SUPPLEMENT

For

Supply, Demand and Monetary Policy Shocks in a Multi-Country New Keynesian Model

By

S. Dees, M.H. Pesaran, L.V. Smith and R.P. Smith

June, 2011

1 Introduction

This supplement provides additional results, tables, figures and computational details for Dees, Pesaran, Smith and Smith (2010): Supply, Demand and Monetary Policy Shocks in a Multi-Country New Keynesian Model. It should be read in conjunction with that paper.

2 GVAR data 1979Q1-2006Q4 used to construct the GVAR deviations

This version of the GVAR data revises and extends the data set used in Dees, di Mauro, Pesaran and Smith (2007, DdPS) which covered the period 1979Q1-2003Q4 for a further three years to the end of 2006Q4. The revisions primary affected the GDP series and to a lesser extent the inflation series.

S.1. Real GDP

In extending the GDP series we relied on International Financial Statistics (IFS) database releases of 2006 and 2007. We first used the 2006 release to extend the GDP series used in DdPS to the end of 2005Q4 and subsequently used the 2007 release of IFS data to extend and revise the series to the end of 2006Q4, in the manner to be described below. The first set of data revision/extension to the end of 2005Q4 was used in Pesaran, Schuermann, and Smith (2009, PSS).

For cases where the IFS data was either too volatile relative to the DdPS data or not available, the DdPS data was used and the series were then extended forward by applying the rate of change of the IFS data to the levels of the DdPS GDP series in order to extend the GDP series used in DdPS to the end of 2005Q4. In particular, this process was applied to the GDP series in the case of Germany, Indonesia, Italy, Malaysia, Netherlands, New Zealand, and Spain. For Belgium DdPS data was extended forward by applying the rate of change of the OECD data (BEL.EXPGDP.LNBQRSA).

For the rest of the countries not mentioned above, excluding Brazil (see below), the quarterly IFS 99BVRZF GDP VOL. (having the base of 100 in 2000) series in the case of seasonally adjusted data and 99BVPZF GDP VOL series in the case of seasonally unadjusted data were used to extend the GDP series to 2005Q4, both of which are based on constant domestic prices. For China and Saudi Arabia the annual 99BVPZF GDP series were used to interpolate the quarterly values of the series. For Singapore we used Datastream data (SPGDP...D : SP GDP at 2000 market prices), as IFS data was only available from 1984Q3 onwards, while there were also missing data for 2000Q2 and 2000Q3.

Seasonal adjustments were applied to the GDP series for the following countries: Argentina, Austria, Brazil, Chile, Finland, India, Indonesia, Korea, Malaysia, Mexico, Norway, Peru, Philippines, Sweden, Thailand, Turkey.[1]

Interpolation from annual to quarterly series was conducted for the following countries (time periods) using the procedure described in Supplement A of Dees, di Mauro, Pesaran and Smith (2007): Argentina (1979Q1-1989Q4), Belgium (1979Q1-1979Q4), Brazil (1979Q1-1989Q4), Chile

[1] See PSS (2009) for a description and detailed results of the procedure used to assess the joint significance of the seasonal effects, both for real gdp and consumer price indices.

(1979Q1-1979Q4), China (1979Q1-2005Q4), India (1979Q1-1996Q3), Indonesia (1979Q1-1982Q4), Malaysia (1979Q1-1987Q4), Mexico (1979Q1-1979Q4), Philippines (1979Q1-1980Q4), Saudi Arabia (1979Q1-2005Q4) and Thailand (1979Q1-1986Q4).

The revised and extended dataset over the period 1979Q1-2005Q4 was established as the 2006 vintage. For all series excluding Belgium and Brazil, we then applied the rate of change of the IFS GDP data (1979Q1-2006Q4) collected in 2007, the IFS 99BVRZF GDP VOL. series in the case of seasonally adjusted data and the IFS 99BVPZF GDP VOL. series in the case of seasonally unadjusted data, seasonally adjusted as appropriate, from 2005Q1 to 2006Q4 to further revise and extend the data set used in this study, which we refer to as the 2007 vintage. For Belgium the rate of change of OECD data (BEL.EXPGDP.LNBQRSA) collected in 2007, is applied from 2005Q1 to 2006Q4 to extend the 2006 vintage dataset forward.

For Brazil BISM data (BISM.Q.RHGA.BR.02) is used from 1991Q1-2006Q4. The rate of change of the DdPS data (1979Q1-1990Q4) is applied to backfill the series.

Where IFS data was not available for any part of the 2005Q1-2006Q4 period, alternative data sources were used. This was the case for Philippines and Turkey, as explained below.

For Philippines the rate of change of the non-seasonally adjusted Datastream data (PHGDP...D) for 2006Q4 is used to complete the series, while for Turkey the rate of change of the non-seasonally adjusted Central Bank of The Republic of Turkey data for 2006Q3 and 2006Q4 is used. For both countries seasonal adjustment is then performed on the data over the whole period 1979Q1-2006Q4.

For China and Saudi Arabia the annual 99BVPZF GDP series is used to interpolate the quarterly values of the series and then the rate of change over the period (2005Q1-2006Q4) is applied to extend the 2006 vintage dataset forward.

Seasonal adjustment for the IFS GDP data (1979Q1-2006Q4) collected in 2007 was performed for the same set of countries as for the 2006 vintage dataset.

S.2. Consumer Price Indices

Initially, IFS data was collected in 2006 and DdPS CPI were revised and further extended to 2005Q4 in the manner described below.

The IFS CPI 64zf series is used for all countries except for China (see below) and Germany for which Datastream data (Codes: BDOCP009F or BDCONPRCX, both are identical) is used.[2] For Brazil, IFS 64zf data was available for the period 1980Q1-2005Q4. The average rate of change of prices for 1980 is used to backfill the series to 1979Q1.

Seasonal adjustments were applied to the inflation series in the case of Austria, Belgium, Chile, Finland, Germany, India, Japan, Korea, Malaysia, Netherlands, Norway, Spain, Sweden, Turkey and the UK.[3]

We establish this revised dataset 1979Q1-2005Q4 as our 2006 vintage dataset for CPI. For all series excluding China, we then apply the rate of change of the IFS CPI data collected in 2007, seasonally adjusted as appropriate, from 2005Q1 to 2006Q4 to further revise and extend the data used in this study, the CPI 2007 vintage .

For China HAVER data (Consumer Price Index (SA, 2000=100), source: CNBSH) is used for

[2] These CPI codes from datastream are based on West German data only pre-unification and are exactly the same as the IFS CPI 64zf series post-unification.

[3] Belgium, Chile and Sweden were seasonally adjusted based on the proximity of the F-statistic for the joint significance of the seasonal components to the 5% significance level. See the Supplement to PSS (2009) for details.

1985Q1-2006Q4. The rate of change of the DdPS data (1979Q1-1984Q4) is applied to backfill the series.

Seasonal adjustment for the IFS CPI data (1979Q1-2006Q4) collected in 2007 was performed for the same set of countries as for the 2006 vintage dataset.

S.3. Exchange Rates

For the exchange rate, quarterly averages of daily Datastream GTIS US $ exchange rate data, calculated based on the last Wednesday of each month within the quarter, are used for Brazil (1994Q1-2006Q4), Chile (1994Q1-2006Q4), Peru (1991Q1-2006Q4) and for the rest of the countries (1986Q1-2006Q4) and the rate of change of the IFS rf series is used to backfill the series to 1979Q1. Note that for the 8 Euro Area countries namely - Austria, Belgium, Finland, France, Germany, Italy, Netherlands and Spain - the dollar rates are transformed to euro dollar rates using the IFS euro dollar exchange rate (1999Q1-2006Q4).

S.4. Short-Term Interest Rates

IFS is used as the main source for short term interest rates; the typical maturity is three months. The IFS Deposit Rate 60Lzf series is used for Argentina, Chile, China and Turkey. The IFS Discount Rate 60zf series is used for New Zealand and Peru. The IFS Treasury Bill Rate IFS 60Czf series is used for Canada, Malaysia, Mexico, Philippines, South Africa, Sweden, UK and US. The IFS Money Market Rate 60Bzf series is used for Australia, Brazil, Finland, Germany, Indonesia, Italy, Japan, Korea, Norway, Singapore, Spain, Switzerland, Thailand. For Austria, Belgium, France and the Netherlands as no data is available for these series from 1991Q1 when the euro was introduced, we use the IFS Money Market Rate 60Bzf series (1979Q1-1998Q4) and complete the data to 2006Q4 using the corresponding 60Bzf series for Germany as the representative euro interest rate.

For Sweden, data for 2006Q3 and 2006Q4 are taken from the corresponding OECD Treasury Bill series (SWE.IR3TTS01.ST) to complete the series. For India, quarterly averages of daily Datastream data, calculated based on the last Wednesday of each month within the quarter, are constructed for both the 91 Day T-Bill Primary Middle Rate (code: INPTB91) (1997Q2-2006Q4) and the 91 Day T-Bill Secondary Middle Rate (Code: INTB91D) (1993Q1-2006Q4) series. The average of the two series is then taken for the period 1993Q1-2006Q4 and the rate of change of the OECD Central Bank Discount Rate (IND.IRSTCB01.ST) is used to backfill the series (1979Q1-1992Q4). For Indonesia, DdPS data is used for the IFS missing data of 1986Q1 and 1986Q2.

S.5. Oil Price Index

Monthly averages of the Brent Crude series from Datastream (LCRINDX).

S.6. Trade Matrix

The trade matrix in DdPS (1980-2003) is extended to 2006 using the import and export annual figures provided by the IMF Direction of Trade Statistics. Where data was unavailable as in the case of Brazil, Mexico and Peru with their trading partner Belgium, this was completed from the UN Commodity Trade Statistics Database (UN Comtrade).

S.7. GDP, PPP

GDP, PPP (current international $) annual figures used in DdPS to construct the GDP, PPP weights are extended to 2006 using the data provided in the World Development Indicators series, (code: NY.GDP.MKTP.PP.CD) of the World Bank.

3 Country Specific and GVAR results

In accordance with the theory, all variables in DPSS (2010) are measured as deviations from their steady states, which are estimated as long-horizon forecasts from a reduced-form cointegrating global vector autoregression. The version of the GVAR model used for this purpose has 131 endogenous variables, 82 stochastic trends and 49 cointegrating relations. All the roots of the global VAR model in the 33 countries either lie on or inside the unit circle. The moduli of the largest non unit eigenvalue is 0.926. It has fewer cointegrating relations than DdPS because excluding the long interest rate removes the term structure relationship, which is likely to be I(0). The lag orders for the domestic variables, p_i, and foreign variables, q_i, are selected based on the Akaike criterion with $p_{\max i} = 2$ and $q_{\max i} = 1$. The individual country models are estimated subject to reduced rank restrictions as described in DdPS and the cointegrating relations obtained are based on the trace statistic at the 95% critical value. For estimation, $\mathbf{x}_{it}^*$ are treated as "long-run forcing" or $I(1)$ weakly exogenous with respect to the parameters of the conditional model. This assumption can be tested by regressing $\mathbf{x}_{it}^*$ on the error correction terms for country i and testing whether these terms are statistically significant. Tables S1-S3 and Figures S1-S4 below provide results related to this version of the GVAR and the associated country specific models.

Specification of Individual Country VARX*(p,q) Models

The table below shows the VARX* order and number of cointegrating relationships in the country specific models.

Table S1: VARX* order and number of cointegrating relationships

Country	VARX*(p_i, q_i) p_i	q_i	# Cointegrating Relationships
China	2	1	1
Austria	2	1	1
Belgium	2	1	3
Finland	2	1	1
France	1	1	1
Germany	2	1	1
Italy	1	1	2
Netherlands	2	1	2
Spain	2	1	0
Japan	1	1	3
Argentina	2	1	1
Brazil	2	1	1
Chile	2	1	2
Mexico	1	1	2
Peru	2	1	2
Australia	2	1	1
Canada	2	1	2
New Zealand	2	1	1
Indonesia	2	1	3
Korea	2	1	2
Malaysia	1	1	1
Philippines	2	1	2
Singapore	1	1	1
Thailand	1	1	2
India	2	1	1
SAfrica	2	1	1
Saudi Arabia	2	1	1
Turkey	1	1	1
Norway	2	1	1
Sweden	2	1	2
Switzerland	2	1	1
UK	2	1	2
US	2	1	1

Note: The lag orders for the domestic variables, p_i, and foreign variables, q_i, are selected based on the Akaike criterion with $p_{\max i} = 2$ and $q_{\max i} = 1$. The individual country models are estimated subject to reduced rank restrictions as described in DdPS and the cointegrating relations obtained are based on the trace statistic at the 95% critical value. The number of cointegrating relations for Germany, Peru and Saudi Arabia was reduced by one based on the performance of the persistence profiles of the GVAR and the inspection of impulse responses.

Figure S1: Persistence profiles

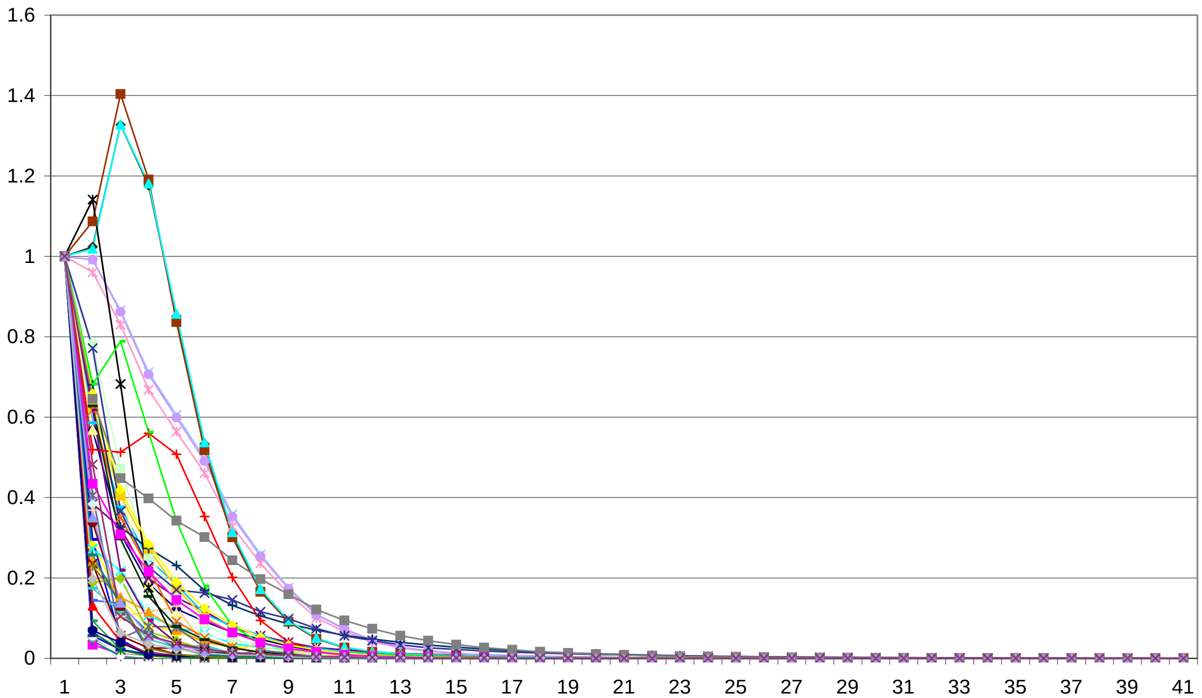


Table S2: Number of rejections of the null of parameter constancy per variable across the country specific models at the 5 percent level

Alternative Test Statistics	Domestic Variables y	π	ep	r	Numbers(%)
PK_{sup}	1(3.0)	1(3.0)	2(6.3)	1(3.1)	5(3.8)
PK_{msq}	1(3.0)	2(6.1)	0(0.0)	1(3.1)	4(3.1)
N	2(6.1)	7(21.2)	2(6.3)	5(15.6)	16(12.2)
robust-N	4(12.1)	5(15.2)	2(6.3)	5(15.6)	16(12.2)
QLR	11(33.3)	20(60.6)	12(37.5)	16(50.0)	59(45.0)
robust-QLR	6(18.8)	3(9.4)	3(9.7)	6(19.4)	19(15.0)
MW	6(18.2)	9(27.3)	5(15.6)	7(21.9)	27(20.6)
robust- MW	4(12.5)	6(18.8)	2(6.5)	3(9.7)	15(11.8)
APW	11(33.3)	19(57.6)	13(40.6)	16(50.0)	59(45.0)
robust- APW	5(15.6)	4(12.5)	2(6.5)	5(16.1)	16(12.6)

Note: The test statistics PK_{sup} and PK_{msq} are based on the cumulative sums of OLS residuals, N is the Nyblom test for time-varying parameters and QLR, MW and APW are the sequential Wald statistics for a single break at an unknown change point. Statistics with the prefix robust denote the heteroskedasticity robust version of the tests. All tests are implemented at the 5% significance level.

