CADF regression to asymptotically eliminate two factors in the y_{it} equation.[13] We consider three combinations of serial dependence in the errors: (A) serially uncorrelated ε_{iyt} and f_{jt} ($\rho_{iy\varepsilon} = \rho_{y\varepsilon} = 0$ and $\rho_{f1} = \rho_{f2} = 0$); (B) serially correlated ε_{iyt} ($\rho_{iy\varepsilon} \sim iidU[0.2, 0.4]$ and $\rho_{f1} = \rho_{f2} = 0$); (C) serially correlated $f_{\ell t}$ ($\rho_{iy\varepsilon} = \rho_{y\varepsilon} = 0$ and $\rho_{f1} = \rho_{f2} = 0.3$). Note that x_{ixjt} are serially correlated in all experiments for $j = 1, 2$, as specified above.

In addition, we consider spatially correlated factor loadings generated as

$$\gamma_{ir} - c_r = \lambda \sum_{j=1}^{N} s_{ij}\left(\gamma_{jr} - c_r\right) + \varphi_{ir}, \varphi_{ir} \sim iidN(0, \sigma_{\varphi i}^2),\ r = y1, y2, x11, x12$$

where s_{ij} is the (i, j) element of an $(N \times N)$ spatial weighting matrix, $\mathbf{S} = \{s_{ij}\}$, which is row standardised with $s_{ij} = 1$ if units i and j are adjacent and $s_{ij} = 0$ otherwise. We set $\lambda = 0.8$. The parameter $\sigma_{\varphi i}^2$ is chosen so that $var(\gamma_{ir}) = 1/3$, and we set $c_{y1} = 1$, $c_{y2} = 1$, $c_{x11} = 1$, $c_{x12} = 0$, for the results to be comparable to our other experimental designs.[14]

[12] Another experiment relating to the specification of the factor loadings is considered, where

$$\mathbf{\Gamma}_i = (\gamma_{iy}, \gamma_{ix1}) = \begin{pmatrix} iidU[0,2] & iidU[0,2] \\ iidU[0,2] & 0 \end{pmatrix} \text{ for } i = 1, 2, ..., N/2$$

but

$$\mathbf{\Gamma}_j = (\gamma_{jy}, \gamma_{jx1}) = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \text{ for } j = N/2 + 1, ..., N$$

so that the rank condition is satisfied. The results are very similar to those using (44).

[13] We have also implemented the experiments with γ_{ix12} replaced by non-zero values, generated as $\gamma_{ix12} \sim iidU[-0.5, 1.5]$. The results are very similar to those with $\gamma_{ix12} = 0$, and are available upon request from the authors.

[14] We further generated bounded factor loadings where $\gamma_{iy1} = \zeta_{iy1}/\sqrt{\sum_{j=1}^{N} \zeta_{jy1}^2}$ and $\gamma_{ix2} = \zeta_{ix2}/\sqrt{\sum_{j=1}^{N} \zeta_{jx2}^2}$ and ζ_{iy1} and ζ_{ix2} are draws from different uniform distributions, $iidU(0, 1)$. The factor loadings γ_{iy2} and γ_{ix1} are generated as in the spatially correlated case with zero expected values. Results were similar to the spatially correlated case.

In another set of experiments, we consider the case in which $m^0 = 2$ is not known but the maximum number of factors is assumed to be three, i.e., $m_{\max} = 3$. Here the value of m^0 is estimated (denoted by $\hat{m}^0$) based on the information criterion IC_1, proposed by Bai and Ng (2002) and used in the simulation exercises of Bai and Ng (2004). Accordingly, $\hat{m}^0$ factors are extracted from y_{it} for the $P_{\hat{u}}$ and t_b^* ($t^{\#}$) statistics and from $(y_{it}, x_{i1t}, x_{i2t})$ for the $P_{\hat{u},z}$ statistic. For the CADF regressions $k = \hat{m}^0 - 1$ additional regressors are included in the augmentations. At most we need $m_{\max} - 1 = 2$ additional regressors. In this experiment we consider uncertainty about the integration properties of the two needed additional regressors. The CIPS test is implemented assuming x_{i1t} and x_{i2t} are $I(1)$ and not cointegrated, but in the DGP x_{i1t} and x_{i2t} are generated as cointegaretd variables. The factor loadings in (42) are generated as $\gamma_{ix11} \sim iidU[0, 2]$, $\gamma_{ix21} \sim iidU[0, 2]$, $\gamma_{ix12} = \gamma_{ix22} = 0$, with ε_{ixjt} replaced by its first difference $\Delta\varepsilon_{ixjt}$ so that the cumulative sum of the idiosyncratic errors in x_{ijt} becomes $\varepsilon_{ixjt} \sim I(0)$, and

$$E(\mathbf{\Gamma}_i) = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 0 & 0 \end{pmatrix}.$$

Under this design $x_{1it} \sim I(1)$ and $x_{2it} \sim I(1)$ but they are cointegrated. When $\hat{m}^0 = 2$, only one regressor is required by CADF augmentation, thus, x_{i1t} is included in the experiment. If $\hat{m}^0 = 1$ or 0, no additional regressors are included. For this set of experiments we confine our attention to case (A) with regard to serial dependence in the errors.[15]

Similar sets of experiments are carried out for the model with a linear time trend. The DGPs corresponding to (40) and (42) are

$$y_{it} = \mu_{iy} + (1 - \rho_i)\delta_i t + \rho_i y_{i,t-1} + \gamma_{iy1} f_{1t} + \gamma_{iy2} f_{2t} + \varepsilon_{iyt}, i = 1, 2, ..., N; t = -49, ..., T$$

$$x_{ijt} = x_{ijt-1} + \lambda_{ixj} + \gamma_{ixj1} f_{1t} + \gamma_{ixj2} f_{2t} + \varepsilon_{ixjt}, \text{ for } j = 1, 2 \quad (45)$$

respectively, where $\mu_{iy} \sim iidU[0.0, 0.02]$, $\delta_i \sim iidU[0.0, 0.02]$, $\lambda_{ixj} \sim iidU[0.0, 0.02]$ for $j = 1, 2$. The rest of the variables are defined as above.

The parameters α_i, $\rho_{iy\varepsilon}$, $\gamma_{iy\ell}$, $\rho_{f\ell}$, ρ_i, $\gamma_{ixj\ell}$, ρ_{ixj}, μ_{iy}, δ_i, and λ_{ixj} are drawn once and fixed over the replications. For size the DGP is given by (40) with $\rho_i = \rho = 1$, and for power with $\rho_i \sim iidU[0.90, 0.99]$. All tests are conducted at the 5% significance level. All combinations of $N, T = 20, 30, 50, 70, 100, 200$ are considered, and all experiments are based on 2,000 replications each.

3.2 Results

Size and power of the tests are summarized in Tables 3 to 8. Recall that for all experiments, the models contain two factors, $m^0 = 2$, and the idiosyncratic errors of additional regressors, v_{ixjt}, can be $I(1)$ or $I(0)$ and are generated as serially correlated variables. Also note that in the case of serially correlated idiosyncratic errors, lag augmentation is required for the asymptotic validity of the CIPS test and the pooled tests of Bai and Ng (2004), while the t_b^* and $t^{\#}$ tests of Moon and Perron (2004) correct for the residual serial correlation in a non-parametric manner.

The results reported in Tables 3 to 7 are obtained assuming that $m^0 = 2$ is known and that the one additional regressor to augment the CIPS test statistic ($k = 1$) is known to be $I(1)$. Table 3 provides the results for the model where the factors, f_{1t} and f_{2t}, and the idiosyncratic components, ε_{iyt}, are serially uncorrelated. Panel A of the table reports the results for the case

[15] Another experiment, in which x_{i1t} and x_{i2t} are generated as $I(1)$ and non-cointegrated, is considered, but the results are very similar and will not be included below to save space.

of an intercept only. The $P_{\hat{u}}^c$ and $P_{\hat{u},z}^c$ tests of Bai and Ng (2004) tend to over-reject the null moderately, with the extent of over-rejection rising as N increases. The same applies to the Moon-Perron test, t_b^*. These results are consistent to those reported in Gengenbach, Palm and Urbain (2009). In contrast, the CIPS test has the correct size for all combinations of sample sizes, even when T is small relative to N. In terms of power, the CIPS test seems less powerful than the other tests for small values of T (which could partly be due to the over-sized nature of the other tests), while in general it tends to be more powerful for larger N and T. In panel B of Table 3 the results for the case with an intercept and a linear trend are reported. Now the $P_{\hat{u}}^\tau$ and $P_{\hat{u},z}^\tau$ tests severely over-reject the null hypothesis in all experiments. Even when $T = 200$ and $N = 200$, the size of these tests is around 13%. The size distortion of the $t^\#$ test is even worse for all experiments. On the other hand, the CIPS test has the correct size for all combinations of sample sizes. Not surprisingly, the power of the CIPS test in the linear trend case is lower than the intercept only case. This is a feature common to all unit root tests in the literature.

Table 4 (Table 5) presents the results for the case where ε_{iyt} are positively (negatively) serially correlated but f_{1t} and f_{2t} are serially uncorrelated. With time series augmentation the size and the power properties of the CIPS test are similar to those reported in Table 3. The $P_{\hat{u}}^c$, $P_{\hat{u},z}^c$, $P_{\hat{u}}^\tau$ and $P_{\hat{u},z}^\tau$ tests display a higher tendency to over-reject the null relative to the case where the idiosyncratic errors are serially uncorrelated. The t_b^* and $t^\#$ tests show slightly less (more) size distortions as compared to the results given in Table 3 when $\varepsilon_{iyt}'s$ are positively (negatively) serially correlated.

Table 6 provides the results for the experiments where f_{1t} and f_{2t} are serially correlated, but ε_{iyt} is not. In this case all the tests exhibit size distortions unless T is sufficiently large relative to N. However, the extent of over-rejection of the CIPS test is less than that of the $P_{\hat{u}}^c$ and $P_{\hat{u},z}^c$ tests. The performance of the t_b^* test is similar to that reported in the previous experiments.

Table 7 displays the size results for the case of spatially correlated factor loadings when the factors, f_{1t} and f_{2t}, and the idiosyncratic components, ε_{iyt}, are serially uncorrelated. The results are similar to those in Table 3.

Table 8 gives the results for the case where $m^0 (= 2)$ is unknown, and is estimated using the selection criterion IC_1 of Bai and Ng (2004), with $m_{\max} = 3$. Recall also that in these experiments x_{i2t} and x_{i1t} are $I(1)$ and cointegrated.[16] The results are similar to those in Table 3, in that the CIPS test has the correct size in all designs considered, maintaining reasonable power. Thus, cointegration between the additional regressors might not be a problem if the there is a sufficient number of $I(1)$ regressors amongst the additional regressors under consideration.

4 Empirical Applications

As an illustration of the proposed test we consider two applications. One to the real interest rates across $N = 32$ economies, and another to the real equity prices across $N = 26$ markets. Under the Fisher parity hypothesis, the real interest rates, the difference between the nominal short-term interest rate and inflation rate, are stationary. For both applications we employ quarterly observations over the period $1979Q2-2003Q4$ (i.e. 99 data points). Existing evidence on the validity of the Fisher parity is rather mixed. The second application is chosen since it is

[16]We found that the results for $\hat{m}^0$ matched those of m^0 in most cases except when T or N were small.

generally believed that real equity prices are non-stationary, and it would be interesting to see if the outcomes of the tests considered in this paper are in line with this belief.

As discussed in Section 2.2, we begin with a choice of the maximum number of factors, $m_{\max}$, as with other panel unit root tests that are based on principal components. We believe that it is reasonable to suppose that both the real interest rates and the real equity prices contain at most six unobserved common factors. As we set $m_{\max} = 6$, our test requires at most five additional regressors ($k = m_{\max} - 1 = 5$), with their cross section averages being $I(1)$ and not cointegrated. The set of regressors that are likely to have common factors with real interest rates, $r_{it}^S - \pi_{it}$, and real equity prices, eq_{it}, are as follows:

	y_{it}	candidates of $\mathbf{x}_{it}$	
Real Interest Rates ($N = 32$)	$r_{it}^S - \pi_{it}$	$(poil_t, r_{it}^L, eq_{it}, ep_{it}, gdp_{it})$	(46)
Real Equity Prices ($N = 26$)	eq_{it}	$(poil_t, r_{it}^L, \pi_{it}, ep_{it}, gdp_{it})$	

where

$$r_{it}^S = 0.25 * \ln(1 + R_{it}^S/100),\ \pi_{it} = p_{it} - p_{it-1} \text{ with } p_{it} = \ln(CPI_{it}),\ poil_t = \ln(POIL_t),$$
$$r_{it}^L = 0.25 * \ln(1 + R_{it}^L/100),\ ep_{it} = e_{it} - p_{it} \text{ with } e_{it} = \ln(E_{it}),\ eq_{it} = \ln(EQ_{it}/CPI_{it}),$$
$$gdp_{it} = \ln(GDP_{it}/CPI_{it})$$

with R_{it}^S the short rate of interest per annum in per cent (chosen to be a three month rate) in country i at time t, CPI_{it} the consumer price index, $POIL_t$ the price of Brent Crude oil, R_{it}^L the long rate of interest per annum in per cent (typically the yields on ten year government bonds), E_{it} the nominal exchange rate of country i in terms of $U.S.$ dollars, EQ_{it} the nominal equity price index, and GDP_{it} the nominal Gross Domestic Product of country i during period t in domestic currency, so that r_{it}^S is the quarterly short-term interest rate, π_{it} is the quarterly inflation rate, $poil_t$ is the logarithm of the nominal oil price, r_{it}^L is the quarterly long-term interest rate, eq_{it} is the logarithm of the real equity price index, ep_{it} is the logarithm of the real exchange rate and gdp_{it} is real log output.[17]

The 32 countries considered are: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, France, Finland, Germany, Indonesia, India, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, New Zealand, Norway, Peru, Philippines, Spain, Sweden, Switzerland, Singapore, South Africa, Thailand, Turkey, UK. Note that not all candidates of $\mathbf{x}_{it}$ variables are available for all countries due to data constraints. In particular, there are 26 series for eq_{it}, 31 series for ep_{it} (excluding the US), and 18 series for r_{it}^L.

For $m_{\max} = 6$, we estimated the true number of common factors in $r_{it}^S - \pi_{it}$ and eq_{it}, using the Bai-Ng information criterion IC_1, since it performs well in the Monte Carlo exercises reported by Bai and Ng (2004). According to IC_1, $\hat{m}_0 = 2$ for the real interest rates and $\hat{m}_0 = 3$ for the real equity prices. Therefore, to apply the CIPS test we require only one additional regressor for testing the unit root hypothesis in the real interests, and two additional regressors for the real equity prices. To check the robustness of the test outcomes to the choice of the additional regressors used in augmentation we present the CIPS test results for all possible combinations of candidate regressors. We consider lag orders $p = 1, 2, 3, 4$.

The test results are reported in Table 9. Panel A of this table reports the results for the real interest rates. As can be seen, the null hypothesis of a panel unit root is strongly rejected at the 5% level for all cases considered across different choices of $\bar{x}_t$ and the lag-augmentation orders, p.

[17] For a detailed description of the data and sources see Supplement A of Dees, di Mauro, Pesaran and Smith (2007).

These results suggest that for a significant number of countries the Fisher parity holds and are in line with recent findings reported in Westerlund (2008) using panel cointegration tests. Panel B of Table 9, summarises the test results for the real equity prices. For all the ten combinations of additional regressors and all the values of p, the null hypothesis of panel unit root cannot be rejected at the 5% level. This result is in line with the generally accepted view that real equity prices approximately follow random walks with a drift.

We also applied the tests proposed by Bai and Ng (2004) and Moon and Perron (2004). Specifically, the tests discussed in section 3, $P_{\hat{u}}$, $P_{\hat{u},z}$ and t_b^* or $t^{\#}$ are computed for the real interest rates and the real equity prices, using the same estimates of $\hat{m}^0$ as above. In the case of the $P_{\hat{u}}$ and $P_{\hat{u},z}$ tests, up to four lags are considered for the underlying ADF regressions. The test results are summarised in Table 10. The results for the real interest rates are summarised in Panel A, and show that the $P_{\hat{u}}^c$, $P_{\hat{u},z}^c$ and t_b^* tests reject the null hypothesis of a panel unit root at the 5% level for all autoregressive lag orders, p, considered, which accord with the results of the CIPS test. Panel B in Table 10 reports the test results for the real equity prices. The results of the $P_{\hat{u}}^{\tau}$ and $P_{\hat{u},z}^{\tau}$ tests are sensitive to the choice of lag orders. When $p = 1$, they do not reject the null of panel unit root. However, when $p > 1$, the null is rejected. This is in contrast to the results of our CIPS test, which could not reject the null for all lag augmentation orders and for all combinations of additional regressors considered. The $t^{\#}$ test also does not reject the null hypothesis. But since $t^{\#}$ lacks power when the regressions include a linear trend, the test outcome might not be reliable.[18]

As a way of dealing with the sampling uncertainty associated with the choice of $\hat{m}^0$, we also consider the application of the CIPS test assuming $m_{\max} = 6$, allowing m^0 to take any value between 0 and 6. Panel A of Table 11 reports the results for the real interest rates, and shows that for all lag orders considered, all the panel unit root tests point to a clear rejection of the null hypothesis. This is in line with the previous results obtained with an estimated value of m^0. The test results for the real equity prices are given in Panel B of the table. For all lag-orders considered, the CIPS test does not reject the null, but as before the results of the $P_{\hat{u}}^{\tau}$ and $P_{\hat{u},z}^{\tau}$ tests are sensitive to the choice of lag orders. But now $t^{\#}$ tests reject the null hypothesis, indicating the sensitivity of this test to the choice of the number of factors.

5 Concluding Remarks

This paper considers a simple panel unit root test that is valid in the presence of cross section dependence induced possibly by m common factors. The proposed test is based on the average of t-ratios from ADF regressions of the variables of interest augmented by the cross section averages of the dependent variable as well as k additional regressors with similar common factor features. Initially we develop a test supposing that m^0, the true number of factors is known, and show that the limit distribution of the test does not depend on any nuisance parameters, so long as $k + 1 \geq m^0$. However, in practice m^0 is not known. Given an assumed maximum number of factors, $m_{\max}$, we suggest two strategies for dealing with uncertainty that surrounds the value of m^0. One is to choose the number of additional regressors as $k = m_{\max} - 1$, which avoids having to estimate m^0. In this case, the true number of factors are allowed to be any integer value between zero and $m_{\max}$. However, for large values of $m_{\max}$, in some situations it can be difficult to find a sufficient number of additional regressors. Another strategy is to

[18] The $t^{\#}$ test has the asymptotic power within a $N^{-1/6}T^{-1}$-neighbourhood of the null hypothesis of a unit root. Moon, Perron and Phillips (2007) show that a full bias correction, rather than just a correction to the numerator of $t^{\#}$, is required to achieve power in $N^{-1/4}T^{-1}$ neighbourhood of the null.

estimate m^0 consistently using suitable selection criteria, as followed by Bai and Ng (2004) and Moon and Perron (2004), amongst others.

Small sample properties of the proposed test are investigated by Monte Carlo experiments, which suggest the test has the correct size and reasonable power for larger values of T and N. In comparing the performance of the proposed test with that of Moon and Perron (2004) and Bai and Ng (2004) when a constant only is included in the data generating process, we find that the CIPS test is somewhat less powerful than the pooled tests of Bai and Ng (2004), which partly could be due to the over-sized nature of the latter. When both an intercept and trend are included, the CIPS test has correct size for all combinations of sample sizes in contrast to the pooled tests of Bai and Ng (2004) and Moon and Perron (2004) that tend to over-reject the null hypothesis, in some cases substantially.

The various panel unit root tests are applied to real interest rates (Fisher's inflation parity) and real equity prices across countries. All tests reject a unit root in real interest rates which is in line with panel cointegration tests of the Fisher equation. However, the pooled test of Bai and Ng (2004) in real equity prices produces rather mixed results. The tests are in favour of the rejection of the unit root hypothesis in real equity prices for moderate lag orders ($p > 1$) while they do not reject the null for lag orders of one ($p = 1$). Also the Moon and Perron test results tend to be sensitive to the choice of the number of factors. This is in contrast to the results of our proposed test that does not reject the null of panel unit root in real equity prices for all lag-orders considered, which accord with the generally accepted view that real equity prices approximately follow random walks with a drift.

