

Supplementary Appendix for
*Herding and Contrarian Behavior
in Financial Markets*

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Abstract

This document contains additional material that was not included in the main version of the paper.

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A Proofs Omitted from the Paper

A.1 Proof of Lemma 1

Observe first that

$$\mathbb{E}[V|S, H^t] - \mathbb{E}[V|H^t] = \mathcal{V}q_2^t \left(\frac{\Pr(S|V_2)}{\Pr(S)} - 1 \right) + 2\mathcal{V}q_3^t \left(\frac{\Pr(S|V_3)}{\Pr(S)} - 1 \right).$$

The RHS of the the above equality has the same sign as

$$\begin{aligned} & q_2^t \left(\Pr(S|V_2) \sum_j q_j^t - \sum_j \Pr(S|V_j)q_j^t \right) + 2 q_3^t \left(\Pr(S|V_3) \sum_j q_j^t - \sum_j \Pr(S|V_j)q_j^t \right) \\ = & q_1^t q_2^t (\Pr(S|V_2) - \Pr(S|V_1)) + q_2^t q_3^t (\Pr(S|V_2) - \Pr(S|V_3)) \\ & + 2 q_3^t (q_1^t (\Pr(S|V_3) - \Pr(S|V_1)) + q_2^t (\Pr(S|V_3) - \Pr(S|V_2))) = \text{expression (2)}. \end{aligned}$$

A.2 Proof of Lemma 2

(i) By standard results on MLRP and stochastic dominance it must be that $\mathbb{E}[V|S_l] < \mathbb{E}[V|S_h]$. By a similar reasoning, at any history H^t , $\mathbb{E}[V|S_l, H^t] < \mathbb{E}[V|S_h, H^t]$ if the following MLRP condition holds at H^t : for any $S_l < S_h$ and any $V_l < V_h$

$$\frac{\Pr(S_h|V_h, H^t)}{\Pr(S_l|V_h, H^t)} > \frac{\Pr(S_h|V_l, H^t)}{\Pr(S_l|V_l, H^t)}. \quad (\text{A-1})$$

To show this note first that $\Pr(V|H^t, S) = \Pr(V|S)\Pr(H^t|V) / \sum_{V' \in \mathbb{V}} \Pr(V'|S)\Pr(H^t|V')$. Then we have by the following manipulations that the MLRP condition $\frac{\Pr(S_h|V_h)}{\Pr(S_l|V_h)} > \frac{\Pr(S_h|V_l)}{\Pr(S_l|V_l)}$ implies the MLRP condition (A-1) at any H^t :

$$\begin{aligned} & \Pr(S_l|V_l)\Pr(S_h|V_h) > \Pr(S_l|V_h)\Pr(S_h|V_l) \\ \Leftrightarrow & \Pr(V_l|S_l)\Pr(V_h|S_h) > \Pr(V_h|S_l)\Pr(V_l|S_h) \\ \Leftrightarrow & \frac{\Pr(V_l|S_l)\Pr(H^t|V_l)}{\sum_{\mathbb{V}} \Pr(V|S_l)\Pr(H^t|V)} \frac{\Pr(V_h|S_h)\Pr(H^t|V_h)}{\sum_{\mathbb{V}} \Pr(V|S_h)\Pr(H^t|V)} > \frac{\Pr(V_h|S_l)\Pr(H^t|V_h)}{\sum_{\mathbb{V}} \Pr(V|S_l)\Pr(H^t|V)} \frac{\Pr(V_l|S_h)\Pr(H^t|V_l)}{\sum_{\mathbb{V}} \Pr(V|S_h)\Pr(H^t|V)} \\ \Leftrightarrow & \Pr(V_l|H^t, S_l)\Pr(V_h|H^t, S_h) > \Pr(V_h|H^t, S_l)\Pr(V_l|H^t, S_h). \end{aligned}$$

(ii) Suppose contrary to the claim, that an informed trader with signal S_1 does not sell at some history H^t . Then by part (i) no informed trader sells at H^t . This implies that at history H^t , $\text{bid}^t = \mathbb{E}[V|H^t]$. But since, by part (i), $\mathbb{E}[V|H^t]$ exceeds $\mathbb{E}[V|S_1, H^t]$, we have $\text{bid}^t > \mathbb{E}[V|S_1, H^t]$. Hence, an informed trader with signal S_1 sells at H^t . This is a contradiction.

