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Exploiting the exogenous and regional nature of the Great East Japan Earthquake of 2011, this paper provides a quantification of the role of input-output linkages as a mechanism for the propagation and amplification of shocks. We document that the disruption caused by the disaster propagated upstream and downstream supply chains, affecting the direct and indirect suppliers and customers of disaster-stricken firms. Using a general equilibrium model of production networks, we then obtain an estimate for the overall macroeconomic impact of the disaster by taking these propagation effects into account. We find that the earthquake and its aftermaths resulted in a 0.47 percentage point decline in Japan's real GDP growth in the year following the disaster.

Supply Chain Disruptions: Evidence from the Great East Japan Earthquake*

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Abstract

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1 Introduction

The production of goods and services in any modern economy is organized around complex, interlocking supply chains, as firms rely on a variety of different inputs for production. The sheer scale of transactions along supply chains attests to their vital role in the economy. For instance, in 2018 alone, firms in the United States spent over 16 trillion dollars on various intermediate goods and services, an amount of the same order of magnitude of the annual U.S. GDP.

Due to the key role of intermediate goods in the production process, disruptions to the orderly flow of goods and services have been increasingly recognized by policymakers as a source of aggregate risk. Overlapping policy initiatives at the international ([World Economic Forum, 2012](#)), regional ([European Commission, 2013](#)), and national levels rely on the premise that firm-level or regional shocks—such as natural disasters, terrorism, or cyber attacks—can propagate through input-output linkages to a wide array of firms and industries, with potentially adverse macroeconomic consequences. For example, the U.S. National Strategy for Global Supply Chain Security is based on the premise that supply chain linkages “serve to propagate risk that arises from a local or regional disruption across a wide geographic area,” which in turn “can adversely impact global economic growth and productivity” ([The White House, 2012](#)). In parallel, a growing academic literature has explored whether the presence of supply chain linkages can translate microeconomic shocks into aggregate, business cycle fluctuations.

Despite the interest of academics and policymakers alike, evidence on the role of input-output linkages as a channel for the propagation of shocks and a source of macroeconomic risk has been scant. In large part, this reflects the dual challenge of identifying plausible exogenous micro shocks in firm-level data and tracing their impact as they spread throughout the economy.

In this paper, we provide a systematic quantification of the role of input-output linkages as a mechanism for propagation and amplification of shocks by exploiting a large, but localized, natural disaster—namely, the Great East Japan Earthquake of 2011.¹ Relying on information on firms’ locations, we exploit the heterogeneous exposure of Japanese firms to the earthquake to obtain measures of firm-level disturbances. We combine this information with extensive micro-data on inter-firm transactions to trace and quantify the extent of shock propagation along supply chains. Using a general equilibrium model of production networks, we then obtain an estimate for the overall macroeconomic impact of the disaster that takes these propagation effects into account.

Our empirical analysis exploits two key features of the March 2011 earthquake in Japan. First, the large-scale destruction caused by the earthquake (which was followed by a massive tsunami and the failure of the Fukushima Dai-ichi Nuclear Power Plant) had a significant negative impact on the economic performance of the affected areas: the real GDP growth rate of the four most severely

¹The much publicized, if anecdotal, reports of disruptions to supply chains following the 2011 earthquake were in fact one of the major triggers for the above-mentioned policy initiatives. For example, a few weeks after the earthquake, Reuters reported that “[s]upply chain disruptions in Japan have forced at least one global automaker to delay the launch of two new models and are forcing other industries to shutter plants. . . . The automaker is just one of dozens, if not hundreds, of Japanese manufacturers facing disruptions to their supply chains as a result of the quake, the subsequent tsunami and a still-unresolved nuclear threat” ([Reuters, 2011](#)).

affected prefectures was 2.2 percentage points lower in the 2011 Japanese fiscal year (April 2011–March 2012) than the previous year. Second, despite their large impact on the coastal areas, the earthquake and its aftermaths were essentially local, regional shocks that directly affected only a small fraction of the Japanese economy, with the four affected prefectures accounting for only 4.6% of the aggregate Japanese output. These two features, together with the exogenous nature of the earthquake, provide us with a natural experiment in which a small subset of firms were exposed to a large negative shock.²

We rely on a proprietary dataset compiled by a major private credit reporting agency that contains information on roughly half of all private and publicly-traded firms in Japan, covering almost all firms with more than five employees across all sectors of the economy. For each firm-year, we observe a set of firm-level covariates as well as the identities of the firm’s suppliers and customers, thus enabling us to construct the network of supply chain relationships for the firms in the sample. We then use an address-matching service to determine the longitude and latitude of each firm’s location. We combine the resulting dataset with information on the geographic distribution of damages in the aftermath of the disaster to determine the set of firms that were directly exposed to the shock.

Based on this information, we examine whether the presence of direct and indirect input-output linkages to firms in the disaster-stricken areas had an impact on firms’ performance in the year after the earthquake. In particular, we compare the post-earthquake sales growth rates of firms at different distances—in the supply chain network sense—from the disaster-area firms to a control group of firms that were relatively more distant. We find significant evidence of both downstream and upstream propagation of the shock. More specifically, we find that the disaster resulted in a 3.6 percentage point decline in growth rate of firms with disaster-hit suppliers and a 2.9 percentage point decline in growth rate of firms with disaster-hit customers. Our results also indicate that the disruption caused by the earthquake led to significant indirect propagation, not only affecting the disrupted firms’ immediate transaction partners, but also their customers’ customers, their suppliers’ suppliers, and so on. For instance, we find that disaster-stricken firms’ customers’ customers experienced a 2.7 percentage point reduction in sales growth, while their suppliers’ suppliers experienced a 2.1 percentage point decline. We find similar, though smaller, effects for indirect suppliers and customers more distant from the source of the shock.

Having documented the significant propagation of the disaster shock over Japanese supply chains, we then outline a general equilibrium model of production networks in the spirit of [Long and Plosser \(1983\)](#) and [Acemoglu et al. \(2012\)](#) to theoretically capture the role of inter-firm linkages as a shock propagation mechanism. In the model, firms use a nested constant-elasticity-of-substitution production technology that combines labor, capital, and a bundle of intermediate inputs into output. Firms are subject to shocks that partially destroy their firm-specific capital

²Taken at face value, these figures also suggest that, solely based on the economic size of the affected areas, the earthquake can account for a $2.2 \times 0.046 \approx 0.1$ percentage point decline in real GDP growth in Japan. In comparison, the actual decline in Japan’s real GDP growth rate in the 2011 fiscal year was more than four times as large (around 0.43 percentage points).

stock. As our main theoretical result, we characterize the extent of the propagation of the shock as a function of the economy's production network and the underlying elasticities of substitution. Besides shedding light on the forces that underpin the propagation mechanism, this result enables us to obtain conditions on the elasticities of substitution under which the model's predictions are broadly consistent with our empirical findings.

We then combine the model with our firm-level data to obtain an estimate for the overall macroeconomic impact of the earthquake and its aftermaths on the Japanese economy. To this end, we first use the variation in firms' sales and the production network data to estimate the elasticities of substitution between various intermediate inputs and between the intermediate input bundle and primary factors of production. We find evidence for weak gross substitutability of various firm-level intermediate inputs, while estimating primary and intermediate inputs to be gross complements. Combining the resulting estimates with the model-implied expression for the aggregate impact of the shock, we estimate that the disaster resulted in a 0.47 percentage point decline in Japan's real GDP growth in the year following the disaster. For comparison, Japan's average growth rate in the decade prior to the disaster was equal to 0.6% (with a standard deviation of 2.4%), while a value-added accounting based on the economic size of the affected prefectures (similar to footnote 2) would have only attributed 0.1 percentage point decline in GDP growth to the disaster.

