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How do banks respond to increased funding uncertainty?

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Abstract

The 2007–9 financial crisis began with increased uncertainty over funding conditions in money markets. We show that funding uncertainty can explain diverse elements of commercial banks' behaviour during the crisis, including: (i) reductions in lending volumes, balance sheets, and profitability; (ii) more intense competition for retail deposits (including deposits turning into a "loss leader"); (iii) stronger lending cuts by more highly extended banks with a smaller deposit base; (iv) weaker pass-through from changes in the central bank's policy rate to market interest rates; and (v) a binding "zero lower bound" as well as a rationale for unconventional monetary policy.

Keywords: Bank lending, financial crises, interbank market, interest rate pass-through, liquidity channel, loan-to-deposit ratio, loss leader, monetary policy, zero lower bound.

JEL classifications: D40 (market structure and pricing), E43 (interest rates), E52 (monetary policy), G21 (banks).

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1 Introduction

The financial crisis began in August 2007 with an extended period of turmoil in money markets. Interbank rates such as Libor disconnected from central banks' policy rates and remained unusually high and volatile for an extended period (Taylor and Williams, 2009). Volumes of interbank lending, too, fell sharply and became more volatile (Afonso et al., 2010; Kuo et al., 2013). Both effects were most pronounced for unsecured (i.e., uncollateralized) term loans between banks with maturities of 3 months or more. Similar effects were felt in wider wholesale funding markets, such as the markets for repos and commercial paper (Adrian and Shin, 2010). Commonly cited reasons for the turmoil are counterparty risk and an increased demand for liquidity (Acharya and Skeie, 2011).

Regarding the behaviour of commercial banks during the crisis, there is significant evidence that customer lending in many countries declined (Campbell et al., 2011; Santos, 2011), and that banks became less profitable (Bank of England, 2008; ECB, 2013). Two important related effects have recently been documented. First, banks with better access to deposits have tended to cut lending by less (Cornett et al., 2011; Ivashina and Scharfstein, 2010; Iyer et al., 2013). Second, the ECB (2013) data summarized in Figure 1 suggests that more extended banking systems, as measured by high loan-to-deposit ratios, experienced stronger drops in bank lending.

Central banks responded to the turmoil by aggressively cutting policy rates, moving close to the "zero lower bound" in many countries. Commentators expressed surprise at the apparently small impact that these rate cuts had on market interest rates. Moreover, central banks used unconventional tools such as asset purchases and quantitative easing in an effort to normalize money market conditions.

This paper develops a theory of the link between increased money market uncertainty, commercial bank behaviour, and monetary policy which can explain and rationalize these empirical observations. We consider a partial-equilibrium model of a payoff-maximizing commercial bank. The bank extends retail loans and funds itself with retail deposits, equity capital, and via the money market.[1] The key feature is uncertainty over money market funding conditions.

We highlight two examples within our general setup. In the first, a risk-neutral bank faces uncertain aggregate liquidity in money markets where lenders are concerned about counterparty risk. This creates convex wholesale borrowing costs: Its (expected) wholesale funding rate increases in the amount borrowed, and also increases in the degree of liquidity uncertainty. In the second, the bank has finite tolerance for risk, (e.g., because of delegation to risk-averse managers), and faces

[1] Our main interest is in banks that are net borrowers in the wholesale market; this corresponds to loan-to-deposit ratios at least modestly above 100%. Figure 1 shows that this condition applied to most European banking systems.

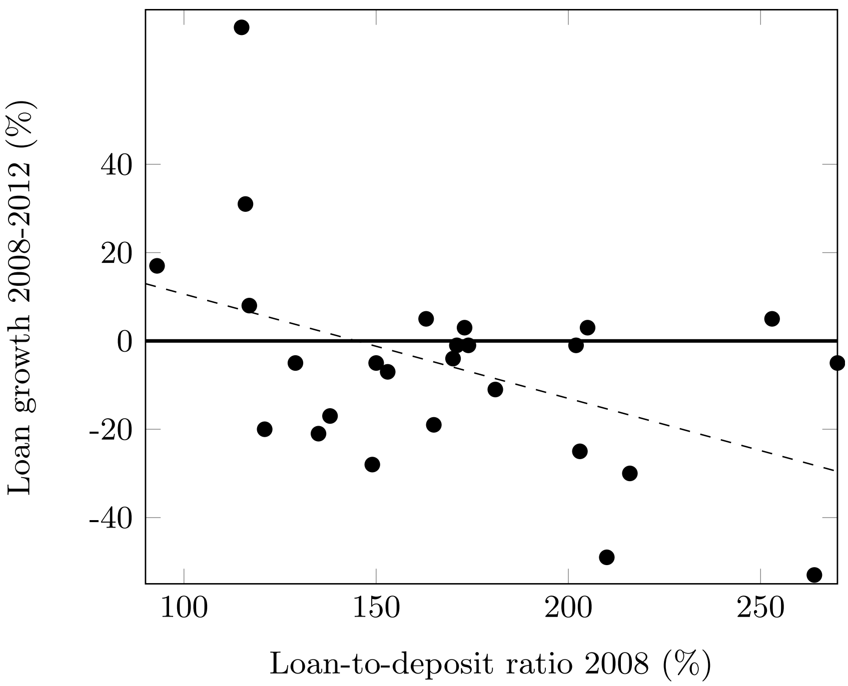

Figure 1: Loan-to-deposit ratios and loan growth in European banking systems. *Source: ECB (2013)*

stochastic money market rates. It can borrow any amount at a constant rate, but raising its interbank market exposure increases its risk-adjusted funding rate.

In line with the empirical findings, we show that a bank responds to higher funding uncertainty by lending less—together with a substitution effect by which it increases its use of deposit finance and raises additional equity capital from investors. Equilibrium market interest rates for loans and deposits both increase. Higher uncertainty also reduces bank profitability, measured, e.g., by return on equity. We show that these "first-order" comparative statics are robust to a wide range of model specifications.[2]

Substitution into deposit finance has striking welfare implications. In our second example, if the bank's risk tolerance is low, its deposit business can become a "loss leader", with the deposit rate exceeding its own wholesale funding cost. This implies that depositor welfare exceeds the level associated with a competitive market.[3]

[2] Our benchmark model focuses on the stylized case where the bank acts as a monopolist, in retail markets (perhaps due to customer lock-in or switching costs), and only faces uncertainty in money markets. We show that our results are robust to (i) different competitive conditions, including oligopoly and price-taking; (ii) multiple sources of risk; and (iii) fixed level of bank equity, e.g. due to binding capital requirements. (See the Appendix, Extensions A–C.)

[3] This risk-based version of loss leaders differs markedly from other mechanisms identified in the literature. These generally rely on product complementarities (e.g., razor and razor blades) or on particular features of the strategic interaction between firms (e.g., related to entry deterrence). By contrast, in our model, loss leaders can occur even in a single-bank setting where loans and deposits are entirely independent in terms of demand and supply conditions.

Our model is also consistent with the "second-order" effects observed in the data. A bank with less favourable access to deposit finance chooses to be more extended *ex ante* (with a higher loan-to-deposit ratio) and cuts lending by more when funding uncertainty rises.[4] A key condition for this result is that wholesale funding costs are *supermodular* in funding uncertainty and the amount borrowed.[5] This means that the cost of raising money market funds rises with uncertainty, and that this effect is more pronounced the greater a bank's exposure.

Turning to monetary policy, we show that heightened funding uncertainty typically dampens the rate of pass-through from policy to retail interest rates. This may help explain the reduced effectiveness of monetary policy observed during the crisis. The key condition, once again, is that a bank's money market funding costs are supermodular in uncertainty and the amount borrowed. In extreme cases where funding uncertainty becomes "large", pass-through tends to zero and market interest rates are completely frozen.

Building on this, we consider the optimal response of an inflation-targeting central bank to higher funding uncertainty. We show that increased uncertainty may lead to a binding "zero lower bound" on policy rates. In our first example with uncertain market liquidity, this can justify unconventional policies such as quantitative easing and asset purchases. These policies dampen the adverse effect of funding uncertainty, using liquidity "as a substitute" for calmer market conditions to stimulate demand. Moreover, we show that the central bank can be more effective if there is a "liquidity channel", i.e., a rate cut increases aggregate liquidity in money markets, for instance, because it is conducted through repo lending or open market operations.

The fact that heightened funding uncertainty can account for such diverse aspects of observed bank behaviour distinguishes this mechanism from others. For example, in a standard banking model, a decrease in the demand for loans typically also leads to a decline in bank lending and bank profitability. However, it is less clear how or why reduced loan demand simultaneously raises deposit rates *and* dampens interest rate pass-through in both loan and deposit markets. Funding uncertainty, by contrast, presents a mechanism that connects all these elements of bank behaviour.

One way of understanding the economic forces at work is in terms of synergies between the loan and deposit sides of a bank's operations. An increase, say, in a bank's deposit bases reduces the funding (risk) exposure of further loan commitments, which in turn makes lending themselves more attractive. As uncertainty over

[4] Our results are also consistent with empirical evidence that low-capital banks tend to charge higher interest rates on loans than well-capitalized banks (Hubbard et al., 2002).

[5] The basic definition is that a function g is supermodular if $g(\inf(x, y)) + g(\sup(x, y)) \geq g(x) + g(y)$ for all x, y. We here work with a differentiable case, for which (strict) supermodularity boils down to a positive cross-partial derivative, i.e., $g_{xy} > 0$.

funding conditions increases, these uncertainty-induced synergies become stronger, and the bank becomes more concerned with asset-liability management. Put this way, our paper contributes to the literature on loan-deposit synergies (Gatev et al., 2009; Kashyap et al., 2002) that focuses on interactions between the two sides of a bank's balance sheet.

Section 2 sets up the baseline model and explains its key properties; it also presents our two examples that fit into the general framework. Section 3 analyzes the impact of funding uncertainty on equilibrium outcomes in credit markets, a bank's balance sheet and its profitability—and relates our findings to the empirical evidence. Section 4 examines monetary policy including interest rate pass-through, the problem of the "zero lower bound", and the potential role of unconventional policy measures. Section 5 offers concluding remarks and some directions for future research. (The Appendix contains details of our robustness analysis.)

2 The baseline model

We study decision-making of a bank that extends retail loans L, and funds itself with customer deposits D, equity capital K and unsecured money market borrowing M. The bank's balance sheet constraint therefore is $L = D + K + M$.

We focus on the case where the bank is a net borrower in money markets ($M > 0$) unless stated otherwise. Defining the bank's loan-to-deposit ratio $\ell \equiv L/D$, note that $M > 0 \Leftrightarrow \ell > (1 + K/D) \equiv \widehat{\ell}$, where $\widehat{\ell}$ is typically only modestly above 100% since most commercial banks' customer deposits are many times larger than equity capital. As discussed in the introduction, our focus on interbank borrowers is motivated by the empirical fact that many countries' banking sectors were highly extended going into the financial crisis, e.g., the UK and much of the Eurozone.

We assume that the bank has a degree of market power in its loan and deposit markets. One could think of this as a model of banks with differentiated products, certain regulatory restrictions, or a "captive" consumer base due to informational lock-in or switching costs, all of which would generate market power. As shown later on, our results are robust to different forms of competition between banks.[6]

The inverse demand curve for loans is given by $r_L = f^L(L)$, where r_L is the market interest rate on loans, and demand is downward-sloping with $f^L_L(\cdot) < 0$.[7] Similarly, the inverse supply curves for deposits and capital are given by $r_D = f^D(D)$

[6] See Sharpe (1990) and Petersen and Rajan (1994) for theoretical and empirical support for informational lock-in as a source of banks' market power, and Kim et al. (2003) for empirical evidence of switching costs in banking. Allen and Gale (2000), Boyd and de Nicoló (2005), Hannan and Berger (1991), Neumark and Sharpe (1992), Neven and Röller (1999), Stein (1998), and Wong (1997) analyze related models of loan and/or deposit markets although none of these consider the impact of funding uncertainty in the money market.

