The Cross-Section of Output and Inflation in a Dynamic Stochastic General Equilibrium Model with Sticky Prices

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Abstract

In a standard dynamic stochastic general equilibrium framework, with sticky prices, the cross sectional distribution of output and inflation across a population of firms is studied. The only form of heterogeneity is confined to the probability that the ith firm changes its prices in response to a shock. In this Calvo setup the moments of the cross sectional distribution of output and inflation depend crucially on the proportion of firms that are allowed to change their prices. We test this model empirically using German balance sheet data on a very large population of firms. We find a significant counter-cyclical correlation between the skewness of output responses and the aggregate economy. Further analysis of sectoral data for the US suggests that there is a positive relationship between the skewness of inflation and aggregates, but the relation with output skewness is less sure. Our results can be interpreted as indirect evidence of the importance of price stickiness in macroeconomic adjustment.

JEL: D12, E52, E43.

1 Introduction

Since the seminal work of, amongst others, Mankiw (1985), Akerlof and Yellen (1985) and Blanchard and Kiyotaki (1987) who (re-)stressed the importance of frictions for at least the short term performance of the economy, the interest in models now termed New-Keynesian or New Neo-classical Synthesis (NNS) has increased dramatically. The workhorse of modern macroeconomics is some form of a DSGE-model incorporating frictions of various kinds, and in particular sticky prices. Those models are used to address a wide variety of macroeconomic questions arising from discussion of monetary policy (e.g.

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Clarida, Galí and Gertler 1999), fiscal policy (e.g. Woodford, 1997) or open economy issues (e.g. Bowman and Doyle 2003).

The models are usually tested empirically by comparing the theoretical impulse response functions with empirical impulse response functions obtained from a VAR-analysis (e.g. Rotemberg and Woodford 1998). These empirical checks all utilize aggregate data for the output gap or the inflation rate. Both are readily available from the simulation of the model and the empirical time series are easy to obtain.

In two recent papers (Higson et al. 2002 and Higson et al. 2004) the emphasis has been to use cross-sectional data to establish some stylised facts of the business cycle at the micro-level. Using large data sets on US and UK firms, the distribution of the growth rate of firms' real sales was examined. The most striking finding was a significant counter-cyclical correlation between the cross sectional skewness of the distribution and the aggregate economy. These stylised facts need some theoretical explanation[1]. In this paper we seek an explanation in a simple New Keynesian model with Calvo-type sticky prices. Price decisions are time dependent but the times between price changes are probabilistic. The probability is set exogenously[2]. The purpose of this paper is to use the simplest form of the New Keynesian model with sticky prices to examine the relationship between the cross sectional moments of output and the aggregate economy and then to test the predictions on firm level data for Germany. It can also be thought of as an indirect way of testing if the assumption of sticky prices[3] is in line with the empirical evidence[4].

The rest of the paper is organized as follows: in the next section a simplified version of a New Keynesian model with sticky prices drawing on Woodford (1997) and Rotemberg and Woodford (1998) is presented and simulated. The pattern of the higher moments of the distribution of output growth and price changes is examined. The results are than compared to empirical results obtained from conducting an analysis in the vein of Higson et al. (2002 and 2004) using a German dataset. This dataset has the advantage of having ten times as many firms per year as the datasets used by Higson et al. And while Higson et al. only have quoted companies in their sample the German sample also includes non-quoted companies. The difficulty with the German data set at the level of the firm is that we do not have information on prices. So we turn to sectoral data for the US.

2 The model

The model presented in this section is a simplified version of Woodford (1997) and Rotemberg and Woodford (1998). Both articles present a general equilibrium model incorporating price stickiness in a Calvo (1983) form. The simplification made in this paper is to neglect the government sector and money-in-utility function. Since we are interested in

[1] A alternative explanation that identifies heterogeneity in the financial state of the firm as important is explored in Holly and Santoro (2008).

[2] Recently Sheedy (2007) has proposed the idea that the probability that a firm changes its price is not set exogenously as in the Calvo model but depends on the time that has elapsed since the last price change. The longer the duration, the higher the probability of a change in prices. The hazard function describing price changes is upward sloping rather than flat as in the Calvo model.

[3] The menu cost model of Ball and Mankiw (1994, 1995) also generates interesting interactions between cross sectional moments and the aggregate economy. However, for a recent challenge to this model see Demery and Duck (2008).

[4] Recently, Gabaix (2005) and Delli Gatti et al. (2007) have also suggested new approaches to modelling aggregate fluctuations based on micro-evidence.

the *intra* cross sectional distribution of output and prices for the aggregate economy and not in specific economic policy questions, those simplifications should be straightforward.

The model consists of a continuum of identical, infinitely living households. Each household $j \in [0,1]$ produces a single differentiated good. Monopolistic competition, which is assumed to prevail follows Dixit and Stiglitz (1977) and Spence (1976) so suppliers can set their prices. Each household maximises lifetime utility over all periods t:

$$E_t \left\{ \sum_{t=0}^{\infty} \beta^t \left[u\left(C_t^j\right) - v\left(y_t\left(j\right)\right)\right] \right\} \tag{1}$$

In equation (1), u is a concave and v a convex utility function with *u',v'>0*. β is a discount factor and $y_t(j)$ denotes the household-produced good of the jth household.. $v\left(y_t\left(j\right)\right)$ represents the disutility of producing the good. C_t^j is the consumption index of the household *j*, which is given by:

$$C_t^j \equiv \left[\int_0^1 c_t^j\left(z\right)^{\frac{\theta-1}{\theta}} dz\right]^{\frac{\theta}{\theta-1}} \tag{2}$$

In equation (2), $c_t^j\left(z\right)$ denotes the consumption of good z by household j in period t. $\theta > 1$ denotes the elasticity of substitution between the individual goods. The price index is defined as:

$$P_t \equiv \left[\int_0^1 p_t\left(z\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}} \tag{3}$$

where $p_t\left(z\right)$ denotes the price of good z in period t.