Finally, we use our general equilibrium framework and estimated parameters as the basis of two counterfactual analyses. First, we quantify the contribution of input-output linkages between firms inside and outside the disaster area to the disaster's aggregate impact by considering a counterfactual economy that is identical to the actual economy, except with no input-output linkages between firms inside and outside the disaster area.³ We find that, in such a counterfactual economy, the disaster would have resulted in a 0.21 percentage point decline in GDP growth, thus indicating that the propagation of the shock to firms outside the disaster area over input-output linkages played a significant role in amplifying the disaster's aggregate impact.

In a second exercise, we use our estimated model to obtain general equilibrium forecasts for the macroeconomic impact of a highly anticipated major earthquake in the Tokai region in central Japan. Combining our estimated model with probabilistic seismic hazard maps sourced from the government, we estimate that, with probability exceeding 50% over a 30-year horizon, such an earthquake would result in a 0.48 percentage point decline in GDP growth, whereas with probability exceeding 10%, it would result in up to 2.17 percentage point decline in GDP growth. These estimates, which indicate a potentially significantly larger impact than that of the Great East Japan Earthquake, reflect the fact that the Tokai region serves as one of the central hubs of manufacturing supply chains in Japan's industrial heartland.

Taken together, our findings provide substantial evidence for the role of input-output linkages as an important mechanism for the propagation and amplification of shocks. They also provide a detailed picture of the nature and intensity of this propagation, suggesting that input-output linkages can play a quantitatively non-trivial role in translating firm-level disturbances into sizable

³By construction, input-output linkages in such an economy cannot propagate the shock to firms outside the disaster area.

fluctuations at the aggregate level.

Related Literature Our paper is most closely related to the growing literature that emphasizes the role of production networks as a mechanism for propagation and amplification of shocks. Building on the multi-sector model of Long and Plosser (1983), papers such as Acemoglu et al. (2012, 2017) and Baqaee and Farhi (2019) characterize the conditions under which propagation of microeconomic shocks over input-output linkages can translate into sizable aggregate fluctuations.^{4,5} Despite its theoretical plausibility, credible identification of the role of the economy’s production network as a propagation and amplification mechanism has remained largely unexplored. While empirical studies, such as Foerster, Sarte, and Watson (2011), di Giovanni, Levchenko, and Méjean (2014), and Atalay (2017) investigate propagation of microeconomic shocks over input-output linkages, they invariably rely on strong identifying assumptions for backing out the shocks from data.

Two exceptions are the works of Barrot and Sauvagnat (2016) and Boehm, Flaaen, and Pandalai-Nayar (2019), who, in exercises similar to ours, leverage natural disasters to study the role of firm-level linkages in propagating input disruptions. Combining county-level data on the occurrence of natural disasters in the U.S. with Compustat data on the identity of customers of large and publicly-traded firms, Barrot and Sauvagnat (2016) find that shocks to suppliers impose substantial output losses on their direct customers. They also document that such shocks propagate to firms that share common customers with the disrupted firms, though only when the latter produce relation-specific inputs that are not easily substitutable. Relatedly, Boehm et al. (2019) provide evidence for cross-country transmission of shocks by documenting that U.S. affiliates of Japanese multinationals suffered large drops in output in the months following the 2011 earthquake in Japan. We contribute to this literature by exploiting the much more detailed nature of firm-level input-output linkages and obtaining a more complete picture of the propagation patterns and their macroeconomic implications. In particular, we provide evidence for the propagation of the natural disaster shock both upstream and downstream the supply chain, as well as to firms that were only indirectly linked to the disaster-stricken firms.⁶ In addition, the large scale of our study at the national level (alongside its focus on both private and publicly-traded firms) enables us to provide an estimate for the overall macroeconomic impact of the earthquake shock as it propagated throughout the Japanese economy.

⁴See Carvalho (2014) and Carvalho and Tahbaz-Salehi (2019) for surveys of the literature on production networks. This literature may in turn be placed in the larger body of works that studies the microeconomic origins of aggregate fluctuations, including early contributions by Jovanovic (1987) and Durlauf (1993); papers such as Carvalho (2010), Jones (2013), Liu (2019), Bigio and La’O (2020), Baqaee and Farhi (2020), and Grassi (2017) studying the role of linkages in propagating shocks and distortions; Gabaix (2011), Amiti and Weinstein (2018), and Carvalho and Gabaix (2013), emphasizing the role of firm size distribution in translating micro shocks into macro fluctuations; and Nirei (2006, 2015) and Guiso et al. (2017), arguing that propagation of lumpy firm-level investments may have non-trivial aggregate implications.

⁵A related set of papers, such as Johnson (2014), Chaney (2014), and Antràs, Fort, and Tintelnot (2017), studies the role of firm-level linkages in the international trade context. Closer to our work, Caliendo et al. (2018) study the role of intersectoral and interregional trade linkages in propagating disaggregated productivity changes across U.S. states.

⁶Also see Acemoglu, Akcigit, and Kerr (2016), who investigate the propagation of various types of shocks over the U.S. input-output network at the sectoral level. We, in contrast, document the role of supply chain linkages in the transmission of shocks at the firm level.

Our work is also related to papers such as [Noy \(2009\)](#), [Raddatz \(2009\)](#), and [Strobl \(2012\)](#), which study the macroeconomic impacts of natural disasters. In line with these papers, we find that the earthquake and its aftermaths had a negative and significant effect on aggregate Japanese output. Relatedly, [Schnell and Weinstein \(2012\)](#) and [Cavallo, Cavallo, and Rigobon \(2014\)](#) perform comparative analyses of the impact of various natural disasters. Comparing the 2011 Great East Japan Earthquake to the 2010 earthquake in Chile, [Cavallo et al. \(2014\)](#) conclude that the pricing behavior and product stockout patterns across Japanese retailers are consistent with a supply-side disruption. [Schnell and Weinstein \(2012\)](#), on the other hand, compare the impact of the 2011 earthquake to that of the 1995 earthquake in Kobe and argue that the 2011 earthquake's much more long-lasting impact on industrial production is linked to the substantial and persistent drop in energy output in the wake of the earthquake. We provide an alternative, and complementary, explanation for the decline in Japanese output in the year immediately after the disaster, highlighting the key role of supply chain linkages as a transmission mechanism in the economy.

Finally, our paper is related to the small literature that analyzes the structure and geographical features of firm-level production networks. [Atalay et al. \(2011\)](#) use firm-level data from Compustat to characterize the buyer-supplier network of the U.S. economy, while [Bernard et al. \(2019a\)](#) and [Dhyne et al. \(2020\)](#) use production network data from Belgium to explore, respectively, the determinants of firm size heterogeneity and firms' role as direct and indirect importers and exporters. Relatedly, [Ohnishi, Takayasu, and Takayasu \(2010\)](#) and [Saito \(2013\)](#) offer detailed overviews of the Japanese firm-level production network. Within this literature, our work is closely related to [Bernard et al. \(2019b\)](#), who, using the same dataset as ours, document that the opening of a high-speed train line in Japan led to the creation of new supplier-customer linkages, as well as significant improvements in firm performance.

Outline The rest of the paper is organized as follows. We start by describing the empirical context and the data in Sections 2 and 3, respectively. Section 4 contains our main empirical results. We present our theoretical model in Section 5, which we then use as the basis of our aggregation and counterfactual analyses in Section 6. All proofs and derivations are presented in the Appendix.

