Business Cycle Effects of Credit and Technology Shocks in a DSGE Model with Firm Defaults

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5 October 2011

CWPE 1159

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October 6, 2011

Abstract

This paper proposes a theoretical framework to analyze the impacts of credit and technology shocks on business cycle dynamics, where firms rely on banks and households for capital financing. Firms are identical *ex ante* but differ *ex post* due to different realizations of firm specific technology shocks, possibly leading to default by some firms. The paper advances a new modelling approach for the analysis of financial intermediation and firm defaults that takes account of the financial implications of such defaults for both households and banks. Results from a calibrated version of the model highlights the role of financial institutions in the transmission of credit and technology shocks to the real economy. A positive credit shock, defined as a rise in the loan to deposit ratio, increases output, consumption, hours and productivity, and reduces the spread between loan and deposit rates. The effects of the credit shock tend to be highly persistent even without price rigidities and habit persistence in consumption behaviour.

Keywords: Bank Credit, Financial Intermediation, Firm Heterogeneity and Defaults, Interest Rate Spread, Real Financial Linkages.

JEL Classification: E32, E44, G21.

*The authors would like to thank Oliver de Groot, Dapeng Gu, Sergejs Saksonovs and Anna Watson for useful discussions.

1 Introduction

The recent financial crisis and the ensuing economic recession have highlighted the importance of inter-linkages between financial markets and the real economy, in particular the role that private credit plays in the transmission of real and financial shocks. Empirical evidence suggests that bank credit plays an important role in explaining business cycle dynamics, in particular output growth, inflation and interest rates in advanced economies since the late 1970s. As shown in Helbling, Huidrom, Kose, and Otrok (2011) and Xu (2010), a negative shock to US real credit has significant adverse effects on output and interest rates in the US, as well as in other advanced economies such as the Euro Area and the UK.

Over the past two decades, there have been important advances in the theoretical literature on the macroeconomic impact of financial frictions. Among others see Kiyotaki and Moore (1997), Bernanke, Gertler, and Gilchrist (1999) and more recently Christiano, Motto, and Rostagno (2010), Carlstrom, Fuerst, and Paustian (2010), Curdia and Woodford (2010) and Gertler and Kiyotaki (2010). By introducing credit market frictions (due to information asymmetry, agency costs or collateral constraints) in the demand and supply for credit, research on the credit and cost channels of monetary policy show that such frictions act as a financial accelerator that leads to an amplification of business cycles, and highlight the mechanisms through which credit market conditions are likely to impact the real economy. In addition, recent literature on modelling the banking sector sheds light on the relationship between bank lending and investment decisions by firms and how credit risks relate to the pricing of bank loans. See, for example, Freixas and Rochet (2008) and Pesaran, Schuermann, Treutler, and Weiner (2006).

The existing literature on financial frictions is largely monetary in nature and is mainly aimed at obtaining a better understanding of the transmission mechanisms for monetary policy shocks. They are motivated by the limitations of traditional demand side monetary models in matching VAR-based empirical evidence on the effects of monetary policy shocks. See, for example Bernanke and Gertler (1995), Bernanke, Gertler, and Gilchrist (1999). Recently, a number of papers have developed models with financial frictions to investigate the effects of unconventional monetary policy such as direct lending by central banks, as observed in the 2008 financial crisis. Notable examples are Christiano, Motto, and Rostagno (2010) and Gertler and Karadi (2011). In this paper we have a different focus and examine the impact of credit shocks on business cycle dynamics, by explicitly allowing for firm defaults, which distinguishes our contribution from the related literature that impose collateral constraints, and hence rule out default in equilibrium. See, for example, Kiyotaki and Moore (1997), Gertler and Kiyotaki (2010) and Carlstrom, Fuerst, and Paustian (2010). Collateral constraints are introduced as a way of ensuring that borrowers can re-pay their debts, which in most cases rule out the possibility of firm defaults almost by design.

Our paper is more closely related to the work by Freixas and Rochet (2008) on the microeconomics of banking, and Bernanke, Gertler, and Gilchrist (1999, BGG), on modelling the producer sector and firm defaults.[1] Our aim is to develop a relatively *parsimonious* theoretical model for the analysis of the impact of credit and technology shocks on the real economy, making a distinction between idiosyncratic and common technology shocks. The proposed framework comprises a large

[1]Other related papers that allow for the possibility of firm defaults include Fiore and Tristani (2009) and Christiano, Motto, and Rostagno (2010).

number of firms, a representative household, and a banking sector that operates competitively. Firms are identical *ex ante* and live from period to period. At the start of each period, firms enter the market and decide on the optimal levels of labour input and capital stock for their operation. Firms receive initial funds from the household sector which can be interpreted as private equity investment from the household, and augment these funds by borrowing from the banking sector. These financing arrangements are made *prior* to the realisation of idiosyncratic and common technology shocks. Some firms may default if the realised technology shocks are unfavourable, such that the firm's revenue is not sufficient to repay the bank loan. We assume that the product market is competitive and while some firms may fail each period, entry is free. The banking sector receives deposits from households before the arrival of a credit shock, which then determines the total level of loanable funds to the firms. The banking sector receives loan repayment from the non-defaulted firms and seizes the revenues of defaulted firms (if any) to partially cover losses. The equilibrium loan rate is in turn affected by the economy wide default probability.

The main contribution of the paper in relation to the literature is fourfold: first, we advance a new modelling framework for the analysis of financial intermediation and firm defaults that take account of the financial implication of such defaults for both households and banks, without using collateral constraints and monitoring. Our modelling of firm defaults differs from BGG in that idiosyncratic shocks affect productivity rather than the return on capital in the economy, which keeps the model tractable and establishes a direct link between credit risk and productivity. The timing of labour and capital decisions and the fact that firms are subject to idiosyncratic technology shocks are essential for modelling firm defaults in equilibrium. Second, we consider the impact of exogenous (but possibly persistent) credit shocks and examine the quantitative importance of such shocks for business cycle fluctuations. A positive credit shock can be viewed as a sudden increase in the level of bank loans relative to bank deposits in the economy, probably due to an increase in liquidity provision by the banking sector or the central bank. We are able to generate theoretical impulse responses that are in line with empirical results on the responses of output and short term interest rates to a US credit shock. Third, the paper contributes to the analysis of steady states in model economies that take account of non-linearities and possible unit roots in the economy wide technology process. Finally, by allowing for a unit root in labour hours and by incorporating a form of non-separable utility function popularised by Greenwood, Hercowitz, and Huffman (1988), the model generates responses in hours and productivity to technology shocks that are consistent with empirical evidence.

The main findings of our paper are as follows. First, a positive credit shock implies an increase in the level of bank loans relative to deposits. The rise in the level of loans leads to an increase in the available capital in the economy and consequently an expansion in investment and output, which is largely consistent with the empirical results in Xu (2010). The increase in the level of loanable funds also drives down the loan rate and the spread between loan and deposit rates. Deposit rate rises following the credit shock, which yields an increase in the level of household deposits with banks, consistent with the zero normal profit condition assumed for the banking sector. Labour hours rises on impact, which lead to higher household income and consequently more consumption. The model also predicts an increase in productivity, since the rise in output is found to be larger compared with that in labour hours.

Second, a positive technology shock increases both the loan and deposit rates, but has no effects on the spread between the two rates. Consumption and output rise initially, and we observe a correction before the economy returns to equilibrium after around 5 quarters. The impulse responses obtained from our model match the empirical evidence that positive technology shocks lead to short-run declines in hours and a rise in productivity (see for example Gali, 1999, Francis and Ramey, 2005 and Canova, Lopez-Salido, and Michelacci, 2010). Using our model we are able to generate responses in hours that are in line with the empirical evidence without introducing sticky price (Gali, 1999) or habit formation in consumption and adjustment cost in investment (Francis and Ramey, 2005 and Smets and Wouters, 2007).

Finally, our calibrated results show that the speed of convergence to equilibrium is four times faster for the technology shock as compared to the credit shock, while the impact of a credit shock on output is twice as large as that of a technology shock, under a benchmark parametrisation. Our finding is consistent with empirical studies on the output effect of financial crisis, which suggest that recessions associated with financial crises have been more severe and more long lasting than recessions associated with other shocks (see, for example, IMF, World Economic Outlook, April 2009, Chapter 3). The prolonged impact of the credit shock also reflects the high persistence in the loan to deposit ratio that we observe empirically.

The rest of the paper is organised as follows. Section 2 provides a review of the relevant literature. Section 3 presents the DSGE model of credit, leverage and default. Section 4 sets out the first order equilibrium conditions, derives the steady states, and the solution of the model. Section 5 discusses the parameterisation of the model for the calibration exercises. Section 6 provides the key results on the impulse responses of positive credit and technology shocks. Section 7 offers some concluding remarks.

2 Literature Review and Motivation

In this section, we provide an overview of two aspects of macroeconomic literature that relates to our contributions, namely modelling of financial intermediation and the effects of technology shocks on hours and labour productivity. We start with an overview of models on financial intermediation that focus on the demand and supply of credit and highlight the relationship between financial frictions and monetary policy. We then review alternative approaches in the literature on modelling credit risk and firm defaults. Finally, to motivate our results on the effects of technology shocks, we also provide an overview of the recent debate on the effects of technology shocks on hours and productivity.

2.1 Modelling of financial frictions

Over the past decade, there has been significant advances in the theoretical literature on the macroeconomic implications of financial imperfections. These advances are partly motivated by the limitations of traditional monetary policy channel in explaining the time series empirical evidence on the effects of monetary policy on the economy.[2] This has led to a search for alternative theories.

[2]For example, Bernanke and Gertler (1995) argue that it is difficult to explain the magnitude, timing and composition of the economy's response to monetary policy shocks solely in terms of conventional interest rate (neoclassical cost of capital) effects. Empirically, the interest rate spike associated with an unanticipated monetary tightening is largely transitory, yet some important components of spending do not begin to react until after most of the interest rate effect has past.

One line of the research focuses on the credit channels of monetary policy and examines the extent to which imperfect information and other financial frictions in credit market affect the transmission of monetary policy.[3] Financial market imperfections could be due to a number of factors: first, the asymmetry of information between lenders and borrowers (see, for example, Bernanke and Gertler, 1995, Bernanke, Gertler, and Gilchrist, 1999 and Gilchrist, 2004), which induces the lenders to engage in costly monitoring activities.[4] The extra cost of monitoring by lenders gives rise to the external finance premium for the firms, which represents a wedge between a firm's own opportunity cost of funds and the cost of external finance. Higher asset prices improve firm balance sheets, reduce the external finance premium, increase borrowing and stimulate investment spending. The rise in investment further increases asset prices and net worth, giving rise to an amplified impact on investment and output in the economy. Financial frictions act as a financial accelerator that leads to an amplification of business cycle fluctuations, working their effects through a "credit channel" of monetary policy.

Financial frictions could also stem from the lending collateral constraints faced by borrowers. Examples of this line of research can be found in Kiyotaki and Moore (1997), Carlstrom, Fuerst, and Paustian (2010), and Gertler and Kiyotaki (2010). Credit constraints arise because lenders cannot force borrowers to repay their debts unless the debts are secured by some form of collateral. Borrowers' credit limits are affected by the prices of the collateralized assets, and these asset prices are in turn influenced by the size of the credit limits, which affects investment and demand for assets in the economy. The dynamic interaction between borrowing limits and the price of assets amplifies the impact of a small initial shock and generate large and persistent fluctuations in output and asset prices in the economy.

In addition to frictions in the *demand* for credit from firms, a number of recent papers argue that banks themselves are also subject to frictions in raising loanable funds and show that the *supply* side of the credit market also contributes to shock propagation, affecting output dynamics in the economy. In Meh and Moran (2010), moral hazard arises as the monitoring activities of banks are not publicly observable. Depositors are concerned that banks may not monitor entrepreneurs adequately and demand that banks invest their own net worth (bank capital) in the financing of entrepreneurial projects. Therefore, the capital position of banks affects their ability to attract loanable funds. The extra financial friction between banks and their depositors constrain the supply of credit and hence the leverage of entrepreneurs in the economy. The "bank capital channel" propagates a negative technology shock through a reduction in the profitability of bank lending, making it more difficult for banks to attract loanable funds. Banks are forced to finance a larger proportion of capital investments using their own capital, and reduce bank lending, since bank capital mostly consists of retained earning and can not adjust immediately. Reduced bank lending in turn lead to a fall in investment and economic activity.[5]

Several papers argue that the degree of competition in the banking sector, or banks' rate setting

[3] According to Bernanke and Gertler (1995), the credit channel is not considered as a distinct, free-standing alternative to the traditional monetary transmission mechanism, but rather a set of factors that amplify and propagate conventional interest rate effects.

[4] For example, costly state verification, in which lenders must pay a fixed "auditing cost" in order to observe an individual borrower's realised return, first introduced in Townsend (1979) and further developed in Bernanke, Gertler, and Gilchrist (1999).

[5] Other papers using a similar approach include Chen (2001), Meh and Moran (2004) and Aikman and Paustian (2006).

strategies contribute to frictions on the supply side of credit markets, which are also important in explaining macroeconomic fluctuations. Gerali, Neri, Sessa, and Signoretti (2010) model an imperfectly competitive banking sector that enjoy some degree of market power in loan and deposit markets and set different rates for households and firms. Based on Bayesian estimation using euro area data, they find that banking sector attenuates the effects of monetary policy shocks, as sticky rates moderate the impact of changes in the policy rate on both consumption and investment. On the other hand, financial intermediation increases the propagation of supply shocks originating in credit markets, which is linked to asset prices and borrowers' balance sheet conditions. In a related paper, Hulsewig, Mayer, and Wollmershaeuser (2006) study the role of banks via the "cost channels" of monetary policy and assume that banks extend loans to firms in an environment of monopolistic competition by setting the loan rate according to a Calvo-type staggered price setting mechanism.[6] These authors find that frictions in the loan market influence the propagation of monetary policy shocks as the pass-through of a change in the money market rate to the loan rate is incomplete. However, the strength of the cost channel is mitigated as banks shelter firms from monetary policy shocks by smoothing the lending rates.[7]

In addition to the above channels of monetary policy, Adrian and Shin (2009) propose that the balance sheet of financial intermediaries also contribute to a "risk-taking channel", involving bank's net interest margin, defined as the difference between the total interest income on the asset side and the interest expense on the liabilities side of bank's balance sheet. A rise in the net interest margin (due to changes in policy rate) raises the profitability of bank lending and increases the present value of bank income, therefore boosting the forward looking measures of bank capital. As banks expand their balance sheets, the market price of risk falls and the supply of credit increases. As a result, financial intermediaries drive the financial cycle and impact the real economy through their influence on the determination of the price of risk.

A number of recent papers also develop quantitative models to explore the effects of unconventional monetary policy instruments such as direct lending by central banks, to capture the policy responses following the financial crisis of 2008. See, for example, Gertler and Kiyotaki (2010) and Gertler and Karadi (2011). Gertler and Karadi (2011) interpret unconventional monetary policy as expanding central bank credit intermediation to offset a disruption of private financial intermediation. They model unconventional monetary policy by allowing the central bank to act as intermediary by borrowing funds from savers and then lending them to investors. The central bank is distinct from private intermediaries (commercial banks) in two aspects. First, the central bank does not face constraints on its leverage ratio. Second, public intermediation is likely to be less efficient than the private intermediation. Their findings suggest that during a financial crisis like the recent one, the balance sheet constraints on private intermediaries tighten, raising the benefits and needs from central bank intermediation. In a related paper, Gertler and Kiyotaki (2010) consider two additional unconventional monetary policy instruments, including discount window lending to banks secured by private credit and direct assistance to large financial institutions including equity

[6] "Cost channels" of monetary policy captures the impact of interest rates and credit conditions on firms' short run ability to produce (by investing in working capital). See, for example, Barth and Ramey (2000), Christiano, Eichenbaum, and Evans (2005), Chowdhury, Hoffmann, and Schabert (2006) and Ravenna and Walsh (2006).

[7] One explanation for the imperfection in the loan market is the existence of long term relationships between banks and customers, which are typical for a bank-based financial system as opposed to a market-based financial system, see for example Fried and Howitt (1980) and Berger and Udell (1992).

injections and examine their impact during crisis period.[8]

As this short review suggests, the existing literature on financial frictions and credit markets is largely monetary in nature and is motivated to examine the impact of monetary policy on macroeconomic fluctuations. However, as noted already, our focus is on the impact of credit shocks on business cycle dynamics, and the role of bank credit in the transmission of technology shocks, allowing for the possibility of firm defaults in the economy. One way to classify credit risks is to make the distinction between microeconomic or idiosyncratic risks, which can be diversified away through the law of large numbers, and macroeconomic or systematic risks, which cannot. Banks generally have to deal with both types of risks. Freixas and Rochet (2008) argue that defining and measuring credit risk is equivalent to determining how the market evaluates the probability of default by a particular borrower, taking into account all the possibilities of diversification and hedging provided by financial markets.

Our analysis is closely related to the modelling approaches of BGG and Christiano, Motto, and Rostagno (2010).[9] In BGG, entrepreneurial loans are risky and returns on the underlying investments are subject to idiosyncratic and common shocks. A sufficiently unfavourable shock can lead to the borrower's bankruptcy. The idiosyncratic shock is observed by the entrepreneur, but not by the bank which, as in Townsend (1979), must pay a fixed monitoring cost in order to observe the entrepreneurs' realised return. To mitigate problems stemming from this source of asymmetric information, entrepreneurs and the bank sign a standard debt contract. Under this contract, the entrepreneurs commits to paying back the loan principal and a interest charge, unless it declares default. In case of default, the bank conducts a costly state verification of the residual value of the entrepreneur's assets and seizes the assets as a partial compensation. Our paper differs from BGG in that the idiosyncratic shocks affect productivity rather than return on capital in the economy, which keeps the model tractable and allows us to establish a direct link between technology shocks and default probability.[10] We also allow households to bear part of the default risk through their equity investments in the firms, in the face of an adverse technology shock, otherwise, the whole burden of default falls on the banking sector, resulting in unexpectedly high spreads between loan and deposit rates. The default settlements and resource transfers in our model will be discussed in details later.

2.2 Effects of technology shocks on hours and productivity

Although, the focus of our analysis is not on the effects of technology shocks, nevertheless it would be of interest to compare our results on technology shocks with those reported in the literature. In the standard RBC model, a positive technology shock results in a temporary rise in hours, because the substitution effect due to higher wages and real interest rates outweighs the wealth effect in the short run. However, as noted by Gali (1999), this prediction from the RBC model is not in accordance with the empirical evidence. Using a structural VAR model, identified by means

[8]Several papers also explore the advantage of incorporating credit variables in the Taylor rule. Christiano, Ilut, Motto, and Rostagno (2008) find that a Taylor rule that is modified to include a response to variations in some measure of aggregate credit would be an improvement upon conventional policy advice. Curdia and Woodford (2010) find that an adjustment for variations in credit spreads can improve upon the standard Taylor rule.

[9]Several other papers models firm defaults in a similar fashion, see for example Fiore and Tristani (2009).

[10]Christiano, Motto, and Rostagno (2010) assume further that the variance of the idiosyncratic shock that hits the entrepreneur's return is the realisation of a time-varying process.

of a long run restriction that only technology shock may have a permanent effect on the level of productivity, Gali (1999) finds that hours decline in response to a positive technology shock, and the estimates of the unconditional correlation of labour input (hours) and productivity are small and slightly negative.

In a subsequent paper, Francis and Ramey (2005) assess the validity of the technology shocks identified using long run restrictions in Gali (1999), by subjecting their model to a host of tests which provide further support to Gali's view. These papers have led to a lively debate on the effects of technology shocks on hours. Francis and Ramey (2009) note that the key to the debate lies in the data generating process assumed for per capital labour input in empirical models. When per capita labour is treated as a unit root process and entered as first differences when estimating a structural VAR, the results predict a fall in labour input in response to a positive shock to technology. However, when per capita labour is treated as a stationary process and included in levels when estimating a structural VAR, the results predict a rise in labour input following a positive innovation in technology. Francis and Ramey (2009) conclude that after controlling for low frequency components in hours to determine the effect of technology shocks, hours decline in the short run in response to a positive technology shock. In a related paper, Canova, Lopez-Salido, and Michelacci (2010) also find that once long cycles in hours are removed, hours robustly fall in response to (neutral) technology shocks, and the percentage of the variation in hours explained by the technology shock is small.[11]

In short, the empirical evidence as documented in the above studies shows that hours tend to fall in response to a positive technology shock, however, this result contradicts standard RBC models where a positive shock to technology is predicted to have a positive effect on all factor inputs. In an attempt to reconcile the empirical evidence with RBC theoretical predictions, Gali (1999), Dotsey (2002) and Basu, Fernald, and Kimball (2006) show that a model with monopolistic competition and sticky prices can potentially explain the observed near zero unconditional correlation between productivity and hours and a positive technology shock can lead to a decline in labour input. In particular, Gali (1999) presents a sticky price model where a positive technology shock can lead to a decline in labor input if the monetary authority is not too accommodative. In his example, the combination of a constant money supply and predetermined prices implies that real balances (and aggregate demand) remain unchanged in the period when the technology shock occurs. Each firm will then meet its demand by producing an unchanged level of output. If the technology shock is positive, producing the same output will require less labour input, and a decline in hours will be observed. Furthermore, unchanged output and lower hours will lead to an increase in measured labour productivity in response to the technology shock. In King and Wolman (1996) and Dotsey (2002), a positive technology shock raises firms' markup and the wedge between marginal productivity of labour and real wage. Because the wedge is expected to decrease over time, real wages are expected to rise in the future, so individuals reduce their labour supply in the short run due to the intertemporal substitution effect.

