To Pool or not to Pool: Revisited

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Abstract

This paper provides a new comparative analysis of pooled least squares and fixed effects estimators of the slope coefficients in the case of panel data models when the time dimension (T) is fixed while the cross section dimension (N) is allowed to increase without bounds. The individual effects are allowed to be correlated with the regressors, and the comparison is carried out in terms of an exponent coefficient, δ, which measures the degree of pervasiveness of the fixed effects in the panel. It is shown that the pooled estimator remains consistent so long as $\delta < 1$, and is asymptotically normally distributed if $\delta < 1/2$, for a fixed T and as $N \to \infty$. It is further shown that when $\delta < 1/2$, the pooled estimator is more efficient than the fixed effects estimator. Monte Carlo evidence provided supports the main theoretical findings and gives some indications of gains to be made from pooling when $\delta < 1/2$. The problem of how to estimate δ in short T panels is not considered in this paper.

Keywords: Short panel, Fixed effects estimator, Pooled estimator, Efficiency

JEL classification: C01, C23, C33

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1 Introduction

This note re-examines the issue of pooling in standard panel data models with exogenous regressors in terms of an exponent coefficient, $0 \leq \delta \leq 1$, which measures the degree of pervasiveness of correlated individual effects, defined by

$$\sum_{i=1}^{N} E\left|\eta_i\right| = O\left(N^{\delta}\right),$$

where N is the cross- section dimension of the panel, and η_i is the mean zero random part of the individual effects. Throughout we allow for non-zero correlations between the individual effects and the regressors, and as a result the pooled estimators will be biased in the standard case where $\delta = 1$. We show that the choice between the pooled least squares (PLS) estimator and the fixed effects (FE) estimator depends on the value of δ, with the PLS estimator being consistent for all values of δ except when $\delta = 1$. For inference, the validity of the PLS estimator requires $\delta < 1/2$. Both of these conditions are significantly weaker than the homogeneity assumption made in the literature requiring that $E\left|\eta_i\right| = 0$ for all i. For example, when $\delta = 0$ we could have a finite number of non-zero $E\left|\eta_i\right|$, or more generally $E\left|\eta_i\right| = K\rho^i$, for a fixed positive constant K, and $0 < \rho < 1$. This corresponds to the sparsity assumption often made in the context of penalized regressions. But our analysis covers non-sparse structures by allowing the number of non-zero $E\left|\eta_i\right|$'s to rise with N but not proportionately. The degree to which the number of units with non-zero $E\left|\eta_i\right|$ is allowed to rise with N is governed by δ. For example, when $\delta = 1/2$ the number of cross-section units with non-zero random effects could rise with $\sqrt{N}$, with the proportion of such units in total declining to zero at the rate of $N^{-1/2}$.

The exponent of pervasiveness of individual effects is also closely related to the exponent of cross-sectional dependence, α, recently introduced in Bailey et al. (2015) to measure the degree of cross-sectional dependence in panels. Both exponents measure the degree of pervasiveness of heterogeneity, δ relates to the heterogeneity of the individual effects , and α the heterogeneity of factor loadings in a panel data model with a factor error structure. In a broad sense, δ can also be viewed as an exponent of cross-sectional dependence applied to the intercepts viewed as a common factor.

Our analysis complements and provides further insights on the discussion of "pool or not to pool" in the panel literature.[1] See for example, Baltagi et al (2000), and Baltagi (2008). More

[1] There is also a related literature that considers the problem of pooling more generally and discusses the issue of pooling in the case of panel data models with heterogenous slopes. As a recent example, see Paap, Wang and Zhang (2015) and references cited therein. In this paper we focus on the issue of pooling in the context of standard panel data models with homogeneous slopes. But our approach and generalization of the concept of

specifically, we derive the asymptotic properties of the pooled least squares estimator when N is large and T is fixed for different values of δ, and derive the bias of PLS when $\delta = 1$, and show that the pooled estimator is more efficient than the fixed effects estimator if $\delta < 1/2$. We also establish the asymptotic equivalence of random effects and PLS estimators when $\delta < 1$. Monte Carlo simulations are conducted to compare the finite sample properties of PLS and FE estimators. The results confirm the main theoretical findings and give some indication of the magnitudes of the gains involved from pooling when $\delta < 1/2$.

The analysis of this paper shows the importance of knowing δ in the choice between PLS (or RE) and FE estimators. In the case of large N and T panels estimation of δ can be carried out using the approach of Bailey et al. (2015). But for short T panels, which is of concern in this paper, such an approach will not be applicable and other suitable techniques will be required. One could, for example, consider the application of the Hausman type tests to the difference between the FE and PLS estimators. However, development of suitable procedures for direct tests on δ in the case of short T panels will be beyond the scope of the present paper.

The rest of the note is organized as follows. Section 2 sets out the model and its assumptions. Section 3 presents the theoretical results. Monte Carlo simulations are provided in Section 4, with some concluding remarks in Section 5.

2 Panel data model

Consider the standard panel data model

$$\begin{aligned} y_{it} &= \alpha_i + \boldsymbol{\beta}'\mathbf{x}_{it} + u_{it}, \text{ for } i = 1, 2, \ldots, N; t = 1, 2, \ldots, T && (2.1) \\ \alpha_i &= \alpha + \eta_i \text{ for } i = 1, 2, ..., N, && (2.2) \end{aligned}$$

where α_i are the individual effects, $\mathbf{x}_{it}$ is a $k \times 1$ vector of regressors which we decompose as

$$\mathbf{x}_{it} = \eta_i \mathbf{g}_t + \mathbf{w}_{it}, \text{ for } i = 1, 2, \ldots, N; t = 1, 2, \ldots, T. \quad (2.3)$$

$\eta_i \mathbf{g}_t$ represents the part of $\mathbf{x}_{it}$ which is correlated with the individual effects , α_i, with $\mathbf{g}_t$ being a $k \times 1$ vector of time effects, and $\mathbf{w}_{it}$ is the part of $\mathbf{x}_{it}$ which is distributed independently of the individual effects . This is a fairly general specification which allows for non-zero, time-varying correlations between $\mathbf{x}_{it}$ and α_i, and allows the regressors to have individual-specific effects and be cross-sectionally correlated. Additional individual-specific effects can be included in $\mathbf{x}_{it}$ through $\mathbf{w}_{it}$. For example, using (2.3), and assuming that $\bar{\mathbf{g}} = T^{-1} \sum_{t=1}^{T} \mathbf{g}_t \neq \mathbf{0}$, then

$$\eta_i = \boldsymbol{\pi}'\bar{\mathbf{x}}_i + v_i, \quad (2.4)$$

cross-sectional heterogeneity can also be applied to panel data models with heterogeneous slopes.

where ,

$$\begin{aligned}\boldsymbol{\pi}' &= \left(\bar{\mathbf{g}}'\bar{\mathbf{g}}\right)^{-1}\bar{\mathbf{g}}',\ v_i = -\left(\bar{\mathbf{g}}'\bar{\mathbf{g}}\right)^{-1}\bar{\mathbf{g}}'\bar{\mathbf{w}}_i, \\ \bar{\mathbf{x}}_i &= T^{-1}\sum_{t=1}^{T}\mathbf{x}_{it}, \text{ and } \bar{\mathbf{w}}_i = T^{-1}\sum_{t=1}^{T}\mathbf{w}_{it},\end{aligned}$$

which is the same as the Mundlak (1978) formulation of the individual effects in standard panel data models.