2 Empirical Context: The 2011 Great East Japan Earthquake

On March 11, 2011, a magnitude 9.0 earthquake occurred off the northeast coast of Japan. This was the largest earthquake in the history of Japan and the fifth largest in the world since 1900. The earthquake brought a three-fold impact on the residents of northeast Japan: (i) the main earthquake and its aftershocks, directly responsible for much of the material damage that ensued; (ii) the resulting tsunami, which flooded 561 square kilometers of the northeast coastline; and (iii) the failure of the Fukushima Dai-ichi Nuclear Power Plant that led to the evacuation of 99,000 residents of the Fukushima prefecture.

In addition to severe damage to infrastructure, the earthquake and its aftermaths resulted in 19,689 confirmed fatalities, a further 2,563 people missing, and complete or partial collapse

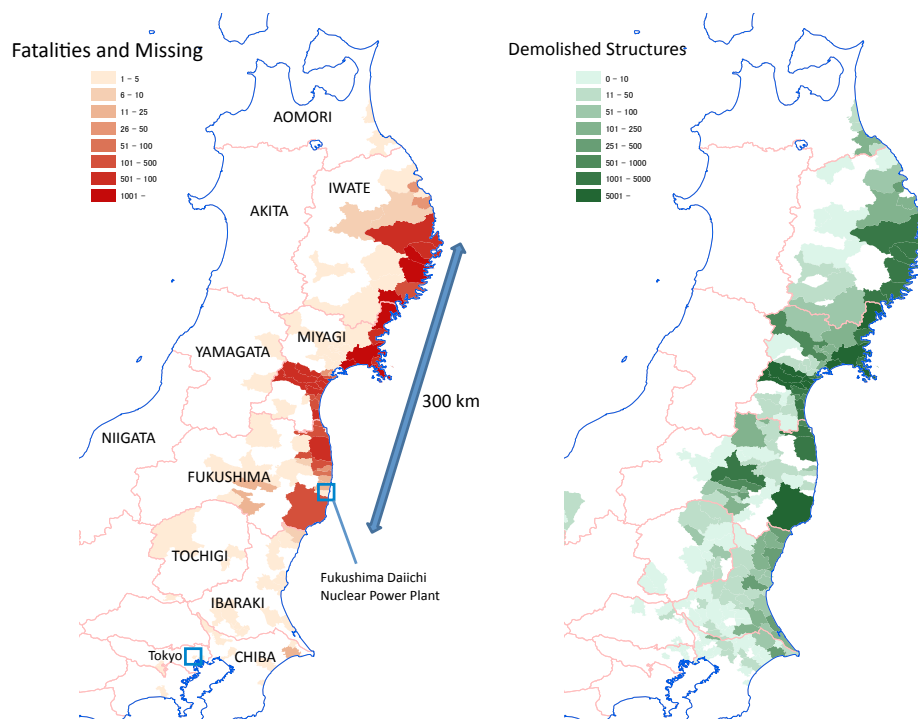


Figure 1. Geographical Distribution of Losses and Damages in Northeast Japan

Source: National Research Institute for Earth Science and Disaster Resilience (2011)

Notes: The figure depicts the distribution of casualties (left panel) and demolished structures (right panel) incurred by the earthquake and its aftermaths at the municipality level.

of 404,934 buildings across 22 prefectures, as of March 8, 2019 (Fire and Disaster Management Agency, 2019). The brunt of the damages, however, was mostly concentrated in the four Pacific coast prefectures of Aomori, Fukushima, Iwate, and Miyagi in the Tohoku region.⁷ According to government estimates, the disaster resulted in 16.9 trillion yen in total capital loss in the affected prefectures, with capital loss due to destruction of buildings (housing, establishment structures, plants, and equipments) amounting to 10.4 trillion yen (Cabinet Office, Director General for Disaster Management, 2011a).

Figure 1 depicts the geographical distributions of casualties and demolished structures. As the figure illustrates, the impact of the shock was far from homogeneous, even within the most severely affected prefectures. In particular, even though the main earthquake resulted in damages in some inland areas, the most severely affected areas were concentrated in the coastal regions that were exposed to the tsunami.

Not surprisingly, this localized, yet large shock had a significant negative impact on the economic performance of the affected areas. The real GDP growth rate of the four disaster-stricken prefectures in the 2011 fiscal year (FY 2011) was -1.5% , revealing weak economic performance in comparison

⁷As we discuss in more detail in Section 3.2, the area officially declared by the government as the disaster area consists of 41 municipalities within these prefectures.

to both their average growth rate in the previous fiscal year and aggregate Japanese GDP growth in FY 2011, which were 0.7% and 2.2%, respectively ([National Accounts of Japan, 2014](#)).⁸ These figures also suggest that, despite its large impact on the local economy of northeast Japan, the earthquake shock cannot, in and of itself, account for the decline in Japan's GDP growth, as the four disaster-stricken prefectures only account for roughly 4.6% of aggregate output in Japan. More specifically, solely based on the economic size of the affected areas, the earthquake can account for at most a $0.046 \times (0.7 - (-1.5)) \approx 0.1$ percentage point decline in real GDP growth. However, the actual decline in Japan's real GDP growth rate was four times as large, dropping from 2.6% in FY 2010 to 2.2% in FY 2011.

Concentrating on manufacturing activity provides a more detailed picture of the economic impact of the disaster. Figure 2 compares the monthly (year-on-year) growth rate of the Index of Industrial Production (IIP) of the disaster-stricken prefectures to that of Japan as a whole. This index, which is constructed by the Ministry of Economy, Trade and Industry of Japan (METI), measures the activity in the manufacturing and mining sectors. As the figure illustrates, the earthquake and its aftermaths resulted in a sharp, but temporary decline in the industrial production of the affected areas: the IIP in the four disaster-stricken prefectures declined on impact by over 40% relative to the previous year, followed by a partial rebound. By February 2012 (that is, one year after the earthquake), industrial production in the affected areas was about 4% lower than the corresponding level on the eve of the earthquake in February 2011. In comparison, industrial production of the entire country experienced a 15% decline in April 2011 and was back to its pre-earthquake growth rate one year after the earthquake.

Another key observation is that earthquake-hit areas were not overly specialized. The three sectors with the largest value-added shares in the four affected prefectures in 2010 were realty, wholesale and retail, and other services, responsible for 10.0%, 10.9%, and 19.6% of the region's aggregate output, respectively. These figures are comparable to the shares of the same sectors in the entire country (9.8%, 11.5%, and 19.4%, respectively). The largest difference among various sectoral shares between the four affected prefectures and Japan as a whole is in the transportation machinery sector, which is responsible for 2.6% and 5.8% of aggregate output, respectively.

We end this discussion with a word on infrastructure. Even though infrastructure (such as roadways, railways, and ports) across northeast Japan was severely affected by the disaster, pre-earthquake levels of activity were largely restored by late March. The one area where activity was disrupted well into the summer of 2011 was electricity supply, as several nuclear—notably the Fukushima Dai-ichi plant—and conventional power plants in northeast Japan went offline. This resulted in rolling (controlled) blackouts throughout March and a power saving edict for the summer months of 2011 that required large-lot users in Kanto and Tohoku regions to reduce power use by 15%.⁹

⁸In Japan, the government's fiscal year runs from April 1 to March 31 of the following calendar year. As a result, FY 2010 contains only 20 post-earthquake days, while FY 2011 falls entirely after the earthquake.

⁹We discuss the implications of the disruption in electricity supply in Subsection 4.3. See Online Appendix C for more details on the disaster's impact on infrastructure.