Table S3: F Statistics for testing the weak exogeneity of the country-specific foreign variables and oil prices

Country	F-test	y^*	π^*	ep^*	r^*	p^o
China	F(1,88)	0.35	$4.65^\dagger$	N/A	0.10	0.00
Austria	F(1,88)	2.70	0.05	N/A	0.76	0.33
Belgium	F(3,86)	0.42	1.09	N/A	$2.77^\dagger$	0.21
Finland	F(1,88)	0.58	0.53	N/A	0.28	3.52
France	F(1,92)	0.11	0.05	N/A	0.72	2.46
Germany	F(1,88)	0.15	0.06	N/A	3.32	0.00
Italy	F(2,91)	1.77	0.42	N/A	0.11	1.10
Netherlands	F(2,87)	0.42	1.62	N/A	1.23	0.60
Spain	N/A	N/A	N/A	N/A	N/A	N/A
Japan	F(3,90)	1.10	1.58	N/A	0.98	1.79
Argentina	F(1,88)	0.70	1.83	N/A	$6.64^\dagger$	0.59
Brazil	F(1,88)	0.00	0.56	N/A	0.34	0.67
Chile	F(2,87)	$7.67^\dagger$	0.10	N/A	0.40	0.34
Mexico	F(2,91)	$4.18^\dagger$	0.13	N/A	0.43	0.20
Peru	F(2,87)	1.14	1.48	N/A	1.31	2.53
Australia	F(1,88)	0.95	0.38	N/A	1.54	0.08
Canada	F(2,87)	0.20	0.37	N/A	$3.14^\dagger$	2.45
New Zealand	F(1,88)	0.14	0.17	N/A	0.01	0.31
Indonesia	F(3,86)	0.51	1.89	N/A	0.77	0.61
Korea	F(2,87)	$3.65^\dagger$	1.12	N/A	0.21	0.34
Malaysia	F(1,92)	0.77	0.24	N/A	0.85	0.07
Philippines	F(2,87)	0.49	1.65	N/A	0.12	$3.64^\dagger$
Singapore	F(1,92)	0.44	1.54	N/A	0.79	0.00
Thailand	F(2,91)	$3.19^\dagger$	1.09	N/A	1.44	1.91
India	F(1,88)	0.78	1.34	N/A	0.11	0.30
South Africa	F(1,88)	0.20	0.14	N/A	1.39	1.09
Saudi Arabia	F(1,90)	0.00	1.38	N/A	0.02	0.00
Turkey	F(1,92)	0.31	0.00	N/A	1.24	0.18
Norway	F(1,88)	0.94	1.01	N/A	0.13	1.19
Sweden	F(2,87)	0.23	1.03	N/A	1.57	0.58
Switzerland	F(1,88)	0.00	0.04	N/A	0.01	0.89
UK	F(2,87)	$3.38^\dagger$	2.67	N/A	0.57	2.52
US	F(1,92)	0.10	1.32	$5.56^\dagger$	N/A	N/A

Note: The lag orders for the domestic and foreign variables, pex_i and qex_i respectively, used in the weak exogeneity tests were set to $pex_i = p_i$ and $qex_i = 2$ for all countries, where p_i is the lag order used in the country specificVARX* models. † denotes statistical significance at the 5% level. Increasing the lag order qex_i, further reduced the number of statistically significant outcomes.

Figure S2a: Generalised impulse responses of a one standard error US interest rate shock on interest rates (per cent per quarter)

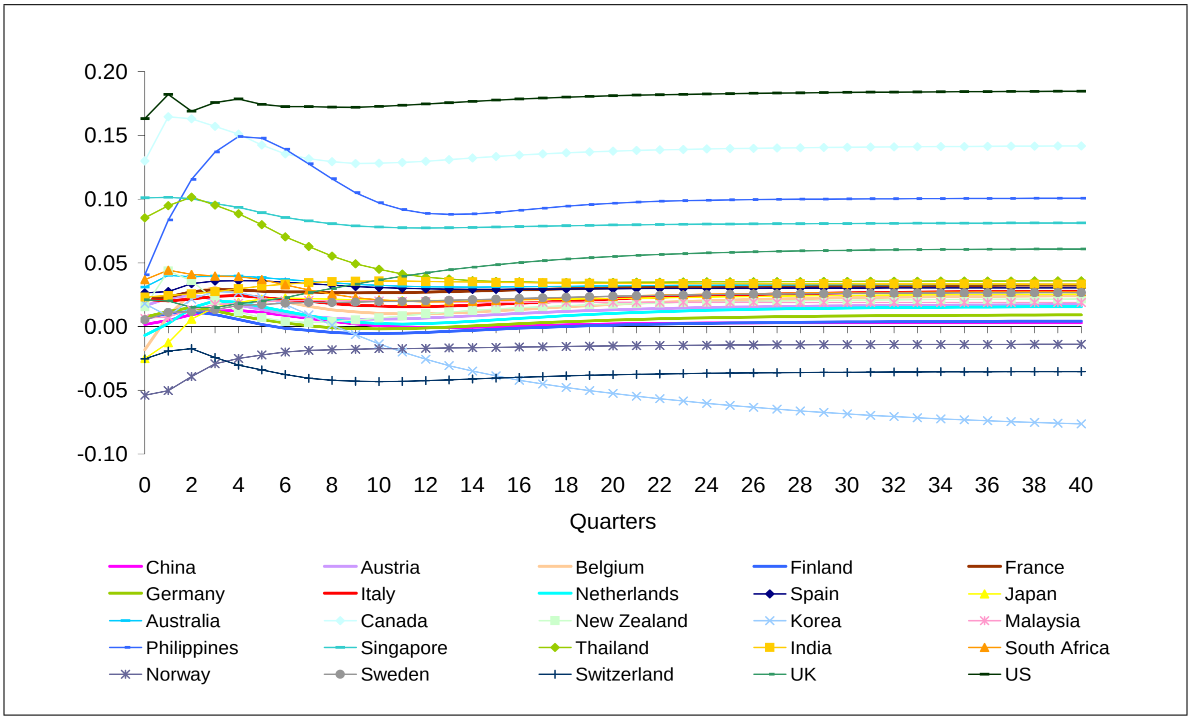


Figure S2b: Generalised impulse responses of a one standard error US interest rate shock on inflation (per cent per quarter)

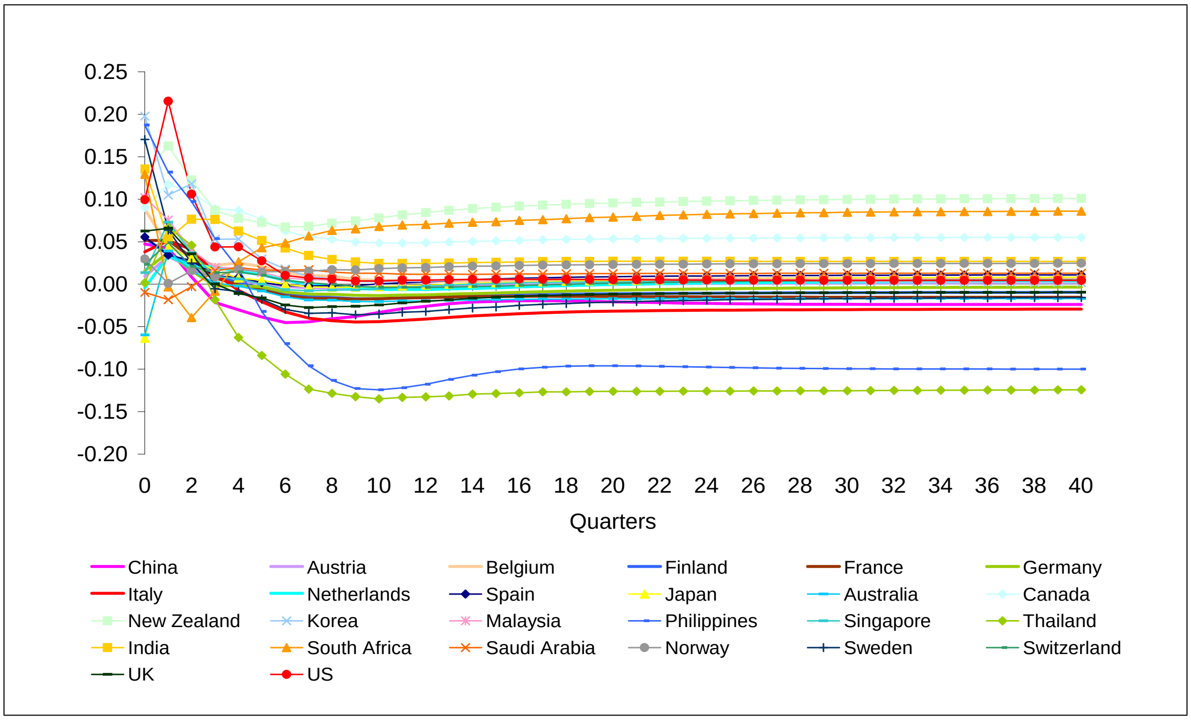


Figure S2c: Generalised impulse responses of a one standard error US interest rate shock on output (per cent per quarter)

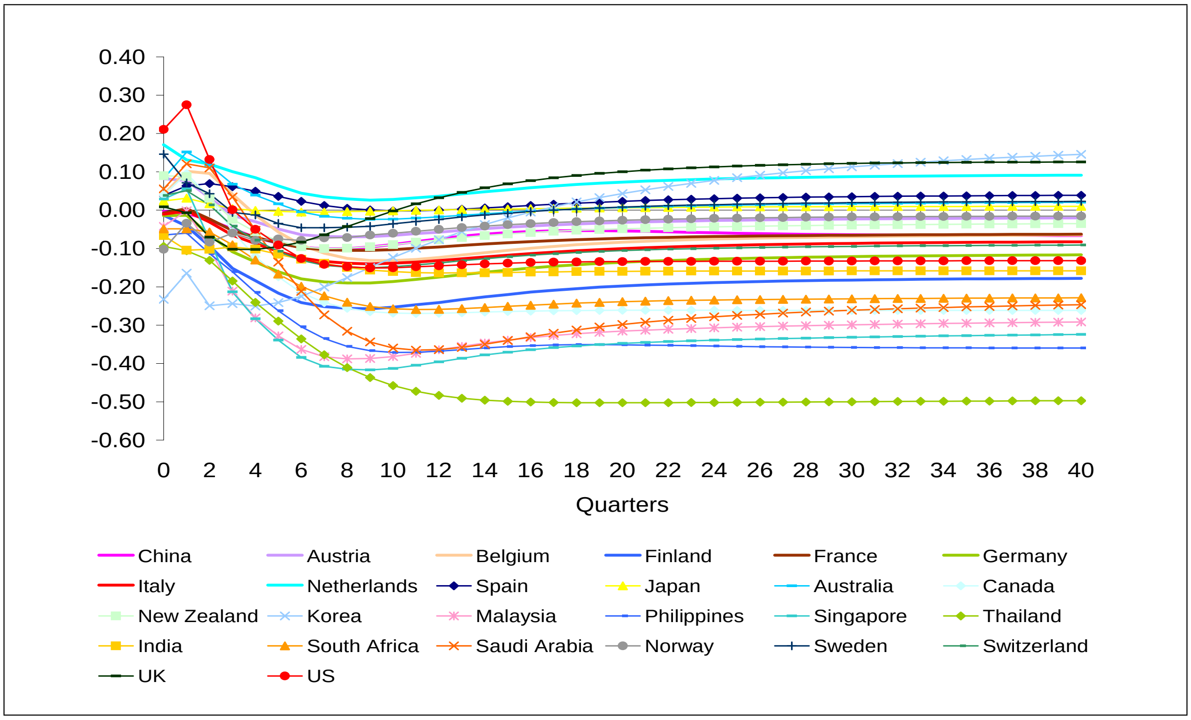


Figure S3a: Generalised impulse responses of a one standard error global inflation shock on inflation (per cent per quarter)