The minimization of expenditure $\int_0^1 p_t\left(z\right) c_t^j\left(z\right) dz$ for a given level of the consumption index (2) leads to the Dixit-Stiglitz demand function for each good z:

$$c_t^j\left(z\right) = C_t^j \left(\frac{p_t\left(z\right)}{P_t}\right)^{-\theta} \tag{4}$$

Equation (4) shows how a given consumption level C_t^j is allocated among the different goods $c_t^j\left(z\right)$ in t. The optimal consumption basket C_t^j for each period t is determined by maximizing the intertemporal utility function (1) subject to the intertemporal budget constraint:

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+i_t}\right)^t P_t C_t^j \leq B_t^j + \sum_{t=0}^{\infty} \left(\frac{1}{1+i_t}\right)^t p_t\left(j\right) y_t\left(j\right) \tag{5}$$

where i_t is the nominal interest rate and B_t^j is the wealth of the household at the start of period t. The left hand is the present value of consumption expenditure while the right hand is the present value of selling the households' own product plus initial wealth.

Solving this gives to the standard Euler-equation (and pricing kernal):

$$\beta \frac{u'\left(C_{t+1}^j\right)}{u'\left(C_t^j\right)} = \frac{1}{1+i_t} \frac{P_{t+1}}{P_t} \tag{6}$$

Assuming a perfect capital market and identical utility functions for all households, equation (6) holds for the whole economy. In a frictionless world identical households would choose identical values for all variables C_t^j etc. Due to sticky price adjustment households differ in income over time. Households can insure themselves against these variations using capital markets, so even with rigidities the path of all variables is equal for all households. Consequently the index j can be omitted from hereon.

Together with a policy rule for the central bank determining the interest rate i_t, equations (4) to (6) represent the demand side of our economy. Turning to the supply side, sticky price adjustment has to be taken into account. In each period a household is able to change the price of its own product with probability $1-\alpha$. After adjusting the price at the beginning of period t, the price will prevail in *t+1* with probability α, in *t+2* with probability α^2 and so on. Each household j sets its price to maximise the present value of future revenues minus the loss of utility stemming from the work necessary to produce the good:

$$\sum_{k=0}^{\infty} \alpha^k \left\{ \Lambda_t E_t \left[\left(\frac{1}{1+i_t} \right)^k p_t(j)\, y_{t+k}(j) \right] - \beta^k E_t \left[v\left(y_{t+k}(j) \right) \right] \right\} \tag{7}$$

Λ_t is household's marginal utility for an additional unit of nominal income in period t. Since the receipts for good j play only an infinitesimal role in the budget constraint (5) marginal utility is assumed to be constant and omitted form the subsequent analysis (Woodford, 1997). The demand function for the good $y_t(j)$ is given by (4) and depends on the price $p_t(j)$. The derivative with respect to $p_t(j)$ yields the first-order condition for the optimal price $p_t^*(j)$:

$$\sum_{k=0}^{\infty} \alpha^k E_t \left\{ \left(\frac{1}{1+i_t} \right)^k C_{t+k} \left(\frac{p_t^*(j)}{P_{t+k}} \right)^{-\theta} \left[p_t^*(j) - \mu S_{t+k,t} \right] \right\} = 0 \tag{8}$$

where $\mu \equiv \frac{\theta}{\theta-1} > 1$ holds. μ denotes a mark-up of prices over marginal costs, with $S_{t+k,t}$.representing the degree of monopoly power. Marginal costs are the cost of producing an additional good in $t+k$, given that the price has been chosen in period t:

$$S_{t+k,t} = \frac{v'\left(C_t \left(\frac{p_t^*(j)}{P_{t+k}} \right)^{-\theta} \right)}{u'\left(C_{t+k} \right)} P_{t+k} \tag{9}$$

The optimal price to be set in period t depends only on expectations of future consumption demand C_{t+k} and the future price level P_{t+k}. Since households are identical they form the same expectations and choose the same optimal price if they are in a position to adjust. Thus the overall price index P_t^* for all prices changed in t equals $p_t^*(j)$. From (3) it follows that the overall price index of the economy is a CES function given by

$$P_t = \left[\alpha P_{t-1}^{1-\theta} + (1-\alpha) P_t^{*\,1-\theta} \right]^{\frac{1}{1-\theta}} \tag{10}$$

In other words, the current price level is a probability-weighted average of last period's prices and the price index of the adjusted prices in t.

It is assumed that the utility function $u(C_t)$ is an iso-elastic function:

$$u(C_t) = \frac{C_t^{1-\sigma}}{1-\sigma} \tag{11}$$

with $\sigma > 0$, the inverse of the intertemporal elasticity of substitution. Since we assume that the economy is closed without government and without private investment the only component of aggregated demand is consumption (Rotemberg and Woodford 1998, p 14). Thus consumption may be replaced by total income, Y. Log-linearising (6) using (11) around the steady state leads to:

$$\hat{Y}_t = E_t \hat{Y}_{t+1} - \frac{1}{\sigma}\left(i_t - E_t \pi_{t+1} - r\right) \tag{12}$$

with π denoting the inflation rate, values denoted with a hat representing percentage deviations from equilibrium and r is the equilibrium real interest rate.

