SUPPLEMENT
for
Business Cycle Effects of Credit and Technology Shocks in a DSGE Model with Firm Defaults
by

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1 Real Interest Rates and Their Time Series Properties

1.1 Computation of real interest rate series

Note that both the loan rate and deposit rate series are measured in percent per annum (nominal interest rates), in order to calibrate the model on a quarterly basis, we first convert the series to percent per quarter (nominal interest rates). Following Dees, di Mauro, Pesaran, and Smith (2007), the nominal interest rate measured in percent per quarter can be obtained using the following expression

$$i_{q,lt} = \frac{1}{4}\ln(1 + i_{a,lt}/100) \approx \frac{i_{a,lt}}{400}$$
$$i_{q,dt} = \frac{1}{4}\ln(1 + i_{a,dt}/100) \approx \frac{i_{a,dt}}{400}$$

where $i_{a,lt}$ and $i_{a,dt}$ are the nominal loan and deposit rates, measured and expressed in percentage per annum, while $i_{q,lt}$ and $i_{q,dt}$ are the nominal loan and deposit rates, measured in percentage per quarter and expressed as a decimal.

Secondly, to calibrate our model, we convert the nominal interest rates to real interest rates. A note must be made on the distinction between the *ex ante* real interest rate and *ex post* real interest rate. The *ex ante* real interest rate is the nominal interest rates minus the expected inflation rate, while the *ex post* real rate is the nominal rate minus actual inflation. As discussed in Neely and Rapach (2008), agents make economic decisions on the basis of their inflation expectation over the decision horizon and the standard Euler equation relates the expected marginal utility of consumption to the expected real return. However, the *ex ante* real interest rate is not directly observable since expected inflation is not directly observable. As a result, we use the actual inflation rate as a proxy for inflation expectations, as is standard in the literature.[1]

If we denote p_t the price level at time t, the real loan rate r_{lt} and real deposit rate r_{dt} series are constructed as follows:

$$r_{lt} = i_{q,lt} - \pi_{qt}$$
$$r_{dt} = i_{q,dt} - \pi_{qt}$$

where π_{qt} is the quarterly inflation rate, computed as the 12 quarter moving average of the changes in the GDP deflator.

1.2 Properties of the US real interest rates series

1.2.1 Sample period: 1985Q1 to 2009Q4

The sample period of US real interest rates used in this paper is from 1985Q1 to 2009Q4. The choice for this sample period is guided by three principles.

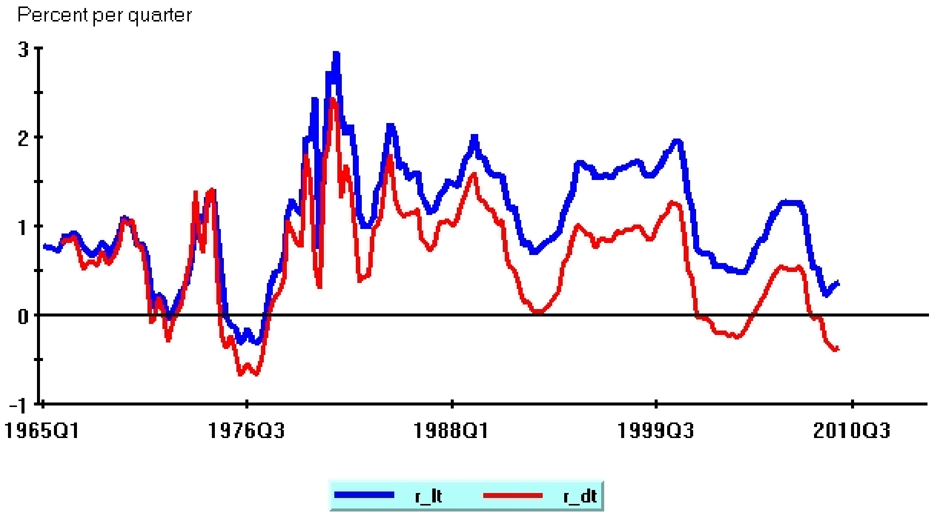
First of all, we choose a sample period when the spread between the loan and deposit rate is positive.[2]