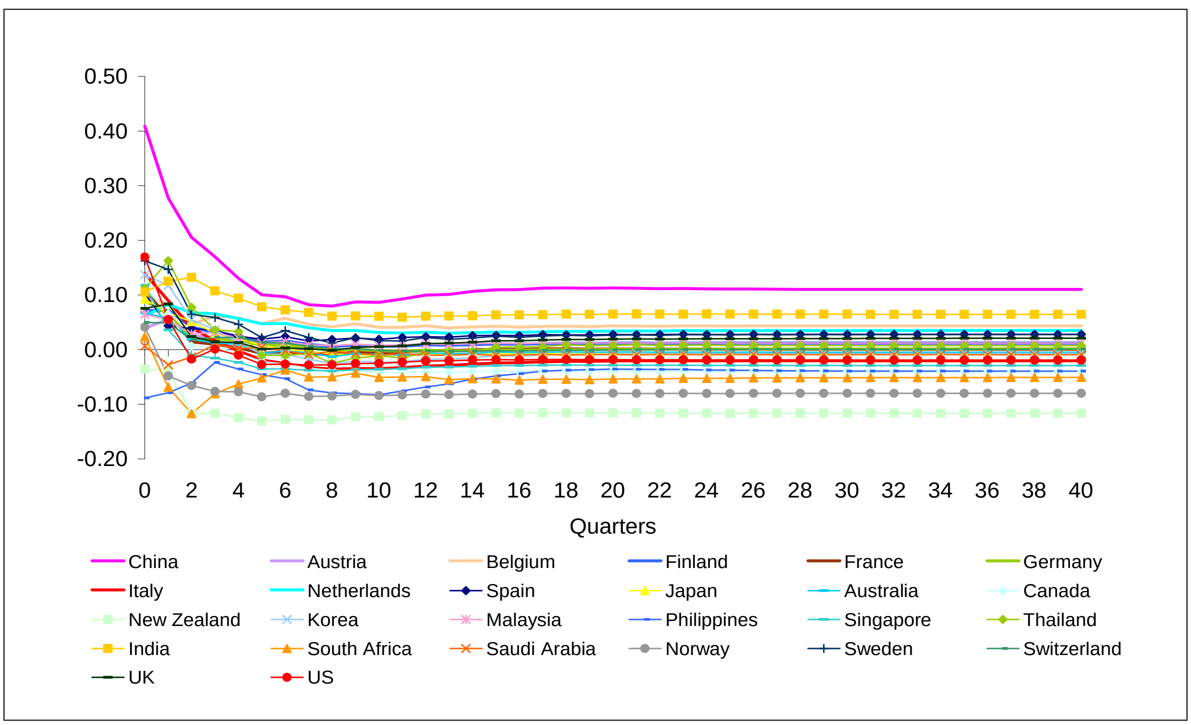


Figure S3b: Generalised impulse responses of a one standard error global inflation shock on output (per cent per quarter)

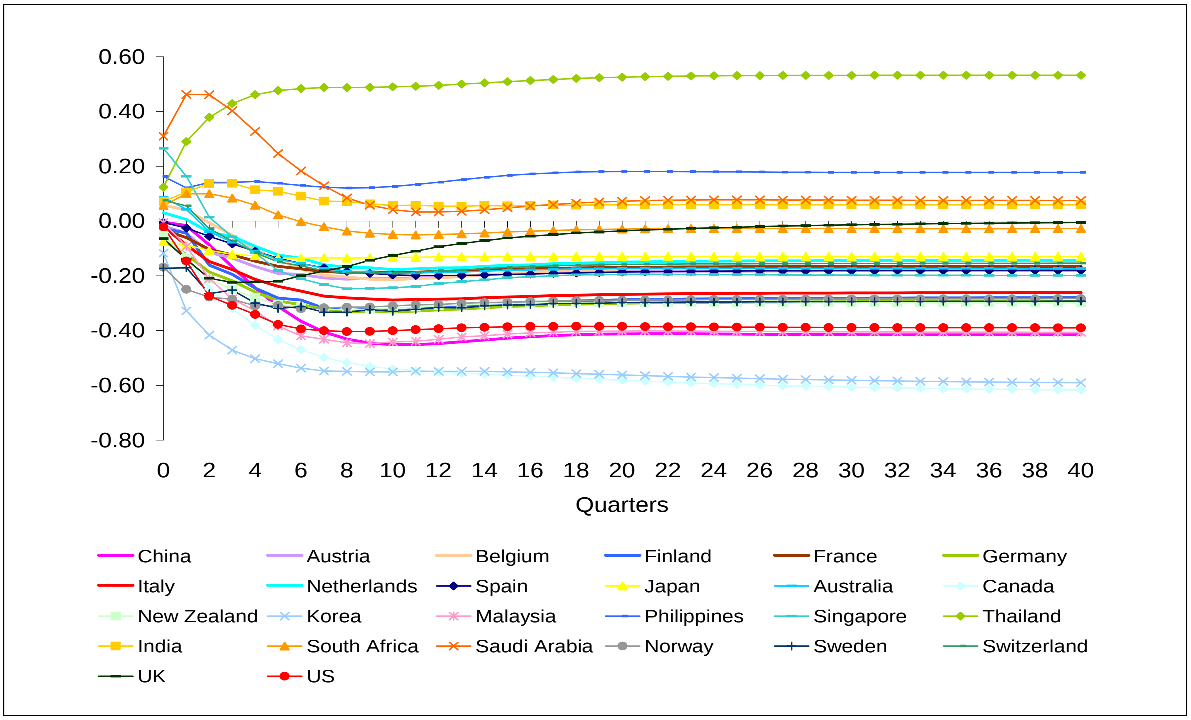


Figure S3c: Generalised impulse responses of a one standard error global inflation shock on interest rates (per cent per quarter)

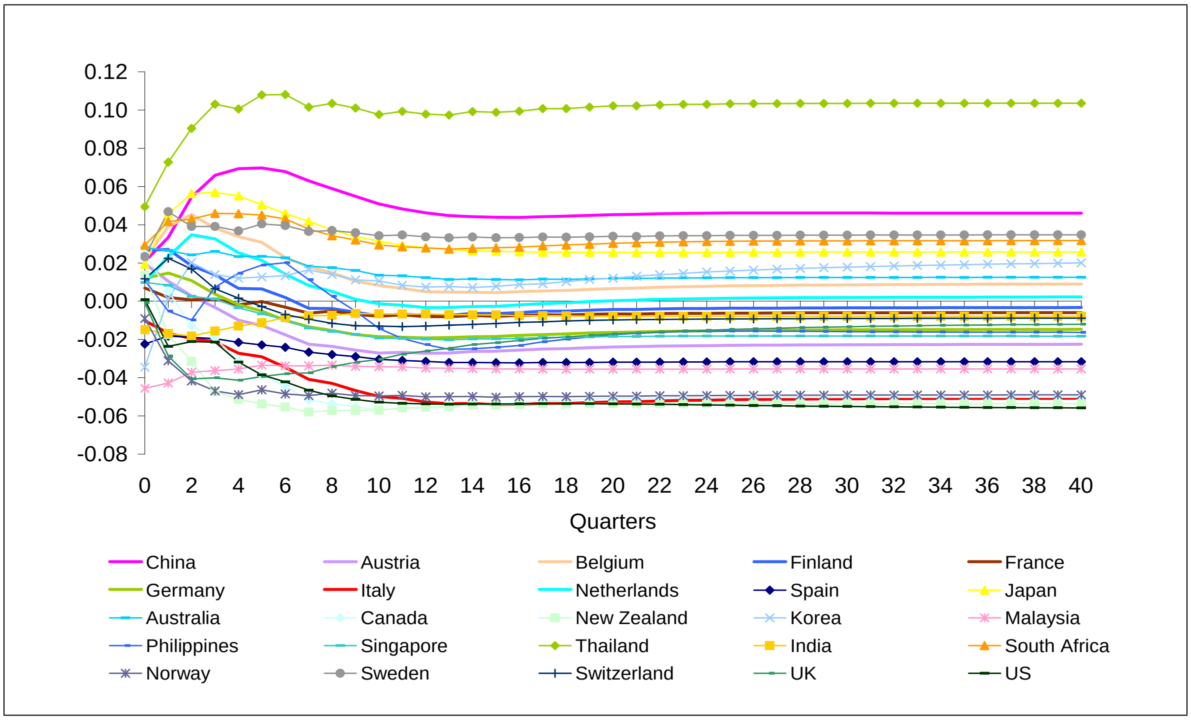


Figure S4a: Generalised impulse responses of a one standard error global output shock on output (per cent per quarter)

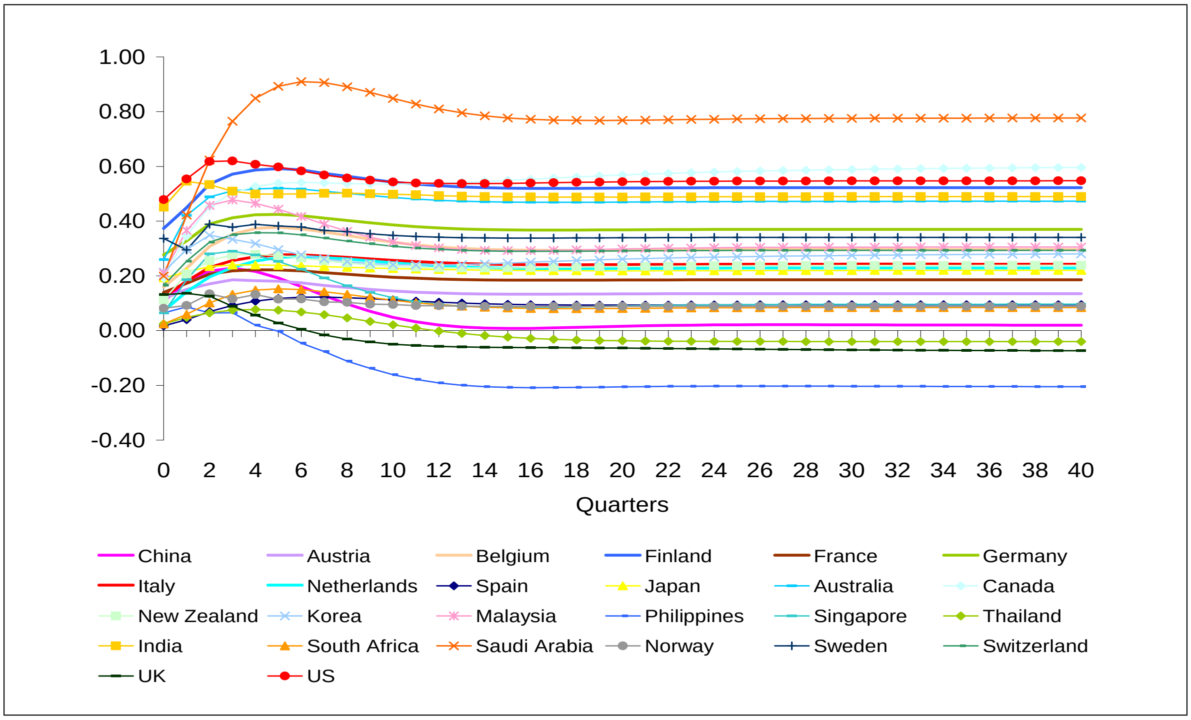


Figure S4b: Generalised impulse responses of a one standard error global output shock on inflation (per cent per quarter)

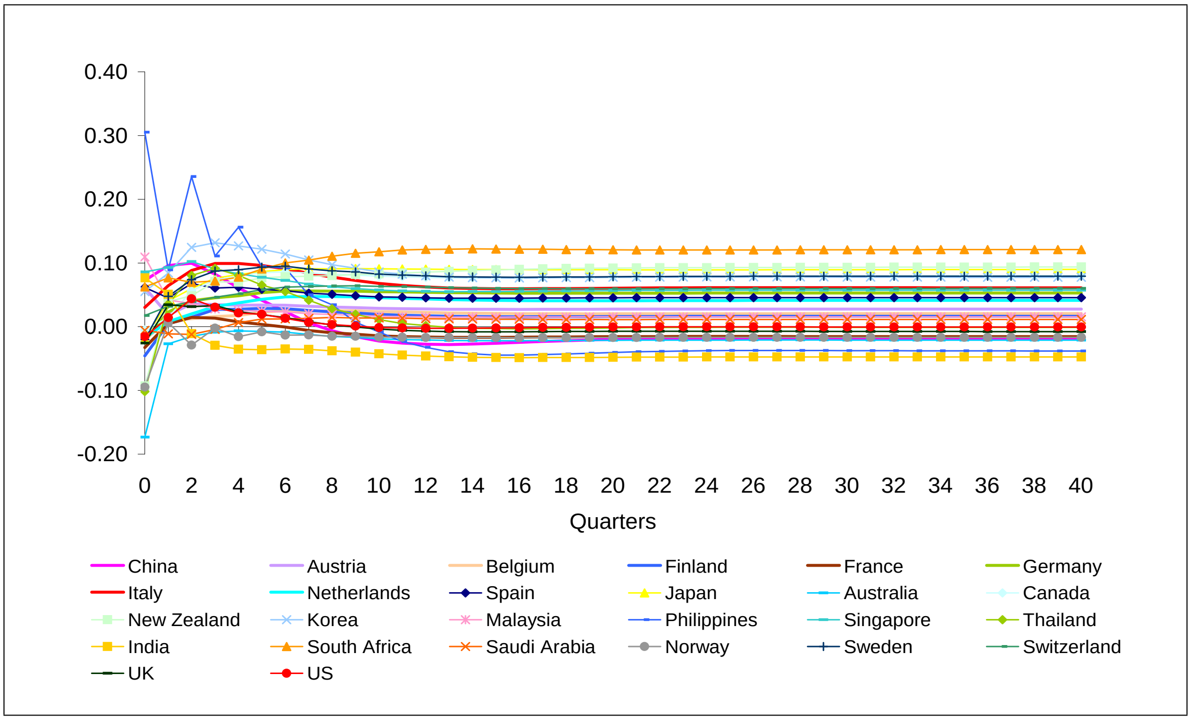
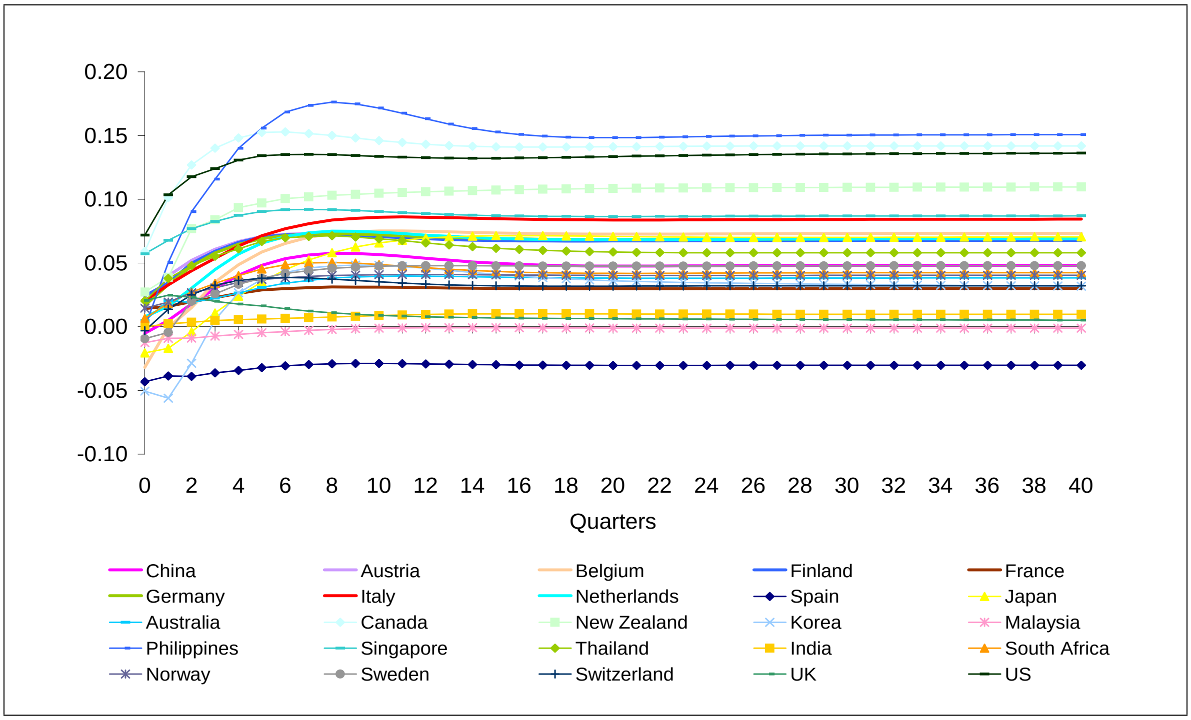


Figure S4c: Generalised impulse responses of a one standard error global output shock on interest rates (per cent per quarter)



4 Additional Results on the MCNK Model

The tables and figures that follow provide additional results associated with the MCNK model, which are not reported in the paper.