The proof that informed traders with signal S_3 always buy is analogous.

(iii) First we show that $\Pr(S_1|V_1) > \Pr(S_1|V_3)$. Suppose otherwise; thus $\Pr(S_1|V_1) \leq$

$\Pr(S_1|V_3)$. Then the two MLRP conditions $\Pr(S_1|V_1)\Pr(S_2|V_3) > \Pr(S_2|V_1)\Pr(S_1|V_3)$ and $\Pr(S_1|V_1)\Pr(S_3|V_3) > \Pr(S_3|V_1)\Pr(S_1|V_3)$ imply respectively that $\Pr(S_2|V_1) < \Pr(S_2|V_3)$ and $\Pr(S_3|V_1) < \Pr(S_3|V_3)$. Hence, since $\Pr(S_1|V_1) \leq \Pr(S_1|V_3)$ we have $\sum_{i=1}^3 \Pr(S_i|V_3) > \sum_{i=1}^3 \Pr(S_i|V_1)$. But this contradicts $\sum_{i=1}^3 \Pr(S_i|V_j) = 1$ for every j .

The same argument can be applied to show that $\Pr(S_1|V_1) > \Pr(S_1|V_2)$ and $\Pr(S_1|V_2) > \Pr(S_1|V_3)$, and also in the reverse direction for $\Pr(S_3|V_1) < \Pr(S_3|V_2) < \Pr(S_3|V_3)$.

A.3 Proof of Lemma 3

This follows from Lemma 1: By the symmetry assumption on the priors ($q_1^1 = q_3^1$), the RHS of (2) is negative (positive) at $t = 1$ if and only if $(\Pr(S|V_3) - \Pr(S|V_1))(q_2^1 + 2q_1^1)q_3^1$ is less (greater) than 0; the latter is equivalent to S having a negative (positive) bias.

A.4 Proof of Lemma 4

The claim follows from $\mathbb{E}[V|H^t] - \mathbb{E}[V] = \mathcal{V}[(1 - q_1^t - q_3^t) + 2q_3^t] - \mathcal{V} = \mathcal{V}(q_3^t - q_1^t)$.

A.5 Proof of Lemma 6

The proof is analogous to the derivation in the proof of Lemma 1. To show (i) note that

$$\mathbb{E}[V|S, H^t] - \text{ask}^t = \mathcal{V}q_2 \left(\frac{\Pr(S|V_2)}{\Pr(S)} - \frac{\beta_2}{\Pr(\text{buy}|H^t)} \right) + 2\mathcal{V}q_3 \left(\frac{\Pr(S|V_3)}{\Pr(S)} - \frac{\beta_3}{\Pr(\text{buy}|H^t)} \right).$$

The RHS of the above has the same sign as

$$\begin{aligned} & q_2 \left(\Pr(S|V_2) \sum_j \beta_j q_j - \beta_2 \sum_j \Pr(S|V_j) q_j \right) + 2 q_3 \left(\Pr(S|V_3) \sum_j \beta_j q_j - \beta_3 \sum_j \Pr(S|V_j) q_j \right) \\ = & q_1 q_2 (\beta_1 \Pr(S|V_2) - \beta_2 \Pr(S|V_1)) + q_2 q_3 (\beta_3 \Pr(S|V_2) - \beta_2 \Pr(S|V_3)) \\ & + 2 q_3 (q_1 (\beta_1 \Pr(S|V_3) - \beta_3 \Pr(S|V_1)) + q_2 (\beta_2 \Pr(S|V_3) - \beta_3 \Pr(S|V_2))) = \text{expression (3)}. \end{aligned}$$

The proof of (ii) is analogous to that of (i).

A.6 Proof of Theorem 3

We first prove that any informed type buys initially if it has a negative bias and if there are enough noise traders.