Secondly, we decided to start the sample after the Volcker period, where the Chow test detects a structural break in the spread between the loan and deposit rate. As we could see in Figure 1, the real lending and deposit rate trace each other closely, the real rates experience a decline during the 1970s, a very sharp rise at the beginning of the 1980s and subsequent gradual decline during the "great disinflation". One possible explanation for the observed pattern of the real loan and

[1] An alternative approach is to use some survey measure of inflation expectations, such as the Livingston survey of professional forecasters, which has been conducted biannually since the 1940s (see for example Carlson, 1977). However, many economists doubt the validity of survey forecasts as expectations (see for example Mishkin, 1984). Further, obtaining survey data at the desired frequency for the desired sample may create other obstacles to the use of survey data (Neely and Rapach, 2008).

[2] The spread between the US loan and deposit rate is computed as the arithmetic difference between the real loan rate and the real deposit rate. The spread is negative for some period in the 70s.

Figure 1: US real loan rate and real deposit rate



deposit rates is the changes in US monetary policy: in the late 60s and throughout the 70s, the federal reserve pursued an expansionary monetary policy, saw the rise of inflation and reduced the real interest rate (see for example Meltzer, 2005 and Romer, 2005). In the early 80s, the federal reserve (under Paul Volcker) sharply raised short term nominal interest rates to reduce inflation from its early 1980 peak of nearly 12 percent and produced a sharp increase in the real interest rate (see Neely and Rapach, 2008).

Finally, starting the sample in 1985Q1 also has the advantage of avoiding the effects of a change in Federal Reserve operating policy from stable interest rates to stable money supply.[3]

Figure 2 presents a chart of the spread between the real loan and real deposit rates, which is computed as the arithmetic difference between the real loan rate and the real deposit rate. Note that we would expect the spread to be stationary under a competitive banking markets, since if the spread ("mark up") between loan and deposit becomes too high, a competitive market will put pressure on the banks to adjust the spread to its equilibrium level.

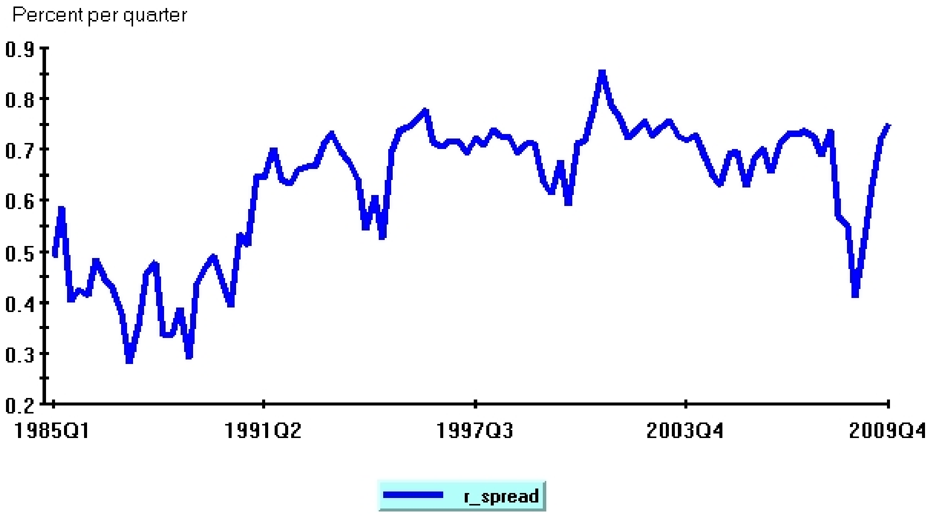
Before proceeding to examine the integration properties of the real loan rate and real deposit rate series, we first test for the existence of seasonality in the real loan rate and deposit rate series. Seasonality is not identified in the real interest rates series for the sample period 1985Q1 to 2009Q4.

