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Uncertainty and Economic Activity: A Global Perspective

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Abstract

The 2007-2008 global financial crisis and the subsequent anemic recovery have rekindled academic interest in quantifying the impact of uncertainty on macroeconomic dynamics based on the premise that uncertainty causes economic activity to slow down and contract. In this paper, we study the interrelation between financial markets volatility and economic activity assuming that both variables are driven by the same set of unobserved common factors. We further assume that these common factors affect volatility and economic activity with a time lag of at least a quarter. Under these assumptions, we show analytically that volatility is forward looking and that the output equation of a typical VAR estimated in the literature is mis-specified as least squares estimates of this equation are inconsistent. Empirically, we document a statistically significant and economically sizable impact of future output growth on current volatility, and no effect of volatility shocks on business cycles, over and above those driven by the common factors. We interpret this evidence as suggesting that volatility is a symptom rather than a cause of economic instability.

Keywords: Uncertainty, Realized volatility, GVAR, Great Recession, Identification, Business Cycle, Common Factors.
JEL Codes: E44, F44, G15.

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1   Introduction

During the 2007-2008 global financial crisis, the world economy experienced a sharp and synchronized contraction in economic activity and an exceptional increase in macroeconomic and financial uncertainty/volatility. Indeed, after the VIX Index (the most commonly used measure of equity market volatility) spiked in the second half of 2008, world growth collapsed dramatically (Figure 1). Once started, the recovery has been unusually weak and uncertain. Many economic commentators and policy makers viewed the widespread and heightened uncertainty as one of the key factors behind the unusual depth, duration, and the degree of synchronization across countries of the ensuing recession, often referred as the "Great Recession" (see for example IMF, 2012). Given this experience, there is strong renewed academic interest in identifying and quantifying the impact of uncertainty on macroeconomic dynamics.
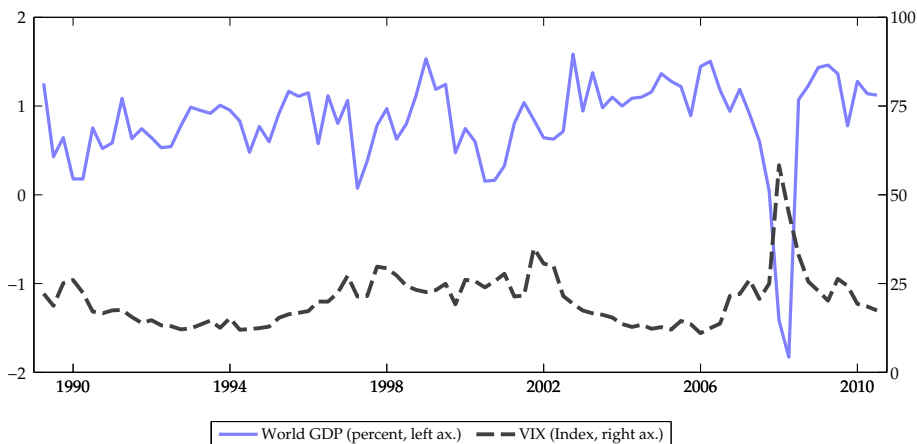
Figure 1 Quarterly World Gdp Growth And Vix Index. World GDP growth (quarter on quarter, in percent) is computed as the weighted average of the GDP of 33 advanced and developing economies—the same used in our empirical application—covering more than 90 percent of world GDP, using PPP-GDP weights. The sample period is 1990.I-2011.II.

In this paper, we approach the problem of modeling the interrelation between uncertainty and macroeconomic dynamics in the world economy as a two-way process. Specifically, we assume that both uncertainty and the business cycle are driven by a similar set of common factors. We then assume that while these common factors can affect financial market volatility contemporaneously, they tend to affect the dynamics of the real economy only with a lag of at least a quarter.[1] Under these assumptions, we find a statistically significant and economically sizable impact of future output growth on current volatility, and no effect of a volatility shock on the business cycle over and above those driven by the common factors. The evidence is clearly compatible with volatility being a symptom rather than a cause of economic instability.

The paper also contributes to the literature in a number of other respects. First, it proposes quarterly measures of global uncertainty constructed using daily returns across 109

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[1]The results of our analysis are unchanged if we were to assume that these common factors affect the macroeconomy contemporaneously, while volatility leads by one period.

asset prices worldwide. We shall consider four asset classes, namely equity prices, exchange rates, bond prices, and commodity prices. Second, it builds an empirical model of volatility and the business cycle for 33 countries representing over 90 percent of the world economy that takes the following stylized facts into account: (i) shocks are transmitted in financial markets faster than in markets for goods and services; (ii) while volatility is well represented by a stationary process, macroeconomic time series are typically found to follow (or being well approximated by) unit root processes; and (iii) neither volatility nor the business cycle can be reduced to a single common component (i.e., they are driven by both common and idiosyncratic factors). Third and finally, using the global model and a number of different realized volatility measures, the paper investigates the interaction between volatility and the business cycle in an interconnected world economy.

To measure economic uncertainty, we build on the contributions of Andersen, Bollerslev, Diebold, and Labys (2001, 2003) and Barndorff-Nielsen and Shephard (2002, 2004), and we compute realized volatility for a given quarter using daily returns on 92 asset prices (in 33 advanced and emerging economies) and 17 commodity indices. Then we study the time series properties of these volatility measures as well as the extent to which they are driven by global or asset-specific factors.

To study the interconnection between volatility and the business cycle, we use the Global Vector Autoregressive (GVAR) methodology, originally proposed by Pesaran, Schuermann, and Weiner (2004) and further developed in Dees, di Mauro, Pesaran, and Smith (2007) and Dees, Hashem Pesaran, Vanessa Smith, and Smith (2014). The GVAR methodology is a relatively novel approach to global macroeconomic modeling that combines time series, panel data, and factor analysis techniques to address the curse of dimensionality problem in modelling the interconnections in the world economy.[2] Augmenting the GVAR framework with a volatility module also allows us to treat the volatility measures we consider as endogenous in a parsimonious yet disaggregated model of the world economy. In this way, we can identify and illustrate the different linkages that might exist between volatility and the idiosyncratic and global components of economic activity. We refer to this combined model as the GVAR-VOL model.

To identify the effects of a volatility shock, we assume that both volatility and real economic activity are affected by the same set of unobserved common factors. These factors could capture general political and economic events that are difficult to measure, but nevertheless have important impacts on volatility and macroeconomic dynamics.[3] We further assume that these common factors affect volatility contemporaneously but have an impact on output growth with a delay: an assumption that rests on the observation that shocks are typically transmitted in financial markets faster than in markets for goods and services. In this way, we can identify global volatility shocks that are orthogonal to the innovations in the cyclical component of real GDP and inflation.

Our main findings are as follows: from a theoretical view point we show that volatility is forward looking and that the output equation of a typical VAR estimated in the literature is mis-specified as least squares estimates of this equation are inconsistent. This implies that,

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[2]For a recent review of the methodology and a number of applications of the GVAR see di Mauro and Pesaran (2013).

[3]Note that while these factors are common across all markets, countries, and variables, they can have differential effects on variables within and across different countries.

if our assumptions are plausible, typical impulse response functions of measures of economic activity to volatility shocks are biased regardless of the structural VAR identification scheme employed.

Empirically, we provide three main sets of results. First, our (unconditional) descriptive analysis shows that volatility is persistent, but is well approximated by a stationary process at business cycle frequency. It behaves countercyclically—consistently with the common wisdom in the literature—and it can significantly lead the business cycle. We also find that realized volatility co-moves significantly within asset classes, but is not as highly correlated across asset classes (especially for commodities).

Second, by using a small open economy assumption and the law of large numbers applied to cross-sectionally weakly correlated processes, our multi-country analysis allows us to consistently estimate the effects of future, contemporaneous, and lagged values of the changes in global (aggregate) activity on volatility. Our results show that there is a strong negative statistical association between future output growth and current volatility.

Third and finally, we find that exogenous changes to volatility have no statistically significant impact on economic activity over and above that of its common component. In other words, we find that volatility shocks have little or no direct effect on real GDP once we condition on a small set of country-specific and global macro-financial factors in the GVAR-VOL model. We do not interpret this evidence as saying that volatility has no effect on economic activity. Instead, we suggest that most of its effect (often found in the literature) may be coming from the fact that volatility itself is driven by the same common factors that affect the business cycle. In other words, volatility seems to be more of a symptom rather than a cause of economic instability.

The above result differs from the ones in literature that typically find volatility to have a statistically significant negative effect on economic activity. This finding primarily emanates from the identifying assumption made in the literature that rules out the existence of a contemporaneous effect from activity on volatility. As a robustness check, we also estimated the GVAR-VOL model excluding future and contemporaneous activity variables from the volatility module. Under these identifying assumptions, and in line with the literature, we do find that volatility has some direct impact on real GDP and a strong association with equity price and exchange rates, which in turn can affect economic activity indirectly via balance sheet and wealth effects. We see our contribution as providing an alternative identifying assumption which allows volatility and activity to be inter-related through a third set of factors.

The rest of the paper is organized as follows. The next section briefly surveys the theoretical and empirical literature on the interconnection between volatility and the business cycle. In Section 3 we sets out a simple factor model for volatility and economic activity. Building on this theoretical framework, Section 4 describes the model that we use for the empirical analysis on the relation between volatility and business cycle. Section 5 gives the details of how we construct our proxy measures of economic uncertainty and the data we use, and Section 6 documents their main time-series properties and comovement with economic activity. Section 7 discusses the specification and estimation of the model. Section 8 reports and comments on the empirical results of the analysis. Section 9 relates our empirical findings to those of the existing literature. Several appendices provide details on the data set we

used and some descriptive statistics on individual volatility series, as well as other technical details and supplemental results.

2   Theory and related empirical literature

Standard macroeconomic theory suggests that an increase in uncertainty may cause a temporary fall in economic activity. From the viewpoint of the firm, irreversible investment provides the traditional mechanism through which changes in uncertainty affect economic activity (see Bernanke (1983), Dixit and Pindyck (1994) and, more recently, Bloom (2009)). In this framework, exogenous changes in volatility lead to the postponement of irreversible investment and hence a fall in the current level of economic activity.[4] But as uncertainty is resolved, investment plans are brought forward and the level of economic activity begins to recover. On the households' side, Leland (1968) and Kimball (1990) show how, under certain assumptions, increased uncertainty regarding the future stream of labour income and dividends induces households to increase their precautionary savings by reducing consumption, and hence demand. But again, as uncertainty recedes, consumption recovers. Financial frictions provide an additional mechanism through which uncertainty may affect the economy, generally *via* an increase in the risk premium (see Christiano, Motto, and Rostagno, 2014, Gilchrist, Sim, and Zakrajsek, 2013, Arellano, Bai, and Kehoe, 2012).[5]

Based on the above theoretical reasoning, a first strand of the empirical literature revisited the relation between uncertainty and the business cycle, mainly focusing on the U.S. economy.[6] Bloom (2009) in particular examines the relationship between volatility and output growth using Hodrick-Prescott filtered data in a recursively identified VAR, where the volatility measure is ordered before economic activity. He shows that in a such a set up, increases in volatility generate a quick drop and rebound in industrial production. Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) show that this result holds using different proxies for uncertainty computed from micro data, such as the cross-sectional dispersion of firms total factor productivity (TFP) and output growth. Baker and Bloom (2013) attempt to identify the causal link between uncertainty and economic activity using an instrumental variable approach.

The available evidence for other countries is consistent with the one for the United States. Carriere-Swallow and Cespedes (2013) estimate a battery of small open economy VARs for 20 advanced and 20 emerging market economies in which the VIX index is assumed to be

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[4]Favero, Pesaran, and Sharma (1994) provide an empirical investigation of this effect in the case of the development of oil fields in the North Sea.

[5]From a theoretical perspective, the impact of uncertainty on economic activity could also be positive. For example Mirman (1971) shows that, if there is a precautionary motive for savings, then higher volatility should lead to higher savings rate, and hence a higher investment rate. Also, Oi (1961), Hartman (1976) and Abel (1983) show that, if labor can be freely adjusted, the marginal revenue product of capital is convex in price; in this case, uncertainty may increase the level of the capital stock and, therefore, investment.

[6]The countercyclical behavior of the U.S. stock market volatility is a well known stylized fact. See, for example, Schwert (1989a) and Schwert (1989b). On the volatility of firm-level stock returns see Campbell, Lettau, Malkiel, and Xu (2001), Bloom, Bond, and Reenen (2007) and Gilchrist, Sim, and Zakrajsek (2013); on the volatility of plant, firm, industry and aggregate output and productivity see Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) and Bachmann and Bayer (2013); on the behavior of expectations' disagreement see Popescu and Smets (2010) and Bachmann, Elstner, and Sims (2013).

determined exogenously. Their results show that emerging market economies suffer deeper and more prolonged impacts from uncertainty shocks, and that a substantial portion of such larger impact can be explained by the presence of credit constraints in the case of emerging market economies, which is in accordance with the recent work of Christiano, Motto, and Rostagno (2014), Gilchrist, Sim, and Zakrajsek (2013) and Arellano, Bai, and Kehoe (2012). Using an unbalanced panel of 60 countries, Baker and Bloom (2013) also provide evidence of the counter-cyclicality of different proxies for uncertainty, such as stock market volatility, sovereign bond yields volatility, exchange rate volatility and GDP forecast disagreement. Finally, Hirata, Kose, Otrok, and Terrones (2012) use a factor-augmented VAR (FAVAR), with factors computed based on data for 18 advanced economies and a recursive identification scheme in which the volatility variable is ordered first in the VAR. They find that, in response to an uncertainty (volatility) shock, GDP falls and then rebounds consistent with Bloom (2009), although the impact is smaller.

The analysis of the interrelation between volatility and economic activity is challenging for a number of reasons. First, and most importantly, the direction of causality between uncertainty and economic activity is difficult to establish empirically and likely runs in both ways. Theoretically, for instance, some papers provide examples of how spikes in uncertainty may be the result of adverse economic conditions rather than being a driving force of economic downturns (see, for example, Van Nieuwerburgh and Veldkamp, 2006, Fostel and Geanakoplos, 2012, Bachmann and Moscarini, 2011, Tian, 2012, Decker, D'Erasmo, and Moscoso Boedo, 2014). While the existing literature typically assumes from the outset of the empirical analysis that uncertainty causes activity to slow and contract, we assume that both uncertainty and activity are driven by the same set of common factors. This is a possibility that is supported by available empirical evidence and that, as we shall see in the next section of the paper, gives rise to estimation issues that can be dealt with only in a the context of a multi-country empirical model like the one we use.

Gilchrist, Sim, and Zakrajsek (2013), for instance, estimate a VAR for the United States with both an aggregate uncertainty measure (computed from firm-level equity returns with the Fama-factor approach) and the 10 years BBB-Treasury credit spread. They find that an increase in uncertainty as measured by stock market volatility leads to an economically and statistically significant drop in detrended GDP (with some mean-reversion but no over-shooting). However, once shocks to uncertainty are orthogonalized with respect to the contemporaneous information from the corporate bond market (i.e., the stock market volatility ordered after credit spread in their recursive identification) uncertainty shocks do not have any statistically significant effect on detrended GDP. This evidence suggests that indeed financial factors (i.e., financial shocks or frictions) could drive both volatility and the business cycle.

Using data from business surveys, Bachmann, Elstner, and Sims (2013) show that positive innovations to business uncertainty (measured as either sectorial business forecasts disagreement or *ex post* forecast errors) have protracted negative effects on the level of economic activity, without any evidence of the drop–and–rebound dynamics documented in the studies mentioned above. The authors suggest as possible explanation for this result that "*uncertainty is driven by some kind of first moment shock that has long-lived effects on production.*" This would imply that uncertainty itself is not the ultimate cause of the long-lasting estimated negative impact found in the data. Again, this evidence is consistent with the idea

that uncertainty may simply be a by-product of "bad" economic times and may be caused by expectations of long-lasting economic downturns.

A second challenge in the analysis of uncertainty and economic activity lies in the fact that standard theory requires a persistent increase in volatility to explain a persistent downturn in activity. In fact in standard theoretical models activity rebounds when uncertainty is resolved. But as we see in Figure 1, and unlike typical macroeconomic variables like real GDP or inflation, volatility is not very persistent. For example, during the recent great recession, uncertainty quickly reverted back to normal levels after spiking in 2008, while world output growth continued to be depressed several years after the onset of the subprime crisis in the United States in early 2007. Partly because of this reason, researcher's attention shifted to a distinct source of uncertainty that is much more persistent, namely measures of "macroeconomic policy uncertainty" (see, for instance Baker, Bloom, and Davis, 2013, Kose and Terrones, 2012, Mumtaz and Surico, 2013). We address this issue specifying an empirical model that takes the different degree of persistence of volatility and macro variables into explicit account and we do not relay on filtering procedures to isolate the business cycle frequencies of economic activity.

Finally, note that both volatility and the business cycle have idiosyncratic (to countries, asset classes, and regions) as well as common components. A separate strand of empirical literature argues that the international business cycle is better characterized by a combination of global and regional cycles rather than a single world business cycle (see, for instance Kose, Otrok, and Whiteman, 2003, Hirata, Kose, and Otrok, 2013). Similar findings extend to financial cycles (see Kose, Otrok, and Prasad, 2013). We take this into account by considering the joint behavior of economic activity in many countries and by allowing for the possibility of multiple sources of global financial volatility.

3 A simple factor model of volatility and macroeconomic dynamics

We begin with a simple model and assume that a small set of common factors characterize the evolution of the world economy. Moreover, given the possible bidirectional relationship between volatility and growth, we allow these factors to drive both asset price volatility and macroeconomic variables. Finally, we assume that these factors affect financial markets faster than they can affect macroeconomic dynamics: while affecting financial market volatility contemporaneously, they can affect macroeconomic dynamics only with a lag of at least one quarter. Note, however, that our basic assumption is the time difference between the way common factors affect volatility and the real economy. For example, the results of our analysis remain qualitatively unchanged if we were to assume that common factors affect the macroeconomy contemporaneously, but with volatility leading the factors by one quarter.

Suppose that there are $N+1$ countries in the global economy, indexed by $i = 0, 1, ..., N$, where country 0 serves as the numeraire. Denote by $\mathbf{v}_t$ a ($m \times 1$) vector of global volatilities and by $\mathbf{y}_{it}$ a ($k_i^y \times 1$) vector of country-specific macroeconomic aggregates that include, for instance, GDP and inflation. Both macroeconomic variables and volatilities are affected by one or more common latent factors, represented by the ($s \times 1$) vector, $\mathfrak{n}_t$. We assume that

$\mathbf{y}_{it}$ is a unit root process, or $I(1)$, and $\mathbf{v}_t$ is stationary, or $I(0)$: assumptions that, as we shall see, are supported by the data. We also assume that m and s are fixed and do not increase with N and/or T.

We shall begin by re-examining the relationship between $\mathbf{v}_t$ and $\Delta \mathbf{y}_{it}$, assuming that these variables are related indirectly through a set of common latent factors, $\mathfrak{n}_t$. In particular, we consider the following dynamic specification (suppressing the deterministic components such as intercepts and higher order lags to simplify the exposition):

$$\begin{aligned}
\mathbf{v}_t &= \mathbf{\Phi}_{1v}\mathbf{v}_{t-1} + \mathbf{\Lambda}\mathfrak{n}_t + \boldsymbol{\xi}_t, \\
\Delta \mathbf{y}_{it} &= \mathbf{\Phi}_{1i}\Delta \mathbf{y}_{i,t-1} + \mathbf{\Gamma}_i\mathfrak{n}_{t-1} + \boldsymbol{\zeta}_{it}, \text{ for } i = 0, 1, ..., N.
\end{aligned} \tag{1}$$

According to (1), the common factors $\mathfrak{n}_t$ affect volatility first, as it realizes contemporaneously, before impacting macroeconomic variables. The same process $\mathfrak{n}_t$ also affects macroeconomic variables in country i with a lag of one quarter. Note here that the process $\mathfrak{n}_t$ represents a *global* factor and it is therefore common across all countries and markets, but it can affect each country in the global economy differently *via* different country-specific loadings, as defined by the elements of $\mathbf{\Gamma}_i$.

The common factors could arise either as a result of the internal dynamics of the global economy or could be the result of political or other external factors such as wars, natural disasters or could even reflect rumors and noisy information. In this paper we do not take specific position regarding the nature of such common factors. But we believe that it is reasonable to suppose that financial markets and their volatility are more immediately affected by such news or events as compared to the real economy where employment and investment decisions are subject to inertia and government regulations, which prevents production firms and households to adapt to news and political events as promptly as it is done by financial firms.