Smets and Wouters (2007) and Francis and Ramey (2009) offer examples of flexible price models that also imply a short run negative correlation between technology shocks and labour input. In a flexible price model with habit formation in consumption and adjustment cost in investment, habit

[11] For additional empirical papers on the impact of technology shocks on labour hours and productivity, see for example Alexius and Carlsson (2007) and Dedola and Neri (2007).

persistence induces a sluggishness in the response of consumption. Consumers prefer not to change their consumption by too much, while the high adjustment cost on investment makes investment a relatively expensive good in the short run. As a result, the households spend the new wealth on the only remaining alternative which is leisure and we observe a fall in hours following a positive technology shock. The present paper provides an alternative explanation for the observed negative effects of technology shocks on hours. This is achieved by using a non-separable utility function, and by allowing a unit root in the technology process, and without the introduction of sticky price and real frictions such as habit formation in consumption and adjustment cost in investment.

3 A Model of Credit and Default

We consider an economy comprised of a large number of firms, one representative household and a competitive banking sector characterized by one representative bank. Firms are identical *ex ante* and operate over a single period. At the beginning of each period, firms enter the market and decide the optimal levels of labour and capital inputs, *before* the technology and credit shocks are realised. The capital investment is financed by borrowing from the banking sector plus a capital injection from the household at the start of the period. The funds invested by the household can be viewed as "private equity". Technology shocks then arrive and firms combine technology with capital and labour to produce a single output. Firms may default if the technology shock is unfavourable, such that the firm's revenue is insufficient to repay its debt (principle and interest charges) to the banking sector. We assume that the product market is fully competitive and while some firms may fail each period, entry is free. The representative household consumes, receives interest payment on their deposits held with the banks, wage payment for their labour services, and an *ex post* lump-sum transfer (could be negative) from firms at the *end* of the period.[12] The banking sector takes deposits from the household at the beginning of the period, before the realisation of a credit shock to the bank's balance sheet that affects the supply of loanable funds available to the firms. The banking sector receives interest and loan repayments from non-defaulted firms at the end of the period and seizes upon the revenue (if any) of defaulted firms to partially cover its losses.

3.1 The household sector

For the household decision, we consider the following standard optimization problem

$$\max_{\{C_{t+j},N_{t+j},j=0,1,2\ldots\}} E\left[\sum_{j=0}^{\infty}\beta^j U(C_{t+j},N_{t+j})|\Omega_{ct}\right], \tag{3.1}$$

subject to the budget constraint

$$D_{t+1} = (1+r_{dt})D_t + W_tN_t - C_t - S_t + \Pi_{tc}, \tag{3.2}$$

where $U(C_t, N_t)$ is the one period (instantaneous) utility function, C_t is the real consumption expenditure, N_t is labour hours, and W_t is the real wage rate paid for household labour. D_t is

[12]Non-defaulted firms transfer any excess profits to the household sector. The transfer from defaulted firms can be negative, depending on the realisation of technology shocks. Resource transfers and default settlements will be discussed in details later.

household's holding of real deposits with the banking sector at the beginning of time t, r_{dt} is the real return on deposits in period t, which is known at time t. S_t is household's real equity investment (private equity) in the firms at the beginning of time t, and Π_{tc} is the household's lump-sum transfer from firms, realised at the end of period t.[13] Finally, β is the discount factor, where $0 < \beta < 1$, and $E(\cdot|\Omega_{ct})$ denotes the mathematical conditional expectations operator with respect to the non-decreasing information set Ω_{ct}, to be defined below. Note that we abstract from the endogenous determination of equity holding for the household sector to keep the model tractable and assume that the household supplies an amount of equity that is determined by an exogenous leverage factor. As we shall see later, it is important to consider equity finance in addition to debt finance in this model, otherwise we shall end up with excessively wide interest rate spreads and unexpectedly high default probability.

We adopt the following specification of the utility function, popularized by Greenwood, Hercowitz, and Huffman (1988, pp.10),

$$U(C_t, N_t) = \frac{1}{1-\gamma}\left[\left(C_t - \frac{\chi_0}{1+\chi}N_t^{1+\chi}\right)^{1-\gamma} - 1\right], \tag{3.3}$$

where $\gamma > 0$ is the coefficient of relative risk aversion and $1/\chi$ corresponds to the intertemporal elasticity of substitution in labour supply, $\chi > 0$. Following Christiano, Eichenbaum, and Evans (1997, pp.1221), we have introduced a scaling parameter $\chi_0 > 0$ in (3.3), which is calibrated with other model parameters.[14] One important property of this form of utility function is that the marginal rate of substitution between consumption and labour effort only depends on labour input, and technically this makes it easy to solve for N_t given the real wage:

$$-\frac{U_N(C_t, N_t)}{U_C(C_t, N_t)} = \chi_0 N_t^{\chi},$$

so that labour effort is determined independently of the inter-temporal consumption–savings choice. As we shall see later, this function implies a labour supply schedule that depends on the real wage only and not on consumption.

The information set available to the household sector at the beginning of period t, Ω_{ct} can be decomposed into a common component Ψ_{t-1}, and a private component Θ_{ct}, which is made up of information that is only known to the consumer at time t (but not necessarily to all the other agents), $\Omega_{ct} = \Psi_{t-1} \cup \Theta_{ct}$, where $\Theta_{ct} = \{\Pi_{tc}, \Pi_{t-1,c}, ...; S_t, S_{t-1}, ...; C_t; D_{t+1}, D_t; W_t; N_t; r_{dt}\}$. The common information set Ψ_{t-1} is publicly available and will be specified later.

The solution to consumer's optimisation problem is obtained using the first order conditions with respect to C_t, D_{t+1} and N_t. Specifically, we end up with (3.2) and the following equations:

$$E\left[\beta\left(\frac{C_{t+1} - \frac{\chi_0}{1+\chi}N_{t+1}^{1+\chi}}{C_t - \frac{\chi_0}{1+\chi}N_t^{1+\chi}}\right)^{-\gamma}(1 + r_{d,t+1})|\Omega_{ct}\right] = 1, \tag{3.4}$$

$$W_t = \chi_0 N_t^{\chi}. \tag{3.5}$$

[13]The implicit rate of return on household's private equity investment is given by $\Pi_{tc}/S_t - 1$.

[14]For other examples of this form of utility function, see Meng and Velasco (2003) and Chapter 3 of Heer and Maussner (2005).

3.2 Firms

3.2.1 Firm's optimisation problem

Each firm i is endowed with the production technology

$$Y_{it} = Z_{it}^{\varphi} {N_{it}}^{1-\alpha} K_{it}^{\alpha}, \text{ for } i = 1, 2, ..., n, \tag{3.6}$$

where K_{it} and N_{it} are capital and labour inputs for firm i in period t, Y_{it} is output for firm i in period t, α is the share of capital, and φ is a constant to be determined subsequently.

The technology variable, Z_{it}, is decomposed into an idiosyncratic component, Λ_{it}, and a common business cycle component, A_t. That is

$$Z_{it} = \Lambda_{it} A_t. \tag{3.7}$$

It is further assumed that

$$A_t = A_{t-1} \exp(\mu + u_t), \tag{3.8}$$

where u_t is a serially correlated common technology shock that follows the first-order autoregressive process

$$u_t = \rho_u u_{t-1} + \varepsilon_t, \qquad \text{where } \varepsilon_t \sim \mathbb{N}(0, \sigma_\varepsilon^2), \text{ and } |\rho_u| < 1. \tag{3.9}$$

The degree of serial correlation in the common technology shock, u_t, is determined by the autoregressive parameter, ρ_u.

Let $a_t = \ln A_t$, then the business cycle component of the technology shock can be written as

$$a_t = a_{t-1} + \mu + \rho_u u_{t-1} + \varepsilon_t. \tag{3.10}$$

Also let $\lambda_{it} = \ln \Lambda_{it}$, and assume that λ_{it} is serially uncorrelated and independently and identically distributed across firms, $\lambda_{it} \sim iid(0, \sigma_\lambda^2)$. Without loss of generality we also assume that ε_t and λ_{it} are independently distributed. Then $z_{it} = \ln Z_{it} = \lambda_{it} + a_t$, can be viewed as a single factor model where the common factor, a_t, is assumed to follow a unit root process. In this sense the specification of technology is quite general and encompasses many other specifications entertained in the theoretical macroeconomic literature.

Firms decide on capital and labour inputs *before* the arrival of the technology shock, Z_{it}. Further, part of the capital is financed through the equity investment from the household sector at the beginning of each period, denoted by S_{it}, and the rest is borrowed from the banking sector, $L_{it} \geq 0$, namely

$$K_{it} = L_{it} + S_{it}. \tag{3.11}$$

The consumer's contribution to capital acquisition can be viewed as private equity investment with possible gains/losses to be settled at the end of the period, once the shocks are realised. Note that we assume that firms are owned by the household. From the household's view point, the leverage ratio of firm i is given by $\upsilon_i = K_{it}/S_{it}$, and equation (3.11) imply that $\upsilon_i \geq 1$ for non-negative L_{it}.

The share of capital financed by the banking sector is then given by

$$L_{it} = \left(\frac{\upsilon_i - 1}{\upsilon_i} \right) K_{it}. \tag{3.12}$$

We assume that the firm leverage ratio is exogenously given and is time-invariant in this version of the paper. It is easy to allow for time variation in υ_i, so long as it is assumed exogenous. An endogenous formulation of the leverage ratio is also of interest but will not be attempted here, as it falls outside the scope of the present paper.

Note that banks can not observe the idiosyncratic technology shocks Λ_{it}, as a result firms are treated the same *ex ante* and receive an equal amount, L_{it}, from the banking sector. We also assume that technology shock, Z_{it}, is not known to firm i when choosing the optimal level of labour and capital. The sequence of events are as follows: firms enter at the beginning of each period t, with commitment from the household regarding private equity finance, borrow from the banking sector, acquire capital, then technology shocks arrive, firms produce, sell output and pay wages to the households. Firms either default or do not default depending on the size of technology shocks, which we will discuss in details later.

Having the firms acquire their entire capital stock K_{it} at the beginning of each period t (together with the assumption of full depreciation of capital) is a modelling device to ensure that firms are identical *ex ante* in each period t. It will be also assumed that firms transfer any excess profits to the household sector, so that a favourable technology shock to firm i at time $t-1$ does not make firm i better off at the beginning of time t, compared with the other firms. The one period nature of the firms' problem enables us to model firm defaults in a tractable manner.

For each firm i, K_{it} and N_{it} are derived by solving the following optimisation problem

$$\max_{\{K_{i,t+s}, N_{i,t+s}, , s=0,1,2...\}} E\left(\sum_{s=0}^{\infty} m_{t+s} \Pi_{f,i,t+s} | \Omega_{f,it} \right), \tag{3.13}$$

where $\Omega_{f,it}$ is the information set available to firm i at the beginning of time t, m_{t+s} is the stochastic discount factor (under the assumption that the representative household owns the firms).[15] The firm's profit function, $\Pi_{f,it}$, is given by

$$\Pi_{f,it} = Y_{it} - W_t N_{it} - (1 + r_{kt}) K_{it},$$

where r_{kt} is the real interest rate on capital in period t, which is known to the firm at the beginning of time t and output Y_{it} is given by equation (3.6).

Decompose the information set of firm i at the beginning of period t, $\Omega_{f,it}$ into the common component, Ψ_{t-1}, and a private (or firm-specific) component $\Theta_{f,it}$. For each firm i, namely $\Omega_{f,it} = \Psi_{t-1} \cup \Theta_{f,it}$, where $\Theta_{f,it}$ is given by

$$\begin{aligned} \Theta_{f,it} \;\; = \;\; & \{\Lambda_{it-1}, \Lambda_{it-2}, ...; Y_{i,t-1}, Y_{i,t-2}, ...; K_{it}, K_{i,t-1}, ...; \\ & N_{it}, N_{i,t-1}, ...; L_{it}, L_{i,t-1}, ...; W_t; r_{lt}; r_{kt}; \upsilon_i\}. \end{aligned}$$

The first order conditions for firm i's optimisation problem yields the optimal levels of capital

[15] The stochastic discount factor associated with the household utility function is given by $m_{t+s} = \beta^s \frac{U_C(C_{t+s}, N_{t+s})}{U_C(C_t, N_t)}$.

and labour inputs and are given by

$$E\left[\alpha\left(\Lambda_{it}A_t\right)^{\varphi}\left(\frac{N_{it}}{K_{it}}\right)^{1-\alpha}|\Omega_{f,it}\right] = 1+r_{kt}, \tag{3.14}$$

$$E\left[(1-\alpha)\left(\Lambda_{it}A_t\right)^{\varphi}\left(\frac{K_{it}}{N_{it}}\right)^{\alpha}|\Omega_{f,it}\right] = W_t. \tag{3.15}$$

These equations state that the expected marginal product of capital and labour are equal to the return on capital and the wage rate, respectively.

Given the independence of λ_{it} and the innovation to a_t, we have

$$E(\Lambda_{it}^{\varphi}A_t^{\varphi}|\Omega_{f,it}) = E(e^{\varphi\lambda_{it}}|\Omega_{f,it})E(e^{\varphi a_t}|\Omega_{f,it}).$$

Define the moment generating functions of λ and ε by $M_{\lambda}(\varphi) = E(e^{\varphi\lambda_{it}}|\Omega_{f,it})$ and $M_{\varepsilon}(\varphi) = E(e^{\varphi\varepsilon_t}|\Omega_{f,it})$ respectively, assuming $M_{\lambda}(\varphi)$ and $M_{\varepsilon}(\varphi)$ exist. Using equations (3.10) and (3.14), it can be shown that the optimal capital to labour ratio is identical for all firms, and is given by

$$\frac{K_{it}}{N_{it}} = \left(\frac{\alpha M_{\lambda}M_{\varepsilon}}{1+r_{kt}}\right)^{\frac{1}{1-\alpha}}\exp\left[\frac{\varphi(a_{t-1}+\mu+\rho_u u_{t-1})}{1-\alpha}\right], \qquad \forall\ i, \tag{3.16}$$

where we denote $M_{\lambda} = M_{\lambda}(\varphi)$ and $M_{\varepsilon} = M_{\varepsilon}(\varphi)$ to simplify the notation. Recall that we have assumed firms to be identical *ex ante*, that is N_{it} and K_{it} are independent of i and only depend on last period's technology shock, since N_{it} and K_{it} are chosen before the realisation of this period's technology shock. In equilibrium we must have

$$K_{it} = K_t,\ N_{it} = N_t,\ L_{it} = L_t,\ \upsilon_i = \upsilon \qquad \forall\ i, \tag{3.17}$$

where $K_t = m^{-1}\Sigma_{i=1}^{m}K_{it}$, $L_t = m^{-1}\Sigma_{i=1}^{m}L_{it}$ and $N_t = m^{-1}\Sigma_{i=1}^{m}N_{it}$.

To determine the optimal level of capital and labour, respectively, note that equation (3.5) in the household optimisation problem and the ratio between the first order conditions (3.14) and (3.15) imply that

$$K_t = \frac{\alpha\chi_0}{1-\alpha}\cdot\frac{N_t^{1+\chi}}{1+r_{kt}}. \tag{3.18}$$

Using equations (3.16), (3.17) and (3.18), we also derive the following expression for optimal labour hours

$$N_t = \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_{\lambda}M_{\varepsilon}\right)^{\frac{1}{1-\alpha}}\left(1+r_{kt}\right)^{-\frac{\alpha}{1-\alpha}}\right]^{\frac{1}{\chi}}\exp\left[\frac{\varphi(a_{t-1}+\mu+\rho_u u_{t-1})}{\chi(1-\alpha)}\right]. \tag{3.19}$$

It is assumed that the rate of return on capital is identical to the rate of return on loans, $r_{kt} = r_{lt}$. A wedge can be introduced between the two rates of returns by introducing information asymmetries and monitoring costs. However, to keep the analysis simple and tractable, we abstract from these complications.

3.2.2 Firm's default condition

We allow for the possibility of firm defaults in our model economy. Firm i is expected to default if the technology shock to the ith firm is unfavourable, such that the value of the firm after wage payments, which we take as $Y_{it} - W_t N_{it}$ (since price is normalised to 1) falls below a threshold value determined by its callable liabilities, which we take as the repayment of loan $R_{lt} L_{it}$, where $R_{lt} = 1 + r_{lt}$. See, for example, Merton (1974), and Pesaran, Schuermann, Treutler, and Weiner (2006). Our set up avoids the need for collateral or monitoring by banks since all firms are *ex ante* identical and the bank relies on diversification of idiosyncratic shocks across firms as a form of insurance. The default condition is such that firm i defaults if and only if

$$Y_{it} - W_t N_{it} < R_{lt} L_{it}. \tag{3.20}$$

To determine the probability of default, first define $\zeta_{it} = \lambda_{it} + \varepsilon_t$, and note that under our assumption $\zeta_{it} \sim iid(0, \sigma_\zeta^2)$, where $\sigma_\zeta^2 = \sigma_\varepsilon^2 + \sigma_\lambda^2$, and ζ_{it} has the following moment generating function

$$M_\zeta = M_\zeta(\varphi) = E(e^{\varphi \zeta_{it}} | \Omega_{f,it}) = M_\lambda M_\varepsilon.$$

Equations (3.5), (3.6), (3.12), (3.14), (3.17) and (3.18) imply that firm i defaults if and only if

$$\frac{e^{\varphi \zeta_{it}}}{M_\zeta} - (1 - \alpha) - \alpha(1 - \frac{1}{\upsilon}) < 0, \tag{3.21}$$

since K_{it}, $1 + r_{lt}$ and α are positive. Alternatively the default condition can be written as

$$\zeta_{it} < \frac{\ln\left(1 - \frac{\alpha}{\upsilon}\right) + \ln M_\zeta}{\varphi} \equiv \varpi_1. \tag{3.22}$$

Let d_{it} denote the default indicator, defined as

$$d_{it} = I(\zeta_{it} < \varpi_1), \tag{3.23}$$

where $I(A)$ takes the value of unity if A holds or zero otherwise. Default occurs if the combined technology shock (idiosyncratic and common) falls below a certain threshold ϖ_1, defined in (3.22), which is common to all firms.

The probability of default depends on the probability distribution of ζ_{it}. Under the assumption that the shocks are normally distributed we have

$$M_\varepsilon = \exp\left(\frac{\varphi^2 \sigma_\varepsilon^2}{2}\right), \quad M_\lambda = \exp\left(\frac{\varphi^2 \sigma_\lambda^2}{2}\right), \quad M_\zeta = \exp\left(\frac{\varphi^2 \sigma_\zeta^2}{2}\right), \tag{3.24}$$

and the default probability is given by

$$\kappa = P(\zeta_{it} < \varpi_1 | \Omega_{f,it}) = \Phi\left[\frac{\ln\left(1 - \frac{\alpha}{\upsilon}\right)}{\varphi \sigma_\zeta} + \frac{\varphi \sigma_\zeta}{2}\right], \tag{3.25}$$

where $\Phi(\cdot)$ is the cumulative distribution function of a standard normal.[16] Under our assumptions

[16]The moment generating function of a random variable X is defined as $M_X(t) = \mathbf{E}(e^{tX}), \quad t \in \mathbb{R}$, wherever this

the probability of default κ is time-invariant, but it is clear that time variation in κ can be allowed for by introducing time variation in the volatility of technological shocks. This is in line with the recent literature by Bloom (2009). The economy wide default probability is dependent on the following deep structural parameters in the model: α, the share of capital; υ, the leverage ratio of firms; φ, the exponent of technological process; and σ_ζ, which depends on σ_ε and σ_λ, the standard deviation of common and idiosyncratic technology shocks, respectively.