Lemma II *Let S be negatively biased. Then $\mathbb{E}[V|S] < \mathbb{E}[V]$. Hence, there exists $\mu^{in} \in (0, 1]$ such that S sells at the initial history if $\mu < \mu^{in}$.*

Proof of Lemma II: Without loss of generality, we present the proof only for the case when the number of states $n > 2$ is even so that $n = 2k$ for some integer k . Then by the symmetry of the prior $\mathbb{E}[V] = (2k - 1)/2$. Also, $\mathbb{E}[V|S] = \sum_{i=1}^{2k} (i - 1)\Pr(V_i|S)$. Thus, we need to show

$$\sum_{i=1}^{2k} (i - 1)\Pr(V_i|S) < \frac{2k - 1}{2}. \quad (\text{A-2})$$

Next note that by $\Pr(S|V_i) > \Pr(S|V_{n+1-i})$ together with the symmetry of the initial prior, we have $\Pr(V_i|S) > \Pr(V_{n+1-i}|S)$ for all $i < (2k + 1)/2$. Using this and $\sum_{i=1}^{2k} \Pr(V_i|S) = 1$, we have $\sum_{i=1}^k \Pr(V_i|S) > \frac{1}{2} > \sum_{i=k+1}^{2k} \Pr(V_i|S)$. Therefore

$$(k - 1) + \sum_{i=k+1}^{2k} \Pr(V_i|S) < (k - 1) + \frac{1}{2} = \frac{2k - 1}{2}. \quad (\text{A-3})$$

Then by (A-2) it is sufficient to show that

$$\sum_{i=1}^k (i - 1)\Pr(V_i|S) + \sum_{i=k+1}^{2k} (i - 1)\Pr(V_i|S) < (k - 1) + \sum_{i=k+1}^{2k} \Pr(V_i|S). \quad (\text{A-4})$$

But the second term on the left hand side of (A-4) is

$$\begin{aligned} & \sum_{i=k+1}^{2k} (i - 1)\Pr(V_i|S) = \sum_{i=k+1}^{2k} \Pr(V_i|S) + (k - 1)\Pr(V_{k+1}|S) + \dots + (2k - 2)\Pr(V_{2k}|S) \\ & < \sum_{i=k+1}^{2k} \Pr(V_i|S) + (k - 1)\Pr(V_{k+1}|S) + [(k - 1)\Pr(V_{k+2}|S) + \Pr(V_{k-1}|S)] \\ & \quad + [(k - 1)\Pr(V_{k+3}|S) + 2\Pr(V_{k-2}|S)] + \dots + [(k - 1)\Pr(V_{2k}|S) + (k - 1)\Pr(V_1|S)] \\ & = \sum_{i=k+1}^{2k} \Pr(V_i|S) + (k - 1) \sum_{i=k+1}^{2k} \Pr(V_i|S) + \sum_{j=1}^k (k - j)\Pr(V_j|S). \end{aligned}$$

Therefore,

$$\begin{aligned} \text{LHS of (A-4)} & < \sum_{i=1}^k (i - 1)\Pr(V_i|S) + \sum_{i=k+1}^{2k} \Pr(V_i|S) + (k - 1) \sum_{i=k+1}^{2k} \Pr(V_i|S) + \sum_{i=1}^k (k - i)\Pr(V_i|S) \\ & = (k - 1) + \sum_{i=k+1}^{2k} \Pr(V_i|S) = \text{RHS of (A-4)}. \end{aligned}$$

This demonstrates that $\mathbb{E}[V|S] < \mathbb{E}[V]$. To complete the proof of the lemma we also need to show that there exists $\mu^{in} \in (0, 1]$ such that $\mathbb{E}[V|S] < \text{bid}^1$ if $\mu < \mu^{in}$. As in Lemma 5 this follows immediately from $\mathbb{E}[V|S] < \mathbb{E}[V]$ and from $\lim_{\mu \rightarrow 0} \mathbb{E}[V] - \text{bid}^1 = 0$. This completes the proof of Lemma II.

Next, we turn to the switching of behavior. Before doing that note that with MLRP the probability of a buy is increasing in the liquidation values and probability of a sale is decreasing in the liquidation values. To show this assume without any loss of generality that $S_1 < S_2 < \dots < S_n$. Then we have the following.