Throughout we assume T is fixed and carry out our analysis for N large. Except for the assumption regarding the individual effects, η_i, we make the following standard assumptions:

Assumption 1: The individual effects, η_i for $i = 1, 2, ..., N$, are either deterministic and bounded (i.e. $|\eta_i| < K$), or stochastic with second order moments, $E\left(\eta_i^2\right) < K$, and distributed independently of $\mathbf{g}_t$ and $\mathbf{w}_{jt}$ for all i, j and t; satisfying the conditions[2]

$$N^{-1}\sum_{i=1}^{N}E\left|\eta_i\right|^s = O\left(N^{\delta-1}\right), \text{ for } s = 1 \text{ and } 2, \text{ where } 0 \leq \delta \leq 1. \tag{2.5}$$

Remark 2.1 *The conditions of Assumption 1 are satisfied, for example, if there exists an ordering of the individual units such that for δ in the range $[0, 1]$*

$$\begin{aligned}\eta_i &= \varepsilon_i,\ \textit{for } i = 1, 2, ..., [N^\delta], \\ &= 0,\ \textit{for } i = [N^\delta] + 1, [N^\delta] + 2, ..., N\end{aligned}$$

where $\{\varepsilon_i\ , i = 1, 2, ..., N\}$ is a sequence of random variables with zero means and finite variances such that

$$\lim_{M\to\infty}\left(M^{-1}\sum_{i=1}^{M}E\left|\varepsilon_i\right|^s\right) = O(1),\ \textit{for } s = 1 \textit{ and } 2.$$

Then,

$$N^{-1}\sum_{i=1}^{N}E\left|\eta_i\right|^s = N^{\delta-1}\left(N^{-\delta}\sum_{i=1}^{[N^\delta]}E\left|\varepsilon_i\right|^s\right) = O\left(N^{\delta-1}\right).$$

Note that the above result holds even if $\varepsilon_i's$ are cross-sectionally correlated. Furthermore, the condition that $\eta_i = 0$, for $i = [N^\delta] + 1, [N^\delta] + 2, ..., N$, can be relaxed by requiring that (see also Bailey et al. (2015))

$$\sum_{i=[N^\delta]+1}^{N}E\left|\eta_i\right|^s = O(1),\ \textit{for } s = 1 \textit{ and } 2\,.$$

This condition holds, for example, if $E\left|\eta_i\right|^s = \kappa_{is}\rho_s^i$ for $i = [N^\delta] + 1, [N^\delta] + 2, ..., N$, where κ_{is} are finite positive constants and $0 \leq \rho_s < 1$.

[2] K represents a generic finite positive constant.

Remark 2.2 *Conditions (2.5) also imply*

$$N^{-1}\sum_{i=1}^{N}\eta_i^2 = O_p\left(N^{\delta-1}\right),\ \textit{and}\ N^{-1}\sum_{i=1}^{N}|\eta_i| = O_p\left(N^{\delta-1}\right).$$

These results follow by application of the Markov inequality to (2.5).

Assumption 2: (a) u_{it} is distributed independently of η_j and $\mathbf{w}_{jt'}$ for all i, j, t, and t'. (b) $u_{it} \sim IID(0, \sigma_u^2)$, $0 < \sigma_u^2 < K$, and $E|u_{it}|^{4+\epsilon} < K$, for some small positive ϵ.

Assumption 3: The time effects, $\mathbf{g}_t$, are bounded such that $\|\mathbf{g}_t\mathbf{g}_t'\| < K < \infty$, if $\mathbf{g}_t$ is deterministic and $E\|\mathbf{g}_t\mathbf{g}_t'\| < K < \infty$, if $\mathbf{g}_t$ is stochastic. $\|\mathbf{A}\|$ represents the Frobenius norm of $\mathbf{A}$ defined by $Tr\left(\mathbf{A}\mathbf{A}'\right)^{1/2}$.

Assumption 4: The variables, $\mathbf{w}_{it}$, are either deterministic and bounded, namely $\|\mathbf{w}_{it}\| < K < \infty$, or they satisfy the moment conditions $E\|\mathbf{w}_{it} - \bar{\mathbf{w}}_i\|^2 < K < \infty$, for all i and t, where $\bar{\mathbf{w}}_i = T^{-1}\sum_{t=1}^{T}\mathbf{w}_{it}$. Similarly, $E\|\bar{\mathbf{w}}_i - \bar{\mathbf{w}}\|^2 < K < \infty$, for all i, where $\bar{\mathbf{w}} = N^{-1}\sum_{i=1}^{N}\bar{\mathbf{w}}_i$.

Assumption 5: The $k \times k$ matrices

$$\begin{aligned}
\mathbf{\Omega}_{P,N} &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it} - \bar{\mathbf{w}}\right)\left(\mathbf{w}_{it} - \bar{\mathbf{w}}\right)', \\
\mathbf{\Omega}_{FE,N} &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right)\left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right)',
\end{aligned}$$

are positive definite for all N, and as $N \to \infty$. The probability limits of $\mathbf{\Omega}_{P,N}$ and $\mathbf{\Omega}_{FE,N}$, as N tends to infinity, will be denoted by $\mathbf{\Omega}_P$ and $\mathbf{\Omega}_{FE}$, respectively.

Remark 2.3 *Note that* $E\|\mathbf{w}_{it} - \bar{\mathbf{w}}_i\| \leq \left[E\|\bar{\mathbf{w}}_i - \bar{\mathbf{w}}\|^2\right]^{1/2} < K < \infty$, *and* $E\|\bar{\mathbf{w}}_i - \bar{\mathbf{w}}\| \leq \left[E\|\bar{\mathbf{w}}_i - \bar{\mathbf{w}}\|^2\right]^{1/2} < K < \infty$. *Hence under Assumption 4 we also have*

$$E\|(\mathbf{w}_{it} - \bar{\mathbf{w}})\| = E\|(\mathbf{w}_{it} - \bar{\mathbf{w}}_i + \bar{\mathbf{w}}_i - \bar{\mathbf{w}})\| \leq E\|(\mathbf{w}_{it} - \bar{\mathbf{w}}_i)\| + E\|(\bar{\mathbf{w}}_i - \bar{\mathbf{w}})\| < K < \infty. \quad (2.6)$$

3 Pooled least squares and FE estimators

The PLS and FE estimators, $\hat{\boldsymbol{\beta}}_P$ and $\hat{\boldsymbol{\beta}}_{FE}$, respectively, can be written as

$$\hat{\boldsymbol{\beta}}_P = \mathbf{Q}_{P,N}^{-1}\mathbf{q}_{P,N}, \quad (3.1)$$

and

$$\hat{\boldsymbol{\beta}}_{FE} = \mathbf{Q}_{FE,N}^{-1}\mathbf{q}_{FE,N}, \quad (3.2)$$

where

$$\mathbf{Q}_{P,N} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)', \;\; \mathbf{q}_{P,N} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)\left(y_{it}-\bar{y}\right), \tag{3.3}$$

$$\mathbf{Q}_{FE,N} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it}-\bar{\mathbf{x}}_i\right)\left(\mathbf{x}_{it}-\bar{\mathbf{x}}_i\right)', \;\; \mathbf{q}_{FE,N} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it}-\bar{\mathbf{x}}_i\right)\left(y_{it}-\bar{y}_i\right), \tag{3.4}$$

and

$$\begin{aligned}
\bar{\mathbf{x}} &= (NT)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T}\mathbf{x}_{it},\; \bar{y} = (NT)^{-1}\sum_{i=1}^{N}\sum_{t=1}^{T} y_{it}, && (3.5)\\
\bar{\mathbf{x}}_i &= T^{-1}\sum_{t=1}^{T}\mathbf{x}_{it},\; \bar{y}_i = T^{-1}\sum_{t=1}^{T} y_{it}. && (3.6)
\end{aligned}$$

To derive the properties of these estimators, using (2.3), we first note that

$$\mathbf{x}_{it}-\bar{\mathbf{x}} = \left(\eta_i\mathbf{g}_t - \bar{\eta}\bar{\mathbf{g}}\right) + \left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right), \tag{3.7}$$

and

$$y_{it}-\bar{y} = \eta_i - \bar{\eta} + \boldsymbol{\beta}'\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right) + (u_{it}-\bar{u}), \tag{3.8}$$

where

$$\bar{\mathbf{g}} = T^{-1}\sum_{t=1}^{T}\mathbf{g}_t,\;\; \bar{\eta} = N^{-1}\sum_{i=1}^{N}\eta_i, \text{ and } \bar{u} = N^{-1}\sum_{i=1}^{N}u_i.$$