Log-linarizing equations (9) to (11) yields:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa \hat{Y}_t \tag{13}$$

with

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} . \frac{\omega + \sigma^{-1}}{\sigma^{-1}(\omega + \theta)} \tag{14}$$

and $\omega \equiv \frac{v'(Y)}{v''(Y)Y}$. This is the New-Keynesian Phillips curve: deviations of inflation from its equilibrium depend on the level of the output gap and expectations of future inflation.

Equation (13) can also be written as:

$$\hat{P}_t^* = \frac{\kappa\alpha}{1-\alpha} \sum_{k=0}^{\infty} (\alpha\beta)^k E_t \hat{Y}_{t+k} + \sum_{k=1}^{\infty} (\alpha\beta)^k E_t \hat{\pi}_{t+k} \tag{15}$$

Equation (15) states that the deviation $(\hat{P}_t^*)$ of the optimal relative price of firm j, $p_t^*(j)/P_t$, from its equilibrium value depends on the probability weighted expectation of future output gaps $(\hat{Y}_{t+k})$ and the future deviation of inflation from its equilibrium $(\hat{\pi}_{t+k})$[5].

The last remaining building block of the model is the determination of the interest rate. An interest rate rule can also be derived from a microfounded approach[6] but since the main focus of the paper is not central bank behaviour we take a rule that has empirical support. In particular we assume a sort of a Taylor-rule with some degree of interest rate smoothing (see eg Clarida, Galí, Gertler 2000):

$$i_t = (1-\rho)\left(\gamma + b_{\hat{Y}} \hat{Y}_t + b_{\hat{\pi}} \hat{\pi}_t\right) + \rho\, i_{t-1} \tag{16}$$

With the specification of an interest rate the model is closed. The model, the workhorse of modern monetary analysis (McCallum 2001)[7] can be summarised as:

$$E_t \hat{Y}_{t+1} + \frac{1}{\sigma} E_t \pi_{t+1} - \frac{1}{\sigma} i_t = \hat{Y}_t - \varepsilon_t$$

[5] This equation will be important for calculating the behaviour of the single firm later on.

[6] See Corrado and Holly (2007).

[7] Of course there have been numerous extensions to the vanilla version of the NKM since the initial efforts. However, for our purposes it is unnecessary to introduce further complications when our main concern is to examine the cross sectional implications of heterogeneous price and output adjustment over the business cycle.

$$\beta E_t \pi_{t+1} = \pi_t - \kappa \hat{Y}_t - \chi_t \tag{17}$$

$$i_t = (1-\rho)\left(b_{\hat{Y}} \hat{Y}_t + b_\pi \pi_t\right) + \rho\, i_{t-1} + \eta_t$$

For the sake of simplicity, the equilibrium real interest rate is set to zero. If the target inflation rate is also set to zero, this implies an equilibrium nominal interest rate of zero. ε_t, χ_t and η_t are independently, normally distributed error terms with zero mean and time-invariant variance. ε_t can be interpreted as a demand shock. χ_t is the supply shock and. η_t is a monetary shock.

3 Model Solution

The common approach in the literature is to use the symmetric equilibrium features of the model to solve for deviations of the aggregate level of output and inflation from a steady state. However, we are also interested in the cross sectional distribution of price and output adjustment. The problem then is how do we simulate a disaggregated version of the NK model? One way would be to iterate over the population of m firms, aggregate using equations (4) and (10) and determine the expected aggregate price and output and re-iterate[8]. An alternative procedure is to take advantage of the fact the Dixit-Stiglitz expression can be thought of as an exact or perfect aggregator[9]. It follows that if we know the aggregates, the Dixit-Stiglitz expression is also the perfect disaggregator (Grossman and Stiglitz, 1976).

We can write the companion form of the aggregate model as:

$$A_0 z_t = A_1 z_{t-1} + A_2 E_t z_{t+1} + \xi_t \tag{18}$$

where $z_t = (\hat{Y}_t, \hat{\pi}_t, i_t)'$, $\xi_t = (\varepsilon_t, \chi_t, \eta_t)'$, and:

$$A_0 = \begin{bmatrix} 1 & 0 & \frac{1}{\sigma} \\ -\kappa & 1 & 0 \\ -(1-\rho)\, b_{\hat{Y}} & -b_\pi & 1 \end{bmatrix},\ A_1 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \rho \end{bmatrix},\ A_2 = \begin{bmatrix} 1 & \frac{1}{\sigma} & 0 \\ 0 & \beta & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

We then use the algorithm from Stock and Watson (2002) which generalises Blanchard and Kahn (1980) to derive the reduced form as a vector autoregression from which standard impulse functions can be obtained. This makes it possible to simulate the model for n periods resulting in aggregate paths for $\hat{Y}_t$,$\hat{\pi}_t$ and i_t.

In order to derive the price and the output response for each household/firm j in t, $y_t(j)$, we have to disaggregate the aggregate response using equation (15) in which the optimal relative price of the firm j is calculated as a deviation from the optimal relative price. The latter is normalised to 1. To do this the expected future values for $\hat{Y}_t$ and $\hat{\pi}_t$ have to be calculated. Assuming that individuals are rational and shocks have an expected value of zero these values are obtained from the impulse response functions. To take the present state of the economy into account the impulse responses have to be calculated with the period t value of the state variable i_t. After calculating the optimal price in t, we

[8]See also den Haan (1997).