Lemma III *When signals satisfy the MLRP, $\beta_1^t < \beta_2^t < \dots < \beta_n^t$ and $\sigma_1^t > \sigma_2^t > \dots > \sigma_n^t$.*

Proof of Lemma III: We will show only $\beta_1^t < \beta_2^t < \dots < \beta_n^t$, the result on sales σ_i follows analogously. To show the former, observe that with MLRP signals, expectations are ordered in signals: for $i > j$, $\mathbb{E}[V|H^t, S_i] > \mathbb{E}[V|H^t, S_j]$. Thus, if signal type S_k buys, so will all $S_l > S_k$. Thus for $i > j$, $\beta_i - \beta_j$ has the same sign as

$$\begin{aligned} \sum_{l=m}^n \Pr(S_l|V_i) - \sum_{l=m}^n \Pr(S_l|V_j) &= 1 - \sum_{l=1}^{m-1} \Pr(S_l|V_i) - \left(1 - \sum_{l=1}^{m-1} \Pr(S_l|V_j)\right) \\ &= \sum_{l=1}^{m-1} \Pr(S_l|V_j) - \sum_{l=1}^{m-1} \Pr(S_l|V_i), \text{ for some } m \leq n. \end{aligned}$$

This latter expression is positive since MLRP implies First Order Stochastic dominance. This completes the proof of Lemma III.

Proof of part (a) of Theorem 3: Fix S . Analogously to Lemma 6, by simple calculations, it can be shown that $\mathbb{E}[V|S, H^t] - \text{ask}^t$ has the same sign as

$$q_n^t q_{n-1}^t \cdot \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t} (\beta_i^t \Pr(S|V_{i+j}) - \beta_{i+j}^t \Pr(S|V_i)). \quad (\text{A-5})$$

Next consider the infinite path consisting of only buys at every date. By MLRP and Lemma III, $\beta_1^t < \beta_2^t < \dots < \beta_n^t$. Then we have that q_i^t/q_n^t converges to zero for all $i < n$. But this implies that along this path

$$\lim_t \mathbb{E}[V|H^t] = V_n > \mathbb{E}[V] \quad (\text{A-6})$$

Also, since for any $i < n - 1$ and $j \geq 1$ and for any t ,

$$\frac{q_i^{t+1} q_{i+j}^{t+1}}{q_{n-1}^{t+1} q_n^{t+1}} = \frac{\beta_i^t \beta_{i+j}^t}{\beta_{n-1}^t \beta_n^t} \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t} < \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t},$$

we have that $(q_i^t q_{i+j}^t)/(q_{n-1}^t q_n^t)$ converges to zero along this infinite path of buys. Thus, as $t \rightarrow \infty$

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i^t q_{i+j}^t}{q_{n-1}^t q_n^t} (\beta_i^t \Pr(S|V_{i+j}) - \beta_{i+j}^t \Pr(S|V_i)) \rightarrow \lim_t [\beta_{n-1}^t \Pr(S|V_n) - \beta_n^t \Pr(S|V_{n-1})]. \quad (\text{A-7})$$

Let S_i be the lowest type that buys at t . Then with MLRP all S_j with $j \in \{i, \dots, n\}$ will buy. This implies that

$$\begin{aligned} & \beta_{n-1}^t \Pr(S|V_n) - \beta_n^t \Pr(S|V_{n-1}) > 0 \\ \Leftrightarrow & (\gamma + \mu \sum_{j=i}^n \Pr(S_j|V_{n-1})) \Pr(S|V_n) > (\gamma + \mu \sum_{j=i}^n \Pr(S_j|V_n)) \Pr(S|V_{n-1}) \\ & \Leftrightarrow (\Pr(S|V_n) - \Pr(S|V_{n-1})) > \frac{\mu}{\gamma} \sum_{j=i}^n \{\Pr(S_j|V_n) \Pr(S|V_{n-1}) - \Pr(S_j|V_{n-1}) \Pr(S|V_n)\}. \end{aligned} \quad (\text{A-8})$$

Since $\Pr(S|V_n) > \Pr(S|V_{n-1})$, the left hand side of the last inequality in (A-8) is positive. Therefore, by (A-8), for μ sufficiently small, $\beta_{n-1}^t \Pr(S|V_n) - \beta_n^t \Pr(S|V_{n-1}) > 0$. Since there is a finite number of types, it then follows from (A-5), (A-6), and (A-7) that there exist a critical level of informed trading $\mu^{ch} > 0$ and a history H^t along the infinite path of buys such that $\mathbb{E}[V|S, H^t] - \text{ask}^t > 0$ and $\mathbb{E}[V|H^t] > \mathbb{E}[V]$. This together with Lemma II completes the proof.