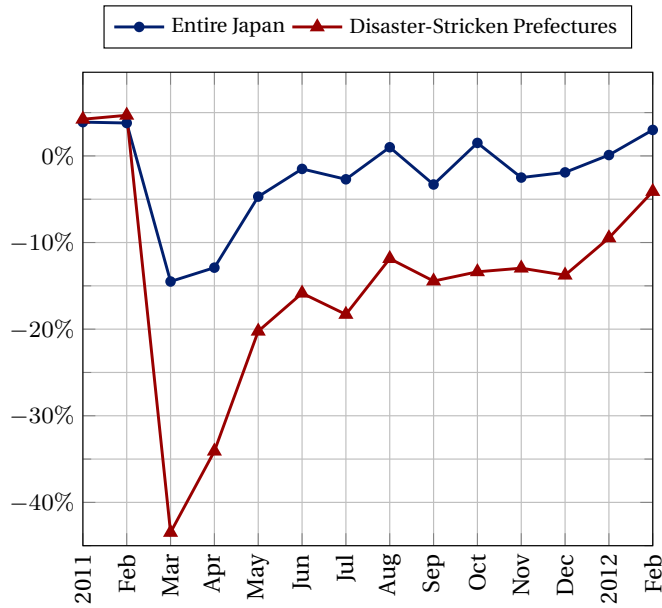


Figure 2. Growth Rate of Index of Industrial Production

Notes: The figure plots the monthly year-on-year growth rate of Index of Industrial Production (IIP) from January 2011 to February 2012. The growth rate at each month is relative to the corresponding month in the previous year. The solid-circle line (in blue) plots IIP growth for entire Japan. The solid-triangle line (in red) plots the weighted average IIP growth rate of the four disaster-stricken prefectures of Aomori, Fukushima, Iwate, and Miyagi, with the weights set as each prefecture's respective GDP share. IIP data is obtained from [Ministry of Economy, Trade and Industry \(2016\)](#).

3 Data

Our empirical analysis relies on identifying (i) firm-level supply chains throughout Japan and (ii) the subset of firms that were directly exposed to the triple shocks of earthquake, tsunami, and nuclear disaster. We rely on a proprietary dataset assembled by the private credit reporting agency Tokyo Shoko Research Ltd. (henceforth, TSR) to construct a firm-level production network of supplier-customer linkages. We then combine the TSR dataset with data on the geographic distribution of damages in the aftermath of the disaster—as officially designated by the government of Japan—to determine the set of disaster-stricken firms.

3.1 Firm-Level and Production Network Data

Firms provide information to TSR in the course of obtaining credit reports on potential suppliers and customers or when attempting to qualify as a supplier. This information consists of a set of firm-level characteristics (such as sales figures and number of employees), as well as the identities of the firms' suppliers and customers. The resulting (raw) database contains information on roughly one million firms in all 47 prefectures across Japan, spanning all sectors of the economy. The dataset provided to us by the Research Institute of Economy, Trade and Industry (RIETI) consists of an unbalanced panel of firms covering years 2010, 2011, and 2012.¹⁰

¹⁰The year in the TSR sample refers to the sample's release date. Therefore, for example, firm-level data corresponding to FY 2011 is reported as part of the 2012 sample.

Table 1. Firm Size Distribution

	Number of Employees									
	0–4	5–9	10–19	20–29	30–49	50–99	100–299	300–999	1000–1999	2000+
TSR	0.40	0.25	0.16	0.06	0.05	0.04	0.03	0.009	0.001	0.001
Census	0.59	0.17	0.11	0.04	0.04	0.03	0.02	0.005	0.001	0.001

Notes: The table reports the fraction of firms with the number of employees in each of the respective bins. “TSR” refers to the 2010 TSR dataset. “Census” refers to the 2009 Economic Census for Business Frame.

Firm-Level Characteristics TSR collects information on employment, the number of establishments, up to three (Japanese Industrial Classification 4-digit) industries the firm may belong to, sales and profits for the past two years, the resulting credit score, and a physical address for the firm’s headquarters. Each firm in the TSR database also reports the date on which its fiscal year ends.

The TSR sample is neither a census nor a representative survey, as the entry of any particular firm takes place at the request of TSR’s clients. This means that TSR does not update the information on all firms on an annual basis. We therefore restrict our sample to the subset of firms for which we observe (i) sales figures for all three years between 2010 and 2012 and (ii) firm-level covariates for the first year of our sample. This procedure leaves us with a panel of 750,237 firms.

To check for biases in the sample, we compare the 2010 TSR dataset with 2009 Economic Census, which contains information on 1,805,545 firms.¹¹ Table 1 reports the distribution of the number of employees in the two datasets. As the table indicates, the firm size distribution in the TSR dataset closely matches that of the census data for firms with five or more employees (though it underestimates the fraction of very small firms with four or less employees). Similarly, Figure 3 illustrates that the geographic distribution and industrial composition of firms in the TSR sample matches those of firms in the 2009 census, with the only major difference being in the fraction of firms that are active in the construction sector.

One limitation of TSR’s data on firm-level characteristics is that it only contains information on firms’ headquarters locations, as opposed to the location of their plants. Even though in principle this may create a bias in our estimates, as part of our robustness checks in Subsection 4.3, we verify that our estimates remain unchanged if we restrict our analysis to the subsample of single-plant firms (for which headquarters and plant locations coincide).

Supplier-Customer Information Each firm in the TSR dataset also provides a list of its suppliers and customers, thus enabling us to construct the production network of supplier-customer linkages for the firms in the sample. Given the occurrence of earthquake in March 2011, we construct this network using the 2010 TSR sample.

¹¹The census is conducted by the Statistics Bureau in the Ministry of Internal Affairs and Communications. The data is from the survey entitled “The Economic Census for Business Frame,” which identifies the basic structure of establishments and enterprises and is available at: <http://www.stat.go.jp/english/data/e-census.htm>.

