An Augmented Anderson-Hsiao Estimator for Dynamic Short-T Panels*

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Abstract

This paper introduces the idea of self-instrumenting endogenous regressors in settings when the correlation between these regressors and the errors can be derived and used to bias-correct the moment conditions. The resulting bias-corrected moment conditions are less likely to be subject to the weak instrument problem and can be used on their own or in conjunction with other available moment conditions to obtain more efficient estimators. This approach can be applied to estimation of a variety of models such as spatial and dynamic panel data models. This paper focuses on the latter, and proposes a new estimator for short T dynamic panels by augmenting Anderson and Hsiao (AAH) estimator with bias-corrected quadratic moment conditions in first differences which substantially improve the small sample performance of the AH estimator without sacrificing on the generality of its underlying assumptions regarding the fixed effects, initial values, and heteroskedasticity of error terms. Using Monte Carlo experiments it is shown that AAH estimator represents a substantial improvement over the AH estimator and more importantly it performs well even when compared to Arellano and Bond and Blundell and Bond (BB) estimators that are based on more restrictive assumptions, and continues to have satisfactory performance in cases where the standard GMM estimators are inconsistent. Finally, to decide between AAH and BB estimators we also propose a Hausman type test which is shown to work well when T is small and n sufficiently large.

Keywords: Short-T Dynamic Panels, GMM, Bias-Corrected Moment Conditions, BMM, Self-Instrumenting, Nonlinear Moment Conditions, Panel VARs, Hausman Test, Monte Carlo Evidence.

JEL Classification: C12, C13, C23

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1 Introduction

Analysis of linear dynamic panel data models where the time dimension (T) is short relative to the cross section dimension (n), plays an important role in applied research. The estimation of such panels is carried out predominantly by the application of the Generalized Method of Moments (GMM) after first-differencing.[1] This approach utilizes instruments that are uncorrelated with the errors but are potentially correlated with the target variables (the included regressors). A number of well-known GMM estimation methods have been advanced in the literature.[2] The GMM methods apply to autoregressive (AR) panels as well as to AR panels augmented with strictly or weakly exogenous regressors and are developed under fairly general moment conditions, which is important for applied work. However, the GMM methods are subject to a number of well-known drawbacks. Anderson and Hsiao (1981 and 1982)'s estimator of AR(1) panels has poor small sample performance due to weak correlations between the regressors and the instruments when the autoregressive coefficient is moderately large (see, e.g. Arellano, 1989). Subsequently proposed GMM estimators have better small sample performance but at the cost of more restricted assumptions. The popular first-difference GMM estimator due to Arellano and Bond (1991) uses lagged levels rather than first-differences as instruments, and the system GMM approach by Blundell and Bond (1998) considers additional moment conditions that help identification but impose stronger requirements on the initialization of the dynamic processes. In particular, as discussed in Section 2, the system GMM approach does not allow for the initial values to differ systematically from the long-run means.

This paper proposes a novel idea of self-instrumenting the endogenous regressors in settings where the correlation between the regressors and the errors can be derived instead of searching for instruments that are uncorrelated with the error terms. The resulting 'bias-corrected' moment conditions are less likely to be subject to the weak instrument problem and can be used on their own and/or augmented with other available moment conditions to obtain more efficient estimators. Our idea differs from the wide variety of the bias-corrected estimation methods in the literature, which correct a first-stage estimator for small-T bias and tend to be applicable under more restrictive

[1] Other approaches in the literature include the likelihood-based methods (Hsiao et al., 2002, Lancaster, 2002, Moral-Benito, 2013, Hayakawa and Pesaran, 2015, and Dhaene and Jochmans, 2016), X-differencing method (Han et al., 2014), factor-analytical method (Bai, 2013), and bias-correction methods mentioned below.

[2] Anderson and Hsiao (1981 and 1982), Holtz-Eakin et al. (1988), Arellano and Bond (1991), Ahn and Schmidt (1995), Arellano and Bover (1995), Blundell and Bond (1998), and Hayakawa (2012), among others. A recent contribution by Breitung, Hayakawa, and Kripfganz (2019) is also an interesting addition to the GMM literature. Their bias-corrected methods of moments estimator requires homoskedastic errors over time.

assumptions.[3] Instead of correcting the bias of standard GMM estimators, we consider correcting the 'bias' of the moment conditions *before* estimation. The idea of self-instrumenting has wide-ranging applications for robust estimation and inference in settings where the correlation between the regressors and the errors can be derived. This paper focuses on dynamic panels. Another application is the estimation of spatial panel data models which is pursued in Pesaran and Yang (2021).

By self-instrumenting lagged differences, we develop a simple bias-corrected methods of moment (BMM) estimator under general conditions on initialization of the underlying dynamics, individual effects, with (possibly) heteroskedastic error variances over time as well as cross-sectionally. The resultant moment conditions turn out to be quadratic, and only reduce to linear moment conditions if the underlying AR processes are stationary. In this special case we show the BMM estimator to be identical to the first difference least square estimator proposed by Han and Phillips (2010). We establish consistency and asymptotic normality of the BMM estimator under general conditions and discuss its relation to a variety of GMM estimators proposed in the literature. These results help illustrate the important role played by the initialization of the AR processes in the case of short T panels.

We also consider augmenting the bias-corrected moment conditions with other moment conditions available in the literature, and for maximum robustness to assumptions regarding individual effects and initial values we focus on Anderson and Hsiao type moment conditions obtained from using appropriately lagged first differences as instruments. Accordingly, we propose a new augmented Anderson and Hsiao (AAH) estimator which substantially improve the small sample performance of the AH estimator without sacrificing on the generality of its underlying assumptions. The AAH estimator holds under less restrictive conditions imposed by other prevalent GMM estimators proposed by Arellano and Bond (AB), and Blundell and Bond (BB) in the literature, and is more generally applicable. To test the validity of the BB moment conditions, we propose a Hausman type test based on the difference between BB and AAH estimators, not previously considered in

[3] See, for example, methods based on exact analytical bias formula or its approximation, Bruno (2005), Bun (2003), Bun and Carree (2005, 2006), Bun and Kiviet (2003), Hahn and Kuersteiner (2002), Hahn and Moon (2006), Juodis (2013), and Kiviet (1995, 1999); simulation-based bias-correction methods by Everaert and Ponzi (2007), and Phillips and Sul (2003, 2007); the jackknife bias corrections by Dhaene and Jochmans (2015), and Chudik, Pesaran, and Yang (2018); or the recursive mean adjustment correction procedures, Choi et al. (2010)). Most of these bias-correction techniques do not apply to short-T type panels where the error variances are heteroskedastic (over i and t), with the exception of Juodis (2013), and the simulation-based bias-correction method of Everaert and Ponzi (2007). A comparative analysis of GMM estimators considered in this paper and bias correction estimators is a welcome addition to the literature but lies beyond the scope of the present paper.

the literature.

Monte Carlo (MC) experiments document AAH's good small sample performance in comparison with a number of GMM estimators. Perhaps not surprisingly the AAH estimator represents a substantial improvement over the AH estimator across all designs considered. When compared to AB and BB estimators, the AAH is less efficient in designs that satisfy the more restrictive assumption that underlie BB estimators, but continues to perform well uniformly across the various designs including in cases where the system-GMM type estimators are not consistent. The robustness of the AAH estimator is an important advantage since in practice it is not known if the additional restrictions of the AB and BB estimators are met.

The remainder of this paper is organized as follows. Section 2 sets up the baseline panel AR(1) model and discusses AH and subsequent GMM moment conditions. Section 3 introduces the main idea and presents a simple BMM estimator. Section 4 introduces the AAH estimator and discusses the related literature, in particular Ahn and Schmidt (1995, 1997). Section 5 discusses extensions of AAH estimator to ARX and VAR short-T panel data models. Section 6 discusses the problem of moment proliferation and adopts the One Covariate at the time Multiple Testing approach by Chudik, Kapetanios, and Pesaran (2018) for selection of relevant subset of AAH moments for estimation and inference. Section 7 presents MC evidence, and the last section concludes and discusses avenues for future research. Further results and discussions are provided in an Appendix, including additional Monte Carlo evidence for panel ARX designs, and an empirical application to earning dynamics using Panel Study of Income Dynamics dataset of Meghir and Pistaferri (2004).

2 Panel AR(1) model

We begin with a simple panel AR(1) model to set out the main idea. Specifically, consider the following dynamic panel data model

$$y_{it} = \alpha_i + \phi y_{i,t-1} + u_{it}, \text{ for } i = 1, 2, ..., n, \tag{1}$$

where $\{\alpha_i, 1 \leq i \leq n\}$ are unobserved unit-specific effects, u_{it} is the idiosyncratic error term, and y_{it} are generated from the initial values, $y_{i,-m_i}$ for $m_i \geq 0$, and $t = -m_i + 1, -m_i + 2..., 1, 2, ..., T$.

Using (1) to solve for the initial observations y_{i0}, we obtain

$$y_{i0} = \phi^{m_i} y_{i,-m_i} + \alpha_i \left(\frac{1-\phi^{m_i}}{1-\phi} \right) + \sum_{\ell=0}^{m_i-1} \phi^{\ell} u_{i,-\ell}. \tag{2}$$

It is assumed that available observations for estimation and inference are y_{it}, for $i = 1, 2, ..., n$, and $t = 0, 1, 2, ..., T$ (a total $T + 1$ observations on y). For the implementation of the proposed estimator we require $T \geq 3$, although under mean and variance stationarity identification of ϕ could be achieved even if $T = 2$, namely if the panel covers three time periods.

ASSUMPTION 1 *(Parameter of interest) The true value of* ϕ*, denoted by* ϕ_0*, is the parameter of interest, and it is assumed that* $\phi \in \Theta$*, where* $\Theta \subset (-1, 1]$ *is a compact set.*[4]

In the case where $|\phi| < 1$, and $m_i \to \infty$, then $E(y_{it}) = E(\alpha_i)/(1-\phi)$ for all t. We set $\mu_i = \alpha_i/(1-\phi)$ and refer to μ_i as the long-run mean of y_{it}, even if m_i is finite. However ,in the unit-root case ($\phi = 1$), μ_i is not defined and to avoid incidental linear trends we set $\alpha_i = 0$ when $\phi = 1$. .

Taking first differences of (1), we have

$$\Delta y_{it} = \phi \Delta y_{i,t-1} + \Delta u_{it}, \tag{3}$$

for $t = 2, 3, ..., T$, and $i = 1, 2, ..., n$; but Δy_{i1} is given by

$$\Delta y_{i1} = b_{i,m_i} + u_{i1} - (1-\phi) \sum_{\ell=0}^{m_i-1} \phi^{\ell} u_{i,-\ell}, \tag{4}$$

where

$$b_{i,m_i} = -\phi^{m_i} (1-\phi) (y_{i,-m_i} - \mu_i). \tag{5}$$

The relations (4) and (5) show how the deviations of starting values from the long-run means, given by $(y_{i,-m_i} - \mu_i)$, affect Δy_{i1}. The initialization effect is given by b_{i,m_i} and tends to zero if $|\phi| < 1$, $E|y_{i,-m_i} - \mu_i| < C$, and $m_i \to \infty$. We aim for a minimal set of assumptions on the starting values and individual effects, since in practice such assumptions are difficult to ascertain and, as our Monte Carlo results show, can have important consequences for estimation and inference when m_i and T are both small.

[4] Our theory applies for all finite values of ϕ so long as T and m_i are fixed as $n \to \infty$. We focus on $-1 < \phi \leq 1$, since we believe these values are most relevant in empirical applications.

We assume m_i is finite and consider the following assumptions on the errors, u_{it}, and the starting values, $y_{i,-m_i}$.

ASSUMPTION 2 *(Idiosyncratic errors) For each $i = 1, 2, ..., n$, the process $\{u_{it}, t = -m_i + 1, -m_i + 2, ..., 1, 2, ..., T\}$ is distributed with mean 0, $E\left(u_{it}^2\right) = \sigma_{it}^2$, and there exist positive constants c and C such that $0 < c < \sigma_{it}^2 < C$. Moreover, $\bar{\sigma}_{tn}^2 \equiv n^{-1}\sum_{i=1}^n \sigma_{it}^2 \to \bar{\sigma}_t^2$ as $n \to \infty$, and $\sup_{it} E\left|u_{it}\right|^{4+\epsilon} < C$ for some $\epsilon > 0$. For each t, u_{it} is independently distributed over i. For each i, u_{it} is serially uncorrelated over t.*

ASSUMPTION 3 *(Initialization and individual effects) Let $b_{i,m_i} \equiv -\phi^{m_i}\left[(1-\phi)\, y_{i,-m_i} - \alpha_i\right]$ and $\varsigma_i^2 = E\left(b_{i,m_i}^2\right)$. Then $\bar{\varsigma}_n^2 \equiv n^{-1}\sum_{i=1}^n \varsigma_i^2 \to \bar{\varsigma}^2$ as $n \to \infty$, and $\sup_i E\left|b_{i,m_i}\right|^{4+\epsilon} < C$ for some $\epsilon > 0$. In addition, b_{i,m_i} is independently distributed of $\left(b_{j,m_j}, u_{jt}\right)'$ for all $i \neq j$, $i, j = 1, 2, ..., n$, and $t = -m_j + 1, -m_j + 2, ..., 1, 2, ..., T$, and the following conditions hold:*

$$E\left(\Delta u_{it} b_{i,m_i}\right) = 0, \text{ for } i = 1, 2, ..., n, \text{ and } t = 2, 3, ..., T. \tag{6}$$

Remark 1 *Assumption 2 does not allow the errors, u_{it}, to be cross-sectionally dependent, as is customary in the GMM short-T panel data literature, and together with Assumption 3 ensures also that Δy_{it} is cross-sectionally independent. When errors are weakly cross-sectionally correlated, in the sense defined in Chudik, Pesaran, and Tosetti (2011), then the BMM estimators proposed in this paper remain consistent, but the inference based on them will no longer be valid.*

Remark 2 *Assumption 2 allows errors to be unconditionally heteroskedastic over time t and across units i.*

Remark 3 *Assumption 3 allows for $E\left(b_{i,m_i}\right)$ to vary across i, and therefore, in view of (3)-(4), $E\left(\Delta y_{it}\right)$ can vary across both i and t.*

2.1 Assumptions underlying GMM estimators

It is important to compare our assumptions on the individual effects and the starting values with those maintained in the GMM literature. Under Assumptions 2 and 3, initial first-differences, Δy_{i1}, given by (4) have fourth-order moments and the following moment conditions, which are key to our estimation method, hold

$$E\left(\Delta y_{is}\Delta u_{it}\right) = 0, \text{ for } i = 1, 2, ..., n, \; s = 1, 2, ..., t-2, \text{ and } t = 3, 4, ..., T. \tag{7}$$

Anderson and Hsiao (1981, 1982) have been the first to utilize this type of moment conditions. In particular, they consider instrumenting Δy_{it-1} with Δy_{it-2} and obtain a simple estimator by averaging moments $E\left(\Delta y_{it-2}\Delta u_{it}\right) = 0$ over $t = 3, 4, ..., T$.[5]

The subsequent GMM estimators advanced by Arellano and Bond (1991), Arellano and Bover (1995), and Blundell and Bond (1998) require stronger conditions on the initial values and the individual effects as compared to (7). In addition, the subsequent GMM literature does not average individual moment conditions over time, but combine them efficiently. The first-difference GMM approach considered by Arellano and Bond (1991) assumes

$$E\left(y_{is}\Delta u_{it}\right) = 0, \text{ for } i = 1, 2, ..., n, s = 0, 1, 2, ..., t-2, \text{ and } t = 2, 3, ..., T, \tag{8}$$

which imply (7) but are not required for the moment conditions in (7) to hold. It is clear that the estimator based on (8) will depend on the distributional assumptions regarding the individual effects, whereas an estimator based on (7) need not depend on the distributional assumptions regarding the individual effects.[6]

In addition to (8), the system GMM approach proposed by Arellano and Bover (1995) and Blundell and Bond (1998) also requires that[7]

$$E\left[\Delta y_{i,t-1}\left(\alpha_i + u_{it}\right)\right] = 0, \text{ for } i = 1, 2, ..., n; \text{ and } t = 2, 3, ..., T. \tag{9}$$

These additional restrictions impose further requirements on the errors and the initial values. To see this, first note that iterating (3) from $t = 1$ and using (4) we have

$$\Delta y_{it} = \phi^{t-1}\left[b_{i,m_i} + u_{i1} - (1-\phi)\sum_{\ell=0}^{m_i-1}\phi^{\ell}u_{i,-\ell}\right] + \sum_{\ell=0}^{t-2}\phi^{\ell}\Delta u_{i,t-\ell}. \tag{10}$$

[5] In adition to Δy_{it-2} Anderson and Hsiao (1981, 1982) also considered using y_{it-2} as instrument.

[6] Suppose that $|\phi| < 1$, and consider the case where m_i is finite, namely, $0 \leq m_i < K$, and consider the following initial values $y_{i,-m_i} = \mu_i + \upsilon_i$, where $E\left(\upsilon_i\right) = 0$, and $E\left(\upsilon_i\Delta u_{it}\right) = 0$, for $i = 1, 2, ..., n$, and $t = 3, 4, ..., T$. υ_i measures the extent to which the initial values $y_{i,-m_i}$ deviate from the long-run means, μ_i. Under this specification of initial values, Δy_{it}, for $t = 0, 1, ..., T$ and all i does not depend on μ_i, and estimator based on (7) will not depend on the distributional assumptions about μ_i.

[7] The complete set of moment conditions is $E\left[\Delta y_{is}\left(\alpha_i + u_{it}\right)\right] = 0$, for $i = 1, 2, ..., n$, $s = 1, 2, ..., t-1$, and $t = 2, 3, ..., T$. The set of conditions in (9) contains the $T - 2$ moment conditions in the system GMM approach that are not redundant.

Since for all i, u_{it}'s are assumed to be serially uncorrelated, then condition (9) is met if

$$\phi^{t-2}E\left[b_{i,m_i}\left(\alpha_i+u_{it}\right)\right]+\phi^{t-2}E\left(u_{i1}\alpha_i\right)+\left(\phi-1\right)\phi^{t-2}\sum_{\ell=0}^{m_i-1}\phi^{\ell}E\left(\alpha_i u_{i,-\ell}\right)+\sum_{\ell=0}^{t-3}\phi^{\ell}E\left(\alpha_i\Delta u_{i,t-\ell-1}\right)=0,$$

for $i=1,2,...,n$; and $t=2,3,...,T$. In the case where $m_i\to\infty$, the first term vanishes and the moment conditions (9) will be satisfied if $E\left(u_{it}\alpha_i\right)=0$, for all i and $t\leq T-1$. If m_i is finite it is further required that $E\left[b_{i,m_i}\left(\alpha_i+u_{it}\right)\right]=0$, unless $\phi=0$. Now using (5) and noting that $|\phi|<1$, we have[8]

$$\begin{aligned}E\left[b_{i,m_i}\left(\alpha_i+u_{it}\right)\right] &= -\phi^{m_i}\left(1-\phi\right)E\left[\left(y_{i,-m_i}-\mu_i\right)\left(\alpha_i+u_{it}\right)\right]\\ &= -\phi^{m_i}\left(1-\phi\right)E\left[\left(y_{i,-m_i}-\mu_i\right)\alpha_i\right].\end{aligned}$$

Therefore, when m_i is finite for the moment conditions (9) to hold we must have

$$E\left[\mu_i\left(y_{i,-m_i}-\mu_i\right)\right]=0,\text{ for }i=1,2,...,n. \tag{11}$$

This condition requires that for each i, individual effects are uncorrelated with the deviations of initial values from their long-run means, μ_i. These restrictions might not hold in practice. For example, condition (11) is violated when $\mu_i\neq 0$ and $y_{i,-m_i}=0$.

It is true that by imposing additional restrictions on individual effects and starting values it might be possible to obtain a more efficient estimator of ϕ. However, it is also desirable to seek estimators of ϕ that are consistent under reasonably robust set of assumptions on starting values, individual effects, and error variances. Seen from this perspective, Assumption 3 is less restrictive than the assumptions that underlie the moment conditions used in the existing GMM literature.

When comparing GMM estimators, it is also worth noting from (10) that if $|\phi|<1$ and $\{y_{it}\}$ are initialized in a distant past (with $m_i\to\infty$), then Δy_{it} will no longer depend on α_i and renders the BMM and Anderson-Hsiao estimators invariant to the individual effects. However, this is not the case for the GMM estimators that make use of lagged values of y_{it} in construction of their moment conditions. As a result, the performance of such GMM estimators can be affected by the ratio $\sum_{i=1}^{n}Var\left(\alpha_i\right)/\sum_{i=1}^{n}Var\left(u_{it}\right)$. See Blundell and Bond (1998) and Binder et al. (2005) for further discussions. Of course, if it can be assumed that m_i is large for all i, then many of the issues raised

[8]Note that by assumption $E\left(u_{it}\alpha_i\right)=0=E\left(u_{it}y_{i,-m_i}\right)$, for $t=2,3,....$

surrounding the validity of the GMM moments discussed above might not arise. However, in most empirical micro applications where there are entry and exit of firms/households, the assumption that m_i is large across *all* i could still be highly restrictive. In the case of the earnings dynamics regressions presented in Section A6 of the Appendix, BB restrictions are far from innocuous, and the Hausman test, which we propose in Subsection 4.1 below, strongly rejects the validity of the BB restrictions.

3 BMM estimation of short-T AR(1) panels

We consider the first-differenced version of the panel AR model (3), but instead of using instruments for $\Delta y_{i,t-1}$ that are uncorrelated with the error terms, Δu_{it}, we propose a self-instrumenting procedure whereby $\Delta y_{i,t-1}$ is 'instrumented' for itself, but the population bias due to the non-zero correlation between $\Delta y_{i,t-1}$ and Δu_{it} is corrected accordingly. The advantage of using $\Delta y_{i,t-1}$ as an instrument lies in the fact that by construction it has maximum correlation with the target variable (itself), so long as we are able to correct for the bias that arises due to $Cov\left(\Delta y_{i,t-1}, \Delta u_{it}\right) \neq 0$. To summarize, GMM searches for instruments that are uncorrelated with the errors but are sufficiently correlated with the target variables. Instead, we propose using the target variables as instruments but correct the moment conditions for the non-zero correlations between the errors and the instruments. Both approaches employ method of moments, but differ in the way the moments are derived.