[7] Throughout the paper, subscripts on functions are used to denote (partial) derivatives.

and $r_K = f^K(K)$, respectively, where r_D is the market interest rate on deposits and r_K is the expected return on equity. Higher deposit rates attract more depositors and higher returns attract more equity investors, so $f_D^D(\cdot) > 0$ and $f_K^K(\cdot) > 0$.[8]

For clarity of exposition, we assume there are no operational economies of scope between the bank's loan and deposit sides; without much further loss of generality, the bank's operating costs are set to zero.

The key feature of our model is that the bank faces uncertainty over funding conditions in the money market. In particular, the cost of money market funds depends on the realization of a random variable ε. The distribution of ε is $F_\varepsilon(\varepsilon;\sigma)$, where $E[\varepsilon] = 0$ and $E[\varepsilon^2] = \sigma$. The parameter σ is central to our analysis as it captures the degree of funding uncertainty.

The bank's cost of funding is given by $C(M, R, \varepsilon)$, where R is the policy rate controlled by the central bank. The bank's problem is to:

$$\begin{aligned} \max_{(L,D,K,M)\in\mathbb{R}_+^4} U &= f^L(L)L - f^D(D)D - f^K(K)K - E[C(M,R,\varepsilon)] \\ \text{subject to } L &= D + K + M. \end{aligned} \tag{1}$$

Define marginal revenues and cost functions $MR^L(L) \equiv \frac{\partial}{\partial L}\left[f^L(L)L\right]$, $MC^D(D) \equiv \frac{\partial}{\partial D}\left[f^D(D)D\right]$, $MC^K(K) \equiv \frac{\partial}{\partial K}\left[f^K(K)K\right]$ as well as the bank's expected marginal funding cost in the money market $\mu(M,R,\sigma) \equiv \frac{\partial}{\partial M}E[C(M,R,\varepsilon)]$. The first-order conditions for an interior solution are

$$\begin{aligned} U_L &= MR^L(L^*) - \mu(M^*,R,\sigma) = 0 \\ U_D &= -MC^D(D^*) + \mu(M^*,R,\sigma) = 0 \\ U_K &= -MC^K(K^*) + \mu(M^*,R,\sigma) = 0, \end{aligned} \tag{2}$$

where the starred variables $L^*, D^*, K^*, M^* > 0$ are optimal choices.[9,10] We study the well-behaved case where these conditions are sufficient for a maximum. For this

[8] Following Allen and Gale (2000) and Boyd and de Nicoló (2005), were here assume implicitly that deposits are fully insured, so the supply of funds does not depend on risk. The model could easily be extended to incorporate a flat-rate insurance premium per unit of deposits without affecting any of the results presented.

[9] A sufficient condition for the existence of an interior solution is $MR^L(0) > \mu(0,R,\sigma)$ and $MC^D(0) = MC^K(0) < \mu(0,R,\sigma)$. By assuming interior solutions, we are imposing that the Modigliani-Miller conditions are violated so there is a meaningful trade-off between bank capital and debt. A microfoundation for this would include financing frictions in capital and debt markets. Capital may be costly due to asymmetric information as in Myers and Majluf (1984) or because it lacks the disciplining effect of debt as in Calomiris and Kahn (1991). Example 1 gives an explicit foundation of frictions in money market debt funding.

[10] The baseline model assumes that funding uncertainty σ does not *directly* affect the cost of capital $MC^K(K)$, although we shall see that it *indirectly* raises it (since $K_\sigma^* > 0$ and $MC_K^K > 0$). Our robustness analysis allows for a direct effect, that is, $MC_\sigma^K > 0$ (see the Appendix, Extension B), so factors that raise wholesale funding uncertainty here also raise the cost of equity.

purpose, we assume that $MR_L^L < 0$, $MC_D^D > 0$, $MC_K^K > 0$ and $\mu_M \geq 0$ everywhere, where the subscripts again denote partial derivatives.[11]

We now present two examples of how the cost of money market funds $C(M, R, \varepsilon)$, and the measure of funding uncertainty σ, can be interpreted. We argue that uncertainty likely raises the expected marginal cost of wholesale funds, $\mu_\sigma > 0$.

Example 1: Counterparty risk and aggregate liquidity shocks

This example is based on two basic frictions: (i) potential lenders in money markets are concerned about default, and there is asymmetric information between interbank lenders and borrowers; (ii) there is uncertain aggregate liquidity.

Our setup captures some of the main intuitions of the theoretical literature on interbank market turmoil. Asymmetric information is emphasized by Freixas and Holthausen (2005), Freixas and Jorge (2008) and Heider et al. (2009). Uncertain aggregate liquidity can arise due to liquidity hoarding by banks as in Acharya and Skeie (2011), or could be interpreted as a reduced-form model of limited "funding liquidity" in secured debt markets as in Brunnermeier and Pedersen (2008).

This example also fits the recent empirical literature on the causes of money market turmoil. While early contributions such as Taylor and Williams (2009) emphasized counterparty risk, liquidity considerations are identified as a key factor by Acharya and Merrouche (2012), Michaud and Upper (2008) and Schwarz (2009).

External lenders in the money market believe that there is a risk of facing a *toxic bank,* which defaults with probability 1. There is a unit measure of external lenders indexed by $i \in [0, 1]$, each of whom is risk-neutral with an opportunity cost equal to the risk-free rate R. Each lender has X dollars to lend out, where X is a measure of aggregate liquidity satisfying $X = \lambda + \varepsilon$, with $\lambda > 0$ a constant measuring expected aggregate liquidity and ε a random variable. There are two aggregate states for liquidity, which are equally likely: In the high state, $\varepsilon = \sigma$, and in the low state, $\varepsilon = -\sigma$, where $0 < \sigma < \lambda$. The parameter σ thus captures uncertainty about aggregate liquidity.[12]

There are differences of opinion among lenders: Lender i believes that the bank is toxic with probability i. To borrow a dollar from lender i, the bank has to promise

[11] An implicit assumption is that the interest rates on loans and deposits do not depend on ε. If complete contracts contingent on money market conditions were feasible, the nature of the optimal contract would be determined by the risk aversion of banks and their customers. However, if banks are less risk-averse than retail customers, the optimal contract would probably be close to the fixed-rate case analysed here. The assumption of incomplete contracts seems sensible as it is unlikely that retail customers have enough information about money markets to verify ε. It also matches the empirical fact that variable-rate loans and deposits tend to be contingent on central bank policy rates rather than interbank rates such as Libor.

[12] The binary state structure is introduced purely for clarity of exposition. A more general model would yield the same results as long as an increase in σ signified a mean-preserving spread of aggregate liquidity in the sense of Rothschild and Stiglitz (1970).

repayment of $X/(1-i)$ dollars to compensate for the perceived risk of facing a toxic bank. To derive the cost of funds for borrowers, consider the cheapest way of borrowing $M > 0$ dollars. This is to go to the M/X most optimistic lenders, i.e. $i \in [0, M/X]$, and to borrow X from each. The cost function is thus given by

$$C(M, R, \varepsilon) \equiv \int_0^{M/X} \left(\frac{RX}{1-i} \right) di \tag{3}$$

and, by Leibniz rule, the marginal cost is $\partial C(M, R, \varepsilon)/\partial M = R/(1-M/X)$. Assume that the net interest rate is close to zero, and that individual bank borrowing is small relative to aggregate liquidity, i.e., $R \simeq 1$ and $M/X \simeq 0$. By a Taylor expansion, the marginal cost is then approximately $R + M/X$. Banks pay a markup proportional to the amount borrowed and inversely proportional to aggregate liquidity. So the (approximate) expected marginal cost is:

$$\begin{aligned} \mu(M, R, \sigma) &= R + M \times E_\varepsilon [1/X] \\ &= R + \frac{1}{2} \left[\frac{1}{\lambda + \sigma} + \frac{1}{\lambda - \sigma} \right] M = R + \left((1/\lambda) \cdot \frac{1}{[1 - (\sigma/\lambda)^2]} \right) M. \end{aligned}$$

Marginal cost equals the policy rate R plus a spread driven by liquidity and counterparty risk. The spread is influenced by two factors. The first factor $1/\lambda$ measures the average tightness of the money market. When expected aggregate liquidity is low, the bank is forced to borrow from more pessimistic lenders, which drives up the cost of funds. The second factor is an increasing function of σ/λ, which is the coefficient of variation of aggregate liquidity. It is easy to see that funding uncertainty here raises marginal cost, $\mu_\sigma > 0$. This is because the marginal funding cost $R + M/X$ is convex in aggregate liquidity, so a mean-preserving spread of the liquidity distribution increases its expectation.

Intuitively, when uncertainty increases, the additional costs given in a low liquidity state more than offset the savings in a high liquidity state. When liquidity is very low, the bank has to borrow from very pessimistic lenders whose belief that it is toxic is close to 1—and for which the required interest rate becomes large.

Note that this example captures *both* quantity and price uncertainty in money markets, since aggregate liquidity shocks shift the entire supply curve of bank funding. This is consistent with evidence in Afonso et al. (2011) who show quantity rationing and the problem of rolling over interbank debt on a day-to-day basis.

Example 2: Limited risk tolerance

Our second example is based on the premise that banks have some incentive to reduce the riskiness of their profits. In an influential paper, Froot and Stein (1998) argue that banks should be concerned with risk management as they, in practice, cannot frictionlessly hedge all the risks they face. There are many other reasons why banks may act as if they were risk-averse, including costs of financial distress, non-linear tax systems, and delegation of control to risk-averse managers; see also Greenwald and Stiglitz (1990).[13]

Suppose that the interest rate in money markets is given by the policy rate plus a random spread, i.e., $R+\varepsilon$. The bank's profits, contingent on the realization of ε, are thus $\Pi = f^L(L)L - f^D(D)D - f^K(K)K - (R+\varepsilon)\, M$. For simplicity, we assume mean-variance preferences with a coefficient of absolute risk aversion $1/\rho$, so we can interpret $\rho \in (0,\infty)$ as the bank's level of risk tolerance. Hence, the bank's objective function is

$$U = E\left[\Pi\right] - (1/2\rho)Var\left[\Pi\right], \tag{4}$$

where the risk-adjusted cost of wholesale funds is given by $RM + (1/2\rho)Var\left[\Pi\right] = \left\{R + (1/2\rho)\sigma^2 M\right\} M$. So the expected marginal cost of money market funding is:

$$\mu\left(M,R,\sigma\right) = R + (1/\rho)\sigma^2 M. \tag{5}$$

Again, marginal cost equals the policy rate plus a spread which increases in funding uncertainty, so that $\mu_\sigma > 0$ whenever the bank is a (net) interbank borrower with $M > 0$. The sensitivity of marginal funding costs to uncertainty here is driven by the bank's desire to avoid exposure to risky funding instruments.[14]

While we emphasize the role of higher funding uncertainty, our results thus also apply to decreases in the bank's risk tolerance ρ, holding funding uncertainty σ fixed—or to combinations of changes in these two parameters (since risk aversion and funding uncertainty enter multiplicatively into $\mu\left(M,R,\sigma\right)$). We can also think of the bank as initially being risk-neutral (or even slightly risk-loving) but then *becoming* risk-averse, e.g., in the context of a financial crisis.[15]

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[13] On the empirical side, Angelini (2000) shows how intra-day behaviour in the Italian interbank market is consistent with risk aversion; Hughes and Mester (1998), Nishiyama (2007) and Ratti (1980) find evidence for risk-averse behaviour by US commercial banks (or their managers).

[14] This mean-variance setup would (i) generally arise if funding uncertainty is normally distributed, and (ii) holds approximately for any risk-averse utility function if funding uncertainy is "small". To see why, note that the first-order conditions $U'\left(\Pi\right)\Pi_j = 0 \implies E\left[\Pi_j\right] + cov\left(U'\left(\Pi\right),\Pi_j\right)/E\left[U'\left(\Pi\right)\right] = 0$ for $j = \{L,D,K\}$, where the latter term is marginal risk. Now, if (i) if Π and Π_j are normally distributed (by Stein's lemma, see Huang and Litzenberger, 1988) or (ii) σ is small (by Taylor's theorem), then $cov\left(U'\left(\Pi\right),\Pi_i\right)/U'\left(\Pi\right) = -(1/\rho)\cdot cov\left(\Pi,\Pi_j\right)$. It is easy to check that the first-order conditions are identical to those from the mean-variance analysis, with a risk-adjusted marginal funding cost of $\mu\left(M,R,\sigma\right) = R + (1/\rho)\sigma^2 M$.

