Generalized Intergenerational Mobility Regressions

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Abstract

Current research on intergenerational mobility (IGM) is informed by *statistical* approaches based on log-level and rank regressions, whose *economic* interpretations remain largely unknown. We reveal the subjective value-judgements in them: they are represented by weighted-sums over heterogeneous groups, with controversial *economic* properties. For the first time, we derive the rank-regression weights as improper and inconsistent aggregators. We propose a general construction of IGM measures that can incorporate any transparent *economic* preferences. They are interpreted as the marginal effect of parental normalized social welfare on children's normalized welfare. Conventional regressions are special cases with implicit economic preferences that fail inequality-aversion and the Pigou-Dalton principle of transfers. Empirically, we show that the disparate findings in the literature may reflect differences in subjective valuation of different groups, rather than "mobility". A variety of economic preferences, with varying inequality aversion, demonstrate a nuanced view of mobility, and perspectives on geographic-differences and dynamics of it.

JEL Classifications: J62, D63, I31, C43, J13

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1　Introduction

Growing concern over rising inequality, as well as the potentially unequal and limited economic opportunity for children from less advantaged families, have intensified interest in measures of intergenerational mobility (IGM). Current research is dominantly informed by partial correlation measures derived from linear regression models for (logs of) income levels and/or ranks of offsprings incomes and their parents. High "correlation" or "dependence" between incomes of generations indicates inelasticity, "immobility".[1] These regressions are extensively contrasted and debated as they produce a wide range of estimates that lead to drastically different impressions of intergenerational mobility. Surveying the field, Mazumder (2018) notes the estimates range from less than .3 in early studies, to around .5 or higher in a number of papers using the Panel Study of Income Dynamics (PSID) with log regressions, to .34 in the recent, influential study of Chetty et al. (2014) using administrative data with rank regressions. "Where exactly the United States and other countries lie on this spectrum has been a central question in the intergenerational income mobility literature." (Mazumder, 2018).[2]

On the surface, these log and rank regressions appear connected because they are both estimated using linear regression whose *statistical* properties are well known. The discrepancies in the results using these two approaches are interpreted to mean drastically different impressions of intergenerational mobility, with significant policy implications. Current aggregated interpretations are challenged due to heterogeneity and nonlinearities in social mobility.

Our paper proceeds in two steps. Firstly. a systematic analysis is given of regressions as implicit, subjective aggregation of "mobility" over an heterogeneous population. For the first time, we extend to rank-rank regressions a little known aggregation property of level regressions. We also derive the relation between the two regressions. The general problem of non-consensus aggregation over individuals and groups, by any scalar index of an outcome distribution, is manifest and often undesirable in the case of these regressions. Regression estimators serve a different "master", *statistical*

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[1] In other words, the regression coefficients capture the mobility concept of origin-independence, as clarified in Fields (2021)

[2] In their extensive survey of "Family Background, Neighborhoods and Intergenerational Mobility", Mogstad and Torsvik (2021) dig deeper into the causal effects of family background variables and neighborhoods.

estimation and prediction risk functions, not *economic* and welfare-theoretic criteria tied to well known social preferences and policy objectives!

Motivated by these findings and insights, this paper then proposes a general decision-theoretic approach to aggregate measures of IGM that can flexibly incorporate any economic preferences or policy objectives. They subsume the current IGE measures. Our measures can be *economically* interpreted as the marginal effect of parents' normalized social welfare on children's normalized social welfare. This approach is transparent on the subjectivity of dealing with heterogeneity and non-linearities in the transmission of mobility. We reveal that the conventional IGEs correspond to convex economic preferences that are not equality-minded (also fail to meet the Pigou-Dalton principle of transfers). We analyse two prominent cases of mobility measures based on commonly used parametric families of economic evaluation functions in the literature.

We consider a complex, nonlinear relationship connecting child income to parental states as predicted by economic theories. Due to differences in factors such as incentives to invest in human capital across family incomes, neighborhood or peer effects, education, and a myriad of other factors, the relationship between parent and child incomes is likely heterogenous and nonlinear. As a result, the "correlation" between these incomes would vary with parental income and other factors. Such nonlinearity exists and can differ across groups, as is evidenced in recent empirical studies (e.g., Landersø and Heckman (2017), An et al. (2020)). Mogstad and Torsvik (2021) provide a clear discussion of the theoretical underpinnings of the empirical intergenerational models. The prototypical relation is $Y = g(X) + \epsilon$, where Y and X are the logarithm of child and parental income, respectively, $g(\cdot)$ is an unknown function. The derivative, $g'(x)$ is the slope which measures persistence, interpreted as the inverse of mobility at a particular level of parental income $X = x$, and ϵ is an error term.

In the presence of heterogeneity and nonlinearity, income immobility, the gradient $g'(\cdot)$, varies with parental income. Any scalar measure, such as currently popular IGEs, would be a *summary* or aggregate of the values $g'(x)$, represented by $\int w(x)g'(x)dx$. Partial correlations and weights $w(x)$, depend on the parental income, $X = x$, and some risk/decision function. Traditional least squares methods impose weights that are "optimal" in that statistical sense, and vary over x with or without a linear $g(.)$. These weights are different for (log) levels and ranks.

Are such weighting functions *economically* desirable and justifiable? For the first

time, we present a transparent and systematic analysis of the relation between the rank-rank and (log) level regressions and their respective weighting schemes in this context. Our analysis reveals the otherwise implicit, subjective weighting of subgroup mobility in the regression measures. Different, low, middle or high income groups and observations, for instance, receive weights to obtain "Least Squares" or satisfy *statistical* orthogonality assumptions, as opposed to any desirable *economic* criteria. These *statistical* weights correspond to implicit welfare evaluations of individuals or groups. Importantly, we show that the implicit weights underlying the rank-rank regressions are not even necessarily proper weights (do not sum to one). Rank-rank IGEs are inconsistent and can indicate immobility that is larger or smaller than subgroup IGEs.

Depending on the summary measure, or estimation method, one can have drastically different assessments of mobility with different policy implications! Comparative analysis of mobility such as between-groups is therefore fragile, and can be particularly misleading, concerning features of mobility, for example, for children from the disadvantaged families (see Section 2 for numerical examples).[3] Our insights also present a great challenge for mobility-related research. Consider the example of the influential Great Gatsby Curve. The literature typically links regression-based mobility measures to the *Gini Coefficients*, a popular inequality measure. The latter is based on a weighted average of heterogenous incomes with the weighting scheme (*economically*) motivated by the Gini welfare function. On the other hand, the corresponding (*statistical*) weighting scheme in the IGM is unknown, and as shown here, is in fact in contrast to the Gini welfare function. This incompatibility, and the implicitness of regression weights questions the meaning and value implications of using these different weight schemes in the same assessment, and hence the validity of the

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[3] A comparison of *multiple* correlation estimates of mobility between groups, over time, and across space is also required for a better and deeper understanding of mobility, as well as monitoring progress in mobility and assessment of the policies. Indeed, existing empirical studies present such comparisons, from which many influential findings emerge in the literature. For example, by conducting cross-country comparisons, the literature has found that the welfare-state economies such as Denmark and Noway have higher levels of mobility than more market-oriented economies such as the United States (e.g., Landersø and Heckman (2017), Solon (2002)). Cross-country differences in IGMs are also further found to be positively correlated with the differences in inequality, a relationship called "The Great Gatsby Curve" (Krueger (2012)). Chetty and his coauthors document stark differences in IGMs across regions (Chetty et al. (2014)) and between whites and blacks (Chetty et al. (2020)). These influential results have prompted further analysis to investigate the sources of mobility to explain such differences.

Great Gatsby Curve.

One may be easily tempted to avoid "parametric" aggregation and employ estimation by nonparametric methods. This approach too entails implicit weights for different subgroups, however. Nonparametric derivatives are functionals of the dependent covariates, and need to be summarized over groups, such as the lower, middle, and upper tails of the distribution. A nonparametric estimation does not address the challenge of optimal summarization/aggregation, and it can be viewed as having **infinitely** many parameters; in practice only a few estimates can actually be presented. [4] (See footnote 5 for issues pertaining to discrete transition matrix type of analysis).[5]

Aggregation is common and inevitable, especially for a measure to be useful for between-group comparisons (as for the Great Gatsby curve) and as a policy metric.For example, after presenting the 100 by 100 matrix representation of the joint distribution, Chetty et al. (2014) states: "in order to provide a parsimonious summary of the degree of mobility and compare rates of mobility across areas, it is useful to characterize the joint distribution using a small set of statistics." [6] One may consider average derivatives of the nonparametric functions, effectively imposing equal weights across groups. As Sen (1992) notes, however, while such weighting scheme is often considered as a form of egalitarianism, "the effect of ignoring the interpersonal variations can, in fact, be deeply inegalitarian", and "equal consideration for all may demand very unequal treatment" in favor of a particular group.

The general contribution and significance of this paper is then to provide a generalized, decision theoretic, and *one-step* framework that subsumes the current parametric regressions. We propose computationally convenient ways of incorporating subgroup weighting schemes with transparent *economic* preferences with desirable

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[4] Moreover, the traditional nonparametric kernel estimation is computationally intensive and infeasible with large datasets (such as administrative records), and perform poorly and thus unreliably in smaller samples (such as Panel Studies of Income Dynamics, PSID) due to the slow rate of convergence. See Li and Racine (2007) for the relationship between the rate of convergence and the order of smoothness.

[5] The challenge of summarizing pertains to both nonparametric kernel estimation and assessment of transition matrices which represent a wider view of the conditional distribution that underlies the IGE regressions. These alternatives have been criticized for their "overtly disaggregate nature" (Bhattacharya and Mazumder (2011) and Hertz (2005) points out that with transition matrices, "there is no best way to summarize their content", prompting development of an easier-to-interpret summary measure of mobility. These criticisms are valid if over stated. For instance, there do exist several summary measures of transition matrices, albeit also non consensus ones. These include Bartholomeu and Shorrocks, amongst others.

[6] This is also noted by **?** in the program evaluation context.

properties of well known social welfare functions. We consider as special cases both the extended Gini family and the Lorenz family of social welfare functions. Our measures exhibit the property that higher inequality aversion assigns higher weight to lower incomes. Our approach can be readily implemented with standard statistical software, such as Stata. Practicioners choose and reveal their aggregation preferences. The theoretical framework here is rich and encompassing, allowing other covariates. A decomposition analysis, which is common and central to our understanding of the sources of mobility, is also developed.

A substantive empirical study is given with the PSID data, providing a comparison of the conventional methods and our approach. Using the same data and the log and rank-rank regressions, we are able to replicate closely the existing findings in the literature; these results suggest that the disparate results may reflect the differences in their implicit subjective aggregation schemes, rather than the actual differences in the income transmission process.

First, IGM estimates vary considerably with different methods and inequality-aversion parameters, confirming a significant heterogeneity and nonlinearity in the income transmission process.. The weighting schemes matter in forming our overall impression of the mobility. The log-regression estimates suggest substantially less mobility than the rank-rank regression estimates. The results based on both the Gini and Lorenz evaluation functions suggest that, with larger level of aversion to inequality, and focusing more on the individuals from disadvantaged families, smaller IGE coefficients are obtained, implying a more mobile society. The Lorenz results suggest that when placing more weights on the richer families, we also obtain smaller IGE coefficients. These results together indicate that children from both disadvantaged and richer families may have a higher level of mobility than those in the "middle class"!

Second, the traditional perspectives on geographic disparities in mobility are challenged when considering different underlying weighting schemes. Both the log and rank-rank regressions suggest a significant disparity in IGMs between the South and the West. However, the former implies a smaller geographic difference than the latter. Varying the inequality aversion parameter revises not only the magnitudes of the geographic disparities in IGMs, but also the patterns. For example, when using the Gini mobility measures and placing more weights on the children from the disadvantaged families, we actually find that the West becomes less and less mobile,

relative to the rest of the country, including the South. By contrast, when using the Lorenz measures and placing more weights on the richer families, we observe a larger coefficient for the South. These results may suggest that there may exist "affluence trap" in the South, but for the rest of the country, mobility is higher in both tails than in the middle.