3.1 The PLS estimator

Starting with the PLS estimator, using (3.7) in (3.3) we have

$$\begin{aligned}
\mathbf{Q}_{P,N} &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\eta_i\mathbf{g}_t + \mathbf{w}_{it} - \bar{\eta}\bar{\mathbf{g}} - \bar{\mathbf{w}}\right)\left(\eta_i\mathbf{g}_t + \mathbf{w}_{it} - \bar{\eta}\bar{\mathbf{g}} - \bar{\mathbf{w}}\right)'\\
&= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)' + \left(N^{-1}\sum_{i=1}^{N}\eta_i^2\right)\left(T^{-1}\sum_{t=1}^{T}\mathbf{g}_t\mathbf{g}_t'\right) - \bar{\eta}^2\left(\bar{\mathbf{g}}\bar{\mathbf{g}}'\right)\\
&\quad + \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left[\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)\mathbf{g}_t' + \mathbf{g}_t\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)'\right]. && (3.9)
\end{aligned}$$

Similarly, using (3.8) in (3.3) we have

$$
\begin{aligned}
\mathbf{q}_{P,N} &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)\left(y_{it}-\bar{y}\right) \\
&= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)\left[\eta_i-\bar{\eta}+\boldsymbol{\beta}'\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)+u_{it}-\bar{u}\right] \\
&= \mathbf{Q}_{P,N}\boldsymbol{\beta}+\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)\left(\eta_i-\bar{\eta}+u_{it}-\bar{u}\right),
\end{aligned}
$$

which upon using (3.7) can be written as

$$
\mathbf{q}_{P,N}=\mathbf{Q}_{P,N}\boldsymbol{\beta}+\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)\left(\eta_i+u_{it}\right)+\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\eta_i\mathbf{g}_t-\bar{\eta}\bar{\mathbf{g}}\right)\left(\eta_i+u_{it}\right), \quad (3.10)
$$

which in turn yields

$$
\hat{\boldsymbol{\beta}}_P=\boldsymbol{\beta}+\mathbf{Q}_{P,N}^{-1}\left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)\left(\eta_i+u_{it}\right)+\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\eta_i\mathbf{g}_t-\bar{\eta}\bar{\mathbf{g}}\right)\left(\eta_i+u_{it}\right)\right]. \quad (3.11)
$$

Furthermore,

$$
\begin{aligned}
\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\eta_i\mathbf{g}_t-\bar{\eta}\bar{\mathbf{g}}\right)\left(\eta_i+u_{it}\right) &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\eta_i^2\mathbf{g}_t-\bar{\eta}\eta_i\bar{\mathbf{g}}\right)+\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\eta_i\mathbf{g}_t-\bar{\eta}\bar{\mathbf{g}}\right)u_{it} \\
&= \frac{1}{N}\sum_{i=1}^{N}\left(\eta_i^2-\bar{\eta}\eta_i\right)\bar{\mathbf{g}}+\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\eta_i\mathbf{g}_t-\bar{\eta}\bar{\mathbf{g}}\right)u_{it} \\
&= \left[N^{-1}\sum_{i=1}^{N}\left(\eta_i-\bar{\eta}\right)^2\right]\bar{\mathbf{g}}+\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\mathbf{g}_t u_{it}-\bar{\eta}\bar{\mathbf{g}}\bar{u}. \quad (3.12)
\end{aligned}
$$

But under Assumption 1 we have

$$
N^{-1}\sum_{i=1}^{N}E\left|\eta_i\right|^2=O\left(N^{\delta-1}\right), \text{ and } E\left|\bar{\eta}\right|\leq N^{-1}\sum_{i=1}^{N}\left|\eta_i\right|=O\left(N^{\delta-1}\right), \quad (3.13)
$$

and since η_i is distributed independently of $\bar{\mathbf{g}}$ and u_{it}, then

$$
\begin{aligned}
E\left|\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\eta_i\mathbf{g}_t-\bar{\eta}\bar{\mathbf{g}}\right)\left(\eta_i+u_{it}\right)\right| &\leq \left[N^{-1}\sum_{i=1}^{N}E\left(\eta_i-\bar{\eta}\right)^2\right]E\left(\|\bar{\mathbf{g}}\|\right) \\
&\quad +\sup_{i,t}E\left|u_{it}\right|\sup_{t}E\left(\|\mathbf{g}_t\|\right)\left(N^{-1}\sum_{i=1}^{N}E\left|\eta_i\right|\right)+E\left|\bar{\eta}\right|E\left|\bar{u}\right|E\left(\|\bar{\mathbf{g}}\|\right) \\
&= O\left(N^{\delta-1}\right)
\end{aligned}
$$

Similarly

$$\begin{aligned}\left\|\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left[(\mathbf{w}_{it}-\bar{\mathbf{w}})\,\mathbf{g}_t'+\mathbf{g}_t\,(\mathbf{w}_{it}-\bar{\mathbf{w}})'\right]\right\| &\leq \frac{2}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}|\eta_i|\,\|(\mathbf{w}_{it}-\bar{\mathbf{w}})\|\,\|\mathbf{g}_t\| \\ &= \frac{2}{N}\sum_{i=1}^{N}|\eta_i|\left[T^{-1}\sum_{t=1}^{T}\|(\mathbf{w}_{it}-\bar{\mathbf{w}})\|\,\|\mathbf{g}_t\|\right],\end{aligned}$$

and since under Assumption 1, η_i is distributed independently of $\mathbf{g}_t$ and $\mathbf{w}_{it}$, we have

$$E\left\|\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left[(\mathbf{w}_{it}-\bar{\mathbf{w}})\,\mathbf{g}_t'+\mathbf{g}_t\,(\mathbf{w}_{it}-\bar{\mathbf{w}})'\right]\right\| \leq \frac{2}{N}\sum_{i=1}^{N}E\,|\eta_i|\left[\left\{T^{-1}\sum_{t=1}^{T}E\left[\|(\mathbf{w}_{it}-\bar{\mathbf{w}})\|\,\|\mathbf{g}_t\|\right]\right\}\right].$$

However, by the Cauchy–Schwarz inequality and under Assumptions 3 and 4

$$E\left[\|(\mathbf{w}_{it}-\bar{\mathbf{w}})\|\,\|\mathbf{g}_t\|\right] \leq \left[E\,\|(\mathbf{w}_{it}-\bar{\mathbf{w}})\|^2\right]^{1/2}\left[E\,\|\mathbf{g}_t\|^2\right]^{1/2} < K,$$

and

$$E\left\|\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left[(\mathbf{w}_{it}-\bar{\mathbf{w}})\,\mathbf{g}_t'+\mathbf{g}_t\,(\mathbf{w}_{it}-\bar{\mathbf{w}})'\right]\right\| \leq \frac{2K}{N}\sum_{i=1}^{N}E\,|\eta_i| = O\left(N^{\delta-1}\right).$$

Using (3.13) and the above result in (3.9) we obtain

$$\mathbf{Q}_{P,N} = \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{w}_{it}-\bar{\mathbf{w}})\,(\mathbf{w}_{it}-\bar{\mathbf{w}})' + O_p\left(N^{\delta-1}\right),$$

which establishes that under $\delta < 1$ (for a fixed T and as $N \to \infty$)

$$\mathbf{Q}_{P,N} \to_p \mathbf{\Omega}_P = \lim_{N\to\infty}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}E\left[(\mathbf{w}_{it}-\bar{\mathbf{w}})\,(\mathbf{w}_{it}-\bar{\mathbf{w}})'\right] > 0. \tag{3.14}$$

Consider now the second component of (3.11), and note from (3.12) that since by assumption η_i, u_{it}, and $\mathbf{g}_t$ are distributed independently, then

$$\begin{aligned}E\left\|\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\mathbf{g}_t u_{it}\right\| &\leq \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}E\,|\eta_i|\,E\,\|\mathbf{g}_t\|\,E\,|u_{it}| \\ &\leq \frac{K}{N}\sum_{i=1}^{N}E\,|\eta_i| = O\left(N^{\delta-1}\right).\end{aligned}$$