We make the following statistical assumptions:

A. $|\lambda(\mathbf{\Phi}_{1i})| < 1 - \epsilon$, for some strictly positive constant $\epsilon > 0$, where $\lambda(\mathbf{\Phi}_{1i})$ denotes the eigenvalue of $\mathbf{\Phi}_{1i}$;

B. the country-specific coefficients, $\mathbf{\Phi}_{1i}$ and $\mathbf{\Gamma}_i$ are random draws from common distributions with finite moments;

C. the average factor loading matrix $\bar{\mathbf{\Gamma}} = (N+1)^{-1}\sum_{i=0}^{N}\mathbf{\Gamma}_i$, and $\mathbf{\Lambda}$ are full column rank matrices such that $\bar{\mathbf{\Gamma}}'\bar{\mathbf{\Gamma}}$ and $\mathbf{\Lambda}'\mathbf{\Lambda}$ are non-singular. Specifically, we assume that $k_i^y \geq s$, and $m \geq s$, namely that there are at least as many macro variables and volatility measures as common factors;

D. the idiosyncratic errors, $\boldsymbol{\zeta}_{it}$ and $\boldsymbol{\xi}_t$ are serially uncorrelated, with $\boldsymbol{\xi}_t$ being independently distributed of the factors. Specifically, $\mathbb{E}(\boldsymbol{\zeta}_{it}\boldsymbol{\zeta}_{it'}) = 0$, $\mathbb{E}(\boldsymbol{\xi}_t\boldsymbol{\xi}_{t'}) = 0$, and $\mathbb{E}(\mathfrak{n}_t\boldsymbol{\xi}_{t'}') = 0$, for all i, t, and $t' \neq t$.

E. $\boldsymbol{\zeta}_{it}$ are cross-sectionally weakly correlated (in the sense defined by Chudik, Pesaran, and Tosetti, 2011) so that $\bar{\boldsymbol{\zeta}}_t = (N+1)^{-1}\sum_{i=0}^{N}\boldsymbol{\zeta}_{it} = O_p\left[(N+1)^{-1/2}\right]$.

Since $\mathfrak{n}_t$ is unobserved, a direct relationship between $\Delta \mathbf{y}_{it}$ and $\mathbf{v}_t$ can be established if $\mathfrak{n}_t$ is eliminated from the above system of equations. Under assumption C, it is possible to obtain

$\Delta \mathbf{y}_{it}$ in terms of $\mathbf{v}_t$, and *vice versa*. However, due to the presence of the idiosyncratic errors ζ_{it} and ξ_t, it is not possible to identify the common factors from the observables, unless—as we shall see—N is sufficiently large and assumptions A and E hold.

Let's first solve for the volatility variables. Assume for simplicity that the dynamics of the macro equations are homogenous, i.e., $\mathbf{\Phi}_{1i} = \mathbf{\Phi}_1$, for all i. Averaging the macro equations across i, we have:

$$\Delta \bar{\mathbf{y}}_t = \mathbf{\Phi}_1 \Delta \bar{\mathbf{y}}_{t-1} + \bar{\mathbf{\Gamma}} \mathfrak{n}_{t-1} + \bar{\boldsymbol{\zeta}}_t,$$

where $\bar{\mathbf{\Gamma}}$ and $\bar{\boldsymbol{\zeta}}_t$ are defined above, and $\bar{\mathbf{y}}_t = (N+1)^{-1} \sum_{i=0}^N \mathbf{y}_{it}.$[7] Under Assumption C, solving for $\mathfrak{n}_t$, we have:

$$\mathfrak{n}_t = (\bar{\mathbf{\Gamma}}' \bar{\mathbf{\Gamma}})^{-1} \bar{\mathbf{\Gamma}}' \left(\Delta \bar{\mathbf{y}}_{t+1} - \mathbf{\Phi}_1 \Delta \bar{\mathbf{y}}_t - \bar{\boldsymbol{\zeta}}_{t+1} \right),$$

which if used in (1) yields:

$$\mathbf{v}_t = \mathbf{\Phi}_{1v} \mathbf{v}_{t-1} + \mathbf{\Psi}_{1,v} \Delta \bar{\mathbf{y}}_{t+1} + \mathbf{\Psi}_{0,v} \Delta \bar{\mathbf{y}}_t - \mathbf{\Psi}_{1,v} \bar{\boldsymbol{\zeta}}_{t+1} + \boldsymbol{\xi}_t, \tag{2}$$

where

$$\mathbf{\Psi}_{1,v} = \mathbf{\Lambda} (\bar{\mathbf{\Gamma}}' \bar{\mathbf{\Gamma}})^{-1} \bar{\mathbf{\Gamma}}', \text{ and } \mathbf{\Psi}_{0,v} = -\mathbf{\Lambda} (\bar{\mathbf{\Gamma}}' \bar{\mathbf{\Gamma}})^{-1} \bar{\mathbf{\Gamma}}' \mathbf{\Phi}_1.$$

Therefore, under the above set up, volatility is led by macroeconomic dynamics and responds to expected changes in economic activity. For example, during the recent global crisis, one could argue that a few factors were responsible for the evolution of the world economy and those factors affected volatility directly within a given quarter, but were impacting on growth and inflation with a lag of at least one quarter. This means, for instance, that when Lehman Brothers went bankrupt in September 2008, volatility increased within the same quarter while growth and inflation were affected by this shock only in the subsequent quarters.[8]

Equation (2) also raises an important estimation issue. If the number of countries, $N+1$, is fixed, there is an endogeneity problem. Specifically, $\Delta \bar{\mathbf{y}}_{t+1}$ and $\bar{\boldsymbol{\zeta}}_{t+1}$ are correlated and, therefore, consistent estimation of the parameters would require the use of instrumental variables, which in the present context are difficult to find. This endogeneity problem would arise in the case of any volatility-growth regression for an individual country. An example would be the typical bivariate VAR model for the United States estimated in the literature with a measure of volatility and output growth. Under our assumptions, however, for N sufficiently large we have that $\bar{\boldsymbol{\zeta}}_{t+1} \to_p 0$, as $N \to \infty$. In other words, by using a small open economy assumption and the law of large numbers applied to cross-sectionally weakly correlated processes, we can address the endogeneity problem of equation (2). Hence, the parameters of (2) can be consistently estimated by least squares regressions of $\mathbf{v}_t$ on $\mathbf{v}_{t-1}$, $\Delta \bar{\mathbf{y}}_{t+1}$, and $\Delta \bar{\mathbf{y}}_t$. This clearly highlights the value added of taking a multi-country approach to the analysis of the interrelation between volatility and the business cycle.

Note that using a large number of countries permits consistent estimation of (2) even if the macro dynamics are heterogeneous across countries (namely $\mathbf{\Phi}_i$ differ across i). In this case,

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[7]One could also use weighted cross sectional averages so long as the weights are granular, in the sense that they are all of order $(N+1)^{-1}$.

[8]As we noted above, an equivalent assumption is that volatility started to rise in the run up to the Lehman's collapse while growth and inflation were affected during the same quarter in which Lehman collapsed. What matters is to assume that these factors affect financial markets faster than they can affect macroeconomic dynamics.

the derivation of the expression for $\mathfrak{n}_t$ is more complicated and now involves lags of $\Delta\bar{\mathbf{y}}_t$. But Chudik and Pesaran (2013) show that, even with dynamic heterogeneity, under assumption A and E, $\mathfrak{n}_t$ can be approximated by an infinite distributed lag function of $\Delta\bar{\mathbf{y}}_{t+1}$, $\Delta\bar{\mathbf{y}}_t$, and their lagged values. The coefficients of such distributed lag function decay exponentially and can therefore be suitably truncated for estimation. In this heterogeneous setting, the volatility regression equation (2) can be written as:

$$\mathbf{v}_t = \boldsymbol{\Phi}_{1v}\mathbf{v}_{t-1} + \sum_{j=0}^{p_T} \boldsymbol{\Psi}_{1-j,v}\Delta\bar{\mathbf{y}}_{t+1-j} + \boldsymbol{\xi}_t + O_p\left[(N+1)^{-1/2}\right], \tag{3}$$

where $p_T = O(T^{1/3})$. In practice, Chudik and Pesaran (2013) show that one can set $p_T = T^{1/3}$.

We now solve for the macro variables. For each country i we have:

$$\Delta\mathbf{y}_{it} = \boldsymbol{\Phi}_{1i}\Delta\mathbf{y}_{i,t-1} + \boldsymbol{\Xi}_{i1}\mathbf{v}_{t-1} - \boldsymbol{\Xi}_{i2}\mathbf{v}_{t-2} + \mathbf{u}_{it}, \tag{4}$$

where:

$$\boldsymbol{\Xi}_{i1} = \boldsymbol{\Gamma}_i(\boldsymbol{\Lambda}'\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}', \ \boldsymbol{\Xi}_{i2} = \boldsymbol{\Gamma}_i(\boldsymbol{\Lambda}'\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}'\boldsymbol{\Phi}_{1v},$$

and:

$$\mathbf{u}_{it} = \boldsymbol{\zeta}_{it} - \boldsymbol{\Xi}_{i1}\boldsymbol{\xi}_{t-1}. \tag{5}$$

The expression (4) for $\Delta\mathbf{y}_{it}$ has the familiar appearance of the reduced-form equation of a bivariate VAR for $\Delta\mathbf{y}_{it}$ and $\mathbf{v}_t$, as it is typically estimated in the literature. However, due to the dependence of $\mathbf{v}_{t-1}$ on $\boldsymbol{\xi}_{t-1}$, we have that:

$$\mathbb{E}(\mathbf{u}_{it}\mathbf{v}'_{t-1}) = -\boldsymbol{\Gamma}_i(\boldsymbol{\Lambda}'\boldsymbol{\Lambda})^{-1}\boldsymbol{\Lambda}'\mathbb{E}\left(\boldsymbol{\xi}_{t-1}\boldsymbol{\xi}'_{t-1}\right) \neq \mathbf{0},$$

and, therefore, the parameters of (4) can not be consistently estimated by ordinary least squares. This implies that, under the assumption that the factor model (1) is true, any bivariate VAR containing an equation like (4) would produce an inconsistent impulse response of $\Delta\mathbf{y}_{it}$ for shocks to $\mathbf{v}_t$, regardless of the identification assumption made. The analysis therefore shows that, if the factor model (1) holds, we cannot estimate the impact of volatility and growth in a model in which $\mathbf{v}_{t-1}$ enters directly in the equation for $\Delta\mathbf{y}_{it}$, even if we were to take a global perspective, focusing only on global volatility and global activity. Note, moreover, that this result does not depend on the timing assumption that we made at the beginning of this section: the mis-specification of (4) also follows when we assume that the common factors affect contemporaneously both volatility and economic activity.

4 The GVAR-VOL model

Modelling global volatility and world growth is problematic for two more reasons other than the estimation issues discussed in the previous section. First, the stochastic process of most macroeconomic times series, such as real output or the level of nominal variables, has a unit root or has roots that are very close to unity (namely they are best approximated as $I(1)$ processes). In contrast, as we will see later, although persistent, volatility measures are clearly stationary at quarterly frequency and best represented as $I(0)$ variables. Using the HP filter,

as often done in some empirical analysis in the literature, may change the business cycle component of economic activity, or may affect its permanent component when the shocks are large and persistent. Moreover, the use of the HP filter may not be appropriate in cases where the model contains a mixture of $I(0)/I(1)$ variables (see Harvey and Jaeger, 1993, for example).

Second, while the bivariate representation (3) and (4) is appealing for its simplicity, in practice there are many sources of volatility and many countries in the world economy. Neither volatility nor the international business cycle can be satisfactorily modelled by a single factor.[9] For this reason a more general framework where $\mathbf{y}_{it}$ (where $i = 0, 1, ..., N$) and $\mathbf{v}_t$ are modelled jointly is better suited for this type of analysis. We also need to deal with the high dimensional nature of the problem since—as suggested in the previous section—N must be sufficiently large for the effects of future changes in global output on volatility to be correctly estimated.

In what follows we avoid the curse of dimensionality by adopting the global vector autoregressive (GVAR) methodology, where a joint model for $\mathbf{y}_{it}$ (where $i = 0, 1, ..., N$) is developed by estimating separate country-specific models conditional on the global and country specific factors. As shown in Dees, di Mauro, Pesaran, and Smith (2007), Dees, Hashem Pesaran, Vanessa Smith, and Smith (2014), in the GVAR model the unobserved factors are proxied by country-specific foreign variables, and to the extent that such common factors are also the drivers of the volatility variables, $\mathbf{v}_t$, then conditional country-specific models can be estimated consistently without the need to include the volatility variables, $\mathbf{v}_t$. The part of $\mathbf{v}_t$ that can not be explained by the common factors are then absorbed in the residuals of the country-specific models. By construction, these innovations will be weakly cross-sectionally correlated and do not pose any problem for the consistent estimation of the GVAR model. This aspect of the GVAR is particularly convenient since it avoids the estimation pitfalls—discussed in the previous section—that arise if $\mathbf{v}_t$ or its lagged values are included in the individual models for $\mathbf{y}_{it}$, for $i = 0, 1, ..., N$.

Having developed the GVAR model for $\mathbf{y}_{it}$ for $i = 0, 1, ..., N$, the GVAR can then be augmented with a set of volatility equations of the type defined by (3). We label this augmented model the GVAR-VOL model. More specifically, to build the GVAR-VOL model we proceed as follows. First, we estimate a stationary autoregressive distributed lag (ARDL) model for volatility in which we include the future, contemporaneous, and lagged values of the changes in a set of macroeconomic variables for which the assumptions made in the previous section are valid. These variables are $I(0)$ by construction and hence conform with the $I(0)$ nature of the volatility variables. So this system is balanced. We label this ARDL model the "volatility module." Next, we specify and estimate a standard GVAR model in $\mathbf{y}_{it}$ for $i = 0, 1, ..., N$, without $\mathbf{v}_t$. Finally, the standard GVAR and the volatility module are combined and solved simultaneously for simulation purposes. We now describe in more detail each of the two components of the GVAR-VOL model and how they are combined, but first we have to establish some notation.

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[9]See for instance Kose, Otrok, and Whiteman (2003) on the international business cycle and Kose, Otrok, and Prasad (2013) on the international financial cycle.

4.1 Notations

Consider a vector $\mathbf{v}_t$ of $(m \times 1)$ global volatility measures and assume that they are $I(0)$, an assumption that, as we shall see, is supported by the data.

Next, define a $(k_i \times 1)$ vector $\mathbf{x}_{it} = (\mathbf{y}'_{it}, \boldsymbol{\chi}'_{it})'$ of country-specific *domestic* macroeconomic and financial variables. The $(k_i^y \times 1)$ vector $\mathbf{y}_{it}$ includes the macroeconomic variables for which the assumptions made above are likely to hold (such as GDP and inflation), while the $(k_i^\chi \times 1)$ vector $\boldsymbol{\chi}_{it}$ includes typical financial variables for which our assumptions may not hold. Financial variables (such as equity prices, exchange rates, and interest rates) are likely to be affected by the set of common factors $(\mathfrak{n}_t)$ with the same speed with which they affect volatility.

Now define a $(K \times 1)$ vector $\mathbf{x}_t$ of all country-specific *domestic* macroeconomic and financial variables as:

$$\mathbf{x}_t = (\mathbf{x}'_{0t}, \mathbf{x}'_{1t}, ..., \mathbf{x}'_{Nt})', \tag{6}$$

with $K = \Sigma_0^N k_i$. Note here that not all countries need to have the same set of variables, and we can also re-write $\mathbf{x}_{it}$ as follows:

$$\mathbf{x}_{it} = \mathbf{S}_i \mathbf{x}_t, \tag{7}$$

where $\mathbf{S}_i$ is an appropriate $(k_i \times K)$ selection matrix. Then define a $(k \times 1)$ vector $\mathbf{x}^*_{it}$ of country-specific *foreign* macroeconomic and financial variables, with $k = \max_i(k_i)$:

$$\mathbf{x}^*_{it} = \mathbf{W}_i \mathbf{x}_t. \tag{8}$$

where $\mathbf{W}_i$ is an appropriate $(k \times K)$ weighting matrix of predetermined weights, typically constructed using trade or financial weights specific to country i.[10] Finally, also define a $(k^y \times 1)$ vector $\mathbf{y}^*_t$ of *global* macroeconomic variables as:

$$\mathbf{y}^*_t = \mathbf{P}\mathbf{x}_t, \tag{9}$$

where $\mathbf{P}$ is a $(k^y \times K)$ weighting and selection matrix, typically made up of zeros and PPP-GDP weights, so as to select only the macroeconomic variables $\mathbf{y}_{it}$ and not the financial variables $\boldsymbol{\chi}_{it}$.[11]

We assume that $\mathbf{x}_{it}$, $\mathbf{x}^*_{it}$, and $\mathbf{y}^*_t$ all follow $I(1)$ processes.

4.2 Volatility module

Consistently with (3), we estimate a separate ARDL model for the level of the volatility measures $(\mathbf{v}_t)$ augmented with the future, contemporaneous, and the lagged values of the changes in the global macroeconomic variables $(\Delta \mathbf{y}^*_t)$. As noted above, we include only the $\mathbf{y}^*_t$ (and not the $\boldsymbol{\chi}^*_t$) since the assumptions under which we derived the volatility module (3) are likely to hold only for slow moving variables such as GDP and inflation. The volatility module is therefore specified as:

$$\mathbf{v}_t = \boldsymbol{\Phi}_v \mathbf{v}_{t-1} + \boldsymbol{\Psi}_{1,v} \Delta \mathbf{y}^*_{t+1} + \boldsymbol{\Psi}_{0,v} \Delta \mathbf{y}^*_t + \boldsymbol{\Psi}_{-1,v} \Delta \mathbf{y}^*_{t-1} + \boldsymbol{\xi}_t, \tag{10}$$

---

[10]These weights can be fixed or time-varying. But to keep the notations simple here we assume they are time-invariant in the construction of $\mathbf{x}^*_{it}$.

[11]Like in the case of the $\mathbf{W}_i$ matrix, the $\mathbf{P}$ matrix could also be time-varying.

where $\mathbf{\Phi}_v$ is a $(m \times m)$ matrix and $\mathbf{\Psi}_{1,v}, \mathbf{\Psi}_{0,v} \mathbf{\Psi}_{-1,v}$ are $(m \times k^y)$ matrices of constant coefficients.[12] By using the definition of $\mathbf{y}_t^*$ in (9), and noting that $\mathbf{P}$ is a $(k^y \times K)$ matrix of known and time invariant weights, the model in (10) can now be re-written as:

$$\mathbf{v}_t = \mathbf{\Phi}_v \mathbf{v}_{t-1} + \mathbf{\Psi}_{1,v} \mathbf{P} \Delta \mathbf{x}_{t+1} + \mathbf{\Psi}_{0,v} \mathbf{P} \Delta \mathbf{x}_t + \mathbf{\Psi}_{-1,v} \mathbf{P} \Delta \mathbf{x}_{t-1} + \mathbf{\xi}_t. \tag{11}$$

Three remarks are in order here. First, note that the volatility module in (10) is fully consistent with the factor model (1). In fact, in the volatility module, we condition only on those global macroeconomic variables for which our assumptions are likely to hold (i.e., we exclude asset prices and interest rates). Second, the residuals $\mathbf{\xi}_t$ are volatility innovations that are orthogonal to future, current and past changes in global macroeconomic variables by construction, and can be interpreted as exogenous volatility changes with respect to those variables.[13] Third and finally, under the assumptions A–E above, for N sufficiently large, the parameters of (11) can be consistently estimated by OLS despite the presence of $\Delta \mathbf{y}_{t+1}^*$ in the volatility equation, (10).

4.3 The GVAR methodology

There are two stages in specifying and building a standard GVAR model.[14] In the first stage, country-specific vector-autoregression models that relate the domestic variables, $\mathbf{x}_{it}$, to their own lagged values and to the country-specific foreign variables, $\mathbf{x}_{it}^*$, are specified. These augmented vector autoregressive models are labelled VARX* models. Consistent estimation of the VARX* models is achieved by treating the $\mathbf{x}_{it}^*$ variables as *weakly exogenous*, an assumption which is expected to hold on *a priori* grounds assuming countries can be viewed as small open economies, and tend to hold when subjected to econometric testing as in our application.[15] In the second stage, individual country models are combined using link matrices that relate foreign variables to country-specific variables. The link matrices are defined in terms of trade weights, or other suitable international transaction flows data. This yields a high-dimensional VAR without any exogenous variables, which can be used for forecasting and impulse response analysis, controlling for a large set of global and country-specific factors. Note that, with the GVAR modelling approach, we do not filter macroeconomic series to obtain their cyclical component, thus avoiding the perils of contaminating the data with spurious components resulting from filtering procedures.

Formally, for each country i, consider the following country-specific VARX*(1,1) model (with no constants and no time trends for simplicity):

$$\mathbf{x}_{it} = \mathbf{\Phi}_{1i} \mathbf{x}_{i,t-1} + \mathbf{\Psi}_{0i} \mathbf{x}_{it}^* + \mathbf{\Psi}_{1i} \mathbf{x}_{i,t-1}^* + \mathbf{\varepsilon}_{it}, \text{ for } i = 0, 1, ..., N, \tag{12}$$

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[12]Note that additional lags of $\mathbf{v}_t$ and $\Delta \mathbf{y}_t^*$ can be included in (10) so as to ensure that the volatility innovations become approximately serially uncorrelated.

[13]This is a notion of a volatility shock close to the one by Bernanke (1983), Dixit and Pindyck (1994), and Bloom (2009) papers (i.e., volatility shock which is not associated with first moment shocks).

[14]See Pesaran, Schuermann, and Weiner (2004), Dees, di Mauro, Pesaran, and Smith (2007), and di Mauro and Pesaran (2013) for more details on the theory and application of the GVAR methodology.