Finally, we also examine the dynamics of the IGMs across cohorts. Both the level and rank-rank regressions suggest that mobility declines for the cohort born before 1954 and the cohort born after 1968. But the dynamics are different. More importantly, when placing greater weight on disadvantaged families, we observe greater mobility for the cohort born after 1968 than the cohort born before 1954.

Relationship and Contributions to Literature

The U-statistics representation of OLS estimator in levels is scarcely known in econometrics. It was noted by Yitzhaki (1996) who studies the OLS weights at different levels of a covariate x. The economic significance of this econometric insight has been largely overlooked in the empirical literature, due to a dearth of economic examples.[7] Our paper fills this gap. In the IGM context, understanding of the mobility for children from disadvantaged families is undoubtedly the primary focus of policies and academic research. It is important to understand the interplay between nonlinearity and estimation strategies, and their subjective impact on overal estimates of IGE. More importantly, we extend the insight on the weighted representation of a summary measure to intergenerational mobility in unique and novel directions: (1) we present for the first time an intricate relationship between the two dominant approaches and IGE measures; (2) we present a rigorous and comprehensive analysis of the weighting functions underlying both of these IGEs; (3) we provide a general, decision-theoretic approach to consider IGM, with additional estimators and their properties; (4) our theoretical framework is rich and encompassing, allowing someone to further analyze the role of covariates and incorporate them in their analysis with their own policy evaluation function. A decomposition analysis that is common and central to our understanding of the sources of mobility is developed.

Our paper also contributes broadly to two separate literatures other than mobility. First, similar to Maasoumi and Wang (2019), our paper is part of an attempt to

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[7] Yitzhaki (1996) provides an insightful empirical example of the average marginal propensity to consume.

connect and integrate the inequality literature more formally with the literature on IGM. On the other hand, our focus differs drastically from the inequality literature in that the latter focuses on univariate distributions, while we deal with the joint distribution of two incomes/outcomes.

Second, our econometric method is related to a growing interest in the treatment effect literature to understand the underlying meaning of various conventional estimators such as instrumental variable estimators (Mogstad and Wiswall (2016)) and two-way fixed effects (De Chaisemartin and d'Haultfoeuille (2020)) in the presence of heterogeneous treatment effects. Our approach and discussion has clear implications and utility for the analysis of summary treatment effects, and summary measures for defined subgroups. The need to consider the treatment effect estimation as a decision-theoretic problem is also considered in**?**. Our paper is focused on a summary measure for the entire distribution of outcomes.

The rest of the paper is organized as follows. Section 2 provides a numerical example to motivate our analysis. Section 3 exposes the conceptual issues for the traditional regression approaches by analyzing their weighting schemes. Section 4 presents our estimators. We illustrate our proposals using the PSID data in Section 5. Section 6 concludes. Proofs are collected in the Appendix.

2 A Motivating Example

A simplistic example highlights the importance of weighting schemes in constructing a summary measure of mobility. We are interested in measuring mobility in one society, say, group A, and wish to compare it with another, say, group B.

Let there be only two income levels, low-income, l and high-income, h. Let the income functions be $Y_A = g_A(X) + \epsilon_A$ and $Y_B = g_B(X) + \epsilon_B$, where $X \in \{l, h\}$ with $0 < l < h < \infty$. Let mobility at $X = x$ be $g'_A(l) = 0.6$, $g'_A(h) = 0.3$, $g'_B(l) = 0.9$, $g'_B(h) = 0.2$.[8] Income in group A is less persistent and more mobile in the lower tail, while income in group B is less persistent and more mobile in the upper tail. A weighted average measure of mobility for any group k is thus $\overline{m}_k = w_l g'_k(l) + w_h g'_k(h)$, where $w_l + w_h = 1$ and $0 < w_l, w_h < 1$. Consider two different weighting schemes for

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[8] A little abuse of terminology here, referencing derivative at a particular income level as local mobility at that point.

$w_l^j, w_h^j, j = 1, 2.$

Case 1: Let the first weighting scheme be $w_l^1 = .95$ and $w_h^1 = .05$. Then, $\overline{m}_A = .585$. However, if we reverse the weighting scheme instead by placing more weights on the richest families with $w_l^2 = .05$ and $w_h^2 = .95$, then $\overline{m}_A = .315$. These two numbers resemble the current debate on the magnitudes of the IGM for the U.S.. The discrepancy is large since the mobility implied by the first weighting scheme is drastically different from the second weighting scheme.

Case 2: Now consider how varying weighting schemes may impact the conclusions regarding the between-group comparison of mobility. Under the first weighting scheme when we place more weights on the children from the disadvantaged families with $w_l^1 = .95$ and $w_h^1 = .05$, it follows $\overline{m}_A = .585 < .865 = \overline{m}_B$. The implied mobility of group A is actually *greater* than that of group B. By contrast, under the second weighting scheme, $w_l^2 = .05$ and $w^2 = .95$, we actually observe the opposite result $\overline{m}_A = .315 > .235 = \overline{m}_B$. In other words, group A is less mobile than group B.

In practice, the weighting schemes should depend on the underlying income of each group as well as the social weight attached to that group. All weight schemes are subjective. Transparency is needed and provided below.

3 Weighted Average Representations of Traditional Regression-based Mobility Measures

In this section, we characterize the properties of the existing summary measures of mobility based on linear level regression and rank-rank regression IGEs. Both can be expressed as weighted sums of heterogeneous movements/gradients over the distribution of parent's income. The U-statistics representation of OLS estimator in levels is relatively well known in econometrics, and has been noted by Yitzhaki (1996). The results for level regressions follow closely Yitzhaki (1996), while the results for rank-rank regressions are new to the literature. Let the true income-transmission process be given by

$$Y = g(X) + \epsilon, \tag{1}$$

with error term ϵ and $\mathbb{E}(\epsilon|X) = 0$. The implied *rank-rank* relationship is given by

$$V = \mathbb{E}(V|U) + \eta = g_r(U) + \eta, \tag{2}$$

where $U = F_X(X)$ (rank of a child's income), $V = F_Y(Y)$ (rank of parent's income), and the error term η satisfies $\mathbb{E}(\eta|U) = 0$ by the definition $g_r(u) = \mathbb{E}(F_Y(Y)|F_X(x) = u)$.

The following Lemma (see Proposition 2 in Yitzhaki (1996)) clarifies the weighted average representation of the Least Squares (LS) of the projection parameter.[9]

Lemma 1. *[**Weighted Average Representation of Linear Regression**] Let $\mathbb{E}^*(Y|X) = \alpha + \beta X$ denote the best linear predictor (projection) of Y with X. Then*

$$\beta = [Var(X)]^{-1}Cov(Y, X) = \int_{\mathcal{S}_X} w(x)g'(x)dx,$$

where $w(x) = F_X(x)\left(\mu_X - \mathbb{E}\{X|X \leq x\}\right)\sigma_X^{-2}$, $\mu_X = \mathbb{E}(X)$, $\sigma_X^2 = Var(X)$, $w(x) \geq 0$, $\int_{\mathcal{S}_X} w(x)dx = 1$, and $\mathcal{S}_X$ is the support of X.

Yitzhaki (1996) showed that under normal and uniform distributions, the weight function $w(\cdot)$ in Lemma 1 is maximized at the median of X.[10] This implies that LS assigns *smaller* weights to the marginal effects in the lower and higher tails of the parent income distribution than at the middle. This property is not considered desirable under concave utility functions with aversion to inequality (and with concern over lower incomes). Note that this property is about the weight on the derivative of the function, $g'(x)$, different from the familiar concern with higher linear LS weights on the outliers of Y.

Next, we consider the rank-rank regression. Since both U and V are distributed as standard uniform, we follow the same strategy as in Lemma 1 for (2):

Lemma 2. *Let $\mathbb{E}^*(V|U) = \alpha_r + \beta_r U$ denote the best linear predictor of V given U. Then*

$$\beta_r = [Var(U)]^{-1}Cov(U, V) = \int_0^1 w(u)g_r'(u)du, \tag{3}$$

$$w(u) = 6u - 6u^2, \tag{4}$$

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[9] Note that Yitzhaki (1996)'s results can be extended to the case of multiple regression.

[10] For the normal distribution, $w(x)$ is equal to its density function. For the uniform distribution with the support $[\underline{x}, \overline{x}]$, $w(x) = 6(\overline{x} - x)(x - \underline{x})(\overline{x} - \underline{x})^{-3}$.

where $w(u) \geq 0$ for $0 \leq u \leq 1$, and $\int_0^1 w(u)du = 1$.

Lemma 2 implies that the weight attains the maximum at the median $x = F_X^{-1}(1/2)$. The closer to the median the x, the greater the weight.

Lemma 2 presents the weighted average representation of the derivatives of the rank-rank model, $g_r'(u)$, but not the corresponding weighted average representation of the correlation or persistence between incomes levels of different generations, $g'(x)$. Hence, the weighting schemes are not directly comparable between linear level regression and rank-rank regression. By substituting $U = F_X(X)$ into (3) and (4) in Lemma 2, we obtain

$$\beta_r = \int_{\mathcal{S}_X} w(F_X(x))g_r'(F_X(x))f_X(x)dx = \int_{\mathcal{S}_X} 6(F_X(x)-F_X^2(x))g_r'(F_X(x))f_X(x)\frac{g'(x)}{g'(x)}dx,$$

which, as summarized in the following Proposition, clarifies the weighted average representation of the derivative of the true income-transmission process $g(x)$ in (1).

Proposition 1. [Weighted Average Representation of Rank-Rank Regression] *Under conditions in Lemma 2, the summary measure β_r in (3) becomes*

$$\beta_r = \int_{\mathcal{S}_X} w_r(x)g'(x)dx, \tag{5}$$

where

$$w_r(x) = 6(F_X(x) - F_X^2(x))g_r'(F_X(x))f_X(x)(g'(x))^{-1}. \tag{6}$$

Proposition 1 exposes an undesirable property of the weighting scheme under the rank-rank regression. In general, the weighting scheme does not integrate to one ($\int w_r(x)dx \neq 1$) since typically $g_r'(F(x)) \neq g'(x)$ for any x. Analytical solutions derived for normal, lognormal, and uniform distributions in Corollary 1 illustrate and confirm this point. Consequently, the rank-rank measures fail an aggregation consistency required for comparative subgroup analysis. IGM may be larger than maximum subgroup mobility or smaller.[11]

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[11] One exception is for the uniform distributions. As stated in case (*iii*) of Corollary 1, under additional restrictions for the case of uniform distributions, the implied rank-rank regression is also linear with the same measure of mobility as that in the level regression. Moreover, the underlying weighting scheme for this particular case is proper.

Corollary 1. *Weighting Schemes of Rank-Rank Regression for Three Parametric Distributions: We provide the weighting schemes under three specific cases. (i) Normal Distribution. Suppose that the conditional distribution of ϵ given $X = x$ is normal with zero mean and variance σ_ϵ^2, and that Y is normally distributed with mean $\mu_y = \mathbb{E}(g(X))$ and variance $\sigma_y^2 = \sigma_\epsilon^2 + Var(g(X))$. Then,*

$$w_r(x) = c(x)(F_X(x) - F_X^2(x)), \tag{7}$$

where $c(x)$ is defined in the proof of Corollary 1 in Appendix.
(ii) Lognormal Distribution. Under the same conditions as those under the normal distribution except that the conditional distribution of Y given $X = x$ is lognormal and that Y is lognormally distributed, $w_r(x)$ is identical to that in (7).
(iii) Uniform Distribution. Suppose that Y is uniformly distributed on $[\underline{y}, \overline{y}]$. Then,

$$w_r(x) = 6(F_X(x) - F_X^2(x))(\overline{y} - \underline{y})^{-1}. \tag{8}$$

If X is uniformly distributed on $[\underline{x}, \overline{x}]$ with $\overline{x} - \underline{x} = \overline{y} - \underline{y}$ and $g(x) = \alpha + x\beta$,[12] the implied rank regression is linear with $g_r'(u) = \beta_r$, $\beta_r = \beta$, and $\int_{\underline{x}}^{\overline{x}} w_r(x)dx = 1$.