Using $\Delta y_{i,t-1}$ as an instrument, under Assumptions 2 and 3, we have

$$E\left(\Delta u_{it}\Delta y_{i,t-1}\right) + \sigma_{i,t-1}^2 = 0, \text{ for } i = 1, 2, ..., n, \text{ and } t = 2, 3, ..., T-1. \tag{12}$$

Also $E\left(\Delta u_{it}\right)^2 = \sigma_{i,t-1}^2 + \sigma_{it}^2$, where $E\left(\Delta u_{i,t+1}\Delta y_{it}\right) = -\sigma_{it}^2$. Hence, $\sigma_{i,t-1}^2 = E\left(\Delta u_{it}\right)^2 + E\left(\Delta u_{i,t+1}\Delta y_{it}\right)$, which if used in (12) yields the following quadratic moment (QM) condition:

$$E\left(\Delta u_{it}\Delta y_{i,t-1}\right) + E\left(\Delta u_{it}\right)^2 + E\left(\Delta u_{i,t+1}\Delta y_{it}\right) = 0, \tag{13}$$

for $i = 1, 2, ..., n$, and $t = 2, 3, ..., T-1$. It is useful to note that the expression for $\sigma_{i,t-1}^2 = E\left(\Delta u_{it}\right)^2 + E\left(\Delta u_{i,t+1}\Delta y_{it}\right)$ depends on the set of assumptions considered, and different solutions could be obtained under different (stricter) conditions. In this paper, we focus on the general set of conditions summarized by Assumptions 2 and 3, although additional moment conditions can be obtained if one is prepared to make stronger assumptions such as time series homoskedasticity $\sigma_{it}^2 = \sigma_{i,t-1}^2 = \sigma_i^2$. Another possibility is to assume y_{it} is covariance stationary, which will lead to a linear moment condition solution, further discussed in Remark 5 below.

Initially, we use the QM condition (13) alone to obtain an estimator of ϕ, and propose averaging (13) over i and t, which will deliver a simple exactly identified moment estimator. In Section 4, we consider optimally weighting the moment conditions in (13), and augmenting them with Anderson-Hsiao type moment conditions.

Averaging moment condition (13) over t, and substituting (3) for Δu_{it} and $\Delta u_{i,t+1}$, we obtain

$$E\left[M_{iT}(\phi)\right] = 0, \text{ for } i = 1, 2, ..., n, \tag{14}$$

where

$$M_{iT}(\phi) = \frac{1}{T-2}\sum_{t=2}^{T-1}\left[\left(\Delta y_{it} - \phi\Delta y_{i,t-1}\right)\Delta y_{i,t-1} + \left(\Delta y_{it} - \phi\Delta y_{i,t-1}\right)^2 + \left(\Delta y_{i,t+1} - \phi\Delta y_{it}\right)\Delta y_{it}\right]. \tag{15}$$

The BMM estimator is then given by

$$\hat{\phi}_{nT} = \arg\min_{\phi\in\Theta}\left\|\bar{M}_{nT}(\phi)\right\|, \tag{16}$$

where $\|.\|$ denotes the Euclidean norm, $\Theta \subset (-1, 1]$ is a compact set for the admissible values of ϕ defined by Assumption 1, and

$$\bar{M}_{nT}(\phi) = \frac{1}{n}\sum_{i=1}^{n} M_{iT}(\phi). \tag{17}$$

The following theorem summarizes the results for the BMM estimator of ϕ.

Theorem 1 *Suppose y_{it}, for $i = 1, 2, ..., n$, and $t = -m_i+1, -m_i+2, ..., 1, 2, ..., T$, are generated by (1) with starting values $y_{i,-m_i}$, and the true value of the parameter of interest ϕ_0. Let Assumptions 1-3 hold, and suppose $\bar{B}_T \neq 0$ and $n^{-1}\sum_{i=1}^{n} E\left(V_{iT}^2\right) \to S_T > 0$, where $\bar{B}_T$ is given by*

$$\bar{B}_T = \lim_{n\to\infty} E\left(\bar{B}_{nT}\right),\ \bar{B}_{nT} = \frac{1}{n}\sum_{i=1}^{n}\left(Q_{iT} + Q_{iT}^{+} + 2H_{iT}\right), \tag{18}$$

$Q_{iT} = \frac{1}{T-2}\sum_{t=2}^{T-1}\Delta y_{i,t-1}^2$, $Q_{iT}^{+} = \frac{1}{T-2}\sum_{t=2}^{T-1}\Delta y_{it}^2$, $H_{iT} = \frac{1}{T-2}\sum_{t=2}^{T-1}\Delta u_{it}\Delta y_{i,t-1}$, and $V_{iT} = \frac{1}{T-2}\sum_{t=2}^{T-1}\left(\Delta u_{it}\Delta y_{i,t-1} + \Delta u_{it}^2 + \Delta u_{i,t+1}\Delta y_{it}\right)$. Consider the BMM estimator $\hat{\phi}_{nT}$ given by (16).

Let $T \geq 3$ be fixed and $n \to \infty$. Then, the unique $\sqrt{n}$-consistent estimator $\hat{\phi}_{nT}$ satisfies

$$\sqrt{n}\left(\hat{\phi}_{nT} - \phi_0\right) \to_d N\left(0, \Sigma_T\right),$$

where

$$\Sigma_T = \bar{B}_T^{-2} S_T. \tag{19}$$

A $\sqrt{n}$-consistent estimator of ϕ exits if $\bar{B}_T \neq 0$, where $\bar{B}_T = \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} E\left(B_{iT}\right)$, and

$$B_{iT} = \frac{1}{T-2} \sum_{t=2}^{T-1} \left(\Delta y_{i,t-1}^2 + \Delta y_{it}^2 + 2\Delta u_{it} \Delta y_{i,t-1}\right). \tag{20}$$

It is now easily seen that condition $\bar{B}_T \neq 0$ is satisfied when Δy_{it} is a stationary process (for $m_i \to \infty$, $\sigma_{it} = \sigma_i^2$ and $|\phi| < 1$). In this case

$$\bar{B}_T = 2\left(\frac{1-\phi}{1+\phi}\right)\bar{\sigma}^2 > 0,$$

where $\bar{\sigma}^2 = \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} \sigma_i^2$. In the non-stationary case (with m_i finite) $\bar{B}_T \neq 0$ even if $\phi = 1$ so long as σ_{it} is sufficiently variable over t. As a simple example consider the case where $T = 3$, and note that (see Section A.1 of the Appendix)

$$\bar{B}_3 = \bar{\sigma}_2^2 - \bar{\sigma}_1^2 + (1-\phi)^2 \bar{\sigma}_1^2 + \left(1 + \phi^2\right)(1-\phi)\psi_0. \tag{21}$$

where $\bar{\sigma}_t^2 = \lim_{n\to\infty} n^{-1} \sum_{i=1}^{n} \sigma_{it}^2$, and

$$\psi_0 = (1-\phi) \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} E\left(y_{i0} - \mu_i\right)^2 - 2 \lim_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} E\left[u_{i1}\left(y_{i0} - \mu_i\right)\right]. \tag{22}$$

If $\phi = 1$, then $\bar{B}_3 = \bar{\sigma}_2^2 - \bar{\sigma}_1^2$, and $\bar{B}_3 \neq 0$, if and only if $\bar{\sigma}_1^2 \neq \bar{\sigma}_2^2$. When $|\phi| < 1$, $\bar{B}_3 \neq 0$ even if $\bar{\sigma}_1^2 = \bar{\sigma}_2^2$, except for when $(1-\phi)\left(1+\phi^2\right)\psi_0 = \phi(2-\phi)\bar{\sigma}_1^2 - \bar{\sigma}_2^2$. Therefore, time variations in the average error variances, $\bar{\sigma}_t^2$, can help identification under the BMM quadratic moment condition.

Remark 4 *When $\bar{B}_T = 0$, from (A.13) we have,*

$$\left(\hat{\phi}_{nT} - \phi_0\right)^2 \bar{Q}_{nT} = \bar{V}_{nT} + \left(\hat{\phi}_{nT} - \phi_0\right) O_p\left(n^{-1/2}\right), \tag{23}$$

where $\bar{V}_{nT} = n^{-1} \sum_{i=1}^{n} V_{iT}$, $\bar{Q}_{nT} = n^{-1} \sum_{i=1}^{n} Q_{iT}$. Note that $\bar{Q}_{nT} \to \bar{Q}_T > 0$ as $n \to \infty$.

Therefore, there exists a unique $n^{1/4}$-consistent estimator $\hat{\phi}_{nT}$. As noted earlier a leading case when $\bar{B}_T = 0$, is the unit root case ($\phi = 1$) under error variance homogeneity over t.

Σ_T, can be estimated consistently by

$$\hat{\Sigma}_{nT} = \hat{\bar{B}}_{nT}^{-2} \left(\frac{1}{n} \sum_{i=1}^{n} \hat{V}_{i,nT}^2 \right), \tag{24}$$

where

$$\hat{\bar{B}}_{nT} = \frac{1}{n} \sum_{i=1}^{n} \left(Q_{iT} + Q_{iT}^{+} + 2\hat{H}_{i,nT} \right), \tag{25}$$

$\hat{H}_{i,nT} = (T-2)^{-1} \sum_{t=2}^{T-1} \Delta\hat{u}_{it} \Delta y_{i,t-1}$, $\Delta\hat{u}_{it} = \Delta y_{it} - \hat{\phi}_{nT} \Delta y_{i,t-1}$,[9] and

$$\hat{V}_{i,nT} = -\frac{1}{T-2} \sum_{t=2}^{T-1} \left(\Delta\hat{u}_{it} \Delta y_{i,t-1} + \Delta\hat{u}_{it}^2 + \Delta\hat{u}_{i,t+1} \Delta y_{it} \right). \tag{26}$$

Consistency of $\hat{\Sigma}_{nT}$ is established in Proposition 1 in the appendix.

Remark 5 *In the case of covariance stationary panels ($|\phi| < 1$ and $m_i \to \infty$), we have $\Delta y_{it} = \sum_{\ell=0}^{\infty} \phi^{\ell} \Delta u_{i,t-\ell}$, where $E\left(u_{it}^2\right) = \sigma_i^2$. Then $E\left(\Delta y_{it}^2\right) = 2\sigma_i^2 / (1+\phi)$ is time-invariant. Under covariance stationarity $\sigma_i^2 = (1+\phi) E\left(\Delta y_{i,t-1}^2\right)/2$, $E\left(\Delta u_{it} \Delta y_{i,t-1}\right) = E\left(\Delta u_{i,t+1} \Delta y_{it}\right)$, and using (12) the quadratic moment condition, (13), simplifies to the following linear moment condition:*

$$E\left(\Delta y_{it} \Delta y_{i,t-1}\right) + \frac{1}{2}(1-\phi) E\left(\Delta y_{i,t-1}^2\right) = 0,$$

which yields the associated BMM estimator given by

$$\hat{\phi}_n = \frac{\sum_{i=1}^{n} \sum_{t=2}^{T} \left(2\Delta y_{it} \Delta y_{i,t-1} + \Delta y_{i,t-1}^2 \right)}{\sum_{i=1}^{n} \sum_{t=2}^{T} \Delta y_{i,t-1}^2}. \tag{27}$$

In this case ϕ is identified even when $T = 2$. Interestingly enough, the above linear BMM estimator is identical to the first-difference least square (FDLS) estimator proposed by Han and Phillips (2010),[10] who show that $\hat{\phi}_n$ has standard Gaussian asymptotics for all values of $\phi \in (-1, 1]$ and does not suffer from the weak instrument problem. However, when T is fixed the covariance stationarity assumption is rather restrictive for most empirical applications in economics, where typically

[9] $\Delta\hat{u}_{it}$ depends on n and T, but we omit subscripts n, T to simplify the notations.

[10] We are grateful to Kazuhiko Hayakawa for drawing our attention to this fact.

not much is known about the initialization of the dynamic processes over i, and it is not possible to rule out the heteroskedasticity of error variances over t.

4 Augmented Anderson Hsiao (AAH) estimator

The BMM estimator above is useful for illustrative purposes, but it is not asymptotically efficient partly due to averaging of moment conditions over t, and more importantly due to not exploring additional readily available moment conditions that hold under the same set of assumptions. Another reason for augmenting the quadratic BMM moments with additional moment conditions is because the global identification for BMM is not guaranteed.

As noted above, amongst the moment conditions proposed in the literature, only the ones proposed by AH are sufficiently general, and accordingly, we propose to augment the $T-2$ quadratic moment conditions in (13) with the $(T-2)(T-1)/2$ AH moment conditions in (7). Together, they provide $(T-2)(T-1)/2+T-2$ AAH moment conditions. As usual, we can obtain first, second and continuous-updating GMM estimators based on these quadratic-linear moment conditions.

Remark 6 *It is worth noting that conditions (7) and (13) do not imply conditions (8) and/or (9), since (7) and (13) rely only on first differences, whereas (8) and (9) also rely on levels. Hence, it is possible that (7) and (13) can hold whilst (8) and/or (9) might not hold. An example of this case is considered in the Monte Carlo section below.*

The set of AAH moment conditions (7) and (13) is a subset of the conditions in Ahn and Schmidt (1995, 1997), who enumerated a complete set of moment conditions under a stronger set of assumptions than are necessary for AAH alone; see their Assumptions SA1-SA3. Sufficient set of assumptions that give rise to AAH are the following 'basic' assumptions:

(BA1) For all i, the u_{it} are mutually uncorrelated.

(BA2) $E\left[(y_{i0}-\mu_i)\,\Delta u_{it}\right]=0$ for all i and $t=2,3,...,T$,

where $\mu_i=\alpha_i/(1-\phi)$. Assumption BA1 on its own has been considered as Case H of Ahn and Schmidt (1997), which implies $T(T-3)/3$ moment conditions. Assumption BA2 is implied by Assumptions SA1-SA2 of Ahn and Schmidt (1995), but not *vice versa*. The full set of moment conditions based on BA1 and BA2 is the union of AH moment conditions given by (7) and QM moment conditions given by (13). Derivation of the asymptotic distribution and conducting inference requires additional standard high-level regularity conditions routinely used in the GMM literature.[11]

It is of interest to consider the efficiency loss that arises when using AAH moment conditions, whilst in fact the more restrictive system GMM conditions (8)-(9) hold. To shed light on this, we report the ratios of asymptotic variances of the AH, first-difference GMM and system GMM estimators, all relative to that of the AAH estimator. We illustrate the asymptotic efficiency gains and losses in Table 1 in the same way as in Ahn and Schmidt (1995). We are interested in two questions: (i) How much is gained by adding QM conditions to AH, and (ii) how much is lost by not utilizing the additional moment conditions assuming that the DGP satisfies all of the restrictions in (8) and (9). Following Ahn and Schmidt (1995), we tabulate the asymptotic variance ratios for the stationary homoskedastic case for different values of ϕ, and different ratios of $E\left(\alpha_i^2\right)/E\left(u_{it}^2\right) = \sigma_\alpha^2/\sigma_u^2$, for all i and t.

The results, computed by simulations, are summarized in Table 1. As can be seen, augmenting AH moment conditions with the quadratic moment conditions (13) results in substantial efficiency gains for all values of ϕ, $\sigma_\alpha^2/\sigma_u^2$ and the three choices of $T = 3, 6$ and 10, in Table 1. The efficiency gains are particularly pronounced for values of ϕ close to unity. Also as to be expected the two estimators perform equally well for all values of $\sigma_\alpha^2/\sigma_u^2$ since both use first-differences as instruments and hence are not affected by σ_α^2. The efficiency gain of AAH over AH reduces somewhat when T is increased.

[11] These are listed, for example, in Pesaran (2015). In particular, assumptions for consistency are given by Assumptions A1 and A2 in Chapter 10 of Pesaran (2015) and the additional assumptions for asymptotic normality are given by Assumptions A3-A5 of the same chapter. See also Assumptions 1-3 for a set of low-level assumptions required for consistency and asymptotic normality.

Table 1: Asymptotic efficiency of AH, AB and BB estimators relative to the AAH estimator under stationarity

	$var\left(AH\right)/var\left(AAH\right)$			$var\left(AB\right)/var\left(AAH\right)$			$var\left(BB\right)/var\left(AAH\right)$		
	$\sigma_\alpha^2/\sigma_u^2$			$\sigma_\alpha^2/\sigma_u^2$			$\sigma_\alpha^2/\sigma_u^2$		
ϕ	0.5	1	4	0.5	1	4	0.5	1	4
$T=3$									
-0.9	1.5	1.5	1.5	1.0	1.0	1.1	0.9	1.0	1.0
-0.8	1.7	1.7	1.7	1.0	1.1	1.2	0.9	1.0	1.0
-0.5	2.3	2.3	2.3	1.1	1.2	1.6	0.9	0.9	1.0
-0.3	2.9	2.9	2.9	1.2	1.3	1.9	0.9	0.9	1.0
0	4.0	4.0	4.0	1.2	1.5	2.4	0.8	0.9	0.9
0.3	5.3	5.3	5.3	1.2	1.7	3.0	0.6	0.7	0.9
0.5	6.4	6.4	6.4	1.2	1.7	3.4	0.4	0.5	0.7
0.8	8.2	8.2	8.2	1.2	1.7	4.1	0.1	0.1	0.3
0.9	9.2	9.2	9.2	1.9	2.6	3.8	0.05	0.05	0.06
$T=6$									
-0.9	1.3	1.3	1.3	1.0	1.0	1.1	1.0	1.0	1.0
-0.8	1.3	1.3	1.3	1.1	1.1	1.1	1.0	1.0	1.0
-0.5	1.6	1.6	1.6	1.2	1.2	1.4	1.0	1.0	1.0
-0.3	1.8	1.8	1.8	1.3	1.4	1.6	1.0	1.0	1.0
0	2.2	2.2	2.2	1.5	1.6	2.0	1.0	1.0	1.0
0.3	3.0	3.0	3.0	1.7	2.0	2.6	0.9	1.0	1.0
0.5	3.9	3.9	3.9	2.0	2.4	3.3	0.9	0.9	1.0
0.8	6.1	6.1	6.1	2.5	3.5	5.1	0.5	0.6	0.8
0.9	7.6	7.6	7.6	2.5	4.0	6.3	0.2	0.3	0.5
$T=10$									
-0.9	1.2	1.2	1.2	1.0	1.0	1.1	1.0	1.0	1.0
-0.8	1.2	1.2	1.2	1.1	1.1	1.1	1.0	1.0	1.0
-0.5	1.3	1.3	1.3	1.1	1.2	1.2	1.0	1.0	1.0
-0.3	1.5	1.5	1.5	1.2	1.3	1.4	1.0	1.0	1.0
0	1.7	1.7	1.7	1.4	1.5	1.6	1.0	1.0	1.0
0.3	2.2	2.2	2.2	1.6	1.8	2.0	1.0	1.0	1.0
0.5	2.7	2.7	2.7	1.9	2.2	2.5	1.0	1.0	1.0
0.8	4.8	4.8	4.8	2.9	3.6	4.3	0.8	0.9	0.9
0.9	6.5	6.5	6.5	3.6	4.6	5.8	0.5	0.6	0.8

Notes: This table reports ratios of asymptotic variance of the Anderson and Hsiao (AH), Arellano and Bond (AB) and Blundell and Bond (BB) estimators relative to the asymptotic variance of the augmented AH (AAH) estimator in a stationary design with $E\left(\alpha_i^2\right)=\sigma_\alpha^2$ and $E\left(u_{it}^2\right)=\sigma_u^2$, and for different values of the AR coefficient, ϕ. Asymptotic variances are computed by simulations using $n=10^7$. $T=3$ requires $y_{i,0}, y_{i,1}, y_{i,2}$, and $y_{i,3}$ are observed.

Turning now to the second issue, namely efficiency loss of AAH relative to AB and BB estimators, we first note that interestingly enough, the expected efficiency gain of AB over AAH does not materialize and AAH is in fact generally more efficient than the AB estimator, with efficiency gain of AAH increasing substantially as larger values of ϕ and $\sigma_\alpha^2/\sigma_u^2$ are considered. Increasing T does not seem to have much effect on the relative efficiency of the AB estimator. The results in Table 1 also confirm the sensitivity of the AB estimator to the ratio, $\sigma_\alpha^2/\sigma_u^2$. In contrast, the

BB estimator performs favorably relative to the AAH estimator (and by implication relative to the AB estimator) particularly, for values of ϕ close to unity. However, this efficiency gain is achieved assuming that $E\left[\mu_i\left(y_{i,-m_i}-\mu_i\right)\right]=0$, for $i=1,2,...,n$, which might not hold in practice. (see (11) and the related discussions). The cost of using BB estimator is inconsistency if condition (11) is not met. Further evidence on this is provided in the Monte Carlo section.

4.1 Hausman test for the validity of moment conditions

The above simulations suggest that AAH estimator cannot be more efficient than BB estimator when all BB moment conditions are met. This can be seen formally by investigating more closely the relation between the BB condition (9) and the QM moment condition (12), or equivalently (13). Using $u_{it}+\alpha_i=\Delta u_{it}+(\alpha_i+u_{i,t-1})$ in (9) we have

$$E\left[\Delta y_{i,t-1}\left(\alpha_i+u_{it}\right)\right]=E\left(\Delta y_{i,t-1}\Delta u_{it}\right)+E\left[\Delta y_{i,t-1}\left(\alpha_i+u_{i,t-1}\right)\right],$$

and since $\Delta y_{i,t-1}=\phi\Delta y_{i,t-2}+\Delta u_{i,t-1}$, then

$$E\left[\Delta y_{i,t-1}\left(\alpha_i+u_{it}\right)\right]=E\left(\Delta y_{i,t-1}\Delta u_{it}\right)+\phi E\left[\Delta y_{i,t-2}\left(\alpha_i+u_{i,t-1}\right)\right]+E\left(\Delta u_{i,t-1}\alpha_i\right)+E\left(\Delta u_{i,t-1}u_{i,t-1}\right). \tag{28}$$

But under BB moment conditions $E\left(\Delta u_{i,t-1}\alpha_i\right)=0$ and

$$E\left[\Delta y_{i,t-2}\left(\alpha_i+u_{i,t-1}\right)\right]=0. \tag{29}$$

Using these results in (28) we have

$$\begin{aligned}E\left[\Delta y_{i,t-1}\left(\alpha_i+u_{it}\right)\right] &= E\left(\Delta y_{i,t-1}\Delta u_{it}\right)+E\left(\Delta u_{i,t-1}u_{i,t-1}\right)\\ &= E\left(\Delta y_{i,t-1}\Delta u_{it}\right)+\sigma^2_{i,t-1}=0,\end{aligned}$$

which is the same as the QM condition given by (12). Namely, the QM condition is implied by the BB moment conditions, but not *vice versa*. Hence, under BB moment conditions the AAH estimator cannot be more efficient than the BB estimator. Note that (29) is the same as (9) and it is satisfied if $E\left(\Delta u_{i,t-1}\alpha_i\right)=0$ and $E\left[\mu_i\left(y_{i,-m_i}-\mu_i\right)\right]=0$, as discussed in Section 2. However, when the BB conditions (8) and/or (9) are not met the BB estimator becomes inconsistent contrary to the

AAH estimator that continues to be consistent. Therefore, the main two conditions underlying the Hausman test (Hausman, 1978) are met and the validity of BB moment conditions can be tested using the Hausman procedure. Denoting the AAH and BB estimators by $\hat{\phi}_{nT}^{aah}$ and $\hat{\phi}_{nT}^{bb}$, respectively, the Hausman test statistic is defined by

$$H_n = \left(\hat{\phi}_{nT}^{aah} - \hat{\phi}_{nT}^{bb}\right)^2 \left[\widehat{Var}\left(\hat{\phi}_{nT}^{aah}\right) - \widehat{Var}\left(\hat{\phi}_{nT}^{bb}\right)\right]^{-1}, \tag{30}$$

assuming that $\widehat{Var}\left(\hat{\phi}_{nT}^{aah}\right) - \widehat{Var}\left(\hat{\phi}_{nT}^{bb}\right) > 0$, where $\widehat{Var}\left(\hat{\phi}_{nT}^{aah}\right)$ and $\widehat{Var}\left(\hat{\phi}_{nT}^{bb}\right)$ are consistent estimators of the asymptotic variances of $\hat{\phi}_{nT}^{aah}$, and $\hat{\phi}_{nT}^{bb}$, respectively. Under the null hypothesis that the BB conditions are met, H_n is asymptotically distributed as $\chi^2(1)$, for a fixed T and as $n \to \infty$.