The better small sample results reported for the CIPS test as compared to the other tests proposed in the literature comes at a cost, as the test requires the existence of additional $I(1)$ regressors that share the same common factors as in y_{it}. We have argued that this might not be a problem when m_0, the true number of factors in y_{it}, is not too large. For example, if $m^0 \leq 2$, only one additional regressor is needed at most to apply the test, and this is unlikely to be a problem in practice. For larger values of m^0 a more careful consideration of the testing problem is required. In such cases it seems more appropriate if the problem of panel unit root testing is considered as part of a more general problem, where robustness of the panel unit root test outcomes to alternative assumptions regarding the integration and cointegration properties of the additional regressors is considered and evaluated.

Appendix A: Mathematical Proofs

A.2 Preliminary Order Results

The results shown below are for the serially uncorrelated case. For the serially correlated case, analogous order results are obtained.

A.2.1 Order Results A

Under Assumptions 1-5,

$$\frac{\boldsymbol{\varepsilon}_{iy}'\bar{\boldsymbol{\varepsilon}}_y}{T} = O_p\left(\frac{1}{\sqrt{NT}}\right), \ \frac{\boldsymbol{\varepsilon}_{iy}'\bar{\mathbf{E}}}{T} = O_p(\frac{1}{\sqrt{TN}}),$$

$$\frac{\boldsymbol{\tau}_T'\bar{\boldsymbol{\varepsilon}}_y}{T} = O_p\left(\frac{1}{\sqrt{NT}}\right), \ \frac{\boldsymbol{\tau}_T'\bar{\mathbf{E}}}{T} = O_p\left(\frac{1}{\sqrt{NT}}\right),$$

$$\frac{\bar{\boldsymbol{\varepsilon}}_y'\bar{\mathbf{E}}}{T} = O_p\left(\frac{1}{\sqrt{T}N}\right),$$

$$\frac{\bar{\boldsymbol{\varepsilon}}_y'\bar{\boldsymbol{\varepsilon}}_y}{T} = O_p\left(\frac{1}{N}\right), \ \frac{\bar{\mathbf{E}}'\bar{\mathbf{E}}}{T} = O_p\left(\frac{1}{N}\right),$$

$$\frac{\mathbf{F}'\bar{\boldsymbol{\varepsilon}}_y}{T} = O_p\left(\frac{1}{\sqrt{TN}}\right), \ \frac{\mathbf{F}'\bar{\mathbf{E}}}{T} = O_p\left(\frac{1}{\sqrt{TN}}\right).$$

A.2.2 Order Results B

Under Assumptions 1-5,

$$\frac{\bar{\mathbf{S}}_{-1}'\bar{\mathbf{E}}}{T} = O_p\left(\frac{1}{N}\right),$$

$$\frac{\bar{\mathbf{S}}_{-1}'\boldsymbol{\varepsilon}_{iy}}{T} = O_p\left(\frac{1}{\sqrt{N}}\right), \ \frac{\bar{\mathbf{S}}_{-1}'\mathbf{F}}{T} = O_p\left(\frac{1}{\sqrt{N}}\right),$$

$$\frac{\bar{\mathbf{s}}_{y,-1}'\boldsymbol{\tau}_T}{T} = O_p\left(\sqrt{\frac{T}{N}}\right), \ \frac{\bar{\mathbf{S}}_{-1}'\boldsymbol{\tau}_T}{T} = O_p\left(\sqrt{\frac{T}{N}}\right),$$

$$\frac{\bar{\mathbf{S}}_{-1}'\bar{\mathbf{S}}_{-1}}{T^2} = O_p\left(\frac{1}{N}\right), \ \frac{\bar{\mathbf{S}}_{-1}'\bar{\mathbf{s}}_{y,-1}}{T^2} = O_p\left(\frac{1}{N}\right),$$

$$\frac{\mathbf{s}_{iy,-1}'\bar{\mathbf{S}}_{-1}}{T^2} = O_p\left(\frac{1}{\sqrt{N}}\right), \frac{\mathbf{S}_{f,-1}'\bar{\mathbf{S}}_{-1}}{T^2} = O_p\left(\frac{1}{\sqrt{N}}\right),$$

$$\frac{\mathbf{s}_{iy,-1}'\bar{\mathbf{E}}}{T} = O_p\left(\frac{1}{\sqrt{N}}\right), \ \frac{\mathbf{S}_{f,-1}'\bar{\mathbf{E}}}{T} = O_p\left(\frac{1}{\sqrt{N}}\right).$$

A.2.3 Order Results C

Recall that $\boldsymbol{\upsilon}_i = (\boldsymbol{\varepsilon}_{iy} - \bar{\mathbf{E}}\boldsymbol{\delta}_i)/\sigma_i$ and $\mathring{\mathbf{s}}_{i,-1} = \left(\mathbf{s}_{iy,-1}' - \bar{\mathbf{S}}_{-1}\boldsymbol{\delta}_i\right)/\sigma_i$. Thus, from (13) and (19) we have

$$\Delta\bar{\mathbf{Z}} = \mathbf{F}\bar{\boldsymbol{\Gamma}}' + \bar{\mathbf{E}}, \tag{A.1}$$

and

$$\bar{\mathbf{Z}}_{-1} = \boldsymbol{\tau}_T\bar{\mathbf{z}}_0' + \mathbf{S}_{f,-1}\bar{\boldsymbol{\Gamma}}' + \bar{\mathbf{S}}_{-1}. \tag{A.2}$$

Using (A.1), (A.2), Order Results A and B, under Assumptions 1- 5, we obtain the following expressions

$$\frac{\Delta\bar{\mathbf{Z}}'\boldsymbol{\upsilon}_i}{\sqrt{T}} = \bar{\boldsymbol{\Gamma}}\frac{\mathbf{F}'\boldsymbol{\upsilon}_i}{\sqrt{T}} + \frac{\bar{\mathbf{E}}'\boldsymbol{\upsilon}_i}{\sqrt{T}} = \bar{\boldsymbol{\Gamma}}\frac{\mathbf{F}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i\sqrt{T}} + O_p\left(\frac{1}{\sqrt{N}}\right) + O_p\left(\frac{\sqrt{T}}{N}\right),$$

$$\frac{\boldsymbol{\tau}_T'\boldsymbol{\upsilon}_i}{\sqrt{T}} = \frac{\boldsymbol{\tau}_T'\boldsymbol{\varepsilon}_{iy}}{\sigma_i\sqrt{T}} + O_p\left(\frac{1}{\sqrt{N}}\right),$$

$$\frac{\bar{\mathbf{Z}}'_{-1}\boldsymbol{\upsilon}_i}{T} = \bar{\mathbf{z}}_0\left(\frac{\boldsymbol{\tau}'_T\boldsymbol{\upsilon}_i}{T}\right) + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\boldsymbol{\upsilon}_i}{T} + \frac{\bar{\mathbf{S}}'_{-1}\boldsymbol{\upsilon}_i}{T}$$

$$= \bar{\mathbf{z}}_0\left(\frac{\boldsymbol{\tau}'_T\boldsymbol{\varepsilon}_{iy}}{\sigma_i T}\right) + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\boldsymbol{\varepsilon}_{iy}}{\sigma_i T} + O_p\left(\frac{1}{\sqrt{N}}\right) + O_p\left(\sqrt{\frac{1}{NT}}\right),$$

$$\frac{\Delta\bar{\mathbf{Z}}'\mathring{\mathbf{s}}_{i,-1}}{T^{3/2}} = \bar{\boldsymbol{\Gamma}}\frac{\mathbf{F}'\mathring{\mathbf{s}}_{i,-1}}{T^{3/2}} + \frac{\bar{\mathbf{E}}'\mathring{\mathbf{s}}_{i,-1}}{T^{3/2}} = \bar{\boldsymbol{\Gamma}}\frac{\mathbf{F}'\mathbf{s}'_{iy,-1}}{\sigma_i T^{3/2}} + O_p\left(\frac{1}{\sqrt{TN}}\right),$$

$$\frac{\boldsymbol{\tau}'_T\mathring{\mathbf{s}}_{i,-1}}{T^{3/2}} = \frac{\boldsymbol{\tau}'_T\mathbf{s}'_{iy,-1}}{\sigma_i T^{3/2}} + O_p\left(\frac{1}{\sqrt{N}}\right),$$

$$\frac{\bar{\mathbf{Z}}'_{-1}\mathring{\mathbf{s}}_{i,-1}}{T^2} = \bar{\mathbf{z}}_0\left(\frac{\boldsymbol{\tau}'_T\mathring{\mathbf{s}}_{i,-1}}{T^2}\right) + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\mathring{\mathbf{s}}_{i,-1}}{T^2} + \frac{\bar{\mathbf{S}}'_{-1}\mathring{\mathbf{s}}_{i,-1}}{T^2}$$

$$= \bar{\mathbf{z}}_0\left(\frac{\boldsymbol{\tau}'_T\mathbf{s}_{iy,-1}}{\sigma_i T^2}\right) + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\mathbf{s}_{iy,-1}}{\sigma_i T^2} + O_p\left(\frac{1}{\sqrt{N}}\right),$$

$$\frac{\Delta\bar{\mathbf{Z}}'\Delta\bar{\mathbf{Z}}}{T} = \bar{\boldsymbol{\Gamma}}\frac{\mathbf{F}'\mathbf{F}}{T}\bar{\boldsymbol{\Gamma}}' + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{F}'\bar{\mathbf{E}}}{T} + \frac{\bar{\mathbf{E}}'\mathbf{F}}{T}\bar{\boldsymbol{\Gamma}}' + \frac{\bar{\mathbf{E}}'\bar{\mathbf{E}}}{T}$$

$$= \bar{\boldsymbol{\Gamma}}\frac{\mathbf{F}'\mathbf{F}}{T}\bar{\boldsymbol{\Gamma}}' + O_p\left(\frac{1}{\sqrt{NT}}\right) + O_p\left(\frac{1}{N}\right),$$

$$\frac{\boldsymbol{\tau}'_T\Delta\bar{\mathbf{Z}}}{T} = \frac{\boldsymbol{\tau}'_T\mathbf{F}}{T}\bar{\boldsymbol{\Gamma}}' + \frac{\boldsymbol{\tau}'_T\bar{\mathbf{E}}}{T} = \frac{\boldsymbol{\tau}'_T\mathbf{F}}{T}\bar{\boldsymbol{\Gamma}}' + O_p\left(\frac{1}{\sqrt{NT}}\right),$$

$$\frac{\bar{\mathbf{Z}}'_{-1}\Delta\bar{\mathbf{Z}}}{T^{3/2}} = \bar{\mathbf{z}}_0\left(\frac{\boldsymbol{\tau}'_T\mathbf{F}}{T^{3/2}}\bar{\boldsymbol{\Gamma}}' + \frac{\boldsymbol{\tau}'_T\bar{\mathbf{E}}}{T^{3/2}}\right) + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\mathbf{F}}{T^{3/2}}\bar{\boldsymbol{\Gamma}}' + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\bar{\mathbf{E}}}{T^{3/2}} + \frac{\bar{\mathbf{S}}'_{-1}\mathbf{F}}{T^{3/2}}\bar{\boldsymbol{\Gamma}}' + \frac{\bar{\mathbf{S}}'_{-1}\bar{\mathbf{E}}}{T^{3/2}}$$

$$= \bar{\mathbf{z}}_0\left(\frac{\boldsymbol{\tau}'_T\mathbf{F}}{T^{3/2}}\bar{\boldsymbol{\Gamma}}'\right) + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\mathbf{F}}{T^{3/2}}\bar{\boldsymbol{\Gamma}}' + O_p\left(\frac{1}{\sqrt{NT}}\right),$$

$$\frac{\bar{\mathbf{Z}}'_{-1}\boldsymbol{\tau}_T}{T^{3/2}} = \frac{\bar{\mathbf{z}}_0}{\sqrt{T}} + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\boldsymbol{\tau}_T}{T^{3/2}} + \frac{\bar{\mathbf{S}}'_{-1}\boldsymbol{\tau}_T}{T^{3/2}} = \frac{\bar{\mathbf{z}}_0}{\sqrt{T}} + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\boldsymbol{\tau}_T}{T^{3/2}} + O_p\left(\frac{1}{\sqrt{N}}\right),$$

$$\frac{\bar{\mathbf{Z}}'_{-1}\bar{\mathbf{Z}}_{-1}}{T^2} = \frac{\bar{\mathbf{z}}_0\bar{\mathbf{z}}'_0}{T} + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\mathbf{S}_{f,-1}}{T^2}\bar{\boldsymbol{\Gamma}}' + \frac{\bar{\mathbf{S}}'_{-1}\bar{\mathbf{S}}_{-1}}{T^2} + \frac{\bar{\mathbf{S}}'_{-1}\mathbf{S}_{f,-1}}{T^2}\bar{\boldsymbol{\Gamma}}' + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\bar{\mathbf{S}}_{-1}}{T^2}$$

$$+\bar{\mathbf{z}}_0\frac{\boldsymbol{\tau}'_T\mathbf{S}_{f,-1}}{T^2}\bar{\boldsymbol{\Gamma}}' + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\boldsymbol{\tau}_T}{T^2}\bar{\mathbf{z}}'_0$$

$$+\bar{\mathbf{z}}_0\frac{\boldsymbol{\tau}'_T\bar{\mathbf{S}}_{-1}}{T^2} + \frac{\bar{\mathbf{S}}'_{-1}\boldsymbol{\tau}_T}{T^2}\bar{\mathbf{z}}'_0$$

$$= \frac{\bar{\mathbf{z}}_0\bar{\mathbf{z}}'_0}{T} + \bar{\mathbf{z}}_0\frac{\boldsymbol{\tau}'_T\mathbf{S}_{f,-1}}{T^2}\bar{\boldsymbol{\Gamma}}' + \bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\boldsymbol{\tau}_T}{T^2}\bar{\mathbf{z}}'_0$$

$$+\bar{\boldsymbol{\Gamma}}\frac{\mathbf{S}'_{f,-1}\mathbf{S}_{f,-1}}{T^2}\bar{\boldsymbol{\Gamma}}' + O_p\left(\frac{1}{\sqrt{NT}}\right) + O_p\left(\frac{1}{\sqrt{N}}\right),$$

$$\frac{\mathring{\mathbf{s}}'_{i,-1}\boldsymbol{\upsilon}_i}{T^{3/2}} = \frac{\mathbf{s}'_{iy,-1}\boldsymbol{\varepsilon}_{iy}}{\sigma_i^2 T^{3/2}} + O_p\left(\frac{1}{\sqrt{NT}}\right),$$

$$\frac{\mathring{\mathbf{s}}'_{i,-1}\mathring{\mathbf{s}}_{i,-1}}{T^2} = \frac{\mathbf{s}'_{iy,-1}\mathbf{s}_{iy,-1}}{\sigma_i^2 T^2} + O_p\left(\frac{1}{\sqrt{N}}\right).$$

A.3 Proof of Theorem 2.1:

A.3.1 T fixed and $N \to \infty$

Recall equation (23) that can be written as

$$t_i(N,T) = \frac{\frac{\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}\hat{\mathbf{s}}_{i,-1}}{T}}{\left(\frac{\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}_i\boldsymbol{\upsilon}_i}{T-2k-4}\right)^{1/2}\left(\frac{\hat{\mathbf{s}}_{i,-1}'\bar{\mathbf{M}}\hat{\mathbf{s}}_{i,-1}}{T^2}\right)^{1/2}}.$$

Expanding $\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}\hat{\mathbf{s}}_{i,-1}/T$ gives

$$\frac{\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}\hat{\mathbf{s}}_{i,-1}}{T} = \frac{\boldsymbol{\upsilon}_i'\hat{\mathbf{s}}_{i,-1}}{T} - \left(\boldsymbol{\upsilon}_i'\bar{\mathbf{W}}\mathbf{B}\right)\left(\mathbf{B}\bar{\mathbf{W}}'\bar{\mathbf{W}}\mathbf{B}\right)^{-1}\left(\frac{\mathbf{B}\bar{\mathbf{W}}'\hat{\mathbf{s}}_{i,-1}}{T}\right),$$

where

$$\mathbf{B} = \begin{bmatrix} \frac{1}{\sqrt{T}}\mathbf{I}_{k+2} & \mathbf{0} \\ \mathbf{0} & \frac{1}{T}\mathbf{I}_{k+1} \end{bmatrix}$$

$$\mathbf{B}\bar{\mathbf{W}}'\boldsymbol{\upsilon}_i = \begin{pmatrix} \Delta\bar{\mathbf{Z}}'\boldsymbol{\upsilon}_i/\sqrt{T} \\ \boldsymbol{\tau}_T'\boldsymbol{\upsilon}_i/\sqrt{T} \\ \bar{\mathbf{Z}}_{-1}'\boldsymbol{\upsilon}_i/T \end{pmatrix}, \quad \frac{\mathbf{B}\bar{\mathbf{W}}'\hat{\mathbf{s}}_{i,-1}}{T} = \begin{pmatrix} \Delta\bar{\mathbf{Z}}'\hat{\mathbf{s}}_{i,-1}/T^{3/2} \\ \boldsymbol{\tau}_T'\hat{\mathbf{s}}_{i,-1}/T^{3/2} \\ \bar{\mathbf{Z}}_{-1}'\hat{\mathbf{s}}_{i,-1}/T^2 \end{pmatrix},$$

and

$$\mathbf{B}\bar{\mathbf{W}}'\bar{\mathbf{W}}\mathbf{B} = \begin{pmatrix} \frac{\Delta\bar{\mathbf{Z}}'\Delta\bar{\mathbf{Z}}}{T} & \frac{\Delta\bar{\mathbf{Z}}'\boldsymbol{\tau}_T}{T} & \frac{\Delta\bar{\mathbf{Z}}'\bar{\mathbf{Z}}_{-1}}{T^{3/2}} \\ \frac{\boldsymbol{\tau}_T'\Delta\bar{\mathbf{Z}}}{T} & \frac{\boldsymbol{\tau}_T'\boldsymbol{\tau}_T}{T} & \frac{\boldsymbol{\tau}_T'\bar{\mathbf{Z}}_{-1}}{T^{3/2}} \\ \frac{\bar{\mathbf{Z}}_{-1}'\Delta\bar{\mathbf{Z}}}{T^{3/2}} & \frac{\bar{\mathbf{Z}}_{-1}'\boldsymbol{\tau}_T}{T^{3/2}} & \frac{\bar{\mathbf{Z}}_{-1}'\bar{\mathbf{Z}}_{-1}}{T^2} \end{pmatrix}.$$

Next, note that $\bar{\mathbf{M}}_i\boldsymbol{\upsilon}_i$ are the residuals from the regression of $\boldsymbol{\upsilon}_i$ on $\bar{\mathbf{W}}_i = (\bar{\mathbf{W}}, \mathbf{y}_{i,-1})$, but from equation (20) $\mathbf{y}_{i,-1}$ has components $(\bar{\mathbf{Z}}_{-1}, \boldsymbol{\tau}_T, \hat{\mathbf{s}}_{i,-1})$. As $(\boldsymbol{\tau}_T, \bar{\mathbf{Z}}_{-1}) \subset \bar{\mathbf{W}}$, but $\hat{\mathbf{s}}_{i,-1}$ is not contained in $\bar{\mathbf{W}}$, by regression theory

$$\bar{\mathbf{M}}_i\boldsymbol{\upsilon}_i = \bar{\mathbf{M}}_i^*\boldsymbol{\upsilon}_i$$

where

$$\bar{\mathbf{M}}_i^* = \mathbf{I} - \bar{\mathbf{H}}_i\left(\bar{\mathbf{H}}_i'\bar{\mathbf{H}}_i\right)^{-1}\bar{\mathbf{H}}_i',$$

with $\bar{\mathbf{H}}_i = (\bar{\mathbf{W}}, \hat{\mathbf{s}}_{i,-1})$. Thus

$$\boldsymbol{\upsilon}_i\bar{\mathbf{M}}_i^*\boldsymbol{\upsilon}_i = \boldsymbol{\upsilon}_i'\boldsymbol{\upsilon}_i - \left(\boldsymbol{\upsilon}_i'\bar{\mathbf{H}}_i\mathbf{E}'\right)\left(\mathbf{E}\bar{\mathbf{H}}_i'\bar{\mathbf{H}}_i\mathbf{E}'\right)^{-1}\left(\mathbf{E}\bar{\mathbf{H}}_i'\boldsymbol{\upsilon}_i\right),$$

where

$$\mathbf{C} = \begin{pmatrix} \mathbf{B} & \mathbf{0} \\ \mathbf{0} & T^{-1} \end{pmatrix}.$$