Hence, in view of (3.14) and using the above results we have

$$\hat{\boldsymbol{\beta}}_P = \boldsymbol{\beta} + \mathbf{\Omega}_P^{-1}\left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}(\mathbf{w}_{it}-\bar{\mathbf{w}})\,(\eta_i+u_{it})\right] + O_p\left(N^{\delta-1}\right). \tag{3.15}$$

Furthermore

$$\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)\eta_i=\frac{1}{N}\sum_{i=1}^{N}\left(\bar{\mathbf{w}}_i-\bar{\mathbf{w}}\right)\eta_i,$$

and since by Assumption $1, \eta_i$ and $\bar{\mathbf{w}}_i-\bar{\mathbf{w}}$ are independently distributed and by Assumption 4, $E\left\|\bar{\mathbf{w}}_i-\bar{\mathbf{w}}\right\|<K$, then

$$E\left\|\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)\eta_i\right\|\leq\frac{1}{N}\sum_{i=1}^{N}E\left|\eta_i\right|E\left\|\bar{\mathbf{w}}_i-\bar{\mathbf{w}}\right\|\leq\frac{K}{N}\sum_{i=1}^{N}E\left|\eta_i\right|=O\left(N^{\delta-1}\right).$$

Therefore, (3.15) simplifies further to

$$\hat{\boldsymbol{\beta}}_P=\boldsymbol{\beta}+\boldsymbol{\Omega}_P^{-1}\left[\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)u_{it}\right]+O_p\left(N^{\delta-1}\right). \tag{3.16}$$

Using this result and noting that under Assumptions 2 and 4,

$$\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)u_{it}\rightarrow_p\mathbf{0},$$

we have the following proposition.

Proposition 3.1 *Consider the panel data model given by equations (2.1), (2.2) and (2.3) and suppose that Assumptions 1-5 hold. Then the pooled least square estimator defined by (3.1) is consistent for estimation of* β*, as long as* $\delta<1$.

Remark 3.2 *The bias of the pooled least squares estimator in the case of* $\delta=1$ *is given by*

$$p\lim_{N\to\infty}\left(\hat{\boldsymbol{\beta}}_P\right)=\boldsymbol{\beta}+\sigma_\eta^2\mathbf{Q}_P^{-1}\bar{\mathbf{g}},$$

where

$$\sigma_\eta^2=\lim_{N\to\infty}N^{-1}\sum_{i=1}^{N}\left(\eta_i-\bar{\eta}\right)^2,$$

and

$$\mathbf{Q}_P=\lim_{N\to\infty}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}E\left[\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)\left(\mathbf{x}_{it}-\bar{\mathbf{x}}\right)'\right].$$

For a derivation see Section 26.3 in Pesaran (2015). As a corollary it also follows that Hausman's (1978) mis-specification test that compares the pooled and FE estimators will only be consistent if $\delta=1$.

To derive the asymptotic distribution of $\hat{\boldsymbol{\beta}}_P$ we note that

$$\sqrt{N}\left(\hat{\boldsymbol{\beta}}_P - \boldsymbol{\beta}\right) = \boldsymbol{\Omega}_P^{-1}\left[\frac{1}{T\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it} - \bar{\mathbf{w}}\right)u_{it}\right] + O_p\left(N^{\delta-1/2}\right).$$

Also under Assumptions 2, 4 and 5, using standard results from panel data literature, we have (for a fixed T and as $N \to \infty$)

$$\frac{1}{T\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it} - \bar{\mathbf{w}}\right)u_{it} \to_d N\left(\mathbf{0},\sigma_u^2 T^{-1}\boldsymbol{\Omega}_P\right).$$

Hence, for a fixed T and as $N \to \infty$

$$\sqrt{N}\left(\hat{\boldsymbol{\beta}}_P - \boldsymbol{\beta}\right) \to_d N\left(\mathbf{0},\sigma_u^2 T^{-1}\boldsymbol{\Omega}_P^{-1}\right), \text{ if } \delta < 1/2. \tag{3.17}$$

3.2 The FE estimator

Consider now the FE estimator, $\hat{\boldsymbol{\beta}}_{FE}$, defined by (3.2). Then using (3.4) we obtain

$$\sqrt{N}\left(\hat{\boldsymbol{\beta}}_{FE} - \boldsymbol{\beta}\right) = \mathbf{Q}_{FE,N}^{-1}\left[\frac{1}{T\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it} - \bar{\mathbf{x}}_i\right)\left(u_{it} - \bar{u}_i\right)\right]. \tag{3.18}$$

Noting that $\mathbf{x}_{it} - \bar{\mathbf{x}}_i = \left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right) + \eta_i\left(\mathbf{g}_t - \bar{\mathbf{g}}\right)$, and $y_{it} - \bar{y}_i = \boldsymbol{\beta}'\left(\mathbf{x}_{it} - \bar{\mathbf{x}}_i\right) + \left(u_{it} - \bar{u}_i\right)$, we also have

$$\begin{aligned}
\mathbf{Q}_{FE,N} &= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it} - \bar{\mathbf{x}}_i\right)\left(\mathbf{x}_{it} - \bar{\mathbf{x}}_i\right)' \\
&= \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right)\left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right)' + \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right)\left(\mathbf{g}_t - \bar{\mathbf{g}}\right)' \\
&\quad + \frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left(\mathbf{g}_t - \bar{\mathbf{g}}\right)\left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right)' + \left(\frac{1}{N}\sum_{i=1}^{N}\eta_i^2\right)\left(\frac{1}{T}\sum_{t=1}^{T}\left(\mathbf{g}_t - \bar{\mathbf{g}}\right)\left(\mathbf{g}_t - \bar{\mathbf{g}}\right)'\right),
\end{aligned}$$

and

$$\begin{aligned}
\frac{1}{T\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{x}_{it} - \bar{\mathbf{x}}_i\right)\left(u_{it} - \bar{u}_i\right) &= \frac{1}{T\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\left[\left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right) + \eta_i\left(\mathbf{g}_t - \bar{\mathbf{g}}\right)\right]\left(u_{it} - \bar{u}_i\right) \\
&= \frac{1}{T\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(u_{it} - \bar{u}_i\right)\left(\mathbf{w}_{it} - \bar{\mathbf{w}}_i\right) \\
&\quad + \frac{1}{T\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left(u_{it} - \bar{u}_i\right)\left(\mathbf{g}_t - \bar{\mathbf{g}}\right).
\end{aligned}$$

Under Assumptions 1-4, using the above results and following the same line of reasoning as in Section 3.1 we have (for a fixed T and as $N \to \infty$)

$$\begin{aligned}\mathbf{Q}_{FE,N} &= \lim_{N\to\infty}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}E\left[\left(\mathbf{w}_{it}-\bar{\mathbf{w}}_i\right)\left(\mathbf{w}_{it}-\bar{\mathbf{w}}_i\right)'\right] \\ &\quad +\left(\frac{1}{N}\sum_{i=1}^{N}E\left(\eta_i^2\right)\right)\left(\frac{1}{T}\sum_{t=1}^{T}E\left[\left(\mathbf{g}_t-\bar{\mathbf{g}}\right)\left(\mathbf{g}_t-\bar{\mathbf{g}}\right)'\right]\right) \\ &= \mathbf{\Omega}_{FE}+O_p\left(N^{\delta-1}\right),\end{aligned}$$

where

$$\mathbf{\Omega}_{FE}=\lim_{N\to\infty}\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}E\left[\left(\mathbf{w}_{it}-\bar{\mathbf{w}}_i\right)\left(\mathbf{w}_{it}-\bar{\mathbf{w}}_i\right)'\right]. \tag{3.19}$$