1.2.2 Unit root test

In order to verify to what degree the real loan rate and real deposit rate series have univariate integration properties, we perform unit root test over the sample period. In particular, we examine the results from the weighted symmetric estimation of the ADF type regressions (introduced by Park and Fuller, 1995). Dees, di Mauro, Pesaran, and Smith (2007) noted that the weighted symmetric (WS) tests exploit the time reversibility of stationary autoregressive processes and hence possess higher power performance compared with the traditional Dickey-Fuller (DF) tests. Further, Leybourne, Kim, and Newbold (2005) and Pantula, Gonzalez-Farias, and Fuller (1994) provide evidence of superior performance of the weighted symmetric (WS) test statistics compared with the standard ADF tests or the GLS-ADF tests (Elliott, Rothenberg, and Stock, 1996). Similar

[3]Forbes and Mayne (1989) state that the sample period from January 1977 to August 1987 encompasses three distinct intervals in terms of federal reserve policy regime: January 1977 through September 1979 when the operating target for implementing monetary policy was the fed funds rates; October 1979 through October 1982 when the target was non-borrowed reserves; and November 1982 through the remainder of the study period with borrowed reserves serving as the target. Further details on these changes in the implementation of monetary policy can be found in Axilrod (1985).

Figure 2: Spread between lending and deposit rates (1985Q1 to 2009Q4)



to the ADF tests, the lag length employed in the WS unit root tests is selected by the Akaike Information Criterion (AIC) based on the standard ADF regressions.

The results from the Unit root tests suggest that the null hypothesis of a unit root can be rejected for the real loan rate and the spread between the loan and deposit rate at 5% level for 1985Q1 to 2009Q4. However, the null hypothesis of a unit root can not be rejected for the real deposit rate at 5% level for 1985Q1 to 2009Q4. The latter result may be puzzling at first sight, as we would expect the real deposit rate to be stationary, however, it could be a result of the structural breaks due to monetary policy changes in the sample period (1985Q1 to 2009Q4). Accounting for such breaks can substantially reduce the persistence within the regimes defined by those breaks (see for example Perron, 1989). Our findings are consistent with empirical studies on the US real interest rates, which found that postwar real interest rates in the US exhibit substantial persistence, shown by extended periods when the real interest rate is substantially above the sample mean.[4]

One group of studies uses unit root and cointegration tests to analyze whether shocks permanently affect the real interest rate–that is whether the real rate behaves like a random walk. These studies first test the integration properties of the nominal interest rates and the inflation rate, then test whether a cointegration relationship between the two rates hold. Often, such studies report evidence of unit roots, or, at a minimum substantial persistence, see for example Rose (1988). In particular, single equation methods provide weaker evidence of a stationary real interest rate, while the Johansen (1991) system based approach support a stationary real interest rate (see Neely and Rapach, 2008).

Other studies extend standard unit root and cointegration tests by considering whether US real interest rates are fractionally integrated (see for example Granger, 1980) or exhibit significant nonlinear behaviour, such as threshold dynamics or nonlinear cointegration. Fractional integration tests typically indicate that real interest rates revert to their mean very slowly (see for example Tsay, 2000 and Karananos, Sekioua, and Zeng, 2006). Similarly, studies that find evidence of nonlinear behaviour in real interest rates identify regimes in which the real rate behaves like a unit root process (see for example Million, 2004).

Another important group of studies reports evidence of structural breaks in the mean of real interest rates, and find that allowing for such breaks reduces the persistence of deviation from the regime-specific means, that is local persistence.[5] Lai (2008) finds widespread support for a

[4]The persistence in the real interest rates is found to be much higher in comparison to consumption growth. Please refer to survey paper Neely and Rapach (2008) on the evidence and implications of real interest rate persistence.

[5]The structural breaks themselves, however, still produce substantial global persistence in real interest rates, if not a unit root.

stationary real interest rate after allowing for a break in the unconditional mean and shows that structural change in real interest rate dynamics can be responsible for the observed unit root behaviour.

In what follows, we treat the real loan rate, the real deposit rate and the spread between the loan and deposit rates to be I(0), in accordance to the predictions from economic theory, notably, the Fisher equation.