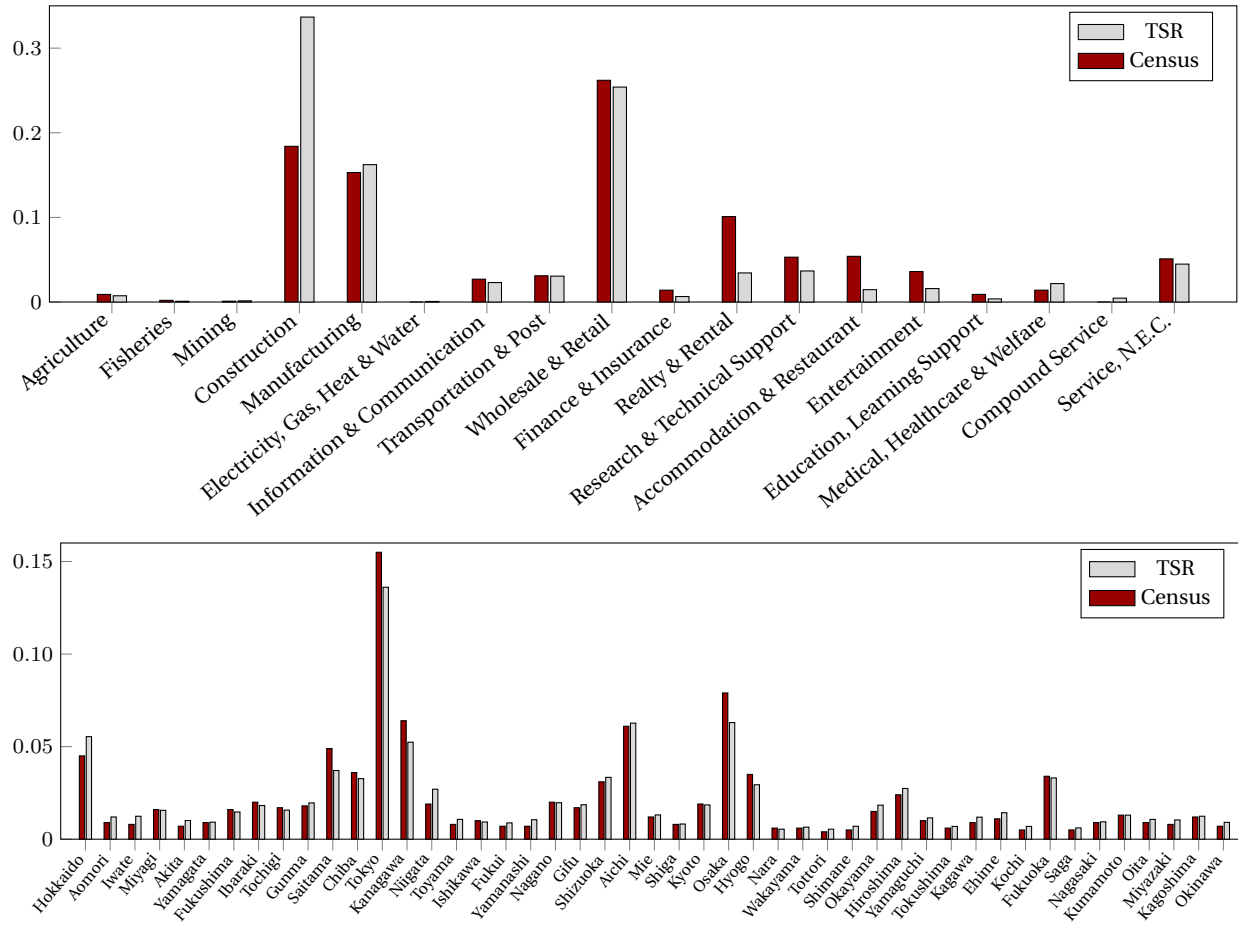


Figure 3. Industrial Composition and Geographical Distribution

Note: The top panel plots industrial composition (single-digit Japanese Standard Industry Codes) of firms. The bottom panel plots the share of firms in each of the 47 prefectures in Japan. “TSR” refers to the 2010 TSR dataset. “Census” refers to the 2009 Economic Census for Business Frame. The vertical axis in each graph represents the fraction of firms.

The TSR dataset on supplier-customer linkages has two limitations. First, it only reports a binary measure of inter-firm supplier-customer relations: even though we observe whether one firm is another firm’s supplier or customer, we do not observe a yen measure associated with their transactions. Second, the forms used by TSR limit the number of suppliers and customers that firms can report to 24 each. Nevertheless, given that each firm in the dataset may also be reported by other firms as a transaction partner, we overcome this limitation by combining the self-reported customer and supplier relations with those reported by other firms. More specifically, we construct a firm’s transaction network by augmenting the list of suppliers (customers) reported by the firm itself with the reports of others that state the firm as their customer (supplier). This procedure enables us to construct the list of suppliers and customers of firms that have more than 24 transaction partners per category, including very large firms that transact with several thousand firms.

Using this information, we then construct the production network of supplier-customer relationships among firms in our sample, with each firm corresponding to a vertex in the network and a directed edge present from vertex j to vertex i if firm i is a supplier of firm j . In constructing

this network, we discard reported transaction partners that fall outside the TSR database. Therefore, a firm may appear to have no customers because all its customers are foreign firms, domestic non-TSR firms, or non-firms (such as final demand customers or the government of Japan). Similarly, a firm may appear to have no suppliers because either all its reported suppliers are foreign or fall outside the TSR database. Throughout, we restrict the sample to the subset of firms with at least one transaction partner (being it a customer or a supplier) within the TSR database, thus discarding firms that are isolated from the rest of the network. We find no evidence of a systematic bias in the subsample of firms with at least one TSR partner.

The resulting production network is highly sparse, with the average number of suppliers and customers equal to 5.0 and 5.6, respectively. However, the distributions of the number of suppliers and customers are highly skewed, spanning several orders of magnitude. For example, firms at the top percentiles of the in-degree and out-degree distributions have 39 suppliers and 49 customers, respectively, while the top 100 firms in each category have more than 4,000 suppliers and 5,000 customers. The large heterogeneity in firms' number of transaction partners is also reflected in parametric estimates of the tail behavior of the production network's in-degree and out-degree distributions, both of which are well-approximated by Pareto distributions with shape parameters estimated as 1.29 (s.e. = 0.019) and 1.25 (s.e. = 0.018), respectively.¹²

3.2 Identifying the Disaster Area

We identify the disaster-hit region by relying on three decrees issued by the government in the aftermath of the earthquake. The first decree, issued on April 28, 2011 by the Ministry of Land, Infrastructure, Transport and Tourism (MLIT) designated 36 municipalities as severely affected municipalities that qualified for special financial aid.¹³ The other two decrees, issued by the Prime Minister's office on April 21 and 22, restricted entrance and residence in 13 municipalities in the aftermath of the failure of the Fukushima Dai-ichi Nuclear Power Plant.¹⁴ Out of the 13 municipalities constituting the evacuation zone, eight were also included in the decree issued by MLIT, thus leaving us with a total of 41 municipalities.¹⁵ We refer to the region covered by these

¹²The reported estimates of Pareto shape parameters are maximum likelihood estimates, with standard errors calculated following Clauset, Shalizi, and Newman (2009). The network statistics reported above are similar to those reported by Ohnishi et al. (2010) and Bernard et al. (2019b) for the 2005 cross-section of TSR data. The skewed nature of the distributions of the number of transaction partners is also present in firm-level production networks constructed from Belgian data (Dhyne et al., 2015; Bernard et al., 2019a). Relative to the Belgian data, the production network constructed from TSR data is an order of magnitude larger but also less dense, with a lower average number of suppliers and customers per firm.

¹³This decree was issued as a notification based on "The Act on Special Financial Support to Deal with the Designated Disaster of Extreme Severity, Article 41-2." The notification is available at http://www.mlit.go.jp/report/press/house03_hh_000054.html (in Japanese).

¹⁴These areas are displayed as "Restricted Area" and "Deliberate Evacuation Area" in the following map issued by METI: http://www.meti.go.jp/english/earthquake/nuclear/roadmap/pdf/evacuation_map_120401.pdf. The decrees issued on April 21 and 22 are available, respectively, at <http://www.kantei.go.jp/saigai/pdf/20110421110001shiji.pdf> and <http://www.kantei.go.jp/saigai/pdf/201104220944siji.pdf> (in Japanese).

¹⁵These municipalities are Hachinohe-city, Miyako-city, Ōfunato-city, Rikuzentakata-city, Kamaishi-city, Ōtsuchi-town, Yamada-town, Iwaizumi-town, Tanohata-village, Noda-village, Sendai-city, Ishinomaki-city, Kesennuma-city, Natori-city, Iwanuma-city, Tome-city, Higashimatsushima-city, Ōsaki-city, Watari-town, Yamamoto-town, Shichigahama-town, Onagawa-town, Minamisanriku-town, Fukushima-city, Kōriyama-city, Iwaki-city, Sukagawa-city, Sōma-city,

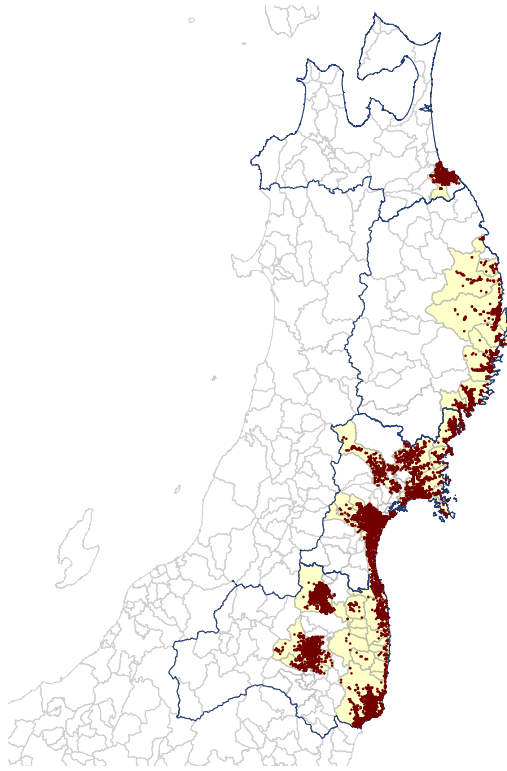


Figure 4. Headquarters Locations of Firms in the Disaster Area

Note: The disaster area, displayed as the shaded area on the map, is defined as the 41 municipalities covered by three decrees issued by Ministry of Land, Infrastructure, Transport and Tourism and the Prime Minister’s Office. Each dot on the map corresponds to the location of a firm in the TSR sample located in the disaster area. The dark blue curve indicates the boundaries of the four prefectures (Aomori, Fukushima, Iwate, and Miyagi) that encompass the disaster area.