[15]Weak exogeneity of the $\mathbf{x}_{it}^*$ variables for the estimation of the reduced form parameters of the VARX* models does not imply any statement on the economic causal relation between $\mathbf{x}_{it}^*$ and $\mathbf{x}_{it}$. It simply states that the parameters of the VARX* model can be estimated consistently conditional on $\mathbf{x}_{it}^*$ without needing to specify or estimate the marginal models for $\mathbf{x}_{it}^*$. See Engle, Hendry, and Richard (1983) for a formal definition.

where $\boldsymbol{\Phi}_{1i}$ is $(k_i \times k_i)$, $\boldsymbol{\Psi}_{0i}$ and $\boldsymbol{\Psi}_{1i}$ are $(k_i \times k)$ matrices. The $(k_i \times 1)$ vector of error terms, $\boldsymbol{\varepsilon}_{it}$, are assumed serially uncorrelated as well as cross-sectionally weakly correlated. Using the identities in (7) and (8) we have:

$$\mathbf{S}_i\mathbf{x}_t = \boldsymbol{\Phi}_{1i}\mathbf{S}_i\mathbf{x}_{t-1} + \boldsymbol{\Psi}_{0i}\mathbf{W}_i\mathbf{x}_t + \boldsymbol{\Psi}_{1i}\mathbf{W}_i\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_{it}, \tag{13}$$

which yields:

$$\mathbf{G}_i\mathbf{x}_t = \mathbf{H}_i\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_{it}, \tag{14}$$

with:

$$\mathbf{G}_i = \left(\mathbf{S}_i - \boldsymbol{\Psi}_{0i}\mathbf{W}_i\right), \qquad \mathbf{H}_i = \left(\boldsymbol{\Phi}_{1i}\mathbf{S}_i + \boldsymbol{\Psi}_{1i}\mathbf{W}_i\right),$$

where $\mathbf{G}_i$ and $\mathbf{H}_i$ are $(k_i \times K)$ matrices, where as before $K = \Sigma_{i=0}^N k_i$.

Stacking all country-specific models, we can now write the above system more compactly as:

$$\mathbf{G}\mathbf{x}_t = \mathbf{H}\mathbf{x}_{t-1} + \boldsymbol{\varepsilon}_t, \tag{15}$$

with:

$$\mathbf{G} = (\mathbf{G}_0', \mathbf{G}_1', ..., \mathbf{G}_N')', \qquad \mathbf{H} = (\mathbf{H}_0', \mathbf{H}_1', ..., \mathbf{H}_N')', \qquad \boldsymbol{\varepsilon}_t = (\boldsymbol{\varepsilon}_{0t}', \boldsymbol{\varepsilon}_{1t}', ..., \boldsymbol{\varepsilon}_{Nt}')',$$

where $\mathbf{G}$ and $\mathbf{H}$ are $(K \times K)$ matrices. Finally, assuming that $\mathbf{G}$ is non-singular we have:

$$\mathbf{x}_t = \mathbf{F}\mathbf{x}_{t-1} + \mathbf{u}_t, \tag{16}$$

where $\mathbf{F} = \mathbf{G}^{-1}\mathbf{H}$ and the residuals of the reduced-form GVAR are given by:

$$\mathbf{u}_t = \mathbf{G}^{-1}\boldsymbol{\varepsilon}_t, \tag{17}$$

where $\mathbf{u}_t = (\mathbf{u}_{0t}', \mathbf{u}_{1t}', ..., \mathbf{u}_{Nt}')'$. Note that $\mathbf{u}_{it}$ refers to the reduced form innovations to the variables $\mathbf{x}_{it}$, which can be further partitioned as $\mathbf{x}_{it} = (\mathbf{y}_{it}', \boldsymbol{\chi}_{it}')'$, where as before $\mathbf{y}_{it}$ refers to the macroeconomic variables of country i, and $\boldsymbol{\chi}_{it}$, the financial variables of country i. This partitioning is important for our identification scheme, since in the underlying factor model (1) we only maintain that latent factors affect the macro variables $(\mathbf{y}_{it})$ with a delay and not the financial variables $(\boldsymbol{\chi}_{it})$. Specifically, for each country i, we select the elements of $\mathbf{u}_t$ associated with the equations of the macroeconomic variables $\mathbf{y}_{it}$ in the $(k_i^y \times 1)$ vector $\mathbf{u}_{it}^y$; and the elements of $\mathbf{u}_t$ associated with the equations of the financial variables $(\boldsymbol{\chi}_{it})$ in the $(k_i^\chi \times 1)$ vector $\mathbf{u}_{it}^\chi$, such that

$$\mathbf{u}_{it}^y = \mathbf{S}_i^y\mathbf{u}_t, \text{ and } \mathbf{u}_{it}^\chi = \mathbf{S}_i^\chi\mathbf{u}_t, \tag{18}$$

where $\mathbf{S}_i^y$ and $\mathbf{S}_i^\chi$ are appropriate $(k_i^y \times k)$ and $(k_i^\chi \times k)$ selection matrices, respectively. Finally we define

$$\mathbf{u}_t^y = (\mathbf{u}_{0t}'^y, \mathbf{u}_{1t}'^y, ..., \mathbf{u}_{Nt}'^y)', \text{ and } \mathbf{u}_t^\chi = \left(\mathbf{u}_{0t}'^\chi, \mathbf{u}_{1t}'^\chi, ..., \mathbf{u}_{Nt}'^\chi\right)'. \tag{19}$$

Two remarks are in order here. First, we note that, the GVAR module in (16) is also consistent with the factor model (1).[16] This is because, as Chudik and Pesaran (2011, 2013) show, the GVAR model can be derived as an approximation to an infinite dimensional VAR (in which all global macro and financial factors are included) that converges to a global

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[16]Note that while (16) is specified in levels, the factor model (1) is specified in first differences.

unobserved common factor model in which $\mathbf{x}_{it}^*$ (and hence $\mathbf{y}_{it}^*$) are proxies for the latent global factors. Importantly, however, as long as the $\mathbf{x}_{it}^*$ variables are weakly exogenous, it is possible to estimate the VARX* models by OLS because we have not included the volatility variables, $\mathbf{v}_t$, directly in the GVAR, unlike the bivariate or panel VARs typically used in the literature in which volatility and activity variables are included jointly.

Second, the vector of all country-specific innovations $\boldsymbol{\varepsilon}_t$ defined by equation (15) are cross-sectionally *weakly correlated* (see Pesaran, Schuermann, and Weiner, 2004, Dees, di Mauro, Pesaran, and Smith, 2007, Dees, Hashem Pesaran, Vanessa Smith, and Smith, 2014). Therefore, no common factor (such as a global volatility shock) could drive them. Differently, the vector of reduced-form residuals $\mathbf{u}_t = \mathbf{G}^{-1}\boldsymbol{\varepsilon}_t$ defined by equation (17) could share a common component. This is because the $\mathbf{G}$ matrix includes all contemporaneous interdependencies in the global economy in the form of a mix between estimated parameters and pre-determined weights in the link matrices, $\mathbf{W}_i$. As a result, a global volatility shock could affect $\mathbf{u}_t$: a possibility that we now discuss in more detail and that we will explore empirically in our application in the last part of the paper.

4.4 Combining the volatility module and the GVAR

The combined GVAR-VOL model is derived in Appendix by stacking the GVAR module (16) and the volatility module (11) in matrix format, yielding a VAR in $\mathbf{v}_t$ and $\mathbf{x}_{t+1}$. Since volatility does not enter directly into the activity equations of the GVAR model, the only way a global volatility innovation $\boldsymbol{\xi}_t$ can have an impact on activity is *via* its correlation with the reduced-form residuals of the GVAR defined in (17). In other words, under our identification assumptions, for the volatility innovations, $\boldsymbol{\xi}_t$, to affect economic activity, over and above that of the unobserved common factors that drive both volatility and the business cycle, they must significant statistical correlations with the elements of $\mathbf{u}_t$ that correspond to macro variables, namely

The factor model (1) provides guidance as to how $\boldsymbol{\xi}_t$ and $\mathbf{u}_t$ can be related under our identifying assumptions. Recall that the factor model (1) assumes that the latent factors, $\mathfrak{n}_t$, can affect financial market volatility contemporaneously, but they tend to affect the dynamics of the real economy ($\mathbf{y}_{it}$) only with a lag of at least a quarter. This assumption has two important implications. First, as we noted already, the timing assumption is less likely to hold for financial variables (such as equity prices or interest rates). Therefore, within our theoretical framework only the relationship (if any) between the GVAR reduced form innovations associated with the macroeconomic variables, namely $\mathbf{u}_t^y$ defined by (19), and $\boldsymbol{\xi}_t$ can be strictly interpreted in terms of causation, while the relation between $\mathbf{u}_t^\chi$ and $\boldsymbol{\xi}_t$ has to be viewed as simple statistical association. Second, equation (5) shows that the volatility innovations $\boldsymbol{\xi}_t$ affect the $\mathbf{u}_t$ residuals only with a lag. With these considerations in mind, in the last part of the paper we will explore empirically the relation between volatility and economic activity by regressing the elements of both $\mathbf{u}_t^y$ and $\mathbf{u}_t^\chi$ on $\boldsymbol{\xi}_{t-1}$.

5 Realized quarterly measures of volatility

This section describes how we construct the variables that we use to measure economic uncertainty at quarterly frequency and the data set we have assembled to compute them.

5.1 Background

We measure economic uncertainty with the "volatility" of asset prices. Asset price volatility has been used extensively in the theoretical and empirical literature to measure uncertainty, and implicitly assumes that uncertainty can be characterized in terms of probability distributions. It therefore abstracts from the Knightian notion of uncertainty that claims that some types of uncertainty can not be as such characterized.

Even if we confine our attention to "volatility," this is not directly observable and like many other economic concepts, such as expectations, demand and supply, it is usually treated as a latent variable and measured indirectly using a number of different proxies. Initially, volatility was measured by standard deviations of output or asset price changes computed over time, typically using a rolling window. But then it was realized that such a historical measure tends to underestimate sudden changes in volatility and is only suitable when the underlying volatility is relatively stable.

To allow for time variations in volatility, Engle (1982) developed the autoregressive conditional heteroskedastic (ARCH) model that relates the (unobserved) volatility to squares of past innovations in price changes. Such a model-based approach only partly overcomes the deficiency of the historical measure and continues to respond very slowly when volatility undergoes rapid changes, as it has been the case during the recent financial crisis (see, for example, Hansen, Huang, and Shek, 2012). The use of ARCH or its various generalizations (GARCHs) in macro-econometric modelling is further complicated by temporal aggregation issues of daily GARCH models for use with quarterly data.

In the finance literature, the focus of the volatility measurement has now shifted to market-based implied volatility obtained from option prices, and realized measures based on the summation of intra-period higher-frequency squared returns (see, for example, Andersen, Bollerslev, Diebold, and Labys (2001, 2003), Barndorff-Nielsen and Shephard (2002, 2004)). The use of implied volatility from option prices in macro-econometric models has thus far been limited both by data availability and the fact that we still need to aggregate daily volatilities to a quarterly frequency. This explains the popularity of the VIX Index, which is an average of the daily option price implied volatility of the S&P 500 index (see Figure 1).

In contrast, the idea of realized volatility can be easily adapted for use in macro-econometric models by summing squares of daily returns within a given quarter to construct a quarterly measure of market volatility. The approach can be extended to include intra-daily return observations when available, but this could contaminate the quarterly realized volatility measures with measurement errors of intra-daily returns due to market micro-structure and jumps in intra-daily returns. In addition intra-daily returns are not available for all markets that we want to consider and, when available, tend to cover a relatively short time period as compared to our data period that begins in 1979.

Note that if we consider a panel of asset prices, a different measure of volatility can be computed as the cross-sectional dispersion of asset prices. As we show in Appendix B, however, given a panel data of asset prices, realized volatility and cross-sectional dispersion are closely related. Indeed, in our application, we obtain similar results when we use the cross-sectional dispersion measures (the results are not reported for sake of brevity).

Realized volatility and cross-sectional dispersion encompass most measures of uncertainty proposed in the macroeconomic literature. For example Schwert (1989b), Ramey and Ramey (1995), Bloom (2009), Fernandez-Villaverde, Guerron-Quintana, Rubio-Ramirez, and Uribe (2011) use aggregate time series volatility (summary measures of dispersion over time of output growth, stock market returns, or interest rates); Leahy and Whited (1996), Campbell, Lettau, Malkiel, and Xu (2001), Bloom, Bond, and Reenen (2007) and Gilchrist, Sim, and Zakrajsek (2013) use dispersion measures of firm-level stock market returns; Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012) use cross-sectional dispersion of plant/firm/industry profits, stocks, or total factor productivity.[17]

In the rest of this section, therefore, we provide precise definitions of the realized volatility measures that we use and briefly describe the data set we assembled to compute them.

5.2   Three types of volatility measures

We construct three types of volatility measures: at the level of individual markets (either country equity markets, foreign exchange markets, country bond markets, or individual commodity markets), at the level of an asset class (i.e., aggregating across individual markets within a given asset class), and at the global level (i.e., aggregating across all asset markets).[18] For exposition purposes, we shall label realized volatility at the level of individual markets, at the level of a whole asset class, and at the global level as *Market-Specific Volatility*, *Asset-Specific Volatility*, and *Global Volatility*, respectively.

5.2.1   Market-specific realized volatility

To construct quarterly measures of realized volatility at the level of individual assets, we begin with the daily price of asset of type κ, in country i, measured on close of day τ in quarter t and we denote it by $P_{\kappa it}(\tau)$. We label the quarterly realized volatility for quarter t at daily rate ($\mathcal{RV}_{\kappa it}$) as "market-specific realized volatility," or market volatility for brevity.

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[17]The literature has also used uncertainty measures based on expectation dispersion: while summarizing the range of disagreement among individual forecasters at a point in time, they do not give information about the uncertainty surrounding the individual's forecast. See, for instance, Zarnowitz and Lambros (1987), Popescu and Smets (2010), and Bachmann, Elstner, and Sims (2013).

[18]One could also consider all asset prices for a given country to construct a country specific measure of volatility. In our application, however, we consider only a small number of asset classes (equities, bonds, exchange rates and commodity prices), and large number of countries for each asset class as well as all commodities for which data are available. This approach is therefore less attractive in our global study with many countries and would be better suited for a country specific study with many different asset classes.

We compute market volatility as:

$$\mathcal{RV}_{\kappa it} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} \left(r_{\kappa it}(\tau) - \bar{r}_{\kappa it}\right)^2} \tag{20}$$

where $r_{\kappa it}(\tau) = \Delta \ln P_{\kappa it}(\tau)$ and $\bar{r}_{\kappa it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{\kappa it}(\tau)$ is the average daily price changes over the quarter t, and D_t is the number of trading days in quarter t. For most time periods, $D_t = 3 \times 22 = 66$, which is larger than the number of data points typically used in the construction of daily realized market volatility in finance.[19] The same market-specific volatility measures can also be computed for real asset prices, with $P_{\kappa it}(\tau)$ in the above expression replaced by $P_{\kappa it}(\tau)/P_{it}$, where P_{it} is the general price level in country i for quarter t, but they yield very similar results and in our application they are not reported.[20]

5.2.2 Asset-specific realized volatility

Market-specific realized volatility—as defined in (20)—can be aggregated across countries for a given asset class such as equity, long term bond, or exchange rate, or across all commodities to construct asset-specific realized volatility measures. This aggregation can be carried out by taking averages using equal weights or PPP-GDP weights or other weighting schemes. Let w_{it} be the weight attached to market (country) i in quarter t, then the realized volatility for asset type κ, in quarter t, denoted by $\mathcal{RV}_{\kappa t}$, is given by:

$$\mathcal{RV}_{\kappa t} = \sum_{i=1}^{N_t} w_{it} \mathcal{RV}_{\kappa it}, \tag{21}$$

where N_t is the number of markets (countries) in quarter t with price data on asset type κ. In this way, we construct measures of realized volatility by different asset classes which we label "asset-specific realized volatility," or asset volatility for brevity. Also, a log-linear aggregate defined by:

$$\mathcal{RV}_{\kappa t}^{L} = \sum_{i=1}^{N_t} w_{it} \ln(\mathcal{RV}_{\kappa it}),$$

could be used.

5.2.3 Global volatility

Finally, a "global realized volatility" measure can be computed by aggregating across different asset classes, namely:

$$\mathcal{RV}_{t} = \frac{1}{M} \sum_{\kappa=1}^{M} \sum_{i=1}^{N_t} w_{it} \mathcal{RV}_{\kappa it}, \tag{22}$$

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[19]In the case of intra-day observations prices are usually sampled at 10-minutes interval which yields around 48 intra-daily returns in an 8 hour-long trading day.

[20]We measure P_{it} by the consumer price index (CPI_{it}) when dealing with equity and bond prices and exchange rates, and use the U.S. producer price index ($PPI_{US,t}$) when measuring realized volatility of real commodity prices. The realized volatility measures of real asset prices are almost identical to the ones computed in equation (20) and are available from the authors on request.

where M is the number of assets that we consider. Alternatively, using a log-specification, we could have:

$$\mathcal{RV}_t^L = \frac{1}{M} \sum_{\kappa=1}^{M} \sum_{i=1}^{N_t} w_{it} \ln(\mathcal{RV}_{\kappa it}).$$

5.3 Data

To construct quarterly measures of market-specific realized volatilities, we first collect daily prices of stock market equity indices, exchange rates, long-term government bonds (whenever available) for 33 advanced and emerging economies, and daily prices of most internationally traded commodities. The data set spans 109 asset prices and, for each asset price, up to 8479 daily observations from 1979 to 2011 (depending on data availability).

After computing the market-specific realized volatility measures as in (20), we re-scale them so as to express them at quarterly rates. We do that by multiplying $\mathcal{RV}_{\kappa it}$ by $\sqrt{D_t}$: in this way we obtain realized volatility measures that are consistent with the remaining macroeconomic time series that we shall use in our empirical analysis (which are at quarterly frequency, too). Therefore all results, charts, and tables presented hereinafter shall refer to the *realized volatility measures expressed at quarterly rates*.

The sources of the data and their sampling information are reported in Appendix C, while a plot of all series—computed as in equation (20)—is reported in Appendix D. Figure 2 plots the quarterly realized volatility of equity prices in the United States and compares it with the quarterly average of the VIX index (already plotted in Figure 1), often considered as a benchmark measure of uncertainty. As Figure 2 shows, the realized volatility of U.S. equity prices co-moves very closely with the VIX Index, with a correlation coefficient of 0.84 and 0.86 in levels and first differences, respectively.
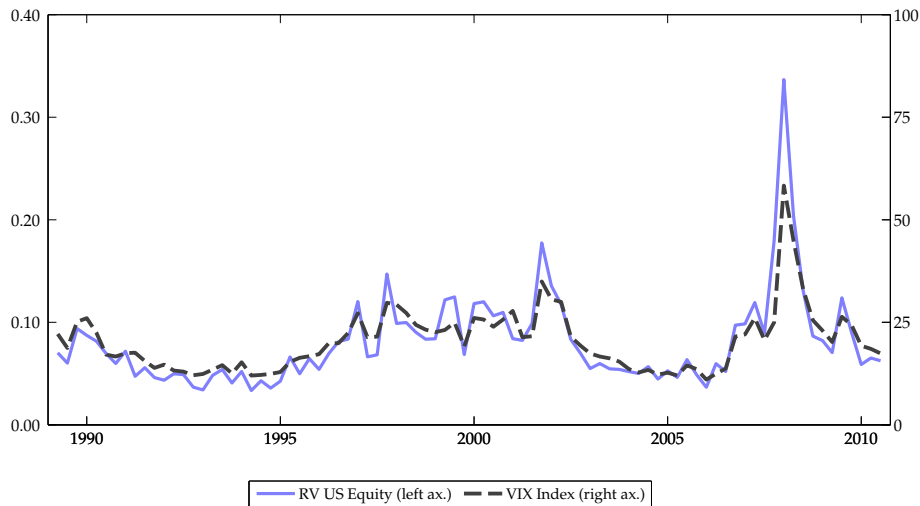
Figure 2 Quarterly U.S. Equity Realized Volatility And The Vix Index. The VIX Index is the quarterly average of the daily Chicago Board Options Exchange Market Volatility Index from Bloomberg. The sample period is 1990.I-2011.II.

6   Properties of realized volatilities at business cycle frequency

In this section we consider some of the key time series properties of the realized volatility measures. These time series properties are of interest themselves but also potentially important for the empirical analysis of the relationship between volatility and the macroeconomy set out in Section 3. First, we focus on market-specific volatility. Then we consider asset-specific volatilities, reporting key time series properties as well as the extent to which individual volatility measures co-move within and between asset classes. Finally, we investigate the (unconditional) relation between realized volatility and economic activity at quarterly frequency.

6.1   Market-specific realized volatility

Individual realized volatility series are positively skewed, fat–tailed, and persistent, even though not persistent enough to be described as $I(1)$ processes. Summary statistics for all 109 market specific realized volatility series are reported in Appendix D, and Table 1 reports the summary statistics of $\mathcal{RV}_{\kappa it}$—computed as in equation (20)—for a few selected advanced economies (United States, Canada, Japan, Germany, U.K., France, Australia, Switzerland, and Norway) and emerging market economies (Thailand, Indonesia, South Korea, China, Brazil, and India).

Considering the summary statistics in Table 1, we see that there is a high degree of similarity across countries and asset classes. But by comparing advanced economies with emerging market economies as a group we can also see important differences: for all three asset classes, standard deviations of realized volatilities for the emerging market economies are larger and their persistence is smaller than in advanced economies.