Similarly, since η_i is distributed independently of u_{it} and $\mathbf{g}_t$, then

$$\begin{aligned}E\left|\frac{1}{\sqrt{N}T}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left(u_{it}-\bar{u}_i\right)\left(\mathbf{g}_t-\bar{\mathbf{g}}\right)\right| &\leq \frac{1}{\sqrt{N}T}\left|\sum_{i=1}^{N}\sum_{t=1}^{T}E\left|\eta_i\right|E\left|\left(u_{it}-\bar{u}_i\right)\left(\mathbf{g}_t-\bar{\mathbf{g}}\right)\right|\right| \\ &\leq \sup_i E\left|\left(u_{it}-\bar{u}_i\right)\left(\mathbf{g}_t-\bar{\mathbf{g}}\right)\right|\left(N^{-1/2}\sum_{i=1}^{N}E\left|\eta_i\right|\right).\end{aligned}$$

But $E\left|\left(u_{it}-\bar{u}_i\right)\left(\mathbf{g}_t-\bar{\mathbf{g}}\right)\right| \leq \left[E\left(u_{it}-\bar{u}_i\right)^2\right]^{1/2}\left[E\left\|\mathbf{g}_t-\bar{\mathbf{g}}\right\|^2\right]^{1/2} < K$, and by Assumptions 1 and 2, it follows that

$$E\left|\frac{1}{\sqrt{N}T}\sum_{i=1}^{N}\sum_{t=1}^{T}\eta_i\left(u_{it}-\bar{u}_i\right)\left(\mathbf{g}_t-\bar{\mathbf{g}}\right)\right| \leq O\left(N^{\delta-1/2}\right).$$

Finally, under Assumptions 2-4, using standard results from panel data literature we have

$$\frac{1}{T\sqrt{N}}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(u_{it}-\bar{u}_i\right)\left(\mathbf{w}_{it}-\bar{\mathbf{w}}_i\right)\to_d N\left(\mathbf{0},\sigma_u^2T^{-1}\mathbf{\Omega}_{FE}\right),$$

where $\mathbf{\Omega}_{FE}$ is already defined by (3.19). Therefore, for a fixed T and as $N \to \infty$, we have

$$\sqrt{N}\left(\hat{\boldsymbol{\beta}}_{FE}-\boldsymbol{\beta}\right)\to_d N\left(\mathbf{0},\sigma_u^2T^{-1}\mathbf{\Omega}_{FE}^{-1}\right),\ \text{for } \delta<1/2. \tag{3.20}$$

Using (3.17) and the above result now yields the following proposition.

Proposition 3.3 *Suppose that the exponent coefficient, δ, defined by Assumption 1, is less than $1/2$, and Assumptions 1-5 hold. Then as $N \to \infty$*

$$\sqrt{NT}\left(\hat{\boldsymbol{\beta}}_P-\boldsymbol{\beta}\right)\to_d N\left(\mathbf{0},\sigma_u^2\mathbf{\Omega}_P^{-1}\right),$$

and

$$\sqrt{NT}\left(\hat{\boldsymbol{\beta}}_{FE}-\boldsymbol{\beta}\right)\rightarrow_d N\left(\mathbf{0},\sigma_u^2\boldsymbol{\Omega}_{FE}^{-1}\right).$$

Furthermore, $\hat{\boldsymbol{\beta}}_P$ is asymptotically more efficient than $\hat{\boldsymbol{\beta}}_{FE}$, as long as $\delta<1/2$.

To establish the relative asymptotic efficiency of $\hat{\boldsymbol{\beta}}_P$ we first note that

$$\left[AsyVar\left(\sqrt{TN}\hat{\boldsymbol{\beta}}_P\right)\right]^{-1}-\left[AsyVar\left(\sqrt{TN}\hat{\boldsymbol{\beta}}_{FE}\right)\right]^{-1}=\sigma_u^{-2}\left[\boldsymbol{\Omega}_P-\boldsymbol{\Omega}_{FE}\right]. \tag{3.21}$$

Also, we note that since

$$\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)\left(\mathbf{w}_{it}-\bar{\mathbf{w}}\right)'=\frac{1}{NT}\sum_{i=1}^{N}\sum_{t=1}^{T}\left(\mathbf{w}_{it}-\bar{\mathbf{w}}_i\right)\left(\mathbf{w}_{it}-\bar{\mathbf{w}}_i\right)'+\frac{1}{N}\sum_{i=1}^{N}\left(\bar{\mathbf{w}}_i-\bar{\mathbf{w}}\right)\left(\bar{\mathbf{w}}_i-\bar{\mathbf{w}}\right)',$$

then

$$\boldsymbol{\Omega}_P=\boldsymbol{\Omega}_{FE}+\boldsymbol{\Omega}_C, \tag{3.22}$$

where

$$\boldsymbol{\Omega}_C=\lim_{N\rightarrow\infty}\frac{1}{N}\sum_{i=1}^{N}E\left[\left(\bar{\mathbf{w}}_i-\bar{\mathbf{w}}\right)\left(\bar{\mathbf{w}}_i-\bar{\mathbf{w}}\right)'\right],$$

and by Assumption 5, $\boldsymbol{\Omega}_C$ is a positive definite matrix. Using (3.22) in (3.21) we have

$$\left[AsyVar\left(\sqrt{TN}\hat{\boldsymbol{\beta}}_P\right)\right]^{-1}-\left[AsyVar\left(\sqrt{TN}\hat{\boldsymbol{\beta}}_{FE}\right)\right]^{-1}=\sigma_u^{-2}\boldsymbol{\Omega}_C>0,$$

and hence

$$AsyVar\left(\sqrt{TN}\hat{\boldsymbol{\beta}}_{FE}\right)>AsyVar\left(\sqrt{TN}\hat{\boldsymbol{\beta}}_P\right).$$

Consistent estimators of $\boldsymbol{\Omega}_P$ and $\boldsymbol{\Omega}_{FE}$ are given by $\mathbf{Q}_{N,p}$ and $\mathbf{Q}_{N,FE}$, respectively.

3.3 Random effects and PLS estimators

Finally, it is easily seen that the random effects (RE) and the pooled least squares estimators of $\boldsymbol{\beta}$ are asymptotically equivalent. The RE estimator is given by (see, for example, Chapter 26 in Pesaran (2015)).

$$\hat{\boldsymbol{\beta}}_{RE}=\left(\mathbf{Q}_{FE,N}+\psi\mathbf{Q}_{C,N}\right)^{-1}\left(\mathbf{q}_{FE,N}+\psi\mathbf{q}_{C,N}\right),$$

where $\mathbf{Q}_{FE,N}$ and $\mathbf{q}_{FE,N}$, are defined by (3.4),

$$\mathbf{Q}_{C,N}=N^{-1}\sum_{i=1}^{N}\left(\bar{\mathbf{x}}_i-\bar{\mathbf{x}}\right)\left(\bar{\mathbf{x}}_i-\bar{\mathbf{x}}\right)',\ \mathbf{q}_{C,N}=N^{-1}\sum_{i=1}^{N}\left(\bar{\mathbf{x}}_i-\bar{\mathbf{x}}\right)\left(\bar{y}_i-\bar{y}\right),$$

and

$$\psi = \frac{\sigma_u^2}{T\sigma_\eta^2 + \sigma_u^2}. \tag{3.23}$$

However, under (2.5), $\sigma_\eta^2 = O\left(N^{\delta-1}\right)$, and for a fixed T, we have $\psi = 1 + O\left(N^{\delta-1}\right)$, and using (3.3) and (3.4) we obtain[3]

$$\begin{aligned}
\mathbf{Q}_{FE,N} + \psi \mathbf{Q}_{C,N} &= (\psi - 1)\,\mathbf{Q}_{C,N} + \mathbf{Q}_{P,N}, \\
\mathbf{q}_{FE,N} + \psi \mathbf{q}_{C,N} &= (\psi - 1)\,\mathbf{q}_{C,N} + \mathbf{q}_{P,N}
\end{aligned}$$

Hence (for a fixed T)

$$\sqrt{N}\left(\hat{\boldsymbol{\beta}}_{RE} - \hat{\boldsymbol{\beta}}_P\right) \to_p \mathbf{0}, \text{ as } N \to \infty, \text{ if } \delta < 1,$$

which establishes the asymptotic equivalence of the random effects and pooled least squares estimators as $N \to \infty$, for $\delta < 1$ and a fixed T.

