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Abstract

Dynamic stochastic general equilibrium models featuring imperfect competition and nominal rigidities have become central for the analysis of the monetary transmission mechanism and for understanding the conduct of monetary policy. However, it is agreed that the benchmark model fails to generate the persistence of output and inflation that is observed in the data. Moreover, it cannot provide a theoretically well-grounded justification for the interest rate smoothing behaviour of monetary authorities. This paper attempts to overcome these deficiencies by embedding a multiplicative habit specification in a New Keynesian model. We show that this particular form of habit formation can explain why monetary authorities smooth interest rates.

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models featuring imperfect competition and nominal rigidities have become central for the analysis of the monetary transmission mechanism and for understanding the conduct of monetary policy.[1] However, it is agreed that the benchmark model fails to generate the persistence of output and inflation[2] that is observed in the data. Moreover, it cannot provide a theoretically well-grounded justification for the interest rate smoothing behavior of monetary authorities. This paper attempts to overcome these deficiencies by embedding a multiplicative habit specification in a New Keynesian (NK) version of the DSGE model.

The role of habits has a long tradition in economics, but the modern approach draws on Becker (1992), and Carroll (2001) where habitual actions imply a positive relation between past and present consumption.[3] The interest in the role of habits has also been driven by its ability to account for a number of anomalies in first generation stochastic general equilibrium models, such as the equity premium puzzle, identified by Mehra and Prescott (1985), Abel (1990), and Campbell and Cochrane (1999). It has also been invoked to account for the excess smoothness of consumption (Muellbauer (1988); Deaton (1992)).

In this paper we put habit formation to another use. In particular we show that a habit formation model, suitably specified, can explain why monetary authorities smooth interest rates. In order to do this, we follow Kozicki and Tinsley (2002) and adopt a geometric form for the way in which the stock of habit accumulates from past consumption. This matters because the use of an additive habit stock otherwise violates reasonable postulates of a utility function. Wendner (2003) has shown that the multiplicative form of the habit term in the utility function, recently employed by Carroll (2000), Amato and Laubach (2004) and Fuhrer (2000) has some undesirable properties if the habit function is itself still additive. This problem does not arise if the subtractive (linear) form of the habit term in the utility function that was originally suggested by Muellbauer (1988) is used in combination with an additive habit formation function. More recently Corrado and Holly (2011) have shown

[1] Examples include Clarida et al. (1999), Goodfriend and King (1997), McCallum and Nelson (1999), and Woodford (2003b) among others.

[2] There have been numerous attempts to correct this shortcoming. Gali and Gertler (1999) argue that if a fraction of firms set the price of their own good equal to the previous period's average reset price plus the lagged inflation rate then there will be inertia in inflation; alternatively, Christiano et al. (2005) let a fraction of firms increase their own prices in line with the lagged inflation rate. Both have been criticized for being ad hoc.

[3] Habit can be internal so it is a household's previous consumption patterns that matter as in Fuhrer (2000) and Christiano et al. (2005), or habit can be external so it depends on what other households are also consuming as in Smets and Wouters (2007). Recently, Ravn et al. (2006) seek to differentiate between 'deep' habits that form at the level of individual goods from those formed from a consumption basket (which they denote 'superficial'). Habits in this paper are of the superficial variety.

that a geometric (multiplicative) process for habit formation addresses all the concerns that Wendner (2003) has raised.

Many empirical studies of central bank behavior suggest that the interest rates set by the monetary authorities move with a certain inertia in response to changes in economic conditions.[4] The Federal Reserve in the U.S., for example, gradually adjusts interest rates to the level that is expected to keep the inflation on target and to close the output gap. While the empirical performance of the New Keynesian model is substantially improved by the inclusion of a lagged-interest-rate in the monetary policy rule, a considerable debate has arisen over the interpretation of such a modification in relation to optimizing behavior. Models that incorporate a lagged interest rate as a determinant of the central bank's reaction function seem to be motivated mainly by a desire to rationalize the observed inertial character of interest rates, rather than by any plausible account of why such an objective is actually appropriate. We find that the multiplicative habit specification in the New Keynesian model can generate endogenous interest rate inertia. This finding is a step forward in explaining why monetary authorities smooth interest rates.

Previous empirical literature, based on vector auto-regressions (VARs), documents persistent hump-shaped responses of output and inflation to monetary policy shocks, see Christiano et al. (2005), and Smets and Wouters (2003) among others. The failure of the NK model to replicate this feature of the data, is referred to as the "persistence problem". As a means of accounting for such a problem, some authors have augmented the benchmark framework with potential sources of endogenous persistence. They have incorporated features such as consumption habits, indexation to lagged inflation in price-setting, rule-of-thumb behavior, or various adjustment costs. Altig et al. (2010), Christiano et al. (2005), and Fuhrer (2000) are prominent examples. As well as generating inertia in the setting of monetary policy, our approach also significantly improves the short run dynamics of the model. Multiplicative habits can give rise to an endogenous backward looking term in the new Keynesian Phillips curve, as well as the IS equation. In contrast to the existing literature, it is not inflation indexation that generates an aggregate supply relation with lagged inflation, but the consumer's desire to smooth consumption in the face of an aspiration level generated by consumption in the past.

One of the advantages of working with optimization-based models is that they facilitate policy evaluation in terms of the welfare of private agents. Amato and Laubach (2004) and Leith et al. (2012) are two examples of such models with habit formation mechanisms. To perform policy analysis, this paper follows the methodology developed by Rotemberg and

[4] See Woodford (1999), Woodford (2003a) and the refrences therein.

Woodford (1997), Benigno and Woodford (2003), and Woodford (2003b) in deriving the central bank's loss function as a second-order Taylor approximation to the habit-adjusted utility of the representative consumer. The resulting quadratic welfare criterion depends on inflation, the current output gap, and lagged output; while the weights given to these terms change with habit formation parameters. We also show that the model-driven loss function includes additional quadratic terms involving quasi-differences of output-gap that are missing from the benchmark NK model. These extra terms capture the welfare losses due to large changes in consumption growth rates at different time horizons.

We then turn to an analysis of the optimal policy problem, both when the monetary authority can commit to a certain future path for inflation and the output gap (optimal policy with commitment) and when such commitment is not feasible (optimal policy with discretion)[5]. Solving the central bank's optimization problem under discretion, it is shown that the monetary policy reaction function depends on lagged values of both inflation and the interest rate. This emerges because the representative consumer dislikes (i) large changes in consumption relative to the level to which they aspire, and (ii) large changes in consumption growth rates at different time horizons. The presence of a multiplicative habit stock alters the standard policy conclusions of the canonical New Keynesian model in important ways. First, the optimal discretionary policy cannot fully insulate the output gap and inflation from demand shocks, whereas under commitment– where a zero inflation policy is optimal– it still can. Secondly, the optimal policy under commitment– in response to a cost-push shock– produces significantly different paths for inflation and output compared to discretion.

The remainder of the paper is organized as follows. Section 2 presents the NK model with multiplicative habits. Section 3 derives a second order approximation to the utility of the representative agent and uses this to define the optimal policy problem under both commitment and discretion. Section 4 describes the simulation exercise under a particular parameterization and analyzes the economy's response to a demand/cost-push shock. Finally, Section 5 summarizes the results and concludes.