Table S4: Inequality-constrained IV estimates using GVAR estimates of deviations from steady states for all countries

	PS			IS				TR			REER
Country	β_{ib}	β_{if}	β_{iy}	α_{ib}	α_{ir}	α_{ie}	α_{iy*}	γ_{ib}	$\gamma_{i\pi}$	γ_{iy}	ρ_i
US	0.22	0.77	0.10	0.21	-0.98	-0.01	0.74	0.79	0.28	0.00	-
China	0.00	0.86	0.14	0.72	-0.47	0.02	0.31	0.98	0.11	0.00	0.78
Japan	0.11	0.84	0.13	0.15	-0.23	0.02	0.15	0.82	0.21	0.15	0.76
UK	0.12	0.87	0.05	0.54	0.00	0.25	0.95	0.74	0.20	0.01	0.53
Austria	0.00	0.85	0.05	-0.02	-0.63	0.06	0.75	0.63	0.38	0.06	0.68
Belgium	0.00	0.99	0.01	0.19	-0.04	-0.04	1.19	0.84	0.05	0.04	0.76
Finland	0.09	0.90	0.03	0.06	-0.56	-0.18	1.06	0.56	0.39	0.02	0.71
France	0.00	0.99	0.08	0.00	-0.02	-0.01	0.64	0.94	0.04	0.03	0.68
Germany	0.10	0.89	0.04	0.03	0.00	-0.04	1.10	0.62	0.27	0.04	0.54
Italy	0.38	0.61	0.04	0.23	0.00	-0.02	0.73	0.82	0.20	0.01	0.73
Netherlands	0.12	0.87	0.03	0.15	0.00	0.12	0.58	0.52	0.58	0.04	0.65
Spain	0.00	0.99	0.07	0.25	-0.11	-0.17	0.32	0.84	0.12	0.00	0.70
Norway	0.00	0.98	0.00	-0.25	0.00	-0.18	0.66	0.98	0.00	0.06	0.68
Sweden	0.01	0.98	0.07	-0.13	0.00	0.23	1.39	0.80	0.17	0.00	0.61
Switzerland	0.10	0.89	0.03	0.17	-0.20	0.03	0.89	0.55	0.00	0.07	0.52
Australia	0.00	0.99	0.04	0.22	-0.05	-0.02	0.82	0.67	0.12	0.05	0.67
Canada	0.22	0.77	0.02	0.37	-1.41	0.12	0.89	0.51	0.42	0.00	0.84
New Zealand	0.05	0.93	0.18	0.02	0.00	0.13	0.90	0.18	0.31	0.15	0.69
Argentina	0.00	0.53	0.00	0.32	0.00	-0.08	0.52	-0.36	0.53	0.00	0.69
Brazil	0.03	0.31	1.19	0.10	0.00	-0.03	1.33	-0.47	1.10	0.00	0.48
Chile	0.28	0.66	0.05	0.80	-0.27	0.16	0.71	0.49	0.58	0.27	0.66
Mexico	0.32	0.67	0.00	0.36	0.00	-0.16	0.32	0.19	0.48	0.32	0.79
Peru	0.40	0.59	0.00	0.89	0.00	0.31	0.00	0.11	0.40	0.00	0.34
Indonesia	0.36	0.63	0.00	0.38	0.00	-0.03	1.78	0.62	0.22	0.11	0.59
Korea	0.00	0.99	0.00	0.07	0.00	0.19	1.07	0.84	0.21	0.10	0.81
Malaysia	0.00	0.99	0.00	-0.02	0.00	-0.05	1.55	0.62	0.00	0.00	0.79
Philippines	0.07	0.58	0.29	0.65	-0.32	0.02	0.00	0.79	0.17	0.00	0.77
Singapore	0.17	0.82	0.04	0.07	-0.88	0.00	1.49	0.85	0.10	0.03	0.82
Thailand	0.18	0.81	0.03	0.70	0.00	-0.05	0.97	0.28	0.15	0.11	0.81
India	0.10	0.77	0.16	0.58	-0.35	0.00	0.01	0.57	0.01	0.00	0.66
South Africa	0.30	0.69	0.00	0.46	0.00	0.12	0.37	0.78	0.02	0.06	0.35
Saudi Arabia	0.11	0.70	0.00	0.52	-	0.00	1.49	-	-	-	0.86
Turkey	0.14	0.85	0.87	0.11	0.00	-0.07	0.46	0.81	0.00	0.23	0.59

Note: The estimation sample is 1980Q1-2006Q3 for the PC and IS equations, except for the PC equation of Argentina where it is 1990Q1-2006Q3. For the TR and exchange rate equations the sample is 1980Q1-2006Q4. For Saudi Arabia interest rate data are not available. The parameters for each country are estimated separately by instrumental variables (IV) subject to the theory restrictions referred to in DPSS (2010). The instruments used are an intercept, the lagged values of the country-specific endogenous variables $\widetilde{y}_{i,t-1}$, $\widetilde{\pi}_{i,t-1}, \widetilde{r}_{i,t-1}, \widetilde{re}_{i,t-1}$, the current values of the country-specific foreign variables $\widetilde{y}^*_{it}, \widetilde{\pi}^*_{it}, \widetilde{r}^*_{it}$, and the log oil price deviation, $\widetilde{p}^o_t$.

Table S5: Long-run estimates based on inequality-constrained IV procedure applied to GVAR deviations

	IS			TR	
Country	κ_{ir}	κ_{ie}	κ_{iy^*}	$\mu_{i\pi}$	μ_{iy}
US	-1.23	-0.01	0.93	1.31	0.00
China	-1.67	0.06	1.11	5.32	0.00
Japan	-0.27	0.02	0.17	1.16	0.80
UK	0.00	0.53	2.04	0.76	0.04
Austria	-0.62	0.06	0.74	1.01	0.17
Belgium	-0.05	-0.05	1.46	0.32	0.28
Finland	-0.60	-0.20	1.13	0.89	0.03
France	-0.02	-0.01	0.64	0.67	0.57
Germany	0.00	-0.04	1.14	0.69	0.11
Italy	0.00	-0.03	0.95	1.12	0.05
Netherlands	0.00	0.14	0.68	1.20	0.08
Spain	-0.15	-0.22	0.43	0.71	0.00
Norway	0.00	-0.15	0.53	0.00	2.36
Sweden	0.00	0.20	1.23	0.84	0.00
Switzerland	-0.24	0.03	1.07	0.00	0.16
Australia	-0.06	-0.03	1.05	0.37	0.16
Canada	-2.25	0.18	1.43	0.86	0.00
New Zealand	0.00	0.13	0.92	0.38	0.19
Argentina	0.00	-0.12	0.76	0.39	0.00
Brazil	0.00	-0.03	1.48	0.75	0.00
Chile	-1.38	0.80	3.59	1.14	0.52
Mexico	0.00	-0.24	0.50	0.60	0.39
Peru	0.00	2.75	0.00	0.45	0.00
Indonesia	0.00	-0.05	2.87	0.58	0.28
Korea	0.00	0.20	1.15	1.31	0.61
Malaysia	0.00	-0.04	1.52	0.00	0.00
Philippines	-0.93	0.07	0.00	0.78	0.00
Singapore	-0.94	0.00	1.60	0.67	0.20
Thailand	0.00	-0.17	3.23	0.21	0.15
India	-0.83	0.01	0.01	0.03	0.00
South Africa	0.00	0.22	0.69	0.07	0.28
Saudi Arabia	-	0.01	3.12	-	-
Turkey	0.00	-0.08	0.52	0.00	1.21

Table S6: Inequality-constrained IV estimates using Hodrick-Prescott (HP) filtered output deviations with smoothing parameter $\lambda = 1600$ and constant steady states for other variables

	PS			IS				TR			REER
Country	β_{ib}	β_{if}	β_{iy}	α_{ib}	α_{ir}	α_{ie}	α_{iy*}	γ_{ib}	$\gamma_{i\pi}$	γ_{iy}	ρ_i
US	0.30	0.69	0.03	0.73	-0.20	-0.01	0.28	0.89	0.12	0.01	-
China	0.36	0.63	0.02	0.91	-0.10	0.00	0.00	0.92	0.05	0.00	0.96
Japan	0.07	0.90	0.04	0.79	0.00	0.00	0.22	0.88	0.23	0.00	0.96
UK	0.29	0.70	0.02	0.86	-0.24	0.01	0.00	0.90	0.08	0.04	0.95
Austria	0.00	0.99	0.05	0.28	-0.15	0.02	0.61	0.85	0.22	0.08	0.98
Belgium	0.00	0.99	0.01	0.40	-0.06	0.00	0.87	0.92	0.05	0.07	0.93
Finland	0.04	0.95	0.01	0.63	-0.65	-0.05	0.56	0.96	0.05	0.03	0.97
France	0.03	0.96	0.02	0.72	0.00	0.02	0.28	0.93	0.05	0.05	0.92
Germany	0.12	0.87	0.00	0.66	0.00	0.01	0.48	0.89	0.10	0.06	0.95
Italy	0.39	0.60	0.02	0.53	-0.08	0.00	0.55	0.94	0.07	0.09	0.95
Netherlands	0.00	0.99	0.00	0.61	-0.11	0.02	0.51	0.93	0.00	0.06	0.95
Spain	0.05	0.94	0.00	0.85	-0.02	0.00	0.21	0.87	0.18	0.03	0.97
Norway	0.00	0.99	0.00	0.25	0.00	0.08	0.47	0.95	0.04	0.11	0.86
Sweden	0.00	0.99	0.03	0.13	0.00	0.02	1.31	0.88	0.13	0.03	0.95
Switzerland	0.11	0.88	0.00	0.58	-0.10	0.02	0.68	0.87	0.11	0.03	0.94
Australia	0.00	0.99	0.00	0.80	-0.05	0.01	0.24	0.91	0.06	0.06	0.92
Canada	0.17	0.82	0.00	0.66	-0.11	0.00	0.52	0.88	0.10	0.07	0.98
New Zealand	0.09	0.90	0.09	0.75	0.00	0.00	0.32	0.76	0.21	0.06	0.96
Argentina	0.19	0.80	0.00	0.91	0.00	0.00	0.00	-0.28	0.77	0.00	0.92
Brazil	0.34	0.65	0.00	0.74	-0.07	0.00	0.62	-0.20	1.18	0.00	0.92
Chile	0.09	0.88	0.01	0.93	-0.89	-0.03	0.41	0.54	0.46	0.15	0.97
Mexico	0.34	0.65	0.00	0.44	-0.51	-0.07	0.95	0.76	0.16	0.08	0.93
Peru	0.34	0.64	0.00	0.82	0.00	0.01	0.30	0.74	0.12	0.42	0.96
Indonesia	0.36	0.63	0.00	0.49	-0.41	-0.01	1.17	0.74	0.28	0.12	0.96
Korea	0.19	0.80	0.00	0.66	-0.28	-0.05	0.00	0.93	0.10	0.10	0.92
Malaysia	0.06	0.93	0.00	0.62	0.00	-0.01	0.89	0.90	0.07	0.01	0.99
Philippines	0.24	0.62	0.00	0.78	-0.17	-0.01	0.00	0.81	0.19	0.00	0.93
Singapore	0.18	0.81	0.02	0.76	-0.76	0.03	0.54	0.84	0.20	0.02	0.97
Thailand	0.20	0.79	0.00	0.83	-0.99	-0.07	0.09	0.89	0.06	0.08	0.96
India	0.26	0.40	0.14	0.73	-0.12	0.00	0.01	0.91	0.02	0.00	0.99
South Africa	0.33	0.65	0.03	0.80	-0.07	-0.01	0.53	0.90	0.05	0.08	0.95
Saudi Arabia	0.10	0.88	0.00	0.85	-	0.00	0.64	-	-	-	0.99
Turkey	0.16	0.83	0.00	0.66	0.00	-0.01	0.00	0.90	0.05	0.24	0.88

Note: The estimation sample is 1980Q1-2006Q3 for the PC and IS equations, except for the PC equation of Argentina where it is 1990Q1-2006Q3. For the TR and exchange rate equations the sample is 1980Q1-2006Q4. For Saudi Arabia interest rate data are not available. The HP filter of log real output is computed using the smoothing parameter of 1600 for all countries. The output deviations based on the HP filter are then computed, which we denote by $\widetilde{y}_{it}^{HP}$, for $i = 0, 1, ..., N$. The country-specific NK models are then estimated by the IV procedure subject to the theoretical restrictions referred to in DPSS (2010), with an intercept included to allow for the assumed constant steady state values. The instruments used are an intercept, the lagged values of the country specific endogenous variables, $\widetilde{y}_{i,t-1}^{HP}$, $\pi_{i,t-1}, r_{i,t-1}, re_{i,t-1}$, the current values of the foreign variables $\widetilde{y}_{it}^{HP*}, \pi_{it}^*, r_{it}^*$, and the first difference of the oil price variable, Δp_{it}^o. The foreign output variable based on HP steady state values are computed as $\tilde{y}_{it}^{*HP} = \Sigma_{j=0}^{N} w_{ij} \tilde{y}_{jt}^{HP}$.