Proof of part (b) of Theorem 3: Analogously to (a), we can rewrite $\mathbb{E}[V|S, H^t] - \text{ask}^t =$

$$q_1 q_2 \cdot \sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i q_{i+j}}{q_1 q_2} (\beta_i^t \Pr(S|V_{i+j}) - \beta_{i+j}^t \Pr(S|V_i)). \quad (\text{A-9})$$

Now consider the infinite path consisting of only sales at every date. By MLRP and Lemma III, $\sigma_1^t > \sigma_2^t > \dots > \sigma_n^t$. Then we have that q_i^t/q_1^t converges to zero for all $i > 1$. But this implies that

$$\lim_t \mathbb{E}[V|H^t] = V_1 < \mathbb{E}[V] \quad (\text{A-10})$$

Also, since for any $i, j \geq 1$ such that either i or $j > 1$, and any t ,

$$\frac{q_i^{t+1} q_{i+j}^{t+1}}{q_1^{t+1} q_2^{t+1}} = \frac{\sigma_i^t \sigma_{i+j}^t}{\sigma_1^t \sigma_2^t} \frac{q_i^t q_{i+j}^t}{q_1^t q_2^t} < \frac{q_i^t q_{i+j}^t}{q_1^t q_2^t},$$

we have that $(q_i^t q_{i+j}^t)/(q_1^t q_2^t)$ converges to zero along this infinite path of buys. Thus, as $t \rightarrow \infty$

$$\sum_{j=1}^{n-1} \sum_{i=1}^{n-j} j \cdot \frac{q_i q_{i+j}}{q_1 q_2} (\beta_i \Pr(S|V_{i+j}) - \beta_{i+j} \Pr(S|V_i)) \rightarrow \lim_t [\beta_1 \Pr(S|V_2) - \beta_2 \Pr(S|V_1)]. \quad (\text{A-11})$$

Let S_i be the highest type that buys at t . Then with MLRP all S_j with $j \in \{i, \dots, n\}$ will buy at t . This implies that $\beta_1^t \Pr(S|V_2) - \beta_2^t \Pr(S|V_1) > 0$ if and only if

$$\Leftrightarrow (\Pr(S|V_2) - \Pr(S|V_1)) > \frac{\mu}{\gamma} \sum_{j=i}^n \{\Pr(S_j|V_2) \Pr(S|V_1) - \Pr(S_j|V_1) \Pr(S|V_2)\}. \quad (\text{A-12})$$

Since $\Pr(S|V_2) > \Pr(S|V_1)$, the left hand side of (A-12) is positive. Thus, for μ sufficiently small, $\beta_1^t \Pr(S|V_2) - \beta_2^t \Pr(S|V_1) > 0$. As there are finite number of types, it then follows from (A-9), (A-10), and (A-11) that there exist a critical level of informed trading $\mu^{ch} > 0$

and a history H^t along the infinite path of sales such that $E[V|S, H^t] - \text{ask}^t > 0$ and $E[V|H^t] < E[V]$. This together with Lemma II completes the proof.

B Additional Results

B.1 Proposition 3a

- (i) *Suppose that S buy herds and there is at most one U shaped signal.*
Then $\mu < \min\{\mu_s^{in}, \mu_2^{ch}\}$.
- (ii) *Suppose that S sell herds and there is at most one U shaped signal.*
Then $\mu < \min\{\mu_b^{in}, \mu_1^{ch}\}$.
- (iii) *Suppose that S acts as a buy contrarian and there is at most one hill shaped signal.*
Then $\mu < \min\{\mu_s^{in}, \mu_1^{ch}\}$.
- (iv) *Suppose that S acts as a sell contrarian and there is at most one hill shaped signal.*
Then $\mu < \min\{\mu_b^{in}, \mu_2^{ch}\}$.