The partial derivatives of κ, with respect to firm's leverage factor υ and the standard deviation of the combined technology shocks σ_ζ are given by

$$\frac{\partial\kappa}{\partial\upsilon} = \frac{\alpha}{\varphi\sigma_\zeta\upsilon(\upsilon-\alpha)}\cdot\phi\left[\frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi\sigma_\zeta}+\frac{\varphi\sigma_\zeta}{2}\right]>0, \tag{3.26}$$

$$\frac{\partial\kappa}{\partial\sigma_\zeta} = \left[-\frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi\sigma_\zeta^2}+\frac{\varphi}{2}\right]\cdot\phi\left[\frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi\sigma_\zeta}+\frac{\varphi\sigma_\zeta}{2}\right]>0, \tag{3.27}$$

since the density function of a standard normal distribution $\phi(\cdot)$ is positive, the firm's leverage factor υ is greater than and equal to 1 (and therefore α), $\ln\left(1-\frac{\alpha}{\upsilon}\right)<0$ and the parameters α, φ and σ_ζ are positive. Therefore, the default probability rises with υ, as firms become more leveraged and dependent on bank finance; and also rises with the volatility of combined technology shocks, σ_ζ, as to be expected.

3.2.3 Resource transfers and default settlements

Two cases can arise after the realisation of technology shocks:

Outcome 1: Firm i does not default ($Y_{it}-W_tN_{it}-R_{lt}L_{it}\geqslant 0$). As we have shown earlier, $Y_{it}-W_tN_{it}-R_{lt}L_{it}\geqslant 0$ if and only if $\zeta_{it}\geqslant\varpi_1$, which occurs with probability $1-\kappa$. When firm i does not default, the bank is repaid the principal and interest on the loan, $R_{lt}L_{it}$, and the household receives a non-negative transfer from firms after wage payment:

$$\begin{aligned}\Pi_{i,tc} &= Y_{it}-W_tN_{it}-R_{lt}L_{it}\geqslant 0, \text{if firm } i \text{ does not default,}\\ \Pi_{i,tb} &= R_{lt}L_{it}, \qquad\qquad\qquad\quad \text{if firm } i \text{ does not default.}\end{aligned}$$

$\Pi_{i,tc}$ is the compensation received by the household for its equity investment in the non-defaulted firm i.

Outcome 2: Firm i defaults ($Y_{it}-W_tN_{it}-R_{lt}L_{it}<0$). When firm i defaults, it is unable to repay $R_{lt}L_{it}$ to the banking sector. The bank instead seizes the revenue of the defaulted firm after wage payments if this value ($Y_{it}-W_tN_{it}$) is positive, otherwise the bank gets no payment. The household bears the rest of default losses and receives zero or negative transfer from the firms after wage payment. More specifically,

$$\begin{aligned}\Pi_{i,tc} &= Min(0,Y_{it}-W_tN_{it}),\ \text{if firm } i \text{ defaults,}\\ \Pi_{i,tb} &= Max(0,Y_{it}-W_tN_{it}),\ \text{if firm } i \text{ defaults.}\end{aligned}$$

expectation exists. For a log-normal distribution where $\ln x\sim N(\mu,\sigma^2)$, all moments exist and $E(x)=e^{\mu+\frac{\sigma^2}{2}}$.

Depending on the realisation of technology shocks, there are two outcomes to distinguish. In the first subcase, firm i defaults and the transfer to the household is negative ($Y_{it} - W_t N_{it} < 0$), in effect, the household does not receive full wage payment.

Using equations (3.5), (3.6), (3.14), (3.17) and (3.18), we have the condition that $Y_{it} - W_t N_{it} < 0$ if and only if

$$\frac{e^{\varphi \zeta_{it}}}{M_\zeta} - (1-\alpha) < 0, \tag{3.28}$$

since K_{it}, R_{lt} and α are positive, which in logarithm is given by

$$\zeta_{it} < \frac{\ln(1-\alpha)}{\varphi} + \frac{\varphi \sigma_\zeta^2}{2} \equiv \varpi_2.$$

The probability τ that the household receives a negative transfer is therefore

$$\tau = P(\zeta_{it} < \varpi_2 | \Omega_{f,it}) = \Phi\left[\frac{\ln(1-\alpha)}{\varphi\sigma_\zeta} + \frac{\varphi\sigma_\zeta}{2}\right], \tag{3.29}$$

which is independent of i and t. The household and the bank receive a negative and zero transfer from the firms, respectively, where $\Pi_{i,tc} = Y_{it} - W_t N_{it} < 0$ and $\Pi_{i,tb} = 0$.

In the second subcase, firm i defaults and the revenue generated is sufficient to cover wage payments, $(0 < Y_{it} - W_t N_{it} < R_{lt} L_{it})$. This scenario arises if and only if the combined technology shock, ζ_{it}, lies within the range given by $\varpi_2 < \zeta_{it} < \varpi_1$,with probability $\kappa - \tau$. The household receives zero transfer after the wage payment and the bank seizes the revenue of the defaulted firm after wage payments, where $\Pi_{i,tc} = 0$ and $\Pi_{i,tb} = Y_{it} - W_t N_{it} > 0$.

3.2.4 Aggregation

To study the equilibrium conditions of the aggregate model economy, we consider the cross sectional average of firm output, defined by

$$Y_t = \frac{\sum_{i=1}^m Y_{it}}{m} = \left(\frac{\sum_{i=1}^m e^{\varphi\lambda_{it}}}{m}\right) A_t^\varphi {N_t}^{1-\alpha} K_t^\alpha.$$

But since, λ_{it} are assumed to be identically and indepednetly distributed and $E(e^{\varphi\lambda_{it}})$ exists, then by law of large numbers we have

$$\frac{\sum_{i=1}^m e^{\varphi\lambda_{it}}}{m} \xrightarrow{p} E_c(e^{\varphi\lambda_{it}}) = M_\lambda(\varphi) = M_\lambda,$$

where $E_c(e^{\varphi\lambda_{it}})$ is the cross section expectation of $e^{\varphi\lambda_{it}}$. Therefore, aggregate output is given by[17]

$$Y_t = M_\lambda A_t^\varphi {N_t}^{1-\alpha} K_t^\alpha. \tag{3.30}$$

Recall that the household and banking sector receive a transfer from the firms after production, the amount of which depends on the realisation of technology shocks. Denote Π_{tc} the average

[17] It is relatively easy to allow M_λ to be time-varying.

transfer to the household and Π_{tb} the average transfer to the banking sector, we have

$$Y_t - W_t N_t = \Pi_{tc} + \Pi_{tb}, \tag{3.31}$$

where Π_{tc} and Π_{tb} comprise of payoff from both defaulted and non-defaulted firms

$$\Pi_{tc} = \frac{\sum_{i=1}^{m}(1-d_{it})(Y_{it} - W_t N_{it} - R_{lt}L_{it}) + \sum_{i=1}^{m} d_{it} Min(0, Y_{it} - W_t N_{it})}{m} \tag{3.32}$$

$$\Pi_{tb} = \frac{\sum_{i=1}^{m} d_{it} Max(0, Y_{it} - W_t N_{it}) + \sum_{i=1}^{m}(1-d_{it})L_{it}R_{lt}}{m}. \tag{3.33}$$

We evaluate Π_{tc} in equation (3.32) by first noting that $Min(0, Y_{it} - W_t N_{it})$ can be written in terms of the following indicator function

$$\begin{aligned} & Min(0, Y_{it} - W_t N_{it}) \\ = \; & I(\zeta_{it} < \varpi_2) \cdot (Y_{it} - W_t N_{it}) + I(\varpi_2 < \zeta_{it} < \varpi_1) \cdot 0. \end{aligned}$$

Recall that $d_{it} = I(\zeta_{it} < \varpi_1)$, therefore

$$d_{it} Min(0, Y_{it} - W_t N_{it}) = I(\zeta_{it} < \varpi_2) \cdot (Y_{it} - W_t N_{it}).$$

By the law of large number, for large m,

$$\frac{\sum_{i=1}^{m} d_{it}}{m} \xrightarrow{p} E_c(d_{it}) = \kappa \quad \forall\ i \text{ and } t,$$

where κ is the probability of default.

The average output of the defaulted firms can be expressed as

$$\frac{\sum_{i=1}^{m} d_{it} Y_{it}}{m} = \frac{\sum_{i=1}^{m} d_{it} e^{\varphi \lambda_{it}}}{m} A_t^{\varphi} N_t^{1-\alpha} K_t^{\alpha}.$$

Note also $d_{it} = I(\lambda_{it} + \varepsilon_t < \varpi_1) = I(\lambda_{it} < \varpi_1 - \varepsilon_t)$. By the law of large number,

$$\frac{\sum_{i=1}^{m} e^{\varphi \lambda_{it}} I(\lambda_{it} < \varpi_1 - \varepsilon_t)}{m} \xrightarrow{p} \int_{-\infty}^{\varpi_1 - \varepsilon_t} e^{\varphi x} f_{\lambda}(x) dx, \text{ as } m \to \infty,$$

where $f_{\lambda}(x)$ is the probability density function of λ_{it}.

Lemma 1 *In the case where $\lambda_{it}/\sigma_{\lambda} \sim N(0,1)$, and hence $f_{\lambda}(x) = \phi(x/\sigma_{\lambda})$ is the normal density, we have*

$$\int_{-\infty}^{\varpi_1 - \varepsilon_t} e^{\varphi x} f_{\lambda}(x) dx = M_{\lambda} \varsigma_1(\varepsilon_t),$$

where M_{λ} is given by (3.24) and

$$\varsigma_1(\varepsilon_t) = \Phi\left(\frac{\varpi_1 - \varepsilon_t - \sigma_{\lambda}^2 \varphi}{\sigma_{\lambda}}\right). \tag{3.34}$$

Proof. *See appendix A.* ∎

Following from Lemma 1, for large m,

$$\frac{\sum_{i=1}^{m} d_{it} Y_{it}}{m} \xrightarrow{p} M_\lambda \varsigma_1(\varepsilon_t) A_t^{\varphi} N_t{}^{1-\alpha} K_t^{\alpha} = \varsigma_1(\varepsilon_t) Y_t.$$

Finally, using the law of large number and Lemma 1, it can be shown that

$$\begin{aligned} \frac{\sum_{i=1}^{m} I(\zeta_{it} < \varpi_2)}{m} &\xrightarrow{p} \tau, \\ \frac{\sum_{i=1}^{m} I(\lambda_{it} < \varpi_2 - \varepsilon_t) Y_{it}}{m} &\xrightarrow{p} \varsigma_2(\varepsilon_t) Y_t, \end{aligned}$$

where τ is given by (3.29) and

$$\varsigma_2(\varepsilon_t) = \Phi\left(\frac{\varpi_2 - \varepsilon_t - \sigma_\lambda^2 \varphi}{\sigma_\lambda}\right). \tag{3.35}$$

The transfer of resources from firms to the household sector, Π_{tc}, and the banking sector, Π_{tb}, are therefore given by

$$\Pi_{tc} = [1 - \varsigma_1(\varepsilon_t)]\, Y_t - (W_t N_t + R_{lt} L_t)(1 - \kappa) + \varsigma_2(\varepsilon_t) Y_t - \tau W_t N_t, \tag{3.36}$$

$$\Pi_{tb} = R_{lt} L_t (1 - \kappa) - (\kappa - \tau) W_t N_t + [\varsigma_1(\varepsilon_t) - \varsigma_2(\varepsilon_t)]\, Y_t, \tag{3.37}$$

where Y_t, κ, $\varsigma_1(\varepsilon_t)$ and $\varsigma_2(\varepsilon_t)$are given in (3.30), (3.25), (3.34) and (3.35), respectively.

3.3 The banking sector

The banking sector acts as the financial intermediary between the household and the firms. It receives deposit, D_t, from the household at the beginning of time t and channels household deposits to loans, L_t, extended to the firms. We postulate the relationship between L_t and D_t by

$$L_t = \theta_t D_t, \tag{3.38}$$

where θ_t is assumed to be exogenously given. Equation (3.38) allows us to introduce shocks that originate on the supply side of credit and to study their propagation to the real economy, in a tractable manner.

One interpretation of (3.38), is that the banking sector is required to deposit some reserves, B_t, with the central bank, through which the central bank is able to influence the amount of bank credit available in the economy (see for example the bank balance sheet in Freixas and Rochet, 2008). Then $L_t + B_t = D_t$, where $B_t = (1-\theta_t) D_t$. The purpose of compulsory reserve requirement as a policy instrument can be two fold. When the economy is overheating and the level of fixed investment is high, central bank can raise reserve ratio $(1-\theta_t)$ to curb credit expansion and reduce the inflationary pressure in the economy, in which case reserve requirement acts as a counter-cyclical policy tool. Alternatively, it could be that, when lending risk is high (for example, owing to an increase in firm default probability), the central bank may raise the reserve requirement ratio, so that the banking sector puts aside sufficient amount of reserve to cushion the impact of higher bank losses due to firm defaults. Part of B_t could also be viewed as bank capital required by the regulatory authority. The importance of bank capital has been highlighted in the recent financial crisis and the debate on Basel III capital regulation to improve capital adequacy in the banking sector.[18] In the United States, the capital requirement of Tier 1 capital is typically set to be at least 5 to 6%. In both cases of capital requirement or reserve requirement, θ_t will be below 1.

However, one could also consider the case where θ_t is greater than 1. This is possible when banks are allowed to issue securities (IOUs) without being backed by deposits, which could potentially be guaranteed by the central bank in case of a bank run (not modelled in our framework). The central bank can also be a source of additional liquidity to the banking sector, as seen in the recent financial crisis. To model this possibility explicitly, one would need to introduce price level and inflation in our framework, as central bank credit provision could lead to inflationary pressure in the economy. Given the relatively simple and canonical characterisation of the banking sector in our model, we abstract from pinpointing the exact source of the credit shock, instead, we investigate all three different scenarios where the *mean* of θ_t, denoted by μ_θ below, is less than, equal to and greater than unity in our calibration and simulation exercises later.

For the banking sector to be solvent, the following condition must be satisfied: the end of period asset position of the banking sector must be greater or equal to the liabilities of the banking sector. Assume that the banking sector makes zero profit and in equilibrium

$$(1 + r_{dt})D_t = \Pi_{tb}, \tag{3.39}$$

where Π_{tb}, the transfer from firms comprises of loan repayment from solvent firms and the confiscation of assets of defaulted firms, is given by equation (3.37).

The exogenous process for the loan to deposit ratio, θ_t, is assumed to follow

$$\ln \theta_t = \rho_\theta \ln \theta_{t-1} + \eta_t, \tag{3.40}$$

where $|\rho_\theta| < 1$, and $\eta_t \sim N(\mu_\eta, \sigma^2_\eta)$. The distribution of $\ln \theta_t$ is therefore given by $\ln \theta_t \sim N(\frac{\mu_\eta}{1-\rho_\theta}, \frac{\sigma^2_\eta}{1-\rho^2_\theta})$. Using the properties of log-normal distribution, we have

$$E(\theta_t) = E(e^{\ln \theta_t}) = \exp\left(\frac{\mu_\eta}{1-\rho_\theta} + \frac{1}{2}\frac{\sigma^2_\eta}{1-\rho^2_\theta}\right) = \mu_\theta,$$

where μ_θ is the mean of the loan to deposit ratio. μ_η and $E(ln\theta_t)$ can be expressed in terms of μ_θ, σ_η and ρ_θ, as follows

$$\mu_\eta = (1-\rho_\theta)ln\,(\mu_\theta) - \frac{1}{2}\frac{\sigma^2_\eta}{1+\rho_\theta}, \tag{3.41}$$

$$E(ln\theta_t) = ln\,(\mu_\theta) - \frac{1}{2}\frac{\sigma^2_\eta}{1-\rho^2_\theta}. \tag{3.42}$$

Finally, for completeness, the information set for the bank Ω_{bt} can be written as $\Omega_{bt} = \Psi_{t-1} \cup \Theta_{bt}$, where Θ_{bt} contains information that is known to the bank at the beginning of time t and $\Theta_{bt} = \{\Pi_{tb}, \Pi_{t-1,b}, ...; r_{dt}; r_{lt}; L_t; D_t; \theta_t\}$. As in Binder and Pesaran (1998, 2001), Ψ_{t-1} is a common information set, containing all the publicly available information at the beginning of period t that

[18] For a discussion on bank capital requirement and risk management, see for example Pelizzon and Schaefer (2007).

is common to the consumer, firms and the bank.

$$\begin{aligned}\Psi_{t-1} &= \{C_{t-1}, C_{t-2}, ...; K_{t-1}, K_{t-2}, ...; Y_{t-1}, Y_{t-2}, ...; L_{t-1}, L_{t-2}, ...; D_{t-1}, D_{t-2}, ...; \\ & N_{t-1}, N_{t-2}, ...; W_{t-1}, W_{t-2}, ...; r_{d,t-1}, r_{d,t-2}, ...; r_{l,t-1}, r_{l,t-2}, ...; \varepsilon_{t-1}, \varepsilon_{t-2}, ...; \\ & \theta_{t-1}, \theta_{t-2}, ...\}.\end{aligned}$$

4 Short Run Equilibrium Conditions and Long Run Steady States

4.1 Equilibrium conditions

The complete set of equations that characterize the equilibrium conditions of the model is given by equations (3.2), (3.4), (3.5), (3.18), (3.19), (3.30), (3.31), (3.37), (3.38), (3.39) and the aggregate version of equations (3.11) and (3.12). We set out below the key equations of the complete macroeconomic framework again for convenience,

$$1 = E\left[\beta\left(\frac{C_{t+1} - \frac{\chi_0}{1+\chi}N_{t+1}^{1+\chi}}{C_t - \frac{\chi_0}{1+\chi}N_t^{1+\chi}}\right)^{-\gamma} R_{d,t+1}|\Omega_{ct}\right], \tag{4.1}$$

$$W_t = \chi_0 N_t^{\chi}, \tag{4.2}$$

$$D_{t+1} = R_{dt}D_t + W_tN_t - C_t - S_t + \Pi_{tc}, \tag{4.3}$$

$$K_t = \frac{\alpha\chi_0}{1-\alpha}\cdot\frac{N_t^{1+\chi}}{R_{lt}}, \tag{4.4}$$

$$N_t = \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_\lambda M_\varepsilon\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}} (R_{lt})^{-\frac{\alpha}{(1-\alpha)\chi}} \exp\left[\frac{\varphi(a_{t-1}+\mu+\rho_u u_{t-1})}{\chi(1-\alpha)}\right], \tag{4.5}$$

$$Y_t = M_\lambda A_t^{\varphi} N_t^{1-\alpha} K_t^{\alpha}, \tag{4.6}$$

$$L_t = (1-\frac{1}{\upsilon})K_t, \tag{4.7}$$

$$\Pi_{tb} = R_{dt}D_t, \tag{4.8}$$

$$K_t = L_t + S_t, \tag{4.9}$$

$$L_t = \theta_t D_t, \tag{4.10}$$

$$\Pi_{tb} = R_{lt}L_t(1-\kappa) - (\kappa-\tau)W_tN_t + \varsigma(\varepsilon_t)Y_t, \tag{4.11}$$

$$\Pi_{tc} = Y_t - W_tN_t - \Pi_{tb}, \tag{4.12}$$

where

$$\begin{aligned}\kappa &= \Phi\left(\frac{\varpi_1}{\sigma_\zeta}\right), \quad \tau = \Phi\left(\frac{\varpi_2}{\sigma_\zeta}\right), \quad \varsigma(\varepsilon_t) = \varsigma_1(\varepsilon_t) - \varsigma_2(\varepsilon_t), \\ \varsigma_1(\varepsilon_t) &= \Phi\left(\frac{\varpi_1 - \varepsilon_t - \sigma_\lambda^2\varphi}{\sigma_\lambda}\right), \quad \varsigma_2(\varepsilon_t) = \Phi\left(\frac{\varpi_2 - \varepsilon_t - \sigma_\lambda^2\varphi}{\sigma_\lambda}\right), \\ \varpi_1 &= \frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi} + \frac{\varphi\sigma_\zeta^2}{2}, \quad \varpi_2 = \frac{\ln(1-\alpha)}{\varphi} + \frac{\varphi\sigma_\zeta^2}{2}, \\ M_\lambda &= \exp\left(\frac{\varphi^2\sigma_\lambda^2}{2}\right), \quad M_\epsilon = \exp\left(\frac{\varphi^2\sigma_\epsilon^2}{2}\right), \\ R_{lt} &= 1 + r_{lt} \text{ and } R_{dt} = 1 + r_{dt}.\end{aligned}$$

There are 12 equations governing the macro economy, (4.1) to (4.12) in 12 endogenous variables C_t, W_t, N_t, D_t, S_t, L_t, K_t, Y_t, Π_{tb}, Π_{tc}, R_{dt} and R_{lt}. The model is subject to two exogenously determined processes, the technological process, a_t, and the credit shock to θ_t, governed by

$$\begin{aligned} a_t &= a_{t-1} + \mu + u_t, \quad \text{where} \quad u_t = \rho_u u_{t-1} + \varepsilon_t, & (4.13) \\ \ln \theta_t &= \rho_\theta \ln \theta_{t-1} + \eta_t. & (4.14) \end{aligned}$$

Combining equations (4.3), (4.8) and (4.12) to obtain the economy wide budget constraint in our model

$$Y_t - C_t = S_t + D_{t+1}, \tag{4.15}$$

which shows the composition of output net of consumption (savings) is in the form of 'private equity' investment, S_t, and deposits, D_{t+1}.