6.1.1   Persistence

In contrast to the typical macroeconomic variable, market-specific volatility appears stationary. As we noted earlier, the persistence and the stationarity (or lack thereof) of volatility is a crucial property for the purpose of modelling the interaction between volatility and the macroeconomy. An Augmented Dickey-Fuller test on individual equity price volatility with a constant and 4 and 8 lags—labelled $ADF(4)$ and $ADF(8)$ in the Table 1—rejects the null of a unit root for all countries, with the exception of South Korea and Indonesia. This conclusion largely holds for the other two asset classes considered. The only cases in which both the $ADF(4)$ and $ADF(8)$ cannot reject the null hypothesis of unit root are: Canada (exchange rate and bond volatility), UK (bond volatility), Germany (bond volatility), Switzerland (bond volatility), and Brazil (bond volatility). Nonetheless, given that these tests have weak power toward rejecting the null hypothesis, this is quite strong evidence in favor of our stationarity assumption.

Table 1 SUMMARY STATISTICS OF QUARTERLY REALIZED VOLATILITY FOR SELECTED COUNTRIES

	United States			Canada			Japan		
	EQ	*FX*	*LB*	*EQ*	*FX*	*LB*	*EQ*	*FX*	*LB*
Obs	130	–	130	130	130	100	130	130	114
Mean	0.08	–	0.09	0.07	0.03	0.08	0.09	0.05	0.13
StDev	0.04	–	0.05	0.04	0.02	0.03	0.04	0.02	0.08
AutoCorr	0.55	–	0.80	0.62	0.78	0.65	0.50	0.44	0.73
Skew	3.32	–	2.02	2.87	2.39	1.62	2.09	1.23	1.72
Kurt	18.25	–	8.55	15.42	11.84	7.37	12.02	5.97	7.00
ADF(4)	-3.44	–	-2.65	-3.39	-1.87*	-2.53*	-3.85	-3.69	-3.45
ADF(8)	-3.32	–	-2.01*	-2.68	-2.06*	-1.45*	-2.84	-3.84	-2.39*

	Germany			UK			France		
	EQ	*FX*	*LB*	*EQ*	*FX*	*LB*	*EQ*	*FX*	*LB*
Obs	130	130	130	130	130	100	130	130	101
Mean	0.09	0.05	0.06	0.08	0.05	0.07	0.09	0.05	0.07
StDev	0.04	0.01	0.03	0.04	0.02	0.03	0.04	0.01	0.02
AutoCorr	0.46	0.51	0.78	0.47	0.62	0.74	0.48	0.49	0.57
Skew	2.64	0.73	1.30	2.46	1.31	2.00	2.05	0.60	0.90
Kurt	16.14	4.09	4.95	11.27	5.09	10.00	8.67	3.73	3.81
ADF(4)	-3.78	-4.98	-1.78*	-3.63	-3.93	-2.48*	-3.45	-5.22	-3.36
ADF(8)	-3.60	-4.03	-1.27*	-3.32	-3.07	-2.10*	-3.32	-4.00	-2.22*

	Australia			Switzerland			Norway		
	EQ	*FX*	*LB*	*EQ*	*FX*	*LB*	*EQ*	*FX*	*LB*
Obs	130	130	129	130	130	69	125	130	74
Mean	0.08	0.05	0.08	0.07	0.06	0.10	0.11	0.05	0.07
StDev	0.04	0.03	0.03	0.04	0.01	0.04	0.05	0.02	0.06
AutoCorr	0.34	0.54	0.57	0.51	0.44	0.75	0.47	0.60	0.07
Skew	4.01	2.82	0.69	2.13	0.47	1.16	2.96	1.84	4.72
Kurt	26.18	19.58	4.69	9.07	2.87	4.43	15.66	9.79	32.99
ADF(4)	-3.65	-3.10	-3.45	-3.36	-5.03	-1.73*	-3.70	-3.62	-4.43
ADF(8)	-3.31	-3.07	-3.58	-3.18	-4.07	-1.48*	-3.04	-2.76	-2.30*

	Thailand			Indonesia			Korea		
	EQ	*FX*	*LB*	*EQ*	*FX*	*LB*	*EQ*	*FX*	*LB*
Obs	97	121	43	94	78	31	130	130	42
Mean	0.13	0.03	0.12	0.14	0.07	0.09	0.12	0.03	0.09
StDev	0.06	0.04	0.06	0.08	0.10	0.06	0.06	0.05	0.04
AutoCorr	0.50	0.59	0.42	0.41	0.80	0.19	0.69	0.53	0.38
Skew	1.23	3.46	0.68	2.37	3.59	2.89	1.43	4.67	1.05
Kurt	3.97	16.15	2.34	11.09	18.46	12.95	5.49	29.42	4.07
ADF(4)	-3.04	-4.37	-2.77	-2.38*	-2.67	-2.62	-2.41*	-3.58	-3.18
ADF(8)	-2.91	-3.15	-1.89*	-2.30*	-2.33*	–	-2.24*	-2.65	-2.57*

	China			Brazil			India		
	EQ	*FX*	*LB*	*EQ*	*FX*	*LB*	*EQ*	*FX*	*LB*
Obs	74	121	24	86	77	17	97	130	–
Mean	0.15	0.01	0.06	0.25	0.07	0.03	0.12	0.03	–
StDev	0.07	0.05	0.04	0.26	0.05	0.04	0.05	0.02	–
AutoCorr	0.58	-0.01	-0.12	0.81	0.52	0.69	0.48	0.37	–
Skew	1.38	6.45	1.11	3.84	1.71	0.88	1.16	2.54	–
Kurt	5.49	50.24	3.57	20.00	6.62	2.83	3.75	14.21	–
ADF(4)	-2.72	-4.36	-2.20*	-3.57	-2.58*	–	-3.16	-4.22	–
ADF(8)	-2.72	-3.26	–	-2.81	-2.24*	–	-3.13	-2.97	–

Note. These summary statistics refer to the realized volatility measures $\mathcal{RV}_{\kappa it}$ at quarterly rates, computed over the 1979.I-2011.II period (subject to data availability). The labels *EQ*, *FX*, and *LB* stand for equity volatility, exchange rate volatility and long-term government bond volatility, respectively. *ADF(4)* and *ADF(8)* are the ADF t-statistics computed with 4 and 8 lags, respectively. The asterisk indicates the cases where the test cannot reject the null hypothesis of $I(1)$ with a confidence level lower than 90 percent.

6.1.2 Synchronization

The degree of synchronization is higher than among typical macroeconomic variables, but varies substantially across asset classes. We measure synchronization by the contemporaneous correlation among market-specific volatilities within each asset class. In order to gauge to what extent our volatility measures co-move across countries we conduct both a standard principal component analysis and pair-wise correlation analysis.

The average pairwise correlation of a volatility series $\mathcal{RV}_{\kappa it}$ is computed over $i = 0, 1, ..., N$ (number of countries), and $\kappa = 1, 2, ..., M$ (number of assets). The average is computed for all pairs of countries and all pairs of assets. This is done for a given asset as well as for a given country. An overall average can also be computed across country pairs and asset pairs. The average pairwise correlation can be interpreted as an average measure of the degree of synchronization of volatilities across markets and asset types. Using principle component (PC) analysis, the degree of synchronization can be measured by the importance of the first PC of volatilities of assets under consideration. In the case of balanced panels both approaches can be used and provide different measures of synchronization. But in the case of unbalanced panels, which is the type of panels we are considering, the average pairwise correlation has the advantage that it can be applied to a larger number of assets/countries.

We start with equity price volatility. In our data set, equity price volatility series covering the full sample period 1979.I-2011.II are available for only 16 countries.[21] The first principal component on these 16 series explains 63 percent of the total variation in the level of equity price volatilities, and 65 percent of the total variation in the first difference of equity price volatilities. The corresponding figures for exchange rates (21 series) are 62 percent and 59 percent, and for commodities (8 series) are 47 percent and 36 percent. Finally, in the case of government bonds, the number of volatility series covering the full sample period are only 3. The application of the PC to bond market volatilities is therefore unlikely to produce reliable estimates. By comparison, the first PC of real GDP explains 97 percent (for log levels) and 18 percent (for log first differences) on 33 available series, and the first principal component of CPI inflation explains 66 and 47 percent of the variations of level and first differences of the inflation rate, respectively, again applied to 33 available series.

The pairwise correlation analysis—which instead uses all the available sample information— yields similar results for real GDP, but somewhat different results for inflation and volatilities. The average pairwise correlations of our volatility measures are: 0.47 and 0.46 for equity prices (in levels and first differences, respectively); 0.23 and 0.21, for exchange rates; 0.42 and 0.33 for long-term government bonds; and 0.24 and 0.16 for commodity prices.[22] By comparison, the average pairwise correlation of real GDP is 0.95 and 0.15 (in levels and first differences, respectively) and the average pairwise correlation of inflation is 0.28 and 0.07, for level and first differences, respectively.

In sum, the comovement of market-specific realized volatilities within asset classes is larger than standard macroeconomic variables (such as real GDP growth and CPI inflation). However, the actual degree of synchronization varies with the specific asset class we consider

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[21]Since principal component analysis can be computed only on balanced panels, we compute the first principal component only on the series with available data covering the full sample period.

[22]All individual–specific average pairwise correlations are reported in Table E.1 in Appendix.

and the measure of synchronization we use. Moreover, as we shall see in the next subsection, asset-specific volatility is not highly correlated across asset classes. In view of this evidence, for the analysis of the relation between uncertainty and macroeconomic activity we will use both asset-specific realized volatility and global volatility (that aggregates all four asset classes in a single measure of global volatility) rather than using highly disaggregated market–specific realized volatilities.

6.2 Asset-specific and global realized volatility

In this subsection, we report and discuss the properties of the asset-specific volatility measures $(\mathcal{RV}_{\kappa t})$ computed as in equation (21) for the four asset classes that we consider in our application. Moreover, we also consider the global volatility measure $(\mathcal{RV}_t)$ computed as in equation (22), i.e. the simple average of our asset–specific volatility measures (with equal weighting). As we already noted, while the aggregation into asset–specific volatility measures can be accomplished in many different ways, for transparency, in the rest of the paper we computed them using equal weights. It is important to note here that aggregating our measures using either PPP-GDP weights, logarithms based series, or principal component techniques give essentially the same results.

6.2.1 Summary statistics

Table 2 reports the summary statistics for asset-specific volatility measures. The results show that asset-specific volatilities, although persistent, tend to be mean reverting. Also, not surprisingly, there are significant departures from normality.

Table 2 SUMMARY STATISTICS OF ASSET–SPECIFIC REALIZED VOLATILITY MEASURES

	Equity	Exch. Rate	Bond	Commodity
Mean	0.10	0.05	0.07	0.13
Median	0.10	0.04	0.07	0.12
Max	0.31	0.12	0.17	0.29
Min	0.06	0.02	0.03	0.08
St. Dev.	0.04	0.01	0.02	0.03
Auto Corr.	0.61	0.58	0.71	0.62
Skew.	2.01	1.49	0.99	2.24
Kurt.	9.44	7.48	4.64	11.10
ADF(4)	-3.55	-4.32	-3.22	-5.12
ADF(8)	-2.91	-3.93	-2.40*	-3.82
Frac. Int.	0.43	0.46	0.42	0.41

Note. Summary statistics are computed over the 1979.I–2011.II period (subject to data availability). *ADF(4)* and *ADF(8)* are the ADF t-statistics computed with 4 and 8 lags, respectively. The asterisk indicates the cases where the test cannot reject the null hypothesis of $I(1)$ with a confidence level lower than 90 percent. *Frac. Int.* refers to the the coefficient of fractional integration term in ARFIMA(0,d,0) estimation.

Spikes in asset-specific volatility are rare events. The strong positive skewness indicates that the tail on the right side of the distribution is longer than the left side, and the bulk of the density lies to the left of the mean. Moreover, the positive excess kurtosis suggests that a high share of the variance is due to infrequent extreme jumps in asset returns. This is particularly the case for equity and commodity price volatilities. Indeed, Table 2 also shows that equity prices and commodity prices tend to be more volatile than exchange rates and bond prices. The average volatility of equity and commodity prices are 10 and 13 percent per quarter, respectively, almost twice as large as the volatilities of exchange rates and long-term government bonds.[23]

Asset-specific realized volatility is persistent, but it is mean reverting. As reported in Table 2, the four series display a similar degree of persistence, with a first order auto-correlation coefficient of about 0.6 for equity, exchange rates and commodity volatility, and about 0.7 for bond volatility. Figure 3 shows that autocorrelation function decays quite rapidly to zero for the four series. Indeed, standard ADF tests reject the null hypothesis that the volatility variables have a unit root. And when we test for fractional integration, for comparison with the finance literature, we find that all four series are indeed stationary.[24] In contrast, macroeconomic variables are typically modeled as unit root processes.
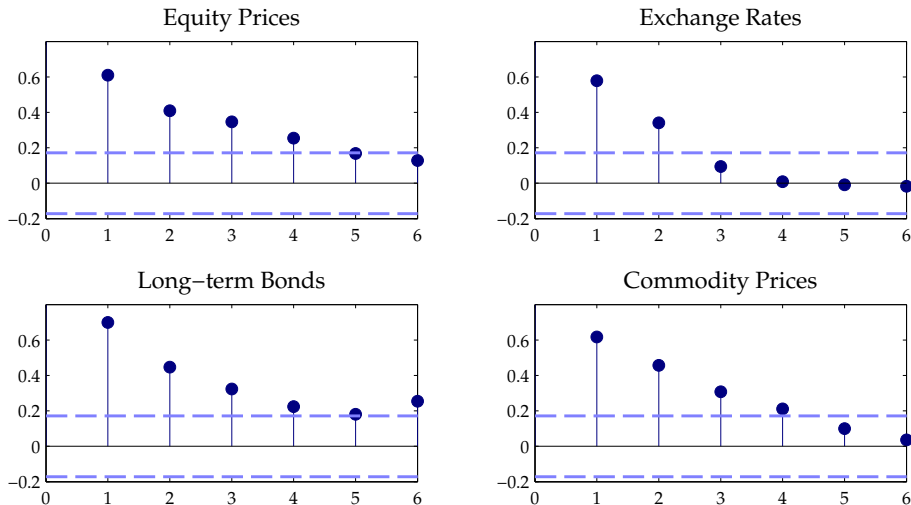
Figure 3 AUTO-CORRELATION OF ASSET–SPECIFIC REALIZED VOLATILITY MEASURES. Auto–correlation coefficients are computed over the 1979.I–2011.II period.

This $I(0)/I(1)$ mismatch poses a challenge for modelling the interaction between volatility and the macroeconomy. For instance, while the "Great Recession" has been very protracted, global volatility subsided in all asset classes. The statistical property of our realized volatility measures is taken into account by augmenting the GVAR with a separate $I(0)$ volatility

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[23]This may reflect the fact that some countries manage the nominal exchange rate and that the sample of bond prices is limited to the most advanced and financially developed economies in the world.

[24]In the finance literature, volatility at higher frequencies has been found to be highly persistent, with the longer-run dependencies well described by a fractionally integrated process (see, e.g., Ding, Granger, and Engle, 1993, Baillie, Bollerslev, and Mikkelsen, 1996, Andersen and Bollerslev, 1997, Comte and Renault, 1998).

module.

6.2.2 Volatility synchronization

Measures of asset-specific realized volatility do not always move together, as we can see in
Figure 4 which compares their behavior to that of global volatility. Specifically, Figure 4
plots the asset-specific volatility measures ($\mathcal{RV}_{\kappa t}$) computed as in equation (21) together
with the global measure of volatility ($\mathcal{RV}_t$) computed as in equation (22). For instance,
the biggest spike in commodity price volatility in the sample coincides with the 1979 energy
crisis, without a matching movement in other asset-specific volatilities. Equity price volatility
increased sharply in 1987 when stock markets around the world crashed, without increases
in bond or commodity price volatility. Bond and exchange rate volatility were relatively high
at the end of the 1990s after the Asian financial crisis in 1997, the default in Russia, and
the near default of a large US Hedge Fund (LTCM) in 1998. Only during the 2008-09 global
financial crisis all asset-specific volatility series moved together.
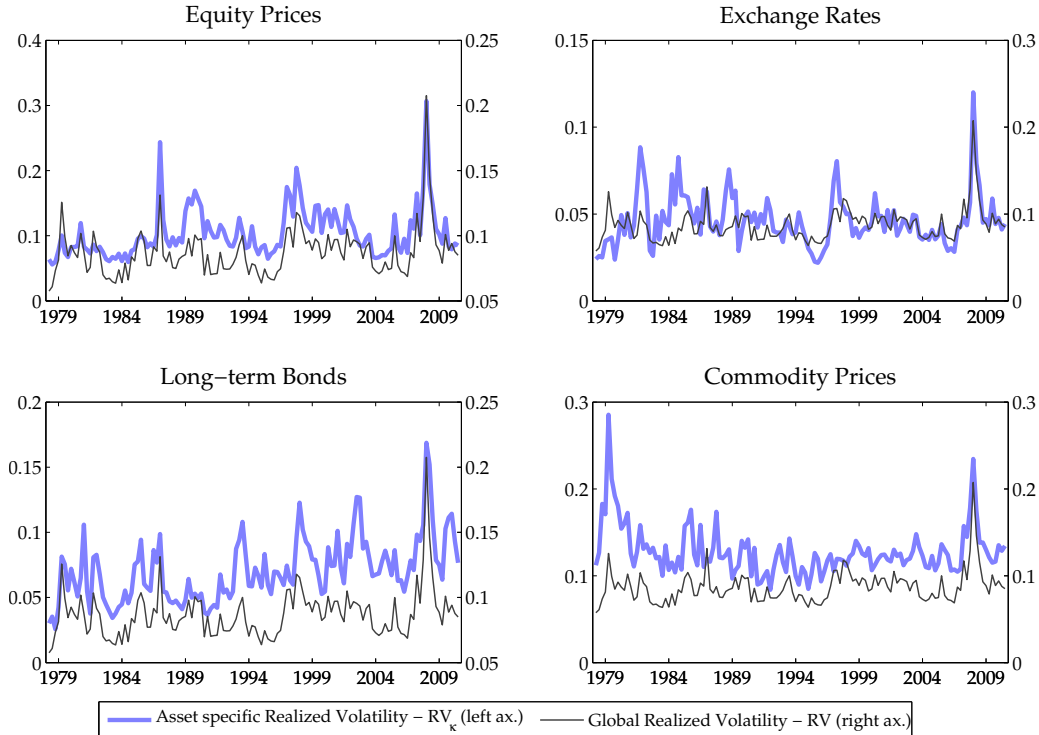
Figure 4 ASSET-SPECIFIC AGGREGATE REALIZED VOLATILITY MEASURE. $\mathcal{RV}_\kappa$ is the
simple average of the volatility measures across countries/commodities computed as in
equation (21) over the 1979.I-2011.II period. $\mathcal{RV}$ is the simple average of the $\mathcal{RV}_\kappa$ com-
puted as in equation (22) over the same sample period.

Table 3 shows that the sample correlation between our asset–specific volatility measures
($\mathcal{RV}_{\kappa t}$) is positive and significant for equity prices, exchange rates, and bonds (between 0.3
and 0.5). However, it is relatively low for commodities, (between 0.15 and 0.25). This
evidence suggests that, although global volatility is a useful concept, it is important to look

also at asset-specific volatilities. More generally, this evidence is consistent with the notion that market-specific realized volatility is not necessarily well represented by a single global factor. In view of this finding, in our empirical analysis we shall consider all our asset-specific volatility measures and we shall jointly model them with a VAR model.

Table 3 CORRELATION BETWEEN ASSET–SPECIFIC REALIZED VOLATILITY MEASURES

	Equity	_Exch. Rate_	_Bond_	_Commodity_
Equity	1.00	–	–	–
Exch. Rate	0.52	1.00	–	–
Bond	0.49	0.32	1.00	–
Commodity	0.16	0.14	0.24	1.00

Note. The correlations are computed over the 1979.I–2011.II period.

6.2.3 Output growth and volatility correlations

One robust stylized fact from the U.S. business cycle literature is that most uncertainty measures are strongly countercyclical, rising in recessions and falling during booms. Does this property holds for all the proxies of uncertainty that we constructed, as well as for other countries? How are our asset-price volatility measures related to the business cycle?

We investigate this issue by examining the comovement between asset-price volatility and the quarterly growth rate of real GDP.[25] In particular, we compute the cross-correlation between the growth rates of real GDP and all volatility series in our data set as:

$$R_{\kappa i}(\pm n) = \mathsf{COR}(\Delta y_{it}, \mathcal{RV}_{\kappa i, t \pm n}) \qquad n = 0, 1, ..., 4,$$

where Δy_{it} is the quarterly growth rate of real GDP in country i, $\mathcal{RV}_{\kappa i, t \pm n}$ is the level of the volatility of asset κ in country i, and n stands for the lead/lag of the generic variable $\mathcal{RV}_{\kappa i, t \pm n}$ for which the correlation coefficient is computed. Figure 5 displays the results. The country-specific cross-correlations are averaged over all countries in our data set (dark dots), and the heterogeneity within each group is taken into account by computing confidence bands (light shaded areas) as in Pesaran, Smith, and Im (1996).[26]

All volatility variables display a negative correlation with the growth rate of real GDP. This is consistent with the empirical evidence on the countercyclicality of stock market volatility documented for the United States, but also puts forth some new evidence concerning the cyclicality of realized volatility of other asset classes. In fact, exchange rate and long-term government bond volatility are as negatively correlated with real GDP growth as stock market volatility, while the correlation between commodity price volatility and growth is slightly lower. On average, over the whole sample, the contemporaneous correlation between all

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[25]We find similar results (not reported but available on request from the authors) when we compute these correlations detrending real GDP with a deterministic quadratic trend or the HP filter.