2 Benchmark Model without the Banking Sector and Firm Defaults

We compare the predictions of our model with a benchmark case where there is no banking sector nor firm defaults. The difference between the credit model in the main body of the paper with the benchmark model is two fold: first, in the simplified benchmark model, we rule out idiosyncratic technology shocks and consider only the impact of common technology shock on the economy; second, the household supplies all the capital demanded by the representative firm. With these two differences, we effectively "switch off" the banking sector and firm defaults in our economy. The benchmark model is outlined as follows:

2.1 Consumer

For the consumer, we again consider the standard optimization problem

$$\max_{\{C_{t+j},N_{t+j},j=0,1,2...\}} E\left(\sum_{j=0}^{\infty}\beta^{j}U(C_{t+j},N_{t+j})|\Omega_{ct}\right),$$

subject to a simplified budget constraint

$$K_{t+1}=(1+r_{kt})K_t+W_tN_t-C_t+\Pi_{tc}, \tag{2.1}$$

in comparison to the full model with banking sector and firm defaults. In the simplified budget constraint, the consumer provides the entire amount of capital demanded by the representative firm. Adopting identical utility function as in the full model, first order conditions of the consumer's optimisation problem are given by

$$E\left[\beta\left(\frac{C_{t+1}-\frac{\chi_0}{1+\chi}N_{t+1}^{1+\chi}}{C_t-\frac{\chi_0}{1+\chi}N_t^{1+\chi}}\right)^{-\gamma}(1+r_{k,t+1})|\Omega_{ct}\right] = 1, \tag{2.2}$$

$$W_t = \chi_0 N_t^{\chi}. \tag{2.3}$$

2.2 Firm

The production technology of the representative firm is given by

$$Y_t=A_t^{\varphi}N_t^{1-\alpha}K_t^{\alpha}, \tag{2.4}$$

where as before $A_t=A_{t-1}\exp(\mu+u_t)$. Firm's optimization problem is given by

$$\max_{\{K_{t+s},N_{t+s},s=0,1,2...\}} E\left(\sum_{s=0}^{\infty}m_{t+s}\Pi_{f,t+s}|\Omega_{ft}\right),$$

where $\Pi_{ft} = Y_t - W_t N_t - (1+r_{kt})K_t$. The first order conditions for the firm's optimisation problem can be written as

$$
\begin{aligned}
E\left[\alpha\left(A_t\right)^{\varphi}\left(\frac{N_t}{K_t}\right)^{1-\alpha}|\Omega_{ft}\right] &= 1+r_{kt}, \\
E\left[(1-\alpha)\left(A_t\right)^{\varphi}\left(\frac{K_t}{N_t}\right)^{\alpha}|\Omega_{ft}\right] &= W_t,
\end{aligned}
$$

and yield the same set of equilibrium conditions as in the full model. Namely

$$
K_t = \frac{\alpha\chi_0}{1-\alpha}\cdot\frac{(N_t)^{1+\chi}}{1+r_{kt}}, \tag{2.5}
$$

and

$$
N_t = \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_{\varepsilon}\right)^{\frac{1}{1-\alpha}}\left(1+r_{kt}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\chi}}\exp\left[\frac{a_{t-1}+\mu+\rho_u u_{t-1}}{1+\chi}\right]. \tag{2.6}
$$

Also as with the full model, the firm sets the optimal levels of capital and labour *ex ante* and transfers the excess profits (or loss), Π_{tc}, to the household *ex post*, where

$$
\Pi_{tc} = Y_t - W_t N_t - (1+r_{kt})K_t. \tag{2.7}
$$

The economy-wide equilibrium condition can be derived from equations (2.1) and (2.7) and we have

$$
K_{t+1} = Y_t - C_t. \tag{2.8}
$$

2.3 Equilibrium conditions

2.3.1 System of equilibrium conditions

The set of equilibrium conditions of the benchmark model is given by equations (2.2), (2.3), (2.4),(2.5), (2.6) and (2.8), which we present again below:

$$
\begin{aligned}
1 &= E\left[\beta\left(\frac{C_{t+1}-\frac{\chi_0}{1+\chi}N_{t+1}^{1+\chi}}{C_t-\frac{\chi_0}{1+\chi}N_t^{1+\chi}}\right)^{-\gamma}(1+r_{k,t+1})|\Omega_{ct}\right], \\
K_{t+1} &= Y_t - C_t, \\
W_t &= \chi_0 N_t^{\chi}, \\
K_t &= \frac{\alpha\chi_0}{1-\alpha}\cdot\frac{(N_t)^{1+\chi}}{1+r_{kt}}, \\
N_t &= \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_{\varepsilon}\right)^{\frac{1}{1-\alpha}}\left(1+r_{kt}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\chi}}\exp\left[\frac{a_{t-1}+\mu+\rho_u u_{t-1}}{1+\chi}\right], \\
Y_t &= A_t^{\varphi}N_t^{1-\alpha}K_t^{\alpha},
\end{aligned}
$$

with 6 equations governing the macro economy in 6 endogenous variables C_t, W_t, N_t, K_t, Y_t and R_{kt}.

2.3.2 Equilibrium conditions in efficiency units

The system in efficiency units is again given by scaling the endogenous variables C_t, W_t, N_t, K_t, Y_t by an appropriate factor of technology A_{t-1} so that they are stationary on a balanced growth path. Denote the variables in efficiency unit by capital letters with a dot, $\mathring{C}_t = \frac{C_t}{A_{t-1}}$, $\mathring{K}_t = \frac{K_t}{A_{t-1}}$,

$\mathring{W}_t = \frac{W_t}{A_{t-1}^{\chi/(1+\chi)}}$, $\mathring{N}_t = \frac{N_t}{A_{t-1}^{1/(1+\chi)}}$ and $\mathring{Y}_t = \frac{Y_t}{A_{t-1}}$.

$$
\begin{aligned}
1 &= E\left[\beta\left(\frac{\mathring{C}_{t+1} - \frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t - \frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}}\right)^{-\gamma} g_t^{-\gamma} R_{k,t+1}|\Omega_{ct}\right], \\
\mathring{K}_{t+1}g_t &= \mathring{Y}_t - \mathring{C}_t, \\
\mathring{W}_t &= \chi_0\mathring{N}_t^{\chi}, \\
\mathring{K}_t &= \frac{\alpha\chi_0}{1-\alpha}\cdot\frac{\mathring{N}_t^{1+\chi}}{R_{kt}}, \\
\mathring{N}_t &= \left[\frac{1-\alpha}{\alpha\chi_0}(\alpha M_\varepsilon)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}} \exp\left[\frac{\mu(1-\rho_u)}{1+\chi}\right](R_{kt})^{-\frac{\alpha}{(1-\alpha)\chi}} g_{t-1}^{\frac{\rho_u}{1+\chi}}, \\
\mathring{Y}_t &= g_t^{\varphi}\mathring{N}_t^{1-\alpha}\mathring{K}_t^{\alpha},
\end{aligned}
$$

where $g_t = (1+g)e^{u_t}$.

2.3.3 Derivation of steady states

Following the same approach of computing steady states as outlined in the main body of the paper, the steady state of the simplified benchmark model is given by

$$
\begin{aligned}
r_k^* &= \gamma\mu + \ln\frac{1}{\beta}, \\
\mathring{n}^* &= -\frac{ln\chi_0}{\chi} + \frac{1}{\chi}ln\left[\frac{1-\alpha}{\alpha}(\alpha M_\varepsilon)^{\frac{1}{1-\alpha}}\right] + \frac{\mu}{1+\chi} - \frac{\alpha}{(1-\alpha)\chi}r_k^*, \\
\mathring{w}^* &= ln\chi_0 + \chi\mathring{n}^*, \\
\mathring{k}^* &= ln\chi_0 + ln\left[\frac{\alpha}{1-\alpha}\right] + (1+\chi)\mathring{n}^* - r_k^*, \\
\mathring{y}^* &= \varphi\mu + (1-\alpha)\mathring{n}^* + \alpha\mathring{k}^*, \\
e^{\mathring{y}^*} - e^{\mathring{c}^*} &= e^{\mathring{k}^*+\mu}.
\end{aligned}
$$