Table S7: Long-run estimates based on inequality-constrained IV procedure applied to Hodrick-Prescott (HP) filtered output deviations

	IS			TR	
Country	κ_{ir}	κ_{ie}	κ_{iy^*}	$\mu_{i\pi}$	μ_{iy}
US	-0.74	-0.05	1.01	1.07	0.12
China	-1.20	0.04	0.00	0.65	0.00
Japan	0.00	0.01	1.08	1.84	0.00
UK	-1.77	0.09	0.00	0.84	0.41
Austria	-0.20	0.02	0.84	1.43	0.53
Belgium	-0.11	0.01	1.45	0.66	0.88
Finland	-1.75	-0.12	1.51	1.24	0.72
France	0.00	0.07	0.98	0.79	0.79
Germany	0.00	0.02	1.41	0.92	0.54
Italy	-0.16	0.00	1.17	1.24	1.45
Netherlands	-0.28	0.05	1.29	0.00	0.84
Spain	-0.13	-0.01	1.45	1.41	0.26
Norway	0.00	0.11	0.62	0.67	2.18
Sweden	0.00	0.02	1.50	1.02	0.21
Switzerland	-0.25	0.06	1.63	0.90	0.21
Australia	-0.27	0.05	1.23	0.71	0.66
Canada	-0.31	0.01	1.52	0.86	0.58
New Zealand	0.00	0.02	1.29	0.88	0.24
Argentina	0.00	0.05	0.00	0.60	0.00
Brazil	-0.26	0.00	2.37	0.98	0.00
Chile	-12.86	-0.47	5.92	1.00	0.33
Mexico	-0.90	-0.12	1.69	0.68	0.33
Peru	0.00	0.04	1.63	0.47	1.65
Indonesia	-0.81	-0.03	2.31	1.08	0.46
Korea	-0.81	-0.15	0.00	1.39	1.49
Malaysia	0.00	-0.03	2.34	0.69	0.12
Philippines	-0.79	-0.06	0.00	1.03	0.01
Singapore	-3.22	0.13	2.27	1.24	0.09
Thailand	-5.95	-0.40	0.55	0.60	0.80
India	-0.45	0.00	0.03	0.22	0.02
South Africa	-0.35	-0.03	2.67	0.54	0.82
Saudi Arabia	-	-0.02	4.26	-	-
Turkey	0.00	-0.02	0.00	0.47	2.33

Figure S5a: Impulse responses of a one standard error US monetary policy shock on interest rates using HP deviations and constant steady states (per cent per quarter)

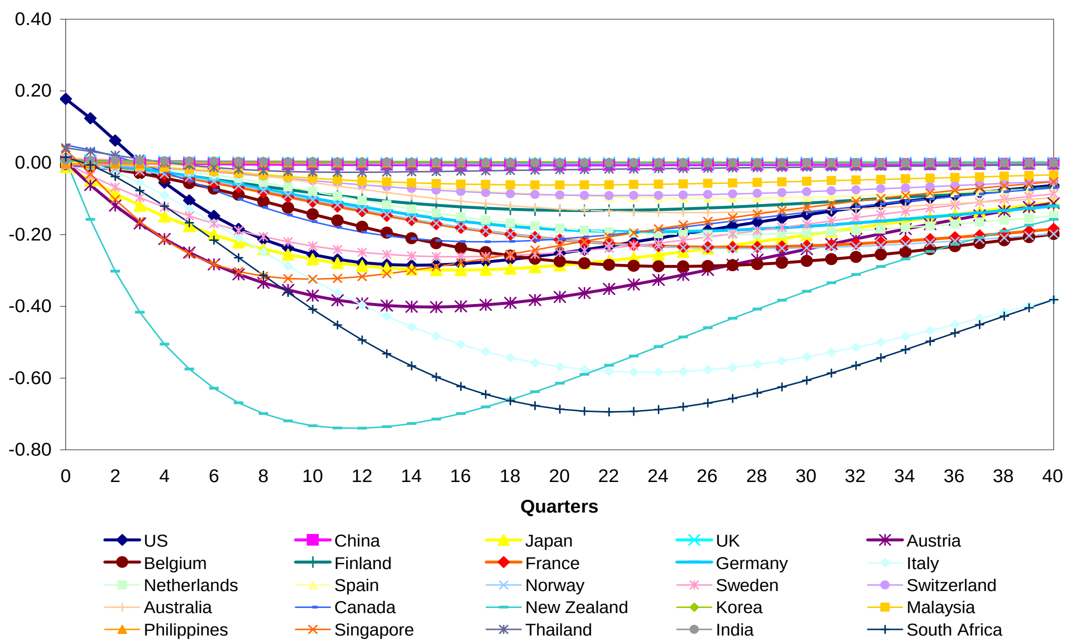


Figure S5b: Impulse responses of a one standard error US monetary policy shock on inflation using HP deviations and constant steady states (per cent per quarter)

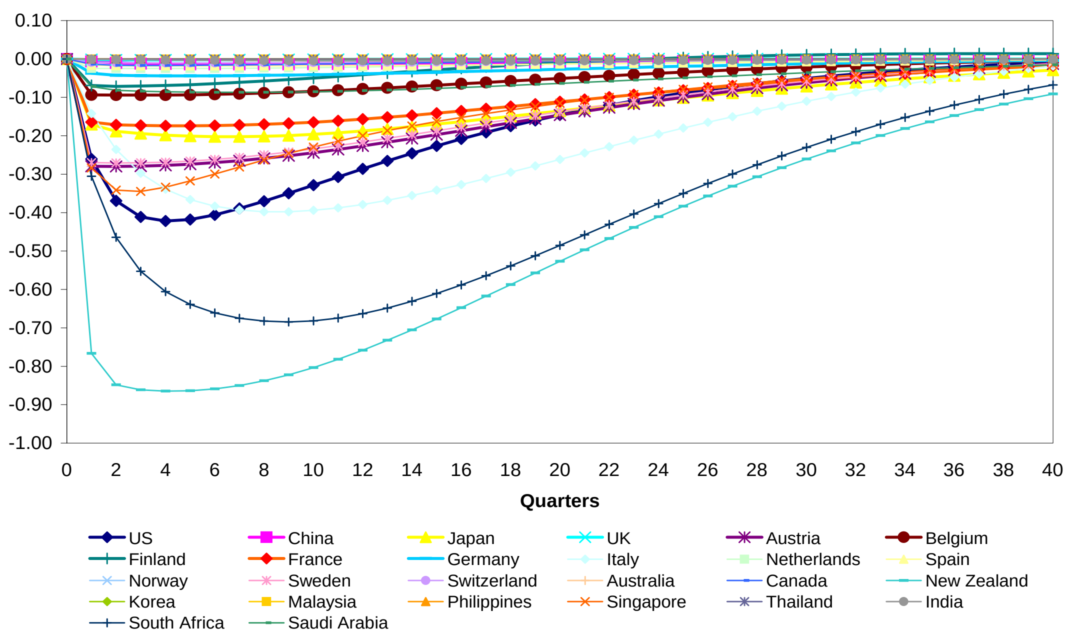


Figure S6a: Impulse responses of a one standard error global supply shock on inflation using HP deviations and constant steady states (per cent per quarter)

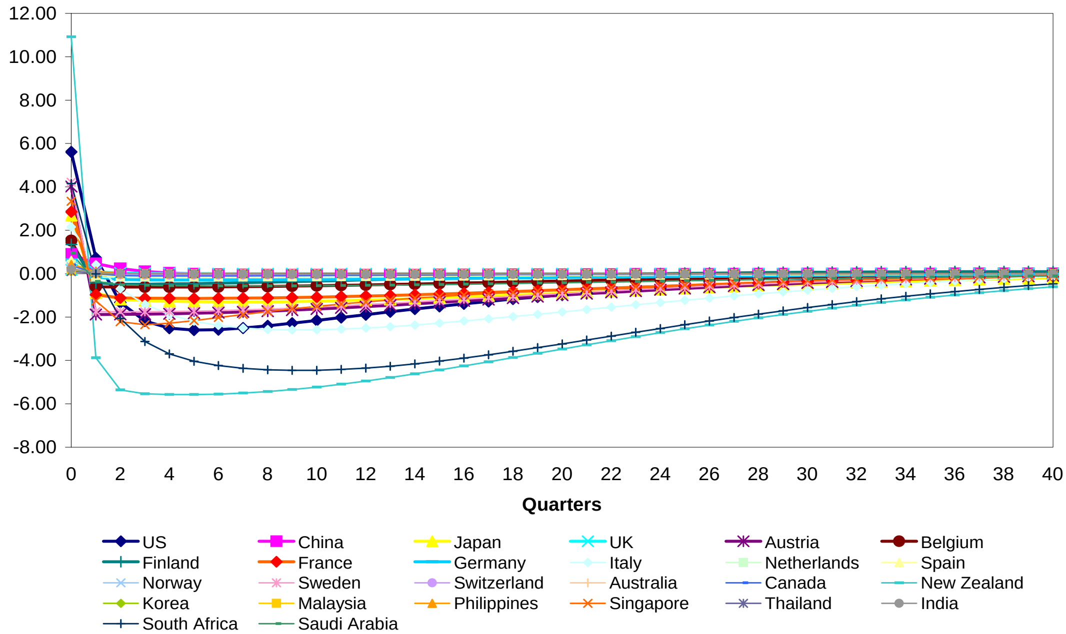


Figure S6b: Impulse responses of a one standard error global supply shock on output using HP deviations and constant steady states (per cent per quarter)

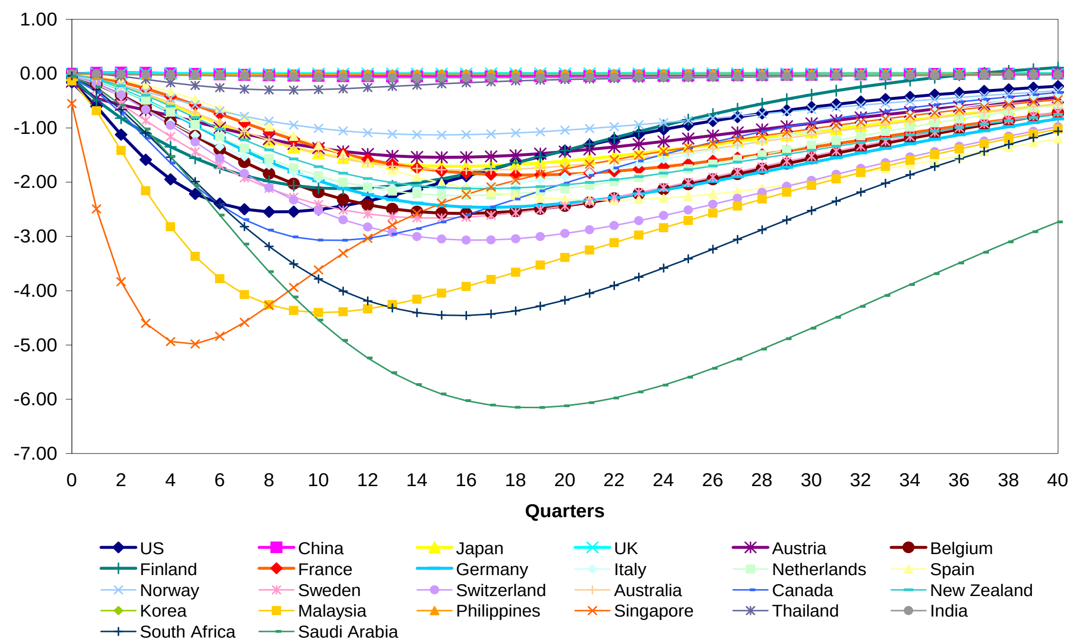


Figure S6c: Impulse responses of a one standard error global supply shock on interest rates using HP deviations and constant steady states (per cent per quarter)

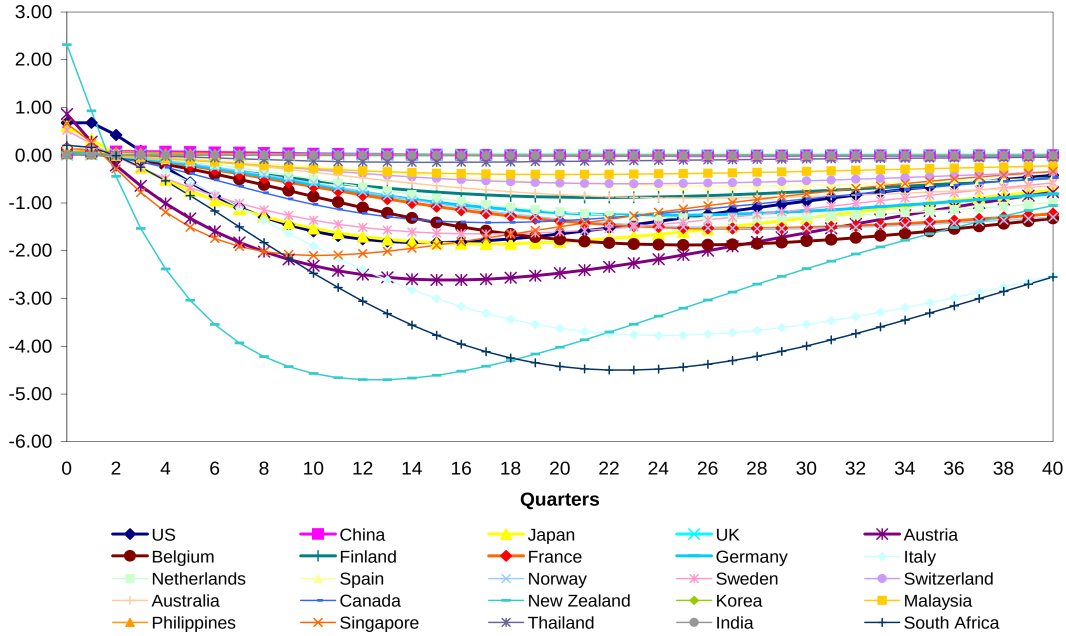


Figure S7a: Impulse responses of a one standard error global demand shock on output using HP deviations and constant steady states (per cent per quarter)

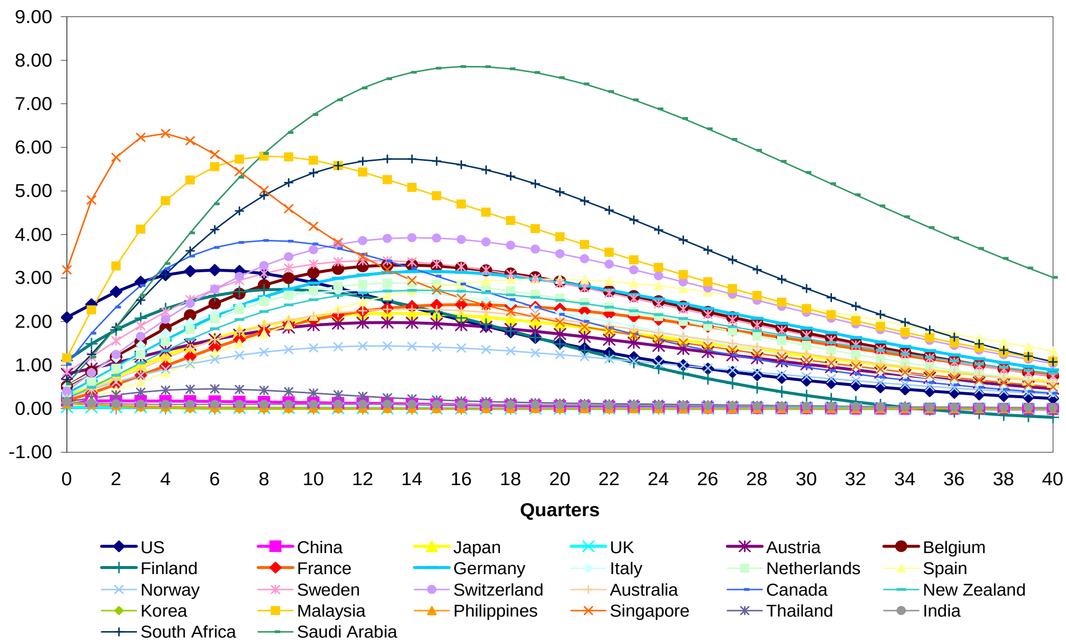


Figure S7b: Impulse responses of a one standard error global demand shock on inflation using HP deviations and constant steady states (per cent per quarter)