[26]The variance of the cross-section can be calculated by taking the variance across countries and dividing it by $(N-1)$, where N is the number of countries. As Pesaran, Smith, and Im (1996) prove, this adjustment yields a consistent estimate of the true cross-section variance.
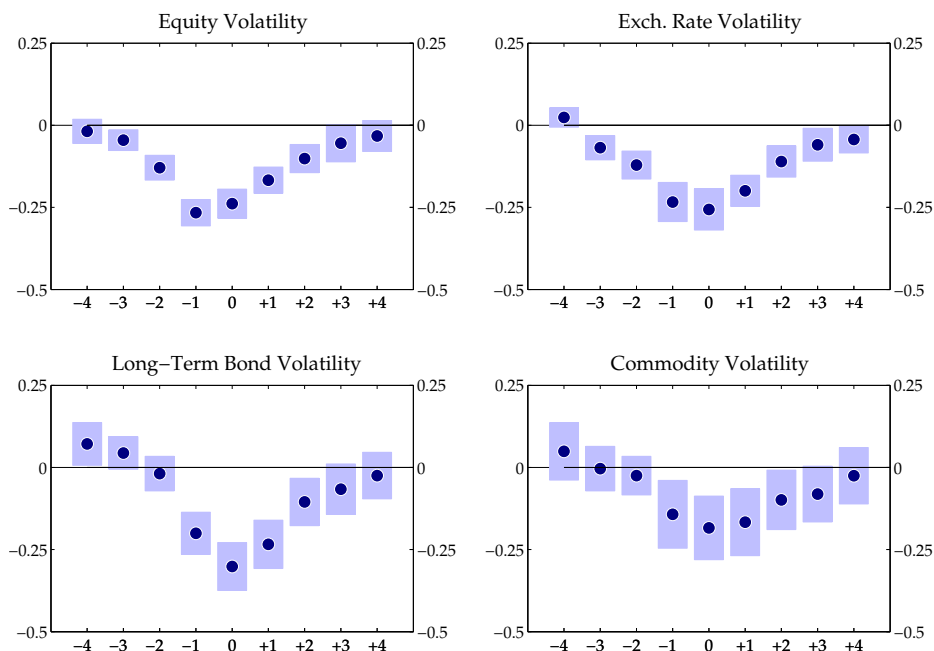
Figure 5 LEAD/LAG CORRELATION WITH QUARTERLY GDP GROWTH. The correlation coefficients are computed over the 1979.I–2011.II period (subject to data availability). Each dot graphs the cross-country average of the correlation coefficient. The shaded areas graph the cross-country two standard deviations confidence bands.

volatility measures and GDP quarterly growth is between -0.2 and -0.3. Moreover, the relation between global volatility and GDP growth is very similar across countries, given the narrow error bands.

Concluding, the lead/lag pattern of volatility and GDP growth shows that—for the four asset classes that we consider—volatility can significantly lead the business cycle. This is particularly true for equity price volatility, for which we obtain the largest correlation coefficient for $\mathcal{RV}_{t-1}$. It is however to bear in mind that these are simple correlations and no causal relation can be inferred.

7 Specifying and estimating the GVAR-VOL model

In this section we briefly describe the specification and estimation of the GVAR-VOL model in our application.

7.1 The standard component of the GVAR model

The GVAR that we use in our application has 26 country-specific VARX* models, including all major advanced and emerging economies in the world and accounting for about 90 percent of world GDP. Core advanced economies are: United States, UK, Japan, Australia,

Canada, New Zealand, Norway, Sweden, Switzerland and the euro area. The latter is made up of its eight largest economies: Austria, Belgium, Finland, France, Germany, Italy, Netherlands, Spain. Emerging markets economies include China, India, and an emerging Asia block excluding China and India (Indonesia, South Korea, Malaysia, Philippines, Singapore, and Thailand), a Latin America block (Argentina, Brazil, Chile, Mexico, and Peru).[27] Thus, the version of the GVAR model that we specify uses data for 33 countries. The models are estimated over the 1979.II–2011.II period.

Table 4 Variables Specification of the Country-specific VARX* Models

Non-US models			US model	
Domestic	Foreign		Domestic	Foreign
y_{it}	y_{it}^*		y_{US}	y_{US}^*
π_{it}	π_{it}^*		π_{US}	π_{US}^*
q_{it}	q_{it}^*		q_{US}	
ρ_{it}^S	ρ_{it}^{S*}		ρ_{US}^S	ρ_{US}^{S*}
ρ_{it}^L	ρ_{it}^{L*}		ρ_{US}^L	-
$e_{it} - p_{it}$	-		-	$e_{US}^* - p_{US}^*$
-	p_t^o		p_t^o	-

Note. In the non-US models the inclusion of the listed variables depends on data availability.

For the specification of the country-specific models we follow Dees, di Mauro, Pesaran, and Smith (2007). All country-specific VARX* models, with the exception of the United States, include the same set of variables (subject to data availability) as summarized in Table 4.[28] The variables included in each country model are log real GDP (y_{it}), the rate of inflation ($\pi_{it} = p_{it} - p_{i,t-1}$), the real exchange rate defined as ($e_{it} - p_{it}$), and, when available, real equity prices (q_{it}), a short rate (ρ_{it}^S) and a long rate of interest (ρ_{it}^L), with: $y_{it} = \ln(GDP_{it}/CPI_{it})$, $p_{it} = \ln(CPI_{it})$, $q_{it} = \ln(EQ_{it}/CPI_{it})$, $e_{it} = \ln(E_{it})$, $\rho_{it}^S = 0.25 \cdot \ln(1 + R_{it}^S/100)$, $\rho_{it}^L = 0.25 \cdot \ln(1 + R_{it}^L/100)$. Here, GDP_{it} is nominal Gross Domestic Product of country i at time t (in domestic currency); CPI_{it} is the Consumer Price Index in country i at time t (equal to 100 in year 2000); EQ_{it} is a nominal Equity Price Index; E_{it} is the nominal exchange rate of country i at time t in terms of U.S. dollars; R_{it}^S is the short rate of interest in percent per year (typically a three-month rate); R_{it}^L is a long rate of interest in percent per year (typically a 10-year rate).

All country models (except the U.S.) also include the log of nominal oil prices (p_t^o) as a weakly exogenous foreign variable. The oil price is determined in the U.S. model as an endogenous variable, but is included in the remaining country-specific VARX models as a weakly exogenous regressor.

The U.S. model is specified differently. First, as we mentioned, the oil price is included as an endogenous variable. In addition, given the importance of the U.S. financial variables in the global economy, the U.S. specific foreign financial variables, $q_{US,t}^*$ and $\rho_{US,t}^{*L}$, are *not* included in the U.S. model. Indeed, when tested, these variables fail to pass weak exogeneity

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[27]The time series data for the euro area are constructed as weighted averages using Purchasing Power Parity GDP weights, averaged over the 2009-2011 period (Source: World Bank). Emerging Latin American and Asian countries enter the model individually.

[28]While the most important details are reported below, the full specification is available from the authors under request.

tests. Note, however, that the real value of the U.S. dollar, by construction, is determined outside the U.S. model, and the U.S. specific real exchange rate (defined as $e^*_{US,t} - p^*_{US,t}$) is included in the U.S. model as a weakly exogenous foreign variable.

The GVAR model hinges on the assumption that the variables included in the country-specific models are integrated of order one (or $I(1)$). We test this assumption by performing the weighted symmetric test for unit root introduced by Fuller and Park (1995). The test largely supports the unit root hypothesis with only a few exceptions, as discussed by Cesa-Bianchi, Pesaran, Rebucci, and Xu (2012) who use the same GVAR specification and data.

Table 5 Lag Specification of the Country-specific VARX* Models and Number of Cointegrating Relations

	p	q	CR		p	q	CR
ARGENTINA	2	2	3	NORWAY	2	1	3
AUSTRALIA	1	1	3	NEW ZEALAND	2	2	2
BRAZIL	2	2	2	PERU	2	2	2
CANADA	2	2	3	PHILIPPINES	2	1	2
CHINA	2	2	2	SOUTH AFRICA	2	2	2
CHILE	2	2	3	SAUDI ARABIA	2	1	1
EURO	2	2	3	SINGAPORE	2	1	3
INDIA	2	2	2	SWEDEN	2	1	3
INDONESIA	2	2	3	SWITZERLAND	2	1	4
JAPAN	2	2	3	THAILAND	2	1	2
KOREA	2	2	3	TURKEY	2	2	1
MALAYSIA	1	1	1	UNITED KINGDOM	1	1	3
MEXICO	2	2	2	UNITED STATES	2	1	2

Note. For each country p is the lag order of the domestic variables, q is the lag order of the foreign variables, and CR is the number of the cointegrating relations.

The lag order of the individual country VARX*(p_i, q_i) models is selected according to the Akaike information criterion under the constraints imposed by data limitations. The rank of the cointegrating space for each country was tested using Johansen's trace and maximal eigenvalue statistics, as set out in Pesaran, Shin, and Smith (2000) for models with weakly exogenous $I(1)$ regressors, in the case where unrestricted constants and restricted trend coefficients are included in the individual country error correction models. Finally, the country-specific models were estimated subject to reduced rank restrictions (Johansen, 1992). The order of the VARX* models, as well as the number of cointegration relations, are presented in Table 5.

The weak exogeneity test results suggest that most of the weak exogeneity assumptions are not rejected by the data. To test for weak exogeneity we follow Johansen (1992) and Harbo, Johansen, Nielsen, and Rahbek (1998), who proposed a test on the joint significance of the estimated error correction terms in auxiliary equations for the country-specific foreign variables ($\mathbf{x}^*_{it}$). Specifically, the test fails to reject the null of weak exogeneity only in 11 out of 153 cases.

Other than the unit roots, the two largest eigenvalues of the GVAR (in *modulus*) are 0.91 and 0.86, implying a satisfactory rate of convergence of the model to its long-run equilibrium.

Moreover, the persistence profiles—the time profiles of the effects of variable specific shocks on the cointegration relations in the GVAR model—return to equilibrium after about 10 quarters for all cointegration relations, providing additional evidence on the stability of the system.

7.2 The volatility module

In this section we present estimates of the volatility module in (10) using our asset-specific volatility measures—labelled $\mathcal{RV}_{\kappa t}$ and defined as in equation (21)—as a proxy for $\mathbf{v}_t$. Specifically, we use realized volatility measures for four asset classes: equity prices, exchange rates, long-term government bonds, and commodity prices. As a result $\mathbf{v}_t$ is a (4×1) vector of realized volatilities.

We model the vector $\mathbf{v}_t$ as a VAR model that includes its own lagged values (where the number of lags has been determined with Akaike criterion), and augmented with the lead, contemporaneous, and the first order lag of global output growth ($\Delta y_{t\pm1}^*$) and global inflation ($\Delta \pi_{t\pm1}^*$). Table 6 reports the OLS estimates and the associated t-ratios in square brackets.

Table 6 Volatility Module

	$v_{EQ,t}$	$v_{FX,t}$	$v_{LB,t}$	$v_{COM,t}$
c	0.09	0.05	0.04	0.08
	[3.91]	[5.25]	[2.97]	[5.50]
$v_{EQ,t-1}$	0.53	-0.08	-0.03	-0.09
	[5.86]	[-2.16]	[-0.55]	[-1.52]
$v_{FX,t-1}$	0.08	0.55	0.00	0.00
	[0.36]	[6.54]	[-0.01]	[0.02]
$v_{LB,t-1}$	-0.01	-0.03	0.71	0.11
	[-0.06]	[-0.64]	[9.37]	[1.37]
$v_{COM,t-1}$	-0.14	-0.01	-0.03	0.48
	[-1.12]	[-0.19]	[-0.37]	[6.02]
Δy_{t+1}^*	-3.37	-0.98	-1.21	-0.99
	[-5.41]	[-4.04]	[-3.17]	[-2.50]
$\Delta \pi_{t+1}^*$	0.60	0.17	0.07	-0.50
	[1.57]	[1.14]	[0.28]	[-2.03]
Δy_t^*	0.63	-0.50	-0.21	-0.71
	[0.85]	[-1.73]	[-0.46]	[-1.52]
$\Delta \pi_t^*$	-0.07	0.23	0.11	0.23
	[-0.17]	[1.50]	[0.44]	[0.94]
Δy_{t-1}^*	-0.01	-0.08	-0.11	0.11
	[-0.02]	[-0.32]	[-0.27]	[0.27]
$\Delta \pi_{t-1}^*$	-0.23	-0.07	0.11	-0.06
	[-0.61]	[-0.48]	[0.48]	[-0.25]

Note. t-Statistics are in brackets. The lag order is determined with Akaike criterion with a maximum of 4 lags. The labels *EQ*, *FX*, *LB*, and *COM* stand for equity volatility, exchange rate volatility, long-term government bond volatility, and commodity volatility respectively. The model is estimated over the 1979.II–2011.II period.

For each volatility variable the coefficient on its own first order lag is positive and statisti-

cally significant. This first-order autoregressive coefficient is around 0.5 for equity, exchange rate, and commodity price volatilities, and is somewhat higher (at 0.71) for the volatility of long-term government bonds. As already documented in Section 6, volatility is persistent but for sure it is not an $I(1)$ process.

Given the multivariate specification adopted, we also allow for a possible interaction between the asset-specific volatility variables. The results in Table 6, however, show that most of the non-diagonal elements of the matrix of coefficients associated with $\mathbf{v}_{t-1}$ are not significantly different from zero, one exception being the volatility of equity prices which significantly affects the volatility of exchange rates with a negative coefficient.

We turn now to the effects of global macroeconomic variables on volatilities. Consistently with our factor model (10), we find a strong negative relation between volatility and the lead of global output growth. As Table 6 shows, the lead of global output growth significantly affects all the volatility variables with a negative sign. Note that the coefficients on Δy_{t+1}^* in the case of exchange rates, long-term government bonds and commodity prices are all close to 1, implying that a one percent increase in future global output growth is associated with a fall of about 1 percent in volatility. Interestingly, the same coefficient for equity volatility equation is much larger, at -3.37, and highly significant, which is consistent with the view that equity markets tend to over-react to news. In contrast, the response of exchange rates and bond markets to news is more muted, arguably because of stabilizing influence of government intervention in these asset markets.

From Table 6 we can also observe that neither the contemporaneous nor the lagged values of global growth are statistically significant for the volatility variables (even though most of them have the expected negative sign). Also, most of the coefficients on global inflation are not statistically significant, an exception being the negative coefficient of global inflation on commodity price volatility.

It is interesting to note that the residuals of the volatility equations remain correlated, as Table 7 shows. The correlation among the reduced-form residuals is lower than the unconditional correlation of the data documented in Table 3 but still statistically significant.

Table 7 CORRELATION OF THE REDUCED-FORM RESIDUALS FROM THE VOLATILITY MODULE

	$\hat{\xi}_{EQ,t}$	$\hat{\xi}_{FX,t}$	$\hat{\xi}_{LB,t}$	$\hat{\xi}_{COM,t}$
$\hat{\xi}_{EQ,t}$	1	–	–	–
$\hat{\xi}_{FX,t}$	0.42	1	–	–
$\hat{\xi}_{LB,t}$	0.42	0.24	1	–
$\hat{\xi}_{COM,t}$	0.23	0.21	0.19	1

Note. Simple correlation between the reduced form residuals of the multivariate volatility module. The labels *EQ*, *FX*, *LB*, and *COM* stand for equity volatility, exchange rate volatility, long-term government bond volatility, and commodity volatility respectively.

8 The macroeconomic impact of volatility innovations

Equipped with the estimates of the two components of our GVAR-VOL model, we are ready to look at the impact of volatility innovations on the macroeconomic dynamics. We first examine whether the volatility innovations have an impact on the reduced-form residuals of the GVAR. This permits us to see the extent to which volatility changes that are not driven by the common factors $\mathfrak{n}_t$ have the potential for explaining movements in macroeconomic variables once the effects of common and country-specific factors (embedded in the GVAR model) are filtered out. If the answer is positive, we then could illustrate and quantify these second round effects by simulating the full GVAR-VOL model. If the answer is negative, we would conclude that this component of volatility cannot drive macroeconomic dynamics.

To analyze how volatility innovations impact on the GVAR residuals we estimate the following country-specific, variable-specific equations:

$$\hat{u}_{i\ell t} = \alpha_{i\ell}\bar{\xi}_{t-1} + \zeta_{i\ell t}, \tag{23}$$

where $\hat{u}_{i\ell t}$ is selected from the vector of the GVAR reduced-form residuals $\hat{\mathbf{u}}_t$ defined by equation (17) so as to pick the residuals of variable $\ell = 1, ..., k_i$ in country $i = 0, 1, ..., N$. The term $\bar{\xi}_t$ is instead constructed as:

$$\bar{\xi}_t = \frac{1}{M}\sum_{\kappa=1}^{M}\hat{\bar{\xi}}_{\kappa t}, \tag{24}$$

where $\hat{\bar{\xi}}_{\kappa t}$ are the residuals of the volatility module and $M = 4$. We label the simple average of the reduced-form residuals $(\bar{\xi}_t)$ a *global volatility shock*. Note here that the lagged specification in the relationship between $\hat{u}_{i\ell t}$ and $\bar{\xi}_{t-1}$ is in line with our discussion of Section 3 that shows that—under our assumptions—the volatility innovations affect the GVAR residuals with a lag.

An alternative approach would have been to aggregate the asset-specific volatility variables to construct a global volatility measure—as in equation (22)—and then estimate a univariate ARDL model in global volatility augmented with macro variables. However, the heterogeneity of the coefficients on global output growth across asset types (see Table 6) suggests that an aggregated approach could result in biased estimates. But as we shall see below, qualitatively similar results are obtained even if we adopt an aggregate volatility model, with the difference that the coefficient of the global output growth in the aggregate volatility model is more difficult to interpret.

Table 8 reports the regression results for output growth and inflation, as well as for equity prices and exchange rates. To evaluate these results, recall that under our identification assumptions, spelled out in Section 3, $\mathfrak{n}_t$ affects financial market volatility contemporaneously and macroeconomic variables with a lag. While this assumption seems reasonable for slow-moving variables such as GDP and inflation, it is less likely to hold for fast moving financial variables such as equity prices and exchange rates. Therefore, only the relation between $\boldsymbol{\xi}_t$ and $\mathbf{u}_t^y$ (i.e., the residuals of GDP and inflation) can be strictly interpreted in terms of causation, while the relation between $\boldsymbol{\xi}_t$ and $\mathbf{u}_t^\chi$ (i.e., the residuals of equity prices and exchange rates) has to be interpreted as simple statistical association.

We describe first the estimation of (23) for GDP and inflation. Starting from the GVAR

Table 8 GVAR-VOL Model: Global Volatility Innovations And Reduced-form GVAR Residuals

	GDP			Inflation			Equity Price			Exchange Rate		
	$\alpha_{i,y}$	t-Stat	R^2	$\alpha_{i,\pi}$	t-Stat	R^2	$\alpha_{i,eq}$	t-Stat	R^2	$\alpha_{i,ep}$	t-Stat	R^2
ARGENTINA	0.10	0.88	0.01	-0.06	-0.08	0.00	4.19	2.33	0.04	-1.70	-1.97	0.03
AUSTRALIA	0.04	0.71	0.00	0.01	0.17	0.00	0.18	0.31	0.00	-0.33	-0.86	0.01
BRAZIL	0.04	0.34	0.00	-0.33	-0.45	0.00	–	–	–	-0.16	-0.29	0.00
CANADA	0.03	0.96	0.01	0.00	0.08	0.00	0.69	1.36	0.02	-0.38	-1.80	0.03
CHINA	0.07	0.89	0.01	0.10	1.38	0.02	–	–	–	0.12	0.40	0.00
CHILE	0.07	0.69	0.00	-0.05	-0.47	0.00	1.08	1.65	0.02	-0.44	-1.36	0.01
EURO	0.04	1.35	0.01	0.03	1.63	0.02	0.97	1.79	0.03	-0.17	-0.51	0.00
INDIA	0.09	1.27	0.01	0.05	0.71	0.00	0.44	0.46	0.00	-0.25	-1.28	0.01
INDONESIA	0.04	0.36	0.00	-0.14	-0.99	0.01	–	–	–	-0.54	-0.83	0.01
JAPAN	0.00	0.03	0.00	0.04	1.57	0.02	1.54	2.66	0.06	-0.81	-2.35	0.04
KOREA	0.24	2.90	0.07	0.07	1.33	0.01	2.11	2.37	0.04	-0.18	-0.50	0.00
MALAYSIA	-0.04	-0.39	0.00	0.06	1.16	0.01	2.08	1.88	0.03	0.10	0.42	0.00
MEXICO	0.05	0.63	0.00	0.06	0.36	0.00	–	–	–	-1.26	-2.71	0.06
NORWAY	-0.07	-0.98	0.01	-0.02	-0.46	0.00	1.26	1.38	0.02	-0.38	-1.14	0.01
NEW ZEALAND	0.00	-0.02	0.00	-0.06	-1.12	0.01	-0.15	-0.28	0.00	-0.35	-0.99	0.01
PERU	-0.06	-0.33	0.00	-0.04	-0.06	0.00	–	–	–	0.71	1.18	0.01
PHILIPPINES	0.09	0.93	0.01	-0.07	-0.59	0.00	0.99	0.87	0.01	-0.42	-1.41	0.02
SOUTH AFRICA	0.05	1.09	0.01	0.02	0.40	0.00	0.64	0.94	0.01	-0.65	-1.36	0.02
SAUDI ARABIA	0.38	3.05	0.07	0.02	0.34	0.00	–	–	–	-0.01	-0.15	0.00
SINGAPORE	-0.06	-0.49	0.00	0.00	0.10	0.00	1.67	1.88	0.03	-0.15	-0.97	0.01
SWEDEN	0.14	1.88	0.03	0.00	-0.01	0.00	1.50	1.87	0.03	-0.22	-0.60	0.00
SWITZERLAND	0.13	3.53	0.09	0.02	0.74	0.00	0.67	1.32	0.01	-0.13	-0.35	0.00
THAILAND	0.07	0.75	0.00	0.12	2.09	0.04	2.09	2.03	0.03	-0.35	-1.29	0.01
TURKEY	0.03	0.19	0.00	0.08	0.37	0.00	–	–	–	-0.26	-0.59	0.00
UNITED KINGDOM	0.05	1.25	0.01	0.02	0.80	0.01	0.33	0.63	0.00	-0.62	-2.10	0.04
USA	0.10	2.32	0.04	0.04	1.04	0.01	0.54	1.14	0.01	–	–	–

Note. Results of the regression $\hat{u}_{i\ell t} = \alpha_{i\ell}\bar{\xi}_{t-1} + \zeta_{i\ell t}$ as in equation (23). The coefficients $\alpha_{i,y}$, $\alpha_{i,\pi}$, $\alpha_{i,eq}$, $\alpha_{i,ep}$ report the impact of the global volatility innovations ($\bar{\xi}_{t-1}$) on the GVAR reduced-form residuals ($\hat{u}_{i\ell t}$) associated with each country's GDP, inflation, equity price, and exchange rate equations, respectively.

residuals of the GDP equations, Table 8 shows that almost all coefficients $(\alpha_{i,y})$ are not statistically significant at the 95 percent level. The only exceptions are Korea, Saudi Arabia, Sweden, Switzerland and the United States, for which the estimates are statistically significant but have a positive sign—which is not consistent with standard theory.