2 A Structural Model with Habit Formation

This section employs a standard New Keynesian framework based on the optimizing behavior of households and imperfectly competitive firms to analyze the consequences for optimal monetary policy of multiplicative habits in consumption. First, the IS equation is derived

[5]The literature has settled upon this particular distinction between a discretionary policy and one with commitment, but there is no reason why in place of a single period solution, we should compute a multiperiod solution but with re-optimisation in each subsequent period. See Holly and Zarrop (1983).

by combining the market clearing relation with the first order optimality condition for consumption, from a constant relative risk aversion (CRRA) utility function with a multiplicative habit stock. Second, a supply equation is derived under the assumption that firms have some monopolistic power, and face a constant probability of resetting prices each period, as in Calvo (1983). The model is closed in Section 3 where we approximate the utility of the representative consumer to obtain a quadratic loss function for the monetary authority and to perform the policy analysis.

2.1 Households

The model economy is inhabited by a measure one continuum of households who consume Dixit-Stiglitz aggregates of consumption goods[6] and supply labor. Each household seeks to maximize its expected lifetime utility from habit-adjusted consumption, $\not{C}_t = (C_t/H_t^\gamma)$, and leisure over time,

$$E_0\sum_{t=0}^{\infty}\beta^t\left[U(\not{C}_t) - V(N_t)\right], \tag{2}$$

subject to a sequence of budget constraints given by

$$P_tC_t + Q_tB_t = W_tN_t + B_{t-1} + T_t, \tag{3}$$

where B_t is the quantity of one-period nominal riskless discount bonds purchased in period t, and maturing in period $t+1$. Each bond is priced at Q_t and pays one unit of the numéraire at maturity. N_t denotes hours of labour, W_t is the nominal wage, and T_t are net lump-sum transfers/taxes. The period utility function in the household's maximization problem takes the following form

$$\frac{1}{1-\sigma}\left(\frac{C_t}{H_t^\gamma}\right)^{1-\sigma} - \frac{{N_t}^{1+\varphi}}{1+\varphi}, \tag{4}$$

where σ measures the curvature of the utility function, φ^{-1} is the Frisch elasticity of labour supply, and H_t represents the stock of habit, against which consumption in period t is evaluated. The parameter γ measures the importance of the habit stock in the utility function:

[6] Such indexes are in turn given by CES aggregators of the quantities consumed of each type of good (j).The optimal allocation of any given expenditure within each category yields the following demand function:

$$C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\varepsilon} C_t, \tag{1}$$

where $P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon}dj\right)^{\frac{1}{1-\varepsilon}}$ is the price index for the consumption basket. The elasticity of substitution between varieties within each category is given by $\varepsilon > 1$.

for example, if $\gamma = 0$, only the absolute level of consumption matters, while if $\gamma = 1$, then consumption relative to the stock of habit matters. This functional form makes the utility function time-non-separable, because a consumption choice today affects future stocks of habit. The reference level of consumption (the habit stock) can be expressed as a geometrically weighted average of past consumption,

$$H_t = C_{t-1}^{1-\delta} H_{t-1}^{\delta}.$$

This functional form corresponds to the multiplicative habit formation proposed by Kozicki and Tinsley (2002). With this formulation the consumer cares about, and forms habits over, consumption growth rates instead of consumption levels. Such a specification is appealing in the light of the findings of Wendner (2003) and Corrado and Holly (2011), who have shown that an additive habit aggregator, $H_t^a = (1-\delta)C_{t-1} + \delta H_{t-1}^a$, violates some reasonable postulates of a utility function.[7] The parameter δ measures the strength with which previous levels of consumption matter for current aspiration levels. In other words, δ indexes the persistence or "memory" in the stock of habit. If $\delta = 0$, then only last period's consumption is important. For $0 < \delta \leq 1$, the larger is δ, the further back in time are levels of consumption important for current consumption.

Corrado and Holly (2011) show that the four properties identified by Wendner (2003) as reasonable features of a utility function are satisfied by the multiplicative form. Firstly, an increase in the strength of habits, γ, with no change in current or past consumption, reduces utility. This happens because the larger is γ the less is the utility generated from current consumption. Hence, habit forming consumers will postpone consumption, given that consumers benefit not only from consumption levels but also from consumption growth (Deaton (1992)). Secondly, an increase in current consumption, with no change in past consumption, and therefore no change in the habit stock, increases utility. Thirdly, an increase in the habit stock with no change in current consumption reduces utility because when a consumer becomes used to a given stock of habit, less utility will be derived from a given amount of current consumption. Finally, an increase in the importance of a given habit stock in period t, as measured by γ, requires a lower marginal rate of substitution of C_t for C_{t+1}. Higher consumption today adds to the future habit stock which then lowers future effective consumption.

If we turn to the household's optimization problem we have the following set of first order

[7]For a recent review of habits in macroeconomics, see Schmitt-Grohe and Uribe (2008). Theoretical foundations for several common representations of intrinsic habit formation are provided in Rozen (2010).

conditions for B_t, C_t, and N_t, respectively (see Appendix A for more details).

$$E_t\left\{-\frac{Q_t}{P_t}\Lambda_t+\frac{\beta}{P_{t+1}}\Lambda_{t+1}\right\}=0, \tag{5}$$

$$E_t\left\{\frac{U_t}{C_t}-\frac{\beta\gamma(1-\delta)}{C_t}\sum_{j=1}^{n}(\beta\delta)^{j-1}U_{t+j}-\Lambda_t\right\}=0, \tag{6}$$

$$-(N_t)^{\varphi}+\Lambda_t\frac{W_t}{P_t}=0, \tag{7}$$

where Λ_t is the Lagrangian multiplier on the t-period flow constraint, with $\Lambda_t>0$. In what follows, lower case letters denote log-deviations from the steady state and capital letters indicate levels. Adopting this notation and log-linearizing (6), one obtains

$$-g_0c_t+\sum_{j=1}^{\infty}g_j(c_{t-j}+\beta^jE_tc_{t+j})=\lambda_t, \tag{8}$$

where the coefficients are defined as

$$\begin{aligned}
g_0 &= \frac{\sigma-\beta\gamma(1-\delta)\left[\frac{\gamma(1-\delta)(1-\sigma)}{1-\beta\delta^2}\right]}{1-\frac{\beta\gamma(1-\delta)}{1-\beta\delta}},\\
g_1 &= \frac{-\gamma(1-\delta)(1-\sigma)\left[1-\beta\gamma\delta(1-\delta)\right]}{1-\frac{\beta\gamma(1-\delta)}{1-\beta\delta}},\\
&\vdots\\
g_j &\simeq \delta^{j-1}g_1.
\end{aligned}$$

We can easily see that equation (5) in its log-linear form reproduces the term structure of interest rates.[8] Formally,

$$\lambda_t=E_t\lambda_{t+1}+i_t-E_t\pi_{t+1}+\log\beta, \tag{9}$$

in which $i_t\equiv-\log Q_t$ is the short-term nominal interest rate. Denoting the equilibrium deviations of the short-term real interest rate by $r_t=i_t-E_t\pi_{t+1}+\log\beta$, and solving (9)

[8] Clearly if there is no habit, $g_0=\sigma$, where σ measures the concavity of the utility function, and $g_j=0$, for $j=1,\infty$.

forward yields

$$\lambda_t = \sum_{i=0}^{\infty} E_t r_{t+i} \simeq \sum_{i=0}^{n-1} E_t r_{t+i} = nE_t\rho_t, \tag{10}$$

where ρ_t is the ex-ante real interest rate on an n-period bond. It is assumed that the infinite sum in (10) is finite for stationary real rate deviations. In other words, after n periods the interest rate converges to its long term value. Combining the log-linearized first order condition for consumption, (8), with (10), and using the market clearing condition $y_t = c_t$,

$$E_t \left\{ g_0 y_t - \sum_{j=1}^{\infty} g_j (y_{t-j} + \beta^j E_t y_{t+j}) + n\rho_t \right\} = 0.$$