[15] Throughout, we have assumed that banks never actually default, so limited liability is not a concern. Adding default and limited liability to the model would introduce complications beyond our scope. If the probability of default were close to one, the results on funding choices below might be offset by risk-seeking when there is limited liability (Jensen and Meckling, 1976). Risk-seeking would also create a case for regulation in the presence of deposit insurance (Merton, 1977) or "too

The common thread between Examples 1 and 2 is that the bank's (approximate) marginal funding cost satisfies $\mu(R, M, \sigma) = R + h(\sigma)M$, where $\mu_M = h(\sigma) > 0$ as well as $\mu_\sigma = h_\sigma M > 0$ (both for any $\sigma > 0$).[16,17]

3 The impact of increased funding uncertainty

We begin our main analysis by exploring the implications of increased funding uncertainty in money markets for a bank's loan and deposit decisions, as well as its optimal choice of equity capital. This also yields predictions regarding its impact on banks' interbank market exposure and equilibrium market interest rates.

3.1 Credit markets and balance sheets

Recall that the bank's equilibrium choices of loans L^*, deposits D^*, and capital K^* are determined by the system of three equilibrium conditions in (2). Let the associated interest rates on loans $r_L^* = f^L(L^*)$ and deposits $r_D^* = f^D(D^*)$.

Proposition 1 *With $\mu_\sigma > 0$, an increase in funding uncertainty induces a bank to:*
(i) extend fewer loans L^* and increase interest rates on loans r_L^*
(ii) take more deposits D^* and increase interest rates on deposits r_D^*
(iii) increase its level of equity capital K^*
(iv) reduce its interbank market borrowing M^*
(v) decrease its loan-to-deposit ratio ℓ^*

Proof. Totally differentiating the first-order conditions (2) yields the system

$$\begin{pmatrix} U_{LL} & U_{LD} & U_{LK} \\ U_{DL} & U_{DD} & U_{DK} \\ U_{KL} & U_{KD} & U_{KK} \end{pmatrix} \times \begin{pmatrix} L_\sigma^* \\ D_\sigma^* \\ K_\sigma^* \end{pmatrix} = - \begin{pmatrix} U_{L\sigma} \\ U_{D\sigma} \\ U_{K\sigma} \end{pmatrix}. \tag{6}$$

big to fail" policies. In abstracting from limited liability, we effectively assume that default is not imminent, so profit concerns dominate the risk-seeking effect.

[16] The examples differ in two more detailed respects. First, without uncertainty, marginal cost in Example 1 is $\mu(R, M, 0) > R$, whereas it is $\mu(R, M, 0) = R$ in Example 2 since zero funding uncertainty makes the bank behave as if it were risk-neutral and able to borrow in the money market at a constant rate. Second, the bank's payoff U equals its expected profits in Example 1, while limited risk tolerance implies a wedge between payoffs and profits in Example 2. This distinction is important only for interpreting Proposition 2 on profitability and return on equity.

[17] We focus on cases where the bank is a net interbank market borrower, $M > 0$. Most of for formal results do not directly rely on this property; rather the proofs use the condition that $\mu_\sigma > 0$. In Example 1, these two conditions are two sides of the same coin. Some of our results also go through for banks with $M < 0$ in the context of Example 2. This includes reduced bank profitability (Proposition 2) and weaker interest rate pass-through (Proposition 8). See the earlier working paper, Ritz (2012).

The left-hand matrix is the Hessian of the objective function U, which we call H. Its determinant is

$$|H| = MR_L^L MC_D^D MC_K^K \left[1 + \mu_M \left(\frac{1}{MC_K^K} + \frac{1}{[-MR_L^L]} + \frac{1}{MC_D^D}\right)\right] < 0. \tag{7}$$

Using Cramer's rule to solve the system, we obtain

$$\begin{aligned} L_\sigma^* &= |H|^{-1} \mu_\sigma MC_D^D MC_K^K < 0 \\ D_\sigma^* &= |H|^{-1} \mu_\sigma MR_L^L MC_K^K > 0 \\ K_\sigma^* &= |H|^{-1} \mu_\sigma MR_L^L MC_D^D > 0 \end{aligned} \tag{8}$$

since $MR_L^L < 0$, $MC_D^D > 0$, $MC_K^K > 0$, and $\mu_\sigma > 0$ by assumption, thus proving parts (i) and (iii). Part (ii) now follows as $f_L^L(L) < 0$ and $f_D^D(D) > 0$, part (iv) since $M^* = L^* - D^* - K^*$ and part (v) since $\ell^* = L^*/D^*$. ■

These results are driven by the property that the bank's expected marginal cost of funding in the interbank market rises with the degree of funding uncertainty, $\mu_\sigma > 0$—which is satisfied by both Examples 1 and 2.

The intuition is straightforward: A higher (risk-adjusted) wholesale funding cost induces the bank to substitute away to other sources of financing. In particular, deposit funds become relatively more attractive, with deposit rates and volumes rising; moreover, the bank raises additional equity capital. Conversely, the rise in μ raises the opportunity cost of extending loans, with loan rates rising and lending volumes falling as a result. All together, the bank reduces its money market exposure and cuts its loan-to-deposit ratio.

This set of predictions is consistent with emerging base of stylized facts on bank behaviour during the financial crisis. It was plain that banks cut back on loans, thereby making it more difficult and costly for retail and corporate customers to borrow. For example, it was noted that "banks have cut overdraft facilities and unused credit lines, withdrawn from lending syndicates and abruptly called in loans. When they do lend, they are charging higher arrangement fees and interest at margins over their cost of funding that are considerably higher than they were" (*The Economist*, 24 January 2009). Campbell et al. (2011) present evidence from a survey of US CFOs which strongly suggests that corporate borrowers became worse off, and Santos (2011) finds that borrowers' interest rate spreads increased.[18]

[18] Of course, there are competing explanations for an observed reduction in bank lending, notably a decrease in the demand for loans. Puri et al. (2011) distinguish between the demand and supply effects of the US-led mortgage crisis on lending by German savings banks, where the particular ownership structure meant that some banks were directly affected whilst others were not. They show that loan demand decreased for both types of banks, but also the supply-side result that affected banks reduced lending significantly more strongly, consistent with our result

There is also significant evidence that banks tried to reduce their exposure to the wholesale market from when the financial crisis began in the second half of 2007.[19] ECB (2012, p. 15) observes that "the financial crisis broke a broad global funding trend characterized by a strong reliance in wholesale funding sources in favour of more stable retail sources of funding. This implies that bank funding strategies needed to be adjusted quickly in order to expand the customer deposit base and reduce the share of wholesale funding." The difference between money market rates and deposit rates in the Eurozone adjusted sharply from being strongly positive (around 150–300 basis points over the 2003–2008) to only slightly positive from late 2008 until 2011 (ECB, 2012). In the UK, many banks sought to raise more funds from retail customers by raising interest rates on existing deposit accounts and introducing various new savings products.[20]

Our results can also rationalize trends in banks' loan-to-deposit ratios. After years of rising loan-to-deposit ratios, the Eurozone unweighted average was 138% in the autumn of 2008, but gradually declined over the following years. Moreover, there was a strong decline near the top of the distribution; while some banks had peak loan-to-deposit ratios of around 250%, the maximum ratio had declined to around 175% by 2011 (ECB, 2012). Indeed, several UK banks, including Royal Bank of Scotland, "set themselves the aim of achieving a loan-to-deposit ratio of no more than 100% over the next five years" (*Financial Times*, 19 June 2009).

Finally, in response to the turmoil, many banks also sought to raise additional equity capital from investors, sometimes apparently under considerable pressure from regulators. Proposition 1 highlights that a payoff-maximizing bank facing increased funding uncertainty would, in fact, find it *privately* optimal to raise more capital—even in the absence of regulatory intervention, or any crisis-induced reductions in available equity due to trading losses or writedowns (Brunnermeier, 2009).

3.2 Bank profitability and return on equity

We now turn to the impact of funding uncertainty on bank profitability.

Proposition 2 *With $\mu_\sigma > 0$, an increase in funding uncertainty decreases a bank's equilibrium expected payoff.*

---

from Proposition 1.

[19] The situation at the time was summarized by a bank manager at Alliance & Leicester: "Lenders are having to examine different funding routes. The increasing rates have no doubt been driven by the turmoil in the wholesale markets" (*Financial Times*, 1 December 2007). Alliance & Leicester is a medium-sized British bank (and former building society) that was subsequently taken over by Banco Santander of Spain (in October 2008).

[20] Our results are also consistent with evidence that banks' interbank liabilities as a proportion of total assets fell substantially from the 3$^{\text{rd}}$ quarter of 2008 onwards, which corresponds to a decline in M^*/L^* in our model (since L^* falls by more than M^* falls).

Proof. By the envelope theorem, $\frac{\partial}{\partial\sigma}U(L^*,D^*,K^*) = -\frac{\partial}{\partial\sigma}E[C(M^*,R,\varepsilon)]$ since L, D, and K are all chosen optimally. Recalling the definition $\mu(M,R,\sigma) \equiv \frac{\partial}{\partial M}E[C(M,R,\varepsilon)]$, we can also write $E[C(M^*,R,\varepsilon)] = \int_0^{M^*}\mu(M',R,\sigma)dM'$. It follows that $\frac{\partial}{\partial\sigma}E[C(M^*,R,\varepsilon)] = \int_0^{M^*}\mu_\sigma(M',R,\sigma)dM' > 0$ given the assumption $\mu_\sigma > 0$, thus proving the claim that $\frac{\partial}{\partial\sigma}U(L^*,D^*,K^*) < 0$. ■

Often a bank's expected payoff is synonymous with its expected profits, including in our Example 1. In such cases, Proposition 2 implies that funding uncertainty reduces a bank's average return on equity (ROE), defined as $U(L^*,D^*,K^*)/K^*$ (by combining Proposition 2 with our earlier result from Proposition 1(iii) that $K^*_\sigma > 0$). In cases without risk-neutrality, as in Example 2, the result tells us that funding uncertainty reduces the bank's expected utility—or that of its managers. However, it is not difficult to show that the same conclusion also applies for such a bank's expected profitability.[21] Intuitively, higher funding uncertainty tightens the "utility constraint" on the bank's expected profits, thus distorting its optimal loan, deposit and equity choices further away from the (profit-maximizing) risk-neutral case.

Proposition 2 thus suggests that increased uncertainty about funding conditions *per se* leads to a reduction in bank profitability. This is consistent with evidence for a sharp drop in UK banks' ROEs in the second half of 2007 when funding uncertainty initially increased (Bank of England, 2008, p. 38), and with low bank profitability levels in the Eurozone since 2008 (ECB, 2013, p. 20). It is also consistent, all else equal, with decreases in banks' stock prices and market capitalizations. More generally, it is possible that a bank could become loss-making overall in the presence of significant fixed costs that need to be covered for it to be operational.