We further checked the significance of the estimates by using a multiple testing procedure due to Holm (1979), which controls for the overall size of the tests, taking account of possible dependence across the 33 t-tests carried out for each of the four variables in Table 8. Application of the Holm procedure yields only one statistically significant coefficient for the GDP growth innovations, namely for Switzerland. Together with the fact that the R^2 of the regressions is very small (averaging 0.01 across all countries) these results suggest that there is virtually no direct effect of a volatility innovation on GDP over and above that of $\mathfrak{n}_t$ that are taken into account under the GVAR methodology through the use of country-specific foreign variables. Similar results also hold for the residuals of the GVAR's inflation equations: as Table 8 shows, the impact of the volatility innovations on the inflation residuals $(\alpha_{i,\pi})$ is largely insignificant, the only exception being Thailand.

These results suggest that there is limited scope for volatility to explain macro dynamics directly after we condition on the set of global and country-specific macroeconomic factors included in the GVAR model. Volatility, however, could be affecting economic activity indirectly via its impact on the level of asset prices and the associated wealth effects on consumption and investment. So we now turn to this indirect channel.

As we noted, it is possible that a volatility innovation is associated with the residual of the equity price equations, the exchange rate equations, or the interest rate equations—that we collected in the vector $\mathbf{u}_t^\chi$ defined in (19). If volatility shocks were to be statistically associated with the residuals of the financial variables in the GVAR, they would have a channel to affect, indirectly, also GDP and inflation. However, the timing assumption in our factor model is less likely to hold for financial variables. Therefore, the relation between the elements of $\hat{\mathbf{u}}_t^\chi$ and the (lagged) volatility innovations, $\bar{\xi}_{t-1}$, cannot be strictly interpreted in terms of causation but has instead to be interpreted as simple statistical association.

We report the estimation of equation (23) for the residuals of the equity price and the exchange rate equations (the results for the interest rates equations are very similar and therefore are not reported here for sake of brevity). The results show that almost all coefficients are statistically insignificant. In the case of equity prices, the exceptions are Argentina, Japan, Korea, and Thailand with a (counter-intuitive) positive coefficient; in the case of exchange rates the exceptions are Japan, Mexico and the United Kingdom. Note, however, that none of these coefficients are statistically significant when using Holm multiple testing procedure.

In summary, we find that a global volatility shock, identified by assuming that common factors drive both volatility and the macroeconomic dynamics of individual countries but affect them with a lag of at least a quarter, has no direct or indirect effect on real GDP once we condition on a small set of country-specific and global macro-financial factors in the GVAR-VOL model. We interpret this evidence as suggesting that most of the effects that volatility has on economic activity documented in the existing literature come from the fact that volatility and the business cycle may share the same set of common factors. In this sense, volatility appears more of a symptom rather a cause of economic stagnation and instability

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during and after the recent global crisis.

9 Reconciling our findings with the literature

In this section we relate our empirical results to those in the existing empirical literature on uncertainty and the business cycle, according to which volatility is often found to have a significant negative effect on economic activity.

As explained in Section 3, in our simple factor model (1), the aggregate volatility and macro shocks are identified through a timing assumption on the relative speed with which the common factors, $\mathfrak{n}_t$, impact the volatility and macroeconomic variables, and the small open economy assumption that allows us to eliminate the effects of country specific shocks on volatility. In contrast, with a few notable exceptions, in the literature identification is typically achieved through a recursive ordering of variables in a VAR framework (or, equivalently, through a Cholesky decomposition of the variance-covariance matrix of the reduced-form residuals in a VAR), with GDP ordered after volatility.[29]

In our factor model, these identification assumptions are equivalent to assuming that:

I. the factors $\mathfrak{n}_t$ affect both volatility and macroeconomic variables contemporaneously;

II. the macroeconomic variables ($\Delta\mathbf{y}_{it}$) are not allowed to affect volatility ($\mathbf{v}_t$) contemporaneously.

According to assumption I, the factor model (1) can be re-written as:

$$
\begin{aligned}
\mathbf{v}_t &= \mathbf{\Phi}_{1v}\mathbf{v}_{t-1} + \mathbf{\Lambda}\mathfrak{n}_t + \boldsymbol{\xi}_t, \\
\Delta\mathbf{y}_{it} &= \mathbf{\Phi}_{1i}\Delta\mathbf{y}_{i,t-1} + \mathbf{\Gamma}_i\mathfrak{n}_t + \boldsymbol{\varepsilon}_{it}, \text{ for } i = 0, 1, ..., N,
\end{aligned}
\tag{25}
$$

where we note that the common factors $\mathfrak{n}_t$ now enter contemporaneously in both equations. It is important to stress here that, in the absence of any additional assumption, the system in (25) is not identified. After substituting $\mathfrak{n}_t$ from the activity equation into the volatility equation, and taking averages over i, we obtain:

$$
\mathbf{v}_t = \mathbf{\Phi}_{1v}\mathbf{v}_{t-1} + \mathbf{\Psi}_{0v}\Delta\bar{\mathbf{y}}_t + \mathbf{\Psi}_{1v}\Delta\bar{\mathbf{y}}_{t-1} - \underbrace{\mathbf{\Psi}_{0v}\bar{\boldsymbol{\varepsilon}}_t}_{O_p((N+1)^{-1/2})} + \boldsymbol{\xi}_t.
\tag{26}
$$

Equation (26) is a modified volatility module in the lags of volatility and the lags and contemporaneous values of changes in macroeconomic variables that resembles the volatility equation of a typical bivariate VAR in volatility and economic activity considered in the literature. In order to achieve identification, the literature generally assumes that $\Delta\mathbf{y}_{it}$ cannot

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[29]See the papers by Gilchrist, Sim, and Zakrajsek (2013), Bachmann, Elstner, and Sims (2013), and Caldara, Fuentes-Albero, Gilchrist, and Zakrajsek (2013). Also, see Baker and Bloom (2013) who try to deal with the contemporaneous determination of volatility and economic activity by using a panel of indicators for natural disasters, terrorist attacks and political shocks as instruments.

affect $\mathbf{v}_t$ contemporaneously (assumption II), which in the case of the above specification requires that $\boldsymbol{\Psi}_{0v} = 0$.

But as noted already in Section 3, the activity equation features the same estimation issues as in our baseline specification: since volatility is correlated with the error terms, OLS would produce inconsistent estimates of the effect of volatility on economic activity.[30] This implies that the mis-specification of the activity equation—one of our main results—does not depend on the timing assumption but requires only the assumption that the same common factors drive both activity and volatility.

We can now quantify empirically the impact of volatility and economic activity under this alternative set of assumptions. Note that, since we do not include volatility directly into the GVAR, our approach does not suffer from the inconsistency bias described above. Similarly to what we did in the previous section, we therefore regress the residuals from the modified volatility module (26) on the reduced-form residuals of the GVAR.

For the purpose of implementing the analysis in manner that is as close as possible to the approach taken in the literature, we re-estimate the volatility module using global volatility—labelled $\mathcal{RV}_t$ and defined as the average of the asset-specific volatility variables as in equation (22)—as a proxy for $\mathbf{v}_t$. As a result, and differently from our baseline where $\mathbf{v}_t$ is a (4×1) vector of realized volatilities, the volatility module has now a univariate representation. According to (26), the specification of the modified volatility module includes lagged global volatility (v_{t-1}), and the first lag of global output growth (Δy_{t-1}^*) and global inflation ($\Delta \pi_{t-1}^*$).[31] Note that we also consider two additional versions of the volatility module: a specification with the future, contemporaneous, and lagged changes in the global variables and a specification with the contemporaneous and lagged changes in the global variables only. Table 9 reports the OLS estimates of these three specifications and the associated t-ratios in square brackets.

The baseline specification (s_1), is presented in the first column of Table 9. Interestingly, the coefficient on lagged global growth is negative but is not statistically significant. In specification (s_2), which includes both contemporaneous and lagged global variables, the co-efficient on contemporaneous global growth is negative as in the baseline specification, but it is now statistically significant. Finally, consistently with the results in our baseline volatility model in Table 6, specification (s_3) shows a strong negative and statistically significant relation between volatility and the future global output growth.

An important consideration is in order here. According to the modified factor model (25), the relation between the volatility innovations and the GVAR innovations is now contemporaneous, whereby the volatility innovations are assumed to be exogenous by virtue of the recursive identification assumption made. We therefore estimate the following regression:

$$\hat{u}_{i\ell t} = \beta_{i\ell} \bar{\xi}_t^0 + \zeta_{i\ell t}^0, \tag{27}$$

where $\bar{\xi}_t^0$ are the volatility innovations from the baseline specification of the modified volatility module—i.e., the residuals from specification (s_1) in Table 6.

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[30]The activity equation can be obtained by substituting $\mathfrak{n}_t$ from the volatility equation into the activity equation, but is not derived here for sake of brevity.

[31]The number of lags of global volatility has been determined with Akaike criterion.

Table 9 MODIFIED VOLATILITY MODULE

	(s_1)	(s_2)	(s_3)
	v_t	v_t	v_t
c	0.04	0.06	0.06
	[4.64]	[5.57]	[6.54]
v_{t-1}	0.54	0.44	0.48
	[6.37]	[4.97]	[5.99]
Δy^*_{t+1}			-1.56
			[-5.54]
$\Delta \pi^*_{t+1}$			0.06
			[0.36]
Δy^*_t		-1.01	-0.15
		[-3.02]	[-0.44]
$\Delta \pi^*_t$		0.04	0.11
		[0.20]	[0.64]
Δy^*_{t-1}	-0.42	-0.03	-0.04
	[-1.33]	[-0.10]	[-0.14]
$\Delta \pi^*_{t-1}$	-0.20	-0.13	-0.07
	[-1.04]	[-0.70]	[-0.38]

Note. t-Statistics are in brackets. The lag order is determined with Akaike criterion with a maximum of 4 lags. The model is estimated over the 1979.II–2011.II period.

Table 10 reports the estimation results. As in the previous section, for each country $i = 0, 1, ..., N$ we consider the GVAR reduced-form residuals from different equations, namely GDP, inflation, equity prices and exchange rates. Starting with the GDP residuals, Table 10 shows that 8 out of 26 coefficients ($\beta_{i,y}$) are now statistically significant at the 95 percent level; and the statistically significant coefficients have a negative sign, implying that an increase in volatility would negatively affect GDP. Similar results are obtained for the inflation residuals: 6 out of 26 coefficients ($\beta_{i,\pi}$) have a statistically significant coefficient, in most cases with a negative sign. Note, however, that according to Holm multiple testing procedure only one coefficient (namely, Brazil GDP) is statistically significant; and the R^2 of the regressions are still small, averaging 0.03 for the GDP regression and 0.02 for the inflation regressions.

We now analyze the association between volatility and asset price residuals. All the coefficients on the equity price residuals ($\beta_{i,eq}$) are negative and statistically significant at the 95 percent level, with only two exception: Korea and the Philippines. Interestingly, this result is robust to the Holm multiple testing procedure: 13 out 19 coefficients are still statistically significant. Moreover, the size of the coefficients is large, with a cross-country average for $\beta_{i,eq}$ of -2.23; and the explanatory power of the volatility innovations is large as well, with a cross-country average of the R^2 of 0.13. Similar (but less clear-cut) results hold for the exchange rate residuals ($\beta_{i,ep}$), for which an increase in volatility generally implies a significant depreciation of currencies *vis-a-vis* the U.S. dollar, which is indicative of "flight to safety" often observed during times of increased economic uncertainty.

This evidence suggests that, when we assume (as it is commonly done in the literature) that activity cannot affect volatility contemporaneously, global volatility has some direct

Table 10 Modified GVAR-VOL Model: Global Volatility Innovations And Reduced-form GVAR Residuals

	GDP			Inflation			Equity Price			Exchange Rate		
	$\beta_{i,y}$	t-Stat	R^2	$\beta_{i,\pi}$	t-Stat	R^2	$\beta_{i,eq}$	t-Stat	R^2	$\beta_{i,ep}$	t-Stat	R^2
ARGENTINA	-0.21	-2.23	0.04	0.18	0.31	0.00	-4.10	-2.74	0.06	0.79	1.08	0.01
AUSTRALIA	-0.03	-0.82	0.01	-0.04	-0.97	0.01	-2.05	-4.65	0.15	1.21	3.95	0.11
BRAZIL	-0.29	-3.41	0.09	0.83	1.37	0.02	–	–	–	1.39	3.01	0.07
CANADA	-0.03	-1.09	0.01	-0.06	-2.24	0.04	-2.21	-5.83	0.22	0.82	4.99	0.17
CHINA	0.02	0.23	0.00	-0.06	-0.95	0.01	–	–	–	0.12	0.47	0.00
CHILE	-0.17	-1.89	0.03	0.00	0.01	0.00	-1.58	-2.94	0.07	0.83	3.12	0.07
EURO	-0.04	-1.88	0.03	-0.03	-1.72	0.02	-2.42	-5.94	0.23	0.08	0.29	0.00
INDIA	0.06	0.90	0.01	0.07	1.01	0.01	-2.54	-3.25	0.08	0.24	1.47	0.02
INDONESIA	-0.16	-1.60	0.02	0.06	0.47	0.00	–	–	–	1.32	2.44	0.05
JAPAN	-0.14	-2.83	0.06	-0.04	-1.91	0.03	-2.23	-4.89	0.16	-0.86	-3.02	0.07
KOREA	-0.21	-3.00	0.07	-0.01	-0.19	0.00	-1.38	-1.82	0.03	1.23	4.50	0.14
MALAYSIA	-0.24	-2.67	0.06	-0.11	-2.78	0.06	-2.73	-3.00	0.07	0.44	2.22	0.04
MEXICO	-0.01	-0.15	0.00	0.19	1.37	0.02	–	–	–	1.27	3.29	0.08
NORWAY	0.00	-0.06	0.00	0.03	0.79	0.01	-4.07	-6.00	0.23	0.68	2.47	0.05
NEW ZEALAND	-0.05	-0.88	0.01	-0.06	-1.32	0.01	-1.89	-4.59	0.15	0.80	2.70	0.06
PERU	0.06	0.38	0.00	-0.36	-0.66	0.00	–	–	–	0.17	0.33	0.00
PHILIPPINES	-0.06	-0.69	0.00	-0.23	-2.39	0.05	-1.17	-1.23	0.01	0.61	2.48	0.05
SOUTH AFRICA	-0.04	-1.02	0.01	0.07	1.90	0.03	-1.94	-3.53	0.09	0.79	1.98	0.03
SAUDI ARABIA	0.04	0.33	0.00	0.02	0.44	0.00	–	–	–	-0.01	-0.17	0.00
SINGAPORE	-0.10	-0.93	0.01	0.07	2.04	0.03	-3.68	-5.42	0.20	0.29	2.35	0.04
SWEDEN	-0.14	-2.14	0.04	-0.01	-0.27	0.00	-2.20	-3.38	0.09	0.65	2.15	0.04
SWITZERLAND	-0.03	-1.03	0.01	-0.01	-0.49	0.00	-2.19	-5.70	0.21	-0.15	-0.47	0.00
THAILAND	-0.22	-2.93	0.07	-0.17	-3.66	0.10	-2.15	-2.51	0.05	0.29	1.27	0.01
TURKEY	-0.20	-1.42	0.02	0.10	0.57	0.00	–	–	–	1.21	3.42	0.09
UNITED KINGDOM	-0.07	-1.93	0.03	-0.03	-1.09	0.01	-2.19	-5.65	0.21	0.43	1.74	0.02
UNITED STATES	-0.10	-2.97	0.07	-0.07	-2.30	0.04	-2.01	-5.70	0.21	–	–	–

Note. Results of the regression $\hat{u}_{i\ell t} = \beta_{i\ell}\bar{\xi}_t^0 + \zeta_{i\ell t}^0$ as in equation (27). The coefficients $\beta_{i,y}$, $\beta_{i,\pi}$, $\beta_{i,eq}$, and $\beta_{i,ep}$ report the impact of the volatility innovations ($\bar{\xi}_t^0$) on the GVAR reduced-form residuals ($\hat{u}_{i\ell t}$) associated with each country's GDP, inflation, equity price, and exchange rate equations, respectively.

impact on real GDP and has a strong association with equity price and exchange rates, which in turn can affect economic activity indirectly *via* balance sheet and wealth effects.

10   Conclusions

The recent global financial crisis has spurred renewed academic interest on quantifying the causal impact of uncertainty on macroeconomic dynamics. In this paper, we study the interrelation between volatility and macroeconomic dynamics in the world economy under the assumption that both uncertainty and business cycle are driven by the same set of common factors. We further assume that while these common factors affect financial market volatility contemporaneously, they can affect macroeconomic dynamics only with a lag of at least one quarter. Under these assumptions, we show analytically that volatility is forward looking and that the output equation of the typical VARs estimated in the literature is mis-specified as least squares estimates of this equation are inconsistent. This implies that, if our identification assumption is plausible, typical impulse response functions of measures of economic activity to volatility shocks are biased regardless of the structural VAR identification scheme employed.

We then construct global measures of uncertainty by using realized volatility at quarterly frequency for 109 asset prices. Empirically, we provide three main sets of results. First, our (unconditional) descriptive analysis shows that volatility is persistent, but is well approximated by a stationary process at business cycle frequency. It behaves countercyclically—consistently with the common wisdom in the literature—and it can significantly lead the business cycle. We also find that realized volatility co-moves significantly within asset classes (equities, bonds, exchange rates and commodities), but it is not as highly correlated across asset classes.

Second, our multi-country analysis allows us to consistently estimate the relation between GDP growth and volatility. Our results show that there is a strong negative and statistically significant association between future output growth and current volatility.

Third, we find that volatility shocks have no statistically significant impact on economic activity over and above that of its common component. In other words, we find that a shock to global volatility has little or no direct effect on real GDP once we condition on a small set of country-specific and global macro-financial factors in the GVAR-VOL model.

We do not interpret this evidence meaning that volatility has no effect on economic activity. We argue that most of its effect (often found in the literature) may be coming from the fact that volatility itself is driven by the same common factors that affect the business cycle. In other words, volatility is likely to be more of a symptom rather than a cause of economic instability.