5 Extensions to panel VARs and to models with covariates

There are two important extensions of model (1). The first extension is to a panel VAR model

$$\mathbf{z}_{it} = \boldsymbol{\alpha}_i + \boldsymbol{\Phi}\mathbf{z}_{i,t-1} + \mathbf{u}_{it},\ t = 0, 1, 2, ..., T;\ \text{and } i = 1, 2, ..., n, \tag{31}$$

where $\mathbf{z}_{it} = (y_{it}, \mathbf{x}_{it}')'$ is the $k \times 1$ vector of endogenous variables, $\boldsymbol{\alpha}_i = (\alpha_{iy}, \boldsymbol{\alpha}_{ix}')'$ is the $k \times 1$ vector of individual effects, $\boldsymbol{\Phi}$ is the $k \times k$ matrix of slope coefficients, and $\mathbf{u}_{it} = \left(u_{y,it}, \mathbf{u}_{x,it}'\right)'$ is the $k \times 1$ vector of idiosyncratic errors. Similarly, to the univariate case, the set of linear AH moment conditions is given by:

$$E\left(\Delta\mathbf{z}_{is}\Delta\mathbf{u}_{it}'\right) = \mathbf{0}_{k\times k},\ \text{for } i = 1, 2, ..., n,\ s = 1, 2, ..., t-2,\ \text{and } t = 3, 4, ..., T, \tag{32}$$

to be augmented with the following QM moment conditions:

$$E\left(\Delta\mathbf{u}_{it}\Delta\mathbf{z}_{i,t-1}'\right) + E\left(\Delta\mathbf{u}_{it}\Delta\mathbf{u}_{it}'\right) + E\left(\Delta\mathbf{u}_{i,t+1}\Delta\mathbf{z}_{it}'\right) = \mathbf{0}_{k\times k}, \tag{33}$$

for $i = 1, 2, ..., n$, and $t = 2, 3, ..., T-1$. AAH estimation of the panel VAR model can proceed based on (32) and (33), which replace (7) and (13), respectively.

The second extension is to augment (1) with the additional $k-1$ regressors in $\mathbf{x}_{it}$, as the conditioning variables, to obtain the ARX model

$$y_{it} = \alpha_i + \phi y_{i,t-1} + \boldsymbol{\beta}' \mathbf{x}_{it} + u_{it}, \text{ for } i = 1, 2, ..., n, \;\; t = 1, 2, ..., T. \tag{34}$$

The regressors in $\mathbf{x}_{it}$ can be strictly or weakly exogenous. AAH moment conditions (7) and (13) can be augmented by the standard orthogonality for the regressors $\mathbf{x}_{it}$, as is standard in the GMM literature. This paper does not have anything new to add regarding instrumenting the regressors $\mathbf{x}_{it}$. But in the case of weakly exogenous regressors, where there are feedbacks from lagged values of y_{it} onto $\mathbf{x}_{it}$, the validity of the ARX specification and the strength of the instruments used for $\Delta \mathbf{x}_{it}$ will depend on the nature and the quantitative importance of such feedbacks. For a general discussion see Chudik, Pesaran, and Yang (2018).

In the case where (34) is derived from an underlying VAR model such as (31), additional restrictions on error variance heteroskedasticity are required. To see this write down the individual equations for y_{it} and $\mathbf{x}_{it}$ in (31) as

$$y_{it} = \alpha_{iy} + \phi_{11} y_{i,t-1} + \boldsymbol{\phi}'_{yx} \mathbf{x}_{i,t-1} + u_{y,it}, \tag{35}$$

$$\mathbf{x}_{it} = \boldsymbol{\alpha}_{ix} + \boldsymbol{\phi}_{xy} y_{i,t-1} + \boldsymbol{\Phi}_{xx} \mathbf{x}_{i,t-1} + \mathbf{u}_{x,it}, \tag{36}$$

where $\boldsymbol{\Phi}$ is partitioned as:

$$\boldsymbol{\Phi} = \begin{pmatrix} \phi_{11} & \boldsymbol{\phi}'_{yx} \\ \boldsymbol{\phi}_{xy} & \boldsymbol{\Phi}_{xx} \end{pmatrix}.$$

Suppose that the errors, $\mathbf{u}_{it}$, are heteroskedastic over i and t, and let

$$E\left(\mathbf{u}_{it}\mathbf{u}'_{it}\right) = \boldsymbol{\Omega}_{it} = \begin{pmatrix} \omega_{yy,it} & \boldsymbol{\omega}'_{xy,it} \\ \boldsymbol{\omega}_{xy,it} & \boldsymbol{\Omega}_{xx,it} \end{pmatrix},$$

for all i and t. Using linear projection of $u_{y,it}$ on $\mathbf{u}_{x,it}$, we have

$$u_{y,it} = \boldsymbol{\theta}'_{it} \mathbf{u}_{x,it} + \eta_{it}, \tag{37}$$

where $\boldsymbol{\theta}_{it} = \boldsymbol{\Omega}^{-1}_{xx,it} \boldsymbol{\omega}_{xy,it}$, and $cov\left(\eta_{it}, \mathbf{u}_{x,it}\right) = \mathbf{0}$. Then using (37) and (36) in (35), we have

$$y_{it} = \alpha_{iy} + \phi_{11} y_{i,t-1} + \boldsymbol{\phi}'_{yx} \mathbf{x}_{i,t-1} + \boldsymbol{\theta}'_{it} \left(\mathbf{x}_{it} - \boldsymbol{\alpha}_{ix} - \boldsymbol{\phi}_{xy} y_{i,t-1} - \boldsymbol{\Phi}_{xx} \mathbf{x}_{i,t-1}\right) + \eta_{it},$$

where $cov\left(\eta_{it}, \mathbf{x}_{is}\right) = \mathbf{0}$ for all i, t and s, and recall that η_{it} is serially uncorrelated. Therefore, for (34) to be compatible with the underlying panel VAR (31) we must have

$$\alpha_i = \alpha_{iy} - \boldsymbol{\theta}_{it}'\boldsymbol{\alpha}_{ix},\ \phi = \phi_{11} - \boldsymbol{\theta}_{it}'\boldsymbol{\phi}_{xy},\ \boldsymbol{\beta} = \boldsymbol{\theta}_{it},\ \text{and } \boldsymbol{\phi}_{yx} = \boldsymbol{\Phi}_{xx}'\boldsymbol{\theta}_{it}.$$

which is possible only if $\boldsymbol{\theta}_{it} = \boldsymbol{\Omega}_{xx,it}^{-1}\boldsymbol{\omega}_{xy,it} = \boldsymbol{\beta}$, for *all* i and t. When this condition is met, the restriction $\boldsymbol{\phi}_{yx} = \boldsymbol{\Phi}_{xx}'\boldsymbol{\theta}_{it} = \boldsymbol{\Phi}_{xx}'\boldsymbol{\beta}$ can be relaxed by considered the autoregressive-distributed lag (ARDL) specification

$$y_{it} = \alpha_i + \phi y_{i,t-1} + \boldsymbol{\beta}_0'\mathbf{x}_{it} + \boldsymbol{\beta}_1'\mathbf{x}_{i,t-1} + \eta_{it}, \tag{38}$$

where

$$\alpha_i = \alpha_{iy} - \boldsymbol{\beta}_0'\boldsymbol{\alpha}_{ix},\ \phi = \phi_{11} - \boldsymbol{\omega}_{xy}'\boldsymbol{\Omega}_{xx}^{-1}\boldsymbol{\phi}_{xy},\ \boldsymbol{\beta}_0 = \boldsymbol{\Omega}_{xx}^{-1}\boldsymbol{\omega}_{xy}\text{ , and } \boldsymbol{\beta}_1 = \boldsymbol{\phi}_{yx} - \boldsymbol{\Phi}_{xx}'\boldsymbol{\beta}_0.$$

The above derivations also show that when $\mathbf{x}_{it}$ is weakly exogenous, it is best to use ARDL specifications to ensure that the conditional model being considered and its underlying VAR are compatible.

6 Problem of many moment conditions

As it is well known, the number of moment conditions that underlie any of the GMM based estimation techniques discussed above (AH, AAH, AB, or BB) grow at the quadratic rate in T. Consequently, the number of moments can get quite large even for moderate values of T. Under their respective set of assumptions, these are all valid moments and their relevance (strength) varies, some of which could be weakly identifying. Unless the number of cross-section dimension, n, is sufficiently large, as compared to the number of moment conditions, $h = h\left(T\right)$, the proliferation of moments will have adverse effects for estimation and inference in finite samples. See, for instance, Anderson and Sorenson (1996), Clark (1996), and Hansen, Heaton, and Yaron (1996). The many moment problem often occurs together with the weak moment problem, but they are not necessarily the same. Han and Phillips (2006) provide a number of asymptotic theoretical results for GMM estimation that allow for the number of moments to increase with the sample size, whilst moment conditions may only be weakly identifying, encompassing earlier contributions by Bekker (1994), Staiger and Stock (1997), Stock and Wright (2000), and Chao and Swanson (2003), among others. GMM estimators utilizing many weak moment conditions may not be consistent and the rate of convergence could depend not only on the sample size, but also on the number and quality of the moment conditions.

Hsiao and Zhang (2015) show that the AB estimator is asymptotically biased if $T/n \to c$, for some $0 < c < \infty$, as $n, T \to \infty$. This bias can be reduced using jackknife instrumental variables estimation (JIVE), which has been considered in a general GMM framework by Angrist, Imbens, and Krueger (1999), Chao, Swanson, Hausman, Newey, and Woutersen (2012), Hansen and Kozbur (2014), Lee, Moon, and Zhou (2017), Phillips and Hale (1977) and Zhang and Zhou (2020).[12] Koenker and Machado (1999) and Donald, Imbens, and Newey (2003) consider GMM estimation under a large number of strong moments, and provide conditions on the number of moments that permits the usual asymptotic theory and inference. In particular, Koenker and Machado (1999) show $h^3/n \to 0$ is sufficient for validity of conventional GMM asymptotic inference.

There are two approaches to dealing with a large number of valid moments. One is to use them all, but combine them in such a way that allows for the number of moments to be large relative to the sample size so that consistency and valid inference are achieved. The second approach is to select and use only a subset of available moments. Contributions to this strand of the literature includes Donald and Newey (2001), Kuersteiner (2002), Hall and Peixe (2003), Inoue (2006), Hall, Inoue, Jana, and Shin (2007), and Donald, Imbens, and Newey (2009).[13] In what follows we propose a new sub-set selection procedure by adapting the One Covariate at the time Multiple Testing (OCMT) recently developed by Chudik, Kapetanios, and Pesaran (2018) for variable selection to the problem of moment selection in the case of the AAH estimator.

6.1 Moment selection using OCMT approach

In the case of AH moments listed in (7), there are $t-2$ instruments for $\Delta y_{i,t-1}$, for $t = 3, 4, ..., T$. We collect them in the set $\mathcal{S}_{i,t-2} = \{\Delta y_{i,1}, \Delta y_{i,2}, ..., \Delta y_{i,t-2}\}$. In general, it is not possible to derive analytical expressions for the correlation of the target variable $\Delta y_{i,t-1}$ and individual instruments in $\mathcal{S}_{i,t-2}$ in the case where the underlying dynamic processes are initialized from finite pasts, and

[12] Monte Carlo findings reported in Zhang and Zhou (2020) suggest very good size performance of JIVE corrected AB GMM estimator. However, the size reported in Zhang and Zhou (2020) is computed using standard deviation of the estimated slope coefficients across Monte Carlo replications, which is not feasible in practice where only one set of realizations is available. Hence, the findings in Zhang and Zhou (2020) are not indicative of inference that can be conducted in empirical applications.

[13] In addition to the literature on selecting relevant moments from a set of valid moments, there is a vast literature on moment validity, and the selection of valid moments, including Andrews (1999), Andrews and Lu (2001), Chatelain (2007), and Liao (2013). The problem of selecting valid as well as relevant moments has been consideed by Cheng and Liao (2015).

there is little known about the data generating processes for the initial values. It is, nevertheless, possible to show that $corr\left(\Delta y_{i,t-1}, \Delta y_{i,t-\ell}\right)$ declines in ℓ at an exponential rate in the case of stationary initial values. This is illustrated in the following example.

Example 1 *Let $y_{it} = \alpha_i + \phi y_{i,t-1} + u_{it}$, for $t = ..., -1, 0, 1, ..., T$ and $i = 1, 2, ..., n$, where $|\phi| < 1$. Then $y_{it} = \mu_i + \sum_{\ell=0}^{\infty} \phi^{\ell} u_{i,t-\ell}$, and*

$$\Delta y_{it} = u_{it} - (1 - \phi) \sum_{\ell=1}^{\infty} \phi^{\ell-1} u_{i,t-\ell},$$

where $\mu_i = \alpha_i / (1 - \phi)$. Provided $E\left(u_{it}u_{it'}\right) = 0$ for $t \neq t'$ and $E\left(u_{it}^2\right)$ is bounded, it follows that $\left|corr\left(\Delta y_{i,t-1}, \Delta y_{i,t-\ell}\right)\right| < C\phi^{|\ell-1|}$.

Hence, it could be the case that some of the $t-2$ instruments in $\mathcal{S}_{i,t-2}$ are rather weak and consequently not very useful in improving the asymptotic variance of the resulting GMM estimator. Our suggestion is to apply OCMT method to select the relevant instruments from the set $\mathcal{S}_{i,t-2}$, for $t = 4, 5, ..., T$. It is desirable to always include $\Delta y_{i,t-2}$, which is likely to have the largest correlation with the target variable $\Delta y_{i,t-1}$, as a conditioning (or pre-selected) variable in the OCMT procedure, as described below.

OCMT algorithm for selecting AH instruments for a given t $(= 4, 5, ..., T)$ is as follows:

1. Estimate the $(t-3)$ individual first stage regressions

$$\Delta y_{i,t-1} = a_{\ell} + \beta_{\ell} \Delta y_{i,t-2} + \theta_{\ell} \Delta y_{i,\ell}, \text{ for } \ell = t-3, t-4, ..., 1 \tag{39}$$

by least squares and compute the associated t-ratios for the coefficients θ_{ℓ} in the above regression, denoted as $t_{\hat{\theta}_{\ell(s)}} = \hat{\theta}_{\ell}/s.e.\left(\hat{\theta}_{\ell}\right)$ for stage $s = 1$. The first stage OCMT selection indicator is given by

$$\widehat{\mathcal{J}}_{\ell,(1)} = I[|t_{\hat{\theta}_{\ell(1)}}| > c_p\left(t-1, \delta\right)], \text{ for } \ell = 1, 2, ..., t-3, \tag{40}$$

where $c_p(t, \delta)$ is a critical value function defined by

$$c_p\left(t, \delta\right) = \Phi^{-1}\left(1 - \frac{p}{2t^{\delta}}\right), \tag{41}$$

$\Phi^{-1}(.)$ is the inverse of standard normal distribution function, $0 < p < 1$, and $\delta > 0$. Following Chudik, Kapetanios, and Pesaran (2018), we set $p = 0.05$ and $\delta = 1$ in the first stage, while another value, $\delta^* = 2$, is used in subsequent stages of OCMT described below. Variables with $\widehat{\mathcal{J}}_{i,(1)} = 1$ are selected as instruments in the first stage. If no variables are selected in the first stage, then OCMT procedure stops. Otherwise, increase s by one.

2. The next stage ($s > 1$) is computed by regressing $\Delta y_{i,t-1}$ on a constant, $\Delta y_{i,t-2}$, all instruments selected from the previous stages, and, one-at-time, the remaining instruments not yet selected. Let $t_{\hat{\theta}_{\ell,(s)}}$ denote the corresponding t-ratio of the instruments considered for selection in the stage $s > 1$. Then the instruments are added to the selected set if the indicator $\widehat{\mathcal{J}}_{\ell,(s)} = I[|t_{\hat{\theta}_{\ell,(s)}}| > c_p(t-1, \delta^*)]$ is one. If no instruments are selected in stage s, then the OCMT procedure stops. Otherwise s is increased by one.

3. Step 2 is repeated until no further instruments are selected.

The outcome of this data-dependent selection of moments is $\hat{h}_{nT}$ selected AH moments, $T-2 \leq \hat{h}_{nT} \leq (T-2)(T-1)/2$.[14]

7 Monte Carlo Evidence

We now provide some evidence on the small sample performance of the AAH estimator as compared to AH, and the two popular AB and BB estimators (also known as first-difference and the system GMM estimators). In addition, we also investigate the small sample performance of the AAH estimator using the subset of AAH moments selected by the OCMT procedure.

7.1 Data generating process (DGP)

The dependent variable is generated as

$$y_{it} = \alpha_i + \phi y_{i,t-1} + u_{it}, \tag{42}$$

[14] This idea can be applied to any of the GMM estimators considered in this paper. Our focus is on the AAH estimator.

for $i = 1, 2, ..., n$, and $t = 1, 2, ..., T$. We consider $\phi = 0.4, 0.6, 0.8, 0.9$ and report results for $\phi = 0.4$ and 0.8 in the body of the paper.[15] Individual effects are generated as

$$\alpha_i = \sum_{\tau=1}^{T} \rho^{\tau} u_{i\tau} + \varepsilon_i,\ \varepsilon_i \sim IIDN\left(1, 1\right). \tag{43}$$

We consider two values for $\rho = 0$ or 0.8. When $\rho \neq 0$ then the individual effects are correlated with errors u_{it}, and AB and BB restrictions implicit in (8)-(9), respectively, are not satisfied. The processes are initialized as

$$y_{i,0} = \mu_i + \kappa\varepsilon_i + \upsilon_i,\ \upsilon_i \sim IIDN\left(0, 1\right), \tag{44}$$

where $\mu_i = \alpha_i / \left(1 - \phi\right)$. We consider two values for $\kappa = 0$ or 1. When $\kappa \neq 0$ the individual effects are correlated with the deviations of initial values from their long-run means μ_i, and BB restrictions implicit in (9) are not satisfied. But setting $\kappa \neq 0$ on its own does not invalidate the AB restrictions implicit in (8). We also need $\rho \neq 0$.

Restriction $\kappa = 0$ rules out any systematic deviations of initial values from their long-run means. It is less likely to hold in empirical applications, where individual dynamic processes over i might have been initialized from a recent past and possibly from non-stationary initial value distributions. In contrast, the restriction $\rho = 0$ appears much less restrictive, since it would be satisfied whenever fixed effects are uncorrelated with innovations.

The idiosyncratic errors, u_{it}, are generated as non-Gaussian processes with heteroskedastic error variances over i and t, namely $u_{it} = \left(e_{it} - 2\right)\sigma_{ia}/2$ for $t \leq [T/2]$, and $u_{it} = \left(e_{it} - 2\right)\sigma_{ib}/2$ for $t > [T/2]$, with $\sigma_{ia}^2 \sim IIDU\left(0.25, 0.75\right)$, $\sigma_{ib}^2 \sim IIDU\left(1, 2\right)$, and $e_{it} \sim IID\chi^2\left(2\right)$, where $[T/2]$ is the integer part of $T/2$. σ_{ia}^2 and σ_{ib}^2 are generated independently of e_{it}. This ensures that the errors have zero means, and heteroskedastic both conditionally and unconditionally, in particular, $V\left(u_{it}|\,\sigma_{ia}\right) = \sigma_{ia}^2$ for $t \leq [T/2]$, and $V\left(u_{it}|\,\sigma_{ib}\right) = \sigma_{ib}^2$ for $t > [T/2]$. We consider a comprehensive set of choices of $T = 3, 4, 5, 6, 8, 10, 12, 14, 16, 18, 20$ and $n = 100, 200, 500, 1000, 2000, 4000, 8000$. Findings for selected sample choices are reported below, whilst the full set of results is available from authors upon request. $2,000$ replications were carried out for each experiment.

Besides the parameter of interest ϕ, the key parameters of the MC design are κ and ρ. AH and AAH estimators are valid for all values of κ and ρ. AB estimators require $\rho = 0$, and the

[15] Findings for the remaining values of ϕ are available from authors upon request.

BB estimator requires $\rho = 0$ and $\kappa = 0$. Consequently, we consider the following three sets of experiments, based on values of ρ and κ: (i) experiments with $\rho = 0$ and $\kappa = 0$ labeled as experiments where both AB and BB restrictions are met; (ii) experiments with $\rho = 0$ and $\kappa \neq 0$ labeled as experiments where BB restrictions are not met whilst AB restrictions are met, and (iii) experiments with $\rho \neq 0$ and $\kappa \neq 0$ labeled as experiments where neither AB nor BB restrictions are met.

7.2 Estimation methods

We consider 2-step GMM estimators based on the AH moment conditions given by (7), the AAH moment conditions given by (7) and (13), the Arellano and Bond's first-difference moment conditions given by (8), and the Arellano and Bover's and Blundell and Bond's system moment conditions given by (8)-(9).[16] These estimators are labeled below as AH, AAH, AB, and BB, respectively. Inference is conducted using the conventional standard errors. In addition to two-step GMM estimator based on AAH moments, we also consider using OCMT to select relevant AAH moments, as discussed in Subsection 6.1. We denote this estimator by AAH-O. In particular, the AAH-O estimator is based on the union of $T-2$ quadratic moments in (13) and $\hat{h}_{nT}$ selected subset of AH moments using the OCMT procedure described in Subsection 6.1. Also since $T-2 \leq \hat{h}_{nT} \leq (T-2)(T-1)/2$, then the number of moments for the AAH-O estimator lies between $2(T-2)$ and $(T-2)(T-1)/2+T-2$.

7.3 Monte Carlo findings

7.3.1 Comparison of AH and AAH estimators

We first focus on the comparison of AH and AAH estimators in experiments where both AB and BB restrictions are met ($\rho = 0$ & $\kappa = 0$).[17] Results for bias and RMSE (both $\times 100$) of estimating ϕ are reported in Table 2, and size and power of the tests at the 5% nominal level are reported in Table 3 and Figure A1 in the Appendix. Table 2 shows very large RMSE values for the AH estimator, especially when $T = 4$. Once the set of AH moment conditions (7) is augmented by the quadratic moment conditions in (13), we see a substantial drop in the reported RMSE values. The small sample improvements in RMSE are about four to five-fold for $T = 4$, and smaller but still

[16] We found that continious-updating (CU) GMM estimators exhibit often worse performance than the 2-step estimators in our experiments. A comparison of two-step and CU GMM estimators is available in an earlier version of this paper, Chudik and Pesaran (2017).