3.3 Robustness of Propositions 1 and 2

Propositions 1 and 2 are robust to many changes in the model's specification. The Appendix contains formal proofs and more detailed discussion of our robustness analysis. In summary, we consider three modifications to the baseline model:

First, we analyze situations where the bank's level of equity capital is fixed, perhaps due to a binding regulatory capital requirement or because the bank is unable to raise additional equity from investors in the short run. This can be thought of as the limiting case of the baseline model where the marginal cost of

[21] To see why, observe that funding uncertainty affects a risk-averse bank's equilibrium expected profits $E[\Pi^*]$ according to $E[\Pi^*_\sigma] = E[\Pi_L]L^*_\sigma + E[\Pi_D]D^*_\sigma + E[\Pi_K]K^*_\sigma$. Now recall the bank's first-order conditions which can be written as $U_L = E[\Pi_L] - (1/\rho)\sigma^2 M^* = 0$, $U_D = E[\Pi_D] + (1/\rho)\sigma^2 M^* = 0$, and $U_K = E[\Pi_K] + (1/\rho)\sigma^2 M^* = 0$, respectively. Since the bank is a net wholesale borrower, $M^* > 0$, the first-order conditions imply $E[\Pi_L] > 0$, $E[\Pi_D] < 0$, and $E[\Pi_K] < 0$. By Proposition 1, we already know that $L^*_\sigma < 0$, $D^*_\sigma > 0$, and $K^*_\sigma > 0$. Putting these elements together shows that the bank's profits $E[\Pi^*_\sigma] < 0$ decline (as does its ROE).

capital is extremely convex, $MC_K^K \to \infty$, such that the bank's choice of capital is effectively inflexible. We show that Propositions 1 and 2 apply exactly as above in such cases.[22]

Second, we consider situations where the bank faces additional risks, such as credit risks in its loan portfolio. As long as these additional risks are not correlated with funding uncertainty, our previous results from Propositions 1 and 2 certainly hold exactly as above. Proposition 1 also goes through under reasonable conditions on the additional (correlated) sources of uncertainty in the loans market and that for equity capital. Finally, we show that higher funding uncertainty very generally reduces the bank's expected payoff, as in Proposition 2.

Third, we analyze different forms of competition between banks in loan and deposit markets, including with differentiated products. We show that the results from Proposition 1 apply very generally with different market structures, ranging from concentrated oligopolies to perfectly competitive markets in banks act as a price-takers over loans and deposits. Proposition 2 continues to hold under fairly general conditions—although the induced cutbacks in lending *by other banks* tend to soften the adverse profit impact of funding uncertainty.

3.4 Deposit access as competitive advantage

One of our key findings so far is the "first-order" comparative static that banks respond to higher funding uncertainty by cutting back their lending volumes. Here we examine a "second-order" comparative static: How does this reduction in lending vary with a bank's access to customer deposits?

We begin by showing that, all else equal, a bank with less favourable deposit access holds more equity capital (to "compensate"), relies more heavily on financing via interbank markets, and chooses to become more extended in terms of its loan-to-deposit ratio. Under reasonable conditions, such a bank also cuts lending more strongly than an otherwise identical bank that has a stronger deposit base.

Formally, suppose there exists a parameter φ which shifts the bank's cost of attracting deposits upwards, in that the cost of deposits $f_D(D, \varphi)D$ with marginal cost $MC^D(D, \varphi)$ satisfying $MC_\varphi^D > 0$ everywhere. We can thus address how deposit access affects a bank's equilibrium choices:

Proposition 3 *With $\mu_M > 0$, a bank with less favourable deposit access will:*
(i) extend fewer loans L^* and increase interest rates on loans r_L^*
(ii) take less deposits D^* and decrease interest rates on deposits r_D^*

[22] Further results from this specialized model are that banks which suffer larger reductions in equity capital, say, due to trading losses increase interest rates on loans by more, and, conversely, an explanation for how recapitalizations (by shareholders or government) can counteract upward pressure on interest rates due to funding uncertainty. See Ritz (2012) for further details.

(iii) choose a higher level of equity capital K^*
(iv) choose a higher level of interbank market borrowing M^*
(v) choose a higher loan-to-deposit ratio ℓ^*

Proof. Performing comparative statics using the same techniques as above, we find

$$\begin{aligned}
L_\varphi^* &= |H|^{-1}\mu_M MC_\varphi^D MC_K^K < 0 \\
D_\varphi^* &= |H|^{-1} MC_\varphi^D \left[-MR_L^L MC_K^K + \mu_M(-MR_L^L + MC_K^K)\right] < 0 \\
K_\varphi^* &= -|H|^{-1} MC_\varphi^D \mu_M(-MR_L^L) > 0 \\
M_\varphi^* &= -|H|^{-1} MC_\varphi^D (-MR_L^L) MC_K^K > 0,
\end{aligned} \tag{9}$$

where the first three inequalities follow given that $\mu_M > 0$. For part (v), note that $\ell_\varphi^* = \frac{\partial}{\partial\varphi}(L^*/D^*) = (1/D^*)\left(L_\varphi^* - D_\varphi^*\ell^*\right)$, so if $D_\varphi^* < L_\varphi^* < 0$ then certainly $\ell_\varphi^* > 0$ (since $M^* > 0 \Leftrightarrow \ell^* > [1 + K^*/D^*] \equiv \widehat{\ell} \gtrsim 1$). Some rearranging shows that indeed

$$D_\varphi^* = L_\varphi^* + |H|^{-1}\left\{MC_\varphi^D(-MR_L^L)MC_K^K + \mu_M MC_\varphi^D(-MR_L^L)\right\} < L_\varphi^* < 0. \tag{10}$$

■

Armed with this result, we can address how the quality of its deposit base affects a bank's response to increased funding uncertainty at the margin.

Proposition 4 *Assume that $\mu_\sigma > 0$, $\mu_M > 0$, $\mu_{M\sigma} > 0$ and $\mu_{MM} \leq 0$, as well as MC_K^K, MR_L^L, and MC_D^D constant. In response to higher funding uncertainty, a bank with less favourable deposit access reduces lending by more, $\frac{\partial}{\partial\varphi}L_\sigma^* < 0$.*

Proof. From the proof of Proposition 1, we know that $L_\sigma = |H|^{-1}\mu_\sigma MC_D^D MC_K^K < 0$ since $\mu_\sigma > 0$. Inserting the expression for the determinant $|H|$ and rearranging gives

$$L_\sigma = -\frac{\mu_\sigma}{[-MR_L^L]}\frac{1}{\left[1 + \mu_M\left(\dfrac{1}{MC_K^K} + \dfrac{1}{[-MR_L^L]} + \dfrac{1}{MC_D^D}\right)\right]} < 0. \tag{11}$$

By Proposition 3(iv), $M_\varphi^* > 0$. Note that a higher M^* raises μ_σ (since $\mu_{M\sigma} > 0$) and lowers μ_M (weakly, since $\mu_{MM} \leq 0$). Together with the assumption that MC_K^K, MR_L^L, and MC_D^D are constant, it follows that $\frac{\partial}{\partial\varphi}L_\sigma^* < 0$ as claimed. ■

By Proposition 3, a bank with worse deposit access is more highly extended and has more interbank market exposure. Proposition 4, in turn, gives *sufficient* conditions under which such a bank cuts lending more strongly in response to a rise in funding uncertainty. In this sense, we find that a bank with better deposit access has a competitive advantage in credit markets.

The linearity assumptions on loan demand, as well as the supply of deposits and capital are thereby made for simplicity and only grossly sufficient for the result.

Loosely put, they mean that a bank's marginal incentive to reoptimize (L, D, K) is unaffected by their levels, and thus directly comparable across banks with different levels of access to deposits.

More important are the assumptions on the funding rate $\mu_\sigma > 0$, $\mu_M > 0$, $\mu_{M\sigma} > 0$ and $\mu_{MM} \leq 0$. The first two of these seem rather weak, and, in any case, are needed to derive the first-order result that $L^*_\sigma < 0$. The key economic assumption is on the cross-partial $\mu_{M\sigma} > 0$—which means that the funding rate is supermodular in (M, σ). The funding rate increases in uncertainty, and this effect is more pronounced at greater levels of interbank market exposure. As long as this bank's decision is not otherwise too different from another bank with a lower cost of capital, this it will thus cut back lending relatively more strongly. These conditions are satisfied for our two leading examples, for which $\mu(R, M, \sigma) = R + h(\sigma)M$, and so $\mu_\sigma = h_\sigma M > 0$, $\mu_M = h(\sigma) > 0$, $\mu_{M\sigma} = h_\sigma > 0$, and $\mu_{MM} = 0$.

These findings explain recent empirical results on bank behaviour during the financial crisis. From a cross-sectional perspective, suppose that banks are identical except for having different costs of raising deposits, as indexed by φ. Proposition 3 implies that $cov(M^*, \ell^*) > 0$. Proposition 4 then shows that under reasonable conditions, $cov(\Delta L^*, \ell^*) < 0$ and $cov(\Delta L^*, M^*) < 0$. The same conclusion also holds in relative terms since by Proposition 3(i) the banks that cut lending more strongly are also those with a smaller loan book to begin with, i.e., $cov(\Delta L^*/L^*, \ell^*) < 0$.

These predictions match the picture presented by Figure 1 above, where banking systems with higher loan-to-deposit ratios experienced sharper reductions in lending, as well as recent more rigorous empirical evidence. Using data on syndicated loans, Ivashina and Scharfstein (2010) document that US banks sharply decreased lending, especially around the height of the crisis in the 4^{th} quarter of 2008. But, importantly, they also find that banks that had higher better access to deposit finance cut their lending by less than other banks. Similarly, in the European context, Iyer et al. (2013) use Portugese bank loan data to show that banks which relied more heavily on interbank borrowing before the crisis, were those that cut their credit supply to firms more strongly during the crisis. As far as we know, our Proposition 4 is the first result in the literature that speaks directly to these empirical findings.

3.5 The role of the cost of capital

We now turn to impact of a bank's cost of capital on its response to funding uncertainty. Similar to before, suppose there exists a parameter τ which shifts the bank's cost of capital upwards. In particular, suppose that the cost of capital is $f_K(K, \tau)K$, so the marginal cost is $MC^K(K, \tau)$, and that $MC^K_\tau > 0$ everywhere.

Proposition 5 *With $\mu_M > 0$, a bank with a higher cost of capital will:*

(i) extend fewer loans L^* and increase interest rates on loans r_L^*
(ii) take more deposits D^* and increase interest rates on deposits r_D^*
(iii) reduce its level of equity capital K^*
(iv) increase its interbank market borrowing M^*
(v) decrease its loan-to-deposit ratio ℓ^*

Proof. Performing comparative statics using the same techniques as above, we find

$$\begin{aligned} L_\tau^* &= |H|^{-1}\,\mu_M MC_\tau^K MC_D^D < 0 \\ D_\tau^* &= |H|^{-1}\,\mu_M MC_\tau^K MR_L^L > 0 \\ K_\tau^* &= -|H|^{-1}\,MC_\tau^K\left[\mu_M\left(MR_L^L - MC_D^D\right) + MR_L^L MC_D^D\right] < 0 \\ M_\tau^* &= |H|^{-1}\,MC_\tau^K MR_L^L MC_D^D > 0, \end{aligned} \tag{12}$$

where the inequalities follow given that $\mu_M > 0$. ■

With convex borrowing costs $\mu_M > 0$, a higher cost of capital thus induces less reliance on equity capital and cutbacks in lending; the bank shrinks the size of its balance sheet and substitutes toward alternative sources of financing in form of deposit and interbank markets.[23] From a cross-sectional perspective, Proposition 5 is consistent with evidence from Hubbard, Kuttner and Palia (2002) that low-capital banks tend to charge higher interest rates on loans to their borrowers (especially when these are small firms) than well-capitalized banks.

Proposition 6 *Assume that $\mu_\sigma > 0$, $\mu_M > 0$, $\mu_{M\sigma} > 0$ and $\mu_{MM} \leq 0$, as well as MC_K^K, MR_L^L, and MC_D^D constant. In response to higher funding uncertainty, a bank with a higher cost of capital reduces lending by more, $\frac{\partial}{\partial\tau}L_\sigma^* < 0$.*

Proof. The argument from the proof of Proposition 4 applies. ■

Taken together, the results from this section are consistent with Cornett et al. (2011) who find evidence that banks which relied more heavily on deposit and equity capital financing maintained relatively higher lending volumes during the crisis.

3.6 "Loss leaders" and consumer welfare

In the baseline model, the bank is a monopolist in the market for loans and a monopsonist in deposits. With $\sigma = 0$, therefore, the equilibrium features too few loans and too few deposits, with monopoly profits in both markets.