In order to get this complicated expression into a more manageable form, we can express the infinite backward and forward summations above in terms of L, the lag operator, so $L^i y_t = y_{t-i}$, and F, the lead operator, so $F^i y_t = y_{t+i}$, as:

$$E_t \left\{ g_0 y_t + g_1 \frac{y_{t-1} + \beta y_{t+1}}{1 + \beta\delta^2 - \delta (L + \beta F)} + n\rho_t \right\} = 0,$$

which can be further simplified to obtain a dynamic IS equation in the presence of multiplicative habits,

$$y_t = \eta_1 (y_{t-1} + \beta E_t y_{t+1}) - \eta_2 n\rho_t + \eta_3 n(\rho_{t-1} + \beta E_t \rho_{t+1}), \tag{11}$$

where $\eta_1 = \frac{1}{1+\beta\delta^2}\left(\delta - \frac{g_1}{g_0}\right)$, $\eta_2 = \frac{1}{g_0}$, and $\eta_3 = \frac{\delta}{g_0\left(1+\beta\delta^2\right)}$. The crucial point here is that current output now depends on lagged output and the lagged interest rate as well as current and forward terms.

It is straight-forward to show that (11) reduces to the standard IS equation in the absence of consumption habits, i.e., when $\gamma = \delta = 0$, then:

$$y_t = E_t y_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} + \log \beta) = -\frac{1}{\sigma} \sum_{i=0}^{\infty} E_t r_{t+i} = -\frac{n}{\sigma} E_t \rho_t. \tag{12}$$

2.2 Firms

The model economy presented here consists of a continuum of households, each of which is a monopolistic supplier of one differentiated product. Suppliers face a downward sloping demand schedule for their goods, given by (1), which they produce using a linear production function.

$$Y_t(j) = N_t(j). \tag{13}$$

Assuming a symmetric equilibrium across all firms, the first order log linear approximation of the aggregate production function can be written as

$$y_t = n_t. \tag{14}$$

Final goods producers are assumed to set prices in a staggered fashion, as in Calvo (1983). Each period a fraction $(1-\theta)$ of suppliers, drawn randomly and independently of their own history, are able to adjust their prices in response to fluctuations in demand; whereas the rest, θ, have to keep their prices unchanged. Denote $P_t^{opt}(j)$ as the optimal price set by agent j in period t. Since all suppliers that reset their prices in any given period will choose the same price as they face similar demand functions, one can drop the j subscript. The optimal price setting then requires solving the following equation:

$$E_t \sum_{k=0}^{\infty} (\beta\theta)^k \left\{ \Lambda_{t+k} \left(\frac{P_t^{opt}}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \frac{P_t^{opt}}{P_{t+k}} - \tilde{V} \left[\left(\frac{P_t^{opt}}{P_{t+k}} \right)^{-\varepsilon} Y_{t+k} \right] \right\}, \tag{15}$$

where $\tilde{V}[Y_t(j)] = V[N_t(j)] = V(f^{-1}[N_t(j)])$. The first term in brackets represents the household's utility from consumption in period $t+k$ if price P_t^{opt} is chosen in the current period. It is the product of revenue (expressed in consumption units) in period $t+k$, conditional on price being P_t^{opt}; and the marginal utility of consumption, Λ_{t+k}. The second term represents the marginal disutility from supplying the amount of products in demand during period $t+k$ if price is still P_t^{opt}. It is easy to convert this term into units of an equivalent quantity of the consumption aggregate to obtain the real marginal cost and subsequently to rewrite (15) in terms of the firm's lifetime profits as in Galí (2008). Finally, since the price chosen in period t will remain unchanged k periods ahead with probability θ^k, the household discounts the stream of future utilities conditional on its choice of price today by $\beta\theta$.

Combining the law of motion for the aggregate price index with (8), and an expression for P_t^{opt} (derived from a supplier's maximization problem) provides the hybrid NKPC. Derivation details are given in Appendix B.

$$\pi_t = \frac{1}{\eta_2 + \beta\eta_3} E_t \left\{ \begin{array}{c} \kappa \left[(1+\varphi\eta_2)\, y_t - (\eta_1 + \varphi\eta_3)\,(y_{t-1} + \beta y_{t+1}) \right] \\ \\ +\, (\eta_2 + \eta_3)\, \beta\pi_{t+1} + \eta_3 \left(\pi_{t-1} - \beta^2 \pi_{t+2} \right) \end{array} \right\}, \tag{16}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\varphi\varepsilon)}$. One can easily verify that the augmented aggregate supply relationship in (16) reduces to the standard NKPC equation in the absence of consumption habits,

$\gamma = \delta = 0$,

$$\pi_t = \beta E_t \pi_{t+1} + \kappa (\sigma + \varphi) y_t .$$

In the canonical model the slope of the Phillips curve depends on both the curvature of the utility function with respect to consumption and leisure/labour supply, the extent of imperfectly competitive markets, and the stickiness of price setting. With an aspiration/habit level of consumption, equation 16 contains lagged and expected future output, because expected marginal revenues are now valued by the shadow-price of consumption, λ_t, which depends on current output and all its historical values. This effect of habit formation on the aggregate supply side is absent from the model of Fuhrer (2000) featuring an additive habit stock. What is also different from the existing literature is that π_{t-1} and $E_t\pi_{t+2}$ now enter the Phillips curve endogenously as a by-product of the Euler equation on the demand side of the model. It is the consumer's intertemporal consumption-labour choice via the marginal utility of consumption that gives rise to this hybrid NKPC.

3 Optimal Monetary Policy

Rotemberg and Woodford (1997) and Woodford (2003b) show that under certain conditions, a second order Taylor approximation to the expected present discounted value of the utility of the representative household is related inversely to a conventional quadratic loss function in inflation and the output gap.

3.1 The Quadratic Approximation to Welfare

We show in Appendix C how the central bank's objective function in the presence of consumption habits is derived as a second order approximation to the representative household's welfare. It is assumed that appropriate subsidies are in place such that the steady-state level of output is efficient despite the presence of imperfect competition (see Galí (2008), Ch. 5).[9] Therefore, monetary policy has no inflationary bias in this model.

$$W \equiv -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\varepsilon}{\kappa (1 + \varphi\varepsilon)} \pi_t^2 - (1 - \sigma)(y_t - \gamma h_t)^2 + (1 + \varphi) y_t^2 \right\} . \tag{17}$$

[9] Raissi (2011) discusses the implications of relaxing this assumption for the conduct of optimal monetary policy in a New Keynesian model with search and matching frictions.

The habit stock, in its log-linear form, is expressed as

$$\begin{aligned} h_t &= (1-\delta)y_{t-1} - \delta h_{t-1} \\ &= \frac{(1-\delta)}{1-\delta L} y_{t-1}. \end{aligned} \tag{18}$$

where L is the lag operator.

In contrast to the time-separable utility case without habits in consumption, the welfare criterion (17) depends not only on current output but also on output in the past. Quasi-differences of output, $y_t - \gamma h_t$, appear in the period loss function because of the intertemporal consumption linkages via multiplicative habits. In a very simple case in which the reference level of habit is tied to only one period of past consumption, $\delta = 0$, and when $\gamma = 1$, the welfare criterion becomes:

$$W \equiv -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\varepsilon}{\kappa (1+\varphi\varepsilon)} \pi_t^2 - (1-\sigma) \Delta y_t^2 + (1+\varphi) y_t^2 \right\},$$

in which the policy maker not only seeks to stabilize the output-gap, y_t, but also it tries to reduce the variability of output growth, Δy_t. The latter reduces welfare because of the dependence of the current shadow price of consumption on past consumption. As in the new Keynesian model, inflation also reduces utility because it leads to an inefficient composition of (habit-adjusted) consumption for a given level of output, due to the dispersion of relative prices. That is, even if total habit-adjusted consumption is equal to the efficient level, up to a first order, the composition of consumption across individual goods is inefficient in the presence of inflation (due to price stickiness).