[17] Findings for the relative performance of AH and AAH estimators are similar for other experiments, available from authors upon request.

substantial for larger values of T, all regardless of n. The relative RMSE differences are somewhat more pronounced when $\phi = 0.8$, as compared to the ones obtained for $\phi = 0.4$. Compared to the AH estimator, the AAH estimator is less biased in almost all reported cases, and has a smaller RMSE even for $T = 14$. This suggests that correcting for bias will be unimportant for the reported sample choices.

In line with the bias and RMSE findings, we see in Table 3 that there are substantial gains in power from the augmentation of the AH moments with the new quadratic moment conditions in (13). These differences can be seen more clearly in Figure A1 in the Appendix, shown for the sample combinations, $n = 1000$, $T = 4$ and 6. The empirical power functions of the AH estimator are rather flat when $\phi = 0.8$, and $T = 4$. As to be expected, the results for the AH estimator improve with a decrease in ϕ (as AH instruments become stronger), and/or a rise in T. In contrast, the empirical power function of the AAH estimator is much more satisfactory. The size of the AH and AAH estimators reported in Table 3 are close to their nominal value of 0.05, in cases where T/n is sufficiently small. For $T = 4$, size is close to 5 per cent for all $n \geq 500$, but it deteriorates when the number of moments is large relative to the number of cross-section units, which is a well-known problem in the GMM literature.

7.3.2 Comparison of AAH and AAH-O estimators

As noted earlier, with an increase in T, the number of moments becomes large, many of which could be relatively weak. In such a case, using a well chosen sub-set of moments could improve the small sample performance. We investigate the small sample benefits and drawbacks of using OCMT procedure described in Section 6.1 to select a subset or relevant AH moments. The AAH-O estimator is based on the union of $T - 2$ quadratic moments (13) and the selected subset of AH moments. We expect that for a fixed T and as $n \to \infty$, all relevant moments will be selected by OCMT procedure and therefore asymptotically AAH, and AAH-O achieve the same variance (for a fixed T), although AAH-O could have lower or higher RMSE compared with AAH in finite samples. These expectations are in line with the reported findings in Tables 4-5 and Figure A2 in the Appendix. First, the average number of moments (reported in the last columns of Table 4) increases in n for a fixed T, since all of the AAH moments are relevant albeit with a varying degree of strength. The differences in RMSE values between AAH and AAH-O estimators are negligible for large values of n, as expected. Second, AAH-O outperforms AAH in cases where T/n

is relatively large. For example, when $n = 100$, and $T > 10$. However, for intermediate cases with more moderate T/n ratio, AAH-O tends to perform less well as compared to the AAH estimator in terms of RMSEs. The size distortion of AAH-O is not as serious as the size distortions of AAH, but still quite substantial in the case of experiments where $T \geq 10$ and $n < 2000$.

7.3.3 Comparison of AAH with AB and BB estimators

We now turn to the small sample performance of AAH relative to the AB and BB estimators. Comparisons for experiments where both AB and BB restrictions are met are reported in Tables 6-7 and Figure A3 in the Appendix. In these experiments AAH is asymptotically less efficient than BB, and this is reflected in the lower values of RMSEs obtained for the BB estimator; although it is interesting to note that these differences are not large in many cases. This result is also in line with the asymptotic relative efficiency of the BB estimator reported in Table 1. The situation is very different when the AAH estimator is compared to the AB estimator. As can be seen from Table 6, in all cases the AAH estimator performs better (in many cases substantially so) than the AB estimator. Size of the tests based on the individual estimators is close to 5 per cent when n is sufficiently large relative to T, otherwise when T is large relative to n inference could be unsafe with substantial over-rejections.[18]

To investigate the factors behind the better performance of the BB estimator, we now consider experiments where individual effects are correlated with the deviations of initial values $y_{i0} - \mu_i$, by setting $\kappa = 1$. In these experiments, reported in Tables 8-9 and Figure A4 in the Appendix, the restrictions underlying the BB estimator are not met. Hence, BB is estimator is no longer consistent, which shows the BB estimator having large biases and close to 100 per cent size rejections. The remaining two estimators (AAH and AB) are consistent and their relative performance is very similar to the previous experiments reported in Tables 6-7, with the proposed AAH estimator generally dominating the AB estimator.

In the last set of experiments, reported in Tables 10-11, we also allow for correlation of errors and fixed effects (by setting the parameter $\rho = 0.8$), in addition to $\kappa = 1$. In these experiments AAH continues to be valid, but the moment conditions of AB and BB are both violated. As a result both of these estimators perform very poorly, and exhibit large biases and substantial size

[18] The size performance can be improved upon by considering alternative estimates of standard errors, such as Windmeijer (2005) finite sample corrections for the standard errors of two-step GMM estimators, or Newey and Windmeijer (2009) standard errors for the CU-GMM estimators. These or other alternative estimators of standard errors are not pursued in this paper.

distortions even when $T = 4$. In contrast, the MC findings for the AAH estimator perform well, and in fact are numerically identical to those reported in Tables 8-9, since due to first-differencing the AAH estimator is not affected by the values of ρ and κ.[19]

Overall, the MC findings show that the AAH estimator is robust and outperform its 'cousin', the AH estimator by a wide margin. The AB and BB estimators are not robust to $\rho \neq 0$, and BB is also not robust to $\kappa \neq 0$. In the case of experiments with $\rho = 0$ & $\kappa = 0$, the AAH estimator continues to outperform the AB estimator, but performs less well when compared to the BB estimator, which is obtained under a much stronger set of restrictions (given by (11)). In practice it is not known whether these additional restrictions on the initialization of dynamic processes are satisfied, and violation of these conditions renders the BB estimator inconsistent.

7.3.4 Hausman test for a comparison of AAH and BB estimators

We now consider the small sample performance of the Hausman test proposed in Subsection 4.1. This test compares AAH and BB estimators. As already noted, under the null hypothesis of BB conditions holding, we have $Var\,(AAH) \leq Var\,(BB)$, whereas BB estimator will be inconsistent if BB conditions are not met. Table 12 shows the rejection rates of Hausman test (defined by (30)) at 5 per cent nominal level under the null that BB conditions are met (namely $H_0 : \rho = \kappa = 0$), as well as the rejection rates under the alternative hypothesis $H_1 : \rho = 0$ and $\kappa = 1$, under which the BB conditions do not hold. Our findings suggest that the Hausman test has relatively good size for n sufficiently large. However, rejection rates increase well beyond the 5 per cent nominal level as T increases and n is not sufficiently large. These distortions can be observed in sample sizes where $Var\,(AAH)$ and $Var\,(BB)$ are not well estimated due to large number of moments and n not being sufficiently large. Under the null hypothesis we also observe a large incidence of cases (reported in the right part of Table 2) where $\widehat{Var}\,(AAH) < \widehat{Var}\,(BB)$ and Hausman test is therefore not applicable. A large number of these cases are reported due to very small differences in RMSE values reported earlier in Table 6, in particular for larger values of T.

Rejection rates under the alternative hypothesis ($H_1 : \rho = 0$ and $\kappa = 1$) are quite large and quickly approach unity as n increases, suggesting relatively good power of the Hausman test for this design. Overall, Hausman test seems to work well when T is small relative to n, but as T is increased we observe size distortions very similar to the ones reported in Table 7.

[19] To make the results in Tables 10 & 11 and Tables 8 & 9 comparable we have used the same seed for generating the random numbers.

8 Concluding remarks

Instead of focusing on instruments that are uncorrelated with the errors, this paper proposes to use the regressors themselves in cases where the non-zero correlation between the regressors and the errors can be derived. This approach will lead to possibly nonlinear bias-corrected moment conditions. In this paper this idea is applied to the estimation of short-T dynamic panel data models, and a new augmented Anderson-Hsiao (AAH) estimator is proposed without making additional restrictions. The basic idea has potential applications in other settings, including spatial panel data models. An application is provided by Pesaran and Yang (2021). The idea can also be exploited to estimate unknown parameters of a known distributional functional form of slope coefficients in short-T autoregressive or vector autoregressive panels with heterogenous slope coefficients, which we leave for future research.

The proposed AAH estimator is applicable under less restrictive conditions on the initialization of the dynamic processes and the individual effects as compared to the leading first-difference and system-GMM methods advanced in the literature. It is, however, acknowledged that AAH estimator can be less efficient asymptotically when the stricter requirements of the system GMM estimator proposed by Blundell and Bond hold. The robustness of the AAH estimators is likely to be an advantage in practice where it is not possible to know if the stronger requirements of the system-GMM estimators are met, and thus avoid possible estimation bias and incorrect inference.

To decide between AAH and BB estimators in empirical applications we also propose a Hausman type test which is shown to work well when T is small and n sufficiently large.

This paper only considered panels with a fixed T. In panels with $n, T \to \infty$ jointly, there is an important issue that pertains to the GMM approach, namely the problem of combining a large number of moment conditions. We have briefly discussed this topic and proposed using OCMT to select a subset of relevant moment conditions as a simple way to mitigate the adverse effects of moments proliferation.

Table 2: Bias and RMSE of AH and AAH estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

		Bias ($\times 100$)				**RMSE**($\times 100$)			
		$\phi_0 = 0.4$		$\phi_0 = 0.8$		$\phi_0 = 0.4$		$\phi_0 = 0.8$	
T	n	AH	AAH	AH	AAH	AH	AAH	AH	AAH
4	100	-11.18	1.22	-39.91	0.22	51.22	12.05	94.44	14.56
4	200	-3.70	0.36	-24.55	0.21	34.55	7.44	75.82	10.65
4	500	-2.43	-0.01	-10.60	-0.01	19.97	4.05	41.66	6.40
4	1000	-0.83	0.13	-3.78	0.13	13.78	2.88	26.43	4.33
4	2000	-0.71	-0.10	-2.21	-0.10	9.78	2.03	17.50	3.04
4	8000	-0.08	-0.01	-0.15	-0.01	4.65	1.06	8.40	1.53
6	100	-5.01	1.08	-23.59	0.14	17.29	9.86	38.13	9.27
6	200	-2.42	0.24	-12.94	-0.06	11.74	6.08	25.62	6.51
6	500	-1.05	-0.14	-5.20	-0.39	7.13	2.64	14.30	3.70
6	1000	-0.37	-0.01	-2.42	-0.14	5.09	1.83	9.91	2.49
6	2000	-0.19	-0.07	-1.28	-0.14	3.51	1.32	6.71	1.75
6	8000	-0.03	-0.02	-0.20	-0.05	1.75	0.63	3.23	0.86
10	100	-3.40	0.48	-14.08	0.22	8.84	5.66	19.59	6.38
10	200	-1.49	0.17	-7.05	-0.01	5.59	3.03	11.61	3.92
10	500	-0.60	-0.03	-2.54	-0.04	3.37	1.82	6.05	2.32
10	1000	-0.22	-0.03	-1.20	-0.12	2.36	1.25	3.97	1.46
10	2000	-0.16	-0.03	-0.60	-0.07	1.65	0.84	2.74	0.98
10	8000	-0.04	-0.01	-0.14	-0.03	0.79	0.41	1.30	0.49

Notes: "AH" is the 2-step GMM estimator based on the $(T-2)(T-1)/2$ Anderson and Hsiao's moment conditions (7), "AAH" is the augmented Anderson and Hsiao 2-step GMM estimator based on the $(T-2)(T-1)/2+T-2$ moment conditions (7) and (13). The DGP is given by $y_{it} = \alpha_i + \phi y_{i,t-1} + u_{it}$, for $i = 1,2,...,n$, and $t = 1,2,...,T$, with $y_{i,0} = \mu_i + \kappa\varepsilon_i + \upsilon_i$, where $\mu_i = \alpha_i/(1-\phi)$, $\alpha_i = \sum_{t=1}^{T} \rho^t u_{it} + \varepsilon_i$, $\varepsilon_i \sim IIDN(1,1)$, and $\upsilon_i \sim IIDN(0,1)$. This table reports findings for experiments where $\kappa = \rho = 0$, namely AB and BB restrictions are met. BB restrictions are not satisfied when $\kappa \neq 0$, and AB restrictions are not satisfied when $\rho \neq 0$. Errors u_{it} are generated to be cross-sectionally heteroskedastic and non-normal, $u_{it} = (e_{it}-2)\sigma_{ia}/2$ for $t \leq [T/2]$, and $u_{it} = (e_{it}-2)\sigma_{ib}/2$ for $t > [T/2]$, with $\sigma_{ia}^2 \sim IIDU(0.25, 0.75)$, $\sigma_{ib}^2 \sim IIDU(1,2)$, $e_{it} \sim IID\chi^2(2)$, and $[T/2]$ is the integer part of $T/2$. See Section 7 for a full description of the MC experiments. The number of datapoints required is $T+1$, namely $y_{i0}, y_{i1}, ..., y_{iT}$, for $i = 1,2,...,n$.

Table 3: Size and Power of AH and AAH estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

		Size (5% level, $\times 100$)				**Power (5% level,** $\times 100$, $H_1: \phi = \phi_0 + 0.1$)			
		$\phi_0 = 0.4$		$\phi_0 = 0.8$		$\phi_0 = 0.4$		$\phi_0 = 0.8$	
T	n	AH	AAH	AH	AAH	AH	AAH	AH	AAH
4	100	11.0	10.2	19.2	12.7	16.0	35.0	23.2	30.6
4	200	7.5	7.4	13.0	9.5	12.3	45.6	17.3	34.0
4	500	6.3	6.1	7.9	6.6	13.9	71.7	12.8	47.6
4	1000	5.2	5.3	5.8	5.9	16.6	91.7	11.9	65.9
4	2000	6.3	5.3	5.3	5.8	23.5	99.5	13.2	88.1
4	8000	5.0	5.6	4.9	5.8	56.8	100.0	23.3	100.0
6	100	18.3	20.5	31.1	20.3	33.9	58.0	43.4	53.9
6	200	11.2	12.0	19.1	13.7	31.4	76.1	31.8	61.8
6	500	7.4	8.0	9.5	9.5	40.0	96.9	26.8	84.7
6	1000	6.5	6.1	7.5	6.5	57.4	100.0	30.5	96.4
6	2000	4.8	5.5	5.5	6.3	82.1	100.0	42.3	100.0
6	8000	4.6	4.1	4.5	5.2	100.0	100.0	87.2	100.0
10	100	40.5	47.3	58.9	49.3	76.6	88.4	83.3	85.4
10	200	20.5	24.1	32.6	27.9	77.9	97.2	74.6	93.4
10	500	10.4	12.1	14.4	15.3	93.7	100.0	75.8	99.8
10	1000	8.3	9.6	8.8	11.3	99.4	100.0	88.8	100.0
10	2000	7.1	6.2	7.4	7.3	100.0	100.0	98.5	100.0
10	8000	5.3	5.2	5.0	5.9	100.0	100.0	100.0	100.0

See the notes to Table 2

Table 4: Bias and RMSE of AAH and AAH-O estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

		Bias ($\times 100$)				**RMSE**($\times 100$)				**Average number of moments**		
		$\phi_0 = 0.4$		$\phi_0 = 0.8$		$\phi_0 = 0.4$		$\phi_0 = 0.8$			AAH-O	
T	n	AAH	AAH-O	AAH	AAH-O	AAH	AAH-O	AAH	AAH-O	AAH	$\phi_0 = 0.4$	$\phi_0 = 0.8$
10	100	0.48	0.12	0.22	-0.39	5.66	5.53	6.38	7.21	44	20	17
10	200	0.17	-0.05	-0.01	-0.56	3.03	3.12	3.92	4.29	44	23	18
10	500	-0.03	-0.07	-0.04	-0.25	1.82	1.90	2.32	2.57	44	28	22
10	1000	-0.03	-0.05	-0.12	-0.26	1.25	1.28	1.46	1.55	44	33	25
10	2000	-0.03	-0.05	-0.07	-0.14	0.84	0.85	0.98	1.02	44	39	29
10	8000	-0.01	-0.01	-0.03	-0.03	0.41	0.41	0.49	0.49	44	43	44
16	100	0.73	0.09	-0.59	-0.11	7.93	4.16	9.24	6.02	119	35	30
16	200	0.22	-0.12	0.42	-0.57	2.92	2.63	3.69	4.00	119	41	32
16	500	0.06	-0.02	0.04	-0.32	1.44	1.45	1.58	1.83	119	56	39
16	1000	0.03	0.00	-0.01	-0.20	0.93	0.97	0.96	1.15	119	71	51
16	2000	-0.03	-0.04	-0.05	-0.16	0.62	0.64	0.63	0.70	119	86	65
16	8000	0.00	0.00	0.00	-0.01	0.31	0.31	0.31	0.31	119	112	103
20	100	-1.73	-0.06	-9.91	-0.34	8.22	3.73	14.22	5.72	189	45	39
20	200	0.55	-0.09	1.07	-0.67	4.63	2.39	5.32	3.51	189	53	41
20	500	0.07	-0.04	0.08	-0.36	1.38	1.33	1.46	1.86	189	72	49
20	1000	0.02	-0.01	0.00	-0.22	0.84	0.86	0.82	1.04	189	96	64
20	2000	0.00	-0.02	0.00	-0.12	0.55	0.57	0.53	0.60	189	121	92
20	8000	0.00	0.00	0.00	-0.02	0.27	0.27	0.25	0.26	189	171	139

Notes: See the notes to Table 2. "AAH-O" estimator is the two-step GMM estimator based on $T-2$ quadratic moment conditions (13) and a subset of $(T-2)(T-1)/2$ AAH moment conditions (7) selected by OCMT. See section 7 for a full description of the MC experiments.

Table 5: Size and power of AAH and AAH-O estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

		Size (5% level, $\times 100$**)**				**Power (5% level,** $\times 100$, $H_1 : \phi = \phi_0 + 0.02$**)**			
		$\phi_0 = 0.4$		$\phi_0 = 0.8$		$\phi_0 = 0.4$		$\phi_0 = 0.8$	
T	n	AAH	AAH-O	AAH	AAH-O	AAH	AAH-O	AAH	AAH-O
10	100	47.3	25.6	49.3	29.6	49.5	32.6	52.3	36.3
10	200	24.1	15.4	27.9	18.7	31.6	25.0	36.4	28.5
10	500	12.1	10.7	15.3	14.6	34.8	30.5	34.2	33.0
10	1000	9.6	9.5	11.3	12.4	46.2	44.1	42.4	44.7
10	2000	6.2	7.0	7.3	8.8	71.0	71.2	61.4	64.9
10	8000	5.2	5.3	5.9	6.1	99.8	99.9	98.0	98.1
16	100	85.9	34.6	85.4	41.9	86.7	41.1	86.3	48.1
16	200	59.4	22.3	61.9	28.5	63.4	36.3	66.6	43.3
16	500	22.0	12.9	27.7	18.8	56.6	44.6	59.3	47.0
16	1000	13.8	10.8	16.1	16.0	71.8	66.3	74.7	69.5
16	2000	8.6	7.9	10.0	11.1	93.1	91.8	93.4	92.8
16	8000	5.7	5.8	6.2	7.2	100.0	100.0	100.0	100.0
20	100	66.8	38.9	78.2	46.1	70.1	48.2	82.8	53.8
20	200	93.1	24.8	92.0	32.3	92.8	41.6	92.7	49.1
20	500	33.4	15.1	39.1	20.3	69.6	54.2	72.0	54.7
20	1000	18.7	11.2	20.8	15.9	82.5	75.7	87.0	75.7
20	2000	10.4	9.0	11.9	11.7	96.5	95.4	98.4	96.7
20	8000	7.3	6.8	6.4	7.5	100.0	100.0	100.0	100.0

See the notes to Tables 2 and 4.

Table 6: Bias and RMSE of AAH, AB and BB estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

		Bias ($\times 100$)						**RMSE**($\times 100$)					
		$\phi_0 = 0.4$			$\phi_0 = 0.8$			$\phi_0 = 0.4$			$\phi_0 = 0.8$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	1.22	-4.92	2.01	0.22	-45.29	2.44	12.05	20.08	9.84	14.56	73.89	11.16
4	200	0.36	-2.18	0.99	0.21	-29.61	1.10	7.44	13.84	6.57	10.65	55.67	7.64
4	500	-0.01	-0.85	0.40	-0.01	-14.09	0.25	4.05	8.65	4.07	6.40	32.95	4.55
4	1000	0.13	-0.41	0.32	0.13	-6.41	0.21	2.88	6.13	2.85	4.33	21.54	3.26
4	2000	-0.10	-0.26	0.01	-0.10	-3.46	-0.07	2.03	4.36	2.01	3.04	14.81	2.40
4	8000	-0.01	-0.03	0.02	-0.01	-0.55	-0.02	1.06	2.13	1.03	1.53	7.25	1.16
6	100	1.08	-4.36	1.00	0.14	-25.52	2.72	9.86	12.77	6.92	9.27	37.03	7.99
6	200	0.24	-2.00	0.44	-0.06	-15.32	1.31	6.08	8.56	4.44	6.51	25.24	5.49
6	500	-0.14	-0.83	0.04	-0.39	-6.53	0.26	2.64	5.16	2.68	3.70	14.21	3.24
6	1000	-0.01	-0.26	0.08	-0.14	-3.03	0.13	1.83	3.63	1.84	2.49	9.43	2.23
6	2000	-0.07	-0.16	-0.03	-0.14	-1.61	-0.03	1.32	2.47	1.31	1.75	6.25	1.54
6	8000	-0.02	-0.06	-0.01	-0.05	-0.36	-0.03	0.63	1.24	0.63	0.86	3.00	0.77
10	100	0.48	-3.14	0.65	0.22	-14.73	2.53	5.66	8.24	5.23	6.38	19.91	6.00
10	200	0.17	-1.44	0.32	-0.01	-7.79	1.40	3.03	5.02	3.04	3.92	11.92	3.84
10	500	-0.03	-0.62	0.03	-0.04	-2.97	0.33	1.82	3.04	1.83	2.32	6.11	2.16
10	1000	-0.03	-0.22	0.00	-0.12	-1.38	0.07	1.25	2.07	1.26	1.46	3.92	1.44
10	2000	-0.03	-0.14	-0.02	-0.07	-0.70	0.01	0.84	1.42	0.84	0.98	2.66	0.95
10	8000	-0.01	-0.04	-0.01	-0.03	-0.17	-0.01	0.41	0.70	0.41	0.49	1.26	0.46

Notes: See the notes to Table 2. "AB" is the 2-step GMM estimator based on the Arellano and Bond's first-difference moment conditions (8), and "BB" is the 2-step GMM estimator based on the Arellano and Bover's and Blundell and Bond's system moment conditions (8)-(9).