41 municipalities as the “disaster area,” depicted as the shaded area in Figure 4. We verify that the physical impact of the shocks (as measured by casualties and demolished structures) were concentrated in these municipalities. In Subsection 4.3, we verify that our results are robust to an alternative definition for the disaster area, encompassing the regions that were flooded following the tsunami.

3.3 Disaster-Area Firms

With the definition of the disaster area in hand, we then identify the set of firms located in this area by using an address-matching service provided by the Center for Spatial Information Science at the University of Tokyo and matching each firm’s headquarters address (given in TSR) to longitude and latitude data. This procedure identifies 20,861 firms in our sample. Figure 4 maps the headquarters locations of these firms.

Table 2 compares the pre-earthquake characteristics of firms inside and outside of the disaster area. It illustrates that, in the year preceding the earthquake, the average disaster-area firm was comparable to the average firm in the rest of the country. This is true both for typical firm

Minamisōma-city, Hirono-town, Naraha-town, Tomioka-town, Ōkuma-town, Futaba-town, Namie-town, Shinchi-town, parts of Tamura-city, parts of Kawamata-town, Kawauchi-village, Katsurao-village, and Iitate-village.

Table 2. Pre-Earthquake Characteristics of Firms Inside and Outside the Disaster Area

	Disaster Area				Rest of Japan
	all firms	sorted by post-earthquake sales growth			
		bottom tercile	middle tercile	top tercile	
Log sales	11.39 (1.67)	11.37 (1.64)	11.48 (1.69)	11.31 (1.68)	11.54 (1.77)
Log No. employees	1.83 (1.27)	1.74 (1.24)	1.88 (1.35)	1.88 (1.22)	1.84 (1.31)
Age	27.41 (15.48)	27.94 (15.45)	27.87 (15.80)	26.41 (15.13)	29.00 (16.54)
No. of suppliers	4.56 (14.68)	4.17 (12.54)	4.59 (12.18)	4.92 (18.17)	5.21 (34.98)
No. of customers	4.56 (30.38)	3.02 (6.36)	4.77 (37.70)	5.82 (36.12)	5.17 (36.13)
Customers' log sales	14.84 (2.43)	14.88 (2.43)	15.32 (2.44)	14.35 (2.32)	14.61 (2.53)
Suppliers' log sales	14.63 (2.39)	14.76 (2.45)	14.63 (2.45)	14.52 (2.30)	14.87 (2.56)

Notes: The table reports summary statistics of pre-earthquake characteristics of firms inside and outside the disaster area. Values are averages across firms in each category with standard deviations reported in parentheses. The first column reports mean and standard deviations for all firms in the disaster area. Second, third, and fourth columns report mean and standard deviations for disaster-area firms in, respectively, the bottom, middle, and top terciles of the distribution of the firms' post-earthquake sales growth rates. The last column reports mean and standard deviation for firms outside of the disaster area.

characteristics such as age and size (as measured by employees and sales), as well as with regards to firms' supply chain characteristics (such as the number and average size of customers and suppliers). Importantly, this observation remains unchanged even when we disaggregate disaster-area firms based on their post-earthquake performance.

We next test whether the earthquake and its aftermaths had an economically significant effect on firms in the disaster area relative to the firms outside of the four disaster-stricken prefectures. To this end, we use a difference-in-differences specification and regress the logarithm of firm sales on a dummy variable indicating disaster-area firms interacted with time dummies for 2010 and 2012, while including firm and industry-year fixed effects. We find that disaster-area firms' growth rate in the year immediately after the earthquake was 3.3 percentage points smaller than that of the rest of the firms in our sample (with a t -statistic of -2.34). In contrast, we find no difference between the growth rates of disaster-area firms and the rest of Japan in the year prior to the earthquake (with a point estimate of 0.3 percentage points and a t -statistic of 0.91).

Taken together, these findings illustrate that while the shock had a large negative impact on the firms in the disaster area, these firms (including the worst performing among them) were not substantially different from the rest of the firms in our sample based on either their pre-earthquake performance or other firm-level characteristics.

4 Propagation of the Disaster Shock over the Production Network

In this section, we use the TSR data to empirically examine the role of input-output linkages in propagating the shock from disaster-area firms to the rest of the firms in Japan. We are specifically interested in the extent of propagation to firms that were direct or indirect suppliers and customers of disaster-area firms.

To this end, we construct a measure of network distance to the set of disaster-area firms for all firms in our sample. Using the 2010 production network data, we first designate the immediate customers and suppliers of disaster-area firms as, respectively, “downstream distance 1” and “upstream distance 1” firms. We then designate a firm as “downstream distance 2” if it was listed in 2010 as a customer of at least one downstream distance 1 firm and was not a distance 1 firm itself.¹⁶ Using a similar recursive procedure, we identify the set of firms that were at various upstream and downstream distances from disaster-area firms in the year prior to the earthquake. Figure A.1 depicts the geographic distribution of firms of various distances across Japan.

4.1 Empirical Specification

Using our measure of network distance constructed from TSR data, we estimate the following difference-in-differences specification:

$$\begin{aligned}
 y_{ipst} = & \gamma_i + \gamma_{pst} + \sum_{k=1}^4 \sum_{\tau \neq 2011} \beta_{k,\tau}^{\text{down}} \times \text{Downstream}_i^{(k)} \times \text{year}_\tau \\
 & + \sum_{k=1}^4 \sum_{\tau \neq 2011} \beta_{k,\tau}^{\text{up}} \times \text{Upstream}_i^{(k)} \times \text{year}_\tau + \sum_{\tau \neq 2011} \delta_\tau \times X_{isp,2010} \times \text{year}_\tau + \varepsilon_{ispt},
 \end{aligned} \tag{1}$$

where y_{ipst} is the logarithm of year t sales of firm i in industry s and prefecture p . $\text{Downstream}_i^{(k)}$ and $\text{Upstream}_i^{(k)}$ are dummy variables that indicate whether firm i is, respectively, a downstream or upstream distance k firm in the 2010 sample. In the above specification, γ_i and γ_{pst} denote firm and prefecture-industry-year fixed effects, respectively, year_τ is a time dummy for year τ which takes value 1 if $t = \tau$, and $X_{isp,2010}$ is a vector of controls that contains firms’ pre-earthquake observable characteristics, consisting of age, number of employees, number of transaction partners, and distance to the disaster area. The main coefficients of interests are $\beta_{k,\tau}^{\text{down}}$ and $\beta_{k,\tau}^{\text{up}}$, which respectively measure the differential growth rates of firms with downstream and upstream network distance k to disaster-area firms relative to firms in the control group between year $\tau \in \{2010, 2012\}$ and the earthquake year of 2011. The control group consists of all firms that, prior to the earthquake, were 5 or more supply chain links away from disaster-area firms. We truncate the set of distance dummies at $k \leq 4$, as enlarging the treatment group further downstream or upstream would reduce the control group to a very small number of firms.