4 Monte Carlo simulations

To compare the performance of the FE and pooled least square estimators when T is fixed as well as $\sum_{i=1}^{N} |\eta_i| = O(N^\delta)$, we conduct several Monte Carlo simulations. The data generating process (DGP) is given by

$$y_{it} = 1 + \eta_i + x_{1,it}\beta_1 + x_{2,it}\beta_2 + u_{it}, \quad i = 1, 2, \ldots, N; t = 1, 2, \ldots, T,$$

with $\beta_1 = 1$ and $\beta_2 = 2$, $N = 100, 500, 1000, 2000$ and $T = 3, 5, 10$. We assume $u_{it} \sim iidN(0, \sigma_i^2)$, with $\sigma_i^2 \sim IID\chi^2(2)$, $\eta_i \sim iidN(0, 2)$ for $i = 1, 2, \ldots, [N^\delta]$ and $\eta_i = 0$, for $i = [N^\delta] + 1, [N^\delta] + 2, \ldots, N$. We let δ to take the following values 1, 0.95, 0.75, 0.5, 0.4, 0.25 and 0. The elements of $\mathbf{x}_{it} = (x_{1,it}, x_{2,it})'$, are generated as

$$x_{j,it} = 1 + \alpha_{j,i} + g_{j,t}\eta_i + w_{j,it}, \text{ for } j = 1, 2,$$

with $\alpha_{j,i} \sim iidN(0, 1)$, $g_{j,t} \sim IIDU[0.1, 0.9]$ and $w_{j,it}$ generated by

$$w_{j,it} = \rho_{j,i} w_{k,it-1} + \varepsilon_{j,it}, \text{ for } j = 1, 2,$$

where $w_{j,i0} = 0$, $\rho_{j,i} \sim IIDU[0.05, 0.95]$, $\varepsilon_{j,i0} = 0$, and $\varepsilon_{j,it} \sim iidN(0, \sigma_{j,\varepsilon i}^2)$ with $\sigma_{j,\varepsilon i}^2 \sim IID\chi^2(2)$ for $j = 1, 2$. For the DGP described above, the first 50 observations are discarded, and the number of replications is set to 1000.

[3]Note tha $\mathbf{Q}_{P,N} = \mathbf{Q}_{FE,N} + \mathbf{Q}_{C,N}$, and $\mathbf{q}_{P,N} = \mathbf{q}_{FE,N} + \mathbf{q}_{C,N}$.

We compute the PLS and FE estimates and the associated bias, absolute bias and RMSE. The results are summarized in Tables 1-6 which are in line with the paper's theoretical findings. As to be expected the RMSE of PLS estimator is much smaller than that of the FE estimator for values of $\delta < 1/2$. However, the PLS estimator starts to show significant bias as δ is allowed to increase beyond the 1/2 threshold, and the RMSE of PLS estimator is much larger than the FE estimator.

5 Conclusion

This paper introduces a new approach to the analysis of the relative efficiency of fixed effects and pooled least square estimators for standard panel data models. We show that the potential benefit from pooling is directly related to the degree with which the heterogeneity of individual effects is pervasive across the individual units in the panel. We characterize this feature by an exponent, δ, and show that the pooled least square estimator is consistent for values of $\delta < 1$. Our specification allows for non-zero correlations between the individual effects and the regressors which renders the pooled least squares and random effects inconsistent if $\delta = 1$. We also derive the asymptotic distributions of the pooled least squares, FE and RE estimators for different values of δ and establish the relative efficiency of the pooled least squares estimator over the FE estimator when $\delta < 1/2$. These results are supported by small sample evidence from Monte Carlo experiments.

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Table 1: Simulation results β_1 when $T = 3$

		δ													
		0		0.25		0.4		0.5		0.75		0.95		1	
N		FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS
	Estim	1.0008	0.9994	0.9999	1.0065	0.9973	1.0312	0.9988	1.0358	0.9952	1.0609	0.9993	1.1271	1.0003	1.1564
100	Bias	0.0008	-0.0006	-0.0001	0.0065	-0.0027	0.0312	-0.0012	0.0358	-0.0048	0.0609	-0.0007	0.1271	0.0003	0.1564
	ABias	0.0737	0.0293	0.0745	0.0288	0.0733	0.0375	0.0714	0.0415	0.0654	0.0623	0.0653	0.1271	0.0626	0.1564
	RMSE	0.0936	0.0366	0.0933	0.0363	0.0922	0.0458	0.0899	0.0500	0.0821	0.0705	0.0821	0.1326	0.0796	0.1602
	Estim	0.9996	0.9997	1.0013	1.0034	1.0006	1.0080	0.9997	1.0134	1.0007	1.0561	1.0012	1.1160	0.9994	1.1357
500	Bias	-0.0004	-0.0003	0.0013	0.0034	0.0006	0.0080	-0.0003	0.0134	0.0007	0.0561	0.0012	0.1160	-0.0006	0.1357
	ABias	0.0318	0.0133	0.0322	0.0133	0.0342	0.0145	0.0341	0.0176	0.0300	0.0561	0.0299	0.1160	0.0269	0.1357
	RMSE	0.0395	0.0164	0.0402	0.0168	0.0427	0.0185	0.0423	0.0215	0.0379	0.0582	0.0375	0.1171	0.0339	0.1365
	Estim	0.9987	1.0000	0.9987	1.0033	1.0013	1.0067	0.9991	1.0088	1.0002	1.0506	0.9994	1.1234	1.0003	1.1423
1000	Bias	-0.0013	0.0000	-0.0013	0.0033	0.0013	0.0067	-0.0009	0.0088	0.0002	0.0506	-0.0006	0.1234	0.0003	0.1423
	ABias	0.0255	0.0094	0.0245	0.0095	0.0252	0.0107	0.0242	0.0118	0.0228	0.0506	0.0213	0.1234	0.0200	0.1423
	RMSE	0.0316	0.0118	0.0308	0.0120	0.0315	0.0135	0.0307	0.0145	0.0284	0.0519	0.0270	0.1239	0.0251	0.1426
	Estim	0.9999	1.0004	0.9994	1.0004	1.0003	1.0052	1.0000	1.0082	0.9996	1.0424	0.9999	1.1217	1.0000	1.1404
2000	Bias	-0.0001	0.0004	-0.0006	0.0004	0.0003	0.0052	0.0000	0.0082	-0.0004	0.0424	-0.0001	0.1217	0.0000	0.1404
	ABias	0.0168	0.0068	0.0171	0.0065	0.0172	0.0078	0.0171	0.0095	0.0163	0.0424	0.0144	0.1217	0.0141	0.1404
	RMSE	0.0210	0.0084	0.0213	0.0082	0.0216	0.0097	0.0216	0.0117	0.0203	0.0432	0.0180	0.1219	0.0175	0.1406

Notes: FE and PLS refer to fixed effects and pooled least squares estimates, respectively. ABias refers to absolute bias for estimators.