Also,

$$\mathbf{C}\bar{\mathbf{H}}_i'\boldsymbol{\upsilon}_i = \begin{pmatrix} \mathbf{B}\bar{\mathbf{W}}'\boldsymbol{\upsilon}_i \\ \hat{\mathbf{s}}_{i,-1}'\boldsymbol{\upsilon}_i/T \end{pmatrix}, \quad \mathbf{C}\bar{\mathbf{H}}_i'\bar{\mathbf{H}}_i\mathbf{C}' = \begin{pmatrix} \mathbf{B}\bar{\mathbf{W}}'\bar{\mathbf{W}}\mathbf{B} & \mathbf{B}\bar{\mathbf{W}}'\hat{\mathbf{s}}_{i,-1}/T \\ \hat{\mathbf{s}}_{i,-1}'\bar{\mathbf{W}}\mathbf{B}/T & \hat{\mathbf{s}}_{i,-1}'\hat{\mathbf{s}}_{i,-1}/T^2 \end{pmatrix}.$$

Under Assumptions 1-5, using the order results in Appendix A.2 and assuming $\bar{\mathbf{z}}_0 = \mathbf{0}$ or re-defining $\mathbf{z}_{it}$ as the deviation from $\bar{\mathbf{z}}_0$, as $N \to \infty$ with T fixed,

$$\mathbf{B}\bar{\mathbf{W}}'\boldsymbol{\upsilon}_i \xrightarrow{N} \boldsymbol{\Xi}\mathbf{q}_{iT}, \ \frac{\mathbf{B}\bar{\mathbf{W}}'\hat{\mathbf{s}}_{i,-1}}{T} \xrightarrow{N} \boldsymbol{\Xi}\mathbf{h}_{iT}, \ \mathbf{B}\bar{\mathbf{W}}'\bar{\mathbf{W}}\mathbf{B} \xrightarrow{N} \boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}',$$

where

$$\boldsymbol{\Xi} = \begin{pmatrix} \boldsymbol{\Gamma} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Gamma} \end{pmatrix} = \underset{N\to\infty}{\text{plim}} \begin{pmatrix} \bar{\boldsymbol{\Gamma}} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \bar{\boldsymbol{\Gamma}} \end{pmatrix}.$$

$$\underset{(2m+1\times 1)}{\mathbf{q}_{iT}} = \begin{pmatrix} \frac{\mathbf{F}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i\sqrt{T}} \\ \frac{\boldsymbol{\tau}_T'\boldsymbol{\varepsilon}_{iy}}{\sigma_i\sqrt{T}} \\ \frac{\mathbf{S}_{f,-1}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i T} \end{pmatrix}, \quad \underset{(2m+1\times 1)}{\mathbf{h}_{iT}} = \begin{pmatrix} \frac{\mathbf{F}'\mathbf{s}_{iy,-1}}{\sigma_i T^{3/2}} \\ \frac{\boldsymbol{\tau}_T'\mathbf{s}_{iy,-1}}{\sigma_i T^{3/2}} \\ \frac{\mathbf{S}_{f,-1}'\mathbf{s}_{iy,-1}}{\sigma_i T^2} \end{pmatrix},$$

$$\mathbf{C}\bar{\mathbf{H}}_i'\boldsymbol{\upsilon}_i \xrightarrow{N} \boldsymbol{\Xi}_*\mathbf{g}_{iT}, \text{ with } \mathbf{g}_{iT} = \begin{pmatrix} \mathbf{q}_{iT} \\ \frac{\mathbf{s}_{iy,-1}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i^2 T} \end{pmatrix},$$

$$\mathbf{C}\bar{\mathbf{H}}_i'\bar{\mathbf{H}}_i\mathbf{C}' \xrightarrow{N} \boldsymbol{\Xi}_*\mathbf{Q}_{iT}\boldsymbol{\Xi}_*', \text{ with } \mathbf{Q}_{iT} = \begin{pmatrix} \boldsymbol{\Upsilon}_{fT} & \mathbf{h}_{iT} \\ \mathbf{h}_{iT}' & \frac{\mathbf{s}_{iy,-1}'\mathbf{s}_{iy,-1}}{\sigma_i^2 T^2} \end{pmatrix},$$

and

$$\boldsymbol{\Xi}_* = \begin{pmatrix} \boldsymbol{\Xi} & \mathbf{0} \\ \mathbf{0} & 1 \end{pmatrix},$$

$$\boldsymbol{\Upsilon}_{fT} = \begin{pmatrix} \frac{\mathbf{F}'\mathbf{F}}{T} & \frac{\mathbf{F}'\boldsymbol{\tau}_T}{T} & \frac{\mathbf{F}'\mathbf{S}_{f,-1}}{T^{3/2}} \\ \frac{\boldsymbol{\tau}_T'\mathbf{F}}{T} & 1 & \frac{\boldsymbol{\tau}_T'\mathbf{S}_{f,-1}}{T^{3/2}} \\ \frac{\mathbf{S}_{f,-1}'\mathbf{F}}{T^{3/2}} & \frac{\mathbf{S}_{f,-1}'\boldsymbol{\tau}_T}{T^{3/2}} & \frac{\mathbf{S}_{f,-1}'\mathbf{S}_{f,-1}}{T^2} \end{pmatrix}.$$

The $(2k+3)\times(2k+3)$ matrix $\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'$ has rank $2m+1 \le 2k+3$ due to rank condition (16), and thus under Assumptions 1-5 we obtain

$$t_i(N,T) \overset{N}{\Longrightarrow} \frac{\frac{\boldsymbol{\varepsilon}_{iy}'\mathbf{s}_{iy,-1}}{\sigma_i^2 T} - \mathbf{q}_{iT}'\boldsymbol{\Xi}'\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)^{-}\boldsymbol{\Xi}\mathbf{h}_{iT}}{\left(\frac{\boldsymbol{\varepsilon}_{iy}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i^2(T-2k-4)} - \frac{\mathbf{g}_{iT}'\boldsymbol{\Xi}_*'\left(\boldsymbol{\Xi}_*\mathbf{Q}_{iT}\boldsymbol{\Xi}_*'\right)^{-}\boldsymbol{\Xi}_*\mathbf{g}_{iT}}{(T-2k-4)}\right)^{1/2}\left(\frac{\mathbf{s}_{iy,-1}'\mathbf{s}_{iy,-1}}{\sigma_i^2 T^2} - \mathbf{h}_{iT}'\boldsymbol{\Xi}'\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)^{-}\boldsymbol{\Xi}\mathbf{h}_{iT}\right)^{1/2}},$$

and as

$$\begin{aligned} \mathbf{q}_{iT}'\boldsymbol{\Xi}'\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)^{-}\boldsymbol{\Xi}\mathbf{h}_{iT} &= \mathbf{q}_{iT}'\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\right)^{-}\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\right)\boldsymbol{\Xi}'\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)^{-}\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right) \\ &\quad \left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)^{-}\boldsymbol{\Xi}\left(\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)\left(\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)^{-}\mathbf{h}_{iT} \\ &= \mathbf{q}_{iT}'\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\right)^{-}\left(\boldsymbol{\Xi}\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)\left(\boldsymbol{\Upsilon}_{fT}\boldsymbol{\Xi}'\right)^{-}\mathbf{h}_{iT} \\ &= \mathbf{q}_{iT}'\boldsymbol{\Upsilon}_{fT}^{-1}\mathbf{h}_{iT}, \end{aligned}$$

where the last line follows using the results of generalised inverse (Magnus and Neudecker, 1999; Miscellaneous Exercises 6, p.38) and similarly for $\mathbf{g}_{iT}'\boldsymbol{\Xi}_*'\left(\boldsymbol{\Xi}_*\mathbf{Q}_{iT}\boldsymbol{\Xi}_*'\right)^{-}\boldsymbol{\Xi}_*\mathbf{g}_{iT}$, it follows that for $rank(\bar{\Gamma}) = m^0 \le k+1$,

$$t_i(N,T) \overset{N}{\Longrightarrow} \frac{\frac{\boldsymbol{\varepsilon}_{iy}'\mathbf{s}_{iy,-1}}{\sigma_i^2 T} - \mathbf{q}_{iT}'\boldsymbol{\Upsilon}_{fT}^{-1}\mathbf{h}_{iT}}{\left(\frac{\boldsymbol{\varepsilon}_{iy}'\boldsymbol{\varepsilon}_{iy}}{\sigma_i^2(T-2k-4)} - \frac{\mathbf{g}_{iT}'\mathbf{Q}_{iT}^{-1}\mathbf{g}_{iT}}{(T-2k-4)}\right)^{1/2}\left(\frac{\mathbf{s}_{iy,-1}'\mathbf{s}_{iy,-1}}{\sigma_i^2 T^2} - \mathbf{h}_{iT}'\boldsymbol{\Upsilon}_{fT}^{-1}\mathbf{h}_{iT}\right)^{1/2}}.$$

A.4 Proof of Theorem 2.2:

A.4.1 Sequential Asymptotics: $N \to \infty$ then $T \to \infty$

Using Proposition 17.1 and 18.1 of Hamilton (1994; p.486, p.547-8), under Assumptions 1-5 we have

$$\mathbf{q}_{iT} \overset{T}{\Longrightarrow} \boldsymbol{\vartheta}_{if} = \begin{pmatrix} \boldsymbol{\Lambda}_f\mathbf{W}_{\mathbf{v},i}(1) \\ \boldsymbol{\Lambda}_f^*\boldsymbol{\omega}_{i\mathbf{v}} \end{pmatrix}, \ \mathbf{h}_{iT} \overset{T}{\Longrightarrow} \boldsymbol{\kappa}_{if} = \begin{pmatrix} \mathbf{0} \\ \boldsymbol{\Lambda}_f^*\boldsymbol{\pi}_{i\mathbf{v}} \end{pmatrix}$$

$$\boldsymbol{\Upsilon}_{fT} \overset{T}{\Longrightarrow} \boldsymbol{\Upsilon}_f = \begin{pmatrix} \mathbf{I}_m & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_f^*\mathbf{G}_{\mathbf{v}}\boldsymbol{\Lambda}_f^{*\prime} \end{pmatrix},$$

where $\boldsymbol{\Lambda}_f$ is defined by (3), $\mathbf{W}_{\mathbf{v},i}(1) = p\lim_{T\to\infty} T^{-1/2}\sum_{t=1}^T \mathbf{v}_t\varepsilon_{iyt}/\sigma_i$ with $\mathbf{v}_t$ as defined in Assumption 2,

$$\boldsymbol{\Lambda}_f^* = \begin{pmatrix} 1 & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Lambda}_f \end{pmatrix}, \tag{A.3}$$

$$\boldsymbol{\omega}_{i\mathbf{v}} = \begin{pmatrix} W_i(1) \\ \int_0^1 \left[\mathbf{W}_{\mathbf{v}}(r)\right] dW_i(r) \end{pmatrix}, \ \boldsymbol{\pi}_{i\mathbf{v}} = \begin{pmatrix} \int_0^1 W_i(r)dr \\ \int_0^1 \left[\mathbf{W}_{\mathbf{v}}(r)\right] W_i(r)dr \end{pmatrix}, \tag{A.4}$$

and

$$\mathbf{G_v} = \begin{pmatrix} 1 & \int_0^1 [\mathbf{W_v}(r)]' \, dr \\ \int_0^1 [\mathbf{W_v}(r)] \, dr & \int_0^1 [\mathbf{W_v}(r)] [\mathbf{W_v}(r)]' \, dr \end{pmatrix}, \tag{A.5}$$

where $W_i(r)$ is a standard Brownian motion and $\mathbf{W_v}(r)$ is an m^0-dimensional standard Brownian motion defined on [0,1], associated with ε_{iyt} and $\mathbf{v}_t$. These two groups of Brownian motions are independent of each other. From the results in Appendix A.2 we have that

$$t_i(N,T) \overset{(N,T)}{\Longrightarrow} \frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}'_{i\mathbf{v}} \boldsymbol{\Lambda}^{*\prime}_f \left(\boldsymbol{\Lambda}^*_f \mathbf{G_v} \boldsymbol{\Lambda}^{*\prime}_f\right)^{-1} \boldsymbol{\Lambda}^*_f \boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r)dr - \boldsymbol{\pi}'_{i\mathbf{v}} \boldsymbol{\Lambda}^{*\prime}_f \left(\boldsymbol{\Lambda}^*_f \mathbf{G_v} \boldsymbol{\Lambda}^{*\prime}_f\right)^{-1} \boldsymbol{\Lambda}^*_f \boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}},$$

$$= \frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}'_{i\mathbf{v}} \mathbf{G_v}^{-1} \boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r)dr - \boldsymbol{\pi}'_{i\mathbf{v}} \mathbf{G_v}^{-1} \boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}}.$$

Recall that $\boldsymbol{\Lambda}^*_f$ defined by (A.3) is non-singular by Assumption 2.

A.4.2 Joint Asymptotics

From the results in Appendix A.2 it follows that

$$t_i(N,T) \overset{(N,T)_j}{\Longrightarrow} \frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}'_{i\mathbf{v}} \mathbf{G_v}^{-1} \boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r)dr - \boldsymbol{\pi}'_{i\mathbf{v}} \mathbf{G_v}^{-1} \boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}},$$

as T and N go to infinity so long as $\sqrt{T}/N \to 0$. This condition is satisfied as $T/N \to \delta$, where δ is a fixed finite non-zero positive constant.

A.5 Proof of Theorem 2.3

Recall equation (37) that can written as

$$t_i(N,T) = \frac{\frac{\boldsymbol{\upsilon}'_i \bar{\mathbf{M}}_{i1} \mathring{\mathbf{s}}_{i\zeta,-1}}{T}}{\left(\frac{\boldsymbol{\upsilon}'_i \bar{\mathbf{M}}_{i1,p} \boldsymbol{\upsilon}_i}{T-3k-6}\right)^{1/2} \left(\frac{\mathring{\mathbf{s}}'_{i\zeta,-1} \bar{\mathbf{M}}_{i1} \mathring{\mathbf{s}}_{i\zeta,-1}}{T^2}\right)^{1/2}},$$

where $\boldsymbol{\upsilon}_i = [\boldsymbol{\eta}_{iy} - (\bar{\mathbf{E}} - \theta \bar{\mathbf{E}}_{-1})\boldsymbol{\delta}_i]/\sigma_{i\eta}$ and $\mathring{\mathbf{s}}_{i\zeta,-1} = (\mathbf{s}_{i\zeta,-1} - \bar{\mathbf{S}}_{-1}\boldsymbol{\delta}_i)/\sigma_{i\eta}$. Expanding $\boldsymbol{\upsilon}'_i \bar{\mathbf{M}}_{i1} \mathring{\mathbf{s}}_{i\zeta,-1}/T$ gives

$$\frac{\boldsymbol{\upsilon}'_i \bar{\mathbf{M}}_{i1} \mathring{\mathbf{s}}_{i\zeta,-1}}{T} = \frac{\boldsymbol{\upsilon}'_i \mathring{\mathbf{s}}_{i\zeta,-1}}{T} - \left(\boldsymbol{\upsilon}'_i \bar{\mathbf{W}}_{i1} \mathbf{B}_1\right) \left(\mathbf{B}_1 \bar{\mathbf{W}}'_{i1} \bar{\mathbf{W}}_{i1} \mathbf{B}_1\right)^{-1} \left(\frac{\mathbf{B}_1 \bar{\mathbf{W}}'_{i1} \mathring{\mathbf{s}}_{i\zeta,-1}}{T}\right),$$

where $\bar{\mathbf{W}}_{i1} = (\Delta \mathbf{y}_{i,-1}, \Delta \bar{\mathbf{Z}}, \Delta \bar{\mathbf{Z}}_{-1}, \boldsymbol{\tau}_T, \bar{\mathbf{Z}}_{-1})$ and

$$\mathbf{B}_1 = \begin{bmatrix} \frac{1}{\sqrt{T}} \mathbf{I}_{2k+4} & \mathbf{0} \\ \mathbf{0} & \frac{1}{T} \mathbf{I}_{k+1} \end{bmatrix}.$$

Using the results set out above, together with the results in Propositions 17.3 and 18.1 of Hamilton (1994), for example, as N and $T \to \infty$ (sequentially and) jointly such that $\sqrt{T}/N \to 0$, we obtain

$$\mathbf{B}_1 \bar{\mathbf{W}}'_{i1} \boldsymbol{\upsilon}_i \overset{(N,T)_j}{\Longrightarrow} \Theta_1 \boldsymbol{\vartheta}_{if1},$$

where

$$\underset{3(k+1)+2\times 3m+2}{\Theta_1} = \begin{pmatrix} 1 & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \boldsymbol{\Gamma} & \mathbf{0} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \boldsymbol{\Gamma} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & 1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \boldsymbol{\Gamma} \end{pmatrix}, \quad \boldsymbol{\vartheta}_{if1} = \begin{pmatrix} \gamma'_i \boldsymbol{\Lambda}_f \mathbf{W}_{\mathbf{v},i}(1) + \sqrt{\frac{\sigma^2_{i\eta}}{1-\theta^2}} W_i(1) \\ \boldsymbol{\Lambda}_f \mathbf{W}_{\mathbf{v},i}(1) \\ \boldsymbol{\Lambda}_f \mathbf{W}_{\mathbf{v},i}(1) \\ \boldsymbol{\Lambda}^*_f \boldsymbol{\omega}_{i\mathbf{v}} \end{pmatrix},$$

where $\mathbf{\Lambda}_f$ and $\mathbf{\Lambda}_f^*$ are defined by (3) and (A.3), respectively, $\mathbf{W}_{\mathbf{v},i}(1) = p\lim_{T\to\infty} T^{-1/2}\sum_{t=1}^{T}\mathbf{v}_t\eta_{iyt}/\sigma_{i\eta}$, $W_i(r)$ is a standard Brownian motion and $\mathbf{W}_{\mathbf{v}}(r)$ is an m^0-dimensional standard Brownian motion defined on [0,1], associated with η_{iyt} and $\mathbf{v}_t$, respectively, and $\boldsymbol{\omega}_{i\mathbf{v}}$ is as defined by (A.4). Also

$$\frac{\mathbf{B}_1\bar{\mathbf{W}}_{i1}'\mathring{\mathbf{s}}_{i\zeta,-1}}{T} \overset{(N,T)_j}{\Longrightarrow} \Theta_1\boldsymbol{\kappa}_{if1},\ \boldsymbol{\kappa}_{if1} = \begin{pmatrix} \mathbf{0}_{2m+1} \\ \frac{1}{1-\theta}\mathbf{\Lambda}_f^*\boldsymbol{\pi}_{i\mathbf{v}} \end{pmatrix},$$

where $\boldsymbol{\pi}_{i\mathbf{v}}$ is defined by (A.4). Next,

$$\mathbf{B}_1\bar{\mathbf{W}}_{i1}'\bar{\mathbf{W}}_{i1}\mathbf{B}_1 \overset{(N,T)_j}{\Longrightarrow} \Theta_1\mathbf{\Upsilon}_{if1}\Theta_1',\ \mathbf{\Upsilon}_{if1} = \begin{pmatrix} \varkappa_{if1} & \mathbf{0}_{2m^0+1\times m^0+1} \\ \mathbf{0}_{2m^0+1\times m^0+1}' & \mathbf{\Lambda}_f^*\mathbf{G}_{\mathbf{v}}\mathbf{\Lambda}_f^{*\prime} \end{pmatrix},$$

where $\mathbf{G}_{\mathbf{v}}$ is defined by (A.5)

$$\varkappa_{if1} = \begin{pmatrix} \gamma_i'\gamma_i + \frac{\boldsymbol{\sigma}_{\eta i}^2}{1-\theta^2} & \gamma_i'\mathbf{\Sigma}_{f1}' & \gamma_i' \\ \mathbf{\Sigma}_{f1}\gamma_i & \mathbf{I}_{m^0} & \mathbf{\Sigma}_{f1} \\ \gamma_i & \mathbf{\Sigma}_{f1}' & \mathbf{I}_{m^0} \end{pmatrix},$$

with $\mathbf{\Sigma}_{f\ell} = E(\mathbf{f}_t\mathbf{f}_{t-\ell}')$, and

$$\frac{\boldsymbol{\upsilon}_i'\mathring{\mathbf{s}}_{i\zeta,-1}}{T} \overset{(N,T)_j}{\Longrightarrow} \frac{1}{1-\theta}\int_0^1 W_i(r)dW_i(r).$$