From simulation exercises, we found that $\varsigma(\varepsilon_t)$ is very small for reasonable parameter values of $\alpha, \upsilon, \chi, \sigma_\varepsilon^2$ and σ_λ^2. As a result, we approximate equation (4.11) by the following expression[19]

$$\Pi_{tb} = R_{lt} L_t (1 - \kappa) - (\kappa - \tau) W_t N_t.$$

We assume that a central planner with a common information set Ω_t solves the system of equilibrium conditions. The common information set Ω_t which is known to the planner at time t is defined by

$$\Omega_t = \Psi_{t-1} \cup \Theta_{bt} \cup \Theta_{ct} \cup \left(\cup_{i=1}^m \Theta_{f,it} \right).$$

Note that the variables C_t, D_{t+1}, L_t, K_t, S_t, N_t, W_t, R_{dt}, R_{lt}, Y_t, Π_{tc}, Π_{tb}, υ, θ_t and ε_t are included in the planner's information set Ω_t at time t.

4.2 Derivation of steady states

Since this model depicts a growing economy where the technological process, a_t, contains a unit root as well as a deterministic growth component, μ, we must scale the endogenous variables C_t, L_t, D_t, S_t, K_t, W_t, N_t and Y_t in the system of equilibrium conditions by an appropriate factor of technology, A_{t-1}, so that the transformed variables are stationary on a balanced growth path, to guarantee that the model possesses steady states. We shall also assume that the real interest rates are stochastically bounded (bounded in probability), that is $R_{lt} = O_p(1)$ and $R_{dt} = O_p(1)$, which is in line with the empirical evidence on US real interest rates series.[20]

Proposition 1 *To guarantee the existence of steady states in the economy defined by equations (4.1) to (4.12), and the processes of the exogenous variables given in equations (4.13) and (4.14), the exponent of the technology process, Z_{it}, in the firm's production function (3.6) must be restricted as*

$$\varphi = \frac{(1-\alpha)\chi}{1+\chi}. \tag{4.16}$$

[19]Under the baseline parametrisation of the model, the average value of $\varsigma(\varepsilon_t)$ is very small, at an average value of 0.0062 for 100,000 simulations.

[20]Boundedness in Probability is defined as follows: We say that the sequence X_n is bounded in probability, written $X_n = O_p(1)$, if for every $\varepsilon > 0$ there exists $\delta(\varepsilon) \in (0, \infty)$ such that $Pr\left[|X_n| > \delta(\varepsilon)\right] < \varepsilon$ for all n.

Proof. *First note that (4.5) can be rewritten as*

$$N_t = \left[\frac{1-\alpha}{\alpha\chi_0}(\alpha M_\lambda M_\varepsilon)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}} (R_{lt})^{-\frac{\alpha}{(1-\alpha)\chi}} \left[A_{t-1}^{\frac{\varphi}{(1-\alpha)\chi}}\right] \exp\left[\frac{\varphi(\mu+\rho_u u_{t-1})}{\chi(1-\alpha)}\right]. \tag{4.17}$$

On the assumption that real interest rates are bounded in probability and u_t is stationary, we have

$$N_t = O_p\left[A_{t-1}^{\frac{\varphi}{(1-\alpha)\chi}}\right]. \tag{4.18}$$

Now scaling K_t by A_{t-1}, and equation (4.4) becomes

$$\frac{K_t}{A_{t-1}} = \left(\frac{\alpha\chi_0}{1-\alpha}\right)\frac{N_t^{1+\chi}A_{t-1}^{-1}}{R_{lt}} \tag{4.19}$$

Therefore, to ensure that $\frac{K_t}{A_{t-1}}$ is bounded in probability, (4.19) implies that

$$N_t = O_p\left(A_{t-1}^{\frac{1}{1+\chi}}\right). \tag{4.20}$$

But for results (4.18) and (4.20) to be compatible, we must have the condition $\frac{\varphi}{(1-\alpha)\chi} = \frac{1}{1+\chi}$ or $\varphi = \frac{(1-\alpha)\chi}{1+\chi}$, as required. ■

Remark 1 *The expressions for the endogenous variables (except for the interest rates) in efficiency units are given by $\mathring{C}_t = \frac{C_t}{A_{t-1}}$, $\mathring{L}_t = \frac{L_t}{A_{t-1}}$, $\mathring{S}_t = \frac{S_t}{A_{t-1}}$, $\mathring{D}_t = \frac{L_t}{A_{t-1}}$, $\mathring{K}_t = \frac{K_t}{A_{t-1}}$, $\mathring{W}_t = \frac{W_t}{A_{t-1}^{\chi/(1+\chi)}}$, $\mathring{N}_t = \frac{N_t}{A_{t-1}^{1/(1+\chi)}}$ and $\mathring{Y}_t = \frac{Y_t}{A_{t-1}}$. In particular, equation (4.2) implies $W_t = \chi_0 N_t^\chi = O_p(A_{t-1}^{\frac{\chi}{1+\chi}})$, which implies that W_t must be scaled by $A_{t-1}^{\frac{\chi}{1+\chi}}$, to ensure that $\mathring{W}_t$ is $O_p(1)$.*

Define the growth rate of technology by

$$g_t = \frac{e^{a_t}}{e^{a_{t-1}}} = e^{\mu+u_t} = (1+g)e^{u_t}, \tag{4.21}$$

where $u_t = \rho u_{t-1} + \varepsilon_t$ and $e^\mu \equiv 1+g$.

The standard macro models assume that all shocks are equal to zero in the steady state analysis, and in effect, abstract from any possible non-linearities in the steady state relations of the model. We propose an alternative method where the steady state relations of the model are derived from unconditional expectations of the model's relations in terms of the variables measured in efficiency units. Using this approach, We are able to express the steady state of shocks as a function of their mean and standard deviation, hence allowing for explicit consideration of risks in the steady state. Denote the natural logarithm of the variables by lower case letters, that is $\mathring{c}_t = ln\mathring{C}_t$, denote the variables in steady state by the lower case letters with a star, for example, the steady state of log consumption in efficiency units is given by $\mathring{c}^* = E(ln\mathring{C}_t)$. Note also that $lnR_{lt} = ln(1+r_{lt}) \approx r_{lt}$ and $lnR_{dt} \approx r_{dt}$.

The steady state of the system of equilibrium conditions are therefore given by[21]

$$\begin{aligned}
E(ln\theta_t) + \mathring{d}^* &= ln\chi_0 + ln\left[\frac{\alpha(1-\frac{1}{\upsilon})}{1-\alpha}\right] + (1+\chi)\mathring{n}^* - r_l^*, && (4.22)\\
\mathring{n}^* &= -\frac{ln\chi_0}{\chi} + \frac{1}{\chi}ln\left[\frac{1-\alpha}{\alpha}(\alpha M_\lambda M_\varepsilon)^{\frac{1}{1-\alpha}}\right] + \frac{\mu}{1+\chi} - \frac{\alpha}{(1-\alpha)\chi}r_l^*, && (4.23)\\
\mathring{y}^* &= ln\left[\frac{M_\lambda}{(1-\frac{1}{\upsilon})^\alpha}\right] + \varphi\mu + (1-\alpha)\mathring{n}^* + \alpha E(ln\theta_t) + \alpha\mathring{d}^*, && (4.24)\\
e^{\mathring{y}^*} - e^{\mathring{c}^*} &= e^{\mathring{d}^*+\mu} + \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*}, && (4.25)\\
r_d^* &= \gamma\mu - \ln\beta, && (4.26)\\
e^{r_d^*+\mathring{d}^*} &= e^{r_l^*+E(ln\theta_t)+\mathring{d}^*}(1-\kappa) - \chi_0(\kappa-\tau)e^{(1+\chi)\mathring{n}^*}, && (4.27)
\end{aligned}$$

where α, υ, χ, μ, β, γ and χ_0 are parameters of the model, which will be calibrated at a later stage. M_λ and M_ε are defined in (3.24), κ and τ are given by equations (3.25) and (3.29), respectively, $E(ln\theta_t)$ is given by equation (3.42), and φ must satisfy equation (4.16) in Proposition 1.

4.3 Log-linearisation

Consistent with the above derivation of steady states, we log-linearise the system of equilibrium equations around the log steady state values obtained by solving (4.22) to (4.27). Denote the log deviations from the steady state as $\widetilde{\mathring{c}}_t = \mathring{c}_t - \mathring{c}^*$, where $\mathring{c}_t = \ln \mathring{C}_t$. Then

$$\mathring{C}_t = e^{\mathring{c}_t} \approx e^{\mathring{c}^*} + e^{\mathring{c}^*}(\mathring{c}_t - \mathring{c}^*) = e^{\mathring{c}^*}(1 + \widetilde{\mathring{c}}_t).$$

Also $\widetilde{\ln g_t} = \ln g_t - E(\ln g_t)$, where using (4.21) we have $\ln g_t = \mu + u_t$. Hence, $\widetilde{\ln g_t} = u_t = \rho_u u_{t-1} + \varepsilon_t$. Similarly, for the logarithm of the loan to deposit ratio, $\widetilde{\ln\theta_t} = \ln\theta_t - E(ln\theta_t)$, and since $\ln\theta_t = \rho_\theta \ln\theta_{t-1} + \eta_t$, then $\widetilde{\ln\theta_t} = \rho_\theta \widetilde{\ln\theta_{t-1}} + \widetilde{\eta}_t$, where $\widetilde{\eta}_t = \eta_t - (1-\rho_\theta)E(ln\theta_t)$. However, using (3.42), and recalling that by assumption $\widetilde{\eta}_t \sim \mathbb{N}(\mu_\eta, \sigma_\eta^2)$, it then readily follows that, $\widetilde{\eta}_t \sim \mathbb{N}(0, \sigma_\eta^2)$.

The log-linearised approximation of the equilibrium conditions of the model are therefore given by[22]

$$\begin{aligned}
\widetilde{\mathring{c}}_t - a_1\widetilde{\mathring{n}}_t &= E\left(\widetilde{\mathring{c}}_{t+1} - a_1\widetilde{\mathring{n}}_{t+1} - a_3\widetilde{r}_{d,t+1}|\Omega_{ct}\right) + a_2u_t, && (4.28)\\
-\widetilde{\mathring{c}}_t + (1+a_4+a_6)\widetilde{\mathring{y}}_t - a_6\widetilde{\mathring{d}}_{t+1} &= a_4\widetilde{\mathring{d}}_t + a_6u_t + a_4\widetilde{\ln\theta_t}, && (4.29)\\
\widetilde{\mathring{n}}_t + \frac{\alpha}{(1-\alpha)\chi}\widetilde{r}_{lt} &= \frac{\rho_u}{1+\chi}u_{t-1}, && (4.30)\\
\widetilde{r}_{dt} - \widetilde{r}_{lt} &= \widetilde{ln\theta_t}, && (4.31)\\
\widetilde{\mathring{y}}_t - (1+\alpha\chi)\widetilde{\mathring{n}}_t + \alpha\widetilde{r}_{lt} &= \varphi u_t, && (4.32)\\
(1+\chi)\widetilde{\mathring{n}}_t - \widetilde{r}_{lt} &= \widetilde{\mathring{d}}_t + \widetilde{\ln\theta_t}, && (4.33)
\end{aligned}$$

[21]The detailed derivation of the steady state conditions can be found in Appendix B.

[22]Note that $E(u_t|\Omega_{ct}) = u_t$. The derivation of the log-linearised approximation of the equilibrium conditions can be found in Appendix B.

where

$$
\begin{aligned}
a_1 &= b_1(1+\chi),\ a_2 = 1-b_1,\ a_3 = \frac{1-b_1}{\gamma}, \\
a_4 &= \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*-\mathring{c}^*},\ a_6 = e^{\mathring{d}^*+\mu-\mathring{c}^*},\ b_1 = \frac{\chi_0}{1+\chi}e^{(1+\chi)\mathring{n}^*-\mathring{c}^*}.
\end{aligned}
$$

There are 6 equations as set out in (4.28) to (4.33) for 6 endogenous variables: $\widetilde{\mathring{c}}_t$, $\widetilde{\mathring{d}}_t$, $\widetilde{\mathring{y}}_t$, $\widetilde{\mathring{n}}_t$, $\widetilde{r}_{dt}$ and $\widetilde{r}_{lt}$, which are all known to the planner at time t.

4.4 Model solution

We use the quadratic determinantal equation (QDE) approach of Binder and Pesaran (1995, 1997) to solve the rational expectations equations given by (4.28) to (4.33). Let $\mathbf{x}_t = (\widetilde{\mathring{c}}_t, \widetilde{\mathring{d}}_{t+1}, \widetilde{r}_{lt}, \widetilde{r}_{dt}, \widetilde{\mathring{y}}_t, \widetilde{\mathring{n}}_t)'$, and write the above system of equations as

$$
\mathbf{H}_0\mathbf{x}_t = \mathbf{H}_1\mathbf{x}_{t-1} + \mathbf{H_2}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{v}_t, \tag{4.34}
$$

where

$$
\mathbf{v}_t = \tilde{\mathbf{G}}_0\boldsymbol{\xi}_t + \tilde{\mathbf{G}}_1\boldsymbol{\xi}_{t-1}, \qquad \boldsymbol{\xi}_t = \begin{pmatrix} u_t \\ \widetilde{\ln\theta_t} \end{pmatrix}, \tag{4.35}
$$

and

$$
\boldsymbol{\xi}_t = \mathbf{R}\boldsymbol{\xi}_{t-1} + \boldsymbol{\psi}_t, \quad \mathbf{R} = \begin{pmatrix} \rho_u & 0 \\ 0 & \rho_\theta \end{pmatrix}, \quad \boldsymbol{\psi}_t = \begin{pmatrix} \varepsilon_t \\ \widetilde{\eta}_t \end{pmatrix}, \tag{4.36}
$$

with $\varepsilon_t \sim \mathbb{N}(0,\sigma_\varepsilon^2)$, and $\widetilde{\eta}_t \sim \mathbb{N}(0,\sigma_\eta^2)$.

The matrices $\mathbf{H}_0$, $\mathbf{H}_1$, $\mathbf{H}_2$, $\tilde{\mathbf{G}}_0$ and $\tilde{\mathbf{G}}_1$ are given by

$$
\mathbf{H}_0 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & -a_1 \\
-1 & -a_6 & 0 & 0 & 1+a_4+a_6 & 0 \\
0 & 0 & \frac{\alpha}{(1-\alpha)\chi} & 0 & 0 & 1 \\
0 & 0 & -1 & 1 & 0 & 0 \\
0 & 0 & \alpha & 0 & 1 & -(1+\alpha\chi) \\
0 & 0 & -1 & 0 & 0 & (1+\chi)
\end{pmatrix}, \mathbf{H}_1 = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & a_4 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0
\end{pmatrix},
$$

$$
\mathbf{H}_2 = \begin{pmatrix}
1 & 0 & 0 & -a_3 & 0 & -a_1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}, \tilde{\mathbf{G}}_0 = \begin{pmatrix}
a_2 & 0 \\
a_6 & a_4 \\
0 & 0 \\
0 & 1 \\
\varphi & 0 \\
0 & 1
\end{pmatrix}, \tilde{\mathbf{G}}_1 = \begin{pmatrix}
0 & 0 \\
0 & 0 \\
\frac{\rho_u}{1+\chi} & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0
\end{pmatrix}, \mathbf{x}_t = \begin{pmatrix}
\widetilde{\mathring{c}}_t \\
\widetilde{\mathring{d}}_{t+1} \\
\widetilde{r}_{lt} \\
\widetilde{r}_{dt} \\
\widetilde{\mathring{y}}_t \\
\widetilde{\mathring{n}}_t
\end{pmatrix}.
$$

Note that $\boldsymbol{\psi}_t$ is a serially uncorrelated vector process with zero mean. Note also that $\mathbf{H}_0$ is non-singular, and (4.34) can be written as (using equation (4.35))

$$
\mathbf{x}_t = \mathbf{H}_0^{-1}\mathbf{H}_1\mathbf{x}_{t-1} + \mathbf{H}_0^{-1}\mathbf{H_2}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{H}_0^{-1}\tilde{\mathbf{G}}_0\boldsymbol{\xi}_t + \mathbf{H}_0^{-1}\tilde{\mathbf{G}}_1\boldsymbol{\xi}_{t-1},
$$

or

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{G}_0\boldsymbol{\xi}_t + \mathbf{G}_1\boldsymbol{\xi}_{t-1}, \tag{4.37}$$

where $\mathbf{A} = \mathbf{H}_0^{-1}\mathbf{H}_1$, $\mathbf{B} = \mathbf{H}_0^{-1}\mathbf{H}_2$, $\mathbf{G}_0 = \mathbf{H}_0^{-1}\tilde{\mathbf{G}}_0$, and $\mathbf{G}_1 = \mathbf{H}_0^{-1}\tilde{\mathbf{G}}_1$.

The rational expectations solution of (4.37) is given by[23]

$$\mathbf{x}_t = \mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1}, \tag{4.38}$$

where

$$\begin{aligned}
\mathbf{BC}^2 - \mathbf{C} + \mathbf{A} &= \mathbf{0}, & (4.39)\\
\mathbf{D}_1 &= (\mathbf{I} - \mathbf{BC})^{-1}\mathbf{G}_1, & (4.40)\\
(\mathbf{I} - \mathbf{BC})\mathbf{D_0} - \mathbf{BD}_0\mathbf{R} &= \mathbf{G}_0 + \mathbf{B}(\mathbf{I} - \mathbf{BC})^{-1}\mathbf{G}_1. & (4.41)
\end{aligned}$$

Following Binder and Pesaran (1995, 1997), we use the quadratic difference equation (QDE) in (4.39) to solve for $\mathbf{C}$. After obtaining $\mathbf{C}$, (4.40) can be used to obtain $\mathbf{D}_1$. To solve for $\mathbf{D}_0$, first write equation (4.41) as

$$\mathbf{D}_0 - \mathbf{Q}_0\mathbf{D}_0\mathbf{R} = \mathbf{Q}_1, \tag{4.42}$$

where

$$\begin{aligned}
\mathbf{Q}_0 &= (\mathbf{I} - \mathbf{BC})^{-1}\mathbf{B},\\
\mathbf{Q}_1 &= (\mathbf{I} - \mathbf{BC})^{-1}\mathbf{G}_0 + (\mathbf{I} - \mathbf{BC})^{-1}\mathbf{B}(\mathbf{I} - \mathbf{BC})^{-\mathbf{1}}\mathbf{G_1}.
\end{aligned}$$

Then using results in Magnus and Neudecker (1988) (pp. 30-31),

$$vec(\mathbf{D}_0) - (\mathbf{R}' \otimes \mathbf{Q}_0)\, vec(\mathbf{D}_0) = vec(\mathbf{Q}_1),$$

which yields

$$vec(\mathbf{D}_0) = \left[\mathbf{I} - (\mathbf{R}' \otimes \mathbf{Q}_0)\right]^{-1} vec(\mathbf{Q}_1).$$

4.5 Impulse responses

In our simulation exercises, we are interested in the impact of (1) a positive technology shock and (2) a positive credit shock on business cycle dynamics, assuming that the two shocks are uncorrelated for identification purposes. We compute the impulse responses to credit and technology shocks following Koop, Pesaran, and Potter (1996) and Pesaran and Shin (1998), as the generalised impulse response functions offer more modelling flexibility and can easily allow for non-zero correlation between the credit and technology shocks if required.

Definition 1 *The "generalised impulse response function"(GIRF) of a vector process $\mathbf{x}_t$ of dimension $p \times 1$ is defined by*

$$GI_{\mathbf{x}}(h, \delta, \Omega_{t-1}) = E(\mathbf{x}_{t+h}|\boldsymbol{\psi}_t = \delta, \Omega_{t-1}) - E(\mathbf{x}_{t+h}|\Omega_{t-1}),$$

[23]The proof that (4.38) is indeed a solution to (4.37) is given in the Appendix B.

where Ω_{t-1} is the common information set at time $t-1$, and δ is a vector of shocks.