[9]This follows from it being homogeneous of degree one. In the econometrics literature a perfect aggregator denotes a situation in which the average of parameters at the micro level is the same as the parameters estimated at the aggregate level. See Pesaran et al (1989) and references therein.

have to determine if a certain firm changes its price in this period. Therefore for each firm a sequence of length n of realisations q_t from a random variable with uniform distribution $Q_t \sim U(0,1)$ is calculated. For each period t the following expression applies:[10]

$$\frac{p_t(j)}{P_t} = \begin{cases} \hat{P}_t^* + 1 \text{ for } q_t \leq 1-\alpha \\ \frac{p_{t-1}(j)}{P_t} \text{ for } q_t > 1-\alpha \end{cases} \tag{19}$$

After determining the relative price for each firm j, the demand function (5) can be used to calculate output $y_t(j)$. By simulating (17) one obtains the output gap. After normalising the equilibrium output gap to 1 it is possible to calculate real output for each firm[11].

3.1 Calibration

To solve the model parameters have to be calibrated. A first crucial assumption is the value of the parameter α which represents the probability of a firm not changing its price in period t. If $\alpha = 0$ all prices are perfectly flexible and each firm can respond immediately to changes in the aggregate environment. Since all firms are by assumption equal they will choose the same price and will end up with the same output level. The result would be perfect homogeneity. A similar argument can be made for the case $\alpha = 1$. Here all firms cannot change their price. If the parameter lies between the two extremes and, for example a negative shock hits the economy, the demand for all goods decreases. Individual firms will be affected by the shock in different ways according to their ability to change prices. Firms that can change their prices will lower them, leading to a smaller reduction in output compared with firms that have to stick to their initial prices. If a large fraction of firms cannot change its prices (i.e. if α is large) a large fraction of the distribution of firms will end up with relatively lower output while only a small fraction will be able to increase their output relative to other firms. The skewness of the distribution would become positive and therefore counter-cyclical. By contrast if many firms are allowed to change prices the mass of the distribution will have relatively small falls in output while a small fraction of firms who are not able to change prices will suffer heavier output falls. The skewness in this case would be negative and pro-cyclical.

The behaviour of prices will provide a mirror image to the response of output. Hence, the value of parameter α is crucial for the predictions of the model concerning the skewness of the cross-section distribution. In the simulations the sensitivity of the results to different values of α and therefore the degree of price stickiness will be examined.

Plausible settings for α may be derived from studies of the average time period between two price changes, which is given by $\frac{1}{1-\alpha}$ (Rotemberg and Woodford 1998: 22). Unfortunately, several studies came up with quite different results[12] ranging from 4 to 30 months implying values for α ranging from 0.03 to 0.67 on an annual base and values between 0.125 and 0.9 on a quarterly basis.

For the other parameters in the model, the literature also uses values within a certain range. To make our results robust we therefore consider three scenarios: one is a baseline

[10] Note that the addition of 1 in the upper part of the expression stems from the normalisation of the equilibrium relative price to 1.

[11] Given that we are using a linearisation of the model it is the case that the sum of each firm's price and output response is not quite equal to the aggregate response calculated from equation (15). However, the difference is very small.

[12] See, e.g., Cecchetti (1986), Carlton (1986), Blinder (1994) and Kashyap (1995).

scenario with parameters taken from Woodford (1997), a second follows Rotemberg and Woodford (1998) and a third uses the settings of McCallum (2001). An overview is given in Table 1.

Table 1: Parameter Values used in Calibration.

Parameter	Baseline-scenario	Rotemberg and Woodford	McCallum
β	0.95	0.99	0.99
θ	19.97	7.88	9.03
σ	1	6.25	2.5
ω	1.66	0.47	0.23

The base-line scenario has the advantage of using parameters for annual data which will be used in the empirical analysis in the next section. Unfortunately the values for θ and ω are not given in Woodford (1997) and have to be inferred by methods used in Rotemberg and Woodford (1998) in the case of θ or plausible values have to be assumed as it is the case for ω. The values for the Rotemberg and Woodford scenario are all estimated but only for quarterly data. They admit that the value for $\sigma$is too high to be plausible. We therefore simulate the results with a more plausible value given by McCallum(2001). Here again the values for θ and ω are not given and have to be inferred. Since none of the parameter settings is entirely satisfactory we use all three to check for the robustness of the results.

Finally the parameters of the interest rate rule have to be specified. Parameters provided by Clarida, Galí and Gertler (1998) are used[13].The parameters of the interest rate rule are set to $b_{\hat{Y}} = 0.25$, b_π =1.3 and $\rho = 0.9$.

4 Simulation results

With the calibrated parameters at hand, we simulate the model for 1000 firms and 1000 periods. We then report results based only on the middle 800 periods of the simulation to make sure that the results are independent of the starting values. Table 2 summarises the standard deviation and the coefficient for the first order autocorrelation for the output gap in all three scenarios and the output gap for annual German GDP[14]. The time series for GDP starts in 1971 and ends in 2004. The output gap was obtained using a Hodrick-Prescott-Filter with lambda set to 6.25 (Ravn and Uhlig 2002).

The theoretical autocorrelation is higher in the baseline and McCallum scenarios than the actual autocorrelation. The standard deviation for the McCallum, as well as our baseline scenario, is also too high. The Rotemberg and Woodford parameterisation leads to the best fit with the actual results. Given the simple structure of the model the weak fit for two of the scenarios should not be treated too seriously.

[13] Note, however, that these settings are broadly in line with the empirical results of Gerberding, Seitz and Worms (2007).The authors show, that Bundesbank policy can best be described in terms of a money supply rule, but that this rule may be transformed into a interest rate rule with some degree of interest rate smoothing.