4   A General Theory of Mobility Measures Based on Social Preferences

Subjectivity in the implicit weights behind the regression-based IGE estimators leads us to consider a general aggregation approach with transparent weighting scheme. We adopt a decision theoretic approach that is well motivated by well founded principles in the literature on inequality, poverty, and mobility.

Following the general approach we consider two special cases and provide a rigorous analysis of their weighting schemes to showcase the flexibility of these measures in accommodating a wide range of economic criteria. We further propose a nonparametric framework to incorporate covariates and discuss how it can be used for decomposition that is needed for understanding the sources of mobility.

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[12] Note that it allows for the lower and upper bounds of X and Y to be different. That is, both the lower and upper bounds for children income can be greater or smaller than that for parents income.

4.1 A General Social Welfare-based Summary Measure of Income Mobility

A general approach to construct summary measures of mobility is based on the axiomatic characterization of desirable social welfare properties mobility. To begin, Yaari (1987, 1988) show that a preference relation, $P(\cdot)$, defined on income distribution, which satisfies a set of standard axioms, can be represented by the following social welfare

$$W_X = \int_{\mathcal{S}_X} x dP(F_X(x)) = \int_0^1 P'(t)F_X^{-1}(t)dt, \tag{9}$$

where $F_X^{-1}(t)$ is the t-th quantile of the income distribution.[13] Social welfare W_X is taken as a weighted average of individual incomes in which the weight $P'(\cdot)$ is a function of income ranks $t \in [0,1]$ with $P(0) = 0, P(1) = 1$ and $P'(t) > 0$. This accommodates interdependencies. The functional form of the preference/evaluation function, $P(\cdot)$, reveals a policy-maker's inequality aversion and determines the weights, $P'(\cdot)$ in measuring social welfare.[14] Due to P's dependence on the rank t, W_X is also called a rank-dependent social welfare (e.g., Aaberge et al. (2021)), and widely adopted in the literature. For example, based on the rank-dependent social welfare, Aaberge (2001) analyzes ordering relations on Lorenz curves, and Aaberge et al. (2021) propose a general approach to ranking intersecting distribution functions. King (1983) analyzes another rank-dependent social welfare in assessing mobility/policy effects as change in ranking between the ex-ante and ex-post income distributions. In the spirit of Atkinson (1970), a normative index of social mobility may be regarded as the proportion of total income which, from a position of zero mobility, one would sacrifice in order to achieve the degree of observed changes in income rankings.

To explicitly incorporate the policy maker's preference function, or her attitude towards inequality in rank-dependent social welfare, the follwing summary measures

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[13] Under a set of standard axioms of the preference relations, such as continuity, completeness, monotonicity, and independence, Yaari (1987, 1988) propose a new theory of choice. This theory is dual to the expected utility theory. The main advantage of it is that it can separate agent's attitude towards risk (increased uncertainty hurts) and attitude towards wealth (the loss hurts the poor relatively more), which are combined in the expected utility theory.

[14] For example, as Aaberge et al. (2021) point out, an inequality neutral social planner would choose $P(t) = t$, which means $W_X = \mu_X$.

of IGmobility are considered:

$$s = \frac{Cov(Y, P'(F_X(X)))}{Cov(X, P'(F_X(X)))} = \frac{Cov(Y, P'(U))}{Cov(X, P'(U))}. \tag{10}$$

where there is no restriction on the functional form of P except for satisfying some standard smoothness conditions, and $P'(F_X(X))$ is the weight in social welfare (9).[15] The proposed mobility measures here comport with "ideal measures" of mobility discussed in the literature (e.g., see Atkinson (1980), Markandya (1982), King (1983), Maasoumi (2020)).[16] An Instrumental Variable (IV) interpretation of (10) is instructive and suggestive of standard econometric estimation and inference methods. The IV here is $P'(F_X(X))$, and its validity is equivalent to assuming no correlation between the *marginal welfare effect* of parent's rank and other random shocks to offspring's outcomes (accounting for other measured covariates, as explained below). This is highly plausible. The IV assumption in $P'(F_X(X))$ is an implication of the one in the current (OLS) IGE regressions which assume children's log income or income rank is uncorrelated with remaining regression errors. We note that these assumptions are required for consistent estimation of the projection value of IGE (pseudo true "beta"), and do not depend on the true mobility function $g(.)$.

The mobility measures in (10) can be interpreted as the marginal effect of parents' (normalized) social welfare on children's (normalized) social welfare , which correspond to the numerator and the denominator in (10), respectively. Specifically, the numerator can be rewritten as

$$\begin{aligned} Cov(Y, P'(F_X(X))) &= \mathbb{E}\{[Y - \mathbb{E}(Y)][P'(F_X(X)) - \mathbb{E}(P'(F_X(X)))]\} \\ &= \mathbb{E}_X\{[\mathbb{E}(Y|X) - \mathbb{E}(Y)][P'(F_X(X)) - \mathbb{E}(P'(F_X(X)))]\} \\ &= \mathbb{E}_X\{[\mathbb{E}(Y|X) - \mathbb{E}(Y)]P'(F_X(X))\} \\ &= \int_{\mathcal{S}_X} \widetilde{y}_x dP(F_X(x)) = \widetilde{W}_Y, \end{aligned} \tag{11}$$

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[15] The connection of our summary measure to social welfare parallels the literature on income inequality, in the spirit of Atkinson (1970) who advocates and proposes a measure of income inequality based on the social welfare in expected utility context. To distinguish, we refer to the social welfare in expected utility theory as level-dependent social welfare.

[16] Atkinson (1980) analyzes how the income mobility measure in a linear regression model affects the inequality of lifetime social welfare. Markandya (1982) clarifies "chexchange mobility", positional changes, as a component of social welfare functions. King (1983) discusses both horizontal equity and social mobility in terms of a non-utilitarian social welfare function.

where $\widetilde{y}_x = \mathbb{E}(Y|X = x) - \mathbb{E}(Y)$ is the normalized conditional expected children income given parents income $X = x$, and the second equality follows the law of iterated expectations. In terms of W_X, we can interpret $\widetilde{W}_Y$ not only as a weighted average of children's normalized conditional incomes given parents' income but also as a normalized children's social welfare. We can similarly show that

$$Cov(X, P'(F_X(X))) = \int_{\mathcal{S}_X} \widetilde{x} dP(F_X(x)) = \widetilde{W}_X, \tag{12}$$

where $\widetilde{x} = x - \mathbb{E}(X)$ is the normalized parents income, and $\widetilde{W}_X$ can be considered a normalized parents social welfare and a weighted average of normalized parents incomes. We summarize the above results in the following proposition.

Proposition 2. *For the mobility measure s, we obtain the following properties: $s = \widetilde{W}_Y \left(\widetilde{W}_X\right)^{-1}$ can be interpreted as the marginal effect of parents' normalized social welfare $\widetilde{W}_Y = \int_{\mathcal{S}_X} \widetilde{y}_x dP(F_X(x))$ on children's normalized social welfare $\widetilde{W}_X = \int_{\mathcal{S}_X} \widetilde{x} dP(F_X(x))$, where $\widetilde{y}_x = \mathbb{E}(Y|X = x) - \mathbb{E}(Y)$ and $\widetilde{x} = x - \mathbb{E}(X)$ are children's normalized conditional income and parents' normalized income, respectively.*

Here, we also provide a general result of the weighted average representation of mobility measure s for a general form of P. Using arguments similar to Yitzhaki (1996), s can be expressed as a weighted average of the individual mobility $g'(x)$.

Proposition 3. *The summary measure of mobility s can be rewritten as*

$$s = \int_{\mathcal{S}_X} w(x)g'(x)dx, \tag{13}$$

where the underlying weight scheme satisfies $w(x) > 0$, $\int_{\mathcal{S}_X} w(x)dx = 1$, and

$$w(x) = \frac{P(F_X(x)) - F_X(x)}{\int_{\mathcal{S}_X} \{P(F_X(t)) - F_X(t)\}dt}. \tag{14}$$

In principle, one may use any evaluation functions in the summary measure. In subsections (4.3) and (4.4), we consider two important special cases of one-parameter family of evaluation functions, indexed by inequality aversion, that are commonly used in the literature: the first is based on the Gini evaluation functions (as in Yitzhaki

(1996)), and the second one is the Lorenz evaluation function. Both functions are widely used in the analyses of social welfare and inequality and characterized by one parameter representing the degree of inequality aversion (e.g., see Aaberge et al. (2021)). Moreover, we provide a rigorous analysis of the properties of their respective weighting schemes. As we will show below, these families of measures are capable of producing most weighting schemes of interest. The two measures vary with degrees of inequality aversion. Our second measure can accommodate even weighting schemes that allow for "efficiency".

4.2 Economic Meaning of Conventional Regression-based Mobility Measures

Using our framework, we can now interpret the economic meaning of the conventional regression-based IGEs. We note that popular linear regression IGEs are special cases of our proposed measure, defined with a constant marginal effect $\beta = Cov(Y, X)/Var(X) = Cov(Y, X)/Cov(X, X)$. Using (10), we can see that the corresponding evaluation function has the derivative of the form, $P'(F_X(x)) = x$, or $P'(t) = F_X^{-1}(t)$. This in turn implies the evaluation function is given by $P(t) = \int_0^t F_X^{-1}(u)du$. This is familiar to a poverty "count measure", the number in population below t.

The evaluation function underlying the IGE is convex due to $P''(t) = 1/f_X(F_X^{-1}(t)) > 0$, where f_X is the density function of X. As a result, the weight $P'(t)$ in (9) increases with the rank t. In other words, the policymaker who employs this IGE cares more about the rich than the poor, which is not a commonly used policy evaluation function. Furthermore, according to the results shown in Yaari (1988), the widely used Pigou-Dalton principle of transfers cannot hold when P is convex; as a result, the policy maker is not *equality minded*.[17] Consequently, the current linear IGEs do not represent desirable metrics.

The undesirable property of the evaluation function $P(t)$ is also consistent with that of the weight function $w(x)$ that we unravel when examining the weighted average representation of the IGE. As shown in Yitzhaki (1996), the weight function $w(\cdot)$ under normal and uniform distributions assigns smaller weights to the individuals

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[17] In Yaari (1988), a policy maker's preference relation is said to be equality minded if it satisfies the Pigou-Dalton principle of transfers. The Pigou-Dalton principle of transfers holds if and only if P is concave. The Pigou-Dalton principle of transfers states that a transfer of income from the rich to the poor is always desirable, so long as it does not affect anybody's position in the income ranking.

in the lower tails of the income distribution than the middle part of the income distribution. These results are summarized in Proposition 4.

Proposition 4. *If the evaluation function is $P(t) = \int_0^t F_X^{-1}(u)du$, then $s = \beta$, where $\beta = Cov(Y, X)/Cov(X, X)$. However, the fact that $P(t)$ is convex implies that the policy maker is not equality minded, and the evaluation fails to satisfy the Pigou-Dalton principle of transfers.*

4.3 Mobility Measures based on Gini Evaluation Functions

We now turn to a special case of our measure that is based on the parametric Gini evaluation function (also considered in Yitzhaki (1996)), given by

$$P_\kappa^I(u) = 1 - (1 - u)^\kappa \ \text{ for } \ u \in [0, 1], \tag{15}$$

where $\kappa > 1$ is the inequality-aversion parameter; $u = F_X(x)$ is the income position or rank. The higher κ, the more inequality-averse a society. On the one extreme case $\kappa = 1$, society is indifferent to inequality; on the other extreme case $\kappa \to \infty$, society cares most about the welfare of the poor. The derivative of the preference function, $dP_\kappa^I(u)/du = \kappa(1 - u)^{\kappa-1} > 0$, reflects the weight placed on a particular income position u in the definition of welfare functions (e.g., Weymark (1981), Yaari (1987, 1988), Aaberge (2000)). Moreover, since $d^2 P_\kappa^I(u)/du^2 = \kappa(1 - \kappa)(1 - \mu)^{\kappa-2} < 0$, the policy maker's evaluation satisfies the principle of transfers (Yaari, 1988), representing the inequality aversion of policy makers.