Recall that model solutions are given by equation (4.38). Using definition 1 of GIRFs and denoting $GI_{\mathbf{x}}(h, \delta, \Omega_{t-1})$ by $GI_{\mathbf{x}}(h)$ for simplicity, we have:

$$\begin{aligned} GI_{\mathbf{x}}(h) &= \mathbf{C}GI_{\mathbf{x}}(h-1) + \mathbf{D}_0 GI_{\xi}(h) + \mathbf{D}_1 GI_{\xi}(h-1), \text{ for } h = 0,1,2,3..., \\ GI_{\xi}(h) &= \mathbf{R}GI_{\xi}(h-1), \quad \text{for } h = 1,2,3..., \\ GI_{\mathbf{x}}(h) &= 0, \quad \text{for } h < 0, \quad \text{and} \quad GI_{\xi}(h) = 0, \quad \text{for } h < 0. \end{aligned}$$

For the technology shock on impact, we have

$$GI_{\xi}(0) = GI_{\psi}(0) = \frac{1}{\sqrt{\mathbf{e}_1' \mathbf{Cov}(\boldsymbol{\psi}_t)\mathbf{e}_1}} \mathbf{Cov}(\boldsymbol{\psi}_t)\mathbf{e}_1 \ , \tag{4.43}$$

where $\mathbf{e}_1 = (1,0)'$ and

$$\mathbf{Cov}(\boldsymbol{\psi}_t) = \begin{pmatrix} \sigma_{\varepsilon}^2 & \rho_{\varepsilon\eta}\sigma_{\varepsilon}\sigma_{\eta} \\ \rho_{\varepsilon\eta}\sigma_{\varepsilon}\sigma_{\eta} & \sigma_{\eta}^2 \end{pmatrix}.$$

To obtain the GIRFs for the credit shock, we need to replace $\mathbf{e}_1$ in (4.43) by $\mathbf{e}_2 = (0,1)'$. In the standard case where technology and credit shocks are assumed to be uncorrelated, we have

$$\mathbf{Cov}(\boldsymbol{\psi}_t) = \begin{pmatrix} \sigma_{\varepsilon}^2 & 0 \\ 0 & \sigma_{\eta}^2 \end{pmatrix},$$

and (4.43) can be simplified such that $GI_{\xi}(0) = (\sigma_{\varepsilon}, 0)'$ for a one standard deviation positive shock to technology, and $GI_{\xi}(0) = (0, \sigma_{\eta})'$ for a one standard deviation positive shock to credit.

Also, recalling that $L_t = \theta_t D_t$, then $\widetilde{\mathring{l}}_t = \widetilde{\ln\theta}_t + \widetilde{\mathring{d}}_t$, and the GIRF of $\widetilde{\mathring{l}}_t$ is given by

$$GI_l(h) = GI_{\ln\theta}(h) + GI_d(h-1),$$

where $GI_{\ln\theta}(h)$ is defined by the second element of $GI_{\xi}(h)$, or $(0,1)GI_{\xi}(h)$, and $GI_d(h-1)$ is given by the second element of $GI_{\mathbf{x}}(h)$.[24]

In log-linearised form, labour productivity is given by $\widetilde{prod}_t = \widetilde{\mathring{y}}_t - \widetilde{n}_t$, and therefore, the GIRF for $\widetilde{prod}_t$ can be computed as

$$GI_{prod}(h) = GI_y(h) - GI_n(h),$$

where $GI_y(h)$ and $GI_n(h)$ are given by the fifth and the sixth elements of $GI_{\mathbf{x}}(h)$, respectively.

Proposition 2 *Under the system of equilibrium conditions set out in equations (4.28) to (4.33), and assuming that credit and technology shocks are uncorrelated, it follows that:*
(a) A technology shock has no impact, at any horizon, on the spread between loan and deposit rates, $\widetilde{r}_{lt} - \widetilde{r}_{dt}$.
(b) A credit shock has a negative impact on the interest rate spread, $\widetilde{r}_{lt} - \widetilde{r}_{dt}$.
(c) The level of labour hours and loan rate respond in opposite directions following a credit shock.

[24] Note that the second element of $GI_{\mathbf{x}}(h)$ refers to the GIRF of $\widetilde{\mathring{d}}_{t+1}$, which is given by $GI_d(h)$, and hence the associate GIRF for $\widetilde{\mathring{d}}_t$ is $GI_d(h-1)$.

(d) Output responds positively to a technology shock on impact, but falls consequently if labour hours fall and loan rate rises following a positive technology shock.

Proof. *The impulse responses implied by equations (4.30), (4.31) and (4.32) can be written as*

$$GI_n(h) + \frac{\alpha}{(1-\alpha)\chi} GI_{r_l}(h) = \frac{\rho_u}{1+\chi} GI_u(h-1), \tag{4.44}$$

$$GI_{r_d}(h) - GI_{r_l}(h) = GI_{ln\theta}(h), \tag{4.45}$$

$$GI_y(h) - (1+\alpha\chi) GI_n(h) + \alpha GI_{r_l}(h) = \varphi GI_u(h). \tag{4.46}$$

Results (a) and (b) follow from equation (4.45), which shows that the impulse response of the interest rate spread does not depend on the technology shock. This equation also establishes that a credit shock has a negative impact on the interest rate spread. Result (c) follows from equation (4.44), where $GI_n(h) + \frac{\alpha}{(1-\alpha)\chi} GI_{r_l}(h) = 0$*, following a credit shock. Finally, (4.46) implies that technology shock leads to a positive response in output on impact, however, one should expect a negative correction in the responses in output if labour hours respond negatively and loan rate responds positively to a technology shock, as stated in result (d).* ■

5 Parameter Calibration

Following much of the literature, the capital share, α, is set to 0.35, the discount rate r to 1.6% per annum (0.4% per quarter) which gives $\beta = 0.996$. Following Greenwood, Hercowitz, and Huffman (1988), Christiano, Eichenbaum, and Evans (1997) and as is standard in business cycle analysis, γ, the coefficient of risk aversion, is set to 1. For $1/\chi$, the intertemporal elasticity of substitution in labour supply, Greenwood, Hercowitz, and Huffman (1988) argue that for a representative household, $1/\chi$ should summarize the variation in labour of all members of such a unit, both at the intensive and extensive margins. These authors suggest that a reasonable value of $1/\chi$ should lie in the range $\{0.3 - 2.2\}$. In our calibrated exercise we select a mid point value in this range and set $1/\chi = 1.4$, or $\chi = 0.7$. The scaling parameter, χ_0, in (3.3) is chosen such that, the steady state value of labour hours in efficiency units is set to unity, namely $\mathring{N}^* = e^{\mathring{n}^*} = 1$.

The remaining parameters, ρ_u, σ_ε, μ, υ, ρ_θ and σ_η are calibrated using US quarterly time series data covering the period 1985Q1 to 2009Q4.[25] Following the literature, we use the Hodrick-Prescott (1997) filter to extract the cyclical components from the data series, with a smoothing parameter value of 1600, which is recommended for quarterly data.[26] To derive the standard deviation of common technology shock, σ_ε, from US data, we first detrend the log of per capital real output series. The standard deviation of the cyclical component (as proxy for the innovation to common technology shock, ε_t) is found to be 0.011, similar to the value used in Romer (2006). The deterministic trend in technology growth μ is obtained by calculating the mean of the growth rate of per capita real GDP in logarithm in the US between 1985Q1 and 2009Q4. μ is found to

[25] A detailed description of the US data series used for calibration can be found in Appendix C.

[26] The HP filter was introduced in Hodrick and Prescott (1980) and discussed in King and Rebelo (1993) and Hodrick and Prescott (1997). The cyclical component y_t^c of the series extracted by an HP filter, is defined by (in the infinite sample version of the HP filter) $y_t^c = \frac{\lambda(1-L)^2(1-L^{-1})^2}{1+\lambda(1-L)^2(1-L^{-1})^2} y_t$, where y_t is the original time series, L is the lag operator and λ is the smoothing parameter. Alternative approaches for permanent–transitory decomposition include the Beveridge-Nelson procedure (see for example Garratt, Lee, Pesaran, and Shin, 2006).

Table 1: Model Parameters

Preference		Value	Source
γ	coefficient of risk aversion	1	Christiano et al. (1997)
χ	inverse of the intertemporal elasticity of substitution in labour supply	0.7	Greenwood et al. (1988)
β	discount factor	0.996	
Technology			
ρ_u	coefficient of autoregression in common technology shock	0.439	US data
σ_ε	standard deviation of common technology shock	0.011	US data
μ	deterministic trend in technology growth	0.003	US data
Production and firm financing			
α	share of capital	0.35	BGG
υ	firm's leverage factor	1.43	US data
κ	probability of default	0.086	Moody (2010) and BGG
Credit			
ρ_θ	coefficient of autoregression in credit shock	0.848	US data
σ_η	standard deviation of credit shock	0.011	US data
μ_θ	mean of loan to deposit ratio	0.95, 1, 1.05	

be around 0.003, which implies a per capita output growth rate of around 1.2%, per annum. The autoregressive coefficient of the growth rate of per capita real GDP (in logarithm) is 0.439, which is taken as the coefficient of autoregression in common technology shock, ρ_u. To match an annualised default probability of around 3.4% (quarterly rate of 0.86%), consistent with BGG and the rolling 12-month US private firm quarterly default rate provided by Moody from 2000 to 2009, we set the standard deviation of idiosyncratic technology shock, σ_λ, to 0.43.[27]

We calibrate the values for the standard deviation and the autoregressive coefficient of the credit shock using US data on loans and deposits between 1985Q1 to 2009Q4. First, we define a series that is the difference between the logarithm of per capita loans and the logarithm of per capita deposits. We then detrend the series using a HP filter (with the smoothing parameter of 1600), and take the cyclical components of the series as a proxy for η_t. The standard deviation of the series, σ_η, is found to be 0.011, and the autoregressive coefficient of the credit shock process, ρ_θ, is estimated to be 0.848.

The leverage factor of firms is derived using the Federal Reserve Flows of Funds data, Table L.102 (levels data) on US non-farm nonfinancial corporate business, following Fiore and Uhlig (2005). Debt is defined as bank loans and corporate bonds (lines 39+26), and equity is defined as the market value of equities outstanding (line 37). The proportion of debt finance in total finance

[27] See, for example, Moody (2010). The data source is Moody's Analytics Credit Research Database (CRD), which collects quarterly data from 15 US lending organizations, representing both large institutions and smaller regional banks. The CRD defines default consistent with the Basel II directive.

between 1985Q1 to 2009Q4 is around 0.3, which implies a firm leverage ratio of around 1.43.[28] Finally, we entertain three different values of μ_θ, the mean of the loan to deposit ratio in our calibration exercises. In particular, we consider $\mu_\theta = \{0.95, 1, 1.05\}$.

Table 2: Steady State Values and Loan to Deposit Ratio

		Steady state values									
		r_d^*	r_l^*	$\mathring{N}^*$	$\mathring{W}^*$	$\mathring{C}^*$	$\mathring{Y}^*$	$\mathring{K}^*$	$\mathring{L}^*$	$\mathring{S}^*$	$\mathring{D}^*$
μ_θ	0.95	0.007	0.121	1	0.350	0.369	0.538	0.167	0.050	0.117	0.053
	1	0.007	0.070	1	0.360	0.373	0.553	0.181	0.054	0.126	0.054
	1.05	0.007	0.021	1	0.369	0.376	0.568	0.195	0.058	0.136	0.056

Note: The steady state values are computed based on the parameter values given in Table 1. The steady state of labour hours in efficiency unit $\mathring{N}^*$ is normalised to 1.

The steady state values of the model variables in efficiency units can be obtained using equations (4.22) to (4.27), and the steady state values are given in Table 2. It is interesting to note that, in the steady state, output per capita, consumption per capita and capital per capita in efficiency units rise with μ_θ, which measures the availability of loans and the extent of leverage in the banking sector. This finding suggests that as the banking sector becomes more leveraged and the extent of financial intermediation increases, the steady state output per capita level in the economy tends to be higher. The result is consistent with empirical studies on the relationship between finance and development, where more developed banking sector is often associated with faster economic development (see for example Levine, 2005). Note also that, when the banking sector becomes more leveraged, the interest rate charged on loans tends to fall. For example, when μ_θ increases from 1 (no leverage) to 1.05, real loan rate reduces from an implausibly high rate of 7% per quarter (28% per annum) to around 8% per annum, yielding an interest rate spread of around 5% per annum, which is more reasonable. The results suggest that, in order to meet the break-even condition, in equilibrium the banking sector could either charge a high loan interest rate to cover the losses resulting from firm defaults, or to take on more risks by increasing leverage, in the form of security issuance, for example.

Since an important focus of this paper is to examine the impact of firm defaults on macroeconomic conditions, we also compute the steady state values of the model economy by varying some of the key parameters that determine the equilibrium default probability, namely, the leverage ratio of the firms, υ, and the standard deviations of common and idiosyncratic technology shocks, σ_ϵ and σ_λ. Recall from (3.26) and (3.27) that default probability rises with firm's leverage ratio and the standard deviations of technology shocks. The results in Table 3 confirm this prediction in the steady states. For firm's leverage ratio, υ, we consider three scenarios with $\upsilon = \{1.25, 1.43, 1.67\}$, where the proportion of debt finance in total finance is 20%, 30% (US data), and 40%, respectively. As the results in Table 3 show, the probability of default rises from 0.086% to 2.33% per quarter when firm's leverage ratio increases from 1.43 to 1.67, with the interest rate on loans doubling from 7% to 14.8%. We also observe a fall in the steady state levels of per capita consumption, output

[28] This result is very similar to Fiore and Uhlig (2005), who find that the debt to total finance ratio is around 0.3 and a leverage ratio of around 1.43 for US non-farm, non-financial corporate business sector, using the shorter sample period of 1997 to 2003.

Table 3: Steady State Values and Default Probability

		Steady state values										
		r_d^*	r_l^*	$\mathring{N}^*$	$\mathring{W}^*$	$\mathring{C}^*$	$\mathring{Y}^*$	$\mathring{K}^*$	$\mathring{L}^*$	$\mathring{S}^*$	$\mathring{D}^*$	κ
Panel A												
	1.25	0.007	0.033	1	0.367	0.373	0.564	0.191	0.038	0.153	0.038	0.0026
υ	1.43	0.007	0.070	1	0.360	0.373	0.553	0.181	0.054	0.126	0.054	0.0086
	1.67	0.007	0.148	1	0.345	0.370	0.531	0.160	0.064	0.096	0.064	0.0233
Panel B												
	0.001	0.007	0.070	1	0.360	0.373	0.554	0.181	0.054	0.126	0.054	0.0086
σ_ϵ	0.011	0.007	0.070	1	0.360	0.373	0.553	0.181	0.054	0.126	0.054	0.0086
	0.110	0.007	0.085	1	0.357	0.372	0.550	0.177	0.053	0.124	0.053	0.0105
Panel C												
	0.33	0.007	0.013	1	0.369	0.372	0.568	0.196	0.059	0.137	0.059	0.0009
σ_λ	0.43	0.007	0.070	1	0.360	0.373	0.553	0.181	0.054	0.126	0.054	0.0086
	0.53	0.007	0.221	1	0.333	0.369	0.513	0.144	0.043	0.101	0.043	0.0281

Note: The steady state values in Panels A, B and C are computed based on the parameter values given in Table 1, with μ_θ=1 and the values of υ, σ_ϵ and σ_λ given in this table, respectively. The steady state of labour hours in efficiency unit $\mathring{N}^*$ is normalised to 1.

and capital, when firm's leverage ratio rises, despite an increase in the level of loans per capita, since a larger proportion of the loans are non-performing and the steady state value of private equity per capita is falling (substitution effect). As to be expected, the probability of default rises with the volatility of common and idiosyncratic technology shocks (see Panels B and C of Table 3). The steady state levels of per capita output, consumption and loans decline with increased volatility. Finally, note that in the steady state $\mathring{Y}^* = \mathring{C}^* + \mathring{S}^* + \mathring{D}^*$, that is output is divided into consumption and savings, comprised of private equity investment, $\mathring{S}^*$, and bank deposits, $\mathring{D}^*$, as implied by equation (4.15).

6 Results of the Calibration Exercise

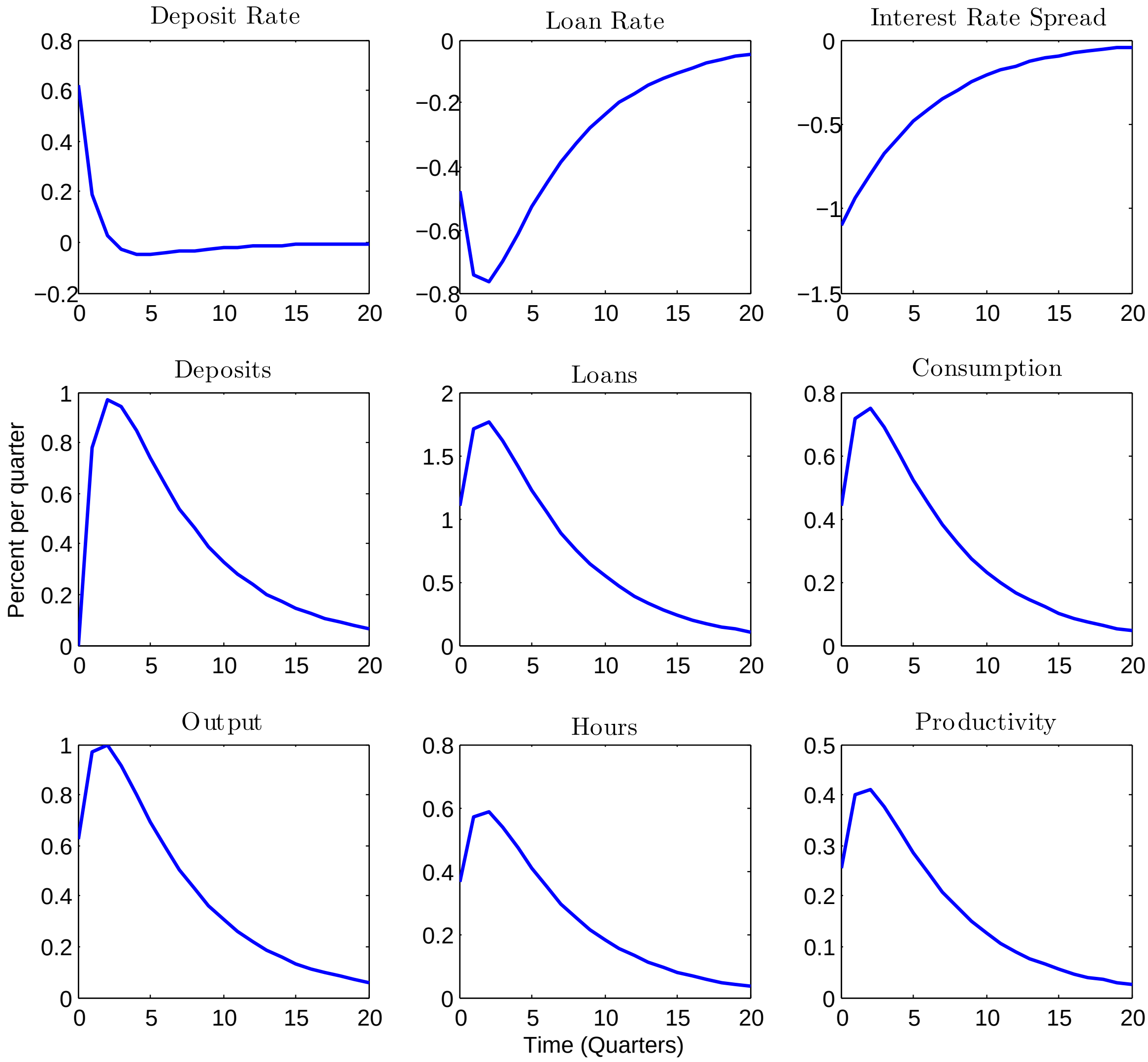
6.1 Effects of credit and technology shocks

Initially, we consider the impacts of a positive credit shock. As can be seen in Figure 1, the impulse responses for a positive credit shock yield an increase in loans of around one percent on impact.[29]

The rise in the level of loans leads to an increase in available capital in the economy and a rise in output level of around 0.6%. The increase in the supply of funds also drives down the interest rate on loans by almost half a percent, and leads to a fall in the interest rate spread by around 1%, as predicted by Proposition 2. Deposit rate rises by around 0.6% on impact, which in turn implies an increase in the level of deposits of around 1% at its peak, consistent with the zero profit condition imposed on the banking sector. In absence of a technology shock, labour hours rise in response to the decline in loan interest rates (see Proposition 2). The resulting increase in labour income raises household consumption, by around 0.4% on impact. Further, productivity rises since output increases more than labour hours on impact. Our findings are consistent with the impulse responses of a shock to bank capital, which predicts that the level of loans, output,

[29] A negative credit shock can be interpreted as a credit crunch.