Table 7: Size and power of AAH, AB and BB estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

		Size (5% level, $\times 100$**)**						**Power (5% level,** $\times 100$, $H_1: \phi = \phi_0 + 0.1$**)**					
		$\phi_0 = 0.4$			$\phi_0 = 0.8$			$\phi_0 = 0.4$			$\phi_0 = 0.8$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	10.2	12.0	15.9	12.7	30.1	28.7	35.0	21.4	32.5	30.6	36.5	32.4
4	200	7.4	9.0	10.3	9.5	21.0	19.8	45.6	21.5	43.5	34.0	27.3	39.0
4	500	6.1	6.5	7.0	6.6	12.0	10.9	71.7	27.9	71.1	47.6	19.8	65.1
4	1000	5.3	6.2	6.0	5.9	8.1	8.5	91.7	41.9	92.7	65.9	16.7	88.7
4	2000	5.3	5.9	5.1	5.8	6.5	7.6	99.5	67.2	99.6	88.1	18.8	99.5
4	8000	5.6	4.7	5.7	5.8	5.8	5.2	100.0	99.8	100.0	100.0	32.6	100.0
6	100	20.5	21.1	27.8	20.3	41.2	46.1	58.0	45.9	61.8	53.9	56.2	56.3
6	200	12.0	14.0	16.5	13.7	25.9	30.1	76.1	44.8	76.5	61.8	43.8	68.2
6	500	8.0	7.9	9.2	9.5	13.4	14.8	96.9	62.6	96.9	84.7	35.3	94.6
6	1000	6.1	7.1	6.2	6.5	8.8	9.0	100.0	83.4	100.0	96.4	37.9	99.8
6	2000	5.5	5.2	6.0	6.3	5.8	7.5	100.0	97.7	100.0	100.0	50.1	100.0
6	8000	4.1	4.6	4.6	5.2	4.6	5.4	100.0	100.0	100.0	100.0	92.6	100.0
10	100	47.3	48.0	56.9	49.3	67.8	72.8	88.4	82.7	90.2	85.4	88.6	86.0
10	200	24.1	23.0	26.2	27.9	39.9	45.1	97.2	87.3	98.1	93.4	81.1	95.1
10	500	12.1	11.9	14.0	15.3	16.8	22.9	100.0	97.1	100.0	99.8	82.4	100.0
10	1000	9.6	8.5	10.3	11.3	10.2	13.7	100.0	100.0	100.0	100.0	91.5	100.0
10	2000	6.2	6.8	6.8	7.3	7.5	9.1	100.0	100.0	100.0	100.0	99.1	100.0
10	8000	5.2	5.3	5.2	5.9	5.3	5.4	100.0	100.0	100.0	100.0	100.0	100.0

See the notes to Tables 2 and 6.

Table 8: Bias and RMSE of AAH, AB and BB estimators when Arellano and Bond (AB) restrictions are met and Blundell and Bond (BB) restrictions are not met

		Bias ($\times 100$)						**RMSE**($\times 100$)					
		$\phi_0 = 0.4$			$\phi_0 = 0.8$			$\phi_0 = 0.4$			$\phi_0 = 0.8$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	1.20	-0.80	23.34	0.54	-7.56	18.94	11.70	8.62	24.96	14.23	23.42	20.44
4	200	0.31	-0.33	23.96	0.32	-3.21	20.34	6.95	6.15	24.80	10.33	15.30	20.93
4	500	-0.02	-0.12	24.75	-0.02	-1.24	21.41	3.30	3.84	25.10	5.89	9.46	21.60
4	1000	0.06	-0.07	24.88	0.09	-0.75	21.82	2.36	2.76	25.05	4.03	6.81	21.91
4	2000	-0.08	-0.03	24.96	-0.10	-0.31	22.03	1.65	1.97	25.05	2.84	4.78	22.07
4	8000	-0.02	0.00	25.08	-0.02	-0.04	22.11	0.85	0.96	25.10	1.44	2.35	22.12
6	100	0.75	-1.41	12.91	0.50	-7.39	15.90	8.12	6.78	14.62	9.35	15.46	16.80
6	200	0.23	-0.57	13.12	0.10	-3.44	17.71	5.41	4.58	14.03	6.33	9.73	18.18
6	500	-0.08	-0.21	13.44	-0.36	-1.31	19.18	2.24	2.78	13.83	3.37	5.65	19.36
6	1000	0.02	-0.02	13.78	-0.11	-0.51	19.87	1.58	2.00	13.98	2.30	3.93	19.95
6	2000	-0.05	-0.04	13.76	-0.12	-0.29	20.14	1.12	1.35	13.86	1.63	2.66	20.18
6	8000	-0.02	-0.02	13.89	-0.05	-0.11	20.33	0.55	0.69	13.91	0.81	1.36	20.34
10	100	0.48	-1.68	6.17	0.54	-6.44	10.01	4.82	5.68	7.92	6.22	10.79	11.13
10	200	0.16	-0.72	6.11	0.08	-2.99	11.15	2.72	3.46	6.95	3.73	6.19	11.80
10	500	-0.02	-0.30	6.21	-0.07	-1.10	12.27	1.67	2.11	6.57	2.01	3.33	12.57
10	1000	-0.01	-0.09	6.30	-0.12	-0.49	12.54	1.13	1.44	6.49	1.36	2.27	12.71
10	2000	-0.02	-0.06	6.36	-0.07	-0.24	12.78	0.77	1.00	6.45	0.92	1.58	12.87
10	8000	-0.01	-0.02	6.46	-0.02	-0.06	12.94	0.38	0.49	6.48	0.45	0.76	12.96

See the notes to Tables 2 and 6.

Table 9: Size and power of AAH, AB and BB estimators in experiments when Arellano and Bond (AB) restrictions are met and Blundell and Bond (BB) restrictions are not met

		Size (5% level, $\times 100$**)**						**Power (5% level,** $\times 100$, $H_1 : \phi = \phi_0 + 0.1$**)**					
		$\phi_0 = 0.4$			$\phi_0 = 0.8$			$\phi_0 = 0.4$			$\phi_0 = 0.8$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	9.3	9.4	95.3	12.1	11.9	92.9	40.7	33.3	82.6	30.8	21.4	73.4
4	200	6.9	8.7	99.6	8.8	8.4	98.1	56.0	45.4	91.7	35.7	20.0	90.4
4	500	6.3	6.4	100.0	6.3	6.0	100.0	84.8	76.7	99.5	50.7	23.8	99.3
4	1000	5.2	5.4	100.0	5.3	5.8	100.0	98.0	96.1	100.0	69.7	38.0	100.0
4	2000	4.8	6.1	100.0	5.6	5.9	100.0	100.0	100.0	100.0	91.4	58.9	100.0
4	8000	5.6	5.1	100.0	5.5	4.8	100.0	100.0	100.0	100.0	100.0	98.7	100.0
6	100	17.5	16.8	90.0	19.9	23.2	96.6	67.4	60.4	61.4	55.3	47.0	78.8
6	200	10.4	10.0	97.5	12.8	12.6	99.0	84.5	73.9	59.0	64.3	44.6	90.4
6	500	7.1	6.3	100.0	8.4	6.4	100.0	99.5	97.3	68.4	87.2	57.3	98.6
6	1000	6.4	6.0	100.0	5.9	5.6	100.0	100.0	99.8	81.9	98.2	78.5	100.0
6	2000	5.4	4.5	100.0	5.8	4.3	100.0	100.0	100.0	93.9	100.0	96.6	100.0
6	8000	4.1	4.6	100.0	4.9	5.1	100.0	100.0	100.0	100.0	100.0	100.0	100.0
10	100	46.1	44.1	84.8	48.1	53.3	95.3	90.4	90.5	77.6	85.0	87.5	77.7
10	200	22.1	21.4	89.4	26.4	26.5	98.8	98.8	97.5	76.7	95.3	88.9	72.1
10	500	12.9	10.7	98.6	14.0	10.9	100.0	100.0	100.0	86.6	99.7	97.8	72.8
10	1000	8.9	8.3	99.9	10.3	7.6	100.0	100.0	100.0	95.6	100.0	99.9	81.5
10	2000	6.7	6.6	100.0	7.1	6.3	100.0	100.0	100.0	99.5	100.0	100.0	91.9
10	8000	5.0	5.0	100.0	5.2	5.7	100.0	100.0	100.0	100.0	100.0	100.0	99.9

See the notes to Tables 2 and 6.

Table 10: Bias and RMSE of AAH, AB and BB estimators in experiments when AB and BB restrictions are not met

		Bias ($\times 100$)						**RMSE**($\times 100$)					
		$\phi_0 = 0.4$			$\phi_0 = 0.8$			$\phi_0 = 0.4$			$\phi_0 = 0.8$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	1.20	-13.32	14.81	0.54	-49.05	12.90	11.70	16.12	16.92	14.23	56.54	16.03
4	200	0.31	-11.97	14.90	0.32	-40.60	14.06	6.95	13.77	15.99	10.33	44.93	15.38
4	500	-0.02	-11.00	15.29	-0.02	-34.86	15.01	3.30	11.83	15.75	5.89	36.98	15.52
4	1000	0.06	-10.57	15.32	0.09	-32.28	15.43	2.36	11.07	15.55	4.03	33.64	15.66
4	2000	-0.08	-10.24	15.33	-0.10	-30.79	15.76	1.65	10.53	15.45	2.84	31.56	15.88
4	8000	-0.02	-9.99	15.42	-0.02	-29.56	15.93	0.85	10.07	15.45	1.44	29.78	15.95
6	100	0.75	-7.79	9.94	0.50	-24.23	12.63	8.12	10.26	11.87	9.35	27.95	13.92
6	200	0.23	-6.59	10.05	0.10	-19.63	13.64	5.41	8.05	11.09	6.33	21.65	14.42
6	500	-0.08	-5.89	10.27	-0.36	-16.82	14.47	2.24	6.56	10.71	3.37	17.70	14.80
6	1000	0.02	-5.49	10.55	-0.11	-15.52	14.93	1.58	5.89	10.78	2.30	16.02	15.10
6	2000	-0.05	-5.42	10.54	-0.12	-15.08	15.08	1.12	5.63	10.65	1.63	15.35	15.16
6	8000	-0.02	-5.24	10.64	-0.05	-14.53	15.20	0.55	5.30	10.67	0.81	14.61	15.22
10	100	0.48	-3.18	5.53	0.54	-9.76	9.00	4.82	6.28	7.42	6.22	13.10	10.21
10	200	0.16	-2.28	5.52	0.08	-6.46	9.78	2.72	4.07	6.41	3.73	8.44	10.48
10	500	-0.02	-1.85	5.63	-0.07	-4.51	10.57	1.67	2.77	6.02	2.01	5.48	10.89
10	1000	-0.01	-1.62	5.74	-0.12	-3.88	10.69	1.13	2.15	5.94	1.36	4.43	10.87
10	2000	-0.02	-1.58	5.80	-0.07	-3.63	10.84	0.77	1.86	5.90	0.92	3.93	10.93
10	8000	-0.01	-1.53	5.91	-0.02	-3.44	10.93	0.38	1.60	5.93	0.45	3.52	10.96

See the notes to Tables 2 and 6.

Table 11: Size and power of AAH, AB and BB estimators in experiments when AB and BB restrictions are not met

		Size (5% level, $\times 100$)						**Power (5% level,** $\times 100$, $H_1 : \phi = \phi_0 + 0.1$)					
		$\phi_0 = 0.4$			$\phi_0 = 0.8$			$\phi_0 = 0.4$			$\phi_0 = 0.8$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	9.3	36.0	84.6	12.1	59.3	81.2	40.7	75.3	55.1	30.8	73.0	50.8
4	200	6.9	46.4	93.8	8.8	63.5	89.3	56.0	89.4	58.3	35.7	79.5	58.7
4	500	6.3	71.5	99.7	6.3	80.7	97.9	84.8	99.7	73.9	50.7	94.3	70.7
4	1000	5.2	90.8	100.0	5.3	93.8	100.0	98.0	100.0	87.0	69.7	98.8	86.4
4	2000	4.8	99.2	100.0	5.6	99.3	100.0	100.0	100.0	96.7	91.4	100.0	96.0
4	8000	5.6	100.0	100.0	5.5	100.0	100.0	100.0	100.0	100.0	100.0	100.0	100.0
6	100	17.5	34.9	83.0	19.9	60.8	91.5	67.4	87.3	53.9	55.3	85.6	66.0
6	200	10.4	37.3	91.4	12.8	62.6	95.3	84.5	96.6	49.6	64.3	92.3	71.3
6	500	7.1	53.7	99.4	8.4	82.8	99.5	99.5	100.0	45.5	87.2	99.6	81.4
6	1000	6.4	74.8	100.0	5.9	95.2	100.0	100.0	100.0	46.1	98.2	100.0	91.2
6	2000	5.4	94.0	100.0	5.8	99.6	100.0	100.0	100.0	45.5	100.0	100.0	97.7
6	8000	4.1	100.0	100.0	4.9	100.0	100.0	100.0	100.0	56.3	100.0	100.0	100.0
10	100	46.1	47.2	83.0	48.1	63.8	93.6	90.4	94.7	78.8	85.0	93.8	77.3
10	200	22.1	27.1	87.1	26.4	44.5	98.0	98.8	99.6	80.8	95.3	97.0	68.4
10	500	12.9	22.8	97.3	14.0	37.5	100.0	100.0	100.0	91.1	99.7	100.0	64.3
10	1000	8.9	24.8	99.9	10.3	46.4	100.0	100.0	100.0	98.0	100.0	100.0	65.1
10	2000	6.7	36.8	100.0	7.1	66.2	100.0	100.0	100.0	99.9	100.0	100.0	67.9
10	8000	5.0	86.9	100.0	5.2	99.6	100.0	100.0	100.0	100.0	100.0	100.0	79.8

See the notes to Tables 2 and 6.

Table 12: Empirical size and power of Hausman test applied to the difference between BB and AAH estimators at the 5% nominal level

		Rejection rates ($\times 100$)				Fraction of replications ($\times 100$) where Hausman test was not applicable due to $\widehat{Var}\left(\hat{\phi}^{aah}\right) - \widehat{Var}\left(\hat{\phi}^{bb}\right) < 0$			
		under H_0		under H_1		under H_0		under H_1	
T	n	$\phi_0 = 0.4$	$\phi_0 = 0.8$	$\phi_0 = 0.4$	$\phi_0 = 0.8$	$\phi_0 = 0.4$	$\phi_0 = 0.8$	$\phi_0 = 0.4$	$\phi_0 = 0.8$
4	100	15.27	17.60	93.54	54.08	28.60	11.65	0.10	2.65
4	200	9.92	13.08	99.65	70.67	28.95	11.70	0.00	0.95
4	500	9.41	8.91	100.00	93.25	27.20	9.05	0.00	0.05
4	1000	7.26	8.34	100.00	99.45	26.30	4.65	0.00	0.00
4	2000	8.88	7.28	100.00	100.00	22.30	2.45	0.00	0.00
4	8000	8.41	5.50	100.00	100.00	9.05	0.05	0.00	0.00
6	100	23.32	33.62	88.25	77.15	25.40	6.30	0.00	0.45
6	200	18.06	23.41	98.70	92.80	30.80	7.95	0.00	0.00
6	500	9.90	15.79	100.00	99.85	36.90	9.75	0.00	0.00
6	1000	6.60	14.47	100.00	100.00	39.40	8.45	0.00	0.00
6	2000	6.38	9.78	100.00	100.00	36.55	3.35	0.00	0.00
6	8000	5.97	7.07	100.00	100.00	30.50	0.30	0.00	0.00
10	100	44.92	58.80	79.58	86.42	20.20	4.50	0.10	0.25
10	200	21.62	41.71	91.05	97.10	34.10	9.25	0.00	0.00
10	500	9.29	25.83	99.55	99.95	39.20	18.70	0.00	0.00
10	1000	5.65	20.95	100.00	100.00	45.10	22.90	0.00	0.00
10	2000	4.29	13.21	100.00	100.00	42.95	21.65	0.00	0.00
10	8000	3.63	11.40	100.00	100.00	44.90	14.45	0.00	0.00

Notes: Reported rejection rates under the null correspond to DGP with $\rho = 0$ and $\kappa = 0$, i.e. both AB and BB conditions are met. Reported rejection rate under alternative correspond to DGP with $\rho = 0$ and $\kappa \neq 0$, i.e. BB conditions are not met, whilst AB conditions still hold. See also the notes to Tables 2 and 4.

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A Appendix

This appendix is organized as follows. Section A.1 derives $\bar{B}_3$ given by (21) and (22). Section A.2 states and proves a number of lemmas used in the rest of this appendix. Additional propositions and proofs are given in Section A.3. Section A.4 presents rejection frequencies for selected estimators considered in the Monte Carlo experiments in Section 7. Section A.5 presents additional Monte Carlo results for the panel ARX(1).model featuring a strictly exogenous covariate. Section A.6 presents an empirical application to earning dynamics using PSID dataset originally analyzed by Meghir and Pistaferri (2004).

A.1 Derivation of $\bar{B}_3$

Using (20), it readily follows that $\bar{B}_3 = \lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} E\left(B_{i3}\right)$, where

$$E\left(B_{i3}\right) = E\left(\Delta y_{i1}^2\right) + E\left(\Delta y_{i2}^2\right) + 2E\left(\Delta u_{i2}\Delta y_{i1}\right). \tag{A.1}$$

Also recall that $\Delta y_{i1} = u_{i1} - (1-\phi)\left(y_{i0}-\mu_i\right)$, and $\Delta y_{i2} = \phi\Delta y_{i1} + \Delta u_{i2}$. Hence $E\left(\Delta u_{i2}\Delta y_{i1}\right) = -\sigma_{i1}^2$,

$$E\left(\Delta y_{i1}^2\right) = \sigma_{i1}^2 + (1-\phi)^2 E\left(y_{i0}-\mu_i\right)^2 - 2\left(1-\phi\right)E\left[u_{i1}\left(y_{i0}-\mu_i\right)\right],$$

and

$$E\left(\Delta y_{i2}^2\right) = E\left(\phi^2\Delta y_{i1}^2 + \Delta u_{i2}^2 + 2\phi\Delta u_{i2}\Delta y_{i1}\right) = \phi^2 E\left(\Delta y_{i1}^2\right) + \left(1-2\phi\right)\sigma_{i1}^2 + \sigma_{i2}^2. \tag{A.2}$$

Using the above results in (A.1) now yields:

$$E\left(B_{i3}\right) = \left(\sigma_{i2}^2 - \sigma_{i1}^2\right) + (1-\phi)^2\sigma_{i1}^2 + \left(1+\phi^2\right)\left\{(1-\phi)^2 E\left(y_{i0}-\mu_i\right)^2 - 2\left(1-\phi\right)E\left[u_{i1}\left(y_{i0}-\mu_i\right)\right]\right\}.$$

Hence, as required, we have

$$\bar{B}_3 = \bar{\sigma}_2^2 - \bar{\sigma}_1^2 + (1-\phi)^2\bar{\sigma}_1^2 + \left(1+\phi^2\right)\left(1-\phi\right)\psi_0, \tag{A.3}$$

where $\bar{\sigma}_t^2 = \lim_{n\to\infty} n^{-1}\sum_{i=1}^{n}\sigma_{it}^2$, for $t = 1, 2$, and

$$\psi_0 = (1-\phi)\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}E\left(y_{i0}-\mu_i\right)^2 - 2\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^{n}E\left[u_{i1}\left(y_{i0}-\mu_i\right)\right].$$

A.2 Lemmas

Lemma A.1 *Suppose y_{it}, for $i = 1, 2, ..., n$, and $t = -m_i + 1, -m_i + 2, ..., 1, 2, ..., T$, are generated by (1) with starting values $y_{i,-m_i}$. Let Assumptions 1-3 hold. Consider*

$$\bar{Q}_{nT} = \frac{1}{n}\sum_{i=1}^{n} Q_{iT}, \text{ and } \bar{B}_{nT} = \frac{1}{n}\sum_{i=1}^{n}\left(Q_{iT} + Q_{iT}^{+} + 2H_{iT}\right),$$

where $Q_{iT} = (T-2)^{-1}\sum_{t=2}^{T-1}\Delta y_{i,t-1}^2$, $Q_{iT}^{+} = (T-2)^{-1}\sum_{t=2}^{T-1}\Delta y_{it}^2$, and $H_{iT} = (T-2)^{-1}\sum_{t=2}^{T-1}\Delta u_{it}\Delta y_{i,t-1}$. Suppose that T is fixed. Then, we have

$$\bar{Q}_{nT} = E\left(\bar{Q}_{nT}\right) + O_p\left(n^{-1/2}\right), \tag{A.4}$$

$$\bar{B}_{nT} = E\left(\bar{B}_{nT}\right) + O_p\left(n^{-1/2}\right). \tag{A.5}$$

Proof. Under Assumptions 1-3, the fourth moments of u_{it} and b_{i,m_i} are bounded, and hence, using Loève's inequality,[20] for each i the fourth moment of Δy_{it} :

$$\Delta y_{it} = \phi^{t-1}\left[b_{i,m_i} + u_{i1} - (1-\phi)\sum_{\ell=0}^{m_i-1}\phi^{\ell}u_{i,-\ell}\right] + \sum_{\ell=0}^{t-2}\phi^{\ell}\Delta u_{i,t-\ell},$$

is also bounded, for all values of $|\phi| \leq 1$ and $m_i \geq 0$. Since T is fixed, it follows that the second moment of $Q_{iT} = (T-2)^{-1}\sum_{t=2}^{T-1}\Delta y_{i,t-1}^2$ must be bounded, and hence there must exist C such that $E\left[Q_{iT} - E\left(Q_{iT}\right)\right]^2 < C$. Consider next the cross-sectional average of $Q_{iT} - E\left(Q_{iT}\right)$. We have $E\left[Q_{iT} - E\left(Q_{iT}\right)\right] = 0$ by construction, and also $Q_{iT} - E\left(Q_{iT}\right)$ is independently distributed across i, since, under Assumptions 1-3, Δy_{it} is independently distributed across i. Hence,

$$Var\left\{n^{-1}\sum_{i=1}^{n}\left[Q_{iT} - E\left(Q_{iT}\right)\right]\right\} \leq n^{-2}\sum_{i=1}^{n}E\left[Q_{iT} - E\left(Q_{iT}\right)\right]^2 < \frac{C}{n},$$

and therefore $n^{-1}\sum_{i=1}^{n}Q_{iT} - n^{-1}\sum_{i=1}^{n}E\left(Q_{iT}\right) = O_p\left(n^{-1/2}\right)$. This completes the proof of (A.4).

Result (A.5) is established similarly. Note that

$$\bar{B}_{nT} = \frac{1}{n}\sum_{i=1}^{n}Q_{iT} + \frac{1}{n}\sum_{i=1}^{n}Q_{iT}^{+} + 2\frac{1}{n}\sum_{i=1}^{n}H_{iT} = \bar{Q}_{nT} + \bar{Q}_{nT}^{+} + 2\bar{H}_{nT}.$$

[20] See equation (9.62) of Davidson (1994).