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A   Appendix: The combined GVAR-VOL model

The combined GVAR-VOL model can be obtained by stacking the system of equations for the GVAR model (16) with those of the volatility module, (11). We have:

$$
\begin{bmatrix} \mathbf{I}_m & \boldsymbol{\Psi}_{1,v}\mathbf{P} \\ \mathbf{0} & \mathbf{I}_K \end{bmatrix} \begin{bmatrix} \mathbf{v}_t \\ \mathbf{x}_{t+1} \end{bmatrix} = \begin{bmatrix} \boldsymbol{\Phi}_v & (\boldsymbol{\Psi}_{0,v} - \boldsymbol{\Psi}_{1,v})\mathbf{P} \\ \mathbf{0} & \mathbf{F} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{t-1} \\ \mathbf{x}_t \end{bmatrix} + \dots
$$
$$
\begin{bmatrix} \mathbf{0} & (\boldsymbol{\Psi}_{-1,v} - \boldsymbol{\Psi}_{0,v})\mathbf{P} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{t-2} \\ \mathbf{x}_{t-1} \end{bmatrix} + \dots
$$
$$
\begin{bmatrix} \mathbf{0} & -\boldsymbol{\Psi}_{-1,v}\mathbf{P} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{bmatrix} \mathbf{v}_{t-3} \\ \mathbf{x}_{t-2} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\xi}_t \\ \mathbf{u}_{t+1} \end{bmatrix},
\tag{28}
$$

or, more compactly:

$$
\begin{bmatrix} \mathbf{v}_t \\ \mathbf{x}_{t+1} \end{bmatrix} = \boldsymbol{\Xi}_0^{-1}\boldsymbol{\Xi}_1 \begin{bmatrix} \mathbf{v}_{t-1} \\ \mathbf{x}_t \end{bmatrix} + \boldsymbol{\Xi}_0^{-1}\boldsymbol{\Xi}_2 \begin{bmatrix} \mathbf{v}_{t-2} \\ \mathbf{x}_{t-1} \end{bmatrix} + \boldsymbol{\Xi}_0^{-1}\boldsymbol{\Xi}_3 \begin{bmatrix} \mathbf{v}_{t-3} \\ \mathbf{x}_{t-2} \end{bmatrix} + \boldsymbol{\Xi}_0^{-1} \begin{bmatrix} \boldsymbol{\xi}_t \\ \mathbf{u}_{t+1} \end{bmatrix}, \tag{29}
$$

where:

$$
\boldsymbol{\Xi}_0 = \begin{bmatrix} \mathbf{I}_m & \boldsymbol{\Psi}_{1,v}\mathbf{P} \\ \mathbf{0} & \mathbf{I}_K \end{bmatrix}, \qquad \boldsymbol{\Xi}_1 = \begin{bmatrix} \boldsymbol{\Phi}_v & (\boldsymbol{\Psi}_{0,v} - \boldsymbol{\Psi}_{1,v})\mathbf{P} \\ \mathbf{0} & \mathbf{F} \end{bmatrix},
$$
$$
\boldsymbol{\Xi}_2 = \begin{bmatrix} \mathbf{0} & (\boldsymbol{\Psi}_{-1,v} - \boldsymbol{\Psi}_{0,v})\mathbf{P} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}, \qquad \boldsymbol{\Xi}_2 = \begin{bmatrix} \mathbf{0} & -\boldsymbol{\Psi}_{-1,v}\mathbf{P} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}.
$$

B   Appendix: Cross-sectional measures of dispersion

B.I   Definition

As noted in the body of the paper, uncertainty can also be measured by the cross-sectional dispersion of returns at any given point in time and for each asset class. We label the quarterly cross-sectional dispersion of asset κ, for quarter t, at daily rates $(\mathcal{CD}_{\kappa t})$ as "asset-specific cross-sectional dispersion." We compute asset-specific cross-sectional dispersion as:

$$
\mathcal{CD}_{\kappa t} = \sqrt{D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N_t} w_{it} \left(r_{\kappa it}(\tau) - \bar{r}_{\kappa t}(\tau) \right)^2} \tag{30}
$$

where $r_{\kappa it}(\tau) = \Delta \ln P_{\kappa it}(\tau)$ and $\bar{r}_{\kappa it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{\kappa it}(\tau)$ is the average daily price changes over the quarter t, and D_t is the number of trading days in quarter t; and w_{it} are weights attached to country i in quarter t.

As for realized volatility, a global cross-sectional dispersion measure can be computed by aggregating across different asset classes, namely:

$$
\mathcal{CD}_t = \frac{1}{M} \sum_{\kappa=1}^{M} \mathcal{CD}_{\kappa t}, \tag{31}
$$

where M is the number of assets that we consider.

The subsection below derives some analytical results on the relationship between realized volatility $(\mathcal{RV}_{\kappa t})$ and cross-sectional dispersion $(\mathcal{CD}_{\kappa t})$.

B.II Relationship realized volatility and the cross sectional dispersion

We note here that there is a close relationship between the asset-specific realized volatility measure $(\mathcal{RV}_{\kappa t})$ computed as in equation (21) and the cross-sectional dispersion measure $(\mathcal{CD}_{\kappa t})$ computed as in equation (30).

For analytical derivations it is easier to compare the squared version of these measures given by:

$$
\mathcal{RV}_{\kappa t}^2 = D_t^{-1} \sum_{i=1}^{N_t} \sum_{\tau=1}^{D_t} w_{it} \left(r_{\kappa it}(\tau) - \bar{r}_{\kappa it} \right)^2,
$$
$$
\mathcal{CD}_{\kappa t}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N_t} w_{it} \left(r_{\kappa it}(\tau) - \bar{r}_{\kappa t}(\tau) \right)^2.
$$

We note that

$$
\mathcal{RV}_{\kappa t}^2 = D_t^{-1} \sum_{i=1}^{N_t} \sum_{\tau=1}^{D_t} w_{it} r_{\kappa it}^2(\tau) - \sum_{i=1}^{N_t} w_{it} \bar{r}_{\kappa it}^2,
$$

and

$$
\mathcal{CD}_{\kappa t}^2 = D_t^{-1} \sum_{\tau=1}^{D_t} \sum_{i=1}^{N_t} w_{it} r_{\kappa it}^2(\tau) - \sum_{i=1}^{N_t} w_{it} \left(D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_{\kappa t}^2(\tau) \right).
$$

Hence, noting that $\sum_{i=1}^{N_t} w_{it} = 1$,

$$
\mathcal{CD}_{\kappa t}^2 - \mathcal{RV}_{\kappa t}^2 = \sum_{i=1}^{N_t} w_{it} \bar{r}_{\kappa it}^2 - D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_{\kappa t}^2(\tau),
$$

where $\bar{r}_{\kappa it} = D_t^{-1} \sum_{\tau=1}^{D_t} r_{\kappa it}(\tau)$, and $\bar{r}_{\kappa t}(\tau) = \sum_{i=1}^{N_t} w_{it} r_{\kappa it}(\tau)$.

Suppose now that daily returns have the following standard single-factor structure:[32]

$$
r_{\kappa it}(\tau) = \beta_{\kappa i} f_{\kappa t}(\tau) + \varepsilon_{\kappa it}(\tau),
$$

where the factor is strong in the sense that (see Bailey, Kapetanios, and Pesaran, 2012):

$$
\lim_{N_t \to \infty} \sum_{i=1}^{N_t} w_{it} \beta_{\kappa i} = \bar{\beta}_{\kappa t} \neq 0,
$$
$$
\lim_{N_t \to \infty} \sum_{i=1}^{N_t} w_{it} \beta_{\kappa i}^2 = \sigma_{\kappa\beta t}^2 + \bar{\beta}_{\kappa t}^2 \neq 0.
$$

---

[32] This factor specification for returns has been used extensively in the finance literature, following the pioneering contributions of Sharpe (1964), Lintner (1965), and Ross (1976). The analysis can be readily extended to a multi-factor specification.

The idiosyncratic components, $\varepsilon_{\kappa it}(\tau)$, are assumed to be independently distributed from $\beta_{\kappa i} f_{\kappa t}(\tau)$, cross-sectionally weakly correlated, and serially uncorrelated with zero means and finite variances. Also let:

$$\lim_{D_t \to \infty} D_t^{-1} \sum_{\tau=1}^{D_t} f_{\kappa t}^2(\tau) = h_{\kappa ft}^2.$$

We now note that

$$
\begin{aligned}
\sum_{i=1}^{N_t} w_{it} \bar{r}_{\kappa it}^2 &= \left(\sum_{i=1}^{N_t} w_{it} \beta_{\kappa i}^2 \right) \bar{f}_{\kappa t}^2 + \left(\sum_{i=1}^{N_t} w_{it} \bar{\varepsilon}_{\kappa it}^2 \right) + 2 \left(\sum_{i=1}^{N_t} w_{it} \beta_{\kappa i} \bar{\varepsilon}_{\kappa it} \right) \bar{f}_{\kappa t} \\
&= \left(\sigma_{\kappa \beta t}^2 + \bar{\beta}_{\kappa t}^2 \right) \bar{f}_{\kappa t}^2 + O_p \left(D_t^{-1/2} \right) + O_p \left(N_t^{-1/2} \right),
\end{aligned}
$$

where $\bar{f}_{\kappa t} = D_t^{-1} \sum_{\tau=1}^{D_t} f_{\kappa t}(\tau)$, and $\bar{\varepsilon}_{\kappa it} = D_t^{-1} \sum_{\tau=1}^{D_t} \varepsilon_{\kappa it}(\tau)$. Also

$$
\begin{aligned}
D_t^{-1} \sum_{\tau=1}^{D_t} \bar{r}_{\kappa t}^2(\tau) &= D_t^{-1} \sum_{\tau=1}^{D_t} \left[\bar{\beta}_{\kappa t} f_{\kappa t}(\tau) + \bar{\varepsilon}_{\kappa t}(\tau) \right]^2 \\
&= \bar{\beta}_{\kappa t}^2 \left[D_t^{-1} \sum_{\tau=1}^{D_t} f_{\kappa t}^2(\tau) \right] + D_t^{-1} \sum_{\tau=1}^{D_t} \bar{\varepsilon}_{\kappa t}^2(\tau) + 2 D_t^{-1} \sum_{\tau=1}^{D_t} \bar{\beta}_{\kappa t} \bar{\varepsilon}_{\kappa t}(\tau) f_{\kappa t}(\tau) \\
&= \bar{\beta}_{\kappa t}^2 h_{\kappa ft}^2 + O_p \left(N_t^{-1/2} \right) + O_p \left(D_t^{-1/2} \right).
\end{aligned}
$$

Hence

$$
\begin{aligned}
\mathcal{CD}_{\kappa t}^2 - \mathcal{RV}_{\kappa t}^2 &= \left(\sigma_{\kappa \beta t}^2 + \bar{\beta}_{\kappa t}^2 \right) \bar{f}_{\kappa t}^2 - \bar{\beta}_{\kappa t}^2 h_{\kappa ft}^2 + O_p \left(N_t^{-1/2} \right) + O_p \left(D_t^{-1/2} \right) \\
&= \sigma_{\kappa \beta t}^2 \bar{f}_{\kappa t}^2 - \bar{\beta}_{\kappa t}^2 \sigma_{\kappa ft}^2 + O_p \left(N_t^{-1/2} \right) + O_p \left(D_t^{-1/2} \right).
\end{aligned}
$$

where $\sigma_{\kappa ft}^2 = \left(h_{\kappa ft}^2 - \bar{f}_{\kappa t}^2 \right) \geq 0$, is the variance of the common factor for asset type κ.

This expression shows that, under fairly general assumptions and for N_t and D_t sufficiently large, we would expect the cross-sectional dispersion measure to be closely related to asset-specific measures of realized volatility when the factor loadings, $\beta_{\kappa i}$, are not too dispersed across countries. The results also show that the relative magnitudes of the cross section dispersion and realized volatility measures depend on the relative values of $\sigma_{\kappa \beta t}^2 \bar{f}_{\kappa t}^2$ and $\bar{\beta}_{\kappa t}^2 \sigma_{\kappa ft}^2$.

Figure B.1 displays a comparison between realized volatility and cross-sectional dispersion using our data set. For this comparison we removed the countries that experienced episodes of high inflation during the sample over which $\mathcal{RV}$ and $\mathcal{CD}$ were computed: Argentina, Brazil, Chile, Mexico, and Turkey.[33] This is because the $\mathcal{CD}$ measure—given the way it is computed—is more affected than the $\mathcal{RV}$ measure by the presence of large outliers, therefore driving a wedge between the two measures. Note, however, that this wedge is not discernible when we use all countries but perform the cross-country aggregation using PPP-GDP weights instead of equal weights (as it is done above).

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[33]Note that Peru—who also experienced periods of consumer price inflation—is not removed form the sample since daily asset price data is available only after the hyperinflation period.
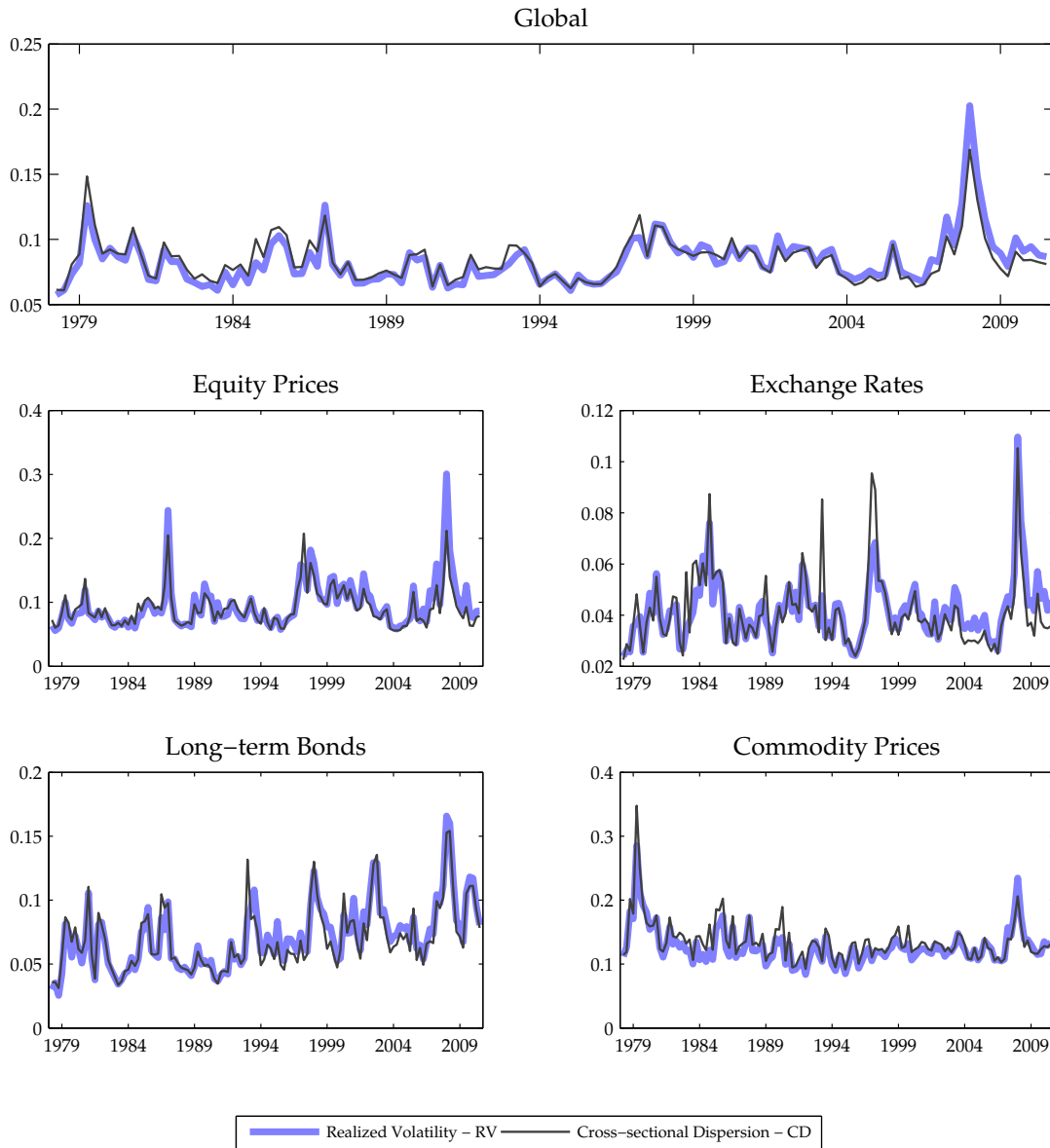
Figure B.1 A COMPARISON BETWEEN REALIZED VOLATILITY AND CROSS-SECTIONAL DISPERSION. The upper panel compares global realized volatility ($\mathcal{RV}_t$, light thick line) and global cross-sectional dispersion ($\mathcal{CD}_t$, dark thin line), computed as in equations (22) and (31) respectively. The lower panels display the same comparison for the asset-specific measures. Specifically, $\mathcal{RV}_{\kappa t}$ is computed as in equation (21) and $\mathcal{CD}_{\kappa t}$ is computed as in equation (30). All measures are expressed at quarterly rates and computed over the 1990.I-2011.II period. High/hyperinflation countries are discarded (as explained in the text).

The upper panel compares global realized volatility ($\mathcal{RV}_t$, light thick line) and global cross-sectional dispersion ($\mathcal{CD}_t$, dark thin line), computed as in equations (22) and (31), respectively. Their sample correlation over the 1979.I to 2011.II period is 0.92. The lower panels display the asset-specific realized volatility measure ($\mathcal{RV}_{\kappa t}$) computed as in equation (21) with equal weights and compare it with the cross-sectional dispersion measure ($\mathcal{CD}_{\kappa t}$) computed as in equation (30) with equal weights. Both series are then re-scaled by the factor $\sqrt{D_t}$ so as to be expressed at quarterly rates. Figure B.1 suggests that the two measures are very closely related, consistently with the evidence provided by Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2012). The simple correlation between the asset-specific measures is 0.92 for equity prices, 0.85 for exchange rates, 0.95 for government bonds, and 0.93 for commodity prices.

C Appendix: Data

This appendix provides additional information on the sources of the data we used to construct the realized volatility measures.[34]

Equity. For equity prices we use the MSCI Index in local currency. The data source for the daily equity price indices is Bloomberg. The countries included in the sample are the following: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Norway, New Zealand, Peru, Philippines, Saudi Arabia, South Africa, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, United Kingdom, and United States. [35]

Bonds. We used 10 years government bonds from Bloomberg. The countries included in the sample are: Australia, Austria, Belgium, Brazil, Canada, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Norway, New Zealand, Peru, Philippines, South Africa, Singapore, Spain, Sweden, Switzerland, Thailand,United Kingdom, and United States. [36]

Exchange rates. Daily exchange rates, measured in terms of the US dollar are obtained from Bloomberg. The countries included in the sample are the following: Argentina, Australia, Austria, Belgium, Brazil, Canada, Chile, China, Finland, France, Germany, India, Indonesia, Italy, Japan, Korea, Malaysia, Mexico, Netherlands, Norway, New Zealand, Peru, Philippines, Saudi Arabia, South Africa, Singapore, Spain, Sweden, Switzerland, Thailand, Turkey, and United Kingdom.[37]

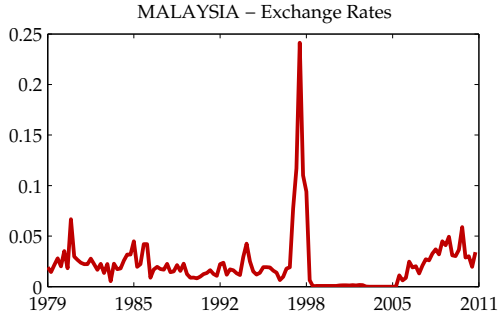[34]A description of the macroeconomic data used for the estimation of the GVAR model can be found at the following web page: https://sites.google.com/site/gvarmodelling/.

[35]The list of Bloomberg tickers is as follows: MSELTAG, MSDLAS, MSDLAT, MSDLBE, MSELTBR, MS-DLCA, MSELTCF, MSELTCH, MSDLFI, MSDLFR, MSDLGR, MSELTIA, MSELTINF, MSDLIT, MSDLJN, MSELTKO, MXMY, MSELTMXF , MSDLNE, MSDLNO, MSDLNZ, MSELTPR, MSELTPHF, MSELTSA, MGCLSA, MSDLSG, MSDLSP, MSDLSW, MSDLSZ, MSELTTHF, MSELTTK, MSDLUK, MSDLUS.

[36]The list of Bloomberg tickers is as follows: GACGB10; GAGB10YR; GBGB10YR; GEBR10Y; GCAN10YR; GCNY10YR BGNC; GFIN10YR; GFRN10; GDBR10; GIDN10YR; GBTPGR10; GJGB10; GVSK10YR; MGIY10Y; GMXN10YR; GNTH10YR; GNOR10YR; GNZGB10; GRPE10Y; PDSF10YR; GSAB10YR; MASB10Y; GSPG10YR; GSGB10YR; GSWISS10; GVTL10YR; GUKG10; USGG10YR.