We shall prove (i); the proof of (ii) – (iv) are analogous.

Since S sells initially it follows from Lemma 5 that $\mu < \mu_s^{in}$. To show that $\mu < \mu_2^{ch}$ first note that by Proposition 1, S must be nU shaped. Next consider the different possibilities separately.

Case A. There is no signal $S' \neq S$ such that $\Pr(S'|V_3) > \Pr(S'|V_2)$. Then it must be that $\mu_2^{ch}(S') = 1$ for all S' and therefore it must be that $\mu < \mu_2^{ch} = 1$.

Case B. There is a signal $S' \neq S$ such that $\Pr(S'|V_3) > \Pr(S'|V_2)$. Since S is U shaped it must be that $\Pr(S|V_3) > \Pr(S|V_2)$ and $\Pr(S''|V_3) \leq \Pr(S''|V_2)$ for $S'' \neq S, S'$. This implies that $\mu_2^{ch}(S'') = 1$ and hence, $\mu_2^{ch}(S') = \mu_2^{ch}$.

Now there are two cases. First, if $\mu_2^{ch}(S')$ also equals 1 then clearly $\mu_2^{ch} = 1$ and the claim is trivially true.

Second, assume that $\mu_2^{ch}(S') = \mu_2^{ch} < 1$. Since S buy herds at H^t , to show that $\mu < \min\{\mu_s^{in}, \mu_2^{ch}\}$ it suffices to show that S' also buys whenever S buys (the alternative is that S' does not buy so that $\mu^{ch} = 1 > \mu_2^{ch}$). When S' buys, $E[V|S', H^t] - \text{ask}^t > 0$. Suppose S' does not buy. As the sign of $E[V|S', H^t] - \text{ask}^t$ is given by equation (3), it must then hold that

$$\begin{aligned} q_1 q_2 [\beta_1 \Pr(S'|V_2) - \beta_2 \Pr(S'|V_1)] + q_2 q_3 [\beta_2 \Pr(S'|V_3) - \beta_3 \Pr(S'|V_2)] \\ + 2 q_1 q_3 [\beta_1 \Pr(S'|V_3) - \beta_3 \Pr(S'|V_1)] \leq 0. \end{aligned} \tag{B-13}$$

Also, since there is at most one U shaped signal it must be that

$$\Pr(S'|V_3) > \Pr(S'|V_2) \geq \Pr(S'|V_1). \tag{B-14}$$

By Proposition 1 this implies that S' does not sell. By supposition S' does not buy and therefore S is the only buyer at H^t (S'' is selling). Since S is nU shaped we must also have $\beta_1^t > \beta_3^t \geq \beta_2^t$. This, together with (B-14) imply that the first and the third term in (B-13) are positive. Furthermore, the second term equals

$$\gamma(\Pr(S'|V_3) - \Pr(S'|V_2)) + \mu(\Pr(S|V_2)\Pr(S'|V_3) - \Pr(S|V_3)\Pr(S'|V_2)). \quad (\text{B-15})$$

By (B-14) the first term in the last expression is positive; furthermore, since S is nU, we have $m^2 = \Pr(S|V_3) > \Pr(S|V_2)$. Since $\mu_2^{ch}(S') < 1$ we must have that $M^2(S') < 1$ is negative. But $-\mu M^2(S')$ is the second term in the last expression and it is thus positive. Consequently, (B-15) is positive. Therefore, the second term in (B-13) must also be positive. Therefore, S' must be buying at any H^t at which S buys and thus $\mu_2^{ch} < 1$ is unique.