The order of $\bar{Q}_{nT} - E\left(\bar{Q}_{nT}\right)$ is given by (A.4). Using the same arguments as in the proof of (A.4), we have

$$\bar{Q}^{+}_{nT} - E\left(\bar{Q}^{+}_{nT}\right) = O_p\left(n^{-1/2}\right), \text{ and } \bar{H}_{nT} - E\left(\bar{H}_{nT}\right) = O_p\left(n^{-1/2}\right).$$

Hence, $\bar{B}_{nT} - E\left(\bar{B}_{nT}\right) = \bar{Q}_{nT} - E\left(\bar{Q}_{nT}\right) + \bar{Q}^{+}_{nT} - E\left(\bar{Q}^{+}_{nT}\right) + 2\left[\bar{H}_{nT} - E\left(\bar{H}_{nT}\right)\right] = O_p\left(n^{-1/2}\right)$, and result (A.5) follows. This completes the proof. ■

Lemma A.2 *Suppose y_{it}, for $i = 1, 2, ..., n$, and $t = -m_i + 1, -m_i + 2, ..., 1, 2, ..., T$, are generated by (1) with starting values $y_{i,-m_i}$. Let Assumptions 1-3 hold. Consider*

$$\bar{V}_{nT} = \frac{1}{n}\sum_{i=1}^{n} V_{iT},$$

where $V_{iT} = \frac{1}{T-2}\sum_{t=2}^{T-1}\left(\Delta u_{it}\Delta y_{i,t-1} + \Delta u_{it}^2 + \Delta u_{i,t+1}\Delta y_{it}\right)$. Suppose that T is fixed. Then, we have

$$\bar{V}_{nT} = O_p\left(n^{-1/2}\right). \tag{A.6}$$

If, in addition, $S_T = \lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} E\left(V_{iT}^2\right)$, and T is fixed as $n \to \infty$, then

$$\sqrt{n}\bar{V}_{nT} \to_d N\left(0, S_T\right). \tag{A.7}$$

Proof. Under Assumptions 2 and 3, V_{iT} is independently distributed of V_{jT} for all $i \neq j$, $i, j = 1, 2, ..., n$. In addition, (using (13))

$$E\left(V_{iT}\right) = \frac{1}{T-2}\sum_{t=2}^{T-1} E\left(\Delta u_{it}\Delta y_{i,t-1} + \Delta u_{it}^2 + \Delta u_{i,t+1}\Delta y_{it}\right) = 0. \tag{A.8}$$

Also, by Assumptions 2 and 3, $\sup_{i,t} E\left|u_{it}\right|^{4+\epsilon} < C$, and $\sup_i E\left|b_{i,m_i}\right|^{4+\epsilon} < C$, for some $\epsilon > 0$, and hence, using Loève's inequality,[21] we have $\sup_{i,t} E\left|\Delta y_{it}\right|^{4+\epsilon} < C$. Using Loève's inequality again, we have

$$E\left|\Delta u_{it}\Delta y_{i,t-1} + \Delta u_{it}^2 + \Delta u_{i,t+1}\Delta y_{it}\right|^{2+\epsilon/2} \leq C\left(E\left|\Delta u_{it}\Delta y_{i,t-1}\right|^{2+\epsilon/2} + E\left|\Delta u_{it}^2\right|^{2+\epsilon/2} + E\left|\Delta u_{i,t+1}\Delta y_{it}\right|^{2+\epsilon/2}\right).$$

But $\sup_{it} E\left|\Delta u_{it}^2\right|^{2+\epsilon/2} = \sup_{it} E\left|\Delta u_{it}\right|^{4+\epsilon} < C$, as well as $\sup_{i,t} E\left|\Delta u_{it}\Delta y_{i,t-1}\right|^{2+\epsilon/2} < C$, and $\sup_{i,t} E\left|\Delta u_{i,t+1}\Delta y_{it}\right|^{2+\epsilon/2} < C$. Hence, $\sup_{it} E\left|\Delta u_{it}\Delta y_{i,t-1} + \Delta u_{it}^2 + \Delta u_{i,t+1}\Delta y_{it}\right|^{2+\epsilon/2} < C$, and

[21] See equation (9.62) of Davidson (1994).

using Loève's inequality again, we have

$$\sup_i E\left(|V_{iT}|^{2+\epsilon/2}\right) < C. \tag{A.9}$$

It follows also that $\sup_i E\left(V_{iT}^2\right) < C$, and given that V_{iT} is independently distributed over i, we have

$$E\left(\bar{V}_{nT}^2\right) = n^{-2}\sum_{i=1}^{n}\sum_{j=1}^{n} E\left(V_{iT}V_{jT}\right) = n^{-2}\sum_{i=1}^{n} E\left(V_{iT}^2\right) < \frac{C}{n},$$

and result (A.6) follows. To establish (A.7), we note that (A.9) holds, and therefore the Lyapunov condition holds (see Theorem 23.12 of Davidson, 1994). Hence, noting also that $n^{-1}\sum_{i=1}^{n} E\left(V_{iT}^2\right) \to S_T$ by assumption, we obtain $\sqrt{n}\bar{V}_{nT} \to_d N\left(0, S_T\right)$, as required. ■

A.3 Propositions and Proofs

First we establish Theorem 1.

Proof of Theorem 1. To derive the asymptotic properties of $\hat{\phi}_{nT}$, let ϕ_0 denote the true value of ϕ, assumed to lie inside Θ, and note that under $\phi = \phi_0$, (3) yields $\Delta y_{it} = \phi_0 \Delta y_{i,t-1} + \Delta u_{it}$, and (15) can be written as

$$\begin{aligned} M_{iT}\left(\phi\right) &= \frac{1}{T-2}\sum_{t=2}^{T-1}\left\{\begin{array}{l} \left[\Delta u_{it} - \left(\phi - \phi_0\right)\Delta y_{i,t-1}\right]\Delta y_{i,t-1} \\ + \left[\Delta u_{it} - \left(\phi - \phi_0\right)\Delta y_{i,t-1}\right]^2 \\ + \left[\Delta u_{i,t+1} - \left(\phi - \phi_0\right)\Delta y_{it}\right]\Delta y_{it} \end{array}\right\} \\ &= \Lambda_{iT} + V_{iT}, \end{aligned} \tag{A.10}$$

where

$$V_{iT} = \frac{1}{T-2}\sum_{t=2}^{T-1}\left(\Delta u_{it}\Delta y_{i,t-1} + \Delta u_{it}^2 + \Delta u_{i,t+1}\Delta y_{it}\right), \tag{A.11}$$

and $\Lambda_{iT} = \left(\phi - \phi_0\right)^2 Q_{iT} - \left(\phi - \phi_0\right)\left(Q_{iT} + Q_{iT}^{+} + 2H_{iT}\right)$, in which

$$Q_{iT} = \frac{1}{T-2}\sum_{t=2}^{T-1}\Delta y_{i,t-1}^2,\ Q_{iT}^{+} = \frac{1}{T-2}\sum_{t=2}^{T-1}\Delta y_{it}^2, \text{ and } H_{iT} = \frac{1}{T-2}\sum_{t=2}^{T-1}\Delta u_{it}\Delta y_{i,t-1}. \tag{A.12}$$

We have one unknown parameter ϕ and one moment condition (14). Suppose there exists $\hat{\phi}_{nT}$ such that $\bar{M}_{nT}\left(\hat{\phi}_{nT}\right)=0$. Then (A.10) evaluated at $\phi=\hat{\phi}_{nT}$ yields

$$\left(\hat{\phi}_{nT}-\phi_0\right)\left[\left(\hat{\phi}_{nT}-\phi_0\right)\bar{Q}_{nT}-\bar{B}_{nT}\right]=-\bar{V}_{nT}, \tag{A.13}$$

where $\bar{V}_{nT}=n^{-1}\sum_{i=1}^{n}V_{iT}$, $\bar{Q}_{nT}=n^{-1}\sum_{i=1}^{n}Q_{iT}$, and

$$\bar{B}_{nT}=\frac{1}{n}\sum_{i=1}^{n}\left(Q_{iT}+Q_{iT}^{+}+2H_{iT}\right). \tag{A.14}$$

Using results (A.4)-(A.5) of Lemma A.1 in the appendix, under Assumptions 1-3, we have (for a fixed T)

$$\bar{Q}_{nT}=E\left(\bar{Q}_{nT}\right)+O_p\left(n^{-1/2}\right),\text{ and }\bar{B}_{nT}=E\left(\bar{B}_{nT}\right)+O_p\left(n^{-1/2}\right), \tag{A.15}$$

where

$$E\left(\bar{Q}_{nT}\right)=\frac{1}{n}\sum_{i=1}^{n}E\left(Q_{iT}\right)>0. \tag{A.16}$$

In addition, using result (A.6) of Lemma A.2 in the appendix, we have

$$\bar{V}_{nT}=O_p\left(n^{-1/2}\right). \tag{A.17}$$

We now use (A.13) to show that there exists a unique $\sqrt{n}$-consistent estimator of ϕ. Suppose that $\hat{\phi}_{nT}$ is a $\sqrt{n}$-consistent estimator of ϕ. Then we establish that such an estimator is in fact unique. Using (A.13), we have

$$\sqrt{n}\left(\hat{\phi}_{nT}-\phi_0\right)^2\bar{Q}_{nT}-\sqrt{n}\left(\hat{\phi}_{nT}-\phi_0\right)\bar{B}_{nT}=-\sqrt{n}\bar{V}_{nT}. \tag{A.18}$$

But, if there exists a $\sqrt{n}$-consistent estimator, then $\sqrt{n}\left(\hat{\phi}_{nT}-\phi_0\right)^2\bar{Q}_{nT}=O_p\left(n^{-1/2}\right)$, and hence

$$\bar{B}_{nT}\sqrt{n}\left(\hat{\phi}_{nT}-\phi_0\right)=\sqrt{n}\bar{V}_{nT}+O_p\left(n^{-1/2}\right). \tag{A.19}$$

Also, using (A.15) the above can be written as

$$E\left(\bar{B}_{nT}\right)\sqrt{n}\left(\hat{\phi}_{nT}-\phi_0\right)=\sqrt{n}\bar{V}_{nT}+O_p\left(n^{-1/2}\right).$$

where by (A.17), $\sqrt{n}\bar{V}_{nT} = O_p(1)$. If

$$\bar{B}_T = \lim_{n\to\infty} E\left(\bar{B}_{nT}\right) \neq 0, \tag{A.20}$$

it then follows that the $\sqrt{n}$-consistent estimator, $\hat{\phi}_{nT}$, must be unique. It also follows that

$$\sqrt{n}\left(\hat{\phi}_{nT} - \phi_0\right) \overset{a}{\sim} \bar{B}_T^{-1}\sqrt{n}\bar{V}_{nT}.$$

Finally, using result (A.7) of Lemma A.2 in the appendix, we have $\sqrt{n}\bar{V}_{nT} \to_d N(0, S_T)$, where $S_T = \lim_{n\to\infty} n^{-1}\sum_{i=1}^{n} E\left(V_{iT}^2\right)$, and it follows that $\sqrt{n}\left(\hat{\phi}_{nT} - \phi_0\right) \to_d N(0, \Sigma_T)$ with $\Sigma_T = \bar{B}_T^{-2} S_T$. ■

We present next propositions for the consistency of $\hat{\Sigma}_{nT}$.

Proposition 1 *Suppose conditions of Theorem 1 hold, and consider $\hat{\Sigma}_{nT}$ defined by (24), namely*

$$\hat{\Sigma}_{nT} = \hat{\bar{B}}_{nT}^{-2}\left(\frac{1}{n}\sum_{i=1}^{n}\hat{V}_{i,nT}^2\right),$$

where $\hat{\bar{B}}_{nT} = n^{-1}\sum_{i=1}^{n}\left(Q_{iT} + Q_{iT}^{+} + 2\hat{H}_{i,nT}\right)$, $\hat{H}_{i,nT} = (T-2)^{-1}\sum_{t=2}^{T-1}\Delta\hat{u}_{it}\Delta y_{i,t-1}$, $\Delta\hat{u}_{it} = \Delta y_{it} - \hat{\phi}_{nT}\Delta y_{i,t-1}$,

$$\hat{V}_{i,nT} = \frac{1}{T-2}\sum_{t=2}^{T-1}\left(\Delta\hat{u}_{it}\Delta y_{i,t-1} + \Delta\hat{u}_{it}^2 + \Delta\hat{u}_{i,t+1}\Delta y_{it}\right),$$

and $\hat{\phi}_{nT}$ is the $\sqrt{n}$-consistent BMM estimator given by (16). Let T be fixed as $n \to \infty$. Then,

$$\hat{\Sigma}_{nT} \to_p \Sigma_T, \tag{A.21}$$

where Σ_T is defined in (19)

Proof. Using Theorem 1, we have $\hat{\phi}_{nT} = \phi_0 + O_p\left(n^{-1/2}\right)$, and therefore $\Delta\hat{u}_{it} = \Delta y_{it} - \hat{\phi}_{nT}\Delta y_{i,t-1}$ is consistent, namely $\Delta\hat{u}_{it} - \Delta u_{it} = -\left(\hat{\phi}_{nT} - \phi_0\right)\Delta y_{i,t-1} = O_p\left(n^{-1/2}\right)$. This implies $\hat{H}_{i,nT}$ is consistent, which in turn implies $\hat{\bar{B}}_{nT} - \bar{B}_{nT} \to_p 0$. But, using result (A.5) of Lemma A.1, we have $\bar{B}_{nT} \to_p E\left(\bar{B}_{nT}\right)$, and $E\left(\bar{B}_{nT}\right) \to B_T$. Therefore $\hat{\bar{B}}_{nT} \to_p \bar{B}_T$. Since $\bar{B}_T > 0$ by assumption, it follows that

$$\hat{\bar{B}}_{nT}^{-2} \to_p \bar{B}_T^{-2}. \tag{A.22}$$

Next consider $n^{-1}\sum_{i=1}^{n}\hat{V}_{i,nT}^{2}$, and note that

$$\hat{V}_{i,nT}^{2}=\left[\left(\hat{V}_{i,nT}-V_{iT}\right)+V_{iT}\right]^{2}=\left(\hat{V}_{i,nT}-V_{iT}\right)^{2}+2\left(\hat{V}_{i,nT}-V_{iT}\right)V_{iT}+V_{iT}^{2},$$

where $V_{iT}=(T-2)^{-1}\sum_{t=2}^{T-1}\left(\Delta u_{it}\Delta y_{i,t-1}+\Delta u_{it}^{2}+\Delta u_{i,t+1}\Delta y_{it}\right)$. Using $\Delta\hat{u}_{n,it}-\Delta u_{n,it}=O_{p}\left(n^{-1/2}\right)$, we have $\hat{V}_{i,nT}-V_{iT}=O_{p}\left(n^{-1/2}\right)$. Noting also that $V_{iT}=O_{p}\left(1\right)$, we then have

$$n^{-1}\sum_{i=1}^{n}\left(\hat{V}_{i,nT}-V_{iT}\right)^{2}\rightarrow_{p}0,\text{ and }n^{-1}\sum_{i=1}^{n}\left(\hat{V}_{i,nT}-V_{iT}\right)V_{iT}\rightarrow_{p}0. \tag{A.23}$$

Finally, to obtain the limiting property of $n^{-1}\sum_{i=1}^{n}V_{iT}^{2}$, note that by assumption V_{iT} is independently distributed over i. Also, as established in (A.9), we have $\sup_{i}E\left|V_{iT}\right|^{2+\epsilon/2}<C$ for some $\epsilon>0$. It follows that $n^{-1}\sum_{i=1}^{n}\left[V_{iT}^{2}-E\left(V_{iT}^{2}\right)\right]\rightarrow_{p}0$, and therefore (noting that $n^{-1}\sum_{i=1}^{n}E\left(V_{iT}^{2}\right)\rightarrow S_{T}$ by assumption) we have

$$n^{-1}\sum_{i=1}^{n}V_{iT}^{2}\rightarrow_{p}S_{T}. \tag{A.24}$$

Result (A.21) now follows from (A.22), (A.23), and (A.24). ■

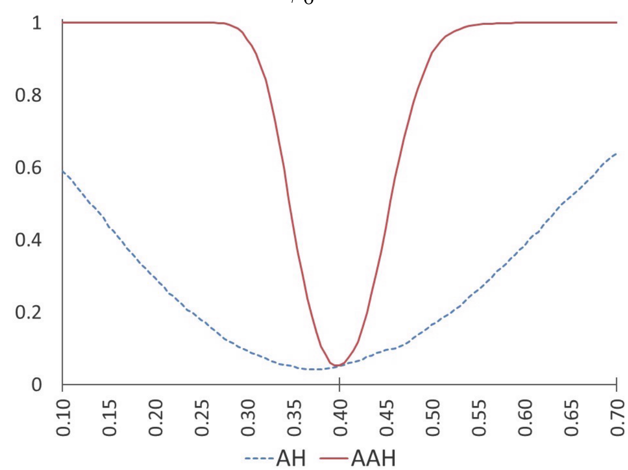
A.4 Rejection frequencies for selected estimators in Monte Carlo experiments

This section presents rejection frequencies for selected estimators considered in the Monte Carlo experiments in Section 7, and selected sample combinations. Figure A1 compares rejection frequencies of AH and AAH estimators in experiments where both AB and BB restrictions are met ($\rho=0$ & $\kappa=0$), for the sample combinations, $n=1000$, $T=4$ and 6. Figure A2 compares rejection frequencies of AAH and AAH-O estimators using the same data generating process ($\rho=0$ & $\kappa=0$), but plotting rejection frequencies for sample sizes with smaller value of $n=200$ and larger values of $T=10$ and 20, where the number of moments is a very important small sample issue. Figure A3 shows the rejection frequencies for AAH, AB, and BB estimators using the same data generating process ($\rho=0$ & $\kappa=0$), using sample combinations $n=1000$, $T=4$ and 6. The last figure (Figure A4) compares rejection frequencies of AAH and AB estimators in the experiments where AB restrictions are met but BB restrictions are not met ($\rho=0$, and $\kappa=1$), and using the sample combinations, $n=1000$, $T=4$ and 6.

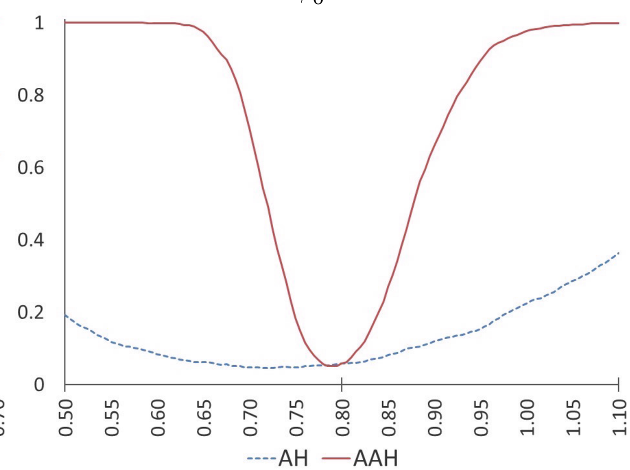
Figure A1: Rejection frequencies (at 5% nominal level) for AH and AAH estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