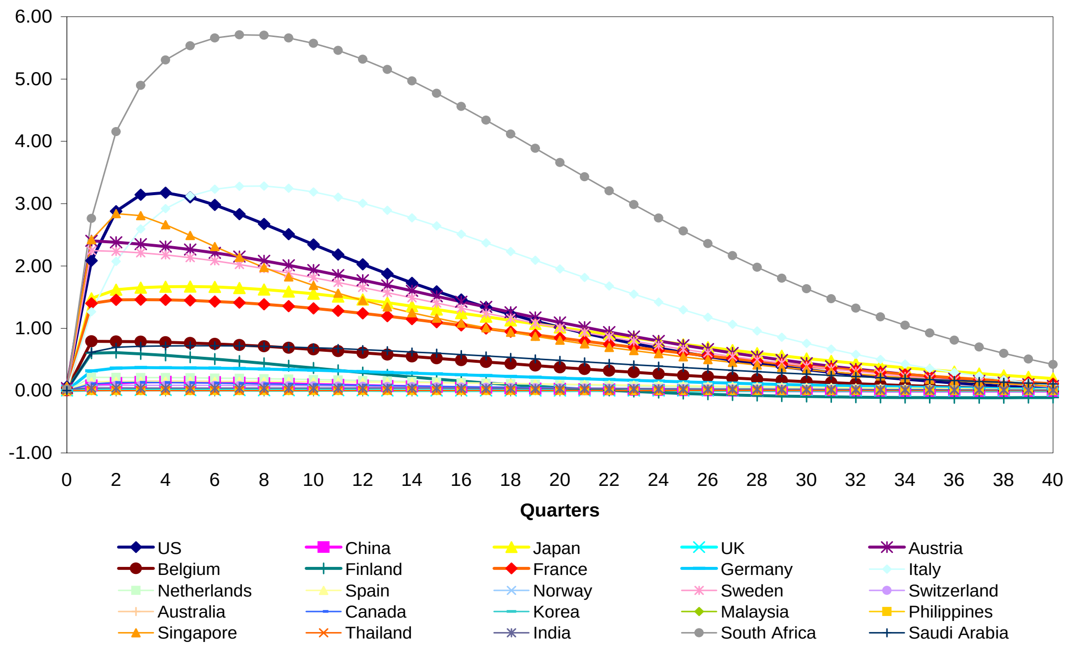


Figure S7c: Impulse responses of a one standard error global demand shock on interest rates using HP deviations and constant steady states (per cent per quarter)

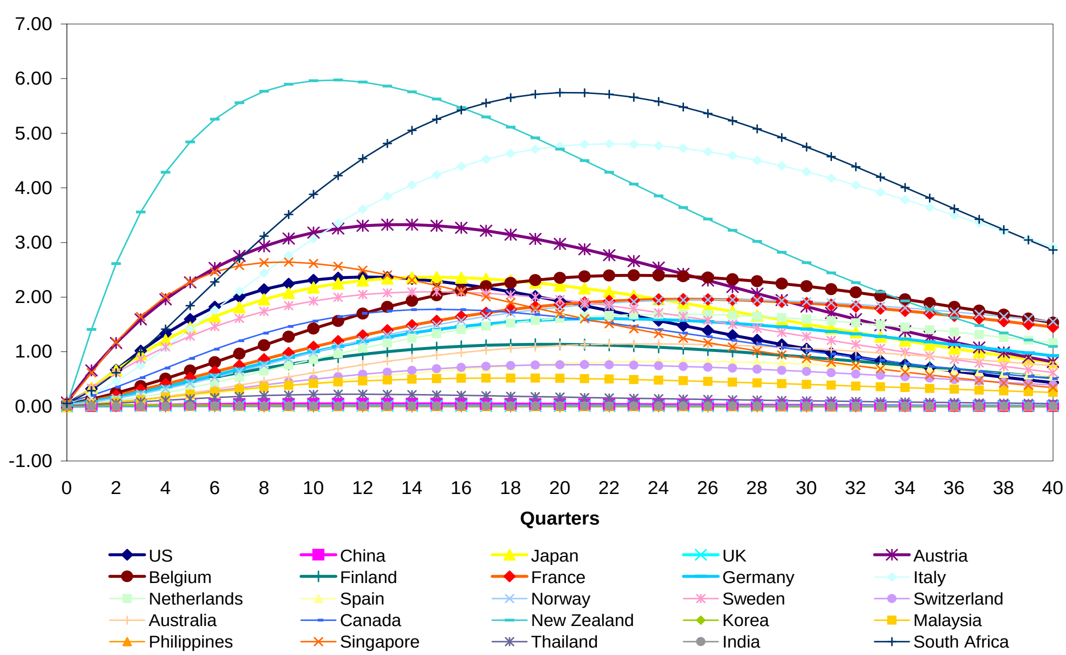


Table S8: Inequality-constrained IV estimates using GVAR estimates of deviations from steady states for all countries. No foreign output included in the IS equation for all countries.

	PS			IS			TR			REER
Country	β_{ib}	β_{if}	β_{iy}	α_{ib}	α_{ir}	α_{ie}	γ_{ib}	$\gamma_{i\pi}$	γ_{iy}	ρ_i
US	0.22	0.77	0.10	0.70	0.00	0.03	0.79	0.28	0.00	-
China	0.00	0.86	0.14	0.77	-0.49	-0.01	0.98	0.11	0.00	0.78
Japan	0.11	0.84	0.13	0.41	-0.36	0.01	0.82	0.21	0.15	0.76
UK	0.12	0.87	0.05	0.82	-0.94	0.11	0.74	0.20	0.01	0.53
Austria	0.00	0.85	0.05	0.37	-0.49	0.32	0.63	0.38	0.06	0.68
Belgium	0.00	0.99	0.01	0.84	0.00	0.01	0.84	0.05	0.04	0.76
Finland	0.09	0.90	0.03	0.57	-0.61	-0.19	0.56	0.39	0.02	0.71
France	0.00	0.99	0.08	0.80	-0.26	-0.01	0.94	0.04	0.03	0.68
Germany	0.10	0.89	0.04	0.73	-1.40	-0.29	0.62	0.27	0.04	0.54
Italy	0.38	0.61	0.04	0.65	-0.62	-0.15	0.82	0.20	0.01	0.73
Netherlands	0.12	0.87	0.03	0.07	0.00	0.30	0.52	0.58	0.04	0.65
Spain	0.00	0.99	0.07	0.22	-0.68	-0.27	0.84	0.12	0.00	0.70
Norway	0.00	0.98	0.00	0.12	-3.68	-0.02	0.98	0.00	0.06	0.68
Sweden	0.01	0.98	0.07	0.57	0.00	-0.22	0.80	0.17	0.00	0.61
Switzerland	0.10	0.89	0.03	0.78	-0.13	-0.03	0.55	0.00	0.07	0.52
Australia	0.00	0.99	0.04	0.59	-0.80	-0.18	0.67	0.12	0.05	0.67
Canada	0.22	0.77	0.02	0.51	-2.69	0.14	0.51	0.42	0.00	0.84
New Zealand	0.05	0.93	0.18	0.63	0.00	0.03	0.18	0.31	0.15	0.69
Argentina	0.00	0.53	0.00	0.21	0.00	-0.16	-0.36	0.53	0.00	0.69
Brazil	0.03	0.31	1.19	0.66	-0.14	0.11	-0.47	1.10	0.00	0.48
Chile	0.28	0.66	0.05	0.66	-0.37	-0.07	0.49	0.58	0.27	0.66
Mexico	0.32	0.67	0.00	0.31	0.00	-0.20	0.19	0.48	0.32	0.79
Peru	0.40	0.59	0.00	0.89	0.00	0.31	0.11	0.40	0.00	0.34
Indonesia	0.36	0.63	0.00	0.59	-0.47	-0.16	0.62	0.22	0.11	0.59
Korea	0.00	0.99	0.00	0.45	-0.59	0.03	0.84	0.21	0.10	0.81
Malaysia	0.00	0.99	0.00	0.65	-1.55	-0.27	0.62	0.00	0.00	0.79
Philippines	0.07	0.58	0.29	0.65	-0.32	0.02	0.79	0.17	0.00	0.77
Singapore	0.17	0.82	0.04	0.55	-1.92	-0.33	0.85	0.10	0.03	0.82
Thailand	0.18	0.81	0.03	0.75	-0.19	-0.11	0.28	0.15	0.11	0.81
India	0.10	0.77	0.16	0.58	-0.36	0.00	0.57	0.01	0.00	0.66
South Africa	0.30	0.69	0.00	0.58	0.00	0.25	0.78	0.02	0.06	0.35
Saudi Arabia	0.11	0.70	0.00	0.60	-	0.02	-	-	-	0.86
Turkey	0.14	0.85	0.87	0.27	0.00	0.06	0.81	0.00	0.23	0.59

Note: The estimation sample is 1980Q1-2006Q3 for the PC and IS equations, except for the PC equation of Argentina where it is 1990Q1-2006Q3. For the TR and exchange rate equations the sample is 1980Q1-2006Q4. For Saudi Arabia interest rate data are not available. The parameters for each country are estimated separately by instrumental variables (IV) subject to the theory restrictions referred to in DPSS (2010) with no foreign output included in the IS equation for all countries. The instruments used are an intercept, the lagged values of the country-specific endogenous variables $\widetilde{y}_{i,t-1}$, $\widetilde{\pi}_{i,t-1}, \widetilde{r}_{i,t-1}, \widetilde{re}_{i,t-1}$, the current values of the country-specific foreign variables $\widetilde{y}^*_{it}, \widetilde{\pi}^*_{it}, \widetilde{r}^*_{it}$, and the log oil price deviation, $\widetilde{p}^o_t$.

Table S9: Long-run estimates based on inequality-constrained IV procedure applied to GVAR deviations. No foreign output included in the IS equation for all countries.

	IS		TR	
Country	κ_{ir}	κ_{ie}	$\mu_{i\pi}$	μ_{iy}
US	0.00	0.09	1.31	0.00
China	-2.10	-0.03	5.32	0.00
Japan	-0.61	0.02	1.16	0.80
UK	-5.33	0.65	0.76	0.04
Austria	-0.77	0.51	1.01	0.17
Belgium	0.00	0.07	0.32	0.28
Finland	-1.42	-0.45	0.89	0.03
France	-1.27	-0.04	0.67	0.57
Germany	-5.16	-1.05	0.69	0.11
Italy	-1.78	-0.42	1.12	0.05
Netherlands	0.00	0.32	1.20	0.08
Spain	-0.87	-0.35	0.71	0.00
Norway	-4.18	-0.02	0.00	2.36
Sweden	0.00	-0.53	0.84	0.00
Switzerland	-0.59	-0.14	0.00	0.16
Australia	-1.93	-0.43	0.37	0.16
Canada	-5.44	0.28	0.86	0.00
New Zealand	0.00	0.08	0.38	0.19
Argentina	0.00	-0.20	0.39	0.00
Brazil	-0.42	0.33	0.75	0.00
Chile	-1.11	-0.22	1.14	0.52
Mexico	0.00	-0.28	0.60	0.39
Peru	0.00	2.75	0.45	0.00
Indonesia	-1.16	-0.40	0.58	0.28
Korea	-1.07	0.05	1.31	0.61
Malaysia	-4.45	-0.77	0.00	0.00
Philippines	-0.93	0.07	0.78	0.00
Singapore	-4.28	-0.73	0.67	0.20
Thailand	-0.74	-0.45	0.21	0.15
India	-0.84	0.01	0.03	0.00
South Africa	0.00	0.60	0.07	0.28
Saudi Arabia	-	0.05	-	-
Turkey	0.00	0.08	0.00	1.21

Table S10: Average pair-wise correlations of shocks using GVAR deviations. No foreign output included in the IS equation for all countries.