[23] The condition $\mu_M > 0$ is needed only for parts (i), (ii), and (v) of the result; if $\mu_M = 0$, the higher cost of capital would only lead to less equity and more interbank market borrowing, without having a knock-on effect for loan and deposit markets. But if $\mu_M > 0$, then the induced increase in M increases the opportunity cost of loans and the implicit return to deposits, thus giving the other parts of the result. (Note also that Proposition 5 does not require $\mu_\sigma > 0$.)

Using Example 2, based on limited risk tolerance, we now argue that funding uncertainty has surprisingly strong implications for the relative level of bank profits and consumer welfare across different credit markets. By Proposition 1, a bank reacts asymmetrically to an increase in funding uncertainty—it cuts lending but expands its deposit base.

The idea is straightforward: Suppose that the market for loans is very attractive relative to the market for deposits (e.g., borrowers have a high willingness-to-pay in a boom). If funding uncertainty is "low", the bank will wish to have a high loan-to-deposit ratio and to borrow heavily in the interbank market. As funding uncertainty increases, the bank reduces its loan-to-deposit ratio, with zero interbank exposure $L^* = D^* + K$ in the limit as $\sigma \to \infty$. The point is that the level of deposits that satisfies this zero-exposure constraint may well be *much* higher than that associated with low levels of funding uncertainty.

This possibility is most easily illustrated by assuming a fixed level of equity capital K, together with linear loan demand and deposit supply functions, $f^L(L) = \alpha_L - L$ and $f^D(D) = \alpha_D + D$.[24] Letting $\psi_L \equiv (\alpha_L - R)$ and $\psi_D \equiv (R - \alpha_D)$, where $\alpha_L > R > \alpha_D$, note that the "first-best", competitive outcome in which both loans and deposits are priced at the bank's expected (risk-free) marginal cost of funding, involves $L^{fb} = \psi_L$ (for which $r_L = R$) and $D^{fb} = \psi_D$ (for which $r_D = R$). By contrast, the two first-order conditions for a risk-averse bank can be written as $(\psi_L - 2L^*) - (1/\rho)\sigma(L^* - D^* - K) = 0$ and $(\psi_D - 2D^*) + (1/\rho)\sigma(L^* - D^* - K) = 0$. These can be solved, in the limit as $\sigma \to \infty$, for

$$L^* = \tfrac{1}{2}\left[\tfrac{1}{2}(\psi_L + \psi_D) + K\right] \text{ and } D^* = \tfrac{1}{2}\left[\tfrac{1}{2}(\psi_L + \psi_D) - K\right] > 0. \tag{13}$$

So, if the market for loans is very attractive (in that ψ_L is high), this increases equilibrium loans, but also increases equilibrium *deposits* due to risk-based "loan-deposit synergies": Higher lending exposes the bank to greater funding risk, which induces it to increase deposits to mitigate that risk. For sufficiently large ψ_L, it is therefore possible that $r_D^* > R \Leftrightarrow D^* > D^{fb}$, so deposits become a "loss leader" for the bank in that the deposit rate exceeds it own wholesale funding cost. Conversely, equilibrium depositor welfare exceeds that of a competitive market.

Proposition 7 *In the presence of funding uncertainty, it is possible for a bank's deposit business to be loss-making (in expectation).*

This result shows that risk-based synergies between the two sides of a bank's balance sheet can lead to cross-subsidization even where a bank's loan and deposit

[24] We assume $K < \frac{1}{2}(\psi_L + \psi_D)$ for an interior solution.

businesses are entirely independent in terms of demand and supply conditions as well as operating costs.[25,26]

While it seems clear that competition for bank deposits has intensified since the beginning of the financial crisis, it can be difficult to tell in practice at what point deposits actually turn into a loss leader. Nonetheless, some recent developments are striking: In the UK, "banks are seeking to attract retail inflows by increasing deposit rates: retail bonds now pay around 200 basis points above the risk-free rate, compared to a sub-zero spread in 2005" (Bank of England, 2009, p. 38). Similarly, in the Eurozone, the difference between deposit rates and money market rates sometimes turned slightly positive between late 2008 until 2011 (ECB, 2012).

The broader point here is that heightened funding uncertainty and loan-deposit synergies can have surprisingly strong implications for consumer welfare.

4 Monetary policy

In this section, we use our model to explore the implications of increased funding uncertainty for monetary policy, along three dimensions.[27] First, we show that funding uncertainty tends to weaken the rate of pass-through from changes in the central bank's policy rate to credit markets. Second, we consider the optimal monetary policy response to an increase in funding uncertainty. This allows us to establish that the zero lower bound can become binding as a result of increased uncertainty, justifying the use of unconventional policies such as quantitative easing. Moreover, we show how policy is affected when there is a liquidity channel.

We initially analyze the policies of a central bank which can determine the policy rate R. While it can change the *effect* of increased funding uncertainty σ by changing R, it cannot influence funding uncertainty directly. This assumption reflects a reality in money markets: While central banks can react to changes in funding conditions, they cannot move quickly enough to eliminate uncertainty altogether, nor remove cross-sectional variation in spreads across banks. Our analysis of unconventional

[25] Note that the bank would not wish to shut down (or sell) its loss-making business as this would expose it to infinite funding uncertainty from a stand-alone operation based only on the other business.

[26] Similar arguments can also be applied to show that loans may become a loss leader for the bank (with positive profits from the deposits business), so $r_L^* < R$, in which case borrower welfare exceeds that of a competitive market. It is, of course, not possible for both sides of the bank to be loss-making at the same time. So either $r_L^* > r_D^* > R$ (deposits are loss-making, but loans are highly profitable) or $R > r_L^* > r_D^*$ (loans are loss-making, but deposit funds are very cheap), while there is always a positive intermediation margin, $(r_L^* - r_D^*) > 0$, in equilibrium.

[27] Of course, monetary policy should consider the entire financial sector, whereas our baseline model only features a single bank. In this section, we thus think of this bank as being representative for a larger banking system. The interaction between the mechanisms we study here and heterogeneity in the banking system remains an important topic for future research.

tools later on partially relaxes this assumption by considering a central bank which can directly inject liquidity into money markets.

4.1 Interest rate pass-through

Central banks around the world responded to the recent turmoil in financial markets by aggressively cutting interest rates to encourage bank lending and stimulate demand more generally. However, many policymakers and commentators expressed surprise at the apparently small impact that this loosening of monetary policy had on interest rates, especially across credit markets. For example, the minutes of the Federal Open Markets Committee (FOMC) noted that "some members were concerned that the effectiveness of cuts in the target federal funds rate may have been diminished by the financial dislocations, suggesting that further policy action might have limited efficacy in promoting a recovery in economic growth" (FOMC Minutes of the Meeting of 28–29 October 2008).

Let $\rho \equiv dr_L^*/dR$ denote *interest rate pass-through*, i.e., the response of the lending rate r_L^* to a change in the policy rate R. Pass through can also be written as $\rho = \frac{d}{dR}f^L(L^*) = f_L^L(L^*)\,(\partial L^*/\partial R)$. Repeating our analysis from Section 3 to determine $\partial L^*/\partial R$, we find that pass-through $\rho = f_L^L\left\{|H|^{-1}\mu_R MC_D^D MC_K^K\right\}$ and so:

$$\rho = \frac{(-f_L^L)}{[-MR_L^L]} \times \frac{\mu_R}{[1+\mu_M(1/MC_K^K + 1/MC_D^D - 1/MR_L^L)]}. \tag{14}$$

We are interested in the effect of funding uncertainty σ on interest pass-through ρ. Determining this generally is cumbersome because it involves the "second-order" comparative static $\partial^2 L^*/\partial R\partial\sigma$. Below, we provide general conditions on the sign and magnitude of this effect.

To guide intuition, we first discuss the effect under simplifying assumptions. In particular, assume that the demand and supply functions f_L, f_D, and f_K are linear, so f_L^L, MR_L^L, MC_D^D, and MC_K^K are constants. Furthermore, suppose that the bank's marginal funding cost is as in Examples 1 and 2, with $\mu(M,R,\sigma) = R + h(\sigma)M$. Under these conditions, $\mu_R = 1$, so the only term in ρ that depends on σ is μ_M, which is increasing in σ given our supermodularity condition $\mu_{M\sigma} > 0$. It follows by inspection that $\partial\rho/\partial\sigma < 0$, implying that pass-through of policy rate changes to bank lending rates is indeed weakened by funding uncertainty.

The simplifying assumptions "switched off" the indirect effect of the bank's balance sheet adjustments. In general, we need to place bounds on these indirect effects in order to characterize the impact of σ on $\rho(\sigma)$. For this purpose, define the curvatures of loan demand $\xi^L(L) \equiv -Lf_{LL}^L/f_{LL}^L$ and deposit and capital supply $\xi^D(D) \equiv Df_{DD}^D/f_D^D$ and $\xi^K(K) \equiv Kf_{KK}^K/f_K^K$. We obtain the following set of results:

Proposition 8 *Suppose there exists a level of funding uncertainty $\overline{\sigma} \leq \infty$ such that $\lim_{\sigma \to \overline{\sigma}} \mu_M = \infty$, and that $\mu_R > 0$ is bounded above.*
(a) Interest rate pass-through tends to zero as $\sigma \to \overline{\sigma}$:

$$\lim_{\sigma \to \overline{\sigma}} \rho = 0.$$

(b) For any $\sigma' < \overline{\sigma}$, there exists a $\sigma'' \in (\sigma', \overline{\sigma})$ such that pass-through becomes weaker when funding uncertainty increases from σ' to σ'':

$$\rho|_{\sigma=\sigma'} < \rho|_{\sigma=\sigma''}.$$

(c) Suppose that the expected marginal funding cost is of the form $\mu(M, R, \sigma) = \Psi(R, \sigma) + h(\sigma) M$, where Ψ and h are strictly positive functions satisfying $\Psi_R > 0$, $\Psi_\sigma \geq 0$, $\Psi_{R\sigma} \leq 0$ and $h_\sigma > 0$. If the curvature of loan demand satisfies $\xi^L(L) \leq 0$ and $\xi^L_L(L) = 0$, and the curvatures of deposit and capital supply satisfy $\xi^D(D) \leq 0$, $\xi^K(K) \leq 0$ and $\xi^D_D(D) \leq 0$, $\xi^K_K(K) \leq 0$, then pass-through decreases in funding uncertainty:

$$\frac{\partial \rho}{\partial \sigma} < 0.$$

Proof. (a) The numerator of ρ is bounded above by assumption, and the denominator becomes infinite as $\sigma \to \overline{\sigma}$. Hence, $\rho \to 0$ as required.
(b) Fix any $\sigma' < \overline{\sigma}$, and note that $\rho|_{\sigma=\sigma'} > 0$. The result is a corollary of part (a).
(c) The direct effect of an increase in σ is strictly negative since $\mu_{R\sigma} \leq 0$ and $\mu_{M\sigma} > 0$ by assumption. So it is sufficient to show that the indirect effect (through the choice variables L^*, D^*, K^* and M^*) is (weakly) negative. The first factor in ρ can be written as $(-f^L_L)/[-MR^L_L] = 1/\left[2 - \xi^L(L)\right]$, which is constant given the assumption $\xi^L_L(L) = 0$. The second factor in ρ does not depend on M^* since $\mu_{MM} = \mu_{RM} = 0$. So if we show that it is increasing in L^* and decreasing in D^*,K^*, the result follows (since $L^*_\sigma < 0$, $D^*_\sigma > 0$, $K^*_\sigma > 0$ by Proposition 1). In particular, we need to show that MR^L_L, MC^D_D and MC^K_K are all non-increasing functions. First, write $MR^L_L = f^L_L\left[2 - \xi^L(L)\right]$ where ξ^L is constant by assumption and $\xi^L \leq 0 \Leftrightarrow f^L_{LL} \leq 0$, so it follows that $MR^L_{LL} \leq 0$. Second, write $MC^D_D = f^D_D\left[2 + \xi^D(D)\right]$ where $\xi^D \leq 0 \Leftrightarrow f^D_{DD} \leq 0$, so $MC^D_{DD} = f^D_{DD}(2 + \xi^D) + f^D_D \xi^D_D \leq 0$ since $\xi^D_D \leq 0$ by assumption and $(2 + \xi^D) > 0$ by the bank's second-order condition. Third, the argument for $MC^K_{KK} \leq 0$ is identical. ■

Although not completely general, Proposition 8 suggests that interest rate pass-through will typically be dampened when uncertainty on funding conditions is high. Put differently, banks' pricing of loans becomes more rigid and less responsive to "shocks." In this sense, monetary policy becomes less effective at influencing a bank's decision-making process—with market interest rates on loans typically completely frozen in the limiting case where funding uncertainty becomes "large".