$n = 1000, T = 4$

$\phi_0 = 0.4$

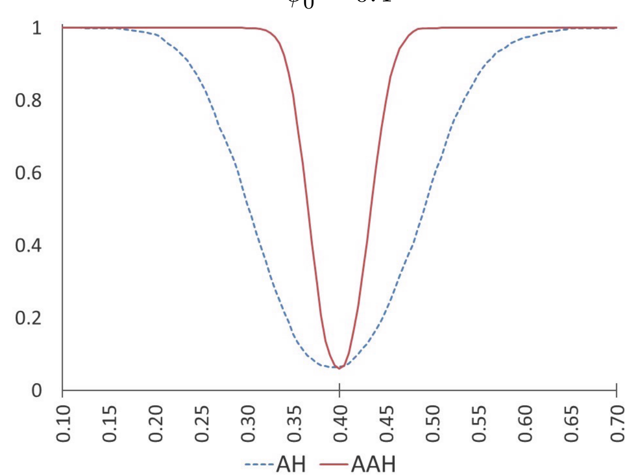


$\phi_0 = 0.8$

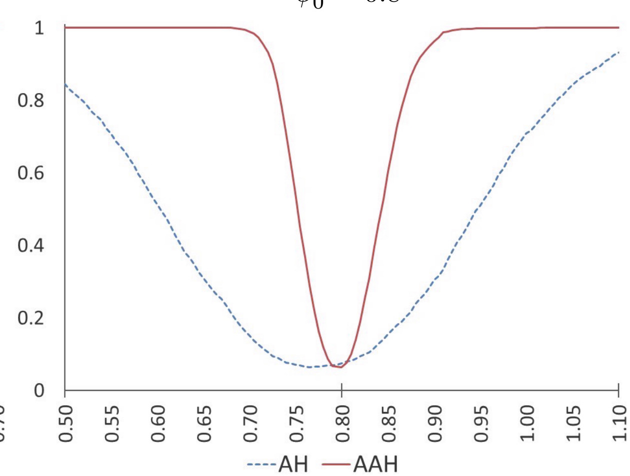


$n = 1000, T = 6$

$\phi_0 = 0.4$

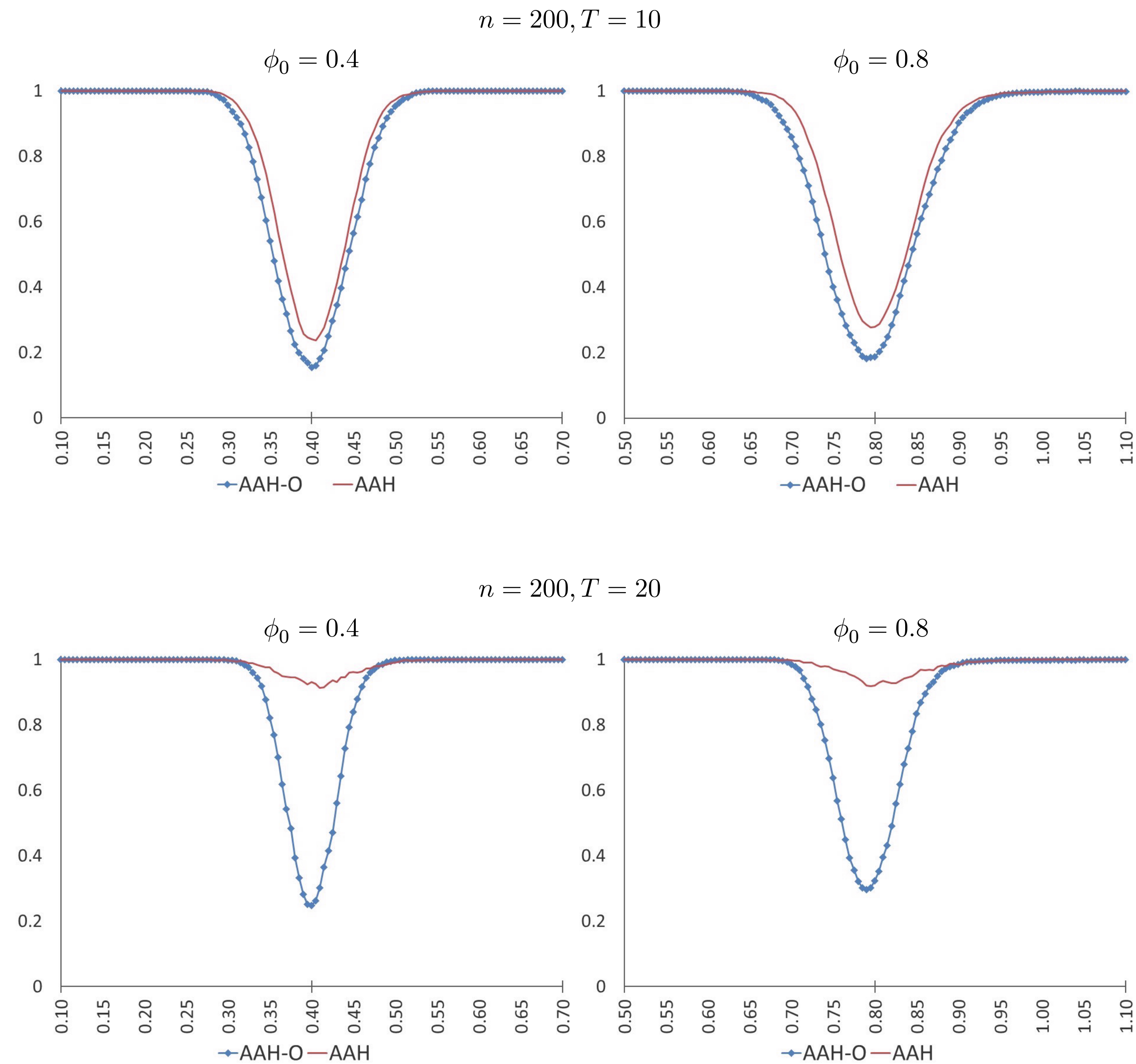


$\phi_0 = 0.8$



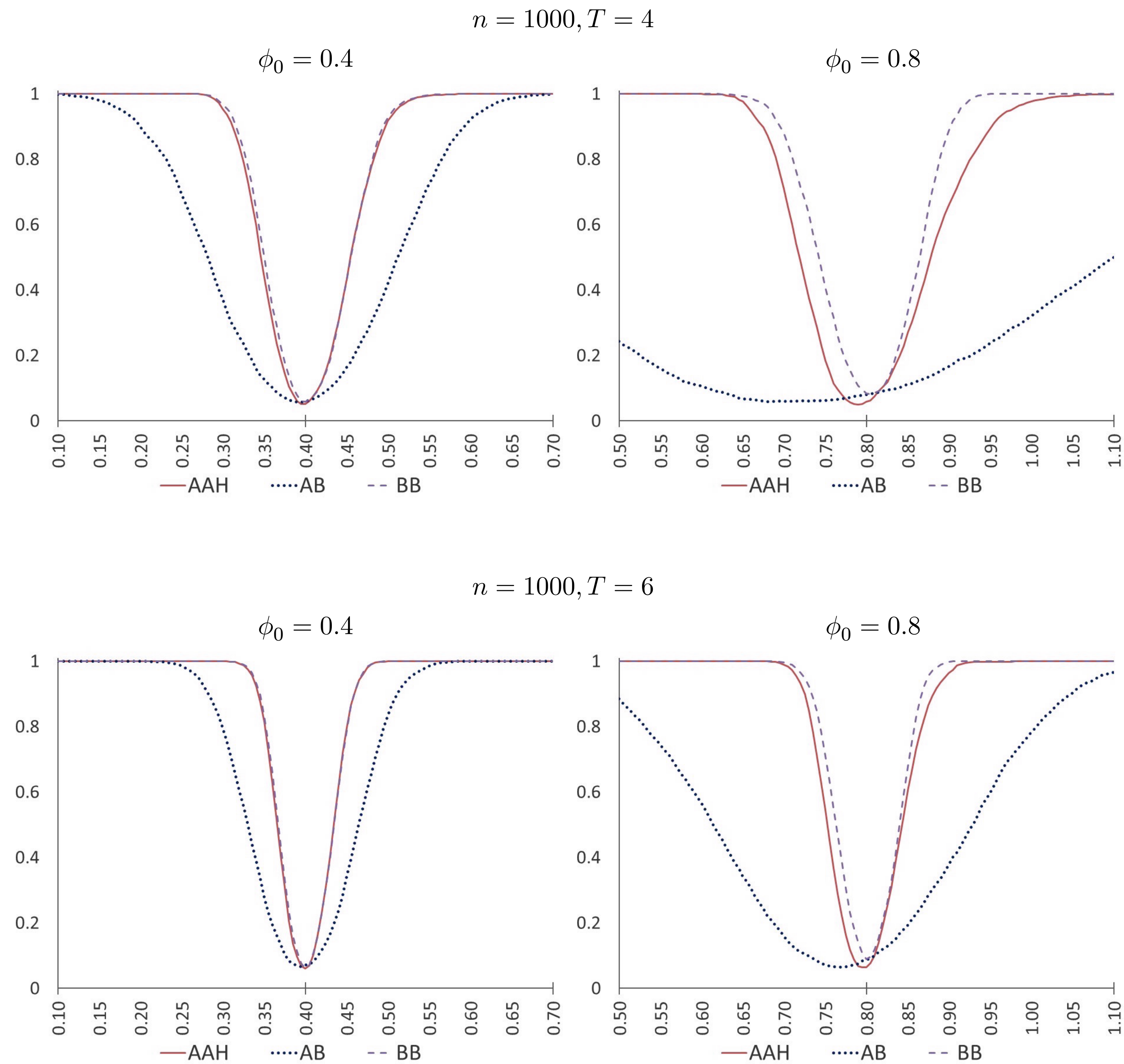
See the notes to Table 2.

Figure A2: Rejection frequencies (at 5% nominal level) for AAH and AAH-O estimators when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met



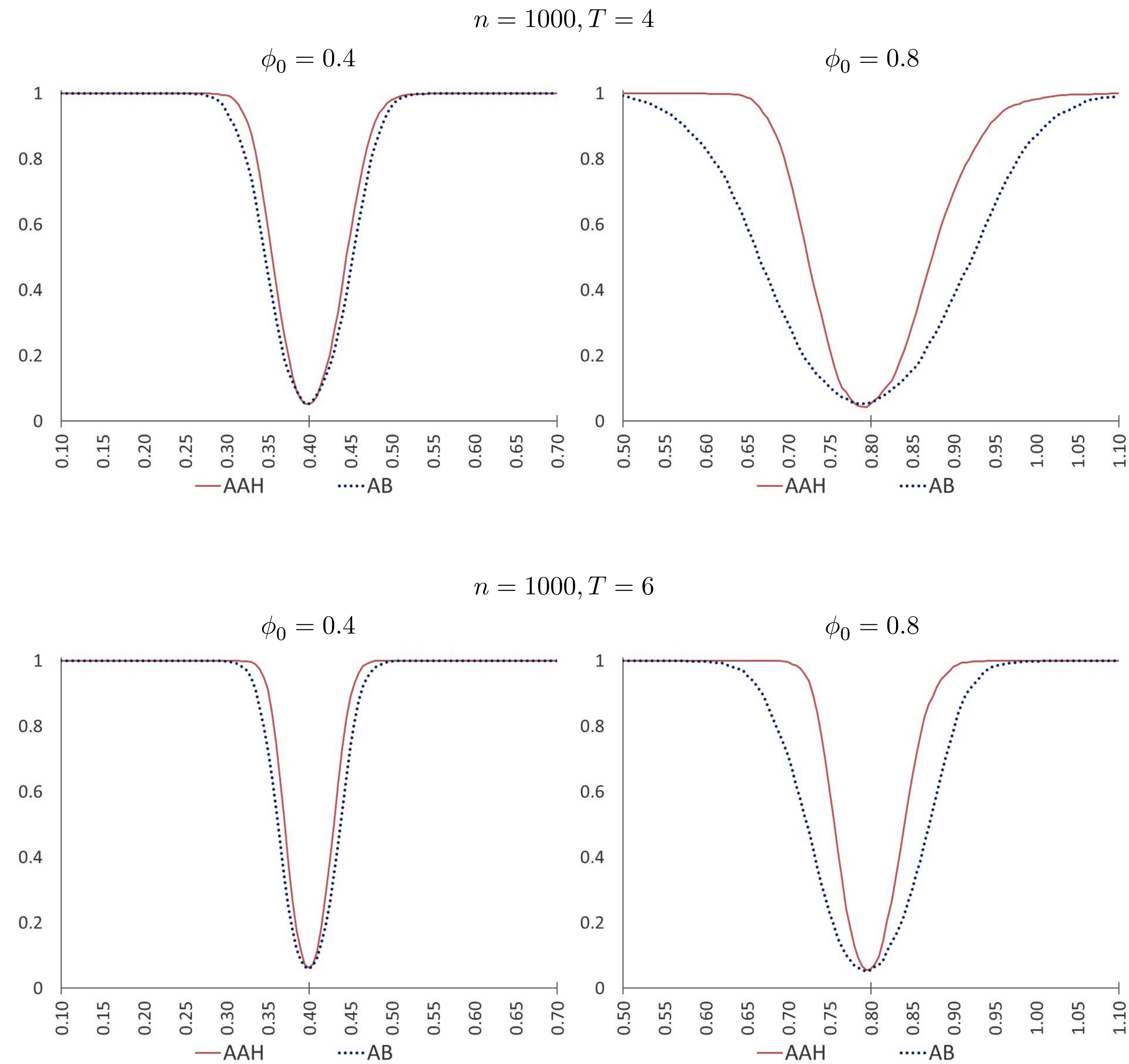
See the notes to Tables 2 and 4.

Figure A3: Rejection frequencies (at 5% nominal level) for AAH, AB, and BB estimators when AB and BB restrictions are met



See notes to Tables 2 and 6.

Figure A4: Rejection frequencies (at 5% nominal level) for AAH and AB estimators when AB restrictions are met and BB restrictions are not met



See the notes to Tables 2 and 6.

A.5 Monte Carlo experiments for panel ARX(1) model

This section presents Monte Carlo evidence on the relative performance of AAH, AB and BB estimators for the panel ARX(1) model.[22]

A.5.1 ARX Monte Carlo Design

We augment the AR(1) DGP in Section 7 with a strictly exogenous regressor:

$$y_{it} = \alpha_i + \phi y_{i,t-1} + \beta x_{it} + u_{it}, \tag{A.25}$$

for $i = 1, 2, ..., n$, and $t = 1, 2, ..., T$. Individual effects, α_i, are generated in the same way as in Section 7, see (43). Similarly, the starting values, $y_{i,0}$, are generated as in (44), namely

$$y_{i,0} = \mu_i + \kappa\varepsilon_i + \upsilon_i,\ \upsilon_i \sim IIDN\left(0,1\right), \tag{A.26}$$

but unlike in Section 7, where $\mu_i = \alpha_i / \left(1-\phi\right)$, the long-run means are generated as $\mu_i = \left(\alpha_i + \mu_{xi}\right) / \left(1-\phi\right)$. The idiosyncratic errors, u_{it}, are generated in the same way as in Section 7. Regressors, x_{it}, are generated as

$$x_{it} = \mu_{x,i}\left(1-\theta\right) + \theta x_{i,t-1} + \epsilon_{it}, \tag{A.27}$$

for $i = 1, 2, ..., n$, and $t = 1, 2, ..., T$, with starting values $x_{i,0} = \mu_{x,i} + IIDN\left(0,1\right)$, where $\mu_{x,i} \sim IIDN(1,1)$, and $\epsilon_{it} \sim IIDN\left(0, 1-\theta^2\right)$.

We set $\phi = 0.8$, $\beta = 0.5$ and $\theta = 0.6$, and consider the same two values for ρ, namely $\rho = 0$ and 0.8, and the same two values for κ, namely $\kappa = 0$ and 1, as in Section 7. Under this design, the covariates x_{it} are strictly exogenous.

Available observations for estimation are (x_{it}, y_{it}) for $t = 0, 1, 2, ..., T$. We consider $T = 4, 6, 10$ and $n = 100, 200, 500, 1000, 2000, 8000$. $R = 2000$ replications were carried out for each experiment.

A.5.2 AB, BB and AAH Estimators for ARX panel

The AB estimator is implemented as a two-step GMM estimator based on "DIF1" set of moment conditions outlined in Hayakawa and Pesaran (2015), comprising the following $T\left(T-1\right)/2+$

[22] We are grateful to an anonymous referee for suggesting these Monte Carlo experiments.

$(T+1)\,T/2-1$ moment conditions:

$$E\left(y_{is}\Delta u_{it}\right)=0,\ \text{ for } s=0,1,...,t-2,\ t=2,3,...,T, \tag{A.28}$$

and

$$E\left(x_{is}\Delta u_{it}\right)=0,\text{ for } s=1,2,...,t,\ t=2,3,...,T. \tag{A.29}$$

The BB estimator is a two-step GMM estimator based on "SYS1" set of moment conditions outlined in Hayakawa and Pesaran (2015), comprising the moment conditions in (A.28)-(A.29) plus the following additional $2\left(T-1\right)$ moment conditions:

$$E\left[\Delta y_{i,t-1}\left(\alpha_i+u_{it}\right)\right]=0,\text{ for } t=2,3,...,T, \tag{A.30}$$

and

$$E\left[\Delta x_{it}\left(\alpha_i+u_{it}\right)\right]=0,\text{ for } t=2,3,...,T. \tag{A.31}$$

Detailed descriptions of these AB and BB estimators are provided in Sections 4 and 5 of Hayakawa and Pesaran (2015).

In addition to AB and BB estimators, we also implement a two-step AAH estimator. This estimator is based on the moment conditions (7) and (13) augmented with additional moment conditions for instrumenting the regressor, x_{it} for $i=1,2,...,n$. As we noted in Section 5, this paper has nothing new to add regarding the moment conditions for the exogenous regressors, and standard moment conditions used in the literature can be considered. In the experiments presented here we consider the same subset of available moment conditions for the regressor x_{it} as chosen for the AB estimator described above. This will make the comparisons between AH and AB methods straightforward and fair. The set of moment conditions for the AAH estimator implemented below is given by the $\left(T-2\right)\left(T-1\right)/2+T+\left(T+1\right)T/2-3$ moment conditions in (7), (13) and (A.29).

For the choices of $T=4,6,10$, we respectively have 15, 35, 99 moment conditions for the AB estimator, 21, 45, 117 moment conditions for the BB estimator, and 14, 34, 98 moment conditions for the AAH estimator.

A.5.3 Results

We consider three sets of experiments, for different values of ρ and κ. Table A1 reports Bias and RMSE (both $\times 100$) of the three estimators in the baseline case where both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met, namely $\rho = \kappa = 0$. The reported values for bias are overall relatively small for estimation of both parameters, ϕ and β, and the bias is reduced quite rapidly with an increase in n. For most experiments RMSEs of estimating β are larger compared to the RMSE obtained for estimating ϕ. This could be due to relatively low variability of Δx_{it} over i and t. For all sample sizes considered, BB estimator has the lowest RMSE values, reflecting that the set of moment conditions underpinning the BB estimator encompasses the moment conditions that underlie the other two estimators. Comparison of RMSE values reported for AAH and AB estimator reveals that AAH has lower RMSE for majority of sample sizes when estimating ϕ, but somewhat larger RMSE when estimating β. Size and power of AAH, AB and BB estimators for the baseline experiments (with $\rho = \kappa = 0$) are reported in Table A2. As in Section 5, we observe large size distortions when the number of moment conditions is relatively large compared with the sample sizes, which can be seen most clearly when $n = 100$ or 200, with the size distortions quickly worsening with as T is increased. For $T = 10$, we need at least $n = 2000$ for the size distortion to be relatively small.

Findings for experiments when AB restrictions are met and but some of the BB restrictions are not met (namely when $\rho = 0$, and $\kappa = 1$) are summarized in Table A3 (for the bias and RMSE) and in Table A4 (for the size and power). We see that BB estimator is subject to bias, which does not decrease with an increase in n, in line with expectation that BB estimator is no longer consistent in these experiments, since some of the BB moment conditions are no longer valid when $\kappa \neq 1$. Consequently, the reported size distortions of the BB estimator in Table A4 are very large and deteriorate rapidly with an increase in n. Comparison of AAH and AB estimators in terms of RMSE (reported in Table A3) reveals that AAH has lower RMSE for all sample sizes with the exception of one sample size ($n = 100$, $T = 4$) when estimating ϕ, whilst the comparisons are more mixed for the estimation of β, where the AAH estimator still outperforms in majority of sample sizes considered.

Last but not least, Tables A5-A6 report Monte Carlo findings for panel ARX(1) experiments when some of AB and BB restrictions are not met ($\rho = 0.8$, and $\kappa = 1$). In the case, not surprisingly, AB and BB estimators being based on invalid moment conditions are biased (Table A5) for most of the sample sizes. It is interesting to note that the bias is quite small for the AB estimator and values of $T = 10$, when parameter β is estimated. Bias distortions of AB and BB estimators manifest in large size distortions that rise in n (Table A8). In contrast, the AAH estimator shows qualitatively similar performance compared with the previous experiments. In particular, there are no serious size distortions when n is sufficiently large relative to T. For example, for $T = 4$ the AAH estimator does not show any size distortions for values of $n \geq 200$, irrespective of whether we consider estimating ϕ and β. But to avoid size distortions when $T = 6$ then we need $n \geq 500$, and so on. The AAH continues to have satisfactory power which rise with n.

Table A1: Bias and RMSE of AAH, AB and BB estimators in panel ARX(1) experiments when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

$\rho = 0$, and $\kappa = 0$

		Bias ($\times 100$)						RMSE($\times 100$)					
		$\phi_0 = 0.8$			$\beta_0 = 0.5$			$\phi_0 = 0.8$			$\beta_0 = 0.5$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	1.51	-0.24	0.47	0.03	0.44	-1.17	7.39	8.24	2.76	8.71	8.19	8.07
4	200	0.79	-0.16	0.21	-0.20	0.05	-0.75	4.71	5.69	1.88	5.74	5.48	5.36
4	500	0.34	0.02	0.03	0.00	0.10	-0.23	2.79	3.58	1.15	3.54	3.49	3.34
4	1000	0.24	0.03	0.05	-0.05	-0.01	-0.20	2.01	2.56	0.81	2.57	2.54	2.40
4	2000	0.10	0.06	0.00	0.03	0.06	-0.05	1.36	1.77	0.57	1.78	1.78	1.69
4	8000	0.01	0.00	0.00	-0.02	-0.02	-0.03	0.68	0.90	0.28	0.90	0.90	0.84
6	100	1.65	0.13	0.88	-0.42	0.02	0.88	5.10	5.63	2.78	7.68	6.48	7.01
6	200	1.03	0.32	0.48	-0.02	0.17	0.48	3.07	3.67	1.71	4.51	4.19	4.21
6	500	0.47	0.27	0.20	-0.05	0.00	0.20	1.78	2.21	1.00	2.72	2.68	2.62
6	1000	0.25	0.06	0.07	0.06	0.08	0.07	1.19	1.50	0.70	1.89	1.87	1.76
6	2000	0.11	0.04	0.03	-0.02	-0.01	0.03	0.82	1.09	0.48	1.28	1.29	1.22
6	8000	0.01	0.00	0.01	-0.01	-0.01	0.01	0.41	0.55	0.24	0.64	0.64	0.60
10	100	3.84	0.36	-	-1.49	1.14	-	10.98	18.90	-	16.13	26.23	-
10	200	1.26	0.71	0.88	-0.08	0.15	0.88	2.63	2.76	1.89	4.13	3.63	3.77
10	500	0.58	0.35	0.37	0.09	0.17	0.37	1.27	1.39	0.89	2.08	1.99	1.94
10	1000	0.31	0.20	0.20	0.05	0.08	0.20	0.80	0.94	0.59	1.34	1.34	1.27
10	2000	0.16	0.10	0.10	0.05	0.06	0.10	0.53	0.66	0.40	0.95	0.96	0.89
10	8000	0.03	0.02	0.02	0.00	0.00	0.02	0.25	0.32	0.19	0.47	0.47	0.44

Notes: "AAH" is the augmented Anderson and Hsiao 2-step GMM estimator based on the $(T-2)(T-1)/2+T+(T+1)T/2-3$ moment conditions in (7), (13), and (A.29), "AB" is 2-step GMM estimator based on the $T(T-1)/2+(T+1)T/2-1$ moment conditions in (A.28)-(A.29), and "BB" is 2-step GMM estimator based on the $T(T-1)/2+(T+1)T/2-1+2(T-1)$ moment conditions in (A.28)-(A.31). See Subsection A.5.2 in Appendix for further details. The DGP is given by $y_{it} = \alpha_i + \phi y_{i,t-1} + \beta x_{it} + u_{it}$, for $i = 1, 2, ..., n$, and $t = 1, 2, ..., T$, with initial values given by $y_{i,0} = \mu_i + \kappa \varepsilon_i + \upsilon_i$, $x_{i,0} = \mu_{xi} + IIDN(0,1)$, where $\mu_i = (\alpha_i + \mu_{xi})/(1-\phi)$, $\mu_{x,i} \sim IIDN(1,1)$, $\alpha_i = \sum_{t=1}^{T} \rho^t u_{it} + \varepsilon_i$, $\varepsilon_i \sim IIDN(1,1)$, and $\upsilon_i \sim IIDN(0,1)$. This table reports findings for experiments where $\kappa = \rho = 0$, namely AB and BB restrictions are met. BB restrictions are not satisfied when $\kappa \neq 0$, and AB restrictions are not satisfied when $\rho \neq 0$. Errors u_{it} are generated to be cross-sectionally heteroskedastic and non-normal, $u_{it} = (e_{it} - 2)\sigma_{ia}/2$ for $t \leq [T/2]$, and $u_{it} = (e_{it} - 2)\sigma_{ib}/2$ for $t > [T/2]$, with $\sigma_{ia}^2 \sim IIDU(0.25, 0.75)$, $\sigma_{ib}^2 \sim IIDU(1,2)$, $e_{it} \sim IID\chi^2(2)$, and $[T/2]$ is the integer part of $T/2$. Errors ϵ_{it} are generated as $\epsilon_{it} \sim IIDN(0, 1-\theta^2)$. See Subsection A.5.1 for a full description of the MC experiments. The number of time periods available for estimation is $T+1$, namely $(x_{i0}, y_{i0}), (x_{i1}, y_{i1}), ..., (x_{iT} y_{iT})$, is available for $i = 1, 2, ..., n$.