B.2 Proof of the statement in condition (7) in Section 7 of the main text

First, note that, by (33) in the proof of Proposition 7, we have

$$\begin{aligned} \sigma_3\sigma_{2,o} - \sigma_{3,o}\sigma_2 &= -\mu^2\rho_{12}^{23} + \mu\gamma(\Pr(S|V_2) - \Pr(S|V_3)) < 0 \\ \sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1 &> \sigma_3\sigma_{1,o} - \sigma_{3,o}\sigma_1 \end{aligned} \quad (\text{B-16})$$

Also, since for herding we require $\mathbb{E}[V|S, H^1] < \text{bid}^1$, it follows from Lemma 6 and (32) that

$$q_2^1 q_1^1 [\sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1] + q_3^1 q_2^1 [\sigma_3\sigma_{2,o} - \sigma_{3,o}\sigma_2] + 2q_3^1 q_1^1 [\sigma_3\sigma_{1,o} - \sigma_{3,o}\sigma_1] > 0.$$

But then by (B-16) we have

$$\sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1 > 0. \quad (\text{B-17})$$

Since $\mathbb{E}[V|H^t] - \mathbb{E}_o[V|H^t]$ has the same sign as the expression in (29), by simple expansion of this expression we have that if $b = 0$ then $\mathbb{E}[V|H^t] - \mathbb{E}_o[V|H^t]$ has the same sign as

$$\begin{aligned} & q_2^r q_1^r \left\{ (\sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1) \sum_{\tau=0}^{s-1} (\sigma_2\sigma_{1,o})^{s-1-\tau} (\sigma_{2,o}\sigma_1)^\tau \right\} \\ & + q_3^r q_2^r \left\{ [(\sigma_3\sigma_{2,o}) - (\sigma_{3,o}\sigma_2)] \sum_{\tau=0}^{s-1} (\sigma_3\sigma_{2,o})^{s-1-\tau} (\sigma_{3,o}\sigma_2)^\tau \right\} \\ & + 2q_3^r q_1^r \left\{ (\sigma_3\sigma_{1,o} - \sigma_{3,o}\sigma_1) \sum_{\tau=0}^{s-1} (\sigma_3\sigma_{1,o})^{s-1-\tau} (\sigma_{3,o}\sigma_1)^\tau \right\}. \end{aligned}$$

Rearranging, we have that for $b = 0$, $\mathbb{E}[V|H^t] - \mathbb{E}_o[V|H^t]$ has the same sign as

$$\begin{aligned} & q_2^r q_1^r \frac{\sum_{\tau=0}^{s-1} (\sigma_2\sigma_{1,o})^{s-1-\tau} (\sigma_{2,o}\sigma_1)^\tau}{\sum_{\tau=0}^{s-1} (\sigma_3\sigma_{2,o})^{s-1-\tau} (\sigma_{3,o}\sigma_2)^\tau} [\sigma_2\sigma_{1,o} - \sigma_{2,o}\sigma_1] + q_3^r q_1^r [\sigma_3\sigma_{2,o} - \sigma_{3,o}\sigma_2] \\ & + 2q_3^r q_1^r \frac{\sum_{\tau=0}^{s-1} (\sigma_3\sigma_{1,o})^{s-1-\tau} (\sigma_{3,o}\sigma_1)^\tau}{\sum_{\tau=0}^{s-1} (\sigma_3\sigma_{2,o})^{s-1-\tau} (\sigma_{3,o}\sigma_2)^\tau} [\sigma_3\sigma_{1,o} - \sigma_{3,o}\sigma_1]. \end{aligned} \quad (\text{B-18})$$

Further manipulations show that

$$\left(\frac{\sigma_1}{\sigma_3}\right)^s > \frac{\sum_{\tau=0}^{s-1} (\sigma_2 \sigma_{1,o})^{s-1-\tau} (\sigma_{2,o} \sigma_1)^\tau}{\sum_{\tau=0}^{s-1} (\sigma_3 \sigma_{2,o})^{s-1-\tau} (\sigma_{3,o} \sigma_2)^\tau} \Leftrightarrow \sum_{\tau=0}^{s-1} \sigma_2^{s-1-\tau} \sigma_{2,o}^\tau (\sigma_1 \sigma_3)^\tau ((\sigma_1 \sigma_{3,o})^{s-1-\tau} - (\sigma_3 \sigma_{1,o})^{s-1-\tau}) > 0.$$