In the absence of habit formation, $\gamma = \delta = 0$, the above welfare function collapses to

$$W = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\varepsilon}{\kappa (1+\varphi\varepsilon)} \pi_t^2 + (\sigma + \varphi) y_t^2 \right\}, \tag{19}$$

which is the utility-based criterion derived in Galí (2008).

3.2 Optimal Policy under Discretion

The optimal monetary policy problem is set out in this section for the case of discretion, i.e. where the central bank makes an optimal decision each period without committing itself to any future paths for inflation and output. Under discretion, the policy maker is unable to influence private sector expectations about future inflation. This implies the policy problem simplifies to a sequence of static optimizations. More specifically, each period the central bank is assumed to choose $(y_t,\, \pi_t)$ in order to minimize the single-period losses

$$\frac{\varepsilon}{\kappa\,(1+\varphi\varepsilon)}\pi_t^2 - (1-\sigma)\,(y_t - \gamma h_t)^2 + (1+\varphi)y_t^2 \tag{20}$$

subject to the inflation adjustment equation (16). The optimality condition for the problem above is given by

$$\begin{aligned} y_t &= -\frac{\varepsilon}{\kappa\,(1+\varphi\varepsilon)\,(\sigma+\varphi)}\pi_t\frac{\partial \pi_t}{\partial y_t}, \\ y_t &= -\psi\pi_t, \end{aligned} \tag{21}$$

where $\psi = \frac{\varepsilon}{(1+\varphi\varepsilon)(\sigma+\varphi)}\left(\frac{1+\varphi\eta_2}{\eta_2+\beta\eta_3}\right)$ and $t = 1,\, 2,\, \ldots$ Condition (21), as noted in Clarida et al. (1999) and Woodford (2003b), has a simple interpretation: in the face of inflationary pressure resulting from a cost-push shock (for example), the central bank must respond by driving output below its efficient level – thus creating a negative output gap – with the objective of dampening the rise in inflation. The central bank carries out such a "lean-against-the-wind" policy up to the point where this condition is satisfied.[10]

By replacing (21) in (11), one can derive a relationship for the ex-ante long-term interest rate as a function of inflation, or the monetary policy rule,

$$\rho_t = \frac{\psi}{n\eta_2}\pi_t - \frac{\psi\eta_1}{n\eta_2}\left(\pi_{t-1} + \beta E_t\pi_{t+1}\right) + \frac{\eta_3}{\eta_2}(\rho_{t-1} + \beta E_t\rho_{t+1}). \tag{22}$$

What is different in (22) from the previous literature is the interest rate smoothing behavior of the central bank. ρ_{t-1} enters the monetary policy reaction function endogenously, resulting in a gradual adjustment of the interest rate in response to shocks. To understand the deriving force behind this result, recall that the utility of the representative household depends on current consumption divided by a stock of multiplicative habit. Endogenous interest rate smoothing arises in the present model as a result of this particular habit specification. This inertia in the interest rate rule will disappear if the habit form is additive. Under Fuhrer's specification, where the reference level of habits is tied to only last period's consumption, $H_t^a = C_{t-1}$, the interest rate does not respond to $(\rho_{t-1} + \beta E_t\rho_{t+1})$ at all, because $\delta = 0$ and so is η_3. Section 4 shows how this modification, $\delta > 0$, affects the dynamics of the model under a standard calibration.

The monetary policy rule can be equivalently formulated as a function of the one-period real interest rate, $r_t = i_t - E_t\,(\pi_{t+1})$, or the infinite horizon sum of expected real policy

[10] Note also that if we think about this optimality condition in terms of the efficient policy frontier of Taylor (1979), then it corresponds to just one point on the frontier determined by the degree of imperfect competition (captured by ε) and the intertemporal elasticity of labour supply (inverse of φ).

rates, $\sum_{i=0}^{\infty} E_t r_{t+i}$, approximated by a long-term ex ante real bond rate, $nE_t\rho_t$.[11] The latter option is adopted in equation (22). It should be noted that under the assumption of rational expectations, the choice of the one-period policy rate or the n-period bond rate is irrelevant because either formulation is consistent with the simulated predictions of the full model.

To find a relationship between the short-term and the long-term interest rate in simulations, this paper follows Fuhrer and Moore (1995), and makes use of the intertemporal arbitrage condition that equalizes the expected real holding-period yields on a long-term bond and the short-term monetary instrument. The latter can be approximated by

$$\rho_t = n\left\{E_t\left(\rho_{t+1}\right) - \rho_t\right\} + \left\{i_t - E_t\left(\pi_{t+1}\right)\right\}, \tag{23}$$

where n represents the duration of the bond, which is assumed to be ten years, i.e. $n = 40$ at a quarterly rate. Using (23), any change in the short-term rate is transmitted to the long-term real interest rate over n periods. Solving equation (23) for ρ_t in terms of ρ_{t+1} and $i_t - E_t(\pi_{t+1})$, then recursively substituting the result into itself, the long term real rate is an exponentially weighted average of the forecast path of the real short term interest rates.

$$\rho_t = \frac{1}{1+n}\sum_{j=0}^{\infty}\left(\frac{n}{1+n}\right)^j E_t\left(i_{t+j} - \pi_{t+j+1}\right) \simeq \frac{1}{n}\sum_{j=0}^{n-1} E_t r_{t+j}.$$

It should be noted that in a canonical new Keynesian model without habits, $y_t = -\frac{\varepsilon}{1+\varphi\varepsilon}\pi_t$, and the monetary policy rule in terms of ρ_t or r_{t+j}, $j = 0, 1, ..., \infty$ can be written as

$$\rho_t = \tfrac{\varepsilon\sigma}{n(1+\varphi\varepsilon)}\pi_t \quad , \quad \textstyle\sum_{j=0}^{\infty} E_t r_{t+j} = \tfrac{\varepsilon\sigma}{1+\varphi\varepsilon}\pi_t.$$

3.3 Optimal Policy under Commitment

Discretion in setting monetary policy can lead to the so-called "stabilization bias" where output is over-stabilized while inflation is too volatile, and thus can result in lower welfare than under commitment. This bias is strictly a dynamic phenomenon, describing the economy's transition path toward its asymptotic equilibrium (from the initial state) and which depends on whether or not the monetary authority can precommit to a policy plan. This section lays out the central bank's optimization problem in the presence of multiplicative habits under the assumption that the monetary authority can commit to a certain future path of inflation and output. It illustrates the effects of "forward-looking expectations" and "the future habit reference levels" for the model dynamics. Under commitment, the

[11] As pointed out by Kozicki and Tinsley (2002).

optimal policy consists in choosing a state contingent sequence $\{y_t, \pi_t\}_{t=0}^{\infty}$ that minimizes the intertemporal model-driven loss function, (17), subject to the private sector's optimal behavior, as summarized in equations (11) and (16), and given a law of motion for the habit stock, (18). As illustrated in Clarida et al. (1999) and Woodford (2003b), the optimal policy under commitment implies some inertia in which the policy maker can take full advantage of the forward-looking plans of the representative agent, as well as the link from current consumption to the future shadow prices of consumption (or habit stocks).