	Supply	Demand	Mon. Pol.	Ex. Rate
Supply	0.298	0.162	0.021	0.032
Demand		0.229	0.093	-0.023
Mon. Pol.			0.139	-0.043
Ex. Rate				0.049

Figure S8a: Impulse responses of a one standard error global supply shock on inflation in model without foreign output (per cent per quarter)

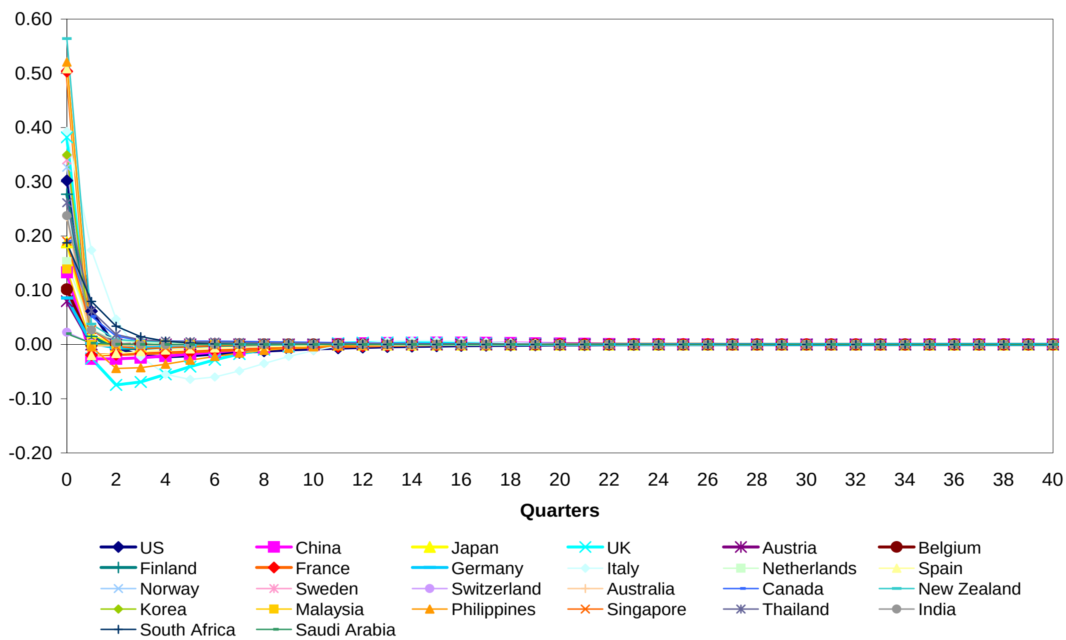


Figure S8b: Impulse responses of a one standard error global supply shock on output in model without foreign output (per cent per quarter)

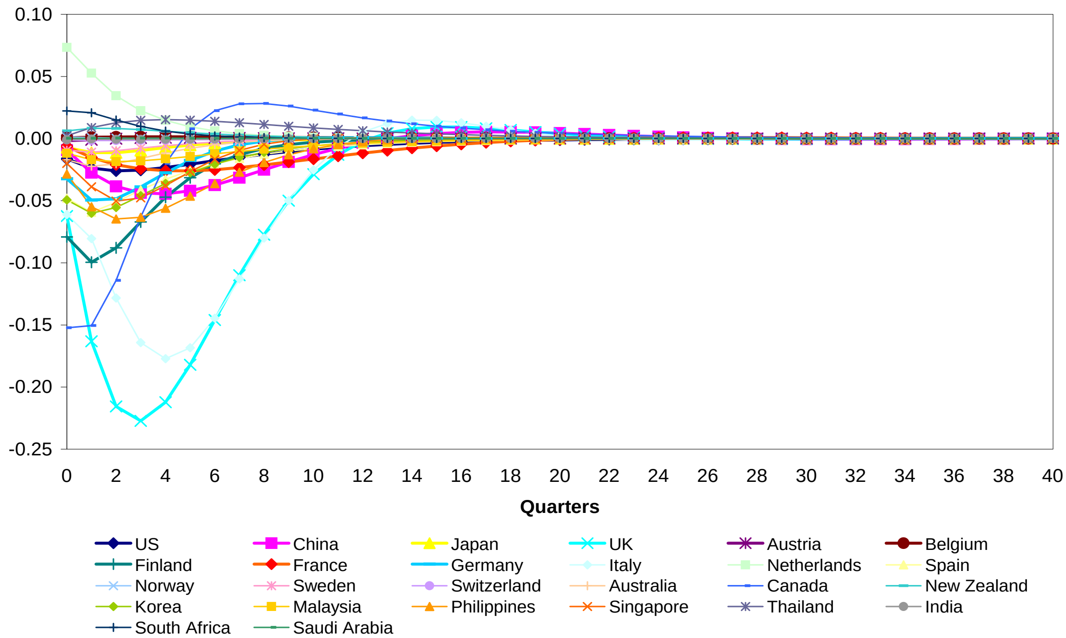


Figure S8c: Impulse responses of a one standard error global supply shock on interest rates in model without foreign output (per cent per quarter)

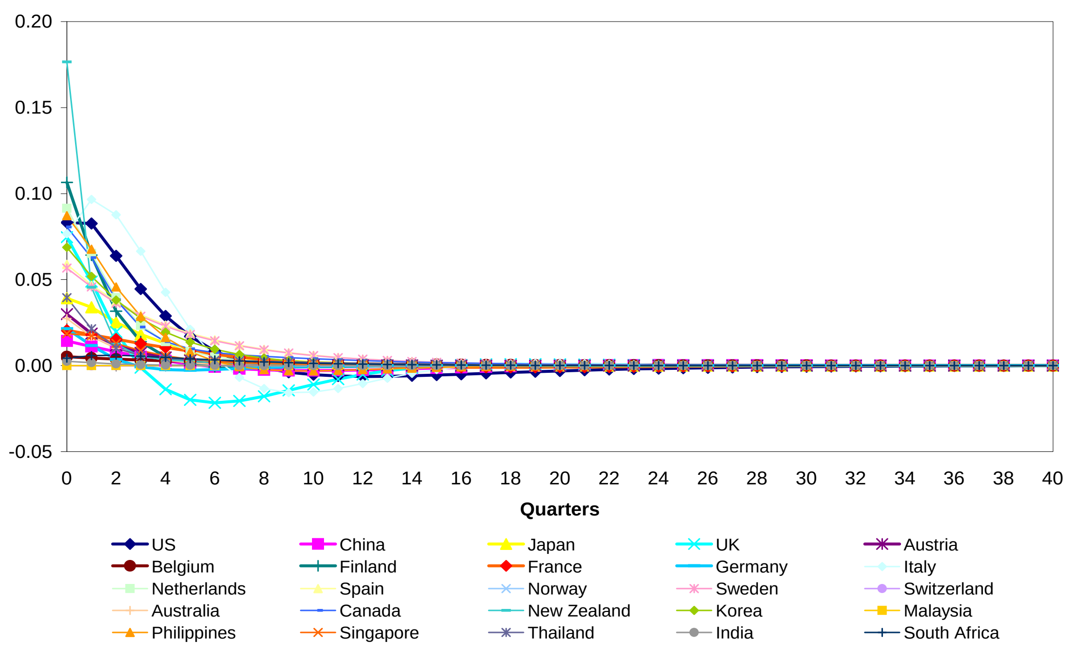


Figure S9a: Impulse responses of a one standard error global demand shock on output in model without foreign output (per cent per quarter)

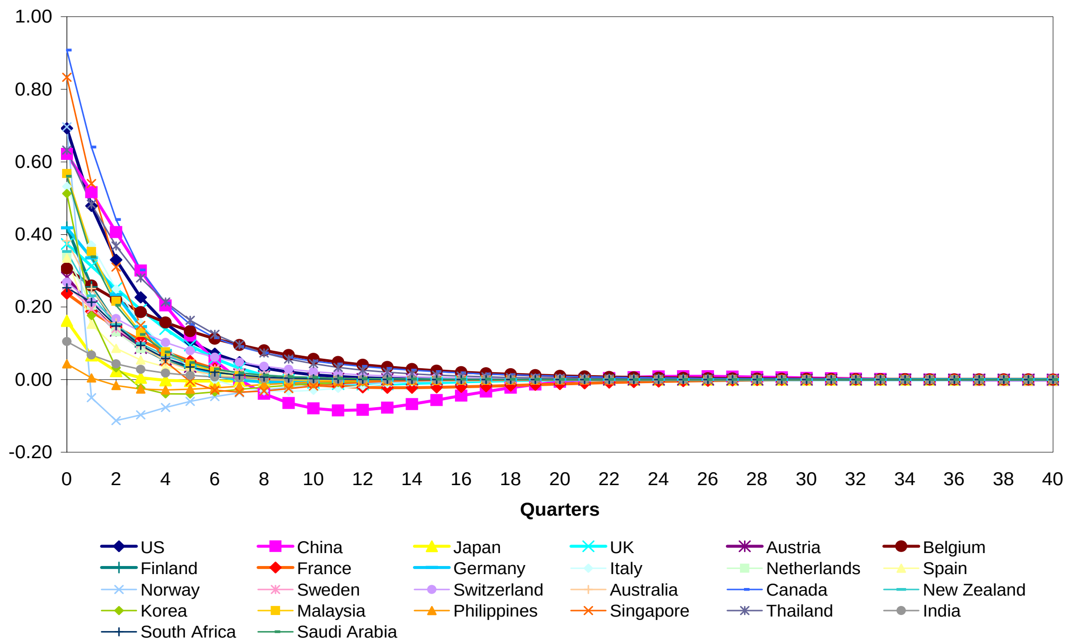


Figure S9b: Impulse responses of a one standard error global demand shock on inflation in model without foreign output (per cent per quarter)

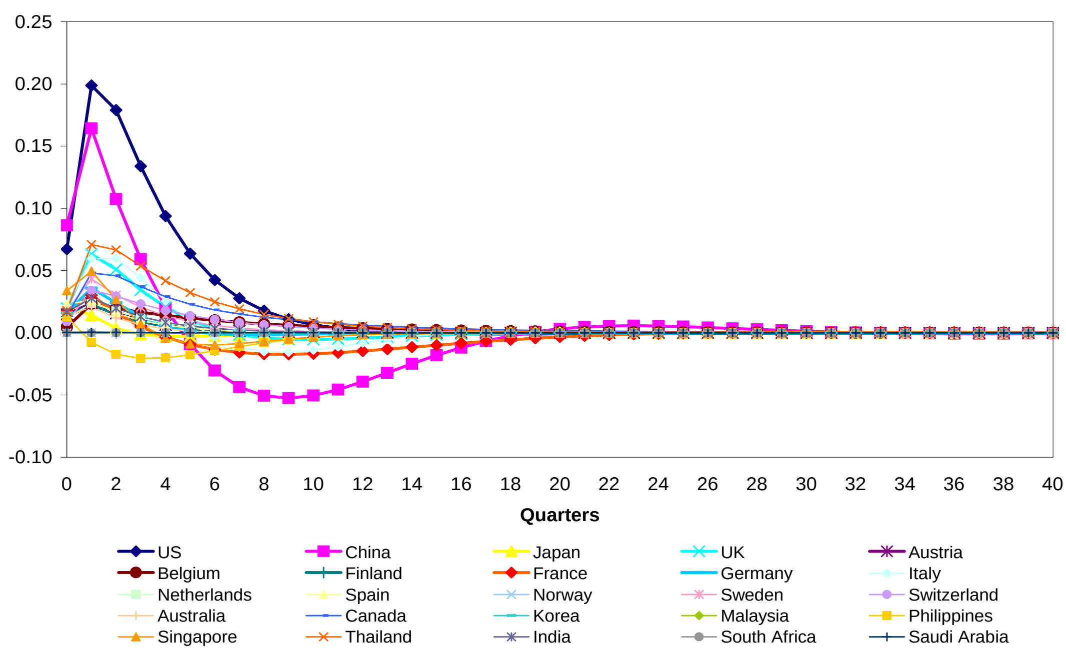
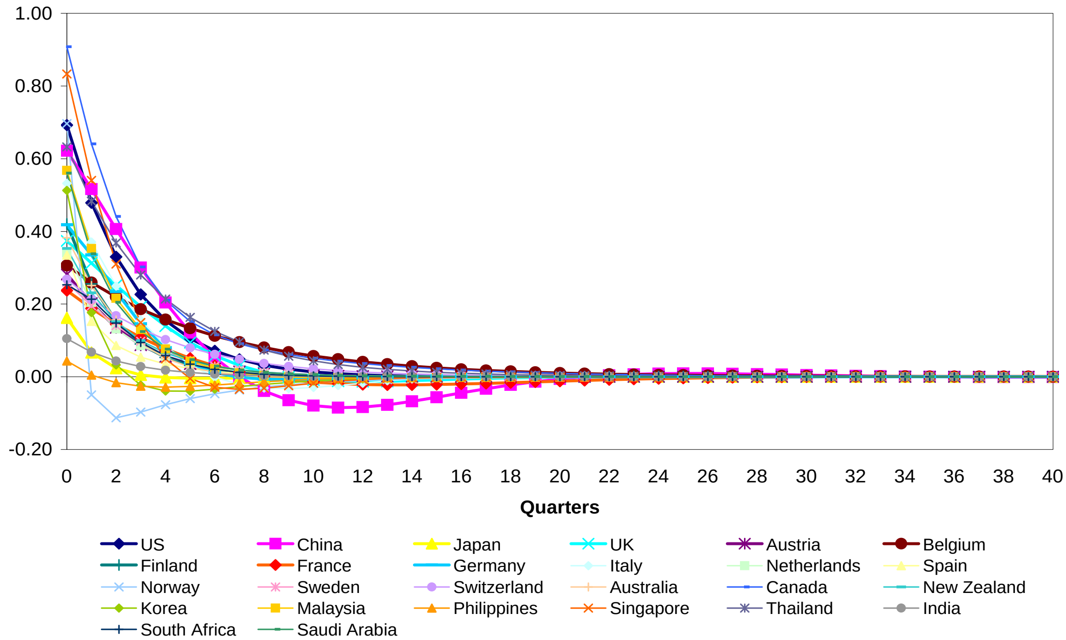


Figure S9c: Impulse responses of a one standard error global demand shock on interest rates in model without foreign output (per cent per quarter)



5 Inequality constrained instrumental variable estimation

The individual country-specific equations are estimated by the inequality constrained instrumental variables (ICIV) method both to obtain the estimates based on historical time series observations and for the bootstrap replications. When the constraints are not satisfied, the parameters are set to their boundary values and the choice between any alternative estimates that satisfy the constraints is based on the sum of squares of in-sample prediction errors. The ICIV is also applied at the bootstrap stage but for the (possibly) constrained specification estimated using the historical observations.

More specifically, ICIV estimation is a non-linear programming problem. In general, if there are n inequality restrictions there are 2^n possible unconstrained and constrained models to consider. In our application the maximum number of constraints is 4, in the PC equation, and the constrained optimization problem can be carried out by searching over all the specifications and then selecting the specification that satisfies all the constraints and has the lowest in-sample mean square prediction errors. Note that since some of the regressors are endogenous, the in-sample prediction errors and the IV residuals would not be the same. See Pesaran and Smith (1994). Consider the regression equation

$$\mathbf{y} = \mathbf{X}\beta + \varepsilon$$

where $\mathbf{y}$ and ε are $T \times 1$ vectors for the dependent variable and unobserved disturbance, $\mathbf{X}$ a $T \times k$ matrix of potentially endogenous regressors and there is also a $T \times s$ matrix of instruments $\mathbf{Z}$, $s \geq k$. Define $\mathbf{P}_z = \mathbf{Z}(\mathbf{Z}'\mathbf{Z})^{-1}\mathbf{Z}'$, $\mathbf{M}_z = (\mathbf{I} - \mathbf{P}_z)$ and $\hat{\mathbf{X}} = \mathbf{P}_z\mathbf{X}$. The IV estimator is $\hat{\beta} = (\mathbf{X}'\mathbf{P}_z\mathbf{X})^{-1}\mathbf{X}'\mathbf{P}_z\mathbf{y}$. The prediction errors are: $\hat{\mathbf{e}} = \mathbf{y} - \hat{\mathbf{X}}\hat{\beta}$; the IV residuals $\mathbf{e} = \mathbf{y} - \mathbf{X}\hat{\beta}$. Then as shown in Pesaran and

Smith (1994), the sum of squared prediction errors equals the IV minimand plus a constant which depends only on the data, not the estimates:

$$\hat{\mathbf{e}}'\hat{\mathbf{e}} = \mathbf{e}'\mathbf{P}_z\mathbf{e} + \mathbf{y}'\mathbf{M_z}\mathbf{y}.$$

Thus minimising the sum of squares of prediction errors, $\hat{\mathbf{e}}'\hat{\mathbf{e}}$, with respect to $\hat{\beta}$ is equivalent to minimising the IV minimand, $\mathbf{e}'\mathbf{P}_z\mathbf{e}$. In some of the constrained cases, there are no endogenous variables so $\mathbf{X} = \mathbf{Z}$, $\hat{\mathbf{X}} = \mathbf{X}$ and IV reduces to least squares; in other cases $\hat{\beta}$ is fully specified by the constraints, so no parameters are estimated. The sum of squared prediction errors remains well defined in all such cases.