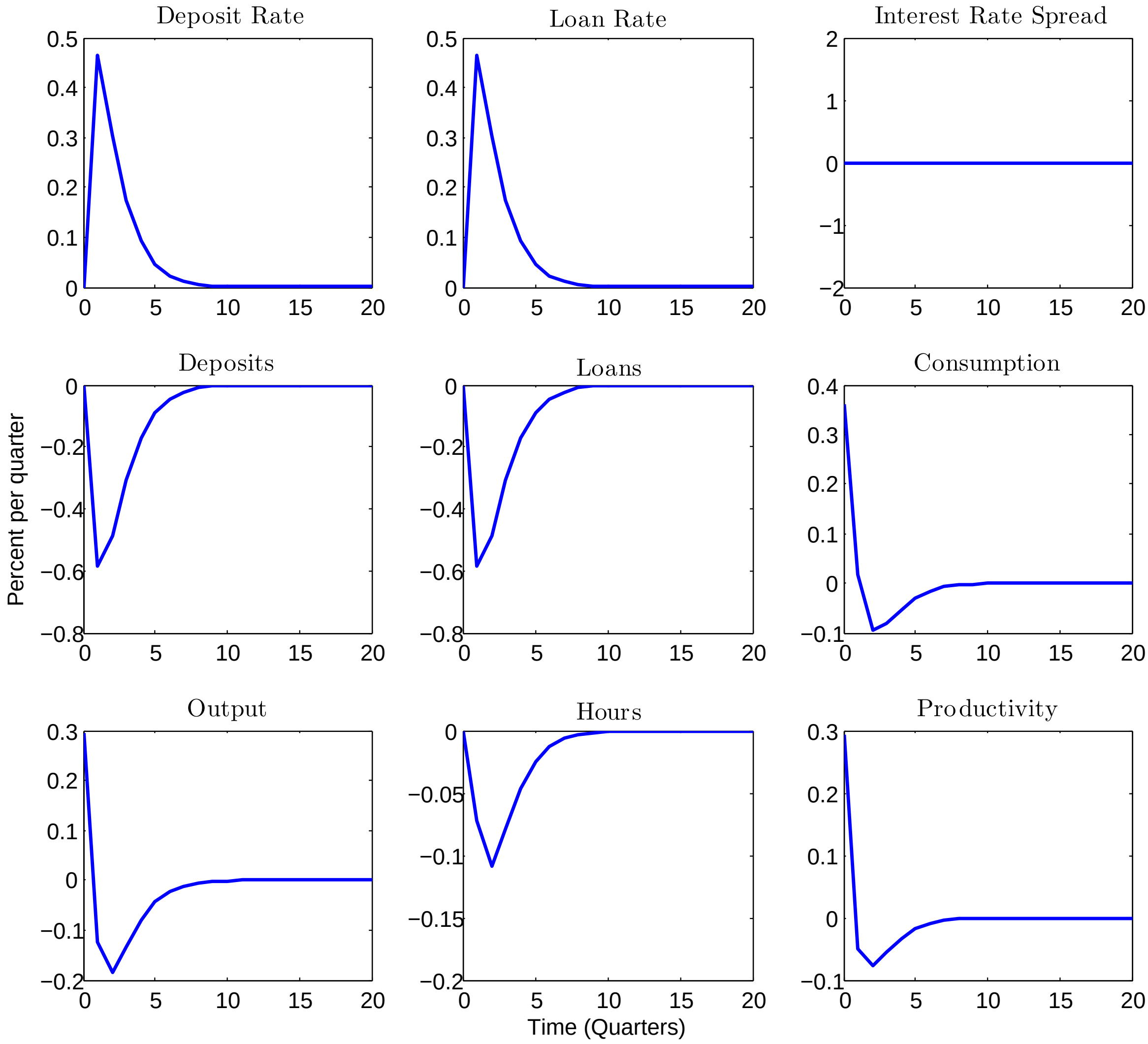
Figure 1: Impulse responses of one s.e positive credit shock (Benchmark Calibration, $\mu_\theta = 1$, percent per quarter)



labour hours and household consumption move in the same direction (see, for example Aikman and Paustian, 2006). It is worth highlighting that our findings are also in line with the empirical evidence of responses to a US credit shock, where output and short term interest rates (deposit rate can be seen as a proxy) move in the same direction as the credit shock (see, for example, Helbling, Huidrom, Kose, and Otrok, 2011 and Xu, 2010).

We then consider the impact of a positive technology shock. As can be seen from Figure 2, the shock results in higher deposit and loan rates, with no impacts on the interest rate spread, as shown in Proposition 2. Loan rate rises as the marginal product of capital increases following a positive technology shock, and the zero profit condition of the banking sector ensues an increase in the deposit rate which reaches to almost 0.5% at its peak. The rise in loan rate leads to a fall in the level of loans in the economy and consequently a fall in the level of deposits, again due to the zero profit condition of the banking sector. The response in hours is negative, which is consistent with the empirical evidence that positive technology shocks lead to short-run declines in hours (see Section 2.2 and references cited therein). Consumption initially increases by 0.4% following the positive technology shock but falls after two quarters, reflecting the reduction in household

Figure 2: Impulse responses of one s.e positive shock to technology (Benchmark Calibration, $\mu_\theta = 1$, percent per quarter)



disposable income due to a decline in hours. Output initially increases by 0.3% on impact and we observe a correction before the economy returns to its equilibrium after around five quarters. We also observe an initial positive response in productivity, which is in line with the empirical evidence that productivity rises following a positive technology shock and that technology shocks induce a negative correlation between productivity and hours.

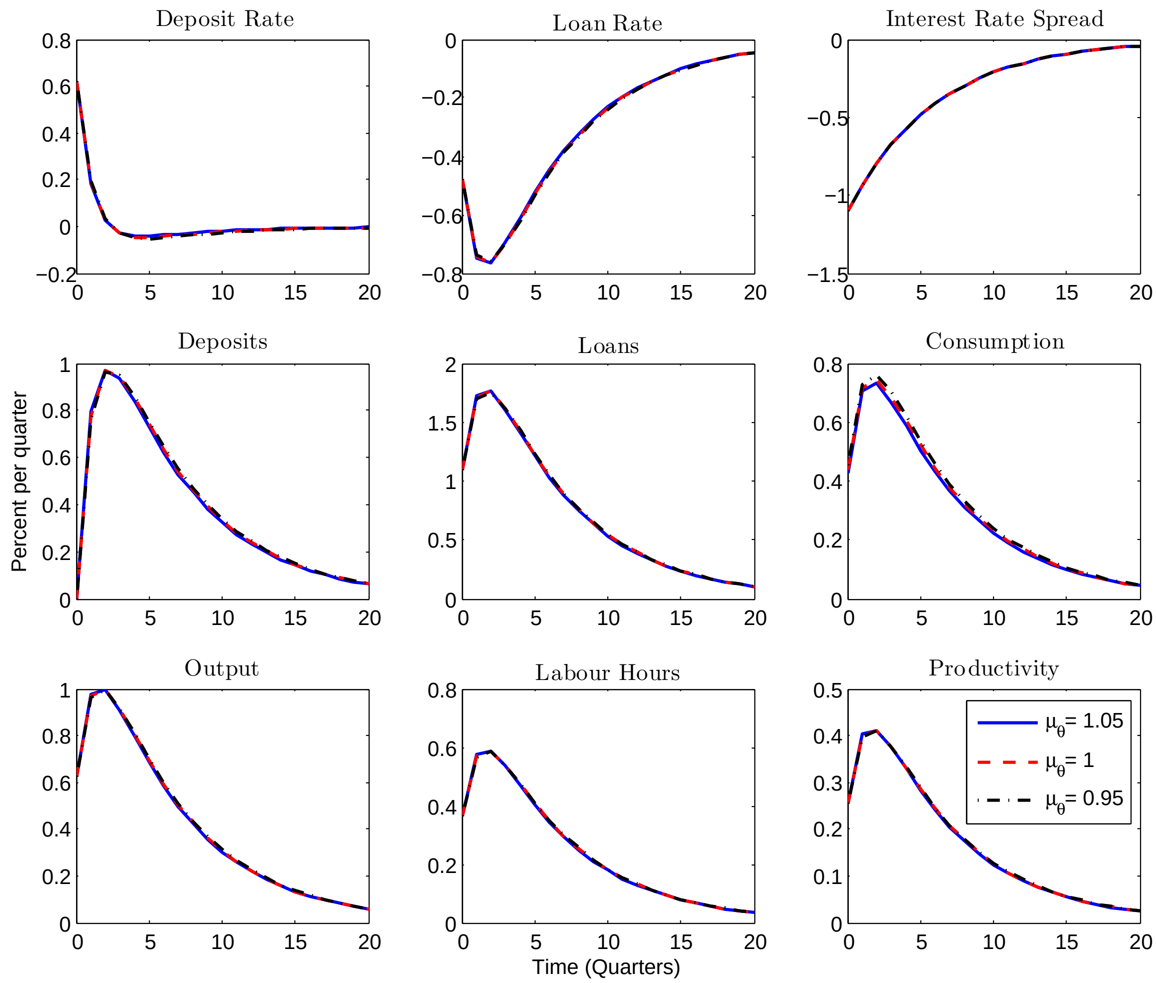
Our calibrated results show that the speed of convergence to equilibrium is much faster for the technology shock as compared to the effects of the credit shock. On average, it takes around five quarters for the effects of the technology shock to vanish, whilst it takes around 20 quarters for the effects of the credit shock to disappear. The peak impacts of a credit shock on output and consumption are around twice as large as those associated with a technology shock. This observation is consistent with empirical studies on the output effect of financial crises, which suggest that recessions associated with financial crises have been more severe and longer lasting than recessions associated with other shocks (see, for example, IMF, World Economic Outlook, April 2009, Chapter 3). The prolonged effects of the credit shock also reflects the high persistence in the loan to deposit ratio that we observe empirically, where the autoregressive coefficient of the credit shock is found to be 0.848, as compared to the autoregressive coefficient of the technology shock which is set to 0.439 in the calibration exercise.

6.2 Robustness of the results

6.2.1 Alternative mean of loan to deposit ratio

In order to check the robustness of our calibrated results, we carried out three further experiments.

Figure 3: Impulse responses of one s.e positive credit shock ($\mu_\theta = 1.05$, 1 and 0.95)



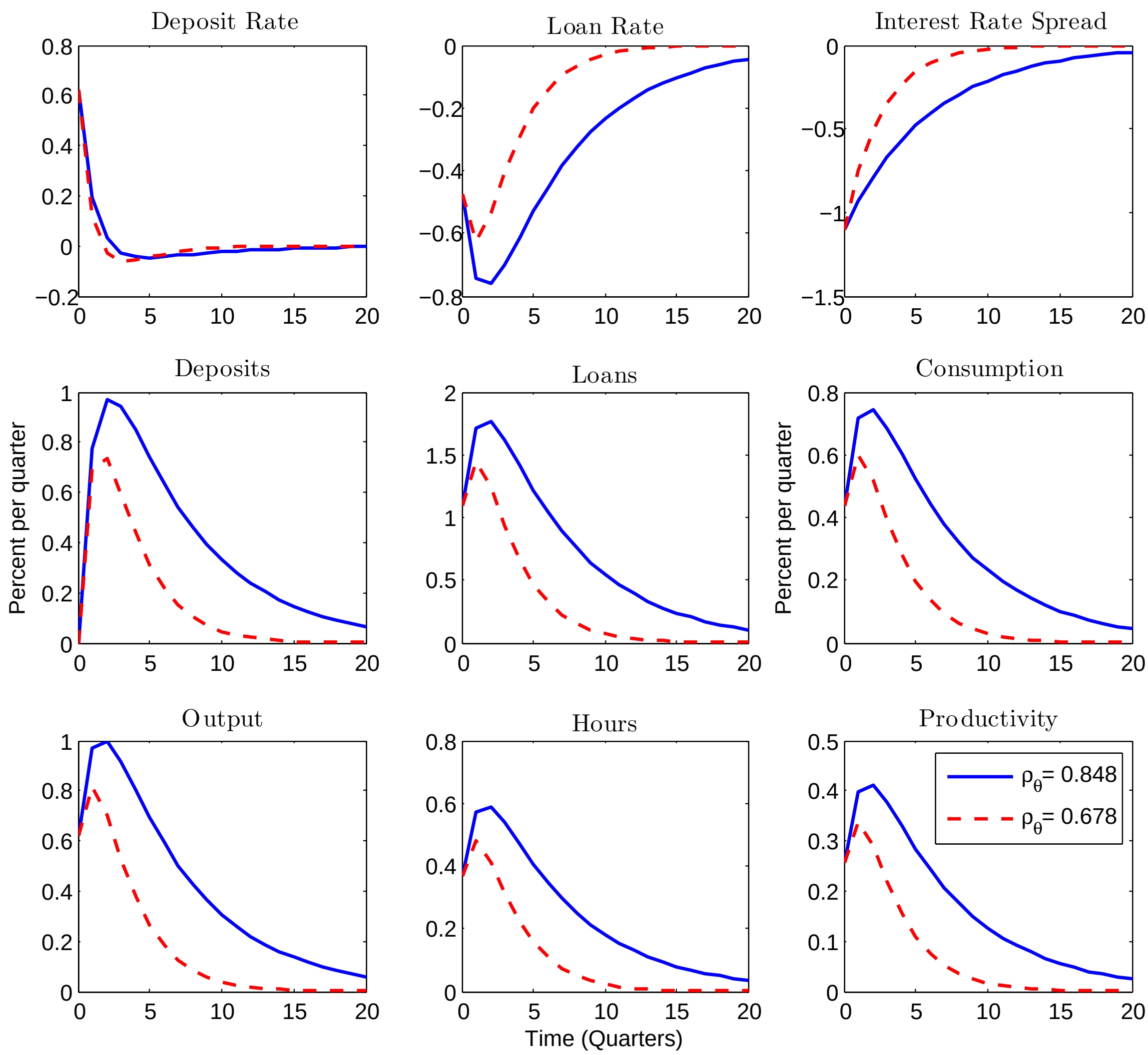
First, we examine the sensitivity of the impulse responses of a positive credit shock to the mean loan to deposit ratio, μ_θ. As we have seen in Table 2, the steady state values of output per capita, consumption per capita and interest rate spreads are sensitive to the choice of μ_θ. The impulse responses in Figure 3 suggest that, while the steady state values are sensitive to the value of μ_θ, the impulse responses to a positive credit shock and the dynamics of the model are robust to the mean of loan to deposit ratio in the economy. We also find similar results for the technology shock.[30]

[30]Due to space considerations, the impulse responses to a positive technology shock are not presented here, but are available upon request.

6.2.2 Lower persistence in the credit shock

In the second experiment, we consider the robustness of our results to the value of ρ_θ, the autoregressive coefficient of the credit shock. We reduce the benchmark estimate of ρ_θ from 0.848 to 0.678 (by 20%). The results are displayed in Figure 4. We observe faster convergence in the impact of the credit shock when the autoregressive coefficient is reduced, in particular, it takes around 8 to 10 quarters for the impact to vanish, as compared to around 20 quarters in the benchmark case. The magnitude of the response in the level of loans, output and consumption is robust on impact, around twice as large as the impact of a technology shock, consistent with the empirical evidence that recessions associated with financial crisis tend to be more severe than recessions associated with other shocks. We also find a lower value for the peak impact of a positive credit shock on output when ρ_θ is reduced.

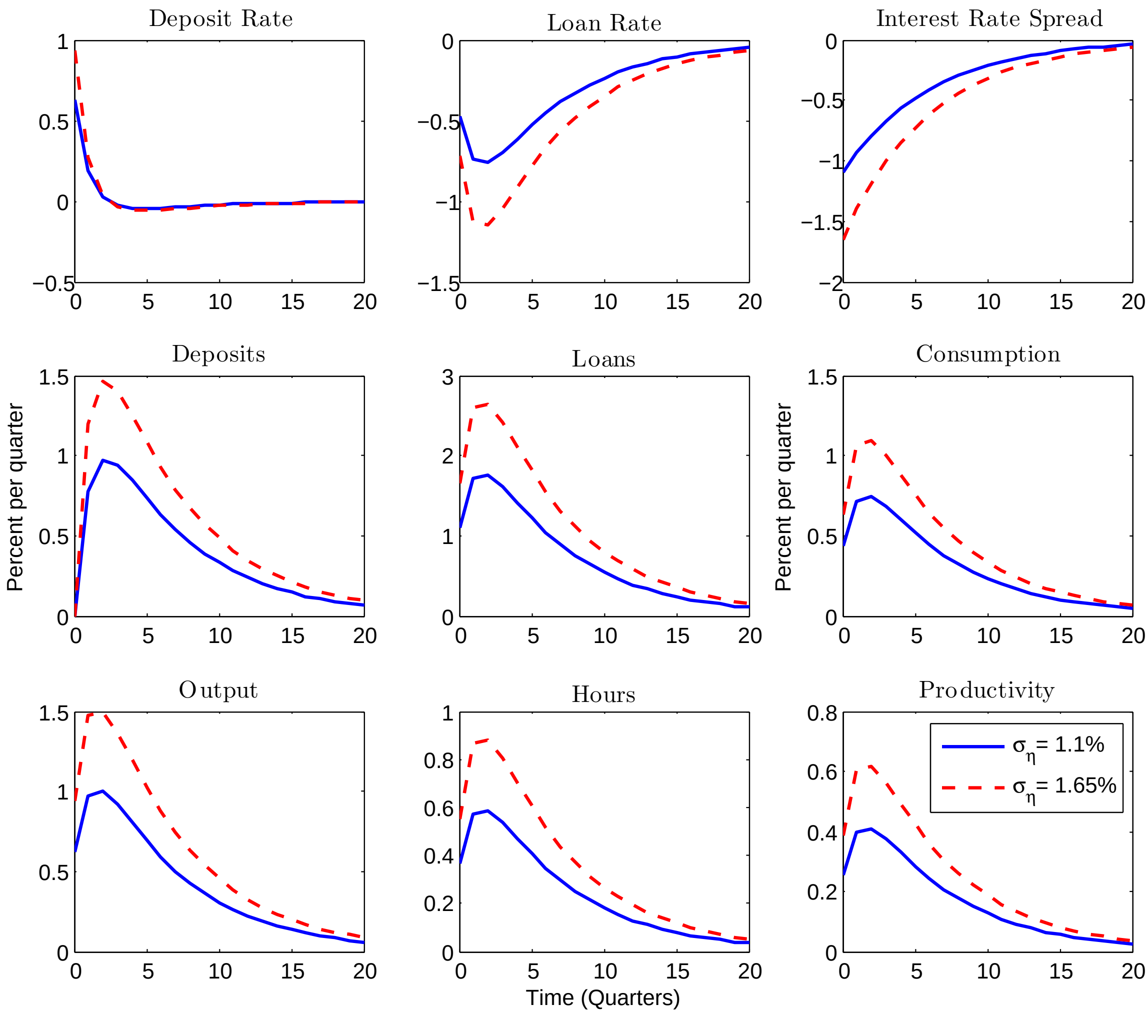
Figure 4: Impulse responses of one s.e positive credit shock
($\rho_\theta = 0.848$ **and** 0.678, $\mu_\theta = 1$)



6.2.3 Higher volatility in the credit shock

In the third experiment, we increase the volatility of the credit shock, as measured by σ_η, by 50% above the value implied by US data, thus increasing σ_η from 1.1% to 1.65%. As can be seen in Figure 5, the peak response in output, consumption and the level of loans is around 50% higher when $\sigma_\eta = 1.65\%$. We also note a slower rate of convergence to equilibrium. The volatility of the credit shock, σ_η, has an impact on the dynamics of the model since it affects the steady state conditions of the model through $E(ln\theta_t)$, given by equation (3.42), and therefore enters the coefficients of the log-linearised approximation of the equilibrium conditions and influences the dynamics of the model. This result confirms our finding that a credit shock could lead to profound impact on the real economy, both in terms of the magnitude and duration of the responses, and the impact could be even more severe during a banking crisis that is coupled with elevated market volatility.