For the term $\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}_{i1,p}\boldsymbol{\upsilon}_i$, following a similar reasoning as in the uncorrelated case we can write

$$\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}_{i1,p}\boldsymbol{\upsilon}_i = \boldsymbol{\upsilon}_i'\boldsymbol{\upsilon}_i - \left(\boldsymbol{\upsilon}_i'\bar{\mathbf{H}}_{i1}\mathbf{C}_1'\right)\left(\mathbf{C}_1\bar{\mathbf{H}}_{i1}'\bar{\mathbf{H}}_{i1}\mathbf{C}_1'\right)^{-1}\left(\mathbf{C}_1\bar{\mathbf{H}}_{i1}'\boldsymbol{\upsilon}_i\right),$$

where $\bar{\mathbf{H}}_{i1} = \left(\bar{\mathbf{W}}_{i1},\mathring{\mathbf{s}}_{i\zeta,-1}\right)$ and

$$\mathbf{C}_1 = \begin{pmatrix} \mathbf{B}_1 & \mathbf{0} \\ \mathbf{0} & T^{-1} \end{pmatrix},$$

so that

$$\mathbf{C}_1\bar{\mathbf{H}}_{i1}'\boldsymbol{\upsilon}_i = \begin{pmatrix} \mathbf{B}_1\bar{\mathbf{W}}_{i1}'\boldsymbol{\upsilon}_i \\ \mathring{\mathbf{s}}_{i\zeta,-1}'\boldsymbol{\upsilon}_i/T \end{pmatrix},\ \mathbf{C}_1\bar{\mathbf{H}}_{i1}'\bar{\mathbf{H}}_{i1}\mathbf{C}_1' = \begin{pmatrix} \mathbf{B}_1\bar{\mathbf{W}}_{i1}'\bar{\mathbf{W}}_{i1}\mathbf{B}_1 & \mathbf{B}_1\bar{\mathbf{W}}_{i1}'\mathring{\mathbf{s}}_{i\zeta,-1}/T \\ \mathring{\mathbf{s}}_{i\zeta,-1}'\bar{\mathbf{W}}_{i1}\mathbf{B}_1/T & \mathring{\mathbf{s}}_{i\zeta,-1}'\mathring{\mathbf{s}}_{i\zeta,-1}/T^2 \end{pmatrix}.$$

It is easily seen that $\boldsymbol{\upsilon}_i'\bar{\mathbf{M}}_{i1,p}\boldsymbol{\upsilon}_i/(T-3k-6) \overset{(N,T)_j}{\Longrightarrow} 1$ using the above conditions and results, and

$$\frac{\mathring{\mathbf{s}}_{i\zeta,-1}'\mathring{\mathbf{s}}_{i\zeta,-1}}{T^2} \overset{(N,T)_j}{\Longrightarrow} \frac{1}{(1-\theta)^2}\int_0^1 W_i^2(r)dr.$$

Thus, under Assumptions 1-5, assuming $\bar{\mathbf{z}}_0 = \mathbf{0}$ or re-defining $\mathbf{z}_{it}$ as the deviation from $\bar{\mathbf{z}}_0$, as N and $T\to\infty$, sequentially or jointly such that $\sqrt{T}/N \to 0$, we obtain (since $\mathbf{\Lambda}_f^*$ is a non-singular matrix)

$$t_i(N,T) \overset{(N,T)_j}{\Longrightarrow} \frac{\frac{1}{1-\theta}\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}_{i\mathbf{v}}'\mathbf{\Lambda}_f^{*\prime}\left(\mathbf{\Lambda}_f^*\mathbf{G}_{\mathbf{v}}\mathbf{\Lambda}_f^{*\prime}\right)^{-1}\frac{1}{1-\theta}\mathbf{\Lambda}_f^*\boldsymbol{\pi}_{i\mathbf{v}}}{\left(\frac{1}{(1-\theta)^2}\int_0^1 W_i^2(r)dr - \frac{1}{1-\theta}\boldsymbol{\pi}_{i\mathbf{v}}'\mathbf{\Lambda}_f^{*\prime}\left(\mathbf{\Lambda}_f^*\mathbf{G}_{\mathbf{v}}\mathbf{\Lambda}_f^{*\prime}\right)^{-1}\frac{1}{1-\theta}\mathbf{\Lambda}_f^*\boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}}$$

$$= \frac{\int_0^1 W_i(r)dW_i(r) - \boldsymbol{\omega}_{i\mathbf{v}}'\mathbf{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}}{\left(\int_0^1 W_i^2(r)dr - \boldsymbol{\pi}_{i\mathbf{v}}'\mathbf{G}_{\mathbf{v}}^{-1}\boldsymbol{\pi}_{i\mathbf{v}}\right)^{1/2}}$$

which is identical to the limit distribution obtained for $\theta = 0$.

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Table 1: Critical Values of Average of Individual Cross-Sectionally Augmented Dickey-Fuller Distribution, Intercept Only

$k = 1$

		1% ($\overline{CADF}$)						5% ($\overline{CADF}$)						10% ($\overline{CADF}$)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
	20	-2.64	-2.56	-2.51	-2.45	-2.42	-2.39	-2.44	-2.38	-2.34	-2.30	-2.29	-2.26	-2.33	-2.28	-2.25	-2.22	-2.21	-2.19
	30	-2.64	-2.55	-2.48	-2.46	-2.44	-2.40	-2.45	-2.39	-2.35	-2.33	-2.31	-2.29	-2.34	-2.30	-2.27	-2.25	-2.24	-2.22
0	50	-2.62	-2.56	-2.49	-2.46	-2.44	-2.42	-2.45	-2.40	-2.36	-2.34	-2.32	-2.30	-2.35	-2.32	-2.28	-2.27	-2.26	-2.25
	70	-2.64	-2.55	-2.49	-2.46	-2.45	-2.43	-2.46	-2.40	-2.37	-2.35	-2.33	-2.32	-2.36	-2.32	-2.30	-2.28	-2.27	-2.26
	100	-2.63	-2.56	-2.51	-2.47	-2.45	-2.43	-2.47	-2.41	-2.38	-2.35	-2.34	-2.32	-2.37	-2.33	-2.31	-2.28	-2.28	-2.26
	200	-2.63	-2.56	-2.50	-2.48	-2.46	-2.43	-2.46	-2.42	-2.38	-2.36	-2.35	-2.33	-2.37	-2.34	-2.31	-2.29	-2.29	-2.27
	20	-2.63	-2.54	-2.45	-2.41	-2.40	-2.35	-2.38	-2.32	-2.27	-2.23	-2.22	-2.19	-2.26	-2.20	-2.17	-2.14	-2.12	-2.10
	30	-2.61	-2.52	-2.46	-2.43	-2.41	-2.36	-2.41	-2.34	-2.30	-2.28	-2.26	-2.24	-2.29	-2.24	-2.22	-2.19	-2.18	-2.17
1	50	-2.62	-2.55	-2.48	-2.44	-2.43	-2.40	-2.43	-2.37	-2.33	-2.31	-2.30	-2.28	-2.33	-2.29	-2.25	-2.24	-2.23	-2.21
	70	-2.62	-2.54	-2.47	-2.45	-2.43	-2.41	-2.45	-2.39	-2.35	-2.33	-2.32	-2.29	-2.35	-2.30	-2.27	-2.26	-2.24	-2.23
	100	-2.62	-2.54	-2.50	-2.46	-2.44	-2.41	-2.45	-2.40	-2.37	-2.34	-2.32	-2.31	-2.36	-2.32	-2.30	-2.27	-2.26	-2.24
	200	-2.62	-2.55	-2.50	-2.47	-2.46	-2.43	-2.46	-2.41	-2.37	-2.36	-2.34	-2.32	-2.37	-2.33	-2.30	-2.28	-2.28	-2.26
	20	-2.44	-2.34	-2.25	-2.21	-2.17	-2.12	-2.18	-2.11	-2.05	-2.01	-1.99	-1.96	-2.03	-1.98	-1.94	-1.91	-1.89	-1.88
	30	-2.50	-2.41	-2.33	-2.30	-2.28	-2.24	-2.27	-2.21	-2.17	-2.14	-2.12	-2.10	-2.15	-2.11	-2.07	-2.05	-2.04	-2.03
2	50	-2.55	-2.49	-2.40	-2.37	-2.35	-2.32	-2.35	-2.31	-2.25	-2.24	-2.22	-2.20	-2.25	-2.22	-2.17	-2.16	-2.15	-2.13
	70	-2.57	-2.49	-2.43	-2.41	-2.39	-2.36	-2.39	-2.34	-2.29	-2.27	-2.26	-2.24	-2.30	-2.25	-2.22	-2.20	-2.19	-2.18
	100	-2.59	-2.51	-2.46	-2.42	-2.40	-2.38	-2.41	-2.37	-2.33	-2.30	-2.29	-2.27	-2.32	-2.28	-2.26	-2.23	-2.22	-2.20
	200	-2.61	-2.54	-2.48	-2.46	-2.43	-2.40	-2.43	-2.39	-2.35	-2.34	-2.33	-2.30	-2.35	-2.31	-2.28	-2.26	-2.27	-2.24
	20	-2.45	-2.32	-2.21	-2.13	-2.09	-2.02	-2.12	-2.05	-1.97	-1.90	-1.89	-1.85	-1.96	-1.90	-1.83	-1.80	-1.77	-1.74
	30	-2.43	-2.34	-2.28	-2.24	-2.21	-2.18	-2.21	-2.13	-2.09	-2.07	-2.05	-2.03	-2.09	-2.02	-1.99	-1.97	-1.96	-1.94
3	50	-2.53	-2.45	-2.38	-2.36	-2.33	-2.30	-2.32	-2.27	-2.22	-2.20	-2.19	-2.17	-2.21	-2.18	-2.13	-2.12	-2.11	-2.09
	70	-2.56	-2.47	-2.42	-2.39	-2.37	-2.35	-2.37	-2.31	-2.27	-2.25	-2.24	-2.22	-2.27	-2.23	-2.19	-2.18	-2.16	-2.15
	100	-2.59	-2.50	-2.46	-2.42	-2.39	-2.37	-2.39	-2.35	-2.32	-2.29	-2.27	-2.25	-2.30	-2.26	-2.24	-2.21	-2.20	-2.19
	200	-2.61	-2.54	-2.47	-2.45	-2.43	-2.40	-2.43	-2.39	-2.34	-2.33	-2.32	-2.30	-2.34	-2.31	-2.27	-2.26	-2.26	-2.24
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.29	-2.20	-2.12	-2.06	-2.05	-2.03	-2.05	-1.99	-1.94	-1.91	-1.89	-1.86	-1.93	-1.87	-1.84	-1.81	-1.79	-1.78
4	50	-2.44	-2.37	-2.30	-2.28	-2.23	-2.21	-2.24	-2.19	-2.14	-2.12	-2.11	-2.08	-2.13	-2.09	-2.05	-2.04	-2.02	-2.01
	70	-2.52	-2.43	-2.37	-2.34	-2.32	-2.30	-2.32	-2.26	-2.22	-2.20	-2.19	-2.17	-2.22	-2.17	-2.14	-2.12	-2.11	-2.10
	100	-2.54	-2.48	-2.42	-2.38	-2.35	-2.33	-2.36	-2.31	-2.28	-2.25	-2.23	-2.22	-2.26	-2.23	-2.20	-2.17	-2.17	-2.15
	200	-2.60	-2.52	-2.46	-2.44	-2.42	-2.39	-2.41	-2.37	-2.33	-2.30	-2.30	-2.28	-2.32	-2.29	-2.26	-2.24	-2.24	-2.22

$k = 2$

		1% ($\overline{CADF}$)						5% ($\overline{CADF}$)						10% ($\overline{CADF}$)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
	20	-2.84	-2.78	-2.68	-2.65	-2.59	-2.57	-2.60	-2.56	-2.49	-2.48	-2.45	-2.43	-2.48	-2.45	-2.40	-2.39	-2.37	-2.35
	30	-2.84	-2.76	-2.68	-2.65	-2.63	-2.59	-2.64	-2.58	-2.53	-2.51	-2.50	-2.47	-2.53	-2.49	-2.45	-2.43	-2.42	-2.40
0	50	-2.84	-2.77	-2.70	-2.67	-2.66	-2.62	-2.66	-2.61	-2.56	-2.55	-2.53	-2.51	-2.57	-2.52	-2.48	-2.48	-2.46	-2.45
	70	-2.86	-2.78	-2.72	-2.68	-2.67	-2.64	-2.67	-2.62	-2.58	-2.56	-2.55	-2.53	-2.58	-2.54	-2.50	-2.49	-2.48	-2.47
	100	-2.85	-2.79	-2.72	-2.69	-2.67	-2.64	-2.68	-2.64	-2.59	-2.57	-2.56	-2.54	-2.59	-2.56	-2.52	-2.50	-2.49	-2.48
	200	-2.87	-2.80	-2.74	-2.70	-2.68	-2.66	-2.69	-2.65	-2.61	-2.58	-2.56	-2.55	-2.60	-2.57	-2.53	-2.52	-2.50	-2.49
	20	-2.78	-2.68	-2.55	-2.53	-2.48	-2.46	-2.47	-2.40	-2.33	-2.32	-2.30	-2.27	-2.33	-2.27	-2.23	-2.21	-2.19	-2.17
	30	-2.76	-2.69	-2.61	-2.57	-2.56	-2.51	-2.54	-2.49	-2.43	-2.41	-2.39	-2.36	-2.42	-2.38	-2.34	-2.32	-2.31	-2.29
1	50	-2.80	-2.74	-2.67	-2.64	-2.61	-2.58	-2.61	-2.56	-2.51	-2.50	-2.47	-2.45	-2.50	-2.47	-2.43	-2.42	-2.40	-2.38
	70	-2.83	-2.75	-2.68	-2.65	-2.64	-2.60	-2.65	-2.59	-2.53	-2.52	-2.51	-2.49	-2.54	-2.49	-2.46	-2.44	-2.44	-2.42
	100	-2.84	-2.78	-2.70	-2.67	-2.65	-2.62	-2.65	-2.61	-2.57	-2.54	-2.53	-2.51	-2.56	-2.52	-2.49	-2.47	-2.46	-2.45
	200	-2.85	-2.80	-2.72	-2.69	-2.67	-2.65	-2.69	-2.64	-2.60	-2.57	-2.56	-2.54	-2.59	-2.56	-2.52	-2.50	-2.49	-2.48
	20	-2.71	-2.51	-2.33	-2.27	-2.21	-2.16	-2.29	-2.17	-2.08	-2.03	-1.99	-1.95	-2.08	-2.01	-1.93	-1.90	-1.87	-1.84
	30	-2.58	-2.50	-2.41	-2.36	-2.34	-2.30	-2.35	-2.30	-2.22	-2.20	-2.18	-2.15	-2.21	-2.17	-2.12	-2.10	-2.10	-2.07
2	50	-2.70	-2.63	-2.55	-2.53	-2.50	-2.47	-2.50	-2.45	-2.39	-2.39	-2.37	-2.34	-2.38	-2.35	-2.30	-2.30	-2.28	-2.26
	70	-2.75	-2.68	-2.61	-2.58	-2.57	-2.53	-2.57	-2.51	-2.46	-2.44	-2.44	-2.41	-2.45	-2.42	-2.38	-2.37	-2.36	-2.34
	100	-2.79	-2.72	-2.65	-2.62	-2.60	-2.57	-2.60	-2.56	-2.51	-2.49	-2.47	-2.46	-2.51	-2.46	-2.44	-2.42	-2.40	-2.39
	200	-2.84	-2.77	-2.69	-2.66	-2.65	-2.62	-2.67	-2.62	-2.57	-2.54	-2.53	-2.51	-2.56	-2.53	-2.49	-2.48	-2.46	-2.45
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.47	-2.37	-2.27	-2.21	-2.19	-2.16	-2.20	-2.14	-2.07	-2.04	-2.02	-2.00	-2.07	-2.02	-1.96	-1.94	-1.93	-1.90
3	50	-2.64	-2.57	-2.49	-2.48	-2.44	-2.41	-2.43	-2.38	-2.32	-2.31	-2.29	-2.27	-2.31	-2.28	-2.23	-2.23	-2.21	-2.19
	70	-2.72	-2.65	-2.56	-2.54	-2.53	-2.49	-2.51	-2.47	-2.42	-2.40	-2.39	-2.36	-2.41	-2.37	-2.33	-2.32	-2.31	-2.30
	100	-2.77	-2.69	-2.63	-2.60	-2.58	-2.54	-2.58	-2.53	-2.49	-2.46	-2.45	-2.43	-2.48	-2.44	-2.41	-2.39	-2.37	-2.36
	200	-2.82	-2.76	-2.68	-2.65	-2.63	-2.61	-2.65	-2.61	-2.55	-2.53	-2.52	-2.50	-2.55	-2.52	-2.48	-2.46	-2.44	-2.44
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.28	-2.15	-2.04	-2.01	-1.97	-1.89	-2.00	-1.90	-1.83	-1.81	-1.78	-1.72	-1.85	-1.77	-1.72	-1.70	-1.68	-1.64
4	50	-2.53	-2.45	-2.38	-2.35	-2.31	-2.28	-2.31	-2.25	-2.20	-2.19	-2.16	-2.14	-2.19	-2.15	-2.10	-2.10	-2.08	-2.06
	70	-2.64	-2.59	-2.50	-2.48	-2.45	-2.41	-2.45	-2.38	-2.34	-2.32	-2.31	-2.29	-2.33	-2.29	-2.25	-2.24	-2.23	-2.21
	100	-2.71	-2.65	-2.58	-2.55	-2.53	-2.49	-2.53	-2.47	-2.44	-2.41	-2.39	-2.37	-2.43	-2.38	-2.35	-2.33	-2.32	-2.30
	200	-2.80	-2.74	-2.66	-2.63	-2.60	-2.58	-2.62	-2.58	-2.53	-2.51	-2.49	-2.47	-2.52	-2.49	-2.45	-2.43	-2.42	-2.41