4 Equilibrium Dynamics (Discretion vs. Commitment)

To explore the monetary policy reaction under optimal discretion and commitment, the model is calibrated and solved numerically.[12] The economy's responses to an exogenous demand shock and a cost-push disturbance are then compared under two different habit forming mechanisms. The first (solid lines) corresponds to the additive habit specification of Fuhrer (2000), in which $\delta = 0$, while the other (dashed lines) represents the multiplicative habit stock used in this paper, where $\delta = 0.5$.

4.1 Calibration

The baseline quarterly calibration of the model parameters to U.S. data is summarized in Table 1. The discount factor β is set to the conventional value of 0.99, which corresponds to a riskless annual return of about 4%. The parameter ε, the elasticity of substitution among differentiated goods, is set equal to 7.88. This reads into a mark-up of prices over marginal costs of 14%. The degree of price stickiness, measured by θ, is set equal to 0.75, which implies an average frequency of price adjustment of four quarters. Rotemberg and Woodford (1997) argue that a 14% markup is a plausible value to be consistent with firms engaging in staggered price adjustments.

The coefficient of relative risk aversion σ is usually assumed to take values in the interval $[1, 6]$, see Chari et al. (1997). Estimates of σ based on aggregate consumption data by Hall (1988) suggests a value of 3; while Rotemberg and Woodford (1997) provide an estimate of 0.16, which they obtain by matching their model's impulse response functions to those obtained from a VAR using U.S. data. As they point out, their low estimate of the relative risk aversion coefficient is related to the fact that σ measures the interest rate sensitivity of total output, not just that of nondurable consumption. In the absence of habits, $\gamma = \delta = 0$,

[12] See Soderlind (1999) for more details. All the simulations are performed in Dynare 4.3.0. and the codes are available upon request.

Table 1: Parameter Values

Discount factor	β	0.99
Coefficient of relative risk aversion	σ	2
Elasticity of substitution across goods	ε	7.88
Price adjustment probability	$1-\theta$	0.25
Inverse elasticity of labour supply	φ	3
Habit strength in utility function	γ	0.8
Habit persistence (memory)	δ	0.5
Duration of long-term bonds	n	40

the parameter σ in the present model has the same interpretation as in Rotemberg and Woodford (1997), implying that a low value would be of greater relevance. However, a value exceeding 1 is necessary for a positive effect of past consumption on current marginal utility, which captures the essence of habit formation. For this reason, in the preferences specification, σ is set to 2.

The parameter φ is set to 3 as in Galí and Monacelli (2005). This parameter measures the curvature of the disutility of labor and implies a Frisch labour supply elasticity of 1/3. Finally, the value of the strength of habits in the utility function, γ, is set to 0.8 based on the estimates of Fuhrer (2000). Since this paper focuses on exploring the effects of multiplicative habits, Sections 4.2 and 4.3 report the impulse responses for two values of δ. Specifically, this paper considers the additive case corresponding to Fuhrer (2000) where, $\delta = 0$, and the multiplicative specification of Kozicki and Tinsley (2002) in which $\delta = 0.5$ (the geometric form used in this paper).

4.2 Impulse Responses under Discretion

To illustrate graphically the effects of multiplicative consumption habits under a discretionary monetary policy, a simulation exercise is performed using the key log-linearized relations derived in Sections 2 and 3.2, namely

$$y_t = \eta_1 \left(y_{t-1} + \beta E_t y_{t+1} \right) - \eta_2 n \rho_t + \eta_3 n (\rho_{t-1} + \beta E_t \rho_{t+1}), \tag{24}$$

$$\pi_t = \frac{1}{\eta_2 + \beta \eta_3} E_t \left\{ \begin{array}{c} \kappa \left[\left(1 + \varphi \eta_2 \right) y_t - \left(\eta_1 + \varphi \eta_3 \right) \left(y_{t-1} + \beta y_{t+1} \right) \right] \\ \\ + \left(\eta_2 + \eta_3 \right) \beta \pi_{t+1} + \eta_3 \left(\pi_{t-1} - \beta^2 \pi_{t+2} \right) \end{array} \right\} \tag{25}$$

$$\rho_t = \frac{\psi}{n \eta_2} \pi_t - \frac{\psi \eta_1}{n \eta_2} \left(\pi_{t-1} + \beta E_t \pi_{t+1} \right) + \frac{\eta_3}{\eta_2} (\rho_{t-1} + \beta E_t \rho_{t+1}) \tag{26}$$

$$\rho_t = n \left\{ E_t \left(\rho_{t+1} \right) - \rho_t \right\} + \left\{ i_t - E_t \left(\pi_{t+1} \right) \right\} \tag{27}$$

where (24) is the dynamic IS relationship, (25) represents the hybrid NKPC, (26) is the discretionary policy rule derived from the utility-based welfare function and (27) is the equilibrium relationship between the short-term and the long-term real interest rates.[13]

Figure 1 plots the impulse responses of inflation, output and interest rates to a positive cost-push disturbance (which is added exogenously to the NKPC) under a discretionary optimal policy for the canonical new Keynesian model as well as for two different habit forming mechanisms. Note that the shock is one-off. There is no autocorrelation in the shock itself, so any persistence is completely endogenous. Clarida et al. (1999) show that the introduction of an ad hoc, exogenous cost-push shock allows the new Keynesian model to generate a meaningful policy problem in which there is a trade-off between stabilizing the inflation rate and reducing the output-gap. Following the shock, output falls and inflation increases initially due to the higher costs of production. However, geometric habits slow down the adjustment of the price level and generates a more muted response in output compared to the additive case. This is reflected in an inflation response that is both smaller on impact and more persistent afterwards. Furthermore, inflation, output, and the short and long term interest rates all display hump-shaped responses. For both habits the initial short term interest rate response is negative, though it is positive for the canonical model. Another feature of the results that stands out is the much smoother and humped-shaped adjustment of interest rates under multiplicative habits reflecting the inertia in the response of the monetary authorities to shocks. It is clear that a habit effect slows adjustment down

[13] It is often argued in the empirical macroeconomics literature that economic theory only tells us something about the long run, and nothing about short run dynamics. But it is clear from equations 24 to 27 that the dynamics of the model are directly a function of the structural parameters.

compared to the canonical new Keynesian model, but that the adjustment process is both much slower with multiplicative habits, and hump-shaped.

Figure 2 plots the impulse responses to a unitary demand shock under the three cases. A common conclusion from the New Keynesian literature is that shocks to the forward-looking IS curve can be neutralized by the central bank, so that neither inflation nor the output gap deviate from their flexible-price equilibrium. But here, by contrast, the optimal discretionary policy, with either additive or multiplicative habits, does not fully insulate the output gap and inflation from demand shocks. This result arises because the future habit stock is affected when current output changes, since the law of motion for habit is given by $h_{t+1} = (1 - \delta)y_t - \delta h_t$. However, h_{t+1} does not appear in the single period, discretionary, loss function. Because of the dampening effect of habit, interest rates cannot respond sufficiently robustly to choke off fully the effects of the demand shock. So both output and inflation rise. The immediate jumps in inflation and output are similar for the two habit formation cases, while the subsequent adjustments are quite different. With additive habits, inflation returns to steady state after two periods, while inflation with multiplicative habits is much more persistent. The monetary authority in both cases responds to the higher rate of inflation by increasing the short-term interest rate. However, under multiplicative habits the responses of the short-term policy rate and the long term rate exhibit more inertia compared to additive habits. With the canonical model, it only requires a one period change in interest rates to neutralize the effect of the shock to demand. Sharp changes in the policy rate are discouraged by the need to smooth output levels and growth rates under the multiplicative habit specification. Finally, the movement in the short term rate is ultimately transmitted to the long-term real interest rate through equation (23).