[14] GDP is annual real GDP for Germany adjusted for the reunification by calculating backward GDP for the unified Germany in 1991 with West-German growth rates (Sachverständigenrat 2001, p 252).

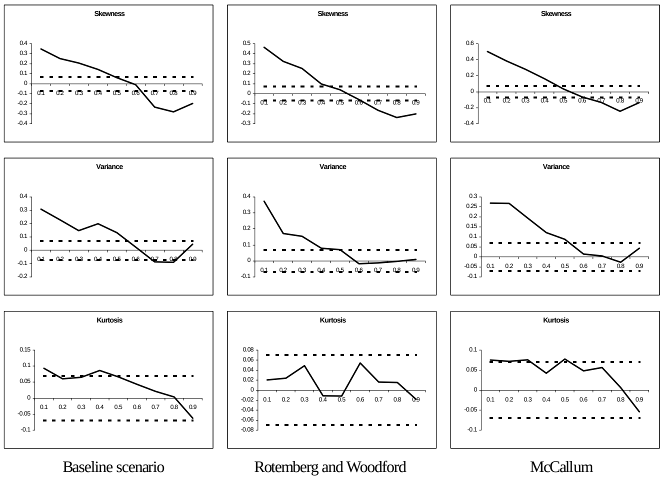


Figure 1: **Correlation of simulated aggregate output growth with the cross section moments for different values of α.**

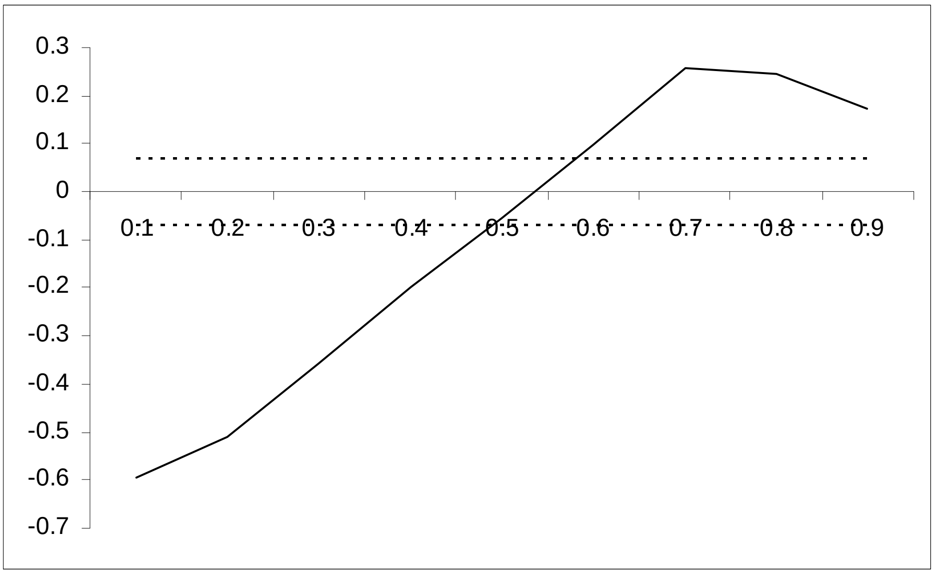
Table 2: Empirical and theoretical standard deviations and autocorrelation.

	output gap	Baseline			Rotemberg and Woodford			McCallum		
α		0.2	0.5	0.8	0.2	0.5	0.8	0.2	0.5	0.8
SD	1.1	4.22	6.21	7.74	1.16	1.86	2.6	2.65	4.78	4.11
AC	0.34	0.53	0.67	0.86	0.33	0.43	0.85	0.54	0.8	0.72

In Figure 1 the results for all three scenarios are summarised. The left column of graphs shows the results for the baseline scenario, the middle column is for the Rotemberg/Woodford values and the right column includes the results for the McCallum scenario. The first graph in each column shows the correlation coefficient (y-axis) of the variance with the growth rate of output for different values of α (x-axis). The second graph shows the correlation coefficient for skewness and the last one for kurtosis. The dotted lines are a confidence band according to the rule-of-thumb for a correlation coefficient significantly different from zero ($2/\sqrt{T}$).

It becomes apparent that for low values of α the moments are markedly pro-cyclical. For higher values of α – about 0.6 or higher – skewness becomes markedly counter-cyclical. For α values of 0.7 or higher the skewness is counter-cyclical while the correlation with variance is negative but not significant. The same holds for kurtosis. When the proportion of firms that can change their prices is similar to to those who cannot (around $\alpha = 0.5$) there is an insignificant correlation between skewness and aggregate growth. The relationship between skewness in the cross section and the aggregate economy may be able, indirectly, to tell us something about the extent of price stickiness at the level of the firm.

As a first robustness check the results are reproduced based on the parameter values from Rotemberg and Woodford (1998). Again, for low values of α the results indicate a non-cyclical or a weak pro-cyclical behaviour of the cross-section moments. With increasing α-values in particular the correlation coefficient for skewness becomes more and



more counter-cyclical as can be seen in the middle column of figure 1. This result is based on parameters obtained from quarterly data therefore the α-values necessary for a counter-cyclical skewness are much more likely.

As a second robustness check the parameter values according to McCallum are applied. Again for parameter values for α of 0.7 or higher skewness becomes pronounced counter-cyclical. The results for variance however are different to the other two scenarios: here the correlation is positive and for $\alpha > 0.6$ even significantly positive. Since these results are for quarterly parameters the counter-cyclical skewness is obtained for plausible α-values.