Also, by assumption we have $\frac{\sigma_1}{\sigma_3} > \frac{\sigma_{1,o}}{\sigma_{3,o}}$. Therefore, we must have

$$\left(\frac{\sigma_1}{\sigma_3}\right)^s > \frac{\sum_{\tau=0}^{s-1} (\sigma_2 \sigma_{1,o})^{s-1-\tau} (\sigma_{2,o} \sigma_1)^\tau}{\sum_{\tau=0}^{s-1} (\sigma_3 \sigma_{2,o})^{s-1-\tau} (\sigma_{3,o} \sigma_2)^\tau}. \quad (\text{B-19})$$

Similar manipulations show that

$$\left(\frac{\sigma_1}{\sigma_2}\right)^s < \frac{\sum_{\tau=0}^{s-1} (\sigma_3 \sigma_{1,o})^{s-1-\tau} (\sigma_3 \sigma_1)^\tau}{\sum_{\tau=0}^{s-1} (\sigma_3 \sigma_{2,o})^{s-1-\tau} (\sigma_{3,o} \sigma_2)^\tau} \Leftrightarrow \sum_{\tau=0}^{s-1} \sigma_3^{s-1-\tau} \sigma_{3,o}^\tau (\sigma_1 \sigma_2)^\tau ((\sigma_2 \sigma_{1,o})^{s-1-\tau} - (\sigma_1 \sigma_{2,o})^{s-1-\tau}) > 0.$$

This together with (B-17), implies that

$$\left(\frac{\sigma_1}{\sigma_2}\right)^s < \frac{\sum_{\tau=0}^{s-1} (\sigma_3 \sigma_{1,o})^{s-1-\tau} (\sigma_3 \sigma_1)^\tau}{\sum_{\tau=0}^{s-1} (\sigma_3 \sigma_{2,o})^{s-1-\tau} (\sigma_{3,o} \sigma_2)^\tau}. \quad (\text{B-20})$$

Also, since $\mathbb{E}[V|S, H^t] - \text{bid}^t > 0$, by Lemma 6, we have that if $b = 0$ then

$$\begin{aligned} q_2^r q_1^r \left(\frac{\sigma_1}{\sigma_3}\right)^s [\sigma_2 \sigma_{1,o} - \sigma_{2,o} \sigma_1] + q_3^r q_2^r [\sigma_3 \sigma_{2,o} - \sigma_{3,o} \sigma_2] \\ + 2q_3^r q_1^r \left(\frac{\sigma_1}{\sigma_2}\right)^s [\sigma_3 \sigma_{1,o} - \sigma_{3,o} \sigma_1] < 0. \end{aligned} \quad (\text{B-21})$$

Then it follows from (B-21), together with $\frac{\sigma_1}{\sigma_3} > \frac{\sigma_{1,o}}{\sigma_{3,o}}$, (B-17), (B-19) and (B-20), that the expression in (B-18) is negative. Thus $\mathbb{E}[V|H^t] - \mathbb{E}_o[V|H^t] < 0$ and (7) follows.

C The Parameters used for Figure 1

Herding Example				Contrarian Example			
$\Pr(S V)$	V_1	V_2	V_3	$\Pr(S V)$	V_1	V_2	V_3
S_1	$\frac{601}{1000}$	$\frac{270}{1000}$	0	S_1	$\frac{7}{10}$	$\frac{1}{4}$	0
S_2	$\frac{399}{1000}$	$\frac{180}{1000}$	$\frac{245}{1000}$	S_2	$\frac{3}{10}$	$\frac{1}{2}$	$\frac{3}{20}$
S_3	0	$\frac{550}{1000}$	$\frac{755}{1000}$	S_3	0	$\frac{1}{4}$	$\frac{17}{20}$
$\mathbb{V} = (0, 10, 20)$				$\mathbb{V} = (0, 10, 20)$			
$\Pr(V) = (1/100, 98/100, 1/100)$				$\Pr(V) = (1/4, 1/2, 1/4)$			
$\mu_b^{ch} = 0.9496, \mu_b^{in} = 0.4294$				$\mu_b^{ch} = 0.4706, \mu_b^{in} = 0.1922$			
$\Rightarrow \mu = 0.4294 - 0.0001$				$\Rightarrow \mu = 0.1922 - 0.001$			