4.3 Impulse Responses under Commitment

We provide a similar set of simulations to section 4.2 but now with optimal commitment where expectations and future habit reference levels are taken into account. In particular, we minimize the intertemporal loss function, (17), subject to the log-linearized equilibrium conditions, (11), (16), and (18).[14] Because, compared to the discretionary case, the optimal policy is forward looking, inflation in Figure 3 is much better anchored when there is a cost shock. However, this is at the expense of a slower adjustment of output. This outcome is achieved by a cut in short term interest rates, with this not been reversed until the third or fourth period. In Figure 4 we show responses to a demand shock. In contrast to the discretionary case, we now find that monetary authorities can neutralize the effect of

[14] In this case, the optimized rule cannot be derived analytically.

the demand shock on inflation and output with habits. This is achieved by a one-period rise in the interest rate for the canonical new Keynesian model and for the additive habit specification. However, there is a considerably smoother response of the short term interest rate with multiplicative habits.

5 Concluding remarks

This paper has incorporated a multiplicative habit specification into a New Keynesian model to study its implications for optimal monetary policy and model dynamics, as well as to explain the interest rate smoothing behavior of monetary authorities. The log-linearized version of the model is shown to consist of a generalized IS equation, a hybrid new-Keynesian Phillips curve, and a monetary policy reaction function with an endogenous backward-looking component. All of these equations contain extra lag and lead terms that produce hump-shaped dynamics typically seen in estimated VARs. Interest rate inertia is also shown to be present when the stock of habits in the consumer's utility function is expressed as a geometrically weighted average of past consumption. The presence of multiplicative habits with a long memory also generates a substantial increase in the persistence of output and inflation following a demand shock.

We also derived an explicit second-order approximation to the welfare of the representative agent in the presence of multiplicative habits. The resulting model-driven loss function includes additional quadratic terms involving quasi-differences of output-gap that are missing from the standard NK loss function. These extra terms capture the welfare losses due to large changes in consumption growth rates at different time horizons and are very important for optimal monetary policy analysis. Multiplicative habits alter the central bank's optimal policy response in important ways. First, under discretion, the output gap and inflation fluctuate in response to demand disturbances even when the central bank is setting policy optimally; whereas, under commitment, shocks to the demand are neutralized (they do not affect neither the output gap nor the inflation). Second, the presence of multiplicative habits may give rise to additional stabilization biases where the optimal policy, either under commitment or discretion, cannot stabilize the output-gap and inflation simultaneously in the face of a cost-push shock.

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Appendix A: First Order Condition for Consumption

Beginning with the definition of period utility

$$U_t = \frac{1}{1-\sigma}\left(\frac{C_t}{H_t^{\gamma}}\right)^{1-\sigma},$$

the overall utility function

$$U = U_t + \beta U_{t+1} + ...$$

and the habit-formation reference consumption level

$$H_t = C_{t-1}^{1-\delta} H_{t-1}^{\delta}.$$

The derivative of U with respect to C_t is

$$\frac{\partial U}{\partial C_t} = \frac{\partial U_t}{\partial C_t} + \frac{\partial U_t}{\partial H_t}\frac{\partial H_t}{\partial C_t} + \beta\frac{\partial U_{t+1}}{\partial H_{t+1}}\frac{\partial H_{t+1}}{\partial C_t} + \beta^2\frac{\partial U_{t+2}}{\partial H_{t+2}}\frac{\partial H_{t+2}}{\partial C_t}...$$

Noting that

$$\begin{aligned}
\frac{\partial U_t}{\partial C_t} &= (1-\sigma)\frac{U_t}{C_t}, \\
\frac{\partial U_t}{\partial H_t} &= -\gamma(1-\sigma)\frac{U_t}{H_t}, \\
\frac{\partial H_{t+i}}{\partial C_t} &= \delta^{i-1}(1-\delta)\frac{H_{t+i}}{C_t},
\end{aligned}$$

then we can write:

$$\frac{\partial U}{\partial C_t} = (1-\sigma)\frac{U_t}{C_t} - (1-\sigma)\frac{\beta\gamma(1-\delta)}{C_t}\sum_{i=1}^{\infty}(\beta\delta)^{i-1}U_{t+i}.$$

Combining this with (3) in a Lagrangian, we obtain the first-order condition (6).

Appendix B: The Hybrid NKPC

The first step in deriving (16) is to take the partial derivative of (15) with respect to P_t^{opt} and approximate it around the steady state,

$$E_t\sum_{k=0}^{\infty}(\beta\theta)^k\left\{\lambda_{t+k} + p_t^{opt} - \sum_{i=1}^{k}\pi_{t+i} - \varphi\left[y_{t+k} - \varepsilon\left(p_t^{opt} - \sum_{i=1}^{k}\pi_{t+i}\right)\right]\right\} = 0, \qquad (28)$$

where $p_t^{opt} = \log\left(\frac{P_t^{opt}}{P_t}\right)$, and $\varphi = \frac{\nu_{yy}(Y;0)Y}{\nu_y(Y;0)}$. In the special case in which prices are flexible, $\theta = 0$, and because all firms choose the same price, p_t^{opt} would be zero. Therefore (28) simplifies to the condition that

$$\lambda_t = \varphi y_t.$$

In the general case where some firms cannot adjust their prices, i.e. $\theta > 0$, an expression for p_t^{opt} can be derived by substituting from (8) for λ_t in (28), or

$$p_t^{opt} = (1-\beta\theta)\, E_t \sum_{k=0}^{\infty} (\beta\theta)^k \left\{ \frac{1}{1+\varphi\varepsilon} \left[\begin{array}{c} g_1 \frac{y_{t+k-1}+\beta y_{t+k+1}}{1+\beta\delta^2-\delta(L+\beta F)} \\ \\ +\left(g_0+\varphi\right) y_{t+k} \end{array} \right] + \sum_{i=1}^{k} \pi_{t+i} \right\}. \tag{29}$$

From

$$P_t = \left(\int_0^1 P_t(j)^{1-\varepsilon} dj \right)^{\frac{1}{1-\varepsilon}},$$

the average price in period t satisfies the following law of motion

$$P_t = \left[\theta \left(P_{t-1}\right)^{1-\varepsilon} + (1-\theta)\left(P_t^{opt}\right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}.$$

Log-linearizing this equation yields

$$p_t^{opt} = \frac{\theta}{1-\theta} \pi_t. \tag{30}$$

Furthermore, the double sum in (29) can be simplified as

$$\sum_{k=0}^{\infty} (\beta\theta)^k \sum_{i=1}^{k} \pi_{t+i} = \frac{1}{(1-\beta\theta)} \left[\sum_{k=0}^{\infty} (\beta\theta)^k \pi_{t+k} - \pi_t \right]. \tag{31}$$

Substituting (30) and (31) into (29), and expressing the resulting expression in recursive form yields

$$\pi_t = \kappa \left[\left(g_0+\varphi\right) y_t + g_1 \frac{y_{t-1}+\beta E_t y_{t+1}}{1+\beta\delta^2-\delta\left(L+\beta F\right)} \right] + \beta E_t \pi_{t+1} + \zeta_t,$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)}{\theta(1+\varphi\varepsilon)}$ and ζ_t is a cost-push shock.