Table 2: Simulation results of β_2 when $T = 3$

		δ													
		0		0.25		0.4		0.5		0.75		0.95		1	
N		FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS
	Estim	2.0029	1.9990	2.0010	2.0062	2.0033	2.0288	2.0009	2.0252	2.0075	2.0725	2.0042	2.1334	2.0042	2.1290
100	Bias	0.0029	-0.0010	0.0010	0.0062	0.0033	0.0288	0.0009	0.0252	0.0075	0.0725	0.0042	0.1334	0.0042	0.1290
	ABias	0.0684	0.0284	0.0741	0.0306	0.0752	0.0385	0.0751	0.0364	0.0631	0.0731	0.0664	0.1334	0.0690	0.1290
	RMSE	0.0854	0.0355	0.0934	0.0385	0.0942	0.0476	0.0950	0.0445	0.0799	0.0801	0.0828	0.1384	0.0875	0.1337
	Estim	1.9988	1.9993	1.9993	2.0048	1.9994	2.0099	1.9996	2.0142	2.0003	2.0598	2.0005	2.1234	2.0000	2.1438
500	Bias	-0.0012	-0.0007	-0.0007	0.0048	-0.0006	0.0099	-0.0004	0.0142	0.0003	0.0598	0.0005	0.1234	0.0000	0.1438
	ABias	0.0347	0.0126	0.0325	0.0134	0.0357	0.0151	0.0339	0.0175	0.0303	0.0598	0.0302	0.1234	0.0282	0.1438
	RMSE	0.0440	0.0160	0.0411	0.0169	0.0448	0.0188	0.0428	0.0215	0.0376	0.0616	0.0375	0.1244	0.0357	0.1446
	Estim	2.0000	1.9991	1.9995	2.0028	2.0002	2.0068	1.9997	2.0116	1.9996	2.0523	2.0004	2.1205	2.0003	2.1392
1000	Bias	0.0000	-0.0009	-0.0005	0.0028	0.0002	0.0068	-0.0003	0.0116	-0.0004	0.0523	0.0004	0.1205	0.0003	0.1392
	ABias	0.0243	0.0099	0.0241	0.0098	0.0235	0.0106	0.0243	0.0137	0.0236	0.0523	0.0206	0.1205	0.0196	0.1392
	RMSE	0.0306	0.0125	0.0307	0.0123	0.0292	0.0133	0.0304	0.0168	0.0294	0.0537	0.0259	0.1210	0.0247	0.1396
	Estim	1.9981	1.9999	2.0001	2.0010	1.9992	2.0023	1.9996	2.0075	2.0010	2.0477	2.0009	2.1190	2.0007	2.1387
2000	Bias	-0.0019	-0.0001	0.0001	0.0010	-0.0008	0.0023	-0.0004	0.0075	0.0010	0.0477	0.0009	0.1190	0.0007	0.1387
	ABias	0.0172	0.0065	0.0172	0.0064	0.0159	0.0067	0.0174	0.0089	0.0158	0.0477	0.0139	0.1190	0.0139	0.1387
	RMSE	0.0216	0.0082	0.0215	0.0081	0.0200	0.0085	0.0220	0.0109	0.0195	0.0484	0.0172	0.1193	0.0174	0.1389

Notes: FE and PLS refer to fixed effects and pooled least squares estimates, respectively. ABias refers to absolute bias for estimators.

Table 3: Simulation results β_1 when $T = 5$

		δ													
		0		0.25		0.4		0.5		0.75		0.95		1	
N		FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS
100	Estim	0.9956	0.9979	0.9977	1.0081	0.9984	1.0370	0.9975	1.0430	1.0010	1.0681	0.9984	1.1053	1.0012	1.1232
	Bias	-0.0044	-0.0021	-0.0023	0.0081	-0.0016	0.0370	-0.0025	0.0430	0.0010	0.0681	-0.0016	0.1053	0.0012	0.1232
	ABias	0.0460	0.0218	0.0481	0.0232	0.0440	0.0382	0.0440	0.0436	0.0380	0.0682	0.0414	0.1053	0.0415	0.1232
	RMSE	0.0594	0.0273	0.0597	0.0288	0.0551	0.0442	0.0552	0.0494	0.0482	0.0726	0.0513	0.1084	0.0521	0.1258
500	Estim	1.0003	1.0003	1.0005	1.0060	0.9997	1.0100	1.0019	1.0193	1.0009	1.0630	1.0007	1.0998	0.9999	1.1082
	Bias	0.0003	0.0003	0.0005	0.0060	-0.0003	0.0100	0.0019	0.0193	0.0009	0.0630	0.0007	0.0998	-0.0001	0.1082
	ABias	0.0207	0.0104	0.0207	0.0109	0.0216	0.0130	0.0226	0.0202	0.0190	0.0630	0.0179	0.0998	0.0178	0.1082
	RMSE	0.0260	0.0129	0.0263	0.0136	0.0271	0.0158	0.0280	0.0232	0.0238	0.0640	0.0226	0.1003	0.0224	0.1087
1000	Estim	0.9986	0.9999	1.0009	1.0052	1.0011	1.0085	1.0012	1.0136	0.9997	1.0586	1.0005	1.1045	0.9993	1.1127
	Bias	-0.0014	-0.0001	0.0009	0.0052	0.0011	0.0085	0.0012	0.0136	-0.0003	0.0586	0.0005	0.1045	-0.0007	0.1127
	ABias	0.0162	0.0071	0.0165	0.0084	0.0157	0.0102	0.0157	0.0140	0.0144	0.0586	0.0130	0.1045	0.0129	0.1127
	RMSE	0.0204	0.0090	0.0206	0.0105	0.0199	0.0124	0.0197	0.0162	0.0180	0.0592	0.0163	0.1048	0.0160	0.1130
2000	Estim	1.0003	1.0003	0.9991	1.0006	0.9993	1.0065	0.9997	1.0110	1.0001	1.0502	1.0001	1.1033	0.9998	1.1118
	Bias	0.0003	0.0003	-0.0009	0.0006	-0.0007	0.0065	-0.0003	0.0110	0.0001	0.0502	0.0001	0.1033	-0.0002	0.1118
	ABias	0.0117	0.0053	0.0113	0.0052	0.0116	0.0076	0.0109	0.0112	0.0102	0.0502	0.0088	0.1033	0.0084	0.1118
	RMSE	0.0146	0.0066	0.0142	0.0064	0.0144	0.0091	0.0138	0.0126	0.0128	0.0506	0.0109	0.1035	0.0106	0.1120

Notes: FE and PLS refer to fixed effects and pooled least squares estimates, respectively. ABias refers to absolute bias for estimators.

Table 4: Simulation results of β_2 when $T = 5$

		δ													
		0		0.25		0.4		0.5		0.75		0.95		1	
N		FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS
100	Estim	2.0002	1.9996	1.9991	2.0080	1.9997	2.0355	1.9995	2.0333	2.0007	2.0752	2.0007	2.1081	1.9994	2.1010
	Bias	0.0002	-0.0004	-0.0009	0.0080	-0.0003	0.0355	-0.0005	0.0333	0.0007	0.0752	0.0007	0.1081	-0.0006	0.1010
	ABias	0.0469	0.0222	0.0458	0.0253	0.0449	0.0387	0.0452	0.0360	0.0396	0.0752	0.0433	0.1081	0.0425	0.1010
	RMSE	0.0595	0.0278	0.0573	0.0314	0.0559	0.0456	0.0567	0.0427	0.0500	0.0787	0.0546	0.1116	0.0527	0.1044
500	Estim	2.0000	2.0000	1.9987	2.0063	2.0001	2.0124	1.9998	2.0209	1.9994	2.0655	1.9998	2.1052	2.0010	2.1152
	Bias	0.0000	0.0000	-0.0013	0.0063	0.0001	0.0124	-0.0002	0.0209	-0.0006	0.0655	-0.0002	0.1052	0.0010	0.1152
	ABias	0.0229	0.0097	0.0211	0.0109	0.0226	0.0141	0.0223	0.0213	0.0189	0.0655	0.0180	0.1052	0.0173	0.1152
	RMSE	0.0285	0.0121	0.0267	0.0136	0.0286	0.0172	0.0282	00243	0.0239	0.0655	0.0227	0.1057	0.0210	0.1157
1000	Estim	1.9994	1.9995	2.0002	2.0042	2.0000	2.0090	2.0002	2.0161	2.0006	2.0601	1.9998	2.1019	2.0005	2.1109
	Bias	-0.0006	-0.0005	0.0002	0.0042	0.0000	0.0090	0.0002	0.0161	0.0006	0.0601	-0.0002	0.1019	0.0005	0.1109
	ABias	0.0161	0.0075	0.0155	0.0078	0.0159	0.0107	0.0165	0.0163	0.0150	0.0601	0.0130	0.1019	0.0123	0.1109
	RMSE	0.0203	0.0094	0.0197	0.0098	0.0201	0.0130	0.0206	0.0184	0.0185	0.0606	0.0162	0.1022	0.0155	0.1112
2000	Estim	2.0000	1.9999	2.0005	2.0015	1.9996	2.0041	1.9994	2.0106	2.0001	2.0548	2.0002	2.1017	1.9999	2.1103
	Bias	0.0000	-0.0001	0.0005	0.0015	-0.0004	0.0041	-0.0006	0.0106	0.0001	0.0548	0.0002	0.1017	-0.0001	0.1103
	ABias	0.0113	0.0050	0.0113	0.005	0.0109	0.0062	0.0106	0.0109	0.0102	0.0548	0.0093	0.1017	0.0083	0.1103
	RMSE	0.0141	0.0063	0.0141	0.0065	0.0137	0.0077	0.0136	0.0124	0.0129	0.0551	0.0115	0.1019	0.0105	0.1104