[37]The list of Bloomberg tickers is as follows: USDARS; USDAUD CMPN; USDATS; USDBEF; USDBRL; USDCAD; USDCNY; USDCLP; USDFIM; USDFRF; USDDEM; USDINR; USDIDR; USDITL; USDJPY; US-

Commodities. The data source for the daily commodity price indices is Bloomberg. The realized volatility measures are computed for the following commodities: Corn, Soybean, Wheat, Coffee, Rice, Sugar, Cocoa, Gold, Silver, Copper, Natural gas, Coal, Oil(CO1), Livestock, Meat and livestock, CRB Commodity Excess Return Index, Linseed oil.[38]

D Appendix: Realized volatility charts and summary statistics

This appendix reports a plot of the realized volatility measures and a full set of country-specific and commodity-specific summary statistics over the period 1979-2011.
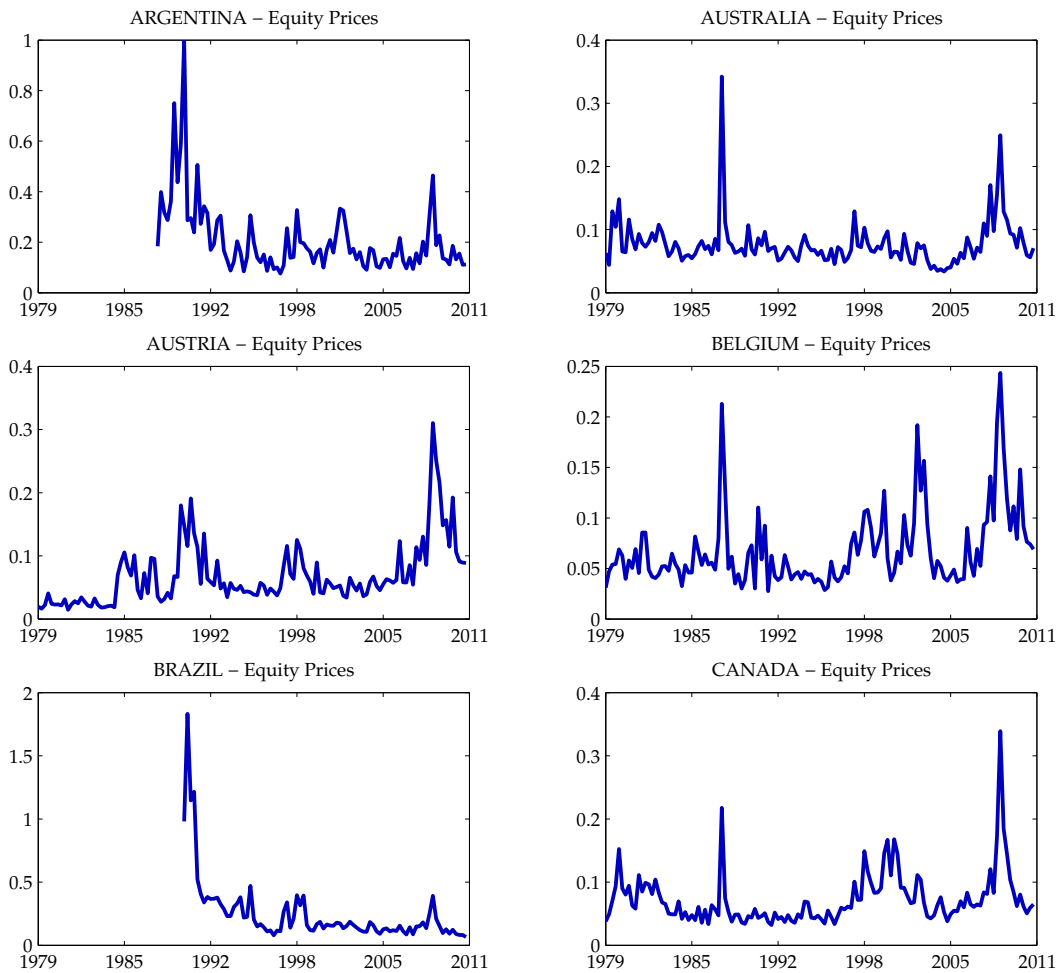
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[38]The list of Bloomberg tickers is as follows: C1; S1; W1; KC1; RR1; Sb1; CC1; GOLDS; SILV; HG1; NGA; QZ1; CO1; CRB LIVS; EYCI; CRY; COMDLINO.

CHINA – Equity Prices

CHILE – Equity Prices

FINLAND – Equity Prices

FRANCE – Equity Prices

GERMANY – Equity Prices

INDIA – Equity Prices

INDONESIA – Equity Prices

ITALY – Equity Prices

JAPAN – Equity Prices

KOREA – Equity Prices

MALAYSIA – Equity Prices

MEXICO – Equity Prices

NETHERLANDS – Equity Prices

NORWAY – Equity Prices

NEW ZEALAND – Equity Prices

PERU – Equity Prices

PHILIPPINES – Equity Prices

SOUTH AFRICA – Equity Prices

SAUDI ARABIA – Equity Prices

SINGAPORE – Equity Prices

SPAIN – Equity Prices

SWEDEN – Equity Prices

SWITZERLAND – Equity Prices

THAILAND – Equity Prices

TURKEY – Equity Prices

UK – Equity Prices

USA – Equity Prices

ARGENTINA – Exchange Rates

AUSTRALIA – Exchange Rates

AUSTRIA – Exchange Rates

BELGIUM – Exchange Rates

BRAZIL – Exchange Rates

CANADA – Exchange Rates

CHINA – Exchange Rates

CHILE – Exchange Rates

FINLAND – Exchange Rates

FRANCE – Exchange Rates

GERMANY – Exchange Rates

INDIA – Exchange Rates

INDONESIA – Exchange Rates

ITALY – Exchange Rates

JAPAN – Exchange Rates

KOREA – Exchange Rates

MALAYSIA – Exchange Rates

MEXICO – Exchange Rates

NETHERLANDS – Exchange Rates

NORWAY – Exchange Rates

NEW ZEALAND – Exchange Rates

PERU – Exchange Rates

PHILIPPINES – Exchange Rates

SOUTH AFRICA – Exchange Rates

SAUDI ARABIA – Exchange Rates

SINGAPORE – Exchange Rates

SPAIN – Exchange Rates

SWEDEN – Exchange Rates

SWITZERLAND – Exchange Rates

THAILAND – Exchange Rates

TURKEY – Exchange Rates

UK – Exchange Rates

AUSTRALIA – Long–term Bonds

AUSTRIA – Long–term Bonds

BELGIUM – Long–term Bonds

BRAZIL – Long–term Bonds

CANADA – Long–term Bonds

CHINA – Long–term Bonds

FINLAND – Long–term Bonds

FRANCE – Long–term Bonds

GERMANY – Long–term Bonds

INDONESIA – Long–term Bonds

ITALY – Long–term Bonds

JAPAN – Long-term Bonds

KOREA – Long-term Bonds

MALAYSIA – Long-term Bonds

MEXICO – Long-term Bonds

NETHERLANDS – Long-term Bonds

NORWAY – Long-term Bonds

NEW ZEALAND – Long-term Bonds

PERU – Long-term Bonds

PHILIPPINES – Long-term Bonds

SOUTH AFRICA – Long-term Bonds

SINGAPORE – Long-term Bonds

SPAIN – Long-term Bonds

SWEDEN – Long-term Bonds

SWITZERLAND – Long-term Bonds

THAILAND – Long-term Bonds

UK – Long-term Bonds

USA – Long-term Bonds

CORN – Commodity Prices

SOYBEAN – Commodity Prices

WHEAT – Commodity Prices

LIVESTOCK – Commodity Prices

MEAT & LIVESTOCK – Commodity Prices

Table D.1 REALIZED VOLATILITY OF EQUITY PRICES

	Obs	Mean	Median	Max	Min	St. Dev.	Auto Corr.	Skew.	Kurt.
ARGENTINA	94	0.21	0.16	1.00	0.08	0.14	0.57	2.89	14.16
AUSTRALIA	130	0.08	0.07	0.34	0.03	0.04	0.34	4.01	26.18
AUSTRIA	130	0.07	0.06	0.31	0.01	0.05	0.76	2.02	8.29
BELGIUM	130	0.07	0.06	0.24	0.03	0.04	0.62	2.13	8.28
BRAZIL	86	0.25	0.16	1.83	0.07	0.26	0.81	3.84	20.00
CANADA	130	0.07	0.06	0.34	0.03	0.04	0.62	2.87	15.42
CHINA	74	0.15	0.13	0.42	0.07	0.07	0.58	1.38	5.49
CHILE	94	0.08	0.07	0.24	0.04	0.03	0.39	1.82	7.42
FINLAND	94	0.15	0.11	0.77	0.04	0.10	0.54	3.03	18.06
FRANCE	130	0.09	0.07	0.28	0.04	0.04	0.48	2.05	8.67
GERMANY	130	0.09	0.08	0.34	0.04	0.04	0.46	2.64	16.14
INDIA	97	0.12	0.10	0.29	0.05	0.05	0.48	1.16	3.75
INDONESIA	94	0.14	0.12	0.57	0.04	0.08	0.41	2.37	11.09
ITALY	130	0.10	0.09	0.28	0.04	0.04	0.55	1.55	6.19
JAPAN	130	0.09	0.09	0.34	0.03	0.04	0.50	2.09	12.02
KOREA	130	0.12	0.10	0.34	0.03	0.06	0.69	1.43	5.49
MALAYSIA	125	0.09	0.08	0.47	0.03	0.06	0.44	3.09	15.99
MEXICO	94	0.12	0.11	0.30	0.05	0.05	0.47	1.68	7.02
NETHERLANDS	130	0.08	0.07	0.30	0.03	0.04	0.59	2.32	9.92
NORWAY	125	0.11	0.10	0.39	0.05	0.05	0.47	2.96	15.66
NEW ZEALAND	94	0.09	0.08	0.24	0.04	0.04	0.49	1.55	6.27
PERU	74	0.13	0.13	0.45	0.06	0.06	0.57	2.10	10.70
PHILIPPINES	101	0.12	0.10	0.44	0.04	0.06	0.51	2.35	11.69
SOUTH AFRICA	130	0.12	0.11	0.41	0.04	0.05	0.35	2.12	10.14
SAUDI ARABIA	24	0.15	0.14	0.34	0.07	0.07	0.19	1.07	3.63
SINGAPORE	130	0.09	0.08	0.39	0.04	0.05	0.42	2.63	13.89
SPAIN	130	0.08	0.07	0.29	0.03	0.04	0.58	1.75	7.60
SWEDEN	117	0.10	0.08	0.30	0.04	0.05	0.55	1.54	5.58
SWITZERLAND	130	0.07	0.06	0.25	0.02	0.04	0.51	2.13	9.07
THAILAND	97	0.13	0.11	0.30	0.06	0.06	0.50	1.23	3.97
TURKEY	94	0.21	0.20	0.42	0.07	0.07	0.57	0.73	3.07
UK	130	0.08	0.07	0.29	0.03	0.04	0.47	2.46	11.27
USA	130	0.08	0.07	0.34	0.03	0.04	0.55	3.32	18.25

Table D.2 Realized Volatility of Exchange Rates

	Obs	Mean	Median	Max	Min	St. Dev.	Auto Corr.	Skew.	Kurt.
ARGENTINA	101	0.08	0.02	0.88	0.00	0.16	0.66	3.19	13.73
AUSTRALIA	130	0.05	0.05	0.24	0.01	0.03	0.54	2.82	19.58
AUSTRIA	130	0.05	0.05	0.11	0.02	0.01	0.44	0.68	3.89
BELGIUM	130	0.05	0.05	0.11	0.02	0.01	0.51	0.59	3.60
BRAZIL	77	0.07	0.06	0.25	0.00	0.05	0.52	1.71	6.62
CANADA	130	0.03	0.02	0.13	0.01	0.02	0.78	2.39	11.84
CHINA	121	0.01	0.00	0.40	0.00	0.05	-0.01	6.45	50.24
CHILE	120	0.05	0.03	0.92	0.00	0.09	0.05	8.56	85.03
FINLAND	130	0.05	0.05	0.17	0.02	0.02	0.51	2.38	15.15
FRANCE	130	0.05	0.05	0.11	0.02	0.01	0.49	0.60	3.73
GERMANY	130	0.05	0.05	0.11	0.02	0.01	0.51	0.73	4.09
INDIA	130	0.03	0.02	0.16	0.00	0.02	0.37	2.54	14.21
INDONESIA	78	0.07	0.04	0.66	0.00	0.10	0.80	3.59	18.46
ITALY	101	0.05	0.05	0.12	0.02	0.02	0.54	1.27	6.14
JAPAN	130	0.05	0.05	0.12	0.02	0.02	0.44	1.23	5.97
KOREA	130	0.03	0.01	0.40	0.00	0.05	0.53	4.67	29.42
MALAYSIA	130	0.02	0.02	0.24	0.00	0.03	0.70	4.72	34.31
MEXICO	130	0.07	0.04	1.08	0.00	0.14	0.56	5.07	32.12
NETHERLANDS	130	0.05	0.05	0.11	0.02	0.01	0.49	0.65	3.81
NORWAY	130	0.05	0.05	0.16	0.02	0.02	0.60	1.84	9.79
NEW ZEALAND	130	0.05	0.05	0.21	0.01	0.03	0.38	2.27	10.64
PERU	80	0.03	0.02	0.12	0.00	0.02	0.74	2.11	7.96
PHILIPPINES	130	0.03	0.02	0.25	0.00	0.04	0.13	3.27	16.83
SOUTH AFRICA	130	0.06	0.05	0.28	0.01	0.04	0.60	1.95	9.06
SAUDI ARABIA	101	0.00	0.00	0.03	0.00	0.01	0.33	2.44	8.12
SINGAPORE	121	0.02	0.02	0.08	0.01	0.01	0.69	2.39	11.15
SPAIN	130	0.05	0.05	0.12	0.01	0.02	0.45	0.89	5.26
SWEDEN	130	0.05	0.05	0.15	0.02	0.02	0.57	1.78	8.00
SWITZERLAND	130	0.06	0.05	0.10	0.03	0.01	0.44	0.47	2.87
THAILAND	121	0.03	0.02	0.21	0.00	0.04	0.59	3.46	16.15
TURKEY	125	0.08	0.05	0.69	0.00	0.10	0.03	3.05	14.70
UK	130	0.05	0.04	0.11	0.02	0.02	0.62	1.31	5.09
USA	–	–	–	–	–	–	–	–	–

Table D.3 REALIZED VOLATILITY OF LONG-TERM BONDS

	Obs	Mean	Median	Max	Min	St. Dev.	Auto Corr.	Skew.	Kurt.
ARGENTINA	–	–	–	–	–	–	–	–	–
AUSTRALIA	129	0.08	0.07	0.19	0.00	0.03	0.57	0.69	4.69
AUSTRIA	99	0.06	0.05	0.17	0.01	0.03	0.73	0.70	3.56
BELGIUM	88	0.06	0.06	0.14	0.01	0.03	0.72	0.46	3.05
BRAZIL	17	0.03	0.01	0.12	0.00	0.04	0.69	0.88	2.83
CANADA	100	0.08	0.07	0.20	0.03	0.03	0.65	1.62	7.37
CHINA	24	0.06	0.05	0.16	0.00	0.04	-0.12	1.11	3.57
CHILE	–	–	–	–	–	–	–	–	–
FINLAND	61	0.08	0.07	0.14	0.04	0.02	0.74	0.70	3.11
FRANCE	101	0.07	0.07	0.15	0.03	0.02	0.57	0.90	3.81
GERMANY	130	0.06	0.06	0.18	0.02	0.03	0.78	1.30	4.95
INDIA	–	–	–	–	–	–	–	–	–
INDONESIA	31	0.09	0.07	0.37	0.04	0.06	0.19	2.89	12.95
ITALY	72	0.07	0.07	0.12	0.03	0.02	0.40	0.62	3.41
JAPAN	114	0.13	0.12	0.45	0.02	0.08	0.73	1.72	7.00
KOREA	42	0.09	0.08	0.20	0.03	0.04	0.38	1.05	4.07
MALAYSIA	24	0.07	0.05	0.18	0.01	0.04	0.42	1.10	3.83
MEXICO	39	0.06	0.06	0.21	0.00	0.04	0.25	1.33	6.40
NETHERLANDS	130	0.06	0.06	0.14	0.02	0.02	0.75	0.85	3.61
NORWAY	74	0.07	0.06	0.46	0.00	0.06	0.07	4.72	32.99
NEW ZEALAND	105	0.06	0.06	0.14	0.02	0.02	0.63	0.84	3.93
PERU	14	0.06	0.05	0.15	0.00	0.04	-0.05	1.28	4.15
PHILIPPINES	50	0.08	0.06	0.29	0.02	0.06	0.33	1.80	5.92
SOUTH AFRICA	57	0.06	0.06	0.17	0.03	0.03	0.44	2.10	7.51
SAUDI ARABIA	–	–	–	–	–	–	–	–	–
SINGAPORE	52	0.12	0.10	0.25	0.04	0.06	0.58	0.68	2.49
SPAIN	73	0.08	0.07	0.17	0.03	0.02	0.50	1.31	6.44
SWEDEN	83	0.08	0.07	0.21	0.04	0.03	0.59	1.90	8.13
SWITZERLAND	69	0.10	0.09	0.23	0.03	0.04	0.75	1.16	4.43
THAILAND	43	0.12	0.10	0.27	0.05	0.06	0.42	0.68	2.34
TURKEY	–	–	–	–	–	–	–	–	–
UK	100	0.07	0.07	0.24	0.03	0.03	0.74	2.00	10.00
USA	130	0.09	0.08	0.31	0.03	0.05	0.80	2.02	8.55

Table D.4 Realized Volatility of Commodity Prices

	Obs	Mean	Median	Max	Min	St. Dev.	Auto Corr.	Skew.	Kurt.
CORN	130	0.12	0.10	0.28	0.04	0.05	0.50	1.04	4.00
SOYBEAN	130	0.11	0.10	0.24	0.04	0.04	0.45	1.10	4.03
WHEAT	130	0.13	0.12	0.30	0.05	0.04	0.59	1.17	4.57
COFFEE	130	0.17	0.16	0.43	0.06	0.06	0.48	1.29	6.01
RICE	90	0.13	0.12	0.30	0.05	0.05	0.39	1.09	4.91
SUGAR	130	0.20	0.18	0.52	0.08	0.07	0.57	1.11	4.94
COCOA	130	0.15	0.15	0.25	0.08	0.04	0.53	0.21	2.33
GOLD	130	0.09	0.08	0.42	0.02	0.05	0.67	2.81	16.87
SILVER	130	0.15	0.13	0.75	0.04	0.09	0.61	3.01	16.32
COPPER	90	0.13	0.12	0.38	0.06	0.05	0.60	2.06	9.82
NATURAL GAS	18	0.08	0.09	0.12	0.04	0.02	0.43	-0.27	2.33
COAL	39	0.12	0.10	0.34	0.06	0.06	0.55	1.95	7.08
OIL (CO1)	117	0.16	0.14	0.55	0.03	0.08	0.63	1.85	8.49
LIVESTOCK	120	0.06	0.06	0.17	0.03	0.03	0.57	1.75	6.70
MEAT & LIVEST.	59	0.05	0.05	0.13	0.02	0.02	0.23	1.51	5.20
CRB INDEX	69	0.08	0.08	0.23	0.04	0.03	0.76	2.58	13.15
LINSEED OIL	19	0.08	0.08	0.15	0.04	0.03	0.05	0.68	2.65

E Appendix: Realized volatility pairwise correlations

In this appendix we report a full set of pairwise correlations. The average pairwise correlation of a volatility series $\mathcal{RV}_{\kappa it}$ (where $i = 1, 2, ..., N$ is the number of countries and $\kappa = 1, 2, ..., M$ is the number of assets) is the cross-sectional average of the correlation between each pair that these series for all i. Hence, the average pairwise correlation can be interpreted as an average measure of synchronization of the volatility measures for a given asset class.

Table E.1 Average Pairwise Correlation Across Countries

	Equity		Exch. Rate		Bond	
	Level	First Diff.	Level	First Diff.	Level	First Diff.
ARGENTINA	0.22	0.29	-0.03	0.07	–	–
AUSTRALIA	0.54	0.56	0.33	0.30	0.47	0.38
AUSTRIA	0.43	0.48	0.39	0.38	0.50	0.37
BELGIUM	0.53	0.50	0.40	0.38	0.57	0.46
BRAZIL	0.16	0.15	0.24	0.23	-0.29	0.08
CANADA	0.55	0.58	0.23	0.32	0.52	0.33
CHINA	0.61	0.51	-0.03	-0.05	0.32	0.27
CHILE	0.42	0.45	0.03	-0.02	–	–
FINLAND	0.34	0.19	0.35	0.30	0.57	0.48
FRANCE	0.56	0.57	0.42	0.39	0.57	0.46
GERMANY	0.60	0.58	0.42	0.41	0.54	0.44
INDIA	0.37	0.28	0.17	0.12	–	–
INDONESIA	0.38	0.25	0.15	0.23	0.49	0.29
ITALY	0.47	0.46	0.42	0.41	0.50	0.43
JAPAN	0.52	0.48	0.25	0.27	0.31	0.23
KOREA	0.46	0.39	0.19	0.11	0.32	0.28
MALAYSIA	0.38	0.39	0.14	0.12	0.30	0.15
MEXICO	0.51	0.52	0.06	0.04	0.40	0.30
NETHERLANDS	0.54	0.56	0.41	0.41	0.56	0.47
NORWAY	0.55	0.59	0.41	0.37	0.36	0.18
NEW ZEALAND	0.49	0.39	0.26	0.24	0.40	0.34
PERU	0.44	0.58	0.17	-0.09	0.40	0.16
PHILIPPINES	0.43	0.33	0.07	-0.02	0.21	0.15
SOUTH AFRICA	0.49	0.53	0.23	0.26	0.29	0.31
SAUDI ARABIA	0.46	0.48	0.10	0.08	–	–
SINGAPORE	0.58	0.52	0.28	0.27	0.49	0.41
SPAIN	0.54	0.55	0.38	0.34	0.49	0.40
SWEDEN	0.57	0.56	0.36	0.31	0.56	0.46
SWITZERLAND	0.56	0.57	0.37	0.36	0.54	0.47
THAILAND	0.46	0.40	0.09	0.07	0.38	0.26
TURKEY	0.36	0.37	0.04	0.04	–	–
UK	0.56	0.57	0.37	0.35	0.52	0.35
USA	0.59	0.58	–	–	0.55	0.42

Note. *Level* indicates that the pairwise correlations have been computed on the level of the volatility variables as in equation 20; *First difference* indicates that the pairwise correlations have been computed on the first difference of the volatility variables.