It is worth discussing some of the technical conditions in part (c) that guarantee $\partial\rho/\partial\sigma < 0$ in more detail. First, a key condition is again that $\mu_{M\sigma} > 0$, so the bank's expected funding rate is supermodular in the amount borrowed M and funding uncertainty σ. Second, funding costs in our Examples 1 and 2 satisfy $\Psi(R, \sigma) = R$, so that the cross-partial $\mu_{R\sigma} = \Psi_{R\sigma} = 0$. (The rate of pass-through is even lower if the initial pass-through from the central bank's policy rate to an individual bank's funding rate is itself also reduced by funding uncertainty, i.e., if $\Psi_{R\sigma} \leq 0$.) Third, the assumptions on the curvature of demand and supply functions are restrictive. As for loan demand, the requirement of constant curvature ($\xi_L^L = 0$) corresponds to a functional form $f^L(L) = \alpha_L - \beta_L L^{\gamma_L}$, and $\xi^L \leq 0$ corresponds to $\gamma_L \geq 1$, that is, a weakly concave demand curve. All these conditions are satisfied for Examples 1 and 2 as long as demand and supply curves are not too non-linear.

To sum up, Proposition 8 may help provide an explanation for the reduced impact that interest rate cuts by central banks in the 2007–9 financial crisis are often said to have had.[28] Clearly, it would be interesting and useful for any future econometric research on pass-through to empirically test this prediction more formally.[29]

4.2 Optimal monetary policy response

A natural conclusion from our analysis is that a central bank seeking to stabilize macroeconomic variables would wish to respond to increased funding uncertainty; it has significant effects on bank lending to the real economy which can, in turn, affect output and inflation, the typical target variables of monetary policy.

A full model of the interactions between bank lending, output and inflation is beyond the scope of this paper; instead, we consider a simplified model where the objective of the central bank is to achieve a certain target level of bank lending, denoted $\widehat{L}$. Our approach can be thought of as a reduced-form version of the textbook macroeconomic model in which the central bank faces an output-inflation trade-off (a short-run Phillips curve), and picks the optimal point along the curve by minimizing a loss function. Hence, optimal monetary implies target levels of output and

[28] It also suggests that any empirical evidence for banks adjusting interest rates by less than they otherwise would have may reflect a rational response to heightened funding uncertainty—rather than being indicative of collusive behaviour, for example.

[29] There is some econometric evidence for reduced pass-through during the recent crisis. Aristei and Gallo (2012) use a regime-switching model to show that pass-through dropped significantly in periods of financial distress in the Euro area. Harbo Hansen and Welz (2011) show decreased pass-through from monetary policy to retail rates in Sweden after 2007, especially for retail loans with long maturities. While these studies are consistent with our results, to our knowledge, no empirical study has directly explored the link between pass-through and funding uncertainty. See Leuvensteijn et al. (2008) for previous evidence on pass-through in the Eurozone, with an emphasis on the role of competition.

inflation at any point in time. If we assume that monetary policy mainly affects output through the bank lending channel, and that output is a strictly increasing function of bank lending, then optimal policy is well-approximated by the bank lending target we stipulate.

We have a simple characterization of optimal policy:

Proposition 9 *Suppose that $\mu_\sigma > 0$, $\mu_R > 0$, and that the central bank is achieving its policy objective, $L^* = \widehat{L}$. Then the optimal monetary policy response to a marginal increase in funding uncertainty satisfies*

$$\frac{dR}{d\sigma} = -\frac{\mu_\sigma (M^*, R, \sigma)}{\mu_R (M^*, R, \sigma)} < 0.$$

Proof. Suppose funding uncertainty increases by $d\sigma$. It is easy to show that to keep lending constant, the central bank must ensure that all of the bank's choices, including M^*, remain constant. Hence, from the first-order conditions, the optimal change in the interest rate dR must satisfies $\mu(M^*, R, \sigma) = \mu(M^*, R + dR, \sigma + d\sigma)$. For small changes, $\mu(M^*, R, \sigma) - \mu(M^*, R + dR, \sigma + d\sigma) = \mu_\sigma d\sigma + \mu_R dR$. Setting this to zero yields the required expression. ■

Not surprisingly, interest rates should be cut in response to increased funding uncertainty to stimulate lending. Moreover, there is a simple rule for the desired interest rate reduction: It is proportional to the ratio of the impact of uncertainty on marginal funding costs ($\mu_\sigma d\sigma$) to the impact of policy rate changes on funding costs (μ_R). This analysis is illustrated in Figure 2. The bank's "iso-lending curves" are plotted in policy-rate/funding-uncertainty space. Lending is high in the south-west of the figure, where interest rates and uncertainty are low. The solid curve is the iso-lending curve corresponding to the central bank's target lending level $\widehat{L}$. Its (negative) slope is equal to the optimal interest rate increase required to hold lending constant when σ increases, $dR/d\sigma$.

In our two examples, this slope $dR/d\sigma$ becomes steeper as σ increases, so iso-lending curves are concave. As a result, the optimal interest rate cut is increasing in current funding uncertainty, e.g., higher at point B than at point A. More generally, moving beyond Examples 1 and 2, iso-lending curves will be concave as long as low interest rates and low uncertainty are *complements* in stimulating bank lending, so that banks "prefer averages to extremes", e.g., moderate interest rates and mid-range funding uncertainty stimulate more lending than low R and high σ.

4.3 The zero lower bound and unconventional policy

We have established that when funding uncertainty increases, the central bank optimally cuts interest rates to achieve its target level of bank lending. We now consider

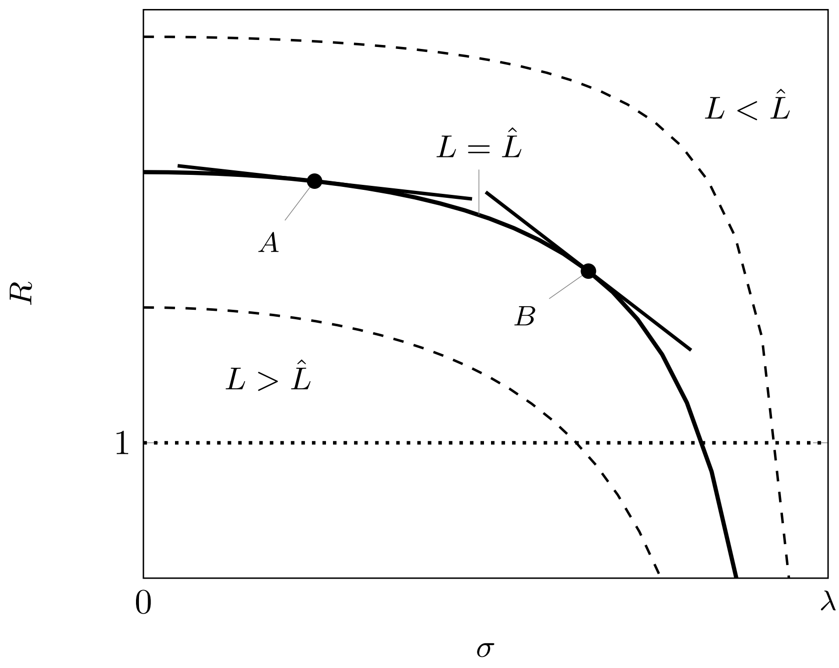

Figure 2: Iso-lending curves in policy-rate/funding-uncertainty space

the possibility that such a cut is not possible due to a "zero lower bound" on policy rates. As R is defined as a *gross* interest rate, the constraint is $R - 1 \geq 0$.

Definition *The zero lower bound on the policy rate is binding if* $L^*|_{R=1} < \widehat{L}$.

Our analysis above implies that even if the central bank is currently achieving its lending target, the zero lower bound can become binding when funding uncertainty increases. This happens precisely when the rate cut prescribed by the logic of Proposition 9 is larger than the current net interest rate $R - 1$:

Lemma 1 *Suppose that* $\sigma = \sigma_0$*, and the central bank achieves its lending target with policy rate* $R_0 > 1$*, i.e.,* $L^*|_{R=R_0} = \widehat{L}$*. If* $\mu_\sigma > 0$*,* $\mu_R > 0$ *and* R_0 *is sufficiently close to* 1*, there exists a* $\sigma_1 > \sigma_0$ *such that the zero lower bound becomes binding when funding uncertainty increases from* σ_0 *to* σ_1*.*

Proof. Let R_1 denote the unconstrained optimal policy rate under σ_1, and write $R_0 = (1 + \delta)$ for some $\delta > 0$. Using Proposition 9, and integrating, we obtain

$$R_1 = R_0 - \int_{\sigma_0}^{\sigma_1} \left(\frac{\mu_\sigma}{\mu_R} \right) d\sigma. \tag{15}$$

Since the integral is strictly positive, we have that $R_1 < 1$ for small enough δ, which implies a binding zero lower bound as required. ■

We now turn to Example 1 to show how unconventional policies—such as large-scale asset purchases and quantitative easing—may allow the central bank to alleviate the zero lower bound problem, at least to some extent. One could argue that such policies allow the central bank to control liquidity in the interbank market independently of its policy rate. By doing so, they can stimulate bank lending without further interest rate cuts. Recall that in Example 1, expected aggregate liquidity λ played a key role in marginal funding costs. To model the effect of unconventional policy, suppose that there is a parameter $\phi \geq 0$ which is controlled directly by the central bank through unconventional policies, and that aggregate liquidity satisfies

$$\lambda = \lambda_0 + \phi. \tag{16}$$

It follows that unconventional policy may become useful after an increase in funding uncertainty:

Proposition 10 *In Example 1, suppose that funding uncertainty increases from σ_0 to σ_1 as in Lemma 1, and that $\phi = 0$ before the increase. Then there exists a unique optimal liquidity injection $\widehat{\phi} > 0$ such that the central bank achieves its lending target after the increase by setting $R = 1$ and $\phi = \widehat{\phi}$.*

Proof. The central bank would like to keep money market borrowing constant at its initial level M^*. Making the dependence of funding costs on liquidity explicit by writing $\mu(R, M, \sigma; \lambda)$, an optimal injection $\phi = \widehat{\phi}$ must satisfy

$$\mu(1, M^*, \sigma_1; \lambda_0 + \phi) = \mu(R_0, M^*, \sigma_0; \lambda_0) \tag{17}$$

Since the zero lower bound binds after the increase, the left-hand side is greater than the right-hand side for $\phi = 0$. Moreover, as $\phi \to \infty$, the left-hand side converges to 1, whereas the right-hand side is bounded below by $R_0 > 1$. Hence, there is a unique $\widehat{\phi}$ satisfying the equation. ■

Because liquidity matters for the real effects of funding uncertainty, unconventional monetary policies which raise liquidity can have an impact that is similar to interest rate cuts. This is because both interest rate cuts and liquidity injections lower the expected marginal cost of funding. Unconventional policies can substitute for interest rate cuts when these are no longer feasible.

Of course, we have simplified things here by assuming that there is no further cost associated with liquidity injections, such as additional inflationary pressures or the danger of creating asset price bubbles. Modelling the impact of unconventional policies in a full framework that takes these costs into account is beyond our scope but may offer a fruitful area for further research.

4.4 The liquidity channel of monetary policy

So far we have assumed that the central bank chooses R without having any further impact on money markets. This is perhaps unrealistic. In practice, monetary policy is conducted through operations affect liquidity in money markets. In the UK and the Eurozone, the central bank lends to commercial banks at the policy rate. A higher policy rate implies less demand for loans from the central bank, which reduces market liquidity. In the US, the Fed uses bond sales and purchases to influence the interest rate in the federal reserve market. When the policy rate is lowered, the Fed sells government bonds. This decreases the aggregate supply of reserves, again implying less liquidity.