For illustration consider the ICIV estimation of the PC equation. There are four inequality constraints:

$$\begin{aligned} A &: \beta_{ib} + \beta_{if} \leq 0.99 \\ B &: \beta_{ib} \geq 0 \\ C &: \beta_{if} \geq 0 \\ D &: \beta_{iy} \geq 0. \end{aligned}$$

The set of possible binding constraints gives 14 cases: two cases are redundant since imposing the boundary conditions under C and D (namely $\beta_{ib} = 0$ and $\beta_{if} = 0$) implies $\beta_{ib} + \beta_{if} \leq 0.99$. All the specifications associated with the 14 cases are estimated by IV. From the set of cases whose coefficients satisfy the inequality constraints the estimates with the lowest sum of squared prediction errors are chosen. The equations for the 14 possible cases are set out in the table below, a cross in the column corresponding to the constraint indicates that the boundary condition in that constraint is imposed.

Case	A	B	C	D	Equation
1					$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + \beta_{if}E_{t-1}(\widetilde{\pi}_{i,t+1}) + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}$
2	X				$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + (0.99 - \beta_{ib})E_{t-1}(\widetilde{\pi}_{i,t+1}) + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}$
3		X			$\widetilde{\pi}_{it} = \beta_{if}E_{t-1}(\widetilde{\pi}_{i,t+1}) + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}$
4			X		$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}$
5				X	$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + \beta_{if}E_{t-1}(\widetilde{\pi}_{i,t+1}) + \varepsilon_{i,st}$
6	X	X			$\widetilde{\pi}_{it} = 0.99E_{t-1}(\widetilde{\pi}_{i,t+1}) + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}$
7	X		X		$\widetilde{\pi}_{it} = 0.99\widetilde{\pi}_{i,t-1} + \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}$
8	X			X	$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi}_{i,t-1} + (0.99 - \beta_{ib})E_{t-1}(\widetilde{\pi}_{i,t+1}) + \varepsilon_{i,st}$
9		X	X		$\widetilde{\pi}_{it} = \beta_{iy}\widetilde{y}_{it} + \varepsilon_{i,st}$
10		X		X	$\widetilde{\pi}_{it} = \beta_{if}E_{t-1}(\widetilde{\pi}_{i,t+1}) + \varepsilon_{i,st}$
11			X	X	$\widetilde{\pi}_{it} = \beta_{ib}\widetilde{\pi} + \varepsilon_{i,st}$
12	X	X		X	$\widetilde{\pi}_{it} = 0.99E_{t-1}(\widetilde{\pi}_{i,t+1}) + \varepsilon_{i,st}$
13	X		X	X	$\widetilde{\pi}_{it} = 0.99\widetilde{\pi}_{i,t-1} + \varepsilon_{i,st}$
14		X	X	X	$\widetilde{\pi}_{it} = \varepsilon_{i,st}$

6 Solving and Bootstrapping the MCNK model

6.1 Solution

We start with the canonical representation of the global model

$$\chi_t = \mathbf{A}\chi_{t-1} + \mathbf{B}E_{t-1}(\chi_{t+1}) + \eta_t,$$

which has the following solution

$$\chi_t = \mathbf{\Phi}\chi_{t-1} + \eta_t,$$

where $\mathbf{\Phi}$ satisfies the quadratic matrix equation

$$\mathbf{B\Phi}^2 - \mathbf{\Phi} + \mathbf{A} = \mathbf{0}. \tag{1}$$

Solving the quadratic equation, we therefore obtain the reduced form solution in terms of $\widetilde{\mathring{\mathbf{x}}}_t$ and the structural shocks, ε_t, as

$$\widetilde{\mathring{\mathbf{x}}}_t = \mathbf{\Phi}_{11}\widetilde{\mathring{\mathbf{x}}}_{t-1} + \mathbf{\Phi}_{12}\widetilde{\mathring{\mathbf{x}}}_{t-2} + \mathbf{H}_0^{-1}\varepsilon_t,$$

where $\mathbf{\Phi}_{11}$ and $\mathbf{\Phi}_{12}$ are defined by

$$\mathbf{\Phi} = \begin{pmatrix} \mathbf{\Phi}_{11} & \mathbf{\Phi}_{12} \\ \mathbf{I}_k & \mathbf{0} \end{pmatrix}.$$

To solve (1) for $\mathbf{\Phi}$, we employ a back-substitution procedure which involves iterating on an initial arbitrary choice of $\mathbf{\Phi}$ and $\mathbf{\Psi}$, say $\mathbf{\Phi}_0$ and $\mathbf{\Psi}_0$, and use the recursive relations

$$\mathbf{\Phi}_r = (\mathbf{I}_k - \mathbf{B\Phi}_{r-1})^{-1}\mathbf{A}, \tag{2}$$

$$\mathbf{\Psi}_r = (\mathbf{I}_k - \mathbf{B\Phi}_{r-1})^{-1}\mathbf{B}, \tag{3}$$

where $\mathbf{\Phi}_r$ and $\mathbf{\Psi}_r$ are the values of $\mathbf{\Phi}$ and $\mathbf{\Psi}$, respectively, at the r^{th} iteration ($r = 1, 2, ...$) and $\mathbf{\Psi}$ is the coefficient matrix in the forward equation

$$\mathbf{z}_t = \mathbf{\Psi}E_{t-1}(\mathbf{z}_{t+1}) + \mathbf{v}_t,$$

with

$$\mathbf{z}_t = \chi_t - \mathbf{\Phi}\chi_{t-1},$$
$$\mathbf{v}_t = (\mathbf{I}_k - \mathbf{B\Phi})^{-1}\eta_t.$$

See Binder and Pesaran (1995, 1997) for further details. Matlab and Gauss code for this procedure is available at http://ideas.repec.org/c/dge/qmrbcd/73.html. This iterative procedure is continued until one of the following convergence criteria is met

$$\|\mathbf{\Phi}_r - \mathbf{\Phi}_{r-1}\|_{\max} \leq 10^{-6} \text{ or } \|\mathbf{\Psi}_r - \mathbf{\Psi}_{r-1}\|_{\max} \leq 10^{-6},$$

where the max norm of a matrix $\mathbf{A} = \{a_{ij}\}$ is defined as $\|\mathbf{A}\|_{\max} = \max_{i,j}\{|a_{ij}|\}$.

In the numerical calculations all unknown parameters are replaced with the restricted IV estimates.

6.2 Computation of bootstrap error bands

We generate B bootstrap samples denoted by $\widetilde{\mathring{\mathbf{x}}}_t^{(b)}$, $b = 1, 2, ..., B$ from the process

$$\widetilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{\Phi}}_{11}\widetilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{\Phi}}_{12}\widetilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \hat{\mathbf{H}}_0^{-1}\hat{\varepsilon}_t^{(b)},\ t = 1, 2, ..., T, \tag{4}$$

by resampling the structural residuals, $\hat{\varepsilon}_t$, and setting $\widetilde{\mathring{\mathbf{x}}}_0^{(b)} = \widetilde{\mathring{\mathbf{x}}}_0$ and $\widetilde{\mathring{\mathbf{x}}}_{-1}^{(b)} = \widetilde{\mathring{\mathbf{x}}}_{-1}$, where $\widetilde{\mathring{\mathbf{x}}}_0$ and $\widetilde{\mathring{\mathbf{x}}}_{-1}$ are the observed initial data vectors that include the US real exchange rate (or equivalently the US price level). Recall that the multi-country rational expectations model is solved in terms of the US price level rather than the US inflation.

We initially orthogonalise the structural shocks, $\hat{\varepsilon}_t$, by using the inverse of the Choleski factor, $\widetilde{\mathbf{P}}$, associated with the Choleski decomposition of the shrinkage covariance matrix, $\hat{\mathbf{\Sigma}}_\varepsilon(0.4)$. This way we obtain the $k \times 1$ orthogonal vector $\hat{\upsilon}_t = \widetilde{\mathbf{P}}^{-1}\hat{\varepsilon}_t$ where its j^{th} element $\hat{\upsilon}_{jt}$, $j = 1, 2, ..., k$, has unit variance. The bootstrap error vector is then obtained as $\varepsilon_t^{(b)} = \widetilde{\mathbf{P}}\hat{\upsilon}_t^{(b)}$, where $\hat{\upsilon}_t^{(b)}$ is the $k \times 1$ vector of re-sampled values from $\{\hat{\upsilon}_{jt}\}_{j=1,2,...,k;t=1,2,...,T}$. Prior to any resampling the structural residuals are recentered to ensure that their bootstrap population mean is zero.

Once a set of $\widetilde{\mathring{\mathbf{x}}}_t^{(b)}$, $b = 1, 2, ..., B$ are generated, US inflation is computed from the US price level so that $\widetilde{\mathbf{x}}_{it}^{(b)}$ is constructed, with the corresponding foreign variables, $\widetilde{\mathbf{x}}_{it}^{*(b)}$, computed using the trade weights. For each bootstrap replication the individual country models are then estimated by the inequality constrained IV procedure, ensuring that any constraint which binds for the estimates based on historical realisations are also imposed on the bootstrap estimates.

The country specific models in terms of $\widetilde{\mathbf{x}}_{it}^{(b)}$ are given by

$$\hat{\mathbf{A}}_{i0}^{(b)}\widetilde{\mathbf{x}}_{it}^{(b)} = \hat{\mathbf{A}}_{i1}^{(b)}\widetilde{\mathbf{x}}_{i,t-1}^{(b)} + \hat{\mathbf{A}}_{i2}^{(b)}E_{t-1}(\widetilde{\mathbf{x}}_{i,t+1}^{(b)}) + \hat{\mathbf{A}}_{i3}^{(b)}\widetilde{\mathbf{x}}_{it}^{*(b)} + \hat{\mathbf{A}}_{i4}^{(b)}\widetilde{\mathbf{x}}_{i,t-1}^{*(b)} + \varepsilon_t^{(b)},$$

and are subsequently combined yielding the MCNK model

$$\widetilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{F}}_1^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{F}}_2^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \hat{\mathbf{F}}_3^{(b)}E_{t-1}(\widetilde{\mathring{\mathbf{x}}}_{t+1}^{(b)}) + \hat{\mathbf{F}}_4^{(b)}E_{t-1}(\widetilde{\mathring{\mathbf{x}}}_t^{(b)}) + \mathbf{u}_t^{(b)}. \tag{5}$$

Solving the quadratic matrix as described earlier, the reduced form solution of (5) follows as

$$\widetilde{\mathring{\mathbf{x}}}_t^{(b)} = \hat{\mathbf{\Phi}}_{11}^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} + \hat{\mathbf{\Phi}}_{12}^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-2}^{(b)} + \mathbf{u}_t^{(b)},$$

with

$$\hat{\mathbf{u}}_t^{(b)} = \widetilde{\mathring{\mathbf{x}}}_t^{(b)} - \hat{\mathbf{\Phi}}_{11}^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-1}^{(b)} - \hat{\mathbf{\Phi}}_{12}^{(b)}\widetilde{\mathring{\mathbf{x}}}_{t-2}^{(b)}$$

and

$$\hat{\varepsilon}_t^{(b)} = \hat{\mathbf{H}}_0^{(b)}\hat{\mathbf{u}}_t^{(b)}.$$

For the first bootstrap replication we begin the iterative back-substitution procedure, using the estimated $\hat{\mathbf{\Phi}}$ from the actual data as an initial value to compute (4) and (5), so that for $b = 1$, $\mathbf{\Phi}_0^{(1)} = \hat{\mathbf{\Phi}}$. For each subsequent bootstrap replication, b, the initial value is set to the solution of (1) obtained under the preceding replication, $b - 1$, so that $\mathbf{\Phi}_0^{(b)} = \hat{\mathbf{\Phi}}^{(b-1)}$ and $\mathbf{\Psi}_0^{(b)} = (\mathbf{I}_k - \hat{\mathbf{B}}^{(b)}\hat{\mathbf{\Phi}}^{(b-1)})^{-1}\hat{\mathbf{B}}^{(b)}$. If for a particular bootstrap replication the iterative back-substitution procedure fails to converge after 500 iterations, the initial values for $\mathbf{\Phi}_0^{(b)}$ and $\mathbf{\Psi}_0^{(b)}$ are set to the identity matrix.

For each bootstrap replication $b = 1, 2, ..., B$, having estimated the individual country NK models using the simulated data $\tilde{\mathbf{x}}_t^{(b)}$, the MCNK model is reconstructed as described above and the impulse responses are calculated $g^{(b)}(n)$, for $n = 0, 1, 2, ...$ These statistics are then sorted in ascending order, and the $(1-\alpha)100\%$ confidence interval is calculated by using the $\alpha/2$ and $(1-\alpha/2)$ quantiles, say $q_{\alpha/2}$ and $q_{(1-\alpha/2)}$, respectively of the bootstrap distribution of $g(n)$.

To compute the upper and lower confidence bounds we use 2000 convergent and stationary bootstrap replications. A convergent replication is defined as one where for the corresponding bootstrap sample, the iterative back-substitution procedure described above converges within 500 iterations, whether the initial values for $\mathbf{\Phi}_0^{(b)}$ and $\mathbf{\Psi}_0^{(b)}$ are set to the identity matrix or otherwise. Having achieved convergence, a bootstrap replication is checked to make sure that it yields a stationary solution. If any of the above two conditions is violated, a new bootstrap sample is computed. For our bootstrap results we had to carry out a total of 2311 bootstrap replications, of which 311 where due to non-convergence of the iterative back-substitution procedure. No bootstrap replications were found to be non-stationary.

References

Binder M, Pesaran MH. (1995). Multivariate rational expectations models and macroeconomic modelling: A review and some new results. In Handbook of Applied Econometrics Vol. 1, Pesaran MH, Wickens W (eds.): 655-673.

Binder M, Pesaran MH. (1997). Multivariate linear rational expectations models: characterization of the nature of the solutions and their fully recursive computation. Econometric Theory 13: 877-888.

Dees S, Pesaran MH, Smith LV, Smith RP. (2010). Supply, Demand and Monetary Policy Shocks in a Multi-Country New Keynesian Model.

Dees S, di Mauro F, Pesaran M.H, Smith, LV. (2007). Exploring the international linkages of the euro area: A global VAR analysis. Journal of Applied Econometrics 22: 1-38.

Pesaran MH, Schuermann T, Smith LV. (2009). Forecasting economic and financial variables with global VARs. International Journal of Forecasting 25: 642-675.

Pesaran MH, Smith RJ. (1994). A generalized R^2 criterion for regression models estimated by the instrumental variables method. Econometrica, 62: 705-710.