Table A2: Size and Power of AAH, AB and BB estimators in panel ARX(1) experiments when both Arellano and Bond (AB) and Blundell and Bond (BB) restrictions are met

$\rho = 0$, and $\kappa = 0$

		Size (5% level, ×100)						**Power (5% level, ×100)**					
		$\phi_0 = 0.8$			$\beta_0 = 0.5$			$H_1 : \phi = \phi_0 + 0.1$			$H_1 : \beta_0 = \beta_0 + 0.1$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	10.60	11.35	17.90	8.75	11.70	17.40	11.40	12.55	20.60	10.20	12.55	19.55
4	200	6.95	7.75	10.95	5.85	7.35	10.10	8.65	10.55	23.60	7.85	9.05	12.80
4	500	5.40	6.45	7.60	4.80	5.85	6.20	10.65	9.10	43.40	8.10	9.85	12.65
4	1000	6.05	6.25	6.25	5.65	6.25	6.75	17.55	13.25	71.55	13.35	13.90	16.70
4	2000	4.85	4.45	5.20	5.20	5.90	5.60	28.90	19.30	94.80	19.60	20.20	25.10
4	8000	5.10	5.00	5.00	5.30	5.40	5.50	82.50	60.35	100.00	62.55	62.40	67.35
6	100	25.60	24.30	39.00	27.60	25.75	37.50	24.10	26.45	40.90	30.20	26.80	42.50
6	200	13.65	13.45	19.95	11.55	11.45	16.40	14.95	16.45	33.20	15.85	15.50	25.65
6	500	8.50	8.40	9.95	7.40	7.90	11.00	20.80	16.40	54.90	15.40	16.55	24.60
6	1000	6.60	5.60	6.90	6.45	5.75	6.70	35.45	25.85	83.60	20.20	20.85	28.00
6	2000	5.90	5.50	6.05	4.10	4.70	5.75	64.05	43.90	98.95	34.90	34.50	43.25
6	8000	5.10	5.65	5.80	4.80	4.80	4.45	99.60	95.60	100.00	86.45	87.05	92.30
10	100	94.90	97.25	-	94.40	97.35	-	95.50	96.80	-	95.45	96.80	-
10	200	44.20	38.25	55.40	40.50	36.65	47.25	40.10	41.65	57.85	44.60	40.35	57.00
10	500	18.90	13.40	18.95	14.30	13.10	17.10	42.50	37.55	73.55	30.25	28.95	43.35
10	1000	10.80	8.70	12.55	8.70	7.80	8.85	72.30	58.40	94.35	37.85	36.85	51.90
10	2000	7.65	6.70	7.80	6.45	6.60	7.00	96.45	85.50	99.85	59.95	58.35	70.65
10	8000	5.65	5.60	5.35	5.35	5.20	5.35	100.00	100.00	100.00	99.35	99.20	99.65

See the notes to Table A1

Table A3: Bias and RMSE of AAH, AB and BB estimators in panel ARX(1) experiments when Arellano Bond (AB) restrictions are met and Blundell and Bond (BB) restrictions are not met

$\rho = 0$, and $\kappa = 1$

		Bias ($\times$100)						**RMSE**($\times$100)					
		$\phi_0 = 0.8$			$\beta_0 = 0.5$			$\phi_0 = 0.8$			$\beta_0 = 0.5$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	1.50	-0.17	3.12	0.04	0.44	-0.04	7.36	7.35	3.77	8.76	8.17	8.08
4	200	0.77	-0.09	3.19	-0.19	0.08	0.45	4.62	5.08	3.52	5.73	5.48	5.40
4	500	0.32	0.02	3.25	0.00	0.10	1.10	2.73	3.19	3.38	3.54	3.49	3.58
4	1000	0.23	0.02	3.32	-0.05	0.00	1.16	1.95	2.27	3.38	2.56	2.54	2.70
4	2000	0.10	0.05	3.32	0.03	0.06	1.33	1.33	1.58	3.35	1.78	1.78	2.18
4	8000	0.01	0.01	3.35	-0.02	-0.01	1.37	0.66	0.80	3.36	0.90	0.89	1.62
6	100	1.58	0.11	3.08	-0.39	0.04	3.08	4.95	5.09	3.74	7.71	6.50	6.81
6	200	1.00	0.26	3.03	0.00	0.20	3.03	3.00	3.31	3.31	4.53	4.21	4.20
6	500	0.47	0.24	3.05	-0.04	0.01	3.05	1.74	1.99	3.16	2.73	2.68	2.84
6	1000	0.24	0.04	3.05	0.06	0.09	3.05	1.16	1.36	3.11	1.89	1.87	2.34
6	2000	0.11	0.03	3.09	-0.02	-0.01	3.09	0.81	0.98	3.12	1.28	1.29	2.03
6	8000	0.01	0.00	3.11	-0.01	-0.01	3.11	0.40	0.50	3.12	0.64	0.65	1.84
10	100	3.82	0.35	-	-1.69	0.78	-	10.72	18.94	-	16.14	27.32	-
10	200	1.24	0.61	2.74	-0.04	0.19	2.74	2.60	2.52	3.07	4.14	3.66	3.69
10	500	0.57	0.29	2.55	0.11	0.20	2.55	1.25	1.28	2.65	2.09	2.01	2.28
10	1000	0.30	0.18	2.55	0.06	0.10	2.55	0.78	0.86	2.60	1.35	1.35	2.00
10	2000	0.16	0.09	2.55	0.06	0.07	2.55	0.52	0.60	2.57	0.95	0.96	1.97
10	8000	0.03	0.02	2.55	0.00	0.00	2.55	0.24	0.29	2.56	0.47	0.47	1.92

Notes: See notes to Table A1.

Table A4: Size and Power of AAH, AB and BB estimators in panel ARX(1) experiments when Arellano Bond (AB) restrictions are met and Blundell and Bond (BB) restrictions are not met

$\rho = 0$, and $\kappa = 1$

		Size (5% level, ×100)						Power (5% level, ×100)					
		$\phi_0 = 0.8$			$\beta_0 = 0.5$			$H_1: \phi = \phi_0 + 0.1$			$H_1: \beta_0 = \beta_0 + 0.1$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	10.00	11.80	56.20	8.65	11.60	16.35	11.20	13.55	25.15	10.15	12.60	18.20
4	200	6.50	8.40	71.75	5.45	7.50	9.40	9.20	10.95	26.10	7.75	9.15	11.05
4	500	5.45	6.15	95.75	4.80	5.80	8.05	10.70	10.55	38.05	8.10	9.70	7.85
4	1000	5.95	6.00	99.95	5.70	6.30	8.60	17.55	15.50	61.45	13.10	13.80	7.60
4	2000	5.10	5.00	100.00	5.10	5.90	13.70	30.10	23.25	87.15	19.55	19.95	7.35
4	8000	5.05	4.75	100.00	5.25	5.25	37.30	83.95	70.15	100.00	62.65	62.40	12.00
6	100	24.90	24.65	74.20	27.20	25.35	34.70	24.35	26.45	43.85	29.45	26.55	37.65
6	200	13.75	14.10	82.10	11.50	11.30	16.25	15.10	17.50	34.35	15.20	16.00	18.85
6	500	8.40	8.55	97.85	7.35	7.85	12.30	20.90	18.20	40.65	15.10	16.30	11.35
6	1000	6.85	5.65	100.00	6.55	5.55	16.35	36.40	31.70	59.20	19.95	20.85	7.30
6	2000	5.75	5.10	100.00	4.00	4.55	26.55	65.75	52.20	85.90	34.30	34.25	6.95
6	8000	4.80	5.15	100.00	4.85	4.85	79.45	99.70	97.85	100.00	86.55	86.50	6.85
10	100	95.95	97.25	-	93.95	97.35	-	94.70	97.00	-	94.70	97.05	-
10	200	45.55	38.55	89.95	40.00	35.85	44.95	40.00	43.95	58.50	44.85	40.55	49.70
10	500	18.65	13.85	98.70	14.75	13.25	24.25	43.75	43.85	36.35	29.80	27.95	19.30
10	1000	11.00	9.15	100.00	8.15	7.70	32.15	73.50	67.25	40.00	37.40	36.10	10.55
10	2000	7.90	6.95	100.00	6.40	6.80	55.75	97.25	91.45	53.60	59.00	57.60	8.00
10	8000	5.65	5.55	100.00	5.10	5.20	98.95	100.00	100.00	94.20	99.20	99.10	6.85

See the notes to Table A1

Table A5: Bias and RMSE of AAH, AB and BB estimators in panel ARX(1) experiments when Arellano Bond (AB) restrictions and Blundell and Bond (BB) restrictions are not met

$\rho = 0.8$, and $\kappa = 1$

		Bias ($\times 100$)						**RMSE**($\times 100$)					
		$\phi_0 = 0.8$			$\beta_0 = 0.5$			$\phi_0 = 0.8$			$\beta_0 = 0.5$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	1.48	-4.16	1.86	0.00	0.46	-0.26	7.37	9.22	2.88	8.78	8.83	8.36
4	200	0.77	-3.56	1.86	-0.21	0.27	0.15	4.62	6.86	2.41	5.73	5.87	5.49
4	500	0.32	-2.80	1.93	0.00	0.43	0.59	2.73	4.66	2.15	3.54	3.76	3.55
4	1000	0.23	-2.58	2.01	-0.05	0.35	0.58	1.95	3.81	2.12	2.56	2.72	2.55
4	2000	0.10	-2.39	2.03	0.03	0.39	0.69	1.33	3.09	2.09	1.78	1.98	1.94
4	8000	0.01	-2.31	2.09	-0.02	0.36	0.76	0.66	2.53	2.10	0.90	1.06	1.20
6	100	1.58	-1.44	2.49	-0.40	0.20	2.49	4.95	5.58	3.25	7.71	6.64	6.96
6	200	1.00	-1.05	2.40	0.00	0.45	2.40	3.00	3.62	2.75	4.53	4.37	4.30
6	500	0.47	-0.97	2.42	-0.04	0.30	2.42	1.74	2.34	2.56	2.73	2.76	2.73
6	1000	0.24	-1.04	2.44	0.06	0.41	2.44	1.16	1.79	2.51	1.89	1.97	2.08
6	2000	0.11	-1.03	2.48	-0.02	0.33	2.48	0.81	1.49	2.51	1.28	1.40	1.70
6	8000	0.01	-1.03	2.51	-0.01	0.33	2.51	0.40	1.17	2.52	0.64	0.75	1.35
10	100	3.82	-0.01	-	-1.69	0.69	-	10.72	18.21	-	16.14	25.94	-
10	200	1.24	0.25	2.56	-0.04	0.18	2.56	2.59	2.50	2.91	4.14	3.69	3.70
10	500	0.57	-0.07	2.38	0.11	0.21	2.38	1.25	1.25	2.48	2.09	2.02	2.19
10	1000	0.30	-0.20	2.38	0.06	0.11	2.38	0.78	0.87	2.43	1.35	1.36	1.84
10	2000	0.16	-0.29	2.38	0.06	0.10	2.38	0.52	0.66	2.41	0.95	0.97	1.76
10	8000	0.03	-0.36	2.38	0.00	0.04	2.38	0.24	0.47	2.39	0.47	0.48	1.68

Notes: See notes to Table A1.

Table A6: Size and Power of AAH, AB and BB estimators in panel ARX(1) experiments when Arellano Bond (AB) restrictions and Blundell and Bond (BB) restrictions are not met

$\rho = 0.8$, and $\kappa = 1$

		Size (5% level, ×100)						**Power (5% level, ×100)**					
		$\phi_0 = 0.8$			$\beta_0 = 0.5$			$H_1 : \phi = \phi_0 + 0.1$			$H_1 : \beta_0 = \beta_0 + 0.1$		
T	n	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB	AAH	AB	BB
4	100	10.05	17.00	35.15	8.65	15.35	17.20	11.25	22.20	19.30	10.15	16.70	18.35
4	200	6.50	15.20	40.10	5.45	9.90	10.25	9.25	23.85	13.15	7.75	12.05	12.10
4	500	5.45	14.85	64.30	4.80	8.95	7.90	10.70	31.30	9.70	8.10	12.10	10.00
4	1000	5.95	21.15	89.10	5.70	9.70	7.60	17.55	48.25	9.35	13.10	14.35	11.40
4	2000	5.10	30.80	99.15	5.10	9.95	9.30	30.10	69.50	8.90	19.55	18.60	14.10
4	8000	5.05	71.55	100.00	5.25	10.85	16.95	83.95	99.15	11.55	62.65	48.40	32.60
6	100	24.95	25.85	63.85	27.15	26.55	36.70	24.40	33.90	39.90	29.40	26.85	39.25
6	200	13.75	15.00	69.80	11.50	13.50	17.00	15.10	26.30	22.85	15.20	16.30	20.35
6	500	8.40	10.60	92.10	7.35	9.40	10.10	20.90	36.40	18.25	15.10	15.65	15.35
6	1000	6.85	12.65	99.55	6.55	7.80	11.35	36.40	59.35	21.50	19.95	17.50	11.50
6	2000	5.75	19.70	100.00	4.00	7.30	16.10	65.75	83.80	32.20	34.30	28.05	12.60
6	8000	4.80	52.35	100.00	4.85	9.45	49.40	99.70	99.90	80.35	86.55	72.35	25.90
10	100	95.90	97.40	-	93.95	96.40	-	94.70	96.90	-	94.70	97.40	-
10	200	45.55	37.20	88.50	39.95	35.90	45.35	40.00	47.65	55.15	44.85	40.45	50.05
10	500	18.65	11.70	98.25	14.75	13.50	21.75	43.75	55.45	29.95	29.80	27.45	20.55
10	1000	11.00	8.75	100.00	8.15	8.10	26.60	73.50	80.35	30.00	37.40	35.75	13.35
10	2000	7.90	9.65	100.00	6.40	6.80	45.30	97.25	97.85	36.00	59.00	56.15	11.50
10	8000	5.65	23.55	100.00	5.10	6.15	95.65	100.00	100.00	73.60	99.20	98.85	15.30

See the notes to Table A1

A.6 Empirical Application: AR(1) model of earning dynamics

This section presents the results of estimating a panel AR(1) model for earnings dynamics using the Panel Study of Income Dynamics (PISD) dataset, originally studied by Meghir and Pistaferri (2004), and more recently by Hospido (2012) and Hayakawa and Pesaran (2015).[23] Hospido argues it is important to account for individual unobserved heterogeneity and dynamics in conditional variance of errors, which can change over time. We note that the AAH estimator of AR(1) panel is robust to any time series and cross section heteroskedasticity of errors, and therefore it is valid for estimation and inference in the presence of such effects. Application by Hayakawa and Pesaran (hereafter HP) compares the estimates of panel AR and panel ARX models of PISD earnings across different GMM estimators and the transformed MLE (TMLE) estimator proposed by Hsiao, Pesaran, and Tahmiscioglu (2002) and further extended by Hayakawa and Pesaran (2015).[24] We build on the application by HP and compare their panel AR(1) estimates with ones obtained using the AAH estimator proposed in this paper.

We estimate panel AR(1) model of earning:

$$y_{it} = \alpha_i + \phi y_{i,t-1} + u_{it}, \text{ for } i = 1, 2, ..., n, \tag{A.32}$$

which is identical to our model (1) extensively discussed in Section 2, where y_{it} is $\log(earnings_{it}/price_{it})$. The annual data is unbalanced with $T_{\min} = 9$ and $T_{\max} = 26$ and $n = 2069$ individuals for the period 1967-1992. Similarly to HP, we consider estimating ϕ using the full panel, as well as subsamples. In particular, we consider dividing the individuals into three groups, based on the years of education: HSD (high school dropouts with less than 12 years of education), HSG (high school graduates with at least 12, but less than 16 years of education), and CLG (college graduates with at least 16 years of education). In addition, we consider three different subperiods: 1977-1987 ($T = 5$ after first-differencing), 1977-1987 ($T = 10$), and 1977-1992 ($T = 15$). We estimate (A.32) using the 2-step AAH estimator based on the moment conditions given by (7) and (13), and compare the AAH estimates with the ones obtained using the AB and BB two-step GMM estimators de-

[23] We have downloaded data from the supplementary materials posted for the Hayakawa and Pesaran (2015) using this link: http://www.econ.cam.ac.uk/people-files/emeritus/mhp1/wp12/Matlab-code-and-data-for-TransML-Hayakawa-and-Pesaran-2012.zip. We are grateful to Kazuhiko Hayakawa for making these codes and data publicly available.

[24] This application is not part of the published paper, but it is available in the supplementary materials available at http://www.econ.cam.ac.uk/people-files/emeritus/mhp1/wp12/Matlab-code-and-data-for-TransML-Hayakawa-and-Pesaran-2012.zip.

scribed in Section 7.2. These estimators are the same ones as the two-step GMM estimators based on "DIF1" and "SYS1" moment conditions considered in HP. In addition, we also compare the GMM estimates of ϕ with the ones obtained using the TMLE approach of Hsiao, Pesaran, and Tahmiscioglu (2002), using the robust standard errors derived in Hayakawa and Pesaran (2015).

A.6.1 Estimation results

Estimation results are reported in Table A7. The left panel of this table reports findings for the sample covering 1977-1982 ($T = 5$ after first differencing), middle panel reports results for 1977-1987 ($T = 10$), and the right panel reports findings for 1977-1992 ($T = 15$). The number of moment conditions depends on T and it is reported in the last row. Each of the three samples is further divided based on the education achievement. The top part show results for all individuals with available data, followed by the high school dropouts (HSD), high school graduates (HSG), and college graduates (CLG). One common theme emerges from these results: persistence of earnings is higher for college graduates compared with high school dropouts and high school graduates. This is generally true for all estimators and samples considered with two notable exception: the TMLE estimator when $T = 5$, where the estimate of ϕ is actually lowest for the college graduates; and the second exception is the unusually large (explosive) value of $\hat{\phi} = 1.0375$ obtained when using the BB estimator for HSG sample with $T = 5$. These results therefore suggest that there is some heterogeneity in persistence of earnings based on the educational achievement, and therefore assuming homogeneity of ϕ in the full sample (sample ALL reported in the top panel of Table A7) is probably not warranted. Second interesting observation from the estimates in Table A7 is the very large disparities that exist across the different estimators. Sample with $T = 15$ reported in the right part of the table A7 is subject to many moments problem (in the case of the GMM estimators) with reported number of moments between 104 and 135 compared with the sample sizes ranging from $n = 72$ to $n = 507$. It is clear that TMLE is more reliable for $T = 15$, with the estimated values 0.4236 (HSD), 0.5488 (HSG) and 0.7352 (CLG), reported in the last column of Table A7. We focus next on the estimates with $T = 5$ and $T = 10$, where the number of moments will be of less consequence compared with $T = 15$ in the case of the GMM estimators (AAH, AB and BB). BB estimates of ϕ range between 0.9292 to 1.0375 in all cases with very tight standard errors. AB estimates range between 0.0910 to 0.8779. TMLE and AAH estimates lie in the narrower range of 0.3610 to 0.7091. It is difficult to reconcile such large heterogeneity of estimates across methods particularly given that all these estimates are rather precisely estimated with small standard errors.

Differences among the requirements for the initial values and fixed effects, discussed in depth in this paper, could be one of the contributing factors explaining such large differences across the estimation methods. Hausman test applied to the difference between BB and AAH estimators (reported in Table A8) show very strong rejection rates for all samples considered. This is a strong indication that the BB restrictions are not met. Other factors could also play a role in such a large differences across individual estimators, such as heterogeneity of slope coefficients, cross-sectional error dependence, and higher order dynamics. It is important that the analysis of this paper is extended along the lines of AAH estimator in search of new estimators that are also robust to slope heterogeneity and cross-sectional error dependence.

Table A7: Estimation results for panel AR(1) model of real earnings using PISD dataset

	1977-1982, $T = 5$				**1977-1987,** $T = 10$				**1977-1992,** $T = 15$			
	AAH	AB	BB	TMLE	AAH	AB	BB	TMLE	AAH	AB	BB	TMLE
All	$n = 994$				$n = 712$				$n = 507$			
$\hat{\phi}$	0.4056	0.0910	0.9417	0.6251	0.4457	0.4636	0.9501	0.5045	0.4321	0.4739	0.9487	0.5899
s.e.	0.0424	0.0811	0.0044	0.1026	0.0264	0.0292	0.0007	0.0294	0.0120	0.0133	0.0005	0.0254
HSD	$n = 237$				$n = 134$				$n = 72$			
$\hat{\phi}$	0.4698	0.3084	1.0375	0.6553	0.4631	0.2537	0.9460	0.4055	-	-	-	0.4236
s.e.	0.0578	0.0938	0.0116	0.2015	0.0106	0.0145	0.0005	0.0459	-	-	-	0.0558
HSG	$n = 514$				$n = 382$				$n = 285$			
$\hat{\phi}$	0.4010	0.1231	0.9292	0.6376	0.3610	0.3241	0.9561	0.4449	0.4335	0.4017	0.9449	0.5488
s.e.	0.0684	0.0993	0.0066	0.1084	0.0217	0.0283	0.0011	0.0344	0.0115	0.0161	0.0004	0.0274
CLG	$n = 243$				$n = 196$				$n = 150$			
$\hat{\phi}$	0.4657	0.6012	0.9949	0.5084	0.5988	0.8779	0.9881	0.7091	0.5561	0.8398	0.9897	0.7352
s.e.	0.0680	0.0913	0.0023	0.1263	0.0191	0.0184	0.0006	0.0784	0.0045	0.0047	0.0001	0.0507
h	9	15	20		44	55	65		104	120	135	

Notes: This table reports estimation of coefficient ϕ in panel AR(1) specification $y_{it} = \alpha_i + \phi y_{i,t-1} + u_{it}$, where y_{it} is $\log(earnings_{it}/price_{it})$ using the PISD dataset. "HSD" refers to high school dropouts with less than 12 years of education, "HSG" refers to high school graduates with at least 12, but less than 16 years of education, and "CLG" refers to college graduates with at least 16 years of education. The last row reports the number of moment conditions (h). AAH is the 2-step GMM estimator based on the moment conditions given by (7) and (13), AB and BB are two-step GMM estimators based on "DIF1" and "SYS1" moment conditions outlined in Hayakawa and Pesaran (2015). DIF1 moment conditions are given by (A.28). SYS1 moment conditions are given by (A.28) and (A.30). Conventional standard errors are reported. TMLE is the transformed ML estimator of Hsiao, Pesaran, and Tahmiscioglu (2002) and Hayakawa and Pesaran (2015). "-" indicates the number of moments (h) exceeds the sample size (n). Reported time dimension T refers to the available time periods after first-differencing.

Table A8: Hausman test applied to AAH and BB estimators in the panel AR(1) model of real earnings using PISD dataset

	1977-1982, $T = 5$	**1977-1987,** $T = 10$	**1977-1992,** $T = 15$
All	$n = 994$	$n = 712$	$n = 507$
Hausman test	161.7	366.0	1866.1
p-value	0.000	0.000	0.000
HSD	$n = 237$	$n = 134$	$n = 72$
Hausman test	100.4	2076.5	-
p-value	0.000	0.000	-
HSG	$n = 514$	$n = 382$	$n = 285$
Hausman test	60.2	753.6	1976.7
p-value	0.000	0.000	0.000
CLG	$n = 243$	$n = 196$	$n = 150$
Hausman test	60.6	416.2	9234.4
p-value	0.000	0.000	0.000

Notes: This table reports Hausman test applied to applied to the AAH and BB estimators. See Section 4.1 for details. Under the null hypothesis that the BB conditions are met, the Hausman test is asymptotically distributed as $\chi^2(1)$, for a fixed T and as $n \to \infty$. Results in this table suggest that the BB restrictions are not met.