Price stickiness in the model arises from both the Calvo pricing rule and from the presence of imperfectly competitive firms. Figure 2 shows how the correlation between the aggregate economy and skewness varies (for α fixed at 0.7) with the degree of imperfect competition (captured by θ). As θ rises and competition increases the relationship between skewness and the aggregate economy, as expected, disappears.

The counterpart to the output decision is also the pricing decision. Figure 3 shows the correlation between the aggregate economy and skewness for different values of α. For values of α greater than 0.2 there is a significant positive relationship between skewness and the aggregate economy[15].

An otherwise standard New Keynesian model with Calvo pricing, where the only form of heterogeneity lies in whether or not a firm changes its prices, has the interesting property that there will be a relationship between the cross sectional moments of inflation and output and the aggregate economy and this relationship varies with the degree of price stickiness in a Calvo economy. This suggests that an indirect test of the degree of price stickiness can be derived from an analysis of the cross section of firms and the aggregate economy. To this we turn in the next section.

[15] Although we do not report the results there is also a significant relationship between inflation skewness and aggregate inflation. Ball and Mankiw (1995) find a similar positive relationship using a menu cost model.

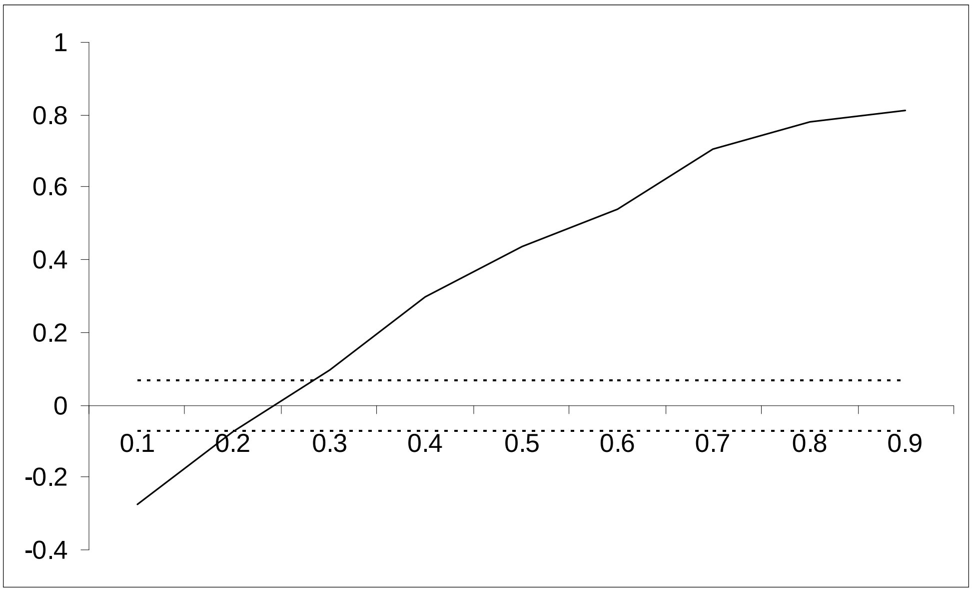


Figure 2: **Correlation between Aggregate growth and skewness for different values of α.**

5 The cross-sectional dynamics of the German business cycle: some empirical evidence

In this section we turn to an empirical examination using data for German companies[16]. We use the Bundesbank's unbalanced corporate balance sheets statistics database (*Unternehmesbilanzstatistik*, UBS for short). This is the largest database of non-financial firms in Germany. The Bundesbank has collected the data when offering rediscounting and lending operations on a strictly confidential basis. To enable the Bundesbank to carry out an extensive evaluation of their creditworthiness, the enterprises submitted their annual accounts to the branch offices of the German State Central Banks (*Landeszentralbanken*). They were then recorded electronically, audited, and evaluated for purposes of trade bill transactions. The Bundesbank received around 60,000 annual accounts per annum. In addition, the Bundesbank performed checks for logical errors and missing data in the database as well as consistency checks and error corrections. According to Stoess (2001), the unbalanced panel dataset comprises only about 4% of the total number of enterprises in Germany but about 60% of the total turnover of the corporate sector. Another key advantage of the database is that it comprises both incorporated and unincorporated firms. This has some appeal since the small and medium-sized firms in Germany ("*Mittelständische Wirtschaft*") show up in our sample[17]. Our micro database therefore gives a faithful representation of the German economy. In contrast to previous

[16] Bryan and Cecchetti (1999) have drawn attention to a potentially serious problem associated with trying to establish a possible relationship between the mean and the skewness of a sampling distribution using panel data. However, given the size of the dataset this bias is unlikely to be significant. For an approach that uses a random cross section sample split see Gerlach and Kuglar (2007).

[17] More than 80% of the included enterprises are small and medium-sized enterprises (SME's) with an annual turnover less than 100 million DM, and more than half of the dataset consists of unincorporated firms.

studies, we were able to use data from 1971 to 1998 for most of the analysis[18]. Even though the number of rediscount lending operations dropped sharply with the start of European Monetary Union at the beginning of 1999, the Bundesbank tries to continue its comprehensive review of the credit standing of German enterprises involved in rediscount transactions. However, eligible enterprises now submit their balance sheets to the European Central Bank. This change of competence is the reason why 1998 is the last year of the period covered.