																			(Continued)
									$k=3$										
		1% ($\overline{CADF}$)						5% ($\overline{CADF}$)						10% ($\overline{CADF}$)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
	20	-2.99	-2.91	-2.82	-2.76	-2.75	-2.71	-2.73	-2.68	-2.63	-2.58	-2.57	-2.54	-2.60	-2.56	-2.52	-2.49	-2.48	-2.45
	30	-3.00	-2.89	-2.85	-2.81	-2.78	-2.73	-2.79	-2.73	-2.69	-2.66	-2.64	-2.61	-2.68	-2.63	-2.60	-2.57	-2.56	-2.54
0	50	-3.01	-2.95	-2.88	-2.86	-2.82	-2.80	-2.83	-2.79	-2.74	-2.72	-2.70	-2.69	-2.73	-2.70	-2.65	-2.64	-2.63	-2.62
	70	-3.04	-2.97	-2.89	-2.86	-2.84	-2.82	-2.86	-2.81	-2.77	-2.74	-2.72	-2.71	-2.76	-2.73	-2.69	-2.67	-2.66	-2.65
	100	-3.05	-2.97	-2.92	-2.88	-2.86	-2.83	-2.87	-2.82	-2.78	-2.76	-2.74	-2.73	-2.78	-2.74	-2.71	-2.69	-2.68	-2.66
	200	-3.07	-2.99	-2.92	-2.90	-2.87	-2.85	-2.89	-2.84	-2.80	-2.79	-2.76	-2.75	-2.80	-2.76	-2.72	-2.71	-2.70	-2.68
	20	-2.91	-2.77	-2.66	-2.58	-2.52	-2.47	-2.55	-2.47	-2.39	-2.34	-2.30	-2.26	-2.37	-2.30	-2.25	-2.20	-2.19	-2.15
	30	-2.86	-2.77	-2.69	-2.67	-2.62	-2.59	-2.63	-2.56	-2.52	-2.49	-2.46	-2.42	-2.50	-2.45	-2.41	-2.39	-2.38	-2.35
1	50	-2.94	-2.88	-2.80	-2.78	-2.74	-2.72	-2.75	-2.69	-2.65	-2.62	-2.61	-2.59	-2.65	-2.60	-2.56	-2.54	-2.53	-2.52
	70	-2.99	-2.90	-2.85	-2.82	-2.79	-2.77	-2.80	-2.75	-2.70	-2.67	-2.67	-2.65	-2.70	-2.66	-2.62	-2.60	-2.59	-2.58
	100	-3.01	-2.93	-2.87	-2.84	-2.83	-2.80	-2.83	-2.78	-2.74	-2.72	-2.70	-2.68	-2.73	-2.70	-2.66	-2.64	-2.63	-2.61
	200	-3.05	-2.96	-2.90	-2.89	-2.86	-2.82	-2.87	-2.82	-2.77	-2.76	-2.74	-2.72	-2.78	-2.74	-2.70	-2.69	-2.68	-2.66
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.59	-2.47	-2.39	-2.35	-2.31	-2.27	-2.34	-2.27	-2.20	-2.17	-2.15	-2.11	-2.20	-2.15	-2.10	-2.07	-2.05	-2.02
2	50	-2.81	-2.74	-2.65	-2.63	-2.58	-2.58	-2.60	-2.55	-2.50	-2.46	-2.45	-2.44	-2.48	-2.45	-2.40	-2.38	-2.37	-2.35
	70	-2.90	-2.81	-2.76	-2.72	-2.69	-2.67	-2.70	-2.65	-2.60	-2.57	-2.57	-2.54	-2.59	-2.55	-2.52	-2.49	-2.49	-2.47
	100	-2.96	-2.88	-2.81	-2.78	-2.75	-2.72	-2.76	-2.71	-2.67	-2.64	-2.63	-2.61	-2.67	-2.62	-2.59	-2.57	-2.55	-2.54
	200	-3.01	-2.94	-2.87	-2.85	-2.83	-2.79	-2.84	-2.79	-2.74	-2.73	-2.71	-2.69	-2.75	-2.70	-2.66	-2.66	-2.65	-2.63
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.51	-2.35	-2.19	-2.14	-2.09	-2.02	-2.15	-2.04	-1.96	-1.92	-1.89	-1.84	-1.98	-1.91	-1.85	-1.81	-1.78	-1.75
3	50	-2.72	-2.62	-2.54	-2.52	-2.48	-2.46	-2.49	-2.43	-2.37	-2.34	-2.34	-2.31	-2.37	-2.33	-2.28	-2.26	-2.25	-2.23
	70	-2.82	-2.77	-2.71	-2.66	-2.63	-2.60	-2.63	-2.57	-2.53	-2.50	-2.49	-2.47	-2.51	-2.48	-2.44	-2.41	-2.41	-2.39
	100	-2.92	-2.83	-2.76	-2.73	-2.71	-2.68	-2.72	-2.67	-2.62	-2.59	-2.58	-2.56	-2.62	-2.58	-2.54	-2.52	-2.51	-2.49
	200	-3.00	-2.94	-2.85	-2.84	-2.82	-2.78	-2.82	-2.77	-2.72	-2.71	-2.69	-2.67	-2.72	-2.68	-2.64	-2.64	-2.62	-2.61
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	50	-2.52	-2.44	-2.36	-2.33	-2.30	-2.28	-2.32	-2.24	-2.19	-2.16	-2.14	-2.12	-2.19	-2.13	-2.09	-2.07	-2.06	-2.03
	70	-2.73	-2.65	-2.58	-2.55	-2.52	-2.50	-2.53	-2.46	-2.41	-2.39	-2.37	-2.35	-2.40	-2.36	-2.32	-2.29	-2.29	-2.27
	100	-2.84	-2.78	-2.71	-2.67	-2.65	-2.61	-2.65	-2.60	-2.54	-2.52	-2.51	-2.48	-2.54	-2.50	-2.46	-2.44	-2.43	-2.41
	200	-2.98	-2.90	-2.82	-2.81	-2.78	-2.74	-2.78	-2.73	-2.68	-2.68	-2.66	-2.63	-2.69	-2.64	-2.61	-2.60	-2.59	-2.57

Notes: The critical values are obtained by stochastic simulation. The data generating process is $y_{it} = y_{it-1} + \varepsilon_{iyt}$, where $\varepsilon_{iyt} \sim iidN(0,1)$, with $y_{i,-p} = 0$, and the j^{th} element of the $k \times 1$ vector of additional regressors, x_{it}, is generated as $x_{ijt} = x_{ijt-1} + \varepsilon_{ixjt}$, where $\varepsilon_{ixjt} \sim iidN(0,1)$ and $x_{ij,-p} = 0$, $i = 1,2,...,N$; $j = 1,2,...,k$; $t = -p,...,T$. The $CADF_i$ statistic is computed as the t-ratio of the coefficient on y_{it-1} of the regression of Δy_{it} on y_{it-1}, $\mathbf{w}'_{it,p} = (\bar{\mathbf{z}}'_{t-1}; \Delta\bar{\mathbf{z}}'_t, \Delta\bar{\mathbf{z}}'_{t-1}, ..., \Delta\bar{\mathbf{z}}'_{t-p}; \Delta y_{i,t-1}, ..., \Delta y_{i,t-p})$, including an intercept, with $\bar{\mathbf{z}}_t = N^{-1}\sum_{i=1}^{N}(y_{it}, \mathbf{x}'_{it})'$, and the average of the individual statistics is computed as $\overline{CADF} = N^{-1}\sum_{i=1}^{N} CADF_i$. $(100 \times \alpha)\%$ critical values are obtained as the α quantiles of $\overline{CADF}$ for $\alpha = 0.01, 0.05, 0.1$. Computations are based on 10000 replications.

Table 2: Critical Values of Average of Individual Cross-Sectionally Augmented Dickey-Fuller Distribution, with an Intercept and a Linear Trend

$k = 1$

		1% ($\overline{CADF}$)						5% ($\overline{CADF}$)						10% ($\overline{CADF}$)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
	20	-3.12	-3.02	-2.94	-2.91	-2.86	-2.82	-2.90	-2.85	-2.78	-2.75	-2.73	-2.70	-2.78	-2.74	-2.70	-2.67	-2.65	-2.63
	30	-3.07	-2.99	-2.92	-2.89	-2.86	-2.82	-2.89	-2.84	-2.78	-2.76	-2.74	-2.72	-2.79	-2.75	-2.71	-2.69	-2.68	-2.66
0	50	-3.06	-2.99	-2.93	-2.88	-2.88	-2.83	-2.89	-2.85	-2.81	-2.77	-2.76	-2.73	-2.80	-2.77	-2.74	-2.71	-2.70	-2.68
	70	-3.07	-3.00	-2.93	-2.89	-2.86	-2.83	-2.91	-2.85	-2.81	-2.79	-2.77	-2.74	-2.82	-2.77	-2.75	-2.72	-2.71	-2.69
	100	-3.06	-3.00	-2.92	-2.90	-2.88	-2.83	-2.91	-2.86	-2.81	-2.79	-2.77	-2.75	-2.83	-2.79	-2.75	-2.73	-2.71	-2.70
	200	-3.06	-2.99	-2.94	-2.91	-2.88	-2.84	-2.91	-2.86	-2.82	-2.79	-2.78	-2.75	-2.83	-2.79	-2.75	-2.74	-2.72	-2.70
	20	-3.09	-2.99	-2.90	-2.88	-2.85	-2.81	-2.83	-2.76	-2.72	-2.67	-2.65	-2.63	-2.69	-2.64	-2.60	-2.57	-2.56	-2.53
	30	-3.06	-2.96	-2.89	-2.88	-2.83	-2.80	-2.85	-2.79	-2.74	-2.71	-2.70	-2.67	-2.73	-2.69	-2.65	-2.63	-2.62	-2.60
1	50	-3.06	-2.96	-2.90	-2.87	-2.86	-2.82	-2.87	-2.82	-2.78	-2.74	-2.73	-2.71	-2.77	-2.73	-2.70	-2.67	-2.67	-2.65
	70	-3.05	-2.99	-2.90	-2.88	-2.84	-2.82	-2.89	-2.83	-2.78	-2.76	-2.75	-2.73	-2.80	-2.75	-2.72	-2.70	-2.69	-2.67
	100	-3.06	-2.99	-2.91	-2.89	-2.87	-2.82	-2.90	-2.85	-2.80	-2.77	-2.76	-2.73	-2.81	-2.77	-2.73	-2.71	-2.70	-2.68
	200	-3.06	-2.99	-2.93	-2.90	-2.87	-2.83	-2.90	-2.86	-2.81	-2.78	-2.77	-2.75	-2.82	-2.78	-2.75	-2.73	-2.71	-2.70
	20	-2.88	-2.74	-2.64	-2.60	-2.56	-2.52	-2.57	-2.49	-2.42	-2.38	-2.37	-2.34	-2.42	-2.36	-2.31	-2.27	-2.26	-2.24
	30	-2.93	-2.81	-2.75	-2.72	-2.68	-2.64	-2.68	-2.62	-2.57	-2.55	-2.53	-2.50	-2.57	-2.51	-2.47	-2.46	-2.45	-2.43
2	50	-2.96	-2.89	-2.83	-2.79	-2.78	-2.74	-2.78	-2.73	-2.70	-2.66	-2.65	-2.62	-2.68	-2.64	-2.61	-2.58	-2.57	-2.55
	70	-3.00	-2.94	-2.84	-2.83	-2.80	-2.77	-2.83	-2.77	-2.72	-2.71	-2.69	-2.67	-2.73	-2.69	-2.66	-2.64	-2.62	-2.61
	100	-3.03	-2.96	-2.88	-2.85	-2.83	-2.79	-2.85	-2.81	-2.76	-2.74	-2.71	-2.70	-2.76	-2.72	-2.69	-2.67	-2.65	-2.64
	200	-3.03	-2.97	-2.91	-2.88	-2.85	-2.81	-2.88	-2.83	-2.79	-2.77	-2.75	-2.72	-2.79	-2.76	-2.73	-2.71	-2.70	-2.68
	20	-2.96	-2.80	-2.65	-2.58	-2.50	-2.42	-2.57	-2.45	-2.36	-2.31	-2.27	-2.22	-2.37	-2.29	-2.21	-2.18	-2.15	-2.11
	30	-2.84	-2.76	-2.66	-2.63	-2.62	-2.56	-2.60	-2.52	-2.48	-2.46	-2.43	-2.41	-2.48	-2.41	-2.38	-2.37	-2.34	-2.33
3	50	-2.94	-2.86	-2.81	-2.75	-2.74	-2.71	-2.73	-2.70	-2.65	-2.61	-2.61	-2.58	-2.63	-2.60	-2.57	-2.54	-2.53	-2.51
	70	-2.98	-2.94	-2.83	-2.81	-2.78	-2.76	-2.81	-2.74	-2.71	-2.68	-2.66	-2.64	-2.71	-2.66	-2.63	-2.61	-2.60	-2.58
	100	-3.00	-2.94	-2.86	-2.84	-2.82	-2.78	-2.84	-2.79	-2.74	-2.72	-2.70	-2.68	-2.74	-2.71	-2.67	-2.65	-2.64	-2.62
	200	-3.03	-2.96	-2.90	-2.87	-2.85	-2.81	-2.88	-2.82	-2.78	-2.76	-2.75	-2.72	-2.78	-2.75	-2.72	-2.70	-2.69	-2.67
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.68	-2.56	-2.47	-2.43	-2.40	-2.35	-2.41	-2.34	-2.29	-2.26	-2.24	-2.20	-2.28	-2.23	-2.18	-2.16	-2.14	-2.12
4	50	-2.83	-2.76	-2.70	-2.67	-2.65	-2.62	-2.66	-2.59	-2.56	-2.52	-2.51	-2.48	-2.54	-2.50	-2.46	-2.44	-2.43	-2.40
	70	-2.93	-2.86	-2.77	-2.75	-2.72	-2.70	-2.75	-2.68	-2.64	-2.62	-2.60	-2.58	-2.65	-2.59	-2.56	-2.55	-2.53	-2.52
	100	-2.96	-2.91	-2.83	-2.81	-2.78	-2.74	-2.79	-2.75	-2.70	-2.68	-2.66	-2.64	-2.70	-2.66	-2.62	-2.61	-2.59	-2.58
	200	-3.02	-2.94	-2.88	-2.85	-2.83	-2.79	-2.86	-2.81	-2.76	-2.74	-2.73	-2.70	-2.77	-2.73	-2.70	-2.68	-2.67	-2.65

$k = 2$

		1% ($\overline{CADF}$)						5% ($\overline{CADF}$)						10% ($\overline{CADF}$)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
	20	-3.26	-3.15	-3.09	-3.04	-3.00	-2.97	-3.01	-2.95	-2.89	-2.86	-2.84	-2.82	-2.89	-2.84	-2.79	-2.77	-2.75	-2.73
	30	-3.25	-3.15	-3.07	-3.03	-3.02	-2.97	-3.04	-2.98	-2.93	-2.90	-2.88	-2.86	-2.93	-2.89	-2.84	-2.82	-2.81	-2.79
0	50	-3.23	-3.16	-3.09	-3.06	-3.03	-3.01	-3.05	-3.01	-2.96	-2.94	-2.92	-2.90	-2.97	-2.93	-2.89	-2.87	-2.86	-2.84
	70	-3.25	-3.17	-3.10	-3.06	-3.04	-3.01	-3.08	-3.03	-2.98	-2.95	-2.93	-2.92	-2.99	-2.94	-2.91	-2.89	-2.88	-2.86
	100	-3.25	-3.17	-3.11	-3.08	-3.06	-3.02	-3.09	-3.04	-2.99	-2.96	-2.95	-2.93	-3.00	-2.96	-2.92	-2.90	-2.89	-2.88
	200	-3.25	-3.18	-3.12	-3.08	-3.06	-3.03	-3.09	-3.04	-3.00	-2.97	-2.96	-2.94	-3.01	-2.97	-2.94	-2.91	-2.91	-2.89
	20	-3.18	-3.06	-2.97	-2.89	-2.87	-2.84	-2.88	-2.78	-2.71	-2.68	-2.66	-2.64	-2.71	-2.64	-2.58	-2.57	-2.55	-2.52
	30	-3.16	-3.08	-3.00	-2.95	-2.93	-2.89	-2.92	-2.87	-2.82	-2.78	-2.77	-2.75	-2.81	-2.76	-2.72	-2.69	-2.68	-2.67
1	50	-3.21	-3.11	-3.04	-3.01	-2.99	-2.96	-3.00	-2.95	-2.90	-2.87	-2.86	-2.84	-2.90	-2.86	-2.82	-2.80	-2.79	-2.77
	70	-3.21	-3.14	-3.07	-3.03	-3.01	-2.98	-3.06	-2.99	-2.94	-2.91	-2.89	-2.87	-2.95	-2.90	-2.86	-2.84	-2.83	-2.82
	100	-3.24	-3.15	-3.09	-3.05	-3.03	-3.00	-3.06	-3.01	-2.96	-2.93	-2.92	-2.90	-2.96	-2.93	-2.89	-2.87	-2.86	-2.85
	200	-3.25	-3.17	-3.12	-3.08	-3.05	-3.02	-3.09	-3.03	-2.99	-2.96	-2.95	-2.93	-3.00	-2.95	-2.92	-2.90	-2.89	-2.87
	20	-3.43	-3.17	-2.92	-2.82	-2.75	-2.61	-2.77	-2.64	-2.51	-2.45	-2.41	-2.34	-2.52	-2.42	-2.33	-2.28	-2.25	-2.21
	30	-2.92	-2.86	-2.76	-2.71	-2.69	-2.64	-2.68	-2.63	-2.55	-2.53	-2.51	-2.49	-2.56	-2.51	-2.46	-2.43	-2.42	-2.40
2	50	-3.09	-3.00	-2.93	-2.90	-2.87	-2.84	-2.88	-2.82	-2.77	-2.75	-2.73	-2.71	-2.77	-2.72	-2.68	-2.67	-2.65	-2.64
	70	-3.14	-3.06	-3.00	-2.95	-2.93	-2.90	-2.96	-2.90	-2.85	-2.82	-2.81	-2.79	-2.85	-2.81	-2.77	-2.75	-2.74	-2.72
	100	-3.18	-3.10	-3.04	-3.00	-2.98	-2.95	-3.01	-2.95	-2.90	-2.88	-2.86	-2.84	-2.91	-2.87	-2.83	-2.81	-2.80	-2.78
	200	-3.23	-3.14	-3.08	-3.04	-3.02	-2.99	-3.05	-3.00	-2.96	-2.93	-2.92	-2.90	-2.96	-2.93	-2.89	-2.87	-2.86	-2.85
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.81	-2.70	-2.59	-2.54	-2.50	-2.48	-2.53	-2.45	-2.37	-2.34	-2.31	-2.30	-2.38	-2.32	-2.26	-2.24	-2.21	-2.20
3	50	-3.00	-2.92	-2.85	-2.83	-2.80	-2.76	-2.80	-2.74	-2.69	-2.66	-2.64	-2.63	-2.68	-2.64	-2.59	-2.57	-2.56	-2.55
	70	-3.09	-3.03	-2.95	-2.91	-2.88	-2.86	-2.91	-2.85	-2.80	-2.77	-2.76	-2.74	-2.81	-2.75	-2.71	-2.70	-2.68	-2.67
	100	-3.16	-3.07	-3.02	-2.97	-2.96	-2.93	-2.97	-2.92	-2.87	-2.84	-2.83	-2.81	-2.87	-2.83	-2.80	-2.78	-2.76	-2.75
	200	-3.21	-3.14	-3.07	-3.03	-3.01	-2.99	-3.04	-2.99	-2.95	-2.92	-2.90	-2.89	-2.95	-2.91	-2.88	-2.85	-2.85	-2.83
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.63	-2.47	-2.34	-2.28	-2.21	-2.18	-2.31	-2.18	-2.10	-2.06	-2.02	-1.98	-2.13	-2.04	-1.98	-1.94	-1.92	-1.89
4	50	-2.86	-2.79	-2.71	-2.69	-2.66	-2.61	-2.65	-2.59	-2.54	-2.52	-2.49	-2.47	-2.54	-2.49	-2.45	-2.42	-2.41	-2.39
	70	-2.99	-2.93	-2.87	-2.83	-2.79	-2.77	-2.81	-2.75	-2.70	-2.67	-2.66	-2.64	-2.71	-2.66	-2.62	-2.60	-2.58	-2.57
	100	-3.10	-3.02	-2.95	-2.92	-2.91	-2.87	-2.91	-2.85	-2.81	-2.79	-2.77	-2.75	-2.80	-2.76	-2.73	-2.71	-2.70	-2.68
	200	-3.19	-3.11	-3.05	-3.01	-2.98	-2.96	-3.01	-2.96	-2.92	-2.89	-2.88	-2.86	-2.92	-2.88	-2.85	-2.83	-2.81	-2.80