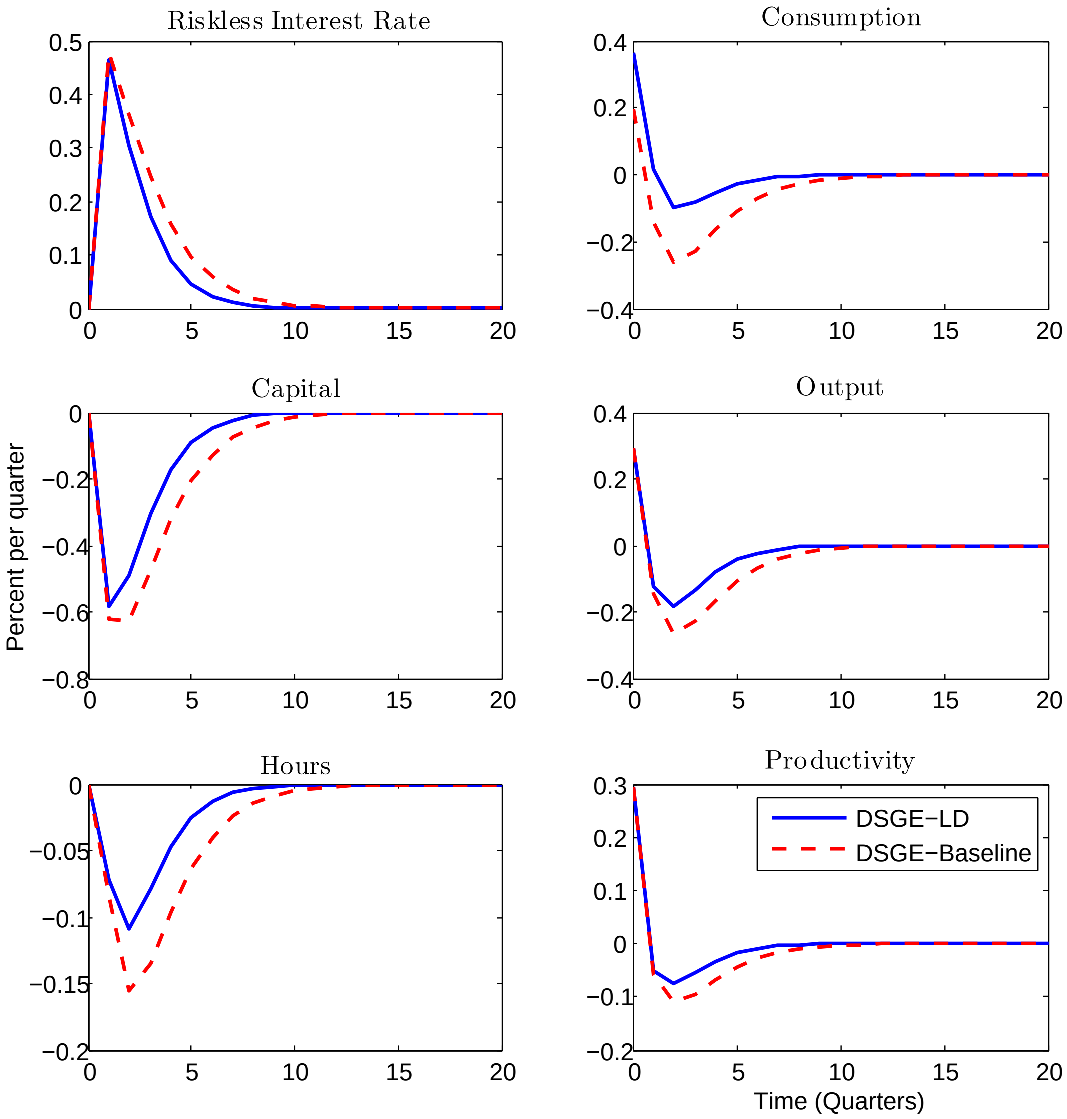
Figure 5: Impulse responses of one s.e positive credit shock ($\sigma_\eta = 1.1\%$ and 1.65%, $\mu_\theta = 1$)



6.3 Comparison to a baseline model

To determine the exact mechanism through which hours fall following a positive technology shock in our model, we compare the responses to a positive technology shock in our model (denoted by DSGE-LD) with a baseline model without the banking sector and firm defaults (denoted by DSGE-Baseline). The difference between the two models is twofold: first, in the simplified baseline model, we rule out idiosyncratic technology shocks and consider only the impact of a common technology shock on the economy; second, the household supplies all the capital demanded by the representative firm. With these two differences, we effectively "switch off" the banking sector and firm defaults in the simplified model economy, DSGE-Baseline.[31] As can be seen from Figure 6, the baseline model with a non-separable utility function and a reasonable coefficient of risk aversion ($\gamma = 1$) is also capable of generating a fall in labour hours in response to a positive technology shock. The comparison between the full model and the baseline model reveals that the existence of banking sector and firm defaults may not be the key features that generate the observed responses in hours and productivity following a positive technology shock.

Figure 6: Impulse responses of one s.e positive shock to technology (comparison with a baseline model, $\mu_\theta = 1$)



Instead, the negative responses in labour hours could possibly result from the specification of unit root in labour hours, the special functional form of non-separable utility in our model and the specification that technology shocks are realised after decisions on labour hours are made by firms. Empirical evidence on US hours data suggests the existence of a unit root in the hours series (see for example Francis and Ramey, 2005, 2009). We allow for a unit root in the technological process

[31] The details of the baseline model can be found in the Supplement, available from the authors upon request.

in our theoretical model, to generate the observed random walk property of US output, labour hours, consumption, loans and deposits.[32] Note also, Francis and Ramey (2009) show that if per capita labour input in empirical models is treated as a unit root process, the results predict a fall in labour input in response to a positive shock to technology. In addition, the special functional form of non-separable utility could contribute to the observed responses in hours and productivity, as the labour supply schedule is a function of real wage only and not of consumption.

It is important to note that the mechanism through which hours fall following a positive technology shock in this model differs from the existing literature. Gali (1999) proposes the use of sticky-price model where technology shocks have negative effects on labour in the short run.[33] Francis and Ramey (2005) and Smets and Wouters (2007) argue that a flexible price model with habit formation in consumption combined with adjustment costs in investment is also able to generate the negative correlation between productivity and hours. In Francis and Ramey (2005), habit persistence induces a sluggishness in the response of consumption and the high adjustment cost on investment makes investment relatively expensive in the short run and leave households with the only remaining alternative that is higher level of leisure, i.e. shorter hours. In our model, labour and capital move in the same direction, since there is no additional friction in the economy (such as adjustment cost or habit persistence in consumption), so that household would not respond differently to leisure and investment as in Francis and Ramey (2005).

Finally, we find that our results of a negative response in hours is robust to the choice of the coefficient of risk aversion. For reasonable values of γ, we observe a negative response in labour hours, with the magnitude of responses decreasing in the coefficient of risk aversion. The findings are consistent with Gali (2008, Chapter 2), which suggests that the responses in labour hours to a positive technology shock depend on the coefficient of risk aversion γ (which also measures the strength of wealth effect of labour supply in Gali's setting). However, the results from our calibrated model suggest that the negative responses in labour hours hold for a much wider range of parameter values. In particular, our model does *not* suggest a positive response in labour hours whenever $\gamma < 1$, as implied by Gali (2008, Chapter 2).[34]

7 Conclusion

This paper develops a *parsimonious* theoretical model for the analysis of the effects of credit and technology shocks on the real economy. It advances a new approach to modelling financial intermediation and firm defaults, and the financial implications of such defaults on behaviour of the household and the banking sector, without requiring collateral constraints and monitoring. It also incorporates growth into a DSGE framework and proposes a new method of computing steady states, which allows the steady state values of the model to depend on the volatility of the

[32]If we were to generate stationary hours in our model, it would require the technology process to be stationary, and results in a stationary output process which contradicts with overwhelming empirical evidence.

[33]Gali (1999) argues that a positive technology shock can lead to a decline in labour input if the monetary authority is not too accommodative. Other papers in favour of sticky price model include Basu, Fernald, and Kimball (2006), King and Wolman (1996) and Dotsey (2002).

[34]Using a model with separable isoelastic utility function and a production function with decreasing scale in labour (no capital), Gali (2008, Chapter 2) finds that when $\gamma < 1$, the substitution effect of labour supply resulting from a higher wage dominates the negative effect caused by a smaller marginal utility of consumption, leading to an increase in employment, with the converse being true whenever $\gamma > 1$.

technology and credit shocks.

We show that a positive credit shock, defined as a rise in the loan to deposit ratio, leads to an increase in available capital in the economy and a rise in output level, which is largely consistent with the empirical findings in the companion paper Xu (2010). The positive credit shock also drives down the spread between loan and deposit rates. The effects of the credit shock are found to be much more persistent and profound than the effects of a technology shock, consistent with empirical studies on the output effects of financial crises, which suggest that recessions associated with financial crisis have been more severe and long lasting than recessions associated with other shocks.

The current modelling framework can be extended and enhanced along several dimensions. First, a more elaborate banking sector including bank capital can be considered, to allow for endogenous credit and leverage shocks. Second, it would be important to consider the risks as well as the benefits of high leverage, the latter being highlighted in the current framework. One way of introducing potential costs of leverage is to augment the model with price rigidities and a central bank operating under a monetary policy rule such as the Taylor rule, to capture possible inflationary pressure from high leverage, and to study the policy implications of credit shocks. Alternatively, one could incorporate leverage costs in the production function, to establish a direct link between excess leverage and low productivity in the economy. Third, the assumption of full depreciation of capital can be relaxed, which would allow for a richer set of dynamic interactions in the economy through investment decisions. Finally, it would be important that the model is confronted with the times series data, using estimation methods along the lines of Smets and Wouters (2007), and Christiano, Motto, and Rostagno (2008).

A Proof of Lemma 1

Proof. Recall that λ_{it} is independently and identically distributed across i and t and $\lambda_{it} \sim N(0, \sigma_\lambda^2)$. Then

$$\begin{aligned}
\int_{-\infty}^{\varpi_1-\varepsilon_t} e^{\varphi x} f_\lambda(x)dx &= \int_{-\infty}^{\varpi_1-\varepsilon_t} \exp(\varphi x)\frac{1}{\sqrt{2\pi\sigma_\lambda^2}}\exp\left(-\frac{x^2}{2\sigma_\lambda^2}\right)dx \\
&= \frac{1}{\sigma_\lambda}\int_{-\infty}^{\varpi_1-\varepsilon_t}\frac{1}{\sqrt{2\pi}}\exp\left(\varphi x-\frac{x^2}{2\sigma_\lambda^2}\right)dx \\
&= \frac{1}{\sigma_\lambda}\int_{-\infty}^{\varpi_1-\varepsilon_t}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{1}{2\sigma_\lambda^2}\left[x^2-2\sigma_\lambda^2\varphi x+(\sigma_\lambda^2\varphi)^2\right]+\frac{\sigma_\lambda^2\varphi^2}{2}\right]dx \\
&= \frac{1}{\sigma_\lambda}\exp\left(\frac{\sigma_\lambda^2\varphi^2}{2}\right)\int_{-\infty}^{\varpi_1-\varepsilon_t}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(x-\sigma_\lambda^2\varphi)^2}{2\sigma_\lambda^2}\right]dx. \qquad \text{(A1)}
\end{aligned}$$

Now let $\varrho = \left(x-\sigma_\lambda^2\varphi\right)/\sigma_\lambda$, then

$$\begin{aligned}
\int_{-\infty}^{\varpi_1-\varepsilon_t}\frac{1}{\sqrt{2\pi}}\exp\left[-\frac{(x-\sigma_\lambda^2\varphi)^2}{2\sigma_\lambda^2}\right]dx &= \int_{-\infty}^{\frac{\varpi_1-\varepsilon_t-\sigma_\lambda^2\varphi}{\sigma_\lambda}}\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{\varrho^2}{2}\right)\sigma_\lambda d\varrho \\
&= \sigma_\lambda\Phi\left(\frac{\varpi_1-\varepsilon_t-\sigma_\lambda^2\varphi}{\sigma_\lambda}\right),
\end{aligned}$$

where $\Phi(\cdot)$ is the cumulative distribution of a standard normal. Then (A1) becomes

$$\int_{-\infty}^{\varpi_1-\varepsilon_t} e^{\varphi x} f_\lambda(x)dx = M_\lambda\Phi\left(\frac{\varpi_1-\varepsilon_t-\sigma_\lambda^2\varphi}{\sigma_\lambda}\right),$$

where $M_\lambda = \exp\left(\frac{\sigma_\lambda^2\varphi^2}{2}\right)$ and $\varpi_1 = \frac{\ln\left(1-\frac{\alpha}{\upsilon}\right)}{\varphi} + \frac{\varphi\sigma_\zeta^2}{2}$ from equations (3.24) and (3.22).
■

B Model Derivation and Solution

B1 Equilibrium conditions in efficiency units

The system of equation in (4.1) to (4.12) can be further simplified into a 9-equation system in 9 unknowns C_t, N_t, D_t, L_t, S_t, K_t, Y_t, R_{lt}, and R_{dt} by eliminating W_t, Π_{tb} and Π_{tc}. As stated earlier, since this model depicts a growing economy where technology grows with a deterministic trend μ, we must scale the endogenous variables C_t, L_t, D_t, S_t, K_t, N_t and Y_t in the system of equilibrium conditions by an appropriate factor of technology A_{t-1} so that they are stationary on a balanced growth path, to guarantee the existence of steady state in solving the model. Denote the variables in efficiency unit by capital letters with a dot, $\mathring{C}_t = \frac{C_t}{A_{t-1}}$, $\mathring{L}_t = \frac{L_t}{A_{t-1}}$, $\mathring{S}_t = \frac{S_t}{A_{t-1}}$, $\mathring{D}_t = \frac{L_t}{A_{t-1}}$, $\mathring{K}_t = \frac{K_t}{A_{t-1}}$, $\mathring{N}_t = \frac{N_t}{A_{t-1}^{1/(1+\chi)}}$, $\mathring{Y}_t = \frac{Y_t}{A_{t-1}}$.

The equilibrium conditions in efficiency units can be written as

$$
\begin{aligned}
1 &= E\left[\beta\left(\frac{\mathring{C}_{t+1}-\frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t-\frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}}\right)^{-\gamma} g_t^{-\gamma}R_{d,t+1}|\Omega_{ct}\right], \\
\mathring{Y}_t-\mathring{C}_t &= \mathring{S}_t+\mathring{D}_{t+1}g_t, \\
\mathring{K}_t &= \frac{\alpha\chi_0}{1-\alpha}\cdot\frac{\mathring{N}_t^{1+\chi}}{R_{lt}}, \\
\mathring{N}_t &= \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_\lambda M_\varepsilon\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}}\exp\left[\frac{\mu(1-\rho_u)}{1+\chi}\right](R_{lt})^{-\frac{\alpha}{(1-\alpha)\chi}}g_{t-1}^{\frac{\rho_u}{1+\chi}}, \\
\mathring{Y}_t &= M_\lambda g_t^{\varphi}\mathring{N}_t^{1-\alpha}\mathring{K}_t^{\alpha}, \\
\mathring{L}_t &= (1-\frac{1}{\upsilon})\mathring{K}_t,\ \ \mathring{K}_t=\mathring{L}_t+\mathring{S}_t, \mathring{L}_t=\theta_t\mathring{D}_t, \\
R_{dt}\mathring{D}_t &= R_{lt}\mathring{L}_t(1-\kappa)-\chi_0(\kappa-\tau)\mathring{N}_t^{1+\chi}.
\end{aligned}
$$

The above system of equations can be further simplified to a system of six equations by eliminating $\mathring{L}_t$, $\mathring{S}_t$ and $\mathring{K}_t$. The equilibrium conditions in terms of $\mathring{C}_t$, $\mathring{D}_t$, $\mathring{N}_t$, $\mathring{Y}_t$, R_{dt} and R_{lt} are then given by

$$
\begin{aligned}
1 &= E\left[\beta\left(\frac{\mathring{C}_{t+1}-\frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t-\frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}}\right)^{-\gamma} g_t^{-\gamma}R_{d,t+1}|\Omega_{ct}\right], && \text{(B1)} \\
\mathring{Y}_t-\mathring{C}_t &= \frac{1}{\upsilon-1}\theta_t\mathring{D}_t+\mathring{D}_{t+1}g_t, && \text{(B2)} \\
\theta_t\mathring{D}_t &= \frac{\alpha\chi_0(1-\frac{1}{\upsilon})}{1-\alpha}\cdot\frac{\mathring{N}_t^{1+\chi}}{R_{lt}}, && \text{(B3)} \\
\mathring{N}_t &= \left[\frac{1-\alpha}{\alpha\chi_0}\left(\alpha M_\lambda M_\varepsilon\right)^{\frac{1}{1-\alpha}}\right]^{\frac{1}{\chi}}\exp\left[\frac{\mu(1-\rho_u)}{1+\chi}\right](R_{lt})^{-\frac{\alpha}{(1-\alpha)\chi}}g_{t-1}^{\frac{\rho_u}{1+\chi}}, && \text{(B4)} \\
\mathring{Y}_t &= M_\lambda g_t^{\varphi}\mathring{N}_t^{1-\alpha}\left[\frac{\theta_t\mathring{D}_t}{(1-\frac{1}{\upsilon})}\right]^{\alpha}, && \text{(B5)} \\
R_{dt}\mathring{D}_t &= R_{lt}\theta_t\mathring{D}_t(1-\kappa)-\chi_0(\kappa-\tau)\mathring{N}_t^{1+\chi}. && \text{(B6)}
\end{aligned}
$$

B2 Derivation of the steady states

Denote the variables in steady state by the lower case letters with a star, for example, the steady state of consumption is given by $\mathring{c}^*=E(ln\mathring{C}_t)$ and the steady state of loan rate is $r_l^*=E(\ln R_{lt})\approx E(r_{lt})$. To derive the steady state, we first take logarithm of the equilibrium conditions (B3) to (B5) and take unconditional expectations of the resulting equations, we have

$$
\begin{aligned}
E(ln\theta_t)+\mathring{d}^* &= ln\chi_0+ln\left[\frac{\alpha(1-\frac{1}{\upsilon})}{1-\alpha}\right]+(1+\chi)\mathring{n}^*-r_l^*, \\
\mathring{n}^* &= -\frac{ln\chi_0}{\chi}+\frac{1}{\chi}ln\left[\frac{1-\alpha}{\alpha}(\alpha M_\lambda M_\varepsilon)^{\frac{1}{1-\alpha}}\right]+\frac{\mu}{1+\chi}-\frac{\alpha}{(1-\alpha)\chi}r_l^*, \\
\mathring{y}^* &= ln\left[\frac{M_\lambda}{(1-\frac{1}{\upsilon})^{\alpha}}\right]+\varphi\mu+(1-\alpha)\mathring{n}^*+\alpha E(ln\theta_t)+\alpha\mathring{d}^*,
\end{aligned}
$$

as $ln(g_t)=\mu+u_t$ and $E(lng_t)=\mu$.

To obtain the steady state conditions for equation (B2), first note that $\mathring{C}_t$ can be approximated as follows

$$\mathring{C}_t = e^{\mathring{c}_t} \approx e^{\mathring{c}^*}(1+\widetilde{\mathring{c}}_t)$$

where $\widetilde{\mathring{c}}_t = \mathring{c}_t - \mathring{c}^*$. Then equation (B2) can be approximated by

$$e^{\mathring{y}^*}(1+\widetilde{\mathring{y}}_t) - e^{\mathring{c}^*}(1+\widetilde{\mathring{c}}_t) = \frac{e^{E(ln\theta_t)+\mathring{d}^*}}{\upsilon-1}(1+\widetilde{ln\theta_t})(1+\widetilde{\mathring{d}}_t) + e^{\mathring{d}^*}(1+\widetilde{\mathring{d}}_{t+1})e^{\mu}(1+\widetilde{lng_t})$$

where $\widetilde{\ln g_t} = \ln g_t - E(\ln g_t)$ and $\widetilde{\ln \theta_t} = \ln\theta_t - E(\ln\theta_t)$. Take unconditional expectation on both sides of the above equation and we have that in steady state

$$e^{\mathring{y}^*} - e^{\mathring{c}^*} = e^{\mathring{d}^*+\mu} + \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*}.$$

Similarly, the steady state condition for equation (B6) is given by

$$e^{r_d^*}e^{\mathring{d}^*} = e^{r_l^*+E(ln\theta_t)+\mathring{d}^*}(1-\kappa) - \chi_0(\kappa-\tau)e^{(1+\chi)\mathring{n}^*}.$$

Finally, to derive the steady state condition for equation (B1), first note that we can approximate $\mathring{C}_t$ and $\mathring{N}_t^{1+\chi}$ as follows

$$\frac{\mathring{C}_{t+1} - \frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t - \frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}} = \frac{1+\frac{1}{1-b_1}\widetilde{\mathring{c}}_{t+1} - \frac{b_1}{1-b_1}(1+\chi)\widetilde{\mathring{n}}_{t+1}}{1+\frac{1}{1-b_1}\widetilde{\mathring{c}}_t - \frac{b_1}{1-b_1}(1+\chi)\widetilde{\mathring{n}}_t},$$

where $b_1 = \frac{\chi_0}{1+\chi}e^{(1+\chi)\mathring{n}^*-\mathring{c}^*}$. Taking first order Taylor expansion, we have that following approximation

$$\begin{aligned} &\left(\frac{\mathring{C}_{t+1} - \frac{\chi_0}{1+\chi}\mathring{N}_{t+1}^{1+\chi}}{\mathring{C}_t - \frac{\chi_0}{1+\chi}\mathring{N}_t^{1+\chi}}\right)^{-\gamma} \\ \approx\ & 1 - \frac{\gamma}{1-b_1}\widetilde{\mathring{c}}_{t+1} + \frac{b_1\gamma}{1-b_1}(1+\chi)\widetilde{\mathring{n}}_{t+1} + \frac{\gamma}{1-b_1}\widetilde{\mathring{c}}_t - \frac{b_1\gamma}{1-b_1}(1+\chi)\widetilde{\mathring{n}}_t. \end{aligned}$$

Furthermore, we can approximate $g_t^{-\gamma}R_{d,t+1}$ in equation (B1) by the following

$$g_t^{-\gamma}R_{d,t+1} \approx b_2(1-\gamma\widetilde{\ln g_t} + \widetilde{r}_{d,t+1})$$

where $b_2 = e^{-\gamma\mu+r_d^*}$. Therefore, equation (B1) is approximated by

$$\frac{1}{\beta b_2} - 1 = E\left[-\frac{\gamma}{1-b_1}\left(\widetilde{\mathring{c}}_{t+1} - \widetilde{\mathring{c}}_t\right) + \frac{b_1\gamma(1+\chi)}{1-b_1}\left(\widetilde{\mathring{n}}_{t+1} - \widetilde{\mathring{n}}_t\right) - \gamma\widetilde{\ln g_t} + \widetilde{r}_{d,t+1}|\Omega_{ct}\right] \tag{B7}$$

Now, take unconditional expectation on both sides of (B7) and by the law of iterated expectations, the right hand side of equation (B7) is equal to zero. In steady state, we then have $\beta b_2 = 1$, which together with $b_2 = e^{-\gamma\mu+r_d^*}$, we obtain $r_d^* = \gamma\mu - \ln\beta$.