Gini-based Mobility Measure, $s^I(\kappa)$. The first class of mobility measure based on Gini evaluation functions (15) is defined as

$$s^I(\kappa) = \frac{Cov(Y, [1 - F_X(X)]^{\kappa-1})}{Cov(X, [1 - F_X(X)]^{\kappa-1})}, \tag{16}$$

where the denominator is the extended Gini variability index, and the numerator is the extended Gini covariance. Note that the constant of the derivative of the preference function (κ) is dropped in the measure since it appears in both denominator and numerator. Below we present a systematic analysis of the weighting scheme behind our measure.

Underlying Weights of $s^{\mathbf{I}}(\kappa)$: $\mathbf{w^I_\kappa(u)}$. We substitute (15) into (14) and obtain the weighting scheme for P^I_κ

$$w^I_\kappa(x) = \frac{[1 - F_X(x)] - [1 - F_X(x)]^\kappa}{\int_{\mathcal{S}_X} \{[1 - F_X(t)] - [1 - F_X(t)]^\kappa\} dt}. \tag{17}$$

Hence, $s^I(\kappa)$ can be expressed as

$$s^I(\kappa) = \int_{\mathcal{S}_X} w^I_\kappa(x) g'(x) dx. \tag{18}$$

Moreover, $w^I_\kappa(\cdot)$ can be rewritten as a function of $u = F_X(x)$, that is, $w^I_\kappa(u) = c^I(\kappa)\left[(1 - u) - (1 - u)^\kappa\right]$, where $c^I(\kappa) = \left(\int_0^1 \{(1 - u) - (1 - u)^\kappa\} dF_X^{-1}(u)\right)^{-1} > 0$ is a positive constant, depending on $F_X(\cdot)$ and κ. As shown below, the expression of $w^I_\kappa(\cdot)$ in terms of u is convenient for analyzing its properties.

Properties of $\mathbf{w^I_\kappa(u)}$. The first- and second-order derivatives of $w^I_\kappa(u)$ are given, respectively, by

$$\frac{dw^I_\kappa(u)}{du} = c^I(\kappa)\left[\kappa(1 - u)^{\kappa-1} - 1\right] \text{ and } \frac{d^2 w^I_\kappa(u)}{du^2} = c^I(\kappa)\kappa(1 - \kappa)(1 - u)^{\kappa-2} < 0. \tag{19}$$
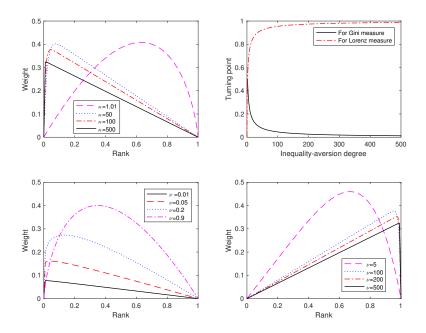
The first-order condition implies that the maximizer (the turning point) of $w^I_\kappa(u)$ is given by $u^I(\kappa) = 1 - \kappa^{\frac{1}{1-\kappa}}$. The second-order condition implies that $w^I_\kappa(u)$ is strictly concave in u. As the upper left panel of Figure 1 illustrates, the weight increases for lower values of u, reaches a maximum, and then declines. The key properties of the weighting scheme are summarized in Proposition 5.

Proposition 5. *We obtain the following properties:*
(i) $u^I(\kappa)$ is strictly decreasing in κ for $\kappa > 1$.
(ii) $\lim_{\kappa \to \infty} u^I(\kappa) \to 0$.
(iii) For a sufficiently large κ, $w^I_\kappa(u) \approx c^I(\kappa)(1 - u)$ for $u \in [u^I(\kappa), 1]$.

Property (i) states that the location of maximum weight (turning point) decreases in the inequality aversion parameter. This property is consistent with the typical policies goals or preferences that are averse to inequality. As a society or policymaker becomes more inequality averse, the individuals from the more disadvantaged families should receive the maximum weight. The pattern is illustrated in the upper right panel of

Figure 1. Property (ii) states that when the inequality aversion tends to infinity, the largest weight is indeed placed on the poorest individuals. More importantly, the relative weight for the poor is larger than that for the rich when we increase the inequality aversion. This later feature is evident in the upper left panel of Figure 1 and formally stated in property (iii). For a sufficiently large inequality aversion parameter κ, the weighting scheme can be approximated by a downward slopping line for almost all the values of u. This result can be of practical importance as well, since this type of weighting schemes is usually consistent with what many empirical researchers have in mind. Moreover, it suggests that the researchers simply need to assign the inequality aversion parameter a relatively large value to obtain such weighting scheme.

Figure 1: Shapes of weight functions and turning points for different inequality-aversions

Note: The lower and upper left panesl are for $w_\nu^{II}(u)$ with $0 < \nu < 1$ and for $w_\kappa^I(u)$ respectively. The lower right panel is for $w_\nu^{II}(u)$ with $\nu > 1$. The upper right panel is for turning points $u^I(\kappa)$ under Gini measure and $u^{II}(\nu)$ under Lorenz measure. The income X has standard normal distribution.

4.4 Mobility Measures based on Lorenz Evaluation Functions

Our second class of mobility measures is based on the Lorenz family of evaluation functions, defined as

$$P_\nu^{II}(u) = (\nu u - u^\nu)(\nu - 1)^{-1} \text{ for } u \in [0,1], \nu > 0, \text{ and } \nu \neq 1 \tag{20}$$

where ν is the preference parameter capturing the extent of inequality aversion (see, e.g., Aaberge (2000)). Note that the inequality aversion *increases* with parameter κ for the Gini measures, while it *decreases* with parameter ν for the Lorenz measures.

Lorenz-based Mobility Measure, $s^{II}(\nu)$. Similar to the first class of mobility measures, based on the first derivative of the Lorenz evaluation function $dP_\nu^{II}(u)/du = \nu(1 - u^{\nu-1})(\nu - 1)^{-1} > 0$, the second class of mobility measures is defined as

$$s^{II}(\nu) = \frac{Cov(Y, 1 - F_X(X)^{\nu-1})}{Cov(X, 1 - F_X(X)^{\nu-1})}, \tag{21}$$

where the constant of the derivative of the preference function $(\nu(\nu-1)^{-1})$ is dropped in the measure since it appears in both denominator and numerator. As in P_κ^I, the evaluation $P_\nu^{II}(u)$ also satisfies the principle of transfers due to $d^2P_\nu^{II}(u)/du^2 = -\nu\mu^{\nu-2} < 0$, representing the inequality aversion of policy makers.

Underlying Weights of $s^{II}(\nu) : w_\nu^{II}(u)$. Using the arguments similar to (18), the second measure $s^{II}(\nu)$ can also be expressed as a weighted average of the individual mobility $g'(x)$

$$s^{II}(\nu) = \int_{\mathcal{S}_X} w_\nu^{II}(x)g'(x)dx, \tag{22}$$

where

$$w_\nu^{II}(x) = \frac{F_X(x) - F_X(x)^\nu}{\int_{\mathcal{S}_X}[F_X(t) - F_X(t)^\nu]dt}. \tag{23}$$

Moreover, $w_\nu^{II}(\cdot)$ can also be rewritten as a function of $u = F_X(x)$, that is, $w_\nu^{II}(u) = c^{II}(\nu)(u - u^\nu)$, where $c^{II}(\nu) = \left(\int_0^1 (u - u^\nu)dF_X^{-1}(u)\right)^{-1}$ is a constant, which is positive for $\nu > 1$ and negative for $0 < \nu < 1$.

Properties of $w_\nu^{II}(u)$. The first- and second-order derivatives of $w_\nu^{II}(u)$ are given,

respectively, by

$$\frac{dw_\nu^{II}(u)}{du} = c^{II}(\nu)\left(1 - \nu u^{\nu-1}\right) \quad \text{and} \quad \frac{d^2 w_\nu^{II}(u)}{du^2} = \nu(1-\nu)c^{II}(\nu)u^{\nu-2} < 0. \quad (24)$$

The first-order condition implies that the maximizer (the turning point) of $w_\nu^{II}(u)$ is given by $u^{II}(\nu) = \nu^{\frac{1}{1-\nu}}$. The second-order condition implies that $w_\nu^{II}(u)$ is strictly concave in u. As the bottom left and right panels of Figure 1 illustrate, the weight increases for lower values of u, reaches a maximum, and then declines. The key properties of the weighting scheme are summarized in Proposition 6.

Proposition 6. *We obtain the following properties:*
(i) $u^{II}(\nu)$ is strictly increasing in ν for $\nu > 0$.
(ii) $\lim_{\nu \to 0} u^{II}(\nu) \to 0$ and $\lim_{\nu \to \infty} u^{II}(\nu) \to 1$.
(iii.a) For a sufficiently small $\nu < 1$, $w_\nu^{II}(u) \approx c^{II}(\nu)(u-1)$ for $u \in [u^{II}(\nu), 1]$.
(iii.b) For a sufficiently large $\nu > 1$, $w_\nu^{II}(u) \approx c^{II}(\nu)u$ for $u \in [0, u^{II}(\nu)]$.

It is useful to compare Proposition 6 vs. Proposition 5 to understand the similarities and differences between the Gini- and Lorenz-based estimators and how they could complement each other in practice.

Meanings of Inequality Aversion Parameters The major difference between $s^I(k)$ (the Gini family of mobility measures) and $s^{II}(\nu)$ (the Lorenz family of mobility measures) is the weighting function. $w_\kappa^I(\cdot)$ varies with $\kappa > 1$ for the former, whereas $w_\nu^{II}(\cdot)$ varies with $\nu > 0$ for the latter. These two parameters have the opposite meaning. Moreover, the parameter space for the inequality-aversion parameter (κ vs ν) is different for the two classes of evaluation functions, as the space for ν, the parameter for the Lorenz family, covers $(0,1) \cup (1, \infty)$. As a result, the behavior of the weighting schemes for the Lorenz measures also differ in the two sub-regions of the space.

Similarities – Economic Implications Three properties in Proposition 6 have economic meanings qualitatively similar to their counterparts in Proposition 5. First, Proposition 6 (i) is similar to its counterpart in Proposition 5. As illustrated by the two left panels and the upper right panel in Figure 1, when the inequality aversion is higher (ν is smaller), the turning point of the income (that is, the income location of the maximum weight) decreases. Note that the opposite patterns of the curves in the upper right panel is due to the opposite meanings of the two parameters κ and ν.

Proposition 6 (ii) again states that at the the limit, the maximum weight (turning point) is placed on the individuals from the poorest families. In addition, Proposition 6 ($iii.a$) is similar to Proposition 5 (iii): for a sufficiently large inequality aversion ($\nu \to 0$), the weights *decrease* monotonically with (log) income levels. In general, individuals from poorer families receive more weight weight than those from richer families.

Differences Despite many qualitative similarities of the weighting schemes between the two estimators, the weights may be quite quantitatively different, which can be critical for relevant policy evaluations. Proposition 6 ($iii.b$) is unique. For a sufficiently *small* inequality aversion ($\nu \to \infty$), the weights can *increase* monotonically with (log) income levels, which may correspond to some form of "efficiency" consideration in measuring mobility. This result suggests the flexibility of our measures in accommodating a wide range of possible weighting schemes that emphasize different policy goals and targets. .

As indicated in the upper left and bottom left panels of Figure 1, at the extreme inequality aversion ($\kappa \to \infty$ and $\nu \to 0$), the weighting schemes are both nearly linear and place the maximum weight on the children from the poorest families. However, the weights differ in the magnitudes and in the exact positions where the maximum weight is placed. For example, the weights behind the Lorenz family seem more homogenous. As we will see from the empirical examples below, these seemingly subtle differences can substantially alter our impressions of mobility and the conclusions for between-group comparisons in the presence of nonlinearity.

Finally, although the weighting schemes exhibit similar shapes when policy makers are more averse to inequality (a larger κ for the Gini measure and a smaller ν for the Lorenz measure), they behave very differently under less inequality-aversions. The lower right panel of Figure 1 implies, the weighting scheme under Lorenz measures is still approximately linear and places the maximum weight on the children from the richest families. In contrast, the upper left panel indicates, the weighting scheme under Gini measures seems concave and places the maximum weight on the children from the middle families. Combining the results of the Gini measures (when κ is large) and those of the Lorenz (when $\nu > 1$ is small), one may gain insights into the possible nature of the nonlinearity of mobility under different underlying evaluation functions. Later in our empirical analysis, we will see an example like this.