¹⁶This construction implies that each firm is assigned to at most one downstream and one upstream network distance group. We allow for a more general specification of network distances in Section 6, accounting for various possible network relationships between firms.

Our main identifying assumption is that, conditional on firm observables $X_{isp,2010}$, the supply chain distance between firm i and disaster-area firms is orthogonal to any of i 's unobservable characteristics that may affect its post-earthquake output dynamics. This exclusion restriction is violated if supply chain partners of disaster-area firms are more likely to be affected by the shock via other channels. For instance, it may have been the case that disruptions to railways, roads, or other local infrastructure negatively impacted production of nearby firms, which were also more likely to transact with disaster-area firms. We address this concern by excluding from our sample all firms whose headquarters were located in the four prefectures of Aomori, Fukushima, Iwate, and Miyagi that encompass the disaster area (see Figure 4). We also note that by deploying prefecture-industry-year fixed effects, we are further addressing the concern that certain areas or industries were simply more affected than others. Finally, to control for any remaining residual effects operating at the firm level, we include the firms' geographic distance to the disaster area as one of the control variables.

4.2 Results

Figure 5 reports the estimated coefficients for equation (1), with the vertical error bars indicating 95 percent confidence intervals based on two-way clustered standard errors at the level of prefecture and industry.¹⁷ The coefficients on all downstream and upstream variables in the year following the earthquake (depicted as blue circles in the figure) are negative and significant, indicating that the disruption caused by the earthquake and its aftermaths propagated to disaster-area firms' direct and indirect customers and suppliers. For instance, the post-earthquake growth rate of firms immediately downstream to disaster-area firms (i.e., those with downstream distance equal to 1) was 3.6 percentage points lower than firms in the same prefecture and industry in the control group. Similarly, firms with a direct customer in the disaster area (i.e., with upstream distance equal to 1) underperformed the control group by 2.9 percentage points in the year after the earthquake.

The results in Figure 5 also indicate that the intensity of the propagation declined in the network distance to disaster-area firms. For instance, whereas the immediate customers of disaster-area firms underperform the control group by 3.6 percentage points, the post-earthquake growth rate of firms at downstream distance 2 was roughly 2.7 percentage points lower than that of the control group, with the same number shrinking to 2.0 percentage points for firms at downstream distance 4. A similar monotonic decline in the post-earthquake growth rates is also evident for upstream firms, with firms at upstream distance 4 to the disaster-area firms experiencing only a 0.9 percentage points lower growth vis-à-vis the control group (compared to 2.9 percentage points for firms at upstream distance 1).

Another important result emerging from Figure 5 is that, in contrast to the post-earthquake coefficients, all coefficients corresponding to the year prior to the earthquake (depicted as red diamonds in the figure) are statistically insignificant. That is, even though firms in various treatment groups experienced slower post-earthquake growth rates compared to the control group firms, there was no statistically significant difference between their growth rates in the year prior to the

¹⁷The figure is based on a single panel regression. Table A.1 presents the results in tabular form.

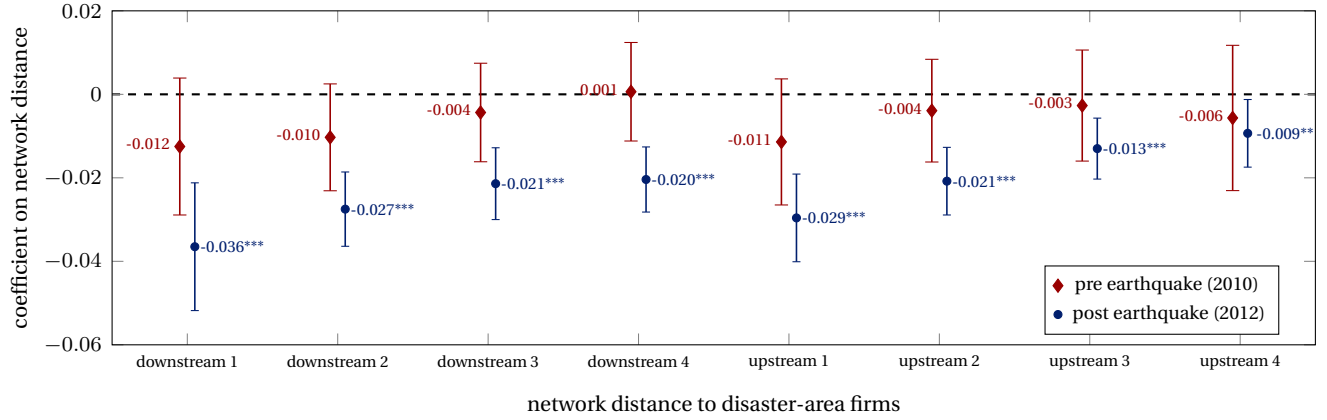


Figure 5. Propagation of the Shock over the Production Network: Baseline Specification

Note: This figure is based on a single panel regression, in which the dependent variable is the logarithm of firms' annual sales. The plotted point estimates are the coefficients on network distance dummies interacted with annual dummy variables. The diamonds (in red) and circles (in blue) indicate the coefficients on network distance dummies interacted with 2010 and 2012 time dummies, respectively. The vertical error bars indicate 95 percent confidence intervals based on two-way clustered standard errors at the level of prefecture and industry. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. The regression includes firm and prefecture-industry-year fixed effects and a set of control variables consisting of the logarithm of the number of transaction partners, age, logarithm of the number of employees, and distance to the disaster area. All control variables are measured for the year 2010 and interacted with 2010 and 2012 time dummies.

earthquake. The mostly negative point estimates for the pre-earthquake coefficients suggest, if anything, that firms in the treatment groups were growing at a faster rate than those in the control group in the year prior to the earthquake.

Taken together, the results in Figure 5 provide evidence for the role of the economy's production network in propagating the disaster shock to firms located outside the four prefectures that encompass the disaster area. Importantly, this propagation was not confined to the disaster-area firms' immediate suppliers and customers, as even firms with indirect supply chain relations to disaster-area firms experienced slower growth rates as a result of the shock's propagation.

4.3 Robustness

We next provide a series of checks to test the robustness of our results. All estimates in this subsection are presented in Figure 7, with further details provided in Table A.1 in the appendix.

Redefining the Disaster Area As a first check, we verify that our estimates are robust to the definition of the disaster area. Given that the large majority of loss of life occurred as a consequence of the tsunami, we rerun the regression in equation (1) while redefining the disaster area as regions that were flooded following the tsunami. These regions, unlike the 41 municipalities in our baseline specification, do not correspond to prefecture or municipality boundaries, and instead are defined based on aerial photos and satellite imagery of flooded areas provided by the Geospatial Information Authority of Japan. Figure 6 maps the headquarters locations of the firms located in this region. Once again, to avoid any possible contamination, we exclude all the firms in the four Pacific coast