Formally, consider Example 1 with the extension that average liquidity is given by $\lambda = \Lambda(R)$, where $\Lambda'(R) < 0$.[30] How does this affect our earlier results? We now have that funding costs satisfy

$$\mu(M, R, \sigma) = R + \frac{\Lambda(R)}{\Lambda(R)^2 - \sigma^2} M^* \Longrightarrow \mu_R = 1 - \Lambda'(R) \frac{\Lambda(R)^2 + \sigma^2}{\left[\Lambda(R)^2 - \sigma^2\right]^2} M^* > 1.$$

Since μ_R is a key factor in the optimal policy response (see Proposition 9), the last equation implies that the response has to be adjusted when monetary policy has liquidity effects.

To analyze this formally, we compare two scenarios. In the first scenario, liquidity is policy-sensitive ($\Lambda'(R) < 0$), and the central bank has set the interest rate to $R = R_0$ and thereby managed to achieve its lending target $L^* = \widehat{L}$. In the second scenario, liquidity λ is held constant at the level arising from optimal policy in the first scenario, i.e. $\lambda = \Lambda(R_0)$. When there is a marginal increase in funding uncertainty σ, we know that the required policy response is $dR = -(\mu_\sigma / \mu_R)\, d\sigma$. Since μ_σ is the same in both scenarios, and we have shown that μ_R is greater when $\Lambda'(R) < 0$, it follows that:

Proposition 11 *In Example 1, the optimal policy monetary response to a marginal increase in funding uncertainty is smaller when in the presence of a liquidity channel ($\Lambda'(R) < 0$) than when λ is constant.*

Intuitively, the impact of monetary policy on funding costs is strengthened through the liquidity channel, and this reduces the interest rate cut required to achieve the policy target.

Furthermore, note that μ_R now depends positively on current money market exposure M^*. Ceteris paribus, an increase in M^* has a smaller effect on the optimal

[30] Since we are interested in the liquidity channel of interest rate policy, we ignore unconventional policy without loss of generality. In order to obtain sensible results, we continue to assume that $\sigma < \Lambda(R)$ for all R.

policy when liquidity is policy-sensitive. This reduced sensitivity to banks' money market exposure is due to two opposing effects in the presence of the liquidity channel. On one hand, banks that are heavily exposed to money markets respond more to increased funding uncertainty, suggesting, as before, that policy should respond more strongly. On the other hand, heavily exposed banks also respond more to the changes in liquidity that a policy change creates. The second effect implies that policy is more effective with heavily exposed banks and has to adjust less strongly.

5 Conclusion

Uncertainty over funding conditions in money markets has important consequences for bank behaviour. Focusing on banking systems with loan-to-deposit ratios above 100%, we have studied a setup with convex wholesale borrowing costs—which arise, e.g., due to counterparty risk and limited aggregate liquidity or finite risk tolerance of a bank's managers. By contrast, a large existing literature using related banking models pays scant attention to funding uncertainty—and thus has little to say about one of the key characteristics of the recent financial crisis.

We have shown that two key conditions are needed to explain a diverse set of stylized facts on bank behaviour during the crisis: (i) funding uncertainty raises a bank's (risk-adjusted) expected wholesale funding cost at the margin ($\mu_\sigma > 0$), and (ii) its wholesale funding rate is supermodular in the funding uncertainty and the amount borrowed ($\mu_{M\sigma} > 0$).

Increased funding uncertainty then leads to contractions in banks' lending volumes, balance sheets, and profitability; by contrast, savers benefit from increased competition in deposit markets—with the possibility of deposits turning into a loss leader. Our results can explain how banks with a stronger deposit base have done better since the crisis, and why other banks such as RBS set themselves the goal of reducing their loan-to-deposit ratios back towards 100%.

Our results on monetary policy show that increased funding uncertainty typically dampens the rate of interest pass-through from the central bank's policy rates to market interest rates. We have derived some guidance for monetary policy following such an increase: Interest rates should be cut according to the banking system's exposure to money markets, and according to current levels of uncertainty. Moreover, reduced pass-through opens up the possibility that funding uncertainty leads to a binding zero lower bound on policy rates, and yields a rationale for unconventional policies such as quantitative easing and other liquidity-enhancing measures.

Of course, thinking about bank behaviour and its implications for the economy is a complex task, especially in the context of a financial crisis, and we have abstracted from some important issues. For example, we have not incorporated general-equilibrium effects across credit markets, and, relatedly, not attempted a full-fledged welfare analysis. Future work might also examine the role of heterogeneity in the banking sector, notably in terms of implications for monetary policy. More research into why funding conditions became so volatile in the first place is clearly also still needed.

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Appendix: Robustness analysis

Extension A: Equity capital requirement

Our benchmark model allows the bank to optimally adjust its level of equity according to market conditions, including the degree of funding uncertainty. An alternative assumption would be that the bank's capital is (exogenously) fixed, perhaps due a binding regulatory capital constraint or because the bank is unable to raise additional equity from investors in the short run.

With fixed equity capital, the bank's program boils down to two first-order conditions for loans and deposits. Just as a above, higher funding uncertainty reduces the attractiveness of wholesale borrowing because $\mu_\sigma > 0$. This induces the bank to substitute away to other funding sources, in this case (only) customer deposits, and to cut back on its loan commitments. As a result, its money market exposure and loan-to-deposit ratio decline, all just as in Proposition 1. Similar reasoning shows that its expected profits decline, as in Proposition 2.

Proposition 1A/2A *Suppose a bank's equity capital is held fixed. Parts (i), (ii), (iv) and (v) of Proposition 1 and Proposition 2 apply.*

Proof. The bank's first-order conditions are reduced to

$$\begin{aligned} U_L &= MR^L - \mu = 0 \\ U_D &= -MC^D + \mu = 0 \end{aligned} \tag{18}$$

Totally differentiating yields the following system of equations:

$$\begin{pmatrix} U_{LL} & U_{LD} \\ U_{DL} & U_{DD} \end{pmatrix} \times \begin{pmatrix} L_\sigma^* \\ D_\sigma^* \end{pmatrix} = -\begin{pmatrix} U_{L\sigma} \\ U_{D\sigma} \end{pmatrix} \tag{19}$$

The left-hand matrix is the Hessian of the objective function U, as defined in Section 2, which we call H, and it satisfies $|H| > 0$. Cramer's rule yields

$$\begin{aligned} L_\sigma^* &= -|H|^{-1} \mu_\sigma MC_D^D < 0 \\ D_\sigma^* &= -|H|^{-1} \mu_\sigma MR_L^L > 0 \end{aligned} \tag{20}$$

which proves part (i) of Proposition 1. Parts (ii), (iv), and (v) follow as in the proof of Proposition 1. By the envelope theorem, the bank's expected equilibrium payoff changes by

$$\begin{aligned} \frac{\partial}{\partial\sigma} U(L^*, D^*) &= -\frac{\partial}{\partial\sigma} E[C(M^*, R, \varepsilon)] \\ &= -\textstyle\int_0^{M^*} \mu_\sigma(M', R, \sigma) dM' < 0, \end{aligned} \tag{21}$$

thus confirming Proposition 2. ■

Extension B: Exposure to multiple risks

To focus sharply on funding uncertainty, the benchmark model assumes that this is the only risk the bank faces. In practice, of course, a bank faces credit risks in its loan portfolio and other uncertainties. Modeling these can make the analysis much more complicated; nonetheless, under fairly mild conditions, we can show that the key insights are preserved in settings with multiple risks.

Begin by observing that, due to the additive separability of the bank's objective function, additional risks in loan, deposit and capital markets have no effect on our results as long as they are not correlated with risk in the money market. We could simply re-define the revenue and cost functions from above as the expectation of revenue and cost in the respective market, and the analysis remains unchanged.

More interesting cases arise when risks in other markets are correlated with the money market. To explore this, without further loss of generality, we allow the marginal revenue and marginal cost functions to depend on our money market risk parameter σ. We here focus on additional sources of uncertainty in the loans market and in the market for equity investment.

Let $f^L(L, \sigma)$ and $f^K(K, \sigma)$ denote the inverse demand and supply functions for loans and capital, and let $MR^L(L, \sigma) = \frac{\partial f^L(L,\sigma)L}{\partial L}$ and $MC^K(K, \sigma) = \frac{\partial f^K(K,\sigma)K}{\partial K}$.

Loans market risk. Suppose first that higher wholesale funding uncertainty also hurts the bank's loans business in that it reduces (expected) marginal revenue, $MR_{\sigma}^{L} < 0$. This could be because factors, such as heightened perceptions of counterparty risk, that raise wholesale funding uncertainty also raise the credit risks associated with commercial banks' customer lending. It is intuitive that this additional credit risk will induce the bank to (further) cut lending, such that our conclusion that $L_{\sigma}^{*} < 0$ from Proposition 1 certainly obtains.

Things are somewhat more complex for the impact on deposits and equity capital. The reason is that the credit-risk-induced decline in lending, all else equal, reduces the bank's needs for financing from these sources; in other words, there is a countervailing effect to the findings from Proposition 1 that $D_{\sigma}^{*} > 0$ and $K_{\sigma}^{*} > 0$. We show that these results continue to hold when the impact of funding uncertainty on the loans business ($MR_{\sigma}^{;L}$) is small relative to its impact on the money market (μ_{σ}).

Proposition 1B/2B (loan market risks) *Suppose that funding uncertainty is correlated with risks in the loans market, with $MR_{\sigma}^{L} < 0$. Part (i) of Proposition 1 as well as Proposition 2 apply; parts (i), (iii), (iv) and (v) of Proposition 1 apply whenever $\mu_{\sigma}/\left|MR_{\sigma}^{L}\right| > \mu_{M}/\left|MR_{L}^{L}\right|$.*

Proof. The system of equations we need to solve is the same as in the proof of Proposition 1. The only change is that now, $U_{L\sigma} = MR_{\sigma}^{L} - \mu_{\sigma}$. Solving, we obtain

$$
\begin{aligned}
L_{\sigma}^{*} &= |H|^{-1}\left\{\mu_{\sigma} MC_{D}^{D} MC_{K}^{K} - MR_{\sigma}^{L}\left[\mu_{M}(MC_{D}^{D} + MC_{K}^{K}) + MC_{D}^{D} MC_{K}^{K}\right]\right\} < 0 \\
D_{\sigma}^{*} &= -|H|^{-1} MC_{K}^{K}\left[\mu_{M} MR_{\sigma}^{L} - \mu_{\sigma} MR_{L}^{L}\right] \\
K_{\sigma}^{*} &= -|H|^{-1} MC_{D}^{D}\left[\mu_{M} MR_{\sigma}^{L} - \mu_{\sigma} MR_{L}^{L}\right], \qquad (22)
\end{aligned}
$$

where $D_{\sigma}^{*}, K_{\sigma}^{*} > 0$ if and only if $\mu_{\sigma}/\left|MR_{\sigma}^{L}\right| > \mu_{M}/\left|MR_{L}^{L}\right|$. The remaining comparative statics follow as before. By the envelope theorem,

$$
\begin{aligned}
\frac{\partial}{\partial\sigma} U(L^{*}, D^{*}, K^{*}) &= \frac{\partial}{\partial\sigma}\left\{f^{L}(L^{*}, \sigma)L^{*} - E[C(M^{*}, R, \varepsilon)]\right\} \\
&= \int_{0}^{L^{*}} MR_{\sigma}^{L}(L', \sigma)\, dL' - \int_{0}^{M^{*}} \mu_{\sigma}(M', R, \sigma) dM' < 0. \qquad (23)
\end{aligned}
$$

■

Equity capital risk. Suppose instead that higher funding risk also leads to a tightening in the market for equity capital such that $MC_{\sigma}^{K} > 0$. Loosely put, this means that the incremental cost of raising an additional dollar of capital also rises, thus making equity less attractive. It is intuitive, based on our earlier discussion, that such a rise in the cost of capital makes lending less attractive, and conversely, means that the bank considers customer deposits relatively more favourable as an alternative financing source. So our earlier findings from Proposition 1 that $L_\sigma^* < 0$ and $D_\sigma^* > 0$ continue to hold, with corresponding increases in loan and deposit rates. Also intuitively, the result that the bank responds by raising more capital becomes less clear-cut due to the cost-of-capital effect. We will show that it continues to hold when the impact of σ on the capital market (MC_σ^K) is small relative to its impact on the money market (μ_σ).