																			(Continued)
$k=3$																			
		1% ($\overline{CADF}$)						5% ($\overline{CADF}$)						10% ($\overline{CADF}$)					
p	(T,N)	20	30	50	70	100	200	20	30	50	70	100	200	20	30	50	70	100	200
	20	-3.39	-3.29	-3.19	-3.13	-3.09	-3.08	-3.10	-3.03	-2.97	-2.94	-2.92	-2.90	-2.96	-2.90	-2.87	-2.84	-2.82	-2.80
	30	-3.38	-3.26	-3.20	-3.16	-3.14	-3.10	-3.16	-3.09	-3.04	-3.01	-3.00	-2.96	-3.05	-2.99	-2.95	-2.93	-2.91	-2.89
0	50	-3.39	-3.31	-3.25	-3.20	-3.18	-3.15	-3.20	-3.15	-3.11	-3.07	-3.06	-3.04	-3.11	-3.07	-3.03	-3.01	-2.99	-2.98
	70	-3.41	-3.32	-3.25	-3.24	-3.19	-3.17	-3.23	-3.17	-3.13	-3.10	-3.09	-3.07	-3.13	-3.09	-3.06	-3.04	-3.02	-3.01
	100	-3.41	-3.33	-3.28	-3.24	-3.22	-3.18	-3.25	-3.20	-3.15	-3.12	-3.10	-3.09	-3.16	-3.12	-3.08	-3.06	-3.04	-3.03
	200	-3.43	-3.34	-3.30	-3.26	-3.23	-3.20	-3.27	-3.21	-3.17	-3.14	-3.13	-3.11	-3.18	-3.13	-3.10	-3.09	-3.07	-3.05
	20	-3.43	-3.26	-3.05	-2.99	-2.93	-2.86	-2.97	-2.85	-2.74	-2.68	-2.65	-2.61	-2.74	-2.66	-2.59	-2.53	-2.51	-2.48
	30	-3.23	-3.13	-3.03	-3.00	-2.97	-2.93	-2.97	-2.90	-2.84	-2.81	-2.79	-2.77	-2.83	-2.79	-2.74	-2.72	-2.70	-2.68
1	50	-3.31	-3.22	-3.17	-3.12	-3.09	-3.07	-3.11	-3.05	-3.00	-2.98	-2.96	-2.94	-3.00	-2.96	-2.92	-2.90	-2.89	-2.87
	70	-3.34	-3.26	-3.21	-3.18	-3.14	-3.11	-3.16	-3.11	-3.06	-3.03	-3.02	-3.00	-3.06	-3.02	-2.99	-2.96	-2.95	-2.94
	100	-3.37	-3.29	-3.23	-3.20	-3.18	-3.15	-3.20	-3.15	-3.10	-3.08	-3.05	-3.04	-3.11	-3.07	-3.03	-3.01	-2.99	-2.98
	200	-3.42	-3.34	-3.27	-3.24	-3.22	-3.19	-3.25	-3.19	-3.15	-3.13	-3.10	-3.08	-3.16	-3.11	-3.08	-3.06	-3.05	-3.03
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.92	-2.77	-2.67	-2.62	-2.60	-2.55	-2.62	-2.54	-2.45	-2.43	-2.41	-2.38	-2.47	-2.41	-2.34	-2.32	-2.31	-2.29
2	50	-3.17	-3.06	-2.99	-2.96	-2.92	-2.90	-2.94	-2.87	-2.83	-2.80	-2.78	-2.76	-2.82	-2.78	-2.73	-2.71	-2.70	-2.68
	70	-3.24	-3.16	-3.09	-3.07	-3.03	-3.00	-3.04	-2.99	-2.94	-2.91	-2.90	-2.88	-2.95	-2.90	-2.86	-2.84	-2.83	-2.81
	100	-3.30	-3.23	-3.16	-3.13	-3.10	-3.07	-3.12	-3.07	-3.02	-3.00	-2.97	-2.96	-3.02	-2.99	-2.95	-2.93	-2.91	-2.90
	200	-3.37	-3.29	-3.24	-3.20	-3.18	-3.15	-3.21	-3.15	-3.11	-3.09	-3.07	-3.05	-3.12	-3.07	-3.04	-3.03	-3.01	-2.99
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-2.85	-2.75	-2.53	-2.45	-2.36	-2.31	-2.46	-2.36	-2.22	-2.18	-2.14	-2.10	-2.27	-2.17	-2.09	-2.05	-2.02	-1.99
3	50	-3.04	-2.94	-2.86	-2.83	-2.79	-2.76	-2.81	-2.74	-2.68	-2.66	-2.64	-2.61	-2.69	-2.63	-2.59	-2.57	-2.55	-2.53
	70	-3.16	-3.11	-3.02	-2.99	-2.95	-2.93	-2.96	-2.91	-2.86	-2.84	-2.81	-2.79	-2.85	-2.81	-2.78	-2.75	-2.74	-2.72
	100	-3.26	-3.19	-3.12	-3.09	-3.06	-3.03	-3.08	-3.02	-2.97	-2.95	-2.93	-2.91	-2.97	-2.93	-2.89	-2.87	-2.86	-2.85
	200	-3.34	-3.28	-3.22	-3.19	-3.16	-3.13	-3.18	-3.13	-3.09	-3.06	-3.05	-3.03	-3.09	-3.05	-3.02	-3.00	-2.98	-2.97
	20	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	30	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
4	50	-2.83	-2.72	-2.66	-2.62	-2.58	-2.54	-2.59	-2.53	-2.47	-2.44	-2.42	-2.38	-2.47	-2.42	-2.37	-2.35	-2.33	-2.30
	70	-3.06	-2.97	-2.89	-2.85	-2.82	-2.80	-2.84	-2.78	-2.73	-2.70	-2.68	-2.66	-2.73	-2.67	-2.64	-2.61	-2.60	-2.58
	100	-3.18	-3.11	-3.05	-3.00	-2.99	-2.95	-2.98	-2.94	-2.89	-2.86	-2.84	-2.83	-2.88	-2.85	-2.81	-2.78	-2.77	-2.76
	200	-3.32	-3.24	-3.18	-3.15	-3.12	-3.10	-3.15	-3.09	-3.05	-3.03	-3.01	-2.99	-3.05	-3.01	-2.98	-2.96	-2.94	-2.93

Notes: The critical values are obtained by stochastic simulation. The data generating process is $y_{it} = y_{it-1} + \varepsilon_{iyt}$, where $\varepsilon_{iyt} \sim iidN(0,1)$, with $y_{i,-p} = 0$, and the j^{th} element of the $k \times 1$ vector of additional regressors, x_{it}, is generated as $x_{ijt} = x_{ijt-1} + \varepsilon_{ixjt}$, where $\varepsilon_{ixjt} \sim iidN(0,1)$ and $x_{ij,-p} = 0$, $i = 1, 2, ..., N$; $j = 1, 2, ..., k$; $t = -p, ..., T$. The $CADF_i$ statistic is computed as the t-ratio of the coefficient on y_{it-1} of the regression of Δy_{it} on y_{it-1}, $\mathbf{w}'_{it,p} = (\bar{\mathbf{z}}'_{t-1}; \Delta\bar{\mathbf{z}}'_t, \Delta\bar{\mathbf{z}}'_{t-1}, ..., \Delta\bar{\mathbf{z}}'_{t-p}; \Delta y_{i,t-1}, ..., \Delta y_{i,t-p})$, including an intercept and a linear trend, with $\bar{\mathbf{z}}_t = N^{-1}\sum_{i=1}^{N}(y_{it}, \mathbf{x}'_{it})'$, and the average of the individual statistics is computed as $\overline{CADF} = N^{-1}\sum_{i=1}^{N} CADF_i$. $(100 \times \alpha)$% critical values are obtained as the α quantiles of $\overline{CADF}$ for $\alpha = 0.01, 0.05, 0.1$. Computations are based on 10000 replications.

Table 3: Size and Power of Panel Unit Root Tests with Two Factors ($m^0 = 2$ and $k = 1$)

PANEL A: With an Intercept Only												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS($p = 0$)												
20	5.10	5.15	4.20	5.15	5.10	5.75	7.35	6.20	5.50	7.30	6.20	6.85
30	5.00	4.60	4.95	3.80	4.25	4.30	9.10	9.05	8.15	8.40	7.70	8.90
50	4.30	4.35	4.60	4.35	5.00	4.65	16.85	17.55	19.45	21.05	23.15	26.95
70	4.80	4.40	4.65	4.05	5.80	5.05	27.10	34.00	42.40	48.05	54.85	63.65
100	4.60	5.05	3.55	4.25	5.00	4.25	52.90	68.70	83.00	89.35	94.85	97.80
200	4.90	4.10	4.15	4.80	4.35	4.05	99.35	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u}}(p = 0)$												
20	9.50	9.35	10.90	10.35	12.85	16.50	18.75	23.80	35.40	36.55	45.90	66.10
30	7.95	8.05	8.55	8.00	11.20	10.95	28.80	36.90	54.30	59.90	71.55	89.75
50	8.20	8.05	7.70	8.95	7.90	8.05	51.90	68.30	89.75	91.30	95.35	98.00
70	6.60	8.45	7.40	7.05	7.40	7.20	75.35	89.00	98.70	98.15	99.25	99.55
100	8.20	8.15	6.20	6.90	5.95	7.10	93.50	98.90	100.00	100.00	99.85	100.00
200	6.75	5.40	6.65	6.10	5.25	6.75	100.00	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u},z}(p = 0)$												
20	8.95	9.60	11.60	10.65	12.60	18.45	14.35	18.20	27.10	29.80	33.85	51.20
30	7.35	6.95	8.45	7.45	10.90	11.45	20.40	23.95	37.10	40.95	48.45	67.85
50	7.55	7.05	7.25	9.20	7.50	8.80	30.90	40.40	63.60	67.95	69.05	80.45
70	5.90	7.95	7.10	6.65	7.40	6.60	44.65	56.15	76.00	77.80	76.65	87.85
100	6.35	7.15	6.10	6.75	6.55	7.50	63.90	72.10	85.35	90.05	85.95	91.25
200	6.05	5.40	5.65	5.60	4.85	6.30	88.20	88.95	94.10	95.75	94.00	97.25
t^*_b												
20	8.90	9.05	9.95	13.80	13.80	20.85	82.70	88.25	95.70	95.35	97.60	97.45
30	10.10	7.15	7.75	11.55	12.35	15.80	93.55	94.50	99.35	98.35	98.95	98.95
50	7.60	7.45	6.80	9.60	8.75	11.65	98.95	98.05	99.70	99.45	99.80	99.65
70	6.95	5.65	6.30	7.85	7.70	9.90	100.00	99.05	99.95	99.80	100.00	99.80
100	7.60	7.50	7.10	8.20	6.85	8.35	100.00	99.95	100.00	100.00	100.00	100.00
200	7.45	7.00	6.20	5.55	5.70	6.85	100.00	100.00	100.00	100.00	100.00	100.00

Notes: y_{it} is generated as $y_{it} = (1 - \rho_i)\alpha_{iy} + \rho_i y_{i,t-1} + \gamma_{iy1} f_{1t} + \gamma_{iy2} f_{2t} + \varepsilon_{iyt}, i = 1, 2, ..., N; t = -49, ..., T$ with $y_{i,-50} = 0$, where $\alpha_{iy} \sim iidN(1, 1)$, $\gamma_{iy\ell} \sim iidU[0, 2]$, $f_{\ell t} = \rho_{f\ell} f_{f\ell,t-1} + v_{\ell t}, v_{\ell t} \sim iidN(0, 1 - \rho^2_{f\ell})$, $f_{\ell,-50} = 0$ for $\ell = 1, 2$, and $\varepsilon_{iyt} = \rho_{iy\varepsilon}\varepsilon_{iyt-1} + \eta_{iyt}, \eta_{iyt} \sim iidN(0, (1 - \rho^2_{iy\varepsilon})\sigma^2_i)$, $\varepsilon_{iy,-50} = 0$, $\sigma^2_i \sim iidU[0.5, 1.5]$. We set $\rho_{iy\varepsilon} = \rho_{\varepsilon y} = 0$ and $\rho_{f1} = \rho_{f2} = 0$. One of two additional regressors is used for augmentation of the CIPS test, which are generated as $x_{ijt} = x_{ijt-1} + \gamma_{ixj1} f_{1t} + \gamma_{ixj2} f_{2t} + \varepsilon_{ixjt}$, $i = 1, 2, ..., N; t = -49, ..., T$ with $x_{ij,-50} = 0$, $\varepsilon_{ixjt} = \rho_{ixj}\varepsilon_{ixjt-1} + \varpi_{ixjt}$, $\varpi_{ixjt} \sim iidN(0, 1-\rho^2_{ixj})$, with $x_{ij,-50} = 0$, and $\rho_{ixj} \sim iidU[0.2, 0.4]$ for $j = 1, 2$. We include only x_{i1t} with $\gamma_{ix1} \sim iidU[0, 2]$ and $\gamma_{ix2} = 0$, so that the rank condition (16) is satisfied. The parameters α_{iy}, $\rho_{iy\varepsilon}$, $\gamma_{iy\ell}$, $\rho_{f\ell}$, ρ_i, $\gamma_{ixj\ell}$, and ρ_{ixj} are drawn once and fixed over the replications. The CIPS(p) test is the proposed panel unit root test, defined by (26), based on cross section augmentation using y_{it} and x_{it} with lag-augmentation of order p. The $P^c_{\hat{u}}(p)$ and $P^c_{\hat{u},z}(p)$ tests are the Bai and Ng (2004) pooled panel unit root tests for the idiosyncratic errors with lag-augmentation of order p based on two extracted factors, where the former uses y_{it}, whilst the latter uses y_{it} and x_{it} for the factor extraction. The t^*_b test is the Moon and Perron (2004) panel unit root test for the idiosyncratic errors based on two extracted factors from y_{it}. This test adopts automatic lag-order selection for the estimation of long-run variances following Andrews and Monahan (1992). All tests are conducted at the 5% significance level, and the CIPS(p) test is based on the critical values for different p and the number of additional regressors, k. All experiments are based on 2000 replications.

(Table 3 continued)

PANEL B: With an Intercept and a Linear Trend												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS($p = 0$)												
20	4.80	3.65	4.40	5.10	5.30	4.95	4.25	4.50	5.35	5.35	5.55	4.35
30	4.90	4.65	4.80	4.45	4.40	5.35	5.90	5.85	5.95	5.35	4.85	5.15
50	4.70	4.30	3.45	4.15	3.90	4.65	7.10	8.35	7.20	8.00	7.75	8.35
70	4.80	5.00	3.75	4.20	4.80	4.30	11.05	15.00	15.55	17.80	18.25	20.55
100	4.50	4.35	4.45	4.60	4.55	4.80	21.85	29.65	43.10	48.65	54.10	68.90
200	4.75	4.10	3.60	5.45	3.40	5.50	89.65	98.30	100.00	100.00	100.00	100.00
$P_{\hat{u}}^{\tau}(p = 0)$												
20	18.85	22.30	28.10	31.70	38.10	51.30	19.40	25.25	30.80	35.45	43.45	59.55
30	16.35	16.60	20.05	22.90	25.55	37.60	19.20	22.10	29.05	31.95	39.05	58.50
50	12.25	12.10	13.60	16.40	16.20	23.15	21.95	28.75	39.75	42.30	55.25	76.75
70	10.25	11.35	11.55	14.00	15.40	17.90	30.10	42.75	59.40	65.15	75.75	92.65
100	10.30	10.15	11.30	11.80	12.40	17.00	46.80	69.45	87.90	86.45	95.00	98.25
200	10.55	8.55	9.30	9.70	9.45	12.65	96.15	99.80	99.95	99.90	100.00	100.00
$P_{\hat{u},z}^{\tau}(p = 0)$												
20	18.30	23.05	28.50	33.80	39.90	53.85	19.70	24.30	29.45	36.80	43.45	58.65
30	14.65	17.00	19.95	22.70	25.45	38.55	16.45	20.40	25.30	29.00	32.40	53.55
50	12.00	11.40	13.65	16.15	16.55	22.90	15.70	23.45	32.00	32.65	42.25	61.80
70	9.50	10.80	10.35	13.55	15.10	18.15	20.70	33.15	43.90	48.35	55.10	76.65
100	8.90	9.05	10.90	11.10	11.55	16.10	33.40	52.70	65.10	68.30	74.80	86.65
200	8.80	7.35	8.30	9.00	9.35	12.70	77.15	92.70	95.55	92.30	95.25	97.30
$t^{\#}$												
20	95.35	96.35	97.90	98.95	99.25	99.55	95.65	97.30	97.90	99.20	99.00	99.90
30	81.85	87.10	93.00	97.15	96.95	99.40	82.00	88.20	94.60	96.50	97.90	99.65
50	43.70	54.60	69.70	78.40	85.75	96.20	49.30	60.95	75.80	80.50	89.10	96.65
70	29.70	37.55	49.30	57.85	68.55	87.45	31.90	43.10	57.65	65.40	74.95	89.65
100	18.95	23.55	31.35	39.00	46.70	69.45	20.85	27.90	41.30	49.40	59.50	77.85
200	9.15	12.75	14.85	16.35	19.55	31.75	9.10	14.00	22.05	27.55	38.70	58.00

Notes: y_{it} is generated as $y_{it} = \mu_{iy} + (1 - \rho_i)\delta_i t + \rho_i y_{i,t-1} + \gamma_{iy1} f_{1t} + \gamma_{iy2} f_{2t} + \varepsilon_{iyt}, i = 1, 2, ..., N; t = -49, ..., T$ with $y_{i,-50} = 0$, where $\mu_{iy} \sim iidU[0.0, 0.02]$, $\delta_i \sim iidU[0.0, 0.02]$, $\gamma_{iy\ell} \sim iidU[0, 2]$, $f_{\ell t} = \rho_{f\ell} f_{f\ell,t-1} + v_{\ell t}, v_{\ell t} \sim iidN(0, 1 - \rho_{f\ell}^2)$, $f_{\ell,-50} = 0$ for $\ell = 1, 2$, and $\varepsilon_{iyt} = \rho_{iy\varepsilon} \varepsilon_{iyt-1} + \eta_{iyt}, \eta_{iyt} \sim iidN(0, (1 - \rho_{iy\varepsilon}^2)\sigma_i^2)$, $\varepsilon_{iy,-50} = 0$, $\sigma_i^2 \sim iidU[0.5, 1.5]$. We set $\rho_{iy\varepsilon} = \rho_{\varepsilon y} = 0$ and $\rho_{f1} = \rho_{f2} = 0$. One of two additional regressors is used for augmentation of the CIPS test, which are generated as $x_{ijt} = x_{ijt-1} + \lambda_{ixj} + \gamma_{ixj1} f_{1t} + \gamma_{ixj2} f_{2t} + \varepsilon_{ixjt}$, $i = 1, 2, ..., N; t = -49, ..., T$ with $x_{ij,-50} = 0$, $\lambda_{ixj} \sim iidU[0.0, 0.02]$, $\varepsilon_{ixjt} = \rho_{ixj} \varepsilon_{ixjt-1} + \varpi_{ixjt}$, $\varpi_{ixjt} \sim iidN(0, 1 - \rho_{ixj}^2)$, with $x_{ij,-50} = 0$, and $\rho_{ixj} \sim iidU[0.2, 0.4]$ for $j = 1, 2$. We include only x_{i1t} with $\gamma_{ix1} \sim iidU[0, 2]$ and $\gamma_{ix2} = 0$, so that the rank condition (16) is satisfied. The parameters μ_{iy}, δ_i, $\rho_{iy\varepsilon}$, $\gamma_{iy\ell}$, $\rho_{f\ell}$, ρ_i, λ_{ixj}, $\gamma_{ixj\ell}$, and ρ_{ixj} are drawn once and fixed over the replications. The CIPS(p) test is the proposed panel unit root tests, defined by (26), based on cross section augmentation using y_{it} and x_{it} with lag-augmentation of order p. The $P_{\hat{u}}^{\tau}(p)$ and $P_{\hat{u},z}^{\tau}(p)$ tests are the Bai and Ng (2004) pooled panel unit root tests for the idiosyncratic errors with lag-augmentation of order p based on two extracted factors, where the former uses y_{it} whilst the latter uses y_{it} and x_{it} for the factor extraction. The $t^{\#}$ test is the Moon and Perron (2004) panel unit root test for the idiosyncratic errors based on two extracted factors from y_{it}. This test adopts automatic lag-order selection for the estimation of long-run variances following Andrews and Monahan (1992). All tests are conducted at the 5% significance level, and the CIPS(p) test is based on the critical values for different p and the number of additional regressors, k. All experiments are based on 2000 replications.