Notes: FE and PLS refer to fixed effects and pooled least squares estimates, respectively. ABias refers to absolute bias for estimators.

Table 5: Simulation results β_1 when $T = 10$

		δ													
		0		0.25		0.4		0.5		0.75		0.95		1	
N		FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS
	Estim	1.0003	1.0003	0.9982	1.0102	0.9978	1.0382	0.9985	1.0434	0.9984	1.0541	1.0003	1.0652	1.0006	1.0755
100	Bias	0.0003	0.0003	-0.0018	0.0102	-0.0022	0.0382	-0.0015	0.0434	-0.0016	0.0541	0.0003	0.0652	0.0006	0.0755
	ABias	0.0288	0.0146	0.0288	0.0168	0.0246	0.0383	0.0250	0.0434	0.0227	0.0541	0.0229	0.0653	0.0245	0.0755
	RMSE	0.0359	0.0183	0.0363	0.0210	0.0309	0.0412	0.0312	0.0459	0.0285	0.0565	0.0287	0.0676	0.0310	0.0773
	Estim	1.0005	1.0004	1.0007	1.0106	1.0007	1.0153	0.9998	1.0269	0.9998	1.0532	1.0002	1.0637	0.9996	1.0653
500	Bias	0.0005	0.0004	0.0007	0.0106	0.0007	0.0153	-0.0002	0.0269	-0.0002	0.0532	0.0002	0.0637	-0.0004	0.0653
	ABias	0.0135	0.0071	0.0132	0.0114	0.0136	0.0156	0.0133	0.0270	0.0108	0.0532	0.0100	0.0637	0.0098	0.0653
	RMSE	0.0168	0.0090	0.0161	0.0135	0.0171	0.0176	0.0166	0.0285	0.0134	0.0537	0.0126	0.0641	0.0123	0.0656
	Estim	1.0008	1.0001	0.9996	1.0073	1.0000	1.0128	0.9997	1.0190	1.0002	1.0517	1.0004	1.0663	1.0002	1.0680
1000	Bias	0.0008	0.0001	-0.0004	0.0073	0.0000	0.0128	-0.0003	0.0190	0.0002	0.0517	0.0004	0.0663	0.0002	0.0680
	ABias	.00099	0.0050	0.0095	0.0080	0.0096	0.0129	0.0093	0.0190	0.0081	0.0517	0.0074	0.0663	0.0073	0.0680
	RMSE	0.0123	0.0063	0.0120	0.0096	0.0120	0.0141	0.0117	0.0199	0.0101	0.0520	0.0093	0.0665	0.0091	0.0682
	Estim	1.0001	1.0002	0.9997	1.0022	0.9999	1.0104	1.0004	1.0169	0.9997	1.0473	0.9999	1.0659	1.0001	1.0677
2000	Bias	0.0001	0.0002	-0.0003	0.0022	-0.0001	0.0104	0.0004	0.0169	-0.0003	0.0473	-0.0001	0.0659	0.0001	0.0677
	ABias	0.0070	0.0036	0.0067	0.0039	0.0069	0.0104	0.0067	0.0169	0.0058	0.0473	0.0049	0.0659	0.0050	0.0677
	RMSE	0.0088	0.0045	0.0085	0.0049	0.0086	0.0113	0.0084	0.0175	0.0073	0.0474	0.0062	0.0660	0.0063	0.0678

Notes: FE and PLS refer to fixed effects and pooled least squares estimates, respectively. ABias refers to absolute bias for estimators.

Table 6: Simulation results of β_2 when $T = 10$

		δ													
		0		0.25		0.4		0.5		0.75		0.95		1	
N		FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS	FE	PLS
	Estim	2.0013	2.0006	2.0016	2.0102	2.0017	2.0380	2.0008	2.0354	2.0003	2.0605	1.9997	2.0670	1.9992	2.0588
100	Bias	0.0013	0.0006	0.0016	0.0102	0.0017	0.0380	0.0008	0.0354	0.0003	0.0605	-0.0003	0.0670	-0.0008	0.0588
	ABias	0.0280	0.0152	0.0286	0.0183	0.0256	0.0382	0.0251	0.0356	0.0221	0.0605	0.0235	0.0670	0.0258	0.0588
	RMSE	0.0352	0.0193	0.0360	0.0226	0.0323	0.0417	0.0318	0.0392	0.0271	0.0623	0.0298	0.0694	0.0320	0.0613
	Estim	1.9999	2.0000	2.0001	2.0111	2.0005	2.0167	2.0007	2.0276	2.0000	2.0550	1.9998	2.0672	2.0002	2.0692
500	Bias	-0.0001	0.0000	0.0001	0.0111	0.0005	0.0167	0.0007	0.0276	0.0000	0.0550	-0.0002	0.0672	0.0002	0.0692
	ABias	0.0138	0.0071	0.0135	0.0120	0.0138	0.0169	0.0138	0.0276	0.0105	0.0550	0.0098	0.0672	0.0100	0.0692
	RMSE	0.0175	0.0088	0.0167	0.0139	0.0175	0.0188	0.0170	0.0289	0.0131	0.0555	0.0123	0.0675	0.0124	0.0695
	Estim	1.9997	1.9998	1.9998	2.0070	2.0001	2.0134	2.0001	2.0221	1.9996	2.0529	2.0000	2.0646	2.0000	2.0665
1000	Bias	-0.0003	-0.0002	-0.0002	0.0070	0.0001	0.0134	0.0001	0.0221	-0.0004	0.0529	0.0000	0.0646	0.0000	0.0665
	ABias	0.0099	0.0053	0.0101	0.0078	0.0097	0.0135	0.0093	0.0221	0.0083	0.0529	0.0076	0.0646	0.0071	0.0665
	RMSE	0.0126	0.0066	0.0126	0.0094	0.0121	0.0148	0.0115	0.0229	0.0102	0.0531	0.0095	0.0648	0.0089	0.0667
	Estim	1.9997	2.0000	2.0002	2.0030	1.9996	2.0079	2.0001	2.0164	2.0003	2.0504	2.0001	2.0647	1.9998	2.0663
2000	Bias	-0.0003	0.0000	0.0002	0.0030	-0.0004	0.0079	0.0001	0.0164	0.0003	0.0504	0.0001	0.0647	-0.0002	0.0663
	ABias	0.0069	0.0035	0.0069	0.0045	0.0066	0.0080	0.0067	0.0164	0.0057	0.0504	0.0049	0.0647	0.0050	0.0663
	RMSE	0.0086	0.0044	0.0086	0.0055	0.0083	0.0090	0.0084	0.0170	0.0071	0.0506	0.0061	0.0648	0.0063	0.0664

Notes: FE and PLS refer to fixed effects and pooled least squares estimates, respectively. ABias refers to absolute bias for estimators.