Proposition 1B/2B (equity capital risk) *Suppose that funding risks are correlated with risks in the capital market, with $MC_\sigma^K > 0$. Parts (i), (ii) and (v) of Proposition 1 as well as Proposition 2 apply; parts (iii) and (iv) of Proposition 1 apply if and only if*

$$\frac{\mu_\sigma}{MC_\sigma^K} > 1 + \mu_M \left(\frac{1}{MC_D^D} + \frac{1}{|MR_L^L|} \right).$$

Proof. The system of equations we need to solve is the same as in the proof of Proposition 1. The only change is that now, $U_{K\sigma} = -MC_\sigma^L + \mu_\sigma$. Solving, we obtain

$$\begin{aligned}
L_\sigma^* &= |H|^{-1} \left\{ \mu_\sigma MC_D^D MC_K^K + \mu_M MC_\sigma^K MC_D^D \right\} > 0 \\
D_\sigma^* &= |H|^{-1} MR_L^L \left\{ \mu_\sigma MC_K^K + \mu_M MC_\sigma^K \right\} > 0 \\
K_\sigma^* &= -|H|^{-1} MC_D^D MR_L^L \left\{ -\mu_\sigma + MC_\sigma^K \left[\mu_M (1/MC_D^D - 1/MR_L^L) + 1 \right] \right\}, \quad (24)
\end{aligned}$$

where $K_\sigma^* > 0$ holds under the proposed condition. The remaining comparative statics follow as before. By the envelope theorem,

$$\begin{aligned}
\frac{\partial}{\partial \sigma} U(L^*, D^*, K^*) &= \frac{\partial}{\partial \sigma} \left\{ -f^K(K^*, \sigma) K^* - E[C(M^*, R, \varepsilon)] \right\} \\
&= -\textstyle\int_0^{K^*} MC_\sigma^K (K', \sigma)\, dK' - \int_0^{M^*} \mu_\sigma(M', R, \sigma) dM' < 0. \quad (25)
\end{aligned}$$

■

Extension C: Competition between banks

The setup underlying the benchmark model is also easily extended to Nash-Cournot competition between $n \geq 2$ banks, which might be offering differentiated savings and loan products.

Suppose that the inverse demand curve for loans from bank $j \in N$ is given by $r_L^j = g^L \left(L^j + \delta_L \sum_{k \neq j} L^k \right)$, where $g_L^L(\cdot) < 0$ similar to above and $\delta_L \in [0, 1]$ is a measure of (symmetric) horizontal product differentiation between the loans associated with different banks. Similarly, deposit supply for bank j is given by

$r_D^j = g^D\left(D^j + \delta_D \sum_{k \neq j} D^k\right)$, where $g_D^D(\cdot) > 0$ and $\delta_D \in [0,1]$. This setup effectively nests all market structures ranging from perfect competition (with $\delta_L = \delta_D = 1$ and $n \to \infty$) to monopoly (with $\delta_L = \delta_D = 0$ or $n = 1$). As before, bank j also faces an inverse supply curve for equity $r_K^j = g^K(K^j)$.

Using its balance sheet constraint, $M^j = L^j - D^j - K^j$, an individual bank's new maximization problem is to $\max_{L^j, D^j, K^j} U^j = g^L(\cdot)L^j - g^D(\cdot)D^j - g^K(\cdot)K^j - E[C^j(L^j - D^j - K^j, R, \varepsilon)]$. We redefine the marginal revenue and marginal cost functions, which are identical across banks, as $MR^L = \frac{\partial}{\partial L^j}\left[g^L(\cdot)L^j\right]$, $MC^D = \frac{\partial}{\partial D^j}\left[g^D(\cdot)D^j\right]$, $MC^K = \frac{\partial}{\partial K^j}\left[g^K(\cdot)K^j\right]$, and its expected wholesale funding rate $\mu(M^j, R, \sigma) = \frac{\partial}{\partial M^j}E[C(M^j, R, \varepsilon)]$.

The first-order conditions for bank j are

$$\begin{aligned} U_L^j &= MR^L\left(L^j + \delta_L \textstyle\sum_{k \neq j} L^k\right) - \mu^j(M^j, R, \sigma) = 0 \\ U_D^j &= -MC^D\left(D^j + \delta_D \textstyle\sum_{k \neq j} D^k\right) + \mu^j(M^j, R, \sigma) = 0 \\ U_K^j &= -MC^K(K^j) + \mu^j(M^j, R, \sigma) = 0. \end{aligned} \tag{26}$$

We assume that $g^L(\cdot)$, $g^D(\cdot)$ and $g^K(\cdot)$ are such that the problem is well-behaved with an interior solution. In symmetric Nash equilibrium with $L^j = L^*$, $D^j = D^*$, $K^j = K^*$, $M^j = M^*$ we thus have

$$\begin{aligned} MR^L\left([1 + \delta_L(n-1)]L^*\right) - \mu(M^*, R, \sigma) &= 0 \\ -MC^D\left([1 + \delta_D(n-1)]D^*\right) + \mu(M^*, R, \sigma) &= 0 \\ -MC^K\left(K^*\right) + \mu(M^*, R, \sigma) &= 0. \end{aligned} \tag{27}$$

These three sector-level equilibrium conditions correspond to those for the monopoly bank from the benchmark model (which are nested with $\delta_L = \delta_D = 0$ or $n = 1$), and take a very similar shape to those presented in Section 3. Therefore, as long as $\mu_\sigma > 0$, the results of Proposition 1 obtain. When funding uncertainty increases, each individual bank in a sector-wide equilibrium substitutes deposits and capital for costly money market funding, and reduces the size of its balance sheet ($L_\sigma^* < 0$) due to the increased overall marginal cost of funds.

A minor complication arises when considering bank profitability. As before, an individual bank's profitability is reduced when σ increases due to the direct cost of funds effect. However, there are two additional effects arising from competitive behaviour. Firstly, there is fiercer competition for deposits, as competitors substitute away from money markets. This reduces profitability by increasing equilibrium deposit rates. Secondly, there is less competition for loans as competitors reduce lending activity. This *raises* equilibrium lending rates and profitability.

In order to establish Proposition 2, we therefore need to show that the last effect is dominated by the other two, leading to reduced profitability overall. *Grossly* sufficient is that the competitiveness of the loan market, i.e., the differentiation parameter δ_L, or the curvature of loan demand $\xi^L = -Lg^L_{LL}/g^L_L$ are sufficiently small compared to their counterparts in the deposit market (δ_D and $\xi^D = Dg^D_{DD}/g^D_D$). A simple set of sufficient conditions is $\delta_L \leq \delta_D$ and $\xi^L \leq \xi^D$.

Proposition 1C/2C *Suppose there are $n \geq 2$ banks engaged in Nash-Cournot competition with differentiated savings and loan products. Proposition 1 applies, and Proposition 2 certainly applies if*

$$\frac{1/\delta_D + (n-1)}{1/\delta_L + (n-1)} \leq \frac{\left(2-\xi^L\right)}{\left(2+\xi^D\right)}.$$

Proof. Define the following functions:

$$\begin{aligned}
MR^L\left([1+\delta_L(n-1)]L^*\right) - \mu(L^*-D^*-K^*,R,\sigma) &= \widetilde{W}_L \\
-MC^D\left([1+\delta_D(n-1)]D^*\right) + \mu(L^*-D^*-K^*,R,\sigma) &= \widetilde{W}_D \\
-MC^K\left(K^*\right) + \mu(L^*-D^*-K^*,R,\sigma) &= \widetilde{W}_K. \qquad (28)
\end{aligned}$$

We have $\widetilde{W}_L = \widetilde{W}_D = \widetilde{W}_K = 0$ by the symmetric equilibrium condition. Totally differentiating this system yields

$$\begin{pmatrix} \widetilde{W}_{LL} & \widetilde{W}_{LD} & \widetilde{W}_{LK} \\ \widetilde{W}_{DL} & \widetilde{W}_{DD} & \widetilde{W}_{DK} \\ \widetilde{W}_{KL} & \widetilde{W}_{KD} & \widetilde{W}_{KK} \end{pmatrix} \times \begin{pmatrix} L^*_\sigma \\ D^*_\sigma \\ K^*_\sigma \end{pmatrix} = -\begin{pmatrix} \widetilde{W}_{L\sigma} \\ \widetilde{W}_{D\sigma} \\ \widetilde{W}_{K\sigma} \end{pmatrix}. \qquad (29)$$

Let $\widetilde{H}$ denote the left-hand side matrix, which has a strictly negative determinant since the demand and supply functions are well-behaved. Furthermore, define the *adjusted* marginal revenue and cost functions

$$\begin{aligned}
\widetilde{MR}^L_L &= [1+\delta_L(n-1)]MR^L_L \\
\widetilde{MC}^D_D &= [1+\delta_D(n-1)]MC^D_D. \qquad (30)
\end{aligned}$$

Cramer's rule yields

$$\begin{aligned}
L^*_\sigma &= \left|\widetilde{H}\right|^{-1}\mu_\sigma \widetilde{MC}^D_D MC^K_K < 0 \\
D^*_\sigma &= \left|\widetilde{H}\right|^{-1}\mu_\sigma \widetilde{MR}^L_L MC^K_K > 0 \qquad (31) \\
K^*_\sigma &= \left|\widetilde{H}\right|^{-1}\mu_\sigma \widetilde{MR}^L_L \widetilde{MC}^D_D > 0.
\end{aligned}$$

The comparative statics of Proposition 1 follow as before. Turning to Proposition 2, by the envelope theorem and symmetry, we have

$$\begin{aligned}
& \frac{\partial}{\partial\sigma} U^j(L^j, D^j, K^j; L^{-j}, D^{-j}) \\
= & -\frac{\partial}{\partial\sigma} E[C(M^j, R, \varepsilon)] + \delta_L g_L'(n-1)L_\sigma^* - \delta_D g_D'(n-1)D_\sigma^* \\
< & -\delta_L g_L'(n-1)L_\sigma^* - \delta_D g_D'(n-1)D_\sigma^* \\
= & (n-1)\left(-\left|\widetilde{H}\right|^{-1}\right)\mu_\sigma\left[-MC_K^K \widetilde{MC}_D^D \widetilde{MR}_L^L\right]\left\{\delta_L \frac{g_L'}{\widetilde{MR}_L^L} - \delta_D \frac{g_D'}{\widetilde{MC}_D^D}\right\} \\
\overset{\text{sign}}{=} & \frac{\delta_L}{1+\delta_L(n-1)} \frac{1}{\left(2-\xi^L\right)} - \frac{\delta_D}{1+\delta_D(n-1)} \frac{1}{\left(2+\xi^D\right)}. \qquad (32)
\end{aligned}$$

The first inequality follows since $\frac{\partial}{\partial\sigma}E[C(\cdot)] > 0$ as before, and the second equality uses the above expressions for L_σ^* and D_σ^*. Hence the proposed condition is sufficient for $\frac{\partial}{\partial\sigma}U^j(\cdot) < 0$. ■

These arguments make clear that our results are quite robust to changes in market structure and the degree of product differentiation; they are rather fundamentally driven by substitution effects away from wholesale funding. Indeed, this analysis could easily be generalized further to allow for (i) asymmetries between the competing banks, for example in terms of operating costs, (ii) for vertical product differentiation with $r_L^j = g_L^L(\cdot) + \eta_L^j$, where $\left|\eta_L^j\right|$ not too large is a bank-specific interest-rate differential which allows for heterogeneity in loan pricing across banks, or (iii) for models where banks set interest rates in Bertrand-Nash competition, rather than choosing quantities as in our benchmark setup.