Table 4: Size and Power of Panel Unit Root Tests with Two Factors ($m^0 = 2$ and $k = 1$), Positively Serially Correlated ε_{iyt}

PANEL A: With an Intercept Only												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS($p = 1$)												
20	3.90	3.20	3.90	2.80	3.05	2.75	5.60	4.75	4.85	3.65	4.50	4.10
30	3.10	3.80	3.75	3.05	2.85	3.45	5.65	7.10	6.80	6.40	6.15	6.85
50	4.00	4.20	4.55	3.75	3.35	4.30	13.10	12.65	15.25	17.10	16.70	19.65
70	4.90	4.60	4.65	4.55	4.55	4.00	20.90	24.80	31.40	35.45	36.95	45.45
100	3.95	4.40	4.15	4.65	4.00	3.75	41.35	56.65	68.90	76.30	81.50	89.70
200	5.65	5.15	5.90	5.05	3.90	5.20	98.45	99.80	100.00	100.00	100.00	100.00
$P^c_{\hat{u}}(p = 1)$												
20	14.55	13.65	17.05	18.00	19.90	25.25	24.10	30.85	43.45	48.50	58.85	80.25
30	9.70	10.75	10.50	12.20	12.85	15.60	30.05	40.55	60.65	69.50	81.15	95.35
50	7.50	8.30	8.40	8.05	9.05	10.45	55.20	72.25	92.85	95.65	98.30	99.85
70	8.20	7.00	7.40	9.35	7.35	7.70	77.55	91.15	99.65	99.65	99.95	100.00
100	6.55	6.00	6.85	6.40	7.60	7.55	95.55	99.35	100.00	100.00	100.00	100.00
200	7.80	6.35	6.20	7.00	5.80	5.25	100.00	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u},z}(p = 1)$												
20	13.55	12.70	17.50	17.80	19.20	24.00	21.00	28.00	36.70	42.00	50.40	71.90
30	10.20	10.00	10.10	11.65	12.85	15.15	26.40	32.65	49.55	58.35	66.80	85.85
50	6.90	8.25	7.50	8.40	10.05	11.05	39.70	54.65	76.50	80.55	84.95	95.30
70	7.85	6.70	8.05	8.50	7.35	7.60	55.65	70.75	87.10	92.75	92.70	97.35
100	7.00	6.65	6.95	6.60	7.60	7.30	76.80	85.25	93.65	96.80	95.50	98.80
200	7.80	6.00	5.85	6.55	5.65	5.30	95.35	95.50	98.00	99.35	98.30	99.65
t^*_b												
20	6.75	6.45	5.80	9.75	9.40	12.95	81.60	91.50	97.85	98.10	99.25	99.65
30	7.15	5.20	5.85	7.15	6.85	8.50	94.15	97.55	99.55	99.65	100.00	99.85
50	5.30	5.20	6.10	6.30	6.10	9.05	99.35	99.70	100.00	100.00	100.00	100.00
70	7.60	6.15	4.80	5.60	5.10	6.35	100.00	100.00	100.00	100.00	100.00	100.00
100	7.25	6.65	5.25	6.15	5.40	6.75	100.00	99.95	100.00	100.00	100.00	100.00
200	7.25	5.25	6.15	6.30	5.20	5.25	100.00	100.00	100.00	100.00	100.00	100.00
PANEL B: With an Intercept and a Linear Trend												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS($p = 1$)												
20	3.45	3.10	3.00	2.40	2.40	2.60	3.80	3.35	3.05	2.50	2.20	2.70
30	3.70	3.55	2.80	3.50	3.40	3.40	4.40	4.90	3.60	3.75	4.20	4.15
50	4.95	3.70	4.05	3.60	4.10	3.10	6.75	6.30	6.35	6.50	7.05	6.00
70	4.70	4.90	3.95	4.25	3.70	3.55	8.35	10.95	12.40	12.00	11.25	15.10
100	4.30	3.90	3.70	4.10	3.55	4.45	16.40	20.60	30.60	34.95	39.30	45.95
200	5.20	4.20	5.55	5.40	3.75	4.85	81.90	94.05	99.60	100.00	100.00	100.00
$P^\tau_{\hat{u}}(p = 1)$												
20	28.25	32.90	41.65	48.20	57.45	70.75	29.10	35.10	45.10	53.55	63.50	78.15
30	18.70	20.55	25.40	30.55	35.45	52.15	21.85	26.10	36.05	42.90	52.40	74.60
50	12.75	14.90	18.65	16.90	22.00	30.05	25.05	32.65	44.65	50.80	63.65	87.90
70	13.00	11.60	12.95	15.20	16.45	23.30	31.35	45.35	65.80	71.85	86.20	97.65
100	10.05	10.40	11.75	12.85	11.90	19.05	53.45	73.65	91.50	92.95	98.10	99.80
200	9.20	9.70	8.60	10.50	11.00	12.65	97.80	99.70	100.00	100.00	100.00	100.00
$P^\tau_{\hat{u},z}(p = 1)$												
20	28.50	31.95	41.35	46.85	57.15	71.05	28.80	34.55	44.90	52.95	63.10	77.40
30	17.30	21.00	25.20	31.20	36.30	52.45	21.95	24.95	35.10	41.75	49.70	72.85
50	12.75	15.60	18.35	17.05	21.35	29.75	22.05	30.55	39.50	45.95	55.05	81.65
70	12.45	12.25	12.50	15.10	16.55	23.35	26.45	39.90	57.95	62.65	74.95	91.80
100	9.80	10.45	10.70	12.95	12.85	19.05	45.05	63.20	80.20	84.65	91.20	97.50
200	9.25	9.95	9.15	10.65	10.90	12.60	90.60	97.65	98.90	99.00	99.55	99.80
$t^\#$												
20	75.60	74.40	78.80	88.60	84.00	89.45	75.70	74.20	78.35	88.30	83.85	90.35
30	44.40	43.30	51.95	64.60	62.40	75.05	43.00	41.35	50.30	63.10	63.45	75.45
50	16.85	18.70	23.20	31.40	33.30	46.50	13.25	15.65	19.95	26.25	30.35	44.85
70	11.65	12.85	14.40	16.05	19.85	31.15	8.50	9.45	11.40	14.35	16.25	27.10
100	7.80	8.85	9.80	12.35	13.65	21.65	4.15	4.80	5.85	8.05	10.20	16.70
200	6.15	6.65	7.75	6.40	8.25	9.50	1.85	2.15	2.45	4.25	3.65	8.15

Notes: See notes to Table 3. The data generating process is the same as the one for Table 3, except $\rho_{iy\varepsilon} \sim iidU[0.2, 0.4]$.

Table 5: Size and Power of Panel Unit Root Tests with Two Factors ($m^0 = 2$ and $k = 1$), Negatively Serially Correlated ε_{iyt}

PANEL A: With an Intercept Only												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS$(p=1)$												
20	4.30	5.00	6.50	5.50	5.80	6.70	6.00	5.60	5.10	5.65	5.05	5.95
30	4.35	5.75	6.05	5.15	4.75	6.40	6.10	7.50	7.10	6.00	5.65	6.00
50	5.30	5.00	5.65	5.35	5.00	6.40	14.10	12.75	15.20	16.75	15.85	18.10
70	5.60	4.35	5.25	4.80	5.25	5.00	23.90	30.45	36.40	39.15	43.70	54.70
100	4.00	3.80	4.75	4.85	4.70	4.60	47.95	63.75	79.50	86.25	92.35	97.85
200	5.35	4.85	5.85	4.95	3.85	5.65	99.45	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u}}(p=1)$												
20	12.50	12.60	15.30	14.50	17.35	22.60	24.70	29.20	39.40	39.70	50.55	67.80
30	9.40	10.20	10.70	11.85	12.25	13.75	29.10	34.90	53.45	57.85	65.55	79.15
50	7.95	8.05	8.80	7.95	8.05	9.55	50.00	60.85	83.70	82.00	85.90	91.60
70	8.10	7.45	8.05	8.40	7.80	7.55	71.15	80.55	96.65	92.00	95.70	97.40
100	6.20	6.20	7.30	6.45	7.60	8.20	90.45	96.45	99.60	98.60	99.25	99.10
200	7.65	6.55	6.55	6.40	5.85	5.85	100.00	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u},z}(p=1)$												
20	11.30	12.60	14.40	14.30	18.10	21.50	15.50	17.30	24.35	26.25	31.80	45.55
30	8.00	9.60	9.55	11.35	11.85	13.60	17.40	19.05	30.70	34.10	37.30	51.05
50	6.50	7.35	7.95	7.40	9.10	9.75	24.55	29.70	43.90	47.55	50.05	62.20
70	7.40	6.25	7.20	7.75	7.10	7.30	34.50	38.45	57.85	58.20	61.35	69.75
100	6.85	5.80	7.15	5.85	7.25	8.20	48.20	51.55	65.15	70.55	67.90	74.60
200	7.05	5.55	6.25	6.50	5.70	5.55	73.80	73.05	79.90	85.65	80.40	86.05
t^*_b												
20	11.40	13.10	15.20	19.70	20.20	28.15	80.70	84.30	92.00	90.75	93.55	94.50
30	11.65	9.65	11.85	15.25	16.10	22.85	90.95	90.40	96.15	93.40	95.45	95.80
50	8.05	8.35	9.60	11.20	11.25	18.30	98.10	95.05	98.75	97.20	99.00	98.70
70	9.40	8.00	6.80	9.30	8.45	14.35	99.60	97.50	99.50	98.95	99.70	99.40
100	8.80	8.35	6.70	9.25	8.15	12.80	99.95	99.05	99.85	99.60	99.90	99.80
200	7.80	6.10	6.75	7.45	6.80	7.50	100.00	99.85	100.00	100.00	100.00	100.00
PANEL B: With an Intercept and a Linear Trend												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS$(p=1)$												
20	5.25	6.70	5.75	5.25	5.75	6.75	5.50	6.15	5.20	4.50	4.85	6.00
30	3.95	5.60	5.15	6.55	6.05	6.60	4.80	5.90	5.00	4.55	5.35	5.65
50	5.50	4.45	5.10	5.00	6.05	5.35	6.25	6.10	6.20	5.85	5.70	4.80
70	4.70	5.00	4.55	4.90	4.75	5.55	8.00	11.55	12.55	9.90	9.55	12.40
100	3.80	4.50	3.80	5.25	4.05	5.25	16.40	23.25	32.65	38.00	46.60	57.45
200	4.70	3.90	5.45	5.60	4.00	5.85	88.25	97.00	99.85	100.00	100.00	100.00
$P^c_{\hat{u}}(p=1)$												
20	18.90	22.25	23.45	26.00	33.30	42.40	19.85	23.10	27.25	29.30	34.25	45.10
30	13.80	14.70	15.10	17.15	18.70	25.35	14.85	17.20	20.00	20.15	26.05	36.75
50	10.30	10.25	11.90	11.45	12.30	14.20	17.35	21.05	26.35	26.20	32.90	48.95
70	10.95	9.35	10.35	9.10	10.95	12.20	21.80	29.65	42.75	41.30	52.90	69.10
100	9.20	9.20	9.25	9.55	7.75	11.65	36.95	53.40	69.80	64.65	78.95	87.75
200	8.10	9.25	7.75	8.60	8.90	10.85	89.40	97.50	99.20	96.60	99.70	99.75
$P^\tau_{\hat{u},z}(p=1)$												
20	16.75	19.55	22.35	25.10	33.45	41.75	17.15	19.85	24.15	25.25	30.50	39.35
30	12.45	13.05	15.00	16.25	19.35	24.15	12.05	13.20	16.40	16.60	21.35	26.10
50	9.05	10.70	10.25	10.70	11.05	13.20	12.20	15.15	17.50	17.00	19.55	30.40
70	9.85	9.20	9.70	9.05	10.55	11.95	14.15	18.55	23.50	24.00	28.90	41.50
100	7.80	8.05	8.40	9.35	8.00	11.00	21.45	32.45	38.65	38.70	44.45	57.25
200	7.15	7.85	7.00	8.45	8.25	10.60	55.30	73.50	74.90	65.50	74.75	81.55
$t^{\#}$												
20	99.60	99.85	99.95	100.00	99.85	99.95	99.75	99.85	100.00	99.95	99.90	99.95
30	98.50	99.75	100.00	99.95	100.00	100.00	98.70	99.55	99.95	99.90	100.00	100.00
50	88.70	95.70	99.15	99.60	99.70	99.90	91.40	97.30	98.15	98.35	98.95	99.25
70	75.95	88.15	95.75	98.30	99.50	99.90	84.80	92.05	95.75	95.10	97.45	98.05
100	57.90	70.75	85.60	92.15	96.15	99.35	74.00	85.80	91.05	90.20	94.50	95.30
200	26.45	35.25	49.75	54.30	66.15	89.00	55.15	72.90	80.80	79.60	88.50	90.40

Notes: See notes to Table 3. The data generating process is the same as the one for Table 3, except $\rho_{iy\varepsilon} \sim iidU[-0.2, -0.4]$.

Table 6: Size and Power of Panel Unit Root Tests with Two Factors ($m^0 = 2$ and $k = 1$), Serially Correlated f_{1t} and f_{2t}

PANEL A: With an Intercept Only												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS($p=0$)												
20	11.40	13.30	13.90	17.20	17.65	22.05	12.05	13.55	13.35	17.45	15.20	19.60
30	8.65	9.40	11.25	10.00	13.65	13.90	13.40	13.70	13.10	14.10	14.25	16.25
50	7.15	8.05	8.30	9.30	9.90	10.95	18.45	20.45	23.00	25.35	27.90	35.05
70	6.35	6.95	7.85	8.25	9.20	8.45	29.90	37.40	46.65	53.70	61.80	73.05
100	5.55	6.55	5.75	6.45	7.55	6.05	55.40	73.85	86.90	93.60	96.80	99.10
200	5.15	4.70	5.10	5.60	5.25	5.10	99.50	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u}}(p=0)$												
20	13.90	17.80	21.10	22.90	28.05	40.60	21.95	27.00	41.95	43.20	51.60	70.25
30	11.35	12.70	14.30	15.50	20.30	26.10	28.70	37.05	54.25	56.95	64.95	81.50
50	9.50	11.35	10.45	13.55	14.05	16.85	48.30	61.70	84.75	82.20	86.85	92.20
70	8.25	10.10	9.35	9.45	10.85	12.80	69.30	82.35	96.70	93.45	95.35	97.00
100	8.30	8.80	7.95	9.25	7.95	10.45	90.75	96.50	99.75	99.25	99.10	99.75
200	6.75	5.80	7.35	6.60	5.90	8.20	100.00	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u},z}(p=0)$												
20	15.65	18.40	22.15	24.35	29.05	43.50	17.55	21.55	31.15	34.40	37.00	53.40
30	11.60	12.70	15.25	14.85	21.40	27.25	18.75	21.00	30.95	34.65	39.55	54.85
50	9.80	10.20	10.35	13.70	13.75	17.75	24.35	28.05	49.00	50.25	50.65	63.25
70	7.30	9.10	9.50	9.55	10.35	12.25	33.35	38.05	58.40	60.20	57.35	69.65
100	7.05	8.35	7.45	8.55	8.65	10.80	47.65	52.70	70.15	73.05	68.40	76.40
200	6.15	6.20	6.20	6.40	5.80	7.35	77.30	74.20	82.45	87.45	80.75	87.50
t^*_b												
20	11.20	10.85	13.15	16.60	16.90	23.75	75.20	77.95	88.45	87.50	89.55	91.20
30	10.50	8.70	9.00	14.25	14.60	18.30	88.75	86.70	95.30	92.95	94.35	94.95
50	7.75	7.60	7.10	10.30	9.60	12.25	96.80	93.85	98.70	96.70	98.45	97.75
70	7.10	5.60	6.00	7.65	7.25	10.00	99.70	96.90	99.40	98.30	99.40	98.85
100	7.45	7.05	6.05	7.10	6.35	7.75	100.00	99.05	99.80	99.60	99.75	99.80
200	7.30	6.35	5.65	5.15	5.00	5.70	100.00	100.00	100.00	100.00	100.00	100.00
PANEL B: With an Intercept and a Linear Trend												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS($p=0$)												
20	12.30	13.05	17.50	18.70	21.30	21.70	11.75	12.60	15.95	18.50	18.60	20.45
30	11.25	12.40	15.15	16.00	15.85	21.30	10.40	12.65	13.80	14.25	13.05	18.45
50	7.80	9.15	8.95	12.45	11.45	14.25	9.35	12.30	10.80	13.65	13.20	14.40
70	7.55	8.15	8.10	9.30	9.80	10.25	12.35	17.25	18.80	21.85	21.20	26.85
100	6.35	6.85	6.85	8.25	7.40	9.20	22.50	30.15	44.40	52.05	60.85	75.65
200	5.55	5.00	5.45	7.05	5.15	7.15	91.10	98.30	100.00	100.00	100.00	100.00
$P^\tau_{\hat{u}}(p=0)$												
20	27.75	36.10	45.55	52.55	62.50	78.85	27.40	36.50	46.25	51.95	64.50	79.25
30	21.95	26.05	31.45	39.50	44.45	63.55	23.45	29.55	37.15	41.60	48.85	69.15
50	15.75	16.90	21.50	23.55	29.30	42.95	21.90	29.95	39.75	39.30	52.05	69.35
70	12.65	14.75	15.60	19.40	22.30	29.85	26.85	37.70	52.40	52.15	65.00	80.85
100	11.45	12.55	14.35	15.35	16.85	24.05	39.55	58.85	76.10	71.60	84.30	90.50
200	11.00	9.30	10.70	11.20	12.25	17.00	90.20	98.40	99.40	97.85	99.95	99.90
$P^\tau_{\hat{u},z}(p=0)$												
20	29.15	37.35	47.55	55.75	65.35	81.35	29.80	37.95	47.20	54.55	63.50	78.70
30	21.80	26.70	33.45	39.50	45.70	64.85	21.45	27.10	33.40	37.40	42.45	61.80
50	15.20	16.35	21.05	24.25	29.30	41.75	15.75	23.45	26.70	27.60	35.15	49.50
70	12.25	13.85	14.30	20.05	21.90	29.65	16.35	26.45	31.65	32.40	37.20	54.75
100	10.55	11.35	14.05	15.15	16.20	24.00	23.55	38.00	44.70	44.65	49.60	63.65
200	9.35	8.40	9.15	10.55	11.50	16.45	57.60	78.40	78.75	71.05	79.65	84.80
$t^\#$												
20	94.80	97.00	98.90	99.30	99.70	99.85	95.05	97.40	98.60	99.60	99.55	100.00
30	77.45	86.95	94.80	97.25	98.85	99.90	78.40	87.90	95.15	96.25	98.15	99.35
50	42.70	55.65	68.70	79.25	87.30	97.35	48.85	61.05	74.75	78.40	86.00	92.60
70	29.15	38.50	50.60	59.40	70.00	89.15	34.00	43.30	57.35	64.60	72.70	84.60
100	18.85	24.50	31.50	40.45	47.65	71.15	23.30	30.70	42.10	49.95	58.80	72.45
200	9.50	12.65	14.65	16.75	20.60	32.35	12.35	17.15	26.40	32.40	42.70	55.35

Notes: See notes to Table 3. The data generating process is the same as the one for Table 3, except $\rho_{f1} = \rho_{f2} = 0.3$.