4.5 Estimation

We provide the estimation of our proposed summary measures of mobility. Suppose that the data consist of an independent sample $\{Y_i, X_i\}_{i=1}^n$ of size n. Given that these two summary measures of mobility have similar expressions, without loss of generality, we focus on the estimation of the summary measure of mobility under the Gini evaluation functions.

Since (16) takes the form of a Wald-IV estimator, we can conveniently estimate $s^I(\kappa)$ using the two steps. Let $Q_i = [1 - F_X(X_i)]^{\kappa-1}$. First, we obtain the estimate of Q_i by $\widehat{Q}_i = [1 - \widehat{F}_X(X_i)]^{\kappa-1}$, where $\widehat{F}_X(X_i) = \frac{1}{n}\sum_{j=1}^n \mathbb{I}(X_j \leq X_i)$, and $\mathbb{I}(\cdot)$ is an indicator function equal to one if the argument is met and zero otherwise. Next, we run an IV regression of Y_i on X_i, with $\widehat{Q}_i$ being the IV for X_i. The IV estimator of the coefficient on X_i is the estimator $\widehat{s}^I(\kappa)$ of $s^I(\kappa)$.

4.6 Estimation in the Presence of Covariates

How to flexibly account for covariates and perform counterfactual analysis in the context of IGM is not trivial and relatively underdeveloped. The importance of accounting for family and neighborhood backgrounds for a deeper understanding of (the nonlinear) nature of mobility profiles is extensively reviewed in Mogstad and Torsvik (2021). Here, we extend our framework to systematically incorporate covariates in estimation, which could prove useful for identification of important covariate effects and counterfactual analysis to understand the sources of mobility, as well as the between-group differences.

Let $Z = (Z_1, \cdots, Z_d)'$ be a d-dimensional vector of continuous variables.[18] The true income-transmission process in (1) becomes $Y = g(X, Z) + \epsilon$, where the error term ϵ satisfies $\mathbb{E}(\epsilon|X, Z) = 0$. In the spirit of (16), we define the summary measure of mobility under the Gini evaluation functions, conditional on $Z = z$, as

$$s^I(\kappa, z) = \frac{Cov(Y, [1 - F_{X|Z}(X|Z)]^{\kappa-1}|Z = z)}{Cov(X, [1 - F_{X|Z}(X|Z)]^{\kappa-1}|Z = z)}. \tag{25}$$

Following Yitzhaki (1996), we obtain the counterpart of (18) in the presence of co-

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[18] For discrete covariates such as gender and race, the estimation is identical to that for the (unconditional) measures for each sub-group. This is often the case for the intergenerational mobility literature (e.g., see the analysis of black-white differences in intergenerational mobility by Bhattacharya and Mazumder (2011)).

variates as

$$s^I(\kappa, z) = \int_{\mathcal{S}_{X|Z=z}} w_\kappa^I(x, z)g'(x, z)dx,$$

where $w_\kappa^I(x, z) > 0$, $\int_{\mathcal{S}_{X|Z=z}} w_\kappa^I(x, z)dx = 1$, $\mathcal{S}_{X|Z=z}$ is the conditional support of X given $Z = z$, and

$$w_\kappa^I(x, z) = \frac{[1 - F_{X|Z}(x|z)] - [1 - F_{X|Z}(x|z)]^k}{\int_{\mathcal{S}_{X|Z=z}}\{[1 - F_{X|Z}(t|z)] - [1 - F_{X|Z}(t|z)]^\kappa\}dt}.$$

Estimation Method #1: Fully Nonparametric Model: we propose the estimation of the heterogeneous mobility measures $s^I(\kappa, z)$ and then obtain the average mobility measure across heterogenous covariates. Let $K_h(\cdot)$ be a d-dimensional kernel and $h = (h_1, \cdots, h_d)$ be a sequence of bandwidths which depend on the sample size n. Let $Q_i^Z = [1 - F_{X|Z}(X_i|Z_i)]^{\kappa-1}$ and $\widehat{Q}_i^Z = [1 - \widehat{F}_{X|Z}(X_i|Z_i)]^{\kappa-1}$, where

$$\widehat{F}_{X|Z}(X_i|Z_i) = \frac{n^{-1}\sum_{j=1}^n \mathbb{I}(X_j \le X_i)K_h(Z_j - Z_i)}{n^{-1}\sum_{j=1}^n K_h(Z_j - Z_i)}$$

is the standard kernel estimator. A natural plug-in estimator of $Cov(Y, Q^Z|Z = z)$ is given by

$$\widehat{Cov}(Y, Q^Z|Z = z) = \frac{n^{-1}\sum_{i=1}^n K_h(Z_i - z)(Y_i - \widehat{\mathbb{E}}(Y_i|Z_i))(Q_i^Z - \widehat{\mathbb{E}}(Q_i^Z))}{n^{-1}\sum_{i=1}^n K_h(Z_i - z)}, \qquad (26)$$

where $\widehat{\mathbb{E}}(Y_i|Z_i) = \left(\frac{1}{n}\sum_{j=1}^n Y_j K_h(Z_j - Z_i)\right)\left(\widehat{f}_Z(Z_i)\right)^{-1}$, $\widehat{\mathbb{E}}(Q_i^Z) = \left(\frac{1}{n}\sum_{j=1}^n Q_j^Z K_h(Z_j - Z_i)\right)\left(\widehat{f}_Z(Z_i)\right)^{-1}$, and $\widehat{f}_Z(Z_i) = \frac{1}{n}\sum_{j=1}^n K_h(Z_j - Z_i)$. The estimator of $Cov(X, Q^Z|Z = z)$ is defined by replacing Y_i with X_i in (26). Based on the estimates $\{\widehat{Q}_i^Z\}_{i=1}^n$, the estimator of $s^I(\kappa, z)$ is given by

$$\widehat{s}^I(\kappa, z) = \frac{\widehat{Cov}(Y, \widehat{Q}^Z|Z = z)}{\widehat{Cov}(X, \widehat{Q}^Z|Z = z)} = \frac{\sum_{i=1}^n K_h(Z_i - z)(Y_i - \widehat{\mathbb{E}}(Y_i|Z_i))(\widehat{Q}_i^Z - \widehat{\mathbb{E}}(\widehat{Q}_i^Z))}{\sum_{i=1}^n K_h(Z_i - z)(X_i - \widehat{\mathbb{E}}(X_i|Z_i))(\widehat{Q}_i^Z - \widehat{\mathbb{E}}(\widehat{Q}_i^Z))}.$$

The asymptotic normality of the kernel estimator $\widehat{s}^I(\kappa, z)$ can be established by following the related literature (e.g., see Li and Racine (2007) and Yin et al. (2010)).

Estimation Method #2: A Semiparametric Approach: In practice, we can adopt a more computationally convenient and efficient way: an alternative semi-

parametric estimation of the summary measure. First, to guarantee that the estimate of Q_i^Z lies between zero and one, we run the logistic regression of $\widehat{Q}_i$ on Z_i to obtain the estimate $\widetilde{Q}_i^Z = \Lambda(Z_i'\widehat{\delta})$, where $\widehat{\delta}$ is the estimator of δ in the logistic specification $\mathbb{E}(\widehat{Q}_i|Z_i) = \Lambda(Z_i'\delta)$, $\Lambda(\cdot)$ is the cumulative distribution of a logistic variable, and δ is the vector of coefficients. Second, we run the OLS regressions of Y_i, X_i, $Y_i\widetilde{Q}_i^Z$, and $X_i\widetilde{Q}_i^Z$ on Z_i to obtain the estimates $\widetilde{\mathbb{E}}(Y_i|Z_i)$, $\widetilde{\mathbb{E}}(X_i|Z_i)$, $\widetilde{\mathbb{E}}(Y_i\widetilde{Q}_i^Z|Z_i)$ and $\widetilde{\mathbb{E}}(X_i\widetilde{Q}_i^Z|Z_i)$, respectively.[19] Then, the estimator of $s^I(\kappa, z)$ is given by

$$\widetilde{s}^I(\kappa, z) = \frac{\widetilde{Cov}(Y, \widetilde{Q}^Z|Z = z)}{\widetilde{Cov}(X, \widetilde{Q}^Z|Z = z)} = \frac{\widetilde{\mathbb{E}}(Y_i\widetilde{Q}_i^Z|Z_i) - \widetilde{\mathbb{E}}(Y_i|Z_i)\widetilde{\mathbb{E}}(\widetilde{Q}_i^Z)}{\widetilde{\mathbb{E}}(X_i\widetilde{Q}_i^Z|Z_i) - \widetilde{\mathbb{E}}(X_i|Z_i)\widetilde{\mathbb{E}}(\widetilde{Q}_i^Z)}. \tag{27}$$

With the conditional estimates, one may obtain the average summary measure of mobility in the presence of covariates as follows

$$\overline{s}^I(\kappa) = \int_{\mathcal{S}_Z} s^I(\kappa, z)dF_Z(z),$$

where $\mathcal{S}_Z$ is the support of Z with the distribution function $F_Z(\cdot)$. Using the empirical distribution function $\widehat{F}_Z(z) = \frac{1}{n}\sum_{i=1}^n \mathbb{I}(Z_i \leq z)$, the average mobility measure $\overline{s}^I(\kappa)$ can be estimated by

$$\widehat{\overline{s}^I}(\kappa) = \int_{\mathcal{S}_Z} \widehat{s}^I(\kappa, z)d\widehat{F}_Z(z) = n^{-1}\sum_{i=1}^n \widehat{s}^I(\kappa, Z_i).$$

4.7   Use of Conditional Mobility $s^I(\kappa, z)$ in Further Analyses

Once we obtain $s^I(\kappa, z)$, we are able to further consider the source(s) of the between-group differences in our mobility measures and perform counterfactual analysis, if desired. Suppose that we are interested in measuring mobility for groups A and B

$$\overline{s}_g^I(\kappa) = \int_{\mathcal{S}_{Z^g}} s_g^I(\kappa, z)f_g(z)dz, \quad g = \{A, B\},$$

where $\mathcal{S}_{Z^g}$ is the support of Z for group $g = \{A, B\}$. One may consider a hypothetical counterfactual state where group A is endowed with the same distribution of observable characteristics among group B, but the income generating process is fixed.

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[19] We can also run these regressions on a polynomial function of Z_i that would well approximate the estimated objects (e.g., see the polynomial series estimations in distributional analysis by Firpo et al. (2009)).

The counterfactual mobility measure is defined as $\overline{s}_c^I(\kappa) = \int_{\mathcal{S}_{Z^B}} s_A^I(\kappa, z) f_B(z) dz$. The difference between these two groups can then be decomposed into two components, process differences (due to the differences in the income-transmission process) and composition differences, as follows

$$\overline{s}_A^I(\kappa) - \overline{s}_B^I(\kappa) = \underbrace{[\overline{s}_A^I(\kappa) - \overline{s}_c^I(\kappa)]}_{\text{Composition Differences}} + \underbrace{[\overline{s}_c^I(\kappa) - \overline{s}_B^I(\kappa)]}_{\text{Process Differences}}.$$

The counterfactual mobility measure $\overline{s}_c^I(\kappa)$ is estimated by $\widehat{\overline{s}_c^I}(\kappa) = n_B^{-1} \sum_{i=1}^{n_B} \widehat{s}_A^I(\kappa, Z_i^B)$, where $\widehat{s}_A^I(\kappa, \cdot)$ is the estimator of $s_A^I(\kappa, \cdot)$ using the sample of size n_A for group A, and $\{Z_i^B\}_{i=1}^{n_B}$ is the sample of size n_B for group B.

5 Empirical Illustration

5.1 Data

Our application is based on data from the Panel Study of Income Dynamics (PSID), which includes information at the household and individual levels for a nationally representative sample of the population of the United States. The data collection began in 1968 and has since continued to update information on the individuals of the original sample and their descendants. The long panel structure allows us to match children to their parents for intergenerational analysis, as well as to obtain their incomes at a wide range of stages over the life-cycle for both generations. See Mazumder (2016); **?** for excellent accounts of the unique advantages of the PSID data for analysis of intergenerational mobility, and over administrative tax data. Because of these advantages, the PSID data are widely used in the literature on estimation of intergenerational mobility. Use of alternative datasets does not impact the illustrative purpose of our analysis, or the central message of the paper.