Although the theoretical and simulation part of the paper was concerned with both output and inflation, data on the price setting of individual firms is not available. Since we are mainly interested in the development of real sales[19] we have relatively few data losses owing to incomplete and inconsistent reporting. Real sales growth is calculated for each firm by deflating the firms' sales with the deflator of real GDP and afterwards taking the difference of the logarithm of real sales.[20] Following Higson et al. (2002, 2004), we take into account outliers by winsorizing the data, that is, by employing several cut-off rates, ie a fraction of, say a $\pm 50\%$ growth rate, is truncated from the data. Some kind of cut-off seems to be necessary as some changes in real sales might be influenced, for example, by mergers. It is clear that a cut-off is a rather crude method to get rid of outliers and mergers. Unfortunately, no variable was included in the dataset to indicate whether a merger had occurred or not[21]. This has the advantage of not being too restrictive while getting rid of most of the outliers and a lot of the mergers.[22] In Table 3 some summary statistics for the dataset using a +/- 50% cut-off are presented.
In Figure 3 the moments of the annual growth rate of real sales (line) are shown together with the growth rate of real GDP (bar) as a measure for business cycle conditions resembling Figure 2.

[18] We thank the Statistics Department of the Deutsche Bundesbank, in particular Tim Körting, for excellent research assistance.

[19] The analysis is in terms of output rather than sales. Gabaix (2005) using the results of Hulten (1978) establishes an equivalance between (value added) output and firm sales.

[20] One might argue that each sector should be deflated with its respective deflator. With only a few exceptions, e.g. computer manufacturing, the sectoral deflators all move closely together so that the GDP-deflator appears to be a good approximation.

[21] The cut-off may also eliminate some newly formed firms as well as firms going bankrupt. Note that numerous other studies suffer from similar problems.

[22] We also did several robustness checks using different cut-offs and checked the dropped observations for some form of regularity which was not the case. For a more thorough discussion see the working paper version of this paper Doepke et al. (2005).

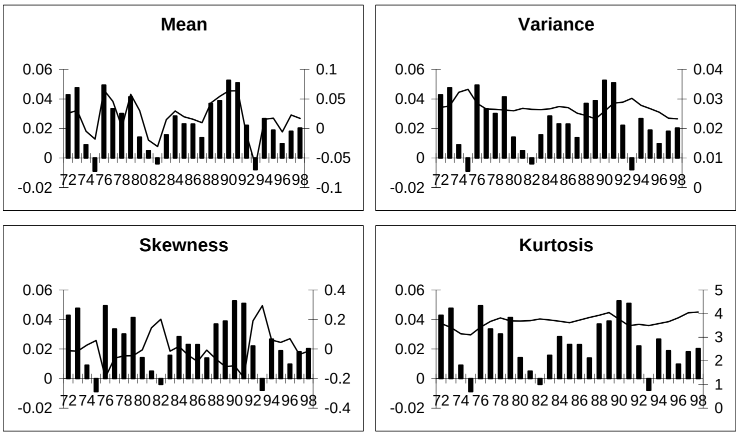


Figure 3: **Moments of truncated (fifty percent) growth cross-section distribution against GDP growth, 1973-1998.**

Table 4 provides regressions of the empirical moments (m) on up to 2 lags of the moments, and current and lagged aggregate output growth. The central moments are very strongly correlated with the aggregate economy. However, the variance is only weakly correlated with lagged output growth. By contrast, and in accordance with the predictions of the model of this paper, skewness is strongly counter-cyclical, suggesting considerable price stickiness at the micro level. Again, consistent with the model, kurtosis is independent of the aggregate business cycle[23].

5.1 Evidence from US Sectoral Data

Although we find a very significant relationship between the skewness of output and the aggregate economy with firm-level data we do not have price data for firms. In this section we use the NBER-CES Manufacturing Industry Database[24]. The database covers all 4-digit manufacturing industries from 1958 to 1996 ordered by 1987 SIC codes, with a total of 459 industries[25]. We take value-added and deflate by the price deflator for shipments. We then compute skewness in each year for the change in real value-added and the change in prices. The results are shown in Table 5. We use two measures of the aggregate economy. We take the median growth rate of the 459 industries in each year. We also use the rate of growth of GDP[26]. The second and third columns report regressions of the skewness of inflation and output against current and lagged values of the median growth rate. The fourth and fifth columns report the same regressions but using GDP. For the median growth rate we find a significant positive effect of (lagged) growth on the skewness of inflation, as suggested by the earlier model. We also find a positive effect when we use GDP but it is now only significant at the 10 per cent level. For the skewness of output the results are much less supportive and appear to contradict

[23]By contrast in the menu costs model of Ball and Mankiw (1994) both varance and skewness should be significant.

[24]For a description of the data see Bartelsman, Becker, and Gray (1996). "NBER-CES Manufacturing Industry Database". Available at http://www.nber.org/nberces/nbprod96.htm.

[25]As in other studies we exclude the "Asbestos Product" industry (SIC 3292) because the time series ends in 1993.

[26]This is taken from the St Louis database. Series GDPCA from the U.S. Department of Commerce: Bureau of Economic Analysis, in Billions of Chained 2000 Dollars

the findings for Germany (and also for firm-level data for the US, the UK and Italy. See Holly and Santoro (2008) for a summary of this empirical evidence).

6 Conclusions

In this paper we have presented a simplified version of Woodford (1997) and Rotemberg and Woodford (1998) using the DSGE framework to simulate the implications for the cross sectional distribution of firms implicitly embedded in the model because of the heterogeneity that arises from Calvo pricing. In each period not all firms are able to change their prices. This property generates a correlation between the skewness of the cross section of price changes and aggregate inflation, and a correlation between the skewness of output changes and the aggregate growth of the economy. The sign of these correlations depends on the proportion of firms that are able to alter prices. For large values of α, where $1-\alpha$ of firms change prices in any period, the correlation with skewness is positive for price changes and negative for output. Moreover, there is no correlation with the variance or the kurtosis of the cross section distribution. These theoretical results go some way to explaining the 'stylised facts' that have been previously identified by Higson et al (2002, 2004) for the US and the UK.