B3 Loglinearisation

We log-linearise the system of equilibrium equations around the steady state of the log of the variables and denote the variables with a tilde the log deviations from the steady state of the log of the variables, e.g. $\widetilde{\mathring{c}}_t = \mathring{c}_t - \mathring{c}^*$. Note again that $\mathring{C}_t$ can be approximated as $\mathring{C}_t = e^{\mathring{c}_t} \approx e^{\mathring{c}^*}(1+\widetilde{\mathring{c}}_t)$.

For equations (B3) to (B5), the log approximations are given by

$$\begin{aligned}
\widetilde{\ln\theta}_t + \widetilde{\mathring{d}}_t + \widetilde{r}_{lt} &= (1+\chi)\,\widetilde{\mathring{n}}_t \\
\widetilde{\mathring{n}}_t &= -\frac{\alpha}{(1-\alpha)\chi}\widetilde{r}_{lt} + \frac{\rho_u}{1+\chi}\widetilde{\ln g}_{t-1} \\
\widetilde{\mathring{y}} &= \varphi\widetilde{\ln g}_t + (1-\alpha)\,\widetilde{\mathring{n}}_t + \alpha\widetilde{\ln\theta}_t + \alpha\widetilde{\mathring{d}}_t
\end{aligned}$$

For equation (B2), first note that it can be approximated by

$$\begin{aligned}
& e^{\mathring{y}^*}(1+\widetilde{\mathring{y}}_t) - e^{\mathring{c}^*}(1+\widetilde{\mathring{c}}_t) \\
= \; & e^{\mathring{d}^*}(1+\widetilde{\mathring{d}}_{t+1})e^{\mu}(1+\widetilde{\ln g}_t) + \frac{1}{\upsilon-1}e^{E(ln\theta_t)}(1+\widetilde{\ln\theta}_t)e^{\mathring{d}^*}(1+\widetilde{\mathring{d}}_t)
\end{aligned}$$

while in the steady state

$$e^{\mathring{y}^*} - e^{\mathring{c}^*} = e^{\mathring{d}^*+\mu} + \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*}$$

then the log approximation is given by

$$e^{\mathring{y}^*-\mathring{c}^*}\widetilde{\mathring{y}}_t - \widetilde{\mathring{c}}_t = e^{\mathring{d}^*+\mu-\mathring{c}^*}\left(\widetilde{\mathring{d}}_{t+1} + \widetilde{\ln g}_t\right) + \frac{1}{\upsilon-1}e^{E(ln\theta_t)+\mathring{d}^*-\mathring{c}^*}\left(\widetilde{\ln\theta}_t + \widetilde{\mathring{d}}_t\right)$$

Similarly, the log-linearised approximation for equation (B6) is given by

$$\begin{aligned}
& e^{r_d^*+\mathring{d}^*}\left(\widetilde{r}_{dt} + \widetilde{\mathring{d}}_t\right) \\
= \; & e^{r_l^*+E(ln\theta_t)+\mathring{d}^*}(1-\kappa)\left(\widetilde{r}_{lt} + \widetilde{\mathring{d}}_t + \widetilde{ln\theta_t}\right) - \chi_0(\kappa-\tau)(1+\chi)e^{(1+\chi)\mathring{n}^*}\widetilde{\mathring{n}}_t.
\end{aligned}$$

Finally, to log-linearise equation (B1), first recall that it can be approximated by equation (B7). Note that $\frac{1}{\beta b_2} - 1 = 0$ in steady state, then we have the following log approximation

$$\widetilde{\mathring{c}}_t - b_1(1+\chi)\widetilde{\mathring{n}}_t = E\left[\widetilde{\mathring{c}}_{t+1} - b_1(1+\chi)\widetilde{\mathring{n}}_{t+1} + (1-b_1)\widetilde{\ln g}_t - \frac{1-b_1}{\gamma}\widetilde{r}_{d,t+1}|\Omega_{ct}\right].$$

The log-linearised approximation of the equilibrium conditions of the model are therefore, noting $\widetilde{\ln g}_t = u_t$,

$$\widetilde{\mathring{c}}_t - a_1\widetilde{\mathring{n}}_t = E\left(\widetilde{\mathring{c}}_{t+1} - a_1\widetilde{\mathring{n}}_{t+1} - a_3\widetilde{r}_{d,t+1}|\Omega_{ct}\right) + a_2 u_t, \tag{B8}$$

$$-\widetilde{\mathring{c}}_t + a_5\widetilde{\mathring{y}}_t - a_6\widetilde{\mathring{d}}_{t+1} = a_4\widetilde{\mathring{d}}_t + a_6 u_t + a_4\widetilde{\ln\theta}_t, \tag{B9}$$

$$\widetilde{\mathring{n}}_t + \frac{\alpha}{(1-\alpha)\chi}\widetilde{r}_{lt} = \frac{\rho_u}{1+\chi}u_{t-1}, \tag{B10}$$

$$a_9\widetilde{\mathring{n}}_t + a_7\widetilde{r}_{dt} - a_8\widetilde{r}_{lt} = -(a_7-a_8)\,\widetilde{\mathring{d}}_t + a_8\widetilde{ln\theta_t}, \tag{B11}$$

$$\widetilde{\mathring{y}}_t - (1-\alpha)\,\widetilde{\mathring{n}}_t = \alpha\widetilde{\mathring{d}}_t + \varphi u_t + \alpha\widetilde{\ln\theta}_t, \tag{B12}$$

$$(1+\chi)\,\widetilde{\mathring{n}}_t - \widetilde{r}_{lt} = \widetilde{\mathring{d}}_t + \widetilde{\ln\theta}_t, \tag{B13}$$

where

$$\begin{aligned}
a_1 &= b_1(1+\chi),\ a_2 = 1-b_1,\ a_3 = \frac{1-b_1}{\gamma}, \\
a_4 &= \frac{1}{\upsilon-1} e^{E(ln\theta_t)+\mathring{d}^*-\mathring{c}^*},\ a_5 = e^{\mathring{y}^*-\mathring{c}^*},\ a_6 = e^{\mathring{d}^*+\mu-\mathring{c}^*}, \\
a_7 &= e^{r_d^*+\mathring{d}^*},\ a_8 = e^{r_l^*+E(ln\theta_t)+\mathring{d}^*}(1-\kappa), \\
a_9 &= \chi_0(\kappa-\tau)(1+\chi)e^{(1+\chi)\mathring{n}^*},\ b_1 = \frac{\chi_0}{1+\chi} e^{(1+\chi)\mathring{n}^*-\mathring{c}^*}.
\end{aligned}$$

Using the expressions for a_4, a_5, a_6, a_7, a_8 and a_9 above, steady state conditions (4.25) and (4.27) can be re-written as

$$a_5 = 1 + a_4 + a_6,\ a_9 = (1+\chi)(a_8 - a_7).$$

Note also the above system of equations can be simplified by substituting $\widetilde{\mathring{d}_t}$ from equation (B13) in equations (B11) and (B12), then the log-linearised equations (B9), (B11) and (B12) can be re-written as the expressions in the main body of the paper.

B4 Solution of the Canonical RE model

Proof. We show that $\mathbf{x}_t = \mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1}$ is indeed a solution of

$$\mathbf{x}_t = \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{G}_0\boldsymbol{\xi}_t + \mathbf{G}_1\boldsymbol{\xi}_{t-1}. \tag{B14}$$

First, note that the left hand side of (B14) can be written as

$$\begin{aligned}
\mathbf{x}_t &= \mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1} \\
&= \mathbf{C}(\mathbf{C}\mathbf{x}_{t-2}+\mathbf{D}_0\boldsymbol{\xi}_{t-1}+\mathbf{D}_1\boldsymbol{\xi}_{t-2}) + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1} \\
&= \mathbf{C}^2\mathbf{x}_{t-2}+\mathbf{D}_0\boldsymbol{\xi}_t+(\mathbf{C}\mathbf{D}_0+\mathbf{D}_1)\boldsymbol{\xi}_{t-1}+\mathbf{C}\mathbf{D}_1\boldsymbol{\xi}_{t-2}.
\end{aligned}$$

To evaluate the right hand side of (B14), note that

$$\begin{aligned}
E(\mathbf{x}_{t+1}|\Omega_t) &= E(\mathbf{C}\mathbf{x}_t + \mathbf{D}_0\boldsymbol{\xi}_{t+1} + \mathbf{D}_1\boldsymbol{\xi}_t|\Omega_t) \\
&= \mathbf{C}\mathbf{x}_t + \mathbf{D}_0 E(\boldsymbol{\xi}_{t+1}|\Omega_t) + \mathbf{D}_1\boldsymbol{\xi}_t \\
&= \mathbf{C}(\mathbf{C}\mathbf{x}_{t-1} + \mathbf{D}_0\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_{t-1}) + \mathbf{D}_0\mathbf{R}\boldsymbol{\xi}_t + \mathbf{D}_1\boldsymbol{\xi}_t \\
&= \mathbf{C}^2\mathbf{x}_{t-1} + (\mathbf{C}\mathbf{D}_0 + \mathbf{D}_0\mathbf{R} + \mathbf{D}_1)\,\boldsymbol{\xi}_t + \mathbf{C}\mathbf{D}_1\boldsymbol{\xi}_{t-1} \\
&= \mathbf{C}^2(\mathbf{C}\mathbf{x}_{t-2}+\mathbf{D}_0\boldsymbol{\xi}_{t-1}+\mathbf{D}_1\boldsymbol{\xi}_{t-2}) + (\mathbf{C}\mathbf{D}_0 + \mathbf{D}_0\mathbf{R} + \mathbf{D}_1)\,\boldsymbol{\xi}_t + \mathbf{C}\mathbf{D}_1\boldsymbol{\xi}_{t-1} \\
&= \mathbf{C}^3\mathbf{x}_{t-2} + (\mathbf{C}\mathbf{D}_0 + \mathbf{D}_0\mathbf{R} + \mathbf{D}_1)\,\boldsymbol{\xi}_t + (\mathbf{C}^2\mathbf{D}_0 + \mathbf{C}\mathbf{D}_1)\boldsymbol{\xi}_{t-1} + \mathbf{C}^2\mathbf{D}_1\boldsymbol{\xi}_{t-2}.
\end{aligned}$$

Therefore the right hand side of (B14) is given by

$$\begin{aligned}
& \mathbf{A}\mathbf{x}_{t-1} + \mathbf{B}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{G}_0\boldsymbol{\xi}_t + \mathbf{G}_1\boldsymbol{\xi}_{t-1} \\
=\ & \mathbf{A}(\mathbf{C}\mathbf{x}_{t-2}+\mathbf{D}_0\boldsymbol{\xi}_{t-1}+\mathbf{D}_1\boldsymbol{\xi}_{t-2}) + \mathbf{B}E(\mathbf{x}_{t+1}|\Omega_t) + \mathbf{G}_0\boldsymbol{\xi}_t + \mathbf{G}_1\boldsymbol{\xi}_{t-1} \\
=\ & \mathbf{A}(\mathbf{C}\mathbf{x}_{t-2}+\mathbf{D}_0\boldsymbol{\xi}_{t-1}+\mathbf{D}_1\boldsymbol{\xi}_{t-2}) + \mathbf{G}_0\boldsymbol{\xi}_t + \mathbf{G}_1\boldsymbol{\xi}_{t-1} \\
& +\mathbf{B}\mathbf{C}^3\mathbf{x}_{t-2} + (\mathbf{B}\mathbf{C}\mathbf{D}_0 + \mathbf{B}\mathbf{D}_0\mathbf{R} + \mathbf{B}\mathbf{D}_1)\,\boldsymbol{\xi}_t + (\mathbf{B}\mathbf{C}^2\mathbf{D}_0 + \mathbf{B}\mathbf{C}\mathbf{D}_1)\boldsymbol{\xi}_{t-1} + \mathbf{B}\mathbf{C}^2\mathbf{D}_1\boldsymbol{\xi}_{t-2} \\
=\ & (\mathbf{A}\mathbf{C}+\mathbf{B}\mathbf{C}^3)\mathbf{x}_{t-2} + (\mathbf{G}_0+\mathbf{B}\mathbf{C}\mathbf{D}_0 + \mathbf{B}\mathbf{D}_0\mathbf{R} + \mathbf{B}\mathbf{D}_1)\boldsymbol{\xi}_t \\
& +(\mathbf{A}\mathbf{D}_0 + \mathbf{G}_1 + \mathbf{B}\mathbf{C}^2\mathbf{D}_0 + \mathbf{B}\mathbf{C}\mathbf{D}_1)\boldsymbol{\xi}_{t-1} + (\mathbf{A}\mathbf{D}_1 + \mathbf{B}\mathbf{C}^2\mathbf{D}_1)\boldsymbol{\xi}_{t-2}.
\end{aligned}$$

Equating the coefficients of $\mathbf{x}_{t-2}$, $\boldsymbol{\xi}_t$, $\boldsymbol{\xi}_{t-1}$ and $\boldsymbol{\xi}_{t-2}$ on both sides of (B14), we have

$$\begin{aligned}
\mathbf{x}_{t-2} &: \mathbf{C}^2 = \mathbf{AC}+\mathbf{BC}^3, \\
\boldsymbol{\xi}_t &: \mathbf{D}_0 = \mathbf{G}_0+\mathbf{BCD_0} + \mathbf{BD_0R} + \mathbf{BD}_1, \\
\boldsymbol{\xi}_{t-1} &: \mathbf{CD}_0+\mathbf{D}_1 = \mathbf{AD_0} + \mathbf{G}_1 + \mathbf{BC}^2\mathbf{D}_0 + \mathbf{BCD_1}, \\
\boldsymbol{\xi}_{t-2} &: \mathbf{CD}_1 = \mathbf{AD_1} + \mathbf{BC}^2\mathbf{D}_1.
\end{aligned}$$

The above conditions can be simplified to

$$\begin{aligned}
\mathbf{BC}^2 + \mathbf{A} - \mathbf{C} &= 0,\ \mathbf{D}_1 = (\mathbf{I}-\mathbf{BC})^{-1}\mathbf{G}_1, \\
(\mathbf{I}-\mathbf{BC})\mathbf{D}_0 - \mathbf{BD}_0\mathbf{R} &= \mathbf{G}_0 + \mathbf{BD}_1,
\end{aligned}$$

that is

$$(\mathbf{I}-\mathbf{BC})\mathbf{D_0} - \mathbf{BD}_0\mathbf{R} = \mathbf{G}_0 + \mathbf{B}(\mathbf{I}-\mathbf{BC})^{-1}\mathbf{G}_1$$

which is the solution given by (4.38). ■

C Data Appendix

C1 Data sources

The main sources of the time series data are Datastream and the Federal Reserve.

C1.1 Deposit rate

The Datastream series "US CD Secondary Market 1 Month - Middle Rate" (FRCDS1M), "US CD Secondary Market 3 Month - Middle Rate" (FRCDS3M), "US CD Secondary Market 6 Month - Middle Rate" (FRCDS6M) are used to constructed the deposit rate series used in the paper. The deposit rate is given by the arithmetic average of the 1M, 3M and 6M CD series. The source for the Datastream series is the Federal Reserve and the series are measured in percent per annum. The middle rate refers to the midpoint between the bid and offered rates.

We have decided to use the Datastream (Federal Reserve) series instead of the IFS series "US Certificate of Deposit rate 3 months (secondary market)" (60LC.ZF), since the Federal Reserve has a broader coverage of CD rates, to include the 1 Month and 6 Month CD rates, which we use to construct the final deposit rates series.

C1.2 Loan rate

We take the Datastream series "US Bank Prime Loan - Middle Rate" (FRBKPRM) as our preferred measure for loan rate. The source of this series is the Federal Reserve. The US Bank Prime Loan Rate is the rate posted by a majority of top 25 (by assets in domestic offices) insured U.S.-chartered commercial banks. Prime is one of the several base rates used by banks to price short-term business loans. Weekly figures are averages of 7 calendar days ending on Wednesday of the current week; monthly figures include each calendar day in the month. The interest rate is annualized using a 360-day year or bank interest and the middle rate refers to the midpoint between the bid and offered rates. We have decided to use the Datastream (Federal Reserve) series instead of the IFS series "Bank prime loan rate" (60P..ZF), in order to be consistent with the source of our deposit rate series.

C1.3 CPI series

Note that, both the series for loan rate and deposit rate are in nominal terms. In order to estimate our model, we would need to convert the nominal loan and deposit rates to real series, using a measure of inflation rate. We take the CPI series “Consumer Price Index for All Urban Consumers: All Items” from the Federal Reserve Bank of St Louis (series ID: CPIAUCSL). The series is seasonally adjusted and indexed at the years 1982-84 (=100).

C1.4 Consumption

We use the data series “US Real Personal Consumption Expenditures” (series ID: PCECC96) from Federal Reserve Bank of St Louis as our measure for consumption. The data source is U.S. Department of Commerce: Bureau of Economic Analysis and the quarterly series is seasonally adjusted in billions of chained 2005 prices.

C1.5 Output

The GDP series is taken from the Federal Reserve Bank of St Louis (series ID: GDPC96). The series is seasonally adjusted in billions of chained 2005 prices.

C1.6 Bank deposits

We use the Federal Reserve series “US commercial bank liabilities–deposits and borrowing” (source: Federal Reserve H8 Table) as a measure of bank deposits. The series is measured in billions of US dollars, current prices and seasonally adjusted. According to the Federal Reserve definition, www.federalreserve.gov/releases/h8/current/default.htm (page 2), deposits is composed of large time deposits and other deposits.

C1.7 Bank credit

The data series on bank credit (“US Commercial bank assets–bank credit”) is taken from the Federal Reserve H8 Table. According to the Federal Reserve definition, bank credit is comprised of securities in bank credit and loans and leases in bank credit.[35] The latter (loans and leases in bank credit) includes commercial and industrial loans, real estate loans and commercial loans. The reason that we are using credit series from the Federal reserve, rather than the IFS measure “Bank credit to the private sector” as in the empirical paper, is that there is no matching deposit series from the IFS, while such series exists in the Federal reserve. The data series on bank credit is seasonally adjusted and expressed in billions of dollars in current prices.

C1.8 Wage

The Federal Reserve series on “Average Hourly Earnings: Total Private Industries” is used as a proxy for wages. The data source is the Federal Reserve Bank of St Louise (Series ID: AHETPI), taken from the U.S. Department of Labor: Bureau of Labor Statistics. The data series is seasonally adjusted and expressed in dollars per hour.

C1.9 Hours worked

The data series on “Average Weekly Hours Private Non-farm United States” is taken from Datastream (USHKIP..O). The primary sources of the data is the Bureau of labor statistics (USDOL). The series is measured in hours and seasonally adjusted. It captures the expected or actual period of employment for the week, usually expressed in number of hours.

[35] See http://www.federalreserve.gov/releases/h8/current/default.htm (page 2) for details.

C1.10 Employment

The employment data ("All Employees: Total Private Industries") is given by the Federal Reserve Bank of St Louise (Series ID: USPRIV). The series is measured in thousands and seasonally adjusted. The source of the data is the U.S. Department of Labor: Bureau of Labor Statistics.

C1.11 US population

The US population data ("US Population: Mid-Month") is given by the Federal Reserve Bank of St Louis (series ID: POPTHM). The series is measured in thousands. The source is the U.S. Department of Commerce: Bureau of Economic Analysis.

C1.12 Liabilities of non-financial non farm corporate business

We take the Federal Reserve Flow of Funds series (levels data, Table L.102) on the liabilities of nonfarm nonfinancial corporate business, in particular, we are interested in the series on corporate bonds (Z1/FL103163003.Q), corporate equities (Z1/FL103164103) and loans and short-term paper (Z1/FL104140005.Q). The series are measured in millions of US dollar.

The data series are available upon request.

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