To facilitate comparison to the literature, especially those studies using the PSID, we follow closely the standard practices in the literature to construct our sample and relevant variables, and therefore provide only limited details here. Following the literature (e.g., Solon (1992); Durlauf et al. (2017)), we include only the Survey Research Center component of the PSID, but exclude the Survey of Economic Opportunity (SEO) component to prevent over-representing the poverty sample. Recent literature has also noted some serious irregularities in the sampling of SEO respondents that

can "preclude easy generalization to any well-defined population" (Bloome (2015); An et al. (2020)).

In our analysis, we use (the logarithm of) permanent incomes for both children and parents. Following the literature (e.g., Durlauf et al., 2017), we define the permanent income as the average of annual family incomes, which include the taxable income of all earners in the family, from all sources, and transfer payments. We exclude zero and negative incomes. These income variables are converted to 2015 dollars using the Consumer Price Index.[20] We also follow **?** to take advantage of the very long panel structure of the PSID and center the average around age 40 (between 30 and 50). The choice of age 40 follows the rule of thumb in the literature that largely overcomes the life cycle bias (Haider and Solon (2006); **?**). The life-cycle bias is due to the heterogenous life cycle earnings profiles, where individuals with high lifetime income often have relatively low income when younger, and use of the incomes when they are young can then bias the estimates downward (Jenkins, 1987). We also restrict the sample to those individuals with at least three observations of annual incomes (e.g., Durlauf et al., 2017).

These standard practices also mitigate some of the known issues such as the issue of zero incomes that typically arise when using the administrative data due to non-employment. First, the family total income in the PSID includes sources of income such as transfers that are not available in the administrative tax record, and it is still reported "even when it may be too low to be filed for tax purposes" (An et al., 2020). Second, the PSID has a better coverage of life-cycles than the administrative records. Therefore, very few instances of zero incomes exist in the PSID, and the instances of multiple years of zero incomes are even rarer. Discussing these issues with the PSID for estimation of intergenerational mobility, Mazumder (2016) concludes that "the concerns about the sensitivity of results around how to handle years of zero income is effectively a non-issue when using family income." See An et al. (2020) for more details on this issue as well.

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[20] Source: `https://fred.stlouisfed.org/series/CPALTT01USA661S`

5.2 Results

5.2.1 Baseline Results

To facilitate the comparison to the literature, we first estimate the IGMs using the traditional regression-based approaches. Panel A of Table 1) reports the results. The level linear regression using the full sample yields an estimate of about 0.54 (Column (1)), consistent with the previous literature using PSID with an average of multiple years of annual incomes. The rank-rank regression, on the other hand, yields an estimate of about .39, similar to .341 reported in Chetty et al. (2014) using the federal income tax records. The estimated correlation using the rank-rank regression is substantially smaller than when using the level regression, suggesting a higher level of mobility in the U.S.. From our theoretical analysis above, the substantial difference between the two approaches is indicative of the nonlinearity of the income transmission process, and stems from the differences in their respective weighting functions. The weighting schemes for both level and rank-rank regressions are generally unknown, except for a few specific parametric distributions. Moreover, the weights are not necessarily proper weights for the rank-rank regressions.[21]

We now turn to our proposed estimators. In Panel B of Table 1, we first present the results based on the Gini evaluation function for inequality aversion $\kappa = 1.1, 2, 10, 50, 100, 500$. The summary measures of mobility vary drastically with respect to the inequality aversion parameter, and so does our impression of the mobility level in the U.S. For example, the correlation coefficient is between .5905 ($\kappa = 2$) and .2828 ($\kappa = 500$), with the difference being more than 100 percent ($\frac{(.5905-.2828)}{.2828} \times 100 \approx 109$). As a larger κ is associated with larger weights on poorer households, evidently, our results suggest a substantially more mobile society when focusing more on the individuals from the disadvantaged families.

The pattern of the changes with respect to κ is, however, not monotonic. We observe that at the relatively low level of inequality aversion, an increase in inequality aversion (from $\kappa = 1.1$ to 2) leads to a larger coefficient and hence a higher level of immobility. On the other hand, for $\kappa > 2$, when the inequality aversion parameter increases and we place more weights on the individuals from the disadvantaged families,

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[21] It is possible that the two variables may follow a normal distribution. To examine this possibility, we also conduct the Jarque-Bera test of normality, and reject the null hypothesis of normality for both child and parental income distributions at the one percent level. The results are available from the authors upon request.

we actually observe that the IGM coefficients decrease substantially in magnitudes, suggesting a more mobile society.

The log-regression coefficient is .5371, falling between the coefficient using $\kappa = 10$ (.5641) and that using $\kappa = 50$ (.4870). While we do not have any idea about the weighting schemes behind the rank-rank regression, its coefficient is closer to the mobility measure suggested by our method between using $\kappa = 100$ and $\kappa = 500$ (when placing more weights on the lower tail than the upper tail of the parental income distribution).

Panels C.1. and C.2. of Table 1 presents the second set of the results based on the Lorenz family of evaluation functions. Recall that the inequality aversion parameter, ν, has the opposite meaning of κ for the Gini-based measures. For $\nu \in (0, 1)$ in Panel C.1., we again find that as the inequality aversion increases (ν decreases), the coefficients decrease in magnitudes. This pattern is similar to the results using the Gini-based measures. However, the variation in the estimates is substantially smaller. The coefficients vary from .5696 to .5804. This is not surprising because Figure 1 suggests that even though the shapes of the underlying weighting functions between the Gini and Lorenz family of measures may appear to be similar, they are not the same and the actual weights also differ drastically. For example, in the case of high inequality aversion (when $\nu = .01, \kappa = 500$), they both place the maximum weight on the children from the lowest-income families, but the variations in the actual weights are smaller for the Lorenz family than for the Gini family. This result again highlights the importance of clarifying the policy objectives or evaluation functions (and hence the weights) in measuring and summarizing IGMs.

For $\nu > 1$ in Panel C.2. of Table 1, we find that as ν increases and the inequality aversion decreases, the magnitudes of the coefficients decrease. As we place more weights on the children from higher-income individuals, our measures suggest a more mobile society. This result is particularly interesting, especially when we view it with the result based on the Gini estimator. As Figure 1 implies, as $\kappa > 1$ increases from a very small value, the Gini-based measure starts from placing the maximum weight on the middle-income families, and gradually place more weights on the *lower*-income families. In contrast, as $\nu > 1$ increases, the maximum weight associated with the Lorenz-based measure also starts the middle-income families, but gradually place more weights on the *higher*-income families. Hence, the estimates of these two measures complement each other by highlighting the features from different parts of

the distribution. Together, our results may suggest that mobility is higher at both ends than at the middle.

In sum, we reach two conclusions. First, our results provide strong evidence that the income transmission process is highly nonlinear, and suggest that both children from disadvantaged and richer families may have a higher level of mobility than those from the "middle class". The latter is only *suggestive* since we pick only a few inequality aversion parameters (hence highlighting only a few selected parts of the distribution as well) and the nonlinear pattern may not be as smooth and straight-forward as the ones suggested here. Second and more importantly, due to the strong presence of nonlinearity, the subjective weighting schemes matter, in fact, a lot when forming a general impression of the mobility in a society. Bearing these results in mind, we now further examine how the potential nonlinearity and varying weighting schemes may impact our understanding of geographic differences in IGMs and that of the evolution of IGMs.

5.2.2 Geographic Disparities in Mobility

We first examine geographic differences in mobility. Following the literature, we compare four regions where an individual grew up: the Northeast, the North Central, the South, and the West. The regression-based results are displayed in Table 2. We again find that the conventional approaches differ drastically across regions, with the rank-rank results suggesting a more mobile society than the level regressions. For example, the difference in the implied mobility can be as large as 55 percent for the West ($55 \approx (.4482 - .2898)/.2898 \times 100$).

Both the level and rank-rank regression results indicate significant geographic disparities in IGMs. A common pattern in geographic heterogeneity emerges for both approaches: the South is less mobile than the West, in line with Chetty et al. (2014).[22] Furthermore, the South is the least mobile region, while the West is the most mobile region. However, the general pattern for the entire country can differ drastically across the methods and their corresponding weighting schemes. For example, the rank-rank regression implies a substantially higher disparity in mobility between the South and the West. Relatively speaking, the rank-rank regression suggests that the South is at least 52 percent less mobile than the West ($52.86 \approx (0.4430 - .2898)/.2898 \times$

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[22] Specifically, both traditional measures find that the coefficients are smaller in magnitude for the children from the West than those from the South.

100), compared to 30 percent suggested by the level regression ($30.63 \approx (0.5855 - 0.4482)/0.4482$). The pattern for the other two regions is not definitive. While the level regression suggests that the North Central is less mobile than the Northeast, the rank-rank regression suggests the opposite.

We now turn to our proposed estimators. The results based on the Gini evaluation functions are presented in Table 3. We start with $k = 1.1$ (the maximum weight is placed roughly at the middle part of the parental income distribution). The result confirms the difference between the South and the West, but the difference between the South and the rest of the country is also not so big as the traditional approaches suggest. The immobility level in the South is about 29 percent higher than the West ($\approx (0.5956 - 0.4611)/.4611$), substantially smaller than the rank-rank results ($.5286 \approx (.4430 - .2898)/.2898$).

Varying the inequality aversion parameters impacts both the size and patterns of IGM across regions. First, the variation of the estimates with respect to the inequality aversion parameters differs across regions, suggesting significant between-group differences in the income transmission process and the extent of nonlinearity. For example, for the Northeast, when we place more weights on the children from the disadvantaged families, the size of the mobility decreases by 80 percent when comparing the largest value ($.1115$, $\kappa = 500$) and the smallest value ($.5481$, $\kappa = 2$). By contrast, the coefficient is only 16 percent smaller for the North Central when comparing the smallest coefficient ($.4941$ when $\kappa = 500$) with the largest ($.5906$ when $\kappa = 2$).

Second, the patterns of the changes with respect to κ differ from the full sample (i.e., when we increase the inequality aversion, the coefficient first increases and then decreases) for the South and the West. For the South, the coefficients *decrease* with respect to the inequality aversion parameter, while for the West, the coefficients *increase*, fluctuating around an increasing trend. When placing more and more weights on the children from the disadvantaged families, we actually find that the West becomes less and less mobile (the coefficient increases from $.4611$ to $.7825$).

Our impression of the *relative* mobility levels is also senstive to the change of the inequality aversion parameter. It starts to change when $\kappa = 10$, and we observe that the West is actually the least mobile region. As inequality aversion increases, a more stable relative ranking emerges. In fact, when $\kappa = 500$ and we place more weights on the individuals from the most disadvantaged families, the Northeast is the most

mobile region and the West is the least mobile region.

We turn to the results based on the Lorenz family of evaluation functions. The results are presented in Table 4. For $0 < \nu < 1$, we find similar patterns to the results based on the Gini family, while the coefficients vary less drastically (as discussed above). However, even a minor change in the coefficients impacts the relative ranking of geographic differences in mobility: when we place more weights on the children from the disadvantaged families (when $\nu = .1, .2, .5,$), the West is the most mobile, while the North Central is the least mobile. For $\nu > 1$, another interesting pattern arises. As we increase ν and place more weights on the individuals from richer families, the coefficients decrease for the Northeast, the North Central, and the West, which is similar to the full-sample results. For the South, we instead observe the opposite. When we increase ν from 1.1 (the maximum weight roughly at the middle-income families) to 500 (the maximum weight roughly at the richest families), the coefficient increases from .6013 to .7690, suggesting a much less mobile region.

The results here highlight the fact that there exists significant heterogeneity in the income transmission processes both within and across regions, which evidently impacts our impression of the mobility for a region and that of the regional disparities in mobility. Moreover, viewing the results based on the Gini and the Lorenz families together, there is some suggestive evidence that the children from richer families may have a higher level of immobility or "affluence trap" in the South, while children from both disadvantaged and richer families may have a higher level of mobility than those from the "middle class" for the rest of the country.