Using a considerably larger dataset for Germany, comprising an average of almost 54,000 firms in each of 27 years, we are able to largely confirm the predictions of the Calvo model (at least for output changes). There is a significant relationship between the skewness of output changes and the aggregate economy. Turning to US sectoral data for 39 years, our results are more mixed. We find - as predicted - a significant positive relationship between the skewness of inflation and output. However, in contrast to the firm level data we are not able to detect a significant negative relationship between the skewness of output and the aggregate economy. Our paper, in essence, offers an indirect way of testing for the causes of frictions that generate price stickiness.

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Table 3: Summary statistics for growth rates of real sales: fifty percent cut-off

Year	Mean	Median	Standard deviation	Skewness	Kurtosis	Obs.
1972	0.026	0.023	0.16	-0.01	3.59	29,319
1973	0.030	0.029	0.17	-0.02	3.42	30,965
1974	-0.005	-0.006	0.18	0.03	3.14	32,987
1975	-0.018	-0.018	0.18	0.06	3.10	37,561
1976	0.065	0.065	0.17	-0.19	3.43	46,596
1977	0.046	0.042	0.16	-0.06	3.67	54,902
1978	0.002	0.002	0.16	-0.05	3.82	61,136
1979	0.058	0.052	0.16	-0.05	3.70	65,630
1980	0.030	0.028	0.16	-0.01	3.69	65,006
1981	-0.020	-0.022	0.16	0.14	3.70	59,974
1982	-0.030	-0.034	0.16	0.20	3.78	60,368
1983	0.015	0.012	0.16	-0.02	3.74	61,871
1984	0.030	0.023	0.16	0.02	3.68	63,408
1985	0.019	0.017	0.17	-0.04	3.62	63,322
1986	0.016	0.014	0.16	-0.08	3.73	63,263
1987	0.010	0.008	0.16	-0.01	3.85	62,059
1988	0.044	0.040	0.16	-0.07	3.94	61,243
1989	0.055	0.051	0.15	-0.12	4.05	59,427
1990	0.064	0.058	0.16	-0.11	3.77	56,991
1991	0.064	0.063	0.17	-0.19	3.50	55,415
1992	-0.011	-0.018	0.17	0.19	3.55	55,218
1993	-0.063	-0.064	0.17	0.29	3.50	55,334
1994	0.016	0.009	0.17	0.06	3.58	55,570
1995	0.017	0.011	0.16	0.04	3.67	55,804
1996	-0.006	-0.009	0.16	0.07	3.83	53,299
1997	0.023	0.018	0.15	-0.04	4.04	49,620
1998	0.017	0.013	0.15	-0.01	4.07	38,796
All	0.019	0.016	0.17	-0.01	3.57	1,455,084

Table 4: Regression of cross section moments on aggregate growth

	Mean	Median	Variance	Skewness	Kurtosis
50 % Cut-off					
Constant	-0.012 (0.021)	-0.015 (0.007)	0.009 (0.001)	0.108 (0.003)	1.407 (0.000)
m_{t-1}	0.070 (0.211)	0.078 (0.223)	1.119 (0.000)	0.087 (0.551)	1.269 (0.000)
m_{t-2}	-0.242 (0.008)	-0.218 (0.021)	-0.485 (0.000)	-0.112 (0.324)	-0.649 (0.000)
y_t	1.556 (0.000)	1.531 (0.000)	-0.031 (0.084)	-5.504 (0.00)	-0.005 (0.979)
y_{t-1}	(-)	(-)	0.049 (0.014)	(-)	(-)
Adj. R^2	0.875	0.858	0.706	0.682	0.735
LM(1)	0.369	0.261	0.360	0.244	0.549
LM(2)	0.649	0.252	0.479	0.135	0.296
het	0.237	0.328	0.167	0.092	0.748
(RESET)	0.927	0.759	0.480	0.505	0.676
Test for $y_t = y_{t-1} = 0$	(-)	(-)	0.028	(-)	(-)
Test for normality	0.885	0.823	0.730	0.301	0.418

Notes: For all tests p-values are reported. The p-values for the t-tests (in brackets) are based on a robust covariance matrix calculated using the Newey and West method. The test for autocorrelation is a Breusch/Godfrey test, the test for heteroskedasticity is a White test, the RESET test is a Ramsey test for functional form. The test for normality is the Jarque/Bera test.

	Inflation skewness	Growth Skewness	Inflation skewness	Growth Skewness
	Median		GDP	
Constant	-0.917 [0.009]	-0.703 [0.0065]	-1.516 [0.0017]	-1.565 [0.0043]
y_t	3.676 [0.417]	6.848 [0.126]	5.360 [0.673]	15.487 [0.118]
y_{t-1}	16.736 [0.001]	-4.819 [0.179]	26.550 [0.0966]	-8.762 [0.386]
y_{t-2}	4.814 [0.221]	9.106 [0.0689]	6.890 [0.516]	28.053 [0.0162]
Adj. R^2	0.226	0.148	0.060	0.193
LM(1)	0.085	0.823	0.068	0.823
LM(2)	0.174	0.218	0.080	0.219
het	0.106	0.261	0.138	0.261
RESET	0.922	0.501	0.172	0.465
Test for normality	0.603	0.122	0.778	0.122

Table 5: Regression of Skewness in output and prices on aggregate economy.

Notes: see Table 4.