2.3.4 Loglinearisation

The log-linearised set of equilibrium conditions is given by

$$
\begin{aligned}
\widetilde{\mathring{c}}_t - a_1\widetilde{\mathring{n}}_t &= E\left[\widetilde{\mathring{c}}_{t+1} - a_1\widetilde{\mathring{n}}_{t+1} + a_2\widetilde{\ln g_t} - a_3\widetilde{r}_{k,t+1}|\Omega_{ct}\right], \\
-\widetilde{\mathring{c}}_t + a_5\widetilde{\mathring{y}}_t - a_6'\widetilde{\mathring{k}}_{t+1} &= a_6'\widetilde{\ln g_t}, \\
\widetilde{\mathring{n}}_t + \frac{\alpha}{(1-\alpha)\chi}\widetilde{r}_{kt} &= \frac{\rho_u}{1+\chi}\widetilde{\ln g_{t-1}}, \\
\widetilde{\mathring{y}}_t - (1-\alpha)\widetilde{\mathring{n}}_t &= \alpha\widetilde{\mathring{k}}_t + \varphi\widetilde{\ln g_t}, \\
(1+\chi)\widetilde{\mathring{n}}_t - \widetilde{r}_{kt} &= \widetilde{\mathring{k}}_t,
\end{aligned}
$$

where

$$
\begin{aligned}
a_1 &= b_1(1+\chi),\ a_2 = 1-b_1,\ a_3 = \frac{1-b_1}{\gamma}, \\
a_5 &= e^{\mathring{y}^*-\mathring{c}^*},\ a_6' = e^{\mathring{k}^*+\mu-\mathring{c}^*},\ b_1 = \frac{\chi_0}{1+\chi}e^{(1+\chi)\mathring{n}^*-\mathring{c}^*}.
\end{aligned}
$$

It can be shown that the coefficients a_1-a_5 do not depend on χ_0.[6]

2.3.5 Model solution

We can rewrite the system of equations in the following form

$$\mathbf{H}_0\mathbf{x}_t = \mathbf{H}_1\mathbf{x}_{t-1} + \mathbf{H_2}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{v}_t$$

where $\mathbf{H}_0 = \begin{pmatrix} 1 & 0 & 0 & 0 & -a_1 \\ -1 & -a_6' & 0 & a_5 & 0 \\ 0 & 0 & \frac{\alpha}{(1-\alpha)\chi} & 0 & 1 \\ 0 & 0 & 0 & 1 & -(1-\alpha) \\ 0 & 0 & -1 & 0 & (1+\chi) \end{pmatrix}$, $\mathbf{H}_1 = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \alpha & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \end{pmatrix}$,

$$\mathbf{H}_2 = \begin{pmatrix} 1 & 0 & -a_3 & 0 & -a_1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \mathbf{v}_t = \begin{pmatrix} E_t a_2 \widetilde{\ln g_t} \\ a_6' \widetilde{\ln g_t} \\ \frac{\rho_u}{1+\chi} \widetilde{\ln g_{t-1}} \\ \varphi \widetilde{\ln g_t} \\ 0 \end{pmatrix}, \mathbf{x}_t = \begin{pmatrix} \widetilde{\mathring{c}}_t \\ \widetilde{\mathring{k}}_{t+1} \\ \widetilde{r}_{kt} \\ \widetilde{\mathring{y}}_t \\ \widetilde{\mathring{n}}_t \end{pmatrix}'.$$

Note that $E_t\widetilde{\ln g_t} = u_t$,and write $\mathbf{v}_t$ as

$$\mathbf{v}_t = \tilde{\mathbf{G}}_0\boldsymbol{\xi}_t + \tilde{\mathbf{G}}_1\boldsymbol{\xi}_{t-1},$$

where

$$\boldsymbol{\xi}_t = u_t, \quad \tilde{\mathbf{G}}_0 = \begin{pmatrix} a_2 \\ a_6' \\ 0 \\ \varphi \\ 0 \end{pmatrix}, \quad \tilde{\mathbf{G}}_1 = \begin{pmatrix} 0 \\ 0 \\ \frac{\rho_u}{1+\chi} \\ 0 \\ 0 \end{pmatrix}.$$

[6] Due to space considerations the proof is not presented here, but is available upon request.

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