5.2.3   Dynamics of Mobility

To examine how the mobility evolves across cohorts, we consider four cohorts (those born before 1954, between 1955 and 1961, between 1962 and 1967, after 1968). The regression-based results are displayed in Table 5. Both the level and rank-rank regression results imply an increase in the magnitudes of the correlation coefficients and a decrease in mobility over time when comparing the (first) cohort born before 1954 and the (last) cohort born after 1968. Specifically, both methods suggest about 13 percent decrease in mobility (level regression: $\frac{(.5597-.4949)}{.4949} \times \approx 13$; rank-rank regression $\frac{(.4197-.3718)}{.3718} \times 100 \approx 13$). However, the dynamics and the magnitudes of the changes between the first and the last cohorts differ across the methods used. For example, the level regressions suggest an increasing trend between cohorts; we find

that relative to the cohort born before 1954, the coefficient is larger for the cohort born during the period 1962-1967. The rank-rank regression suggests the opposite.

We now turn to our proposed estimators. The results based on the Gini evaluation functions are reported in Table 6. Varying the inequality aversion parameter again can drastically revise our view of mobility for a particular cohort, as well as that of the dynamics of the mobility across cohorts. First, the coefficients do not monotonically vary with the inequality aversion parameter (κ), and the patterns differ drastically across cohorts.[23] Second, our relative ranking of the mobility levels across cohorts also depend crucially on the part of the distribution which a particular weighting scheme emphasizes. For example, when $\kappa = 1.1$, we observe the same ranking as the level regression, where it is more mobile for the cohort born before 1954 than for the cohort born after 1968. Such impression is reversed when $\kappa = 50$ (.5747 (Before 1954) vs .4751 (After 1968)).

Turning to our results based on the Lorenz family in Table 7, we again observe a far less variation in the coefficients with respect to the changes in inequality aversions and more importantly, even a reversed trend, compared to the Gini-based results. For example, among those born between 1962 and 1967, when we increase the level of inequality aversion (from $\nu = .8$ to $\nu = .1$), we observe monotonically decreasing coefficients, which suggests a more mobile society. By contrast, for the same cohort, when we increase the level of inequality aversion (from $\kappa = 1.1$ to $\kappa = 500$) for the Gini-based measures, we observe the coefficients fluctuate.

These results imply that we could have a highly nonlinear income transmission process that fluctuates a lot (and hence the derivative and the implied mobility level) at adjacent values and can be drastically different.

5.2.4   Before Conclusions

The results here, both theoretical and empirical, may be uncomfortable for some. The ubiquitous heterogeneity and nonlinearity may imply that any conclusions regarding the mobility can be subjective and sensitive to the varying parameter. That is

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[23] For the cohorts born before 1954, between 1955-1961, and after 1968, we observe that the coefficients first increase and then decrease when we place more and more weights on the children from the low-income families. They peak at different inequality aversion parameters (for the cohort born before 1954, the largest coefficient is .5747 when $\kappa = 50$, while for the cohort born after 1968, the largest coefficient is .6344 when $\kappa = 2$). On the other hand, for the cohort born between 1962 and 1967, we actually find that the coefficients first decrease and then increase, reaching the maximum when the maximum weights are placed on the children from the lowest-income families with $\kappa = 500$.

true. However, some of the *qualitative* conclusions do not have to be. It is important to see what consensus may arise from this kind of analysis. When no uniform conclusions can be reached, our paper points out the need to explicate the commitment to certain policy goals when measuring mobility. For example, many may agree that the measurement of mobility should reflect our care for the poor, and that monitoring the changes or policy effectiveness should place more weights on the children from more disadvantaged families. Only when such qualifying statements are made can our policy discussions be more meaningful and fruitful.

6   Conclusions

In this paper, we consider the decision-theoretic foundation of the intergenerational mobility measures in the presence of nonlinearity and heterogeneity. We first recast the dominant regression-based measures as a weighted average of intergenerational income elasticities at different parts of the parental income distribution. A careful analysis of the weighting schemes underlying these traditional measures exposes some undesirable features of these approaches. The weights for the rank-rank regression fail an aggregation consistency. Two prominent one-parameter families of summary measures of mobility were analyzed to exemplify . Other evaluation functions can easily be accommodated and implemented with a simple method of moments implementation. We provided extention to allow for multiple covariates. This will allow counterfactual analysis for further understanding of the sources of IGM. We apply our method to the PSID: Impression of IGM depends crucially on which part of the distribution we would like to highlight in practice. The sensitivity of the mobility measures for a particular group also has a substantial impact on our between-group comparisons of mobility levels. Any perspective on the geographic differences in mobility and the dynamics of mobility can be challenged, but should be based on plausible transparent, subjective evaluation functions.

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Appendix

A.   Proof of Corollary 1

Proof. First, we consider the case of normal distributions. Since the conditional distribution of ϵ given $X = x$ is normal with zero mean and variance σ_ϵ^2, the conditional distribution of Y given $X = x$ is normal with mean $\mu_{y|x} = g(x)$ and variance $\sigma_{y|x}^2 = \sigma_\epsilon^2$. Given that Y is normally distributed, the mean and variance of Y are $\mu_y = \mathbb{E}(g(X))$ and $\sigma_y^2 = \sigma_\epsilon^2 + Var(g(X))$, respectively. Note that $F_Y(y) = Pr(Y \leq y) = Pr((Y - \mu_y)/\sigma_y \leq (y - \mu_y)/\sigma_y) = \Phi((y - \mu_y)/\sigma_y)$. Then, it can be shown that $g_r'(u)$ in (3) becomes

$$
\begin{aligned}
g_r'(u) &= \frac{d}{du}\left\{\mathbb{E}(F_Y(Y)|F_X(X) = F_X(x) = u)\right\} = \frac{d}{du}\left\{\int_{-\infty}^{\infty} \Phi((y - \mu_y)/\sigma_y)f(y|x)dy\right\} \\
&= a(x)g'(x)(f_X(x))^{-1},
\end{aligned}
\tag{28}
$$

where

$$
a(x) = \frac{1}{\sqrt{2\pi(\sigma_\epsilon^2 + \sigma_y^2)}} \times exp\left\{-\frac{(\sigma_\epsilon^2 + \sigma_y^2)(\sigma_\epsilon^2\mu_y^2 + g^2(x)\sigma_y^2) - (\mu_y\sigma_\epsilon^2 + g(x)\sigma_y^2)^2}{2\sigma_y^2\sigma_\epsilon^2(\sigma_\epsilon^2 + \sigma_y^2)}\right\}.
$$

Therefore, $w_r(x)$ in (6) becomes

$$
w_r(x) = 6(F_X(x) - F_X^2(x))g_r'(F_X(x))f_X(x)(g'(x))^{-1} = c(x)(F_X(x) - F_X^2(x)), \quad (29)
$$

where $c(x) = 6a(x)$.

Second, for the case of lognormal distribution, using arguments similar to the case of normal distributions, it can be shown that $w_r(x)$ remains the same as that in (29). Specifically,

$$
\begin{aligned}
g_r'(u) &= \frac{d}{du}\left\{\mathbb{E}(F_Y(Y)|F_X(X) = F_X(x) = u)\right\} = \frac{d}{du}\left\{\int_0^{\infty} \Phi\left(\frac{log(y) - \mu_y}{\sigma_y}\right)f(y|x)dy\right\} \\
&= \int_{-\infty}^{\infty} \Phi\left(\frac{t - \mu_y}{\sigma_y}\right)\left\{\frac{1}{\sigma_\epsilon\sqrt{2\pi}}exp\left(-\frac{(t - g(x))^2}{2\sigma_\epsilon^2}\right)\frac{t - g(x)}{\sigma_\epsilon^2}\frac{g'(x)}{f(x)}\right\}dt \\
&= a(x)g'(x)(f_X(x))^{-1},
\end{aligned}
$$

where the last equality, which is the same as (28), is obtained by using the same

arguments as those for the case of the normal distribution. Therefore, $w_r(x)$ in (6) is identical to that in (29). Third, we consider the case of uniform distributions. Note that $\mathbb{E}(\epsilon|X) = 0$ implies $\mathbb{E}(\epsilon|F_X(X)) = 0$. Then,

$$g_r(U) = \mathbb{E}[V|U] = \mathbb{E}[F_Y(g(X)+\epsilon)|F_X(X)] = \frac{g(X) + \mathbb{E}[\epsilon|F_X(X)] - \underline{y}}{\overline{y} - \underline{y}} = \frac{g(F_X^{-1}(U)) - \underline{y}}{\overline{y} - \underline{y}}.$$

Therefore, $w_r(x)$ in (6) becomes

$$w_r(x) = 6(F_X(x) - F_X^2(x))(\overline{y} - \underline{y})^{-1}. \tag{30}$$

In addition, we provide a result connecting summary measures between level and rank regressions under additional assumptions. Let $\Delta_y = \overline{y} - \underline{y}$. If X is also uniformly distributed on $[\underline{x}, \overline{x}]$ with $\Delta_x = \overline{x} - \underline{x}$ and $g(x) = \alpha + x\beta$, we obtain

$$g_r(u) = \frac{g(u\Delta_x + \underline{x}) - \underline{y}}{\Delta_y} = \frac{\alpha + \beta(u\Delta_x + \underline{x}) - \underline{y}}{\Delta_y} = \frac{\alpha + \beta\underline{x} - \underline{y}}{\Delta_y} + \frac{\beta\Delta_x}{\Delta_y}u = \alpha_r + \beta_r u,$$

where $\alpha_r = (\alpha + \beta\underline{x} - \underline{y})/\Delta_y$ and $\beta_r = \beta\Delta_x/\Delta_y$. Suppose $\Delta_x = \Delta_y$, we obtain $\beta_r = \beta$. Moreover, $w_r(x)$ in (30) becomes $w_r(x) = 6(x - \underline{x})(\overline{x} - x)\Delta_x^{-3}$, and it can be shown that $\int_{\underline{x}}^{\overline{x}} w_r(x)dx = 1$. $\qquad\square$

B. Proof of Proposition 5

Proof. (*i*) $u^I(\kappa)$ is strictly decreasing in κ for $\kappa > 1$ because

$$\frac{du^I(\kappa)}{d\kappa} = -\frac{\kappa^{\frac{1}{1-\kappa}}(1 - \kappa + \kappa\log\kappa)}{\kappa(1-\kappa)^2} < 0 \text{ for } \kappa > 1. \tag{31}$$

(*ii*) Using L'Hopital's rule leads to

$$\lim_{\kappa\to\infty} \kappa^{\frac{1}{1-\kappa}} = \lim_{\kappa\to\infty} \exp\left(\log\left(\kappa^{\frac{1}{1-\kappa}}\right)\right) = \exp\left(\lim_{\kappa\to\infty} \log\left(\kappa^{\frac{1}{1-\kappa}}\right)\right) = \exp(0) = 1. \tag{32}$$

Therefore, $\lim_{\kappa\to\infty} u^I(\kappa) = \lim_{\kappa\to\infty}\left(1 - \kappa^{\frac{1}{1-\kappa}}\right) = 0$.
(*iii*) Recall the first derivative in (19) is

$$\frac{dw_\kappa^I(u)}{du} = c^I(\kappa)\kappa(1-u)^{\kappa-1} - c^I(\kappa), \tag{33}$$

where $c^I(\kappa) = \left(\int_0^1 \{(1-u) - (1-u)^\kappa\} dF_X^{-1}(u) \right)^{-1}$. First, the first term in (33) converges to zero as $\kappa \to \infty$ because $\kappa(1-u)^{\kappa-1}$ converges to zero for any $u \in (0, 1]$, and $c^I(\kappa)$ converges to a non-zero constant. This implies that for a sufficiently large value of κ, the approximating first derivative in (33) is a constant. Second, the rate of convergence for $\kappa(1-u)^{\kappa-1}$ seems much faster than that for the κ-related component $\int_0^1 (1-u)^\kappa dF_X^{-1}(u)$ in $c^I(\kappa)$. Therefore, for a sufficiently large value of κ, the approximate slope of $w_\kappa^I(u)$ is $-c^I(\kappa)$. As the upper left panel of Figure 1 indicates, when κ is larger, all of the curves become closer to lines, with different slopes corresponding to different values of κ. Last, combining the approximate slope $-c^I(\kappa)$ with the additional condition $w_\kappa^I(1) = 0$, we obtain that for a sufficiently large κ, the approximating expression for $w_\kappa^I(u)$ with $u \in [u^I(\kappa), 1]$ is $c^I(\kappa)(1-u)$. $\qquad\square$

C. Proof of Proposition 6

Proof. (i) $u^{II}(\nu)$ is strictly increasing in $\nu > 0$ because it can be shown that

$$\frac{du^{II}(\nu)}{d\nu} = \frac{\nu^{\frac{1}{1-\nu}}(1 - \nu + \nu log\ \nu)}{\nu(1-\nu)^2} > 0. \tag{34}$$

(ii) It is clear that $u^{II}(\nu) = \nu^{\frac{1}{1-\nu}} \to 0$ as $\nu \to 0$. Using the same proof as in (32), we obtain $u^{II}(\nu) \to 1$ as $\nu \to \infty$.