One can further simplify the above equation to complete the specification of inflation dynamics in (16).

$$\pi_t = \frac{1}{\eta_2 + \beta\eta_3} E_t \left\{ \begin{array}{c} \kappa \left[(1+\varphi\eta_2)\, y_t - (\eta_1 + \varphi\eta_3)\,(y_{t-1} + \beta y_{t+1}) \right] \\ \\ + (\eta_2 + \eta_3)\, \beta\pi_{t+1} + \eta_3 \left(\pi_{t-1} - \beta^2 \pi_{t+2} \right) \\ \\ -\eta_2 \zeta_t + \eta_3 \left(\zeta_{t-1} + \beta \zeta_{t+1} \right) \end{array} \right\}$$

Appendix C: Second Order Expansions

To drive a second order approximation to the representative household's welfare, it is necessary to introduce some additional notation. Let $x = \log\left(\frac{X_t}{X}\right)$ be the log deviation of any variable X_t around its steady-state value X. The following second order approximation of relative deviations in terms of log deviations is frequently used below,

$$X_t - X \simeq X\left(x_t + \frac{1}{2}x_t^2\right) + O^3,$$

where O^k indicates terms of order $k - th$ and higher in the size of the shocks. We assume that the average utility flow of the representative household follows

$$U(\not{C}_t) - V(N_t) = \frac{1}{1-\sigma}\not{C}_t^{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi},$$

where $\not{C}_t = \left(\frac{C_t}{H_t^\gamma}\right)$ is the habit-adjusted level of consumption. Taking a second order Taylor series expansion to the first term, $U(\not{C}_t)$, yields

$$U(\not{C}_t) - U(\not{C}) \simeq U_{\not{C}}(\not{C}_t - \not{C}) + \frac{1}{2}U_{\not{C}\not{C}}(\not{C}_t - \not{C})^2 + O^3.$$

In terms of log deviations,

$$U(\not{C}_t) - U(\not{C}) \simeq U_{\not{C}}\not{C}\left(\not{c}_t + \frac{1}{2}\not{c}_t^2\right) + \frac{1}{2}U_{\not{C}\not{C}}\not{C}^2\left(\not{c}_t + \frac{1}{2}\not{c}_t^2\right)^2 + O^3.$$

Further simplification, using the fact that $\sigma \equiv -\frac{U_{\not{C}\not{C}}}{U_{\not{C}}}\not{C}$, gives a more compact representation,

$$\frac{U(\not{C}_t) - U(\not{C})}{U_{\not{C}}\not{C}} \simeq \not{c}_t + \frac{1}{2}(1-\sigma)\not{c}_t^2 + O^3, \tag{32}$$

where $\not{c}_t = c_t - \gamma h_t = c_t - \frac{\gamma(1-\delta)}{1-\delta L} c_{t-1}$.

The next step is to approximate the disutility of labour following Galí (2008).

$$V(N_t) - V(N) \simeq V_N (N_t - N) + \frac{V_{NN}}{2} (N_t - N)^2 + O^3,$$

which can be rewritten as

$$V(N_t) - V(N) \simeq V_N N (n_t + \frac{1}{2} n_t^2) + \frac{V_{NN} N^2}{2} n_t^2 + O^3. \tag{33}$$

Assuming a constant return to scale technology, the aggregate level of labour, N_t, evolves according to

$$N_t = Y_t \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di.$$

Log linearizing the above expression around the zero-inflation steady state yields

$$n_t = y_t + \Delta_t,$$

where $\Delta_t = \log \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon} di$ is a measure of price dispersion. Galí (2008) shows that Δ_t is proportional to the cross sectional variance of relative prices, $\Delta_t = \frac{\varepsilon}{2} Var_i \left[P_t(i) \right]$ and therefore of second order. Furthermore, the expression $V(N_t) = \frac{N_t^{1+\varphi}}{1+\varphi}$, implies that $\varphi = \frac{V_{NN}}{V_N} N$, and then equation (33) becomes

$$\begin{aligned} V(N_t) - V(N) &\simeq V_N N (n_t + \frac{1}{2}(1+\varphi) n_t^2) + O^3 \\ &\simeq V_N N \left[y_t + \frac{1}{2}(1+\varphi) y_t^2 + \Delta_t \right] + O^3, \end{aligned}$$

where the last line makes use of the fact that Δ_t^2 and $\Delta_t y_t$ are of order four and three respectively. Finally using the steady state relationship $U_{\not{C}} \not{C} = -V_N N$, we arrive at the following expression

$$-\frac{V(N_t) - V(N)}{U_{\not{C}} \not{C}} \simeq y_t + \frac{1}{2}(1+\varphi) y_t^2 + \frac{\varepsilon}{2} Var_i \left[p_t(i) \right] + O^3. \tag{34}$$

Adding (34) to (32) and rearranging terms yields the following utility based welfare criterion

$$\frac{U(\not{C}_t) - V(N_t)}{U_{\not{C}} \not{C}} \simeq -\frac{1}{2} \left\{ \varepsilon Var_i \left[p_t(i) \right] - (1-\sigma) \not{c}_t^2 + \frac{1}{2}(1+\varphi) y_t^2 \right\} + O^3 + t.i.p.,$$

in which $t.i.p.$ represents terms independent of policy. Forming the expected sum $E_0 \sum_{t=0}^{\infty} \beta^t$ and rearranging terms across summands yields a second order approximation to the consumer's welfare losses expressed as a fraction of steady state consumption,

$$W \equiv -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{\varepsilon}{\kappa \left(1 + \varphi \varepsilon\right)} \pi_t^2 - \left(1 - \sigma\right) \not{c}_t^2 + (1 + \varphi) y_t^2 \right\},$$

in which we have used the fact that $\sum_{t=0}^{\infty} \beta^t Var_i \left[p_t(i)\right] = \frac{\theta}{(1-\theta)(1-\beta\theta)} \sum_{t=0}^{\infty} \beta^t \pi_t^2$.

Using the market clearing condition $c_t = y_t$ as well as the definition of $\not{c}_t$, we obtain equation (17) in the main text.

Figure 1: Impulse responses of selected variables under optimal discretionary policy to a one percent cost-push shock for three cases: (i) New Keynesian; (ii) Fuhrer's specification; and (iii) Multiplicative habits.