Table 7: Size and Power of Panel Unit Root Tests with Two Factors ($m^0 = 2$ and $k = 1$) with Spatially Correlated Factor Loadings

PANEL A: With an Intercept Only												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS($p = 0$)												
20	5.35	4.00	4.40	5.45	4.60	5.65	7.85	5.60	4.65	7.30	5.85	7.20
30	5.00	4.25	4.50	4.80	4.45	4.90	9.25	7.90	7.65	6.60	7.40	9.90
50	4.45	4.00	5.20	4.55	4.35	5.50	15.85	16.35	19.85	19.80	20.60	27.30
70	5.90	4.85	5.25	4.85	4.20	4.00	28.15	36.05	46.00	44.55	50.75	62.40
100	4.25	4.30	3.00	6.20	4.70	4.30	50.45	69.95	85.10	88.95	93.85	98.05
200	4.90	4.35	4.25	4.80	4.60	4.35	99.60	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u}}(p = 0)$												
20	11.15	12.05	9.45	10.80	11.45	15.80	17.30	23.00	29.85	35.80	43.25	66.55
30	7.70	7.60	8.40	8.50	9.80	10.50	24.05	35.10	50.65	56.00	70.55	87.40
50	6.85	7.00	7.35	7.00	8.40	7.30	46.80	72.55	82.95	87.40	94.75	97.30
70	6.65	6.10	7.60	7.45	7.30	7.30	68.00	89.85	95.65	96.70	98.80	99.10
100	7.10	8.10	7.25	6.35	6.80	6.45	89.60	98.70	99.55	99.60	99.90	99.90
200	7.20	6.75	6.15	6.75	5.85	5.85	99.80	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u},z}(p = 0)$												
20	9.75	10.00	9.75	11.35	13.05	16.90	14.25	17.10	18.55	27.75	33.05	51.55
30	7.90	7.15	8.25	8.30	9.55	11.25	18.50	21.40	26.05	38.20	47.90	65.65
50	7.00	6.65	7.70	6.75	7.65	7.85	30.35	40.70	43.75	63.10	68.95	79.65
70	6.05	6.05	6.90	7.05	6.75	6.90	45.10	51.25	54.15	74.45	80.15	88.20
100	6.10	6.95	6.75	5.65	7.00	6.25	59.60	67.10	66.55	85.35	86.40	91.90
200	6.20	6.05	5.35	6.45	5.65	5.25	85.25	80.40	80.65	93.10	95.55	98.60
t^*_b												
20	14.30	8.80	10.45	14.65	16.50	23.05	76.50	88.95	93.65	94.65	96.65	97.90
30	12.85	7.90	8.75	12.00	12.35	17.00	87.00	96.90	98.25	97.55	98.25	99.40
50	8.90	6.80	7.60	9.65	8.85	12.80	95.20	99.30	99.60	99.70	99.70	99.70
70	8.65	5.75	7.05	9.25	7.90	9.95	97.15	99.95	99.95	99.80	99.95	99.85
100	10.40	7.45	7.60	8.15	7.50	8.45	99.15	100.00	100.00	99.95	100.00	100.00
200	8.90	6.55	6.85	6.45	5.80	6.45	100.00	100.00	100.00	100.00	100.00	100.00
PANEL B: With an Intercept and a Linear Trend												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS($p = 0$)												
20	4.65	3.60	4.70	4.60	5.05	5.80	4.75	3.50	4.90	4.80	5.30	3.95
30	5.10	4.85	4.20	4.40	4.50	4.30	5.30	5.60	5.30	5.05	5.35	4.75
50	4.90	4.50	3.80	4.85	5.00	4.70	7.50	9.55	7.00	8.40	9.35	8.65
70	4.70	4.80	4.65	4.50	4.10	4.70	8.90	14.45	16.85	14.35	18.35	22.60
100	5.15	4.15	4.45	4.95	4.30	4.55	19.75	31.60	44.15	45.40	54.60	72.55
200	4.55	4.25	3.60	4.35	4.50	4.85	90.70	98.85	100.00	100.00	100.00	100.00
$P^\tau_{\hat{u}}(p = 0)$												
20	20.40	22.50	28.30	33.50	38.45	52.45	21.55	24.70	31.40	36.40	43.75	58.35
30	14.10	18.10	17.60	19.90	26.65	37.20	17.20	23.15	27.60	30.75	40.00	54.65
50	10.80	12.50	15.50	14.65	15.95	24.15	20.05	28.50	36.80	41.60	53.45	70.25
70	9.90	11.45	12.75	13.95	13.55	17.85	26.05	42.40	57.80	63.05	76.25	87.65
100	9.75	10.05	11.85	11.30	11.85	16.20	43.80	67.10	83.70	89.40	94.35	96.15
200	8.40	9.90	9.45	8.60	9.30	12.45	94.10	99.50	99.85	99.85	100.00	99.85
$P^\tau_{\hat{u},z}(p = 0)$												
20	20.35	22.30	28.05	34.45	39.05	55.05	20.10	22.75	30.25	35.75	42.40	57.35
30	12.65	17.85	17.40	21.20	26.30	37.65	14.95	19.05	24.30	26.50	34.50	46.60
50	9.95	12.50	14.15	14.35	16.15	23.65	16.00	22.25	28.05	29.95	38.00	52.40
70	9.35	10.60	12.10	12.85	13.00	17.70	20.05	28.50	41.70	42.30	50.70	63.70
100	9.20	9.20	10.65	11.15	10.65	15.20	32.10	43.25	60.85	59.90	68.35	74.20
200	7.55	8.40	7.80	8.65	9.30	11.70	77.55	80.95	91.65	84.60	89.40	91.30
$t^\#$												
20	93.20	97.95	98.05	98.60	99.55	99.40	93.95	97.85	97.85	98.90	99.80	99.30
30	75.70	89.45	93.85	94.35	98.60	99.10	76.55	90.95	93.60	95.70	98.45	98.80
50	44.10	58.30	68.95	78.75	85.70	95.50	45.80	61.75	74.85	80.25	88.65	94.00
70	28.35	39.45	49.55	58.95	68.10	86.45	32.15	42.75	57.05	66.00	74.90	86.05
100	19.60	23.60	32.80	37.60	50.20	68.75	19.50	29.05	39.40	48.60	60.40	74.30
200	10.35	11.95	13.30	17.95	19.85	30.55	9.65	13.30	21.50	28.80	39.65	54.55

Notes: See notes to Table 3. The data generating process is the same as the one for Table 3, except $\gamma_{ir} - c_r = 0.8 \sum_{j=1}^{N} s_{ij} \left(\gamma_{jr} - c_r\right) + \varphi_{ir}, \varphi_{ir} \sim iidN(0, \sigma^2_{\varphi i})$, $r = y1, y2, x11, x12$, where s_{ij} is the (i,j) element of an $(N \times N)$ row standardised spatial weighting matrix, $\mathbf{S} = \{s_{ij}\}$, with $s_{ij} = 1$ if units i and j are adjacent and $s_{ij} = 0$ otherwise. $\sigma^2_{\varphi i}$ is chosen so that $var(\gamma_{ir}) = 1/3$, and we set $c_{y1} = 1$, $c_{y2} = 1$, $c_{x11} = 1$, $c_{x12} = 0$.

Table 8: Size and Power of Panel Unit Root Tests, $m^0 = 2$ Unknown and Estimated Assuming $m_{\max} = 3$: x_{i1t} and x_{i2t} are Cointegrated

PANEL A: With an Intercept Only												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS$(p=0)$												
20	3.00	5.05	4.70	6.15	5.25	5.70	4.05	4.75	5.95	7.20	6.35	7.10
30	2.90	4.50	5.70	4.25	5.05	4.65	5.80	7.50	7.75	7.85	7.30	8.85
50	4.20	5.70	5.40	5.80	5.55	4.75	12.35	15.60	18.25	20.50	22.10	25.85
70	4.40	6.10	6.00	6.05	6.35	5.25	20.75	33.45	40.10	46.70	54.85	63.30
100	6.15	6.60	6.05	5.60	5.80	4.40	48.80	69.00	84.45	90.00	95.05	98.15
200	6.80	6.60	6.40	6.25	5.30	4.25	99.65	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u}}(p=0)$												
20	9.00	9.15	10.80	10.35	12.80	16.30	15.45	22.35	33.80	36.40	45.85	65.85
30	7.80	8.15	8.55	8.10	11.20	10.95	26.45	36.25	54.10	59.90	71.55	89.75
50	8.65	8.10	7.70	8.95	7.90	8.05	50.90	68.05	89.75	91.30	95.35	98.00
70	6.80	8.45	7.40	7.05	7.40	7.20	73.75	89.00	98.70	98.15	99.25	99.55
100	8.55	8.15	6.20	6.90	5.95	7.10	93.25	98.90	100.00	100.00	99.85	100.00
200	6.90	5.40	6.65	6.10	5.25	6.75	100.00	100.00	100.00	100.00	100.00	100.00
$P^c_{\hat{u},z}(p=0)$												
20	5.70	7.15	8.00	8.35	9.25	9.25	9.45	14.95	22.20	23.80	27.50	42.00
30	6.95	6.45	8.00	6.85	8.75	8.75	17.20	22.15	33.50	38.55	44.05	63.80
50	8.30	8.75	7.45	9.20	7.70	7.20	29.70	37.90	57.55	64.45	63.40	76.85
70	6.25	8.15	7.10	6.70	6.95	6.75	44.00	51.75	69.45	73.05	69.85	83.35
100	7.80	8.30	6.35	7.50	6.85	7.45	61.05	67.25	79.20	85.05	79.45	87.90
200	7.60	5.65	6.80	5.95	5.45	6.60	87.25	83.65	88.20	92.45	88.05	94.60
t^*_b												
20	17.90	13.25	12.05	14.10	13.75	20.80	85.40	89.20	95.80	95.35	97.60	97.45
30	14.65	8.35	8.30	11.55	12.35	15.80	94.40	94.65	99.35	98.35	98.95	98.95
50	11.50	7.80	6.85	9.60	8.75	11.65	99.15	98.05	99.70	99.45	99.80	99.65
70	8.95	5.90	6.35	7.85	7.70	9.90	100.00	99.10	99.95	99.80	100.00	99.80
100	8.45	7.50	7.10	8.20	6.85	8.35	100.00	99.95	100.00	100.00	100.00	100.00
200	7.60	7.00	6.20	5.55	5.70	6.85	100.00	100.00	100.00	100.00	100.00	100.00
PANEL B: With an Intercept and a Linear Trend												
	Size: $\rho_i = \rho = 1$						**Power**: $\rho_i \sim iidU[0.90, 0.99]$					
(T,N)	20	30	50	70	100	200	20	30	50	70	100	200
CIPS$(p=0)$												
20	2.60	3.25	5.05	5.20	5.95	5.10	2.15	4.00	5.40	5.10	5.35	4.40
30	3.30	4.85	5.25	5.00	5.10	5.55	3.30	5.50	6.15	5.35	5.05	5.55
50	4.40	5.20	5.30	5.70	5.05	5.15	5.35	8.20	8.55	8.10	8.15	8.45
70	4.30	6.15	6.00	5.00	5.95	4.75	7.85	14.65	16.10	18.00	17.00	21.05
100	5.95	6.65	7.15	5.75	5.50	5.25	18.00	28.90	44.00	48.65	55.30	69.40
200	7.50	6.50	7.15	6.35	4.75	6.10	89.30	98.65	100.00	100.00	100.00	100.00
$P^\tau_{\hat{u}}(p=0)$												
20	19.80	21.65	27.85	31.50	38.20	51.50	19.40	24.75	30.65	35.15	43.50	59.65
30	15.30	16.25	20.40	23.00	25.55	37.60	17.70	22.20	29.00	31.95	39.05	58.50
50	12.45	12.20	13.65	16.40	16.20	23.15	21.25	28.40	39.55	42.30	55.25	76.75
70	10.55	11.05	11.50	14.00	15.40	17.90	28.70	42.60	59.35	65.15	75.75	92.65
100	10.30	10.15	11.30	11.80	12.40	17.00	46.15	69.50	87.90	86.45	95.00	98.25
200	10.50	8.55	9.30	9.70	9.45	12.65	95.95	99.80	99.95	99.90	100.00	100.00
$P^\tau_{\hat{u},z}(p=0)$												
20	11.80	15.60	20.75	27.15	32.65	41.40	12.55	18.00	22.50	29.35	34.80	47.45
30	13.10	15.30	18.85	20.10	23.50	32.70	14.15	19.40	21.95	27.00	29.45	46.30
50	11.10	11.70	13.55	15.05	16.65	21.00	15.65	23.60	29.45	32.45	39.25	57.75
70	10.40	11.55	11.30	13.70	15.30	17.40	21.25	33.90	40.50	46.90	50.95	71.95
100	10.35	10.75	11.50	11.65	11.90	16.10	32.60	52.15	59.45	67.15	69.85	82.65
200	10.70	8.65	9.50	9.95	10.45	13.35	74.85	89.90	90.80	90.15	92.70	95.25
$t^{\#}$												
20	97.40	97.40	97.75	98.40	99.15	99.55	97.55	97.70	97.85	98.85	98.90	99.90
30	88.80	89.30	93.30	97.15	96.95	99.40	89.90	90.60	94.95	96.50	97.90	99.65
50	57.60	59.50	69.85	78.40	85.75	96.20	61.30	64.35	76.00	80.50	89.10	96.65
70	39.45	40.20	49.45	57.85	68.55	87.45	42.00	44.85	57.90	65.40	74.95	89.65
100	22.40	24.05	31.35	39.00	46.70	69.45	23.70	28.55	41.35	49.40	59.50	77.85
200	9.60	12.75	14.85	16.35	19.55	31.75	9.30	14.05	22.05	27.55	38.70	58.00

Notes: See notes to Table 3. The data generating process is the same as the one for Table 3, except $\gamma_{ixj1} \sim iidU[0,2]$ and $\gamma_{ixj2} = 0$ for $j = 1, 2$, with ε_{ixjt} replaced by $\Delta\varepsilon_{ixjt}$ so that the cumulative sums become $\varepsilon_{ixjt} \sim I(0)$. Under this design $x_{1it} \sim I(1)$ and $x_{2it} \sim I(1)$, and they are cointegrated. The number of factors m^0 is estimated with $m_{\max} = 3$ by IC_1 proposed by Bai and Ng (2002), then, $\hat{m}^0$ factors are extracted from y_{it} for $P_{\hat{u}}$ and t^*_b ($t^{\#}$) statistics and from $(y_{it}, x_{i1t}, x_{i2t})$ for $P_{\hat{u},z}$ statistic. For CADF regressions, when $\hat{m}^0 = 3$ and 2, (x_{i1t}, x_{i2t}) and x_{i1t} are used for augmentation respectively, otherwise no additional regressors are used.

Table 9. Results of CIPS Panel Unit Root Test for Real Interest Rates and Real Equity Prices, for All Combinations of $\hat{m}^0 - 1$ Additional Regressors out of the Five Candidates ($1979Q2 - 2003Q4$)

Panel A					**Panel B**				
Real Interest Rates ($N = 32, T = 94$)					Real Equity Prices ($N = 26, T = 94$)				
With an Intercept					With an Intercept and a Trend				
$\hat{m}^0 = 2$	$CIPS(p)$				$\hat{m}^0 = 3$	$CIPS(p)$			
Included $\bar{x}_t$	$p = 1$	$p = 2$	$p = 3$	$p = 4$	Included $\bar{\mathbf{x}}_t'$	$p = 1$	$p = 2$	$p = 3$	$p = 4$
$poil_t$	-4.94*	-4.04*	-3.06*	-2.95*	$poil_t, \bar{r}_t^L$	-2.56	-2.68	-2.78	-2.72
$\bar{r}_t^L$	-5.31*	-4.38*	-3.40*	-3.21*	$poil_t, \bar{\pi}_t$	-2.34	-2.42	-2.52	-2.48
$\overline{eq}_t$	-5.05*	-4.16*	-3.14*	-3.03*	$poil_t, \overline{ep}_t$	-2.42	-2.54	-2.59	-2.55
$\overline{ep}_t$	-5.17*	-4.19*	-3.23*	-3.07*	$poil_t, \overline{gdp}_t$	-2.01	-2.14	-2.32	-2.30
$\overline{gdp}_t$	-5.27*	-4.33*	-3.22*	-3.05*	$\bar{r}_t^L, \bar{\pi}_t$	-2.19	-2.36	-2.44	-2.25
					$\bar{r}_t^L, \overline{ep}_t$	-2.37	-2.56	-2.68	-2.69
					$\bar{r}_t^L, \overline{gdp}_t$	-2.14	-2.26	-2.33	-2.15
					$\bar{\pi}_t, \overline{ep}_t$	-2.38	-2.55	-2.67	-2.48
					$\bar{\pi}_t, \overline{gdp}_t$	-1.99	-1.98	-2.05	-2.02
					$\overline{ep}_t, \overline{gdp}_t$	-2.73	-2.76	-2.80	-2.70
	5% Critical Values					5% Critical Values			
	-2.39	-2.35	-2.34	-2.30		-3.02	-2.95	-2.92	-2.85

Note: All additional regressors, $\bar{\mathbf{x}}_t$, are assumed to be $I(1)$ and not cointegrated among themselves. * denotes the rejection of the null of the panel unit root hypothesis at the 5% significance level. Critical values are obtained by stochastic simulation as described in section 2.3

Table 10. Bai and Ng and Moon and Perron Panel Unit Root Test Results for Real Interest Rates and Real Equity Prices over the Period $1979Q2 - 2003Q4$

PANEL A				**PANEL B**			
Real Interest Rates ($N = 32$)				Real Equity Prices ($N = 26$)			
With an Intercept				With an Intercept and a Linear Trend			
$\hat{m}^0 = 2$				$\hat{m}^0 = 3$			
	$P_{\hat{u}}^c(p)$	$P_{\hat{u},z}^c(p)$	t_b^*		$P_{\hat{u},z}^\tau(p)$	$P_{\hat{u},z}^\tau(p)$	$t^{\#}$
$p = 1$	15.29*	9.93*	-17.48*	$p = 1$	-0.04	0.12	-1.36
$p = 2$	19.03*	11.73*		$p = 2$	1.71*	1.92*	
$p = 3$	6.33*	2.72*		$p = 3$	3.61*	4.16*	
$p = 4$	7.81*	6.04*		$p = 4$	4.42*	4.71*	

Note: * denotes rejection at the 5% significance level. $P_{\hat{u}}$ ($P_{\hat{u},z}$) is the Bai and Ng (2004) test based on factors extracted from y_{it} (y_{it} and all other five candidate regressors), and t_b^* ($t^{\#}$) is the test of Moon and Perron (2004) with an intercept (and a linear trend). $\hat{m}^0$ is the estimated value of m^0, the assumed true number of common factors and p is the lag order of the ADF regressions. The $P_{\hat{u}}$ and $P_{\hat{u},z}$ tests reject the null hypothesis of a unit root if they are greater than 1.645, and t_b^* and $t^{\#}$ if they are less than -1.645. The Moon-Perron tests adopt automatic lag-order selection for the estimation of long-run variances, which explains why only one value is reported for each choice of $\hat{m}^0$.

Table 11. Results of Panel Unit Root Test for the Real Interest Rates and the Real Equity Prices with $m_{\max} = 6$

Panel A: Real Interest rates $(N = 32, T = 94)$				
With an Intercept				
	$CIPS(p)$ (5% C.V.)	$P^c_{\hat{u}}(p)$	$P^c_{\hat{u},z}(p)$	t^*_b
$p = 1$	-5.64* (-3.06)	16.49*	6.70*	-24.86*
$p = 2$	-4.63* (-2.93)	19.69*	7.72*	
$p = 3$	-3.42* (-2.84)	6.10*	1.92*	
$p = 4$	-3.04* (-2.70)	8.15*	2.74*	
Panel B: Real Equity Prices $(N = 26, T = 94)$				
With an Intercept and a Trend				
	$CIPS(p)$ (5% C.V.)	$P^\tau_{\hat{u}}(p)$	$P^\tau_{\hat{u},z}(p)$	$t^{\#}$
$p = 1$	-2.81 (-3.39)	-0.77	1.07	-1.81*
$p = 2$	-2.62 (-3.26)	0.91	2.57*	
$p = 3$	-2.58 (-3.15)	2.45*	4.92*	
$p = 4$	-2.21 (-2.99)	3.21*	4.63*	

Note: * denotes rejection of the null of panel unit root hypothesis at the 5% significance level. For the CIPS test, the set of additional regressors $\bar{\mathbf{x}}_t$ included for the real interest rates and for the real equity prices are $(poil_t, \bar{r}^L_t, \overline{eq}_t, \overline{ep}_t, \overline{gdp}_t)$ and $(poil_t, \bar{r}^L_t, \bar{\pi}_t, \overline{ep}_t, \overline{gdp}_t)$, respectively. The $P_{\hat{u}}(p)$ and t^*_b $(t^{\#})$ tests are based on six factors extracted from y_{it} and for $P_{\hat{u},z}(p)$ from $(y_{it}, \mathbf{x}'_{it})$. The $P_{\hat{u}}$ and $P_{\hat{u},z}$ tests reject the null hypothesis of a unit root if they are greater than 1.645, and the t^*_b and $t^{\#}$ if they are less than -1.645. Critical values for the CIPS statistic are obtained by stochastic simulation as described in section 2.3.