(iii) Recall the first derivative in (24) is

$$\frac{dw_\nu^{II}(u)}{du} = c^{II}(\nu) - c^{II}(\nu)\nu u^{\nu-1}, \tag{35}$$

where $c^{II}(\nu) = \left(\int_0^1 (u - u^\nu) dF_X^{-1}(u) \right)^{-1}$. First, the second term in (35) converges to zero as $\nu \to 0$ because $\nu u^{\nu-1}$ converges to zero for any $u \in (0, 1]$, and $c^{II}(\nu)$ converges to a non-zero constant. This implies that for a sufficiently small ν, the approximating first derivative in (35) is a constant. Second, the rate of convergence for $\nu u^{\nu-1}$ seems much faster than that for the ν-related component $\int_0^1 u^\nu dF_X^{-1}(u)$ in $c^{II}(\nu)$, in the sense that $\nu u^{\nu-1}$ is negligible relative to $c^{II}(\nu)$. Therefore, for a sufficiently small ν, the approximate slope of $w_\nu^{II}(u)$ is $c^{II}(\nu)$. As the lower left panel of Figure 1 implies, when ν is smaller, all of the curves become closer to lines, with different slopes corresponding to different values of ν. Last, combining the approximate slope $c^{II}(\nu)$ with the additional condition $w_\nu^{II}(1) = 0$, we obtain that for a sufficiently small

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ν, the approximating expression for $w_\nu^{II}(u)$ with $u \in [u^{II}(\nu), 1]$ is $c^{II}(\nu)(u-1)$.

(iv) First, the second term in (35) converges to zero as $\nu \to \infty$ because $\nu u^{\nu-1}$ converges to zero for any $u \in [0, 1)$, and $c^{II}(\nu)$ converges to a non-zero constant. This implies that for a sufficiently large ν, the approximating first derivative in (35) is a constant. Second, similar to the proof of property (iii), when ν is larger, $\nu u^{\nu-1}$ is negligible relative to $c^{II}(\nu)$. Therefore, for a sufficiently large ν, the approximate slope of $w_\nu^{II}(u)$ is $c^{II}(\nu)$. As the lower right panel of Figure 1 shows, when ν is larger, all of the curves become closer to lines, with different slopes corresponding to different values of ν. Last, combining the approximate slope $c^{II}(\nu)$ with the additional condition $w_\nu^{II}(0) = 0$, we obtain that for a sufficiently large ν, the approximating expression for $w_\nu^{II}(u)$ with $u \in [0, u^{II}(\nu)]$ is $c^{II}(\nu)u$. □

Table 1: Measures of Immobility (Full Sample)

Panel A: Conventional Regression-based results					
Level	Rank-Rank				
Regression	Regression				
0.5371***	0.3924***				
(0.0292)	(0.0204)				
Panel B: Gini Family of Measures ($\kappa > 1$)					
		Low $\to$ Higher Inequality Aversion			
$k = 1.1$	$k = 2$	$k = 10$	$k = 50$	$k = 100$	$k = 500$
0.5814***	0.5905***	0.5641***	0.4870***	0.4535***	0.2828**
(0.0311)	(0.0317)	(0.0380)	(0.0547)	(0.0668)	(0.1121)
Panel C.1. Lorenz Family of Measures ($\nu \in (0, 1)$)					
		Low $\to$ Higher Inequality Aversion			
$v = .8$	$v = .5$	$v = .2$	$v = .1$		
0.5804***	0.5796***	0.5736***	0.5696***		
(0.0308)	(0.0306)	(0.0309)	(0.0311)		
Panel C.2. Lorenz Family of Measures ($\nu > 1$)					
		High $\to$ Low Inequality Aversion			
$v = 1.1$	$v = 2$	$v = 10$	$v = 50$	$v = 100$	$v = 500$
0.5786***	0.5680***	0.4965***	0.3648***	0.3135***	0.2700**
(0.0312)	(0.0325)	(0.0412)	(0.0583)	(0.0699)	(0.1142)
Observations		2042			

Table 2: Measures of Immobility: Conventional Regression Approaches (By Region)

	By Region			
	Northeast	North Central	South	West
	(1)	(2)	(3)	(4)
Panel A: Regression Approach				
Log of	0.4650***	0.5193***	0.5855***	0.4482***
Father's Income	(0.0634)	(0.0497)	(0.0544)	(0.0827)
Panel B: Rank-Rank Approach				
Rank of	0.3693***	0.3636***	0.4430***	0.2898***
Father's Income	(0.0462)	(0.0335)	(0.0390)	(0.0531)
Observations	407	776	532	327

Table 3: Measures of Immobility: Gini Family (By Region)

	Parameter	Northeast	North Central	South	West
		(1)	(2)	(3)	(4)
Low	$\kappa = 1.1$	0.5306***	0.5782***	0.5956***	0.4611***
		(0.0692)	(0.0535)	(0.0561)	(0.0870)
	$\kappa = 2$	0.5481***	0.5906***	0.5758***	0.4938***
$\downarrow$		(0.0711)	(0.0552)	(0.0570)	(0.0875)
	$\kappa = 10$	0.5442***	0.5750***	0.5056***	0.5649***
High		(0.0851)	(0.0686)	(0.0701)	(0.1013)
Inequality	$\kappa = 50$	0.4309***	0.5171***	0.4393***	0.4743***
		(0.1127)	(0.0993)	(0.1075)	(0.1516)
Aversion					
	$\kappa = 100$	0.3408***	0.5643***	0.3850***	0.4871**
		(0.1272)	(0.1212)	(0.1396)	(0.2005)
	$\kappa = 500$	0.1115	0.4941**	0.3892*	0.7825*
		(0.1936)	(0.1992)	(0.2358)	(0.4121)

Table 4: Measures of Immobility: Lorenz Family (By Region)

	Parameter	Northeast	North Central	South	West
		(1)	(2)	(3)	(4)
Panel A: $\nu \in (0,1)$					
High	$\nu = .1$	0.5264***	0.5779***	0.5547***	0.5019***
		(0.0689)	(0.0547)	(0.0567)	(0.0856)
↓					
	$\nu = .2$	0.5298***	0.5790***	0.5615***	0.4972***
Low		(0.0685)	(0.0540)	(0.0561)	(0.0851)
Inequality	$\nu = .5$	0.5329***	0.5799***	0.5783***	0.4820***
Aversion		(0.0682)	(0.0532)	(0.0555)	(0.0851)
	$\nu = .8$	0.5301***	0.5780***	0.5912***	0.4664***
		(0.0686)	(0.0532)	(0.0558)	(0.0862)
Panel B: $\nu > 1$					
High	$\nu = 1.1$	0.5249***	0.5745***	0.6013***	0.4516***
		(0.0692)	(0.0535)	(0.0564)	(0.0877)
↓					
	$\nu = 2$	0.5057***	0.5611***	0.6211***	0.4173***
Low		(0.0714)	(0.0550)	(0.0591)	(0.0927)
Inequality	$\nu = 10$	0.3715***	0.4980***	0.6516***	0.3308***
Aversion		(0.0870)	(0.0660)	(0.0798)	(0.1240)
	$\nu = 50$	0.2848**	0.3277***	0.7022***	0.1386
		(0.1212)	(0.0878)	(0.1319)	(0.1930)
	$\nu = 100$	0.3097**	0.2307**	0.7192***	0.0181
		(0.1464)	(0.1043)	(0.1683)	(0.2373)
	$\nu = 500$	0.3027	0.1910	0.7690***	-0.1225
		(0.2270)	(0.1630)	(0.2644)	(0.3894)
Observations		407	776	532	327

Table 5: Measures of Immobility: Conventional Regression Approaches (By Cohort)

| | By Birth Cohort | | | |
| | Before 1954 | 1954-1961 | 1961-1967 | Post 1967 |
	(1)	(2)	(3)	(4)
Panel A: Regression Approach				
Log of	0.4949***	0.5303***	0.5226***	0.5597***
Father's Income	(0.0754)	(0.0602)	(0.0789)	(0.0422)
Panel B: Rank-Rank Approach				
Rank of	0.3718***	0.3965***	0.3260***	0.4197***
Father's Income	(0.0507)	(0.0448)	(0.0496)	(0.0300)
Observations	337	422	365	918

Table 6: Measures of Immobility: Gini Family (By Cohort)

| | Parameter | Northeast | North Central | South | West |
		(1)	(2)	(3)	(4)
Low	$\kappa = 1.1$	0.5054***	0.5706***	0.5505***	0.6183***
		(0.0798)	(0.0638)	(0.0829)	(0.0452)
	$\kappa = 2$	0.5230***	0.5875***	0.5150***	0.6344***
$\downarrow$		(0.0808)	(0.0652)	(0.0859)	(0.0459)
	$\kappa = 10$	0.5691***	0.5916***	0.3741***	0.5914***
High		(0.0963)	(0.0784)	(0.1121)	(0.0541)
Inequality	$\kappa = 50$	0.5747***	0.5171***	0.2493	0.4751***
		(0.1437)	(0.1149)	(0.1758)	(0.0757)
Aversion					
	$\kappa = 100$	0.5436***	0.4492***	0.3184	0.4074***
		(0.1822)	(0.1435)	(0.2195)	(0.0911)
	$\kappa = 500$	0.2716	0.3696*	0.9267*	0.2254
		(0.2824)	(0.2012)	(0.4758)	(0.1488)
	Observations	337	422	365	918

Table 7: Measures of Immobility: Lorenz Family (By Cohort)

	Parameter	Northeast	North Central	South	West
		(1)	(2)	(3)	(4)
Panel A: $\nu \in (0,1)$					
High	$\nu = .1$	0.5308***	0.5766***	0.4795***	0.6018***
		(0.0796)	(0.0642)	(0.0866)	(0.0447)
$\downarrow$					
	$\nu = .2$	0.5274***	0.5770***	0.4904***	0.6077***
Low		(0.0789)	(0.0637)	(0.0853)	(0.0445)
Inequality	$\nu = .5$	0.5174***	0.5752***	0.5194***	0.6165***
Aversion		(0.0785)	(0.0631)	(0.0831)	(0.0444)
	$\nu = .8$	0.5081***	0.5708***	0.5424***	0.6172***
		(0.0791)	(0.0633)	(0.0825)	(0.0448)
Panel B: $\nu > 1$					
High	$\nu = 1.1$	0.5001***	0.5652***	0.5601***	0.6138***
		(0.0801)	(0.0640)	(0.0827)	(0.0454)
$\downarrow$					
	$\nu = 2$	0.4817***	0.5466***	0.5934***	0.5955***
Low		(0.0840)	(0.0665)	(0.0848)	(0.0475)
Inequality	$\nu = 10$	0.4106***	0.4284***	0.6183***	0.5084***
Aversion		(0.1079)	(0.0834)	(0.1034)	(0.0609)
	$\nu = 50$	0.3327**	0.3009**	0.4690***	0.3769***
		(0.1518)	(0.1191)	(0.1432)	(0.0868)
	$\nu = 100$	0.2947	0.2672*	0.4616***	0.2971***
		(0.1807)	(0.1443)	(0.1701)	(0.1042)
	$\nu = 500$	0.2447	0.2151	0.5004*	0.2488
		(0.2546)	(0.1979)	(0.2757)	(0.1749)
Observations		337	422	365	918