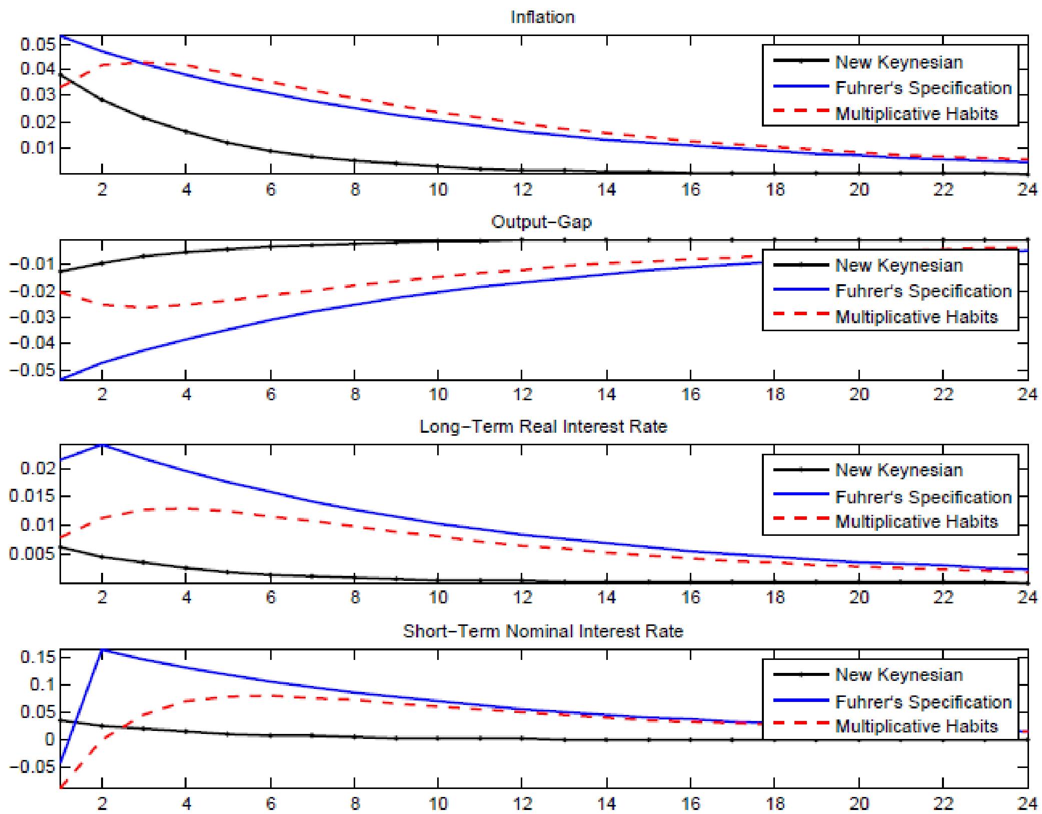

Figure 2: Impulse responses of selected variables under optimal discretionary policy to a unitary demand shock for three cases: (i) New Keynesian; (ii) Fuhrer's specification; and (iii) Multiplicative habits.

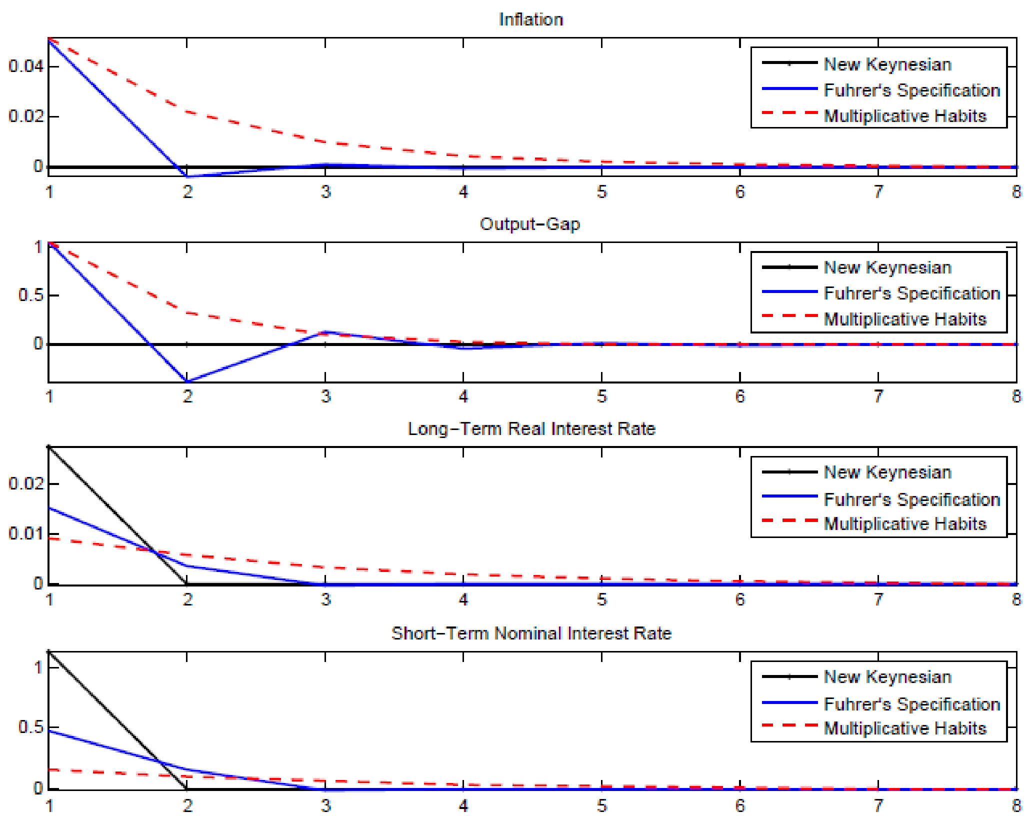

Figure 3: Impulse responses of selected variables under optimal commitment policy to a one percent cost-push shock for three cases: (i) New Keynesian; (ii) Fuhrer's specification; and (iii) Multiplicative habits.

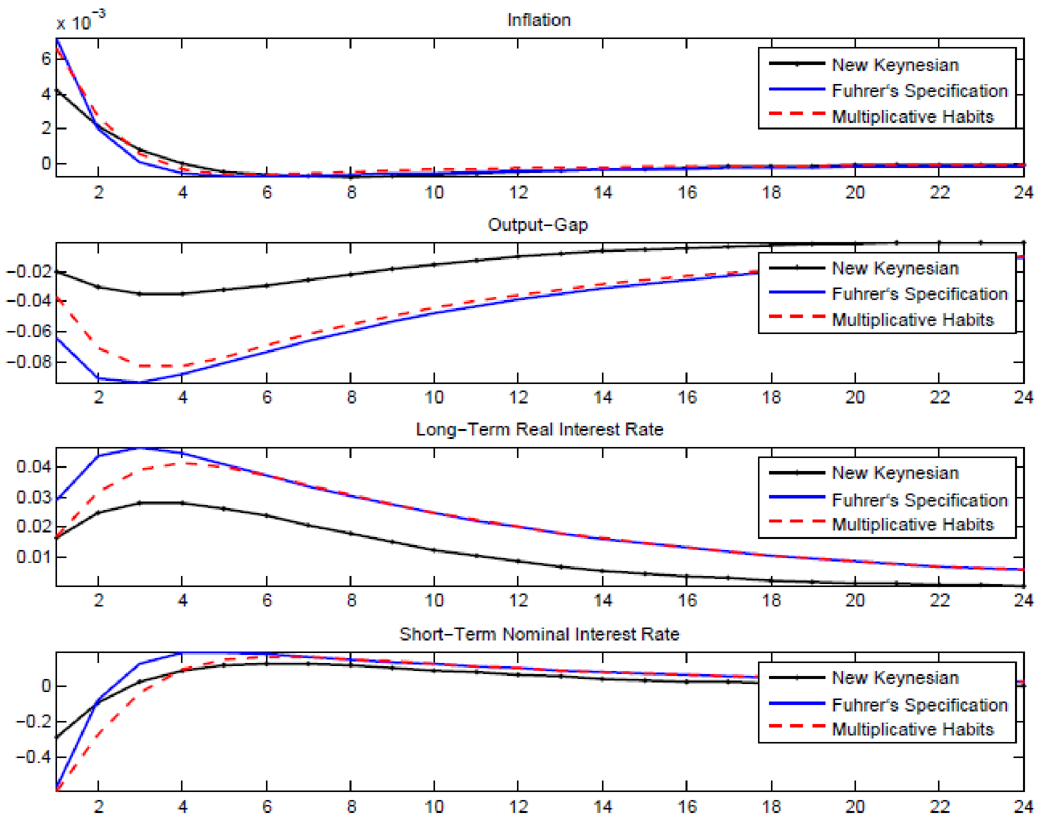

Figure 4: Impulse responses of selected variables under optimal commitment policy to a unitary demand shock for three cases: (i) New Keynesian; (ii) Fuhrer's specification; and (iii) Multiplicative habits.