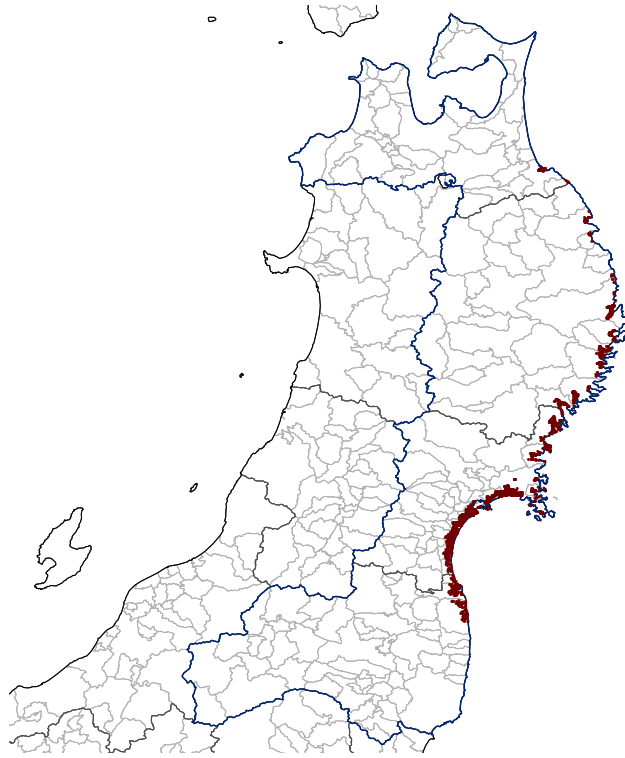


Figure 6. Headquarters Locations of Firms in the Flooded Region

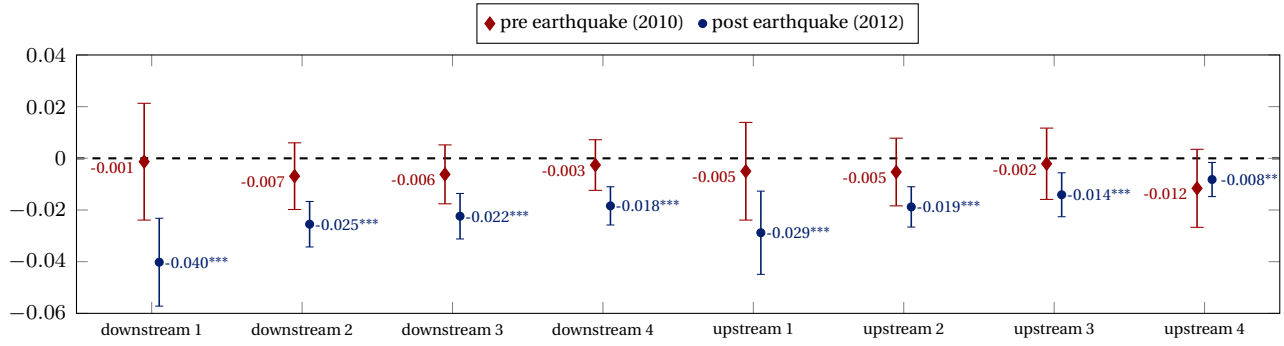
Note: Each dot on the map corresponds to the location of a firm in the TSR sample located in the flooded region. The dark blue curve indicates the boundary of the four prefectures (Aomori, Fukushima, Iwate, and Miyagi) that encompass the disaster area.

prefectures of Aomori, Fukushima, Iwate, and Miyagi from the regressions.

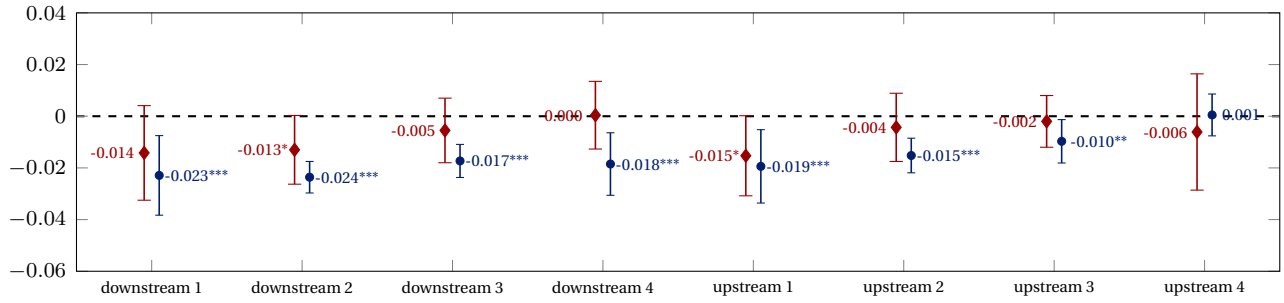
The results, reported in the top panel of Figure 7, are consistent with our baseline estimates, illustrating the presence of both upstream and downstream propagation to the direct and indirect customers and suppliers of firms in the flooded region. As before, the post-earthquake estimates are negative and significant, with magnitudes that decline with the network distance to disaster-area firms, whereas all pre-earthquake estimates are insignificant.

Single-Plant Firms Since our analysis is based on firm-level (as opposed to plant-level) data, one potential concern regarding our baseline estimates is the possibility that the treatment group—i.e., firms with headquarters outside the disaster area that are directly or indirectly linked to disaster-area firms—may include multi-plant firms with plants in the disaster area. For instance, our baseline results would overestimate the intensity of downstream propagation if customers of disaster-area firms are more likely to operate plants in the disaster area.

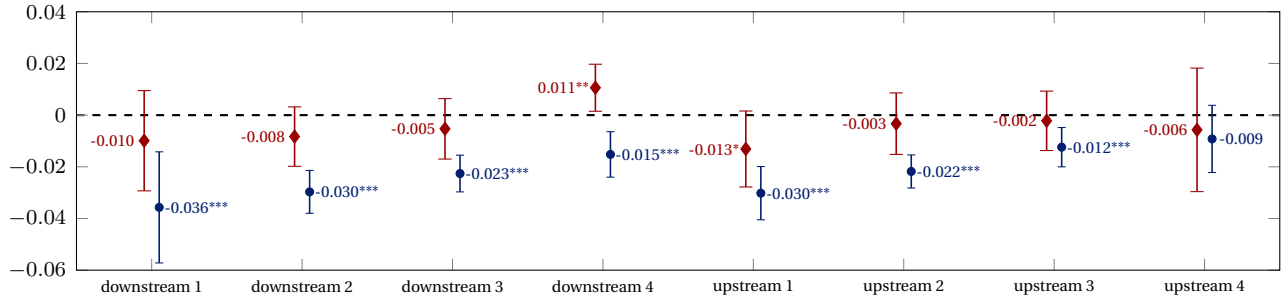
Even though the TSR data does not contain information on plant locations, it reports the number of plants operated by each firm. We thus leverage this information and rerun our baseline specification on the subsample of single-plant firms (for which headquarters and plant locations coincide). This subsample contains roughly 47% of the firms in our sample. We follow the same sample selection criteria as in the rest of our analysis by restricting the treatment and control groups



(a) disaster area redefined as the region flooded after the tsunami



(b) sample restricted to single-plant firms



(c) sample restricted to firms west of the frequency frontier

Figure 7. Propagation of the Shock: Robustness Checks

Note: Each panel in this figure is based on a single panel regression, in which the dependent variable is the logarithm of firms' annual sales. The plotted point estimates are the coefficients on network distance dummies interacted with annual dummy variables. The diamonds (in red) and circles (in blue) indicate the coefficients on network distance dummies interacted with 2010 and 2012 time dummies, respectively. The vertical error bars indicate 95 percent confidence intervals based on two-way clustered standard errors at the level of prefecture and industry. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively. The regression includes firm and prefecture-industry-year fixed effects and a set of control variables consisting of the logarithm of the number of transaction partners, age, logarithm of the number of employees, and distance to the disaster area. All control variables are measured for the year 2010 and interacted with 2010 and 2012 time dummies.

to only consist of firms that are located outside the four prefectures that constitute the broader earthquake-affected area.

The results are reported in panel (b) of Figure 7. Once again, the pattern that emerges resembles our baseline estimates in Figure 5, with negative and statistically significant post-earthquake estimates coupled with mostly insignificant pre-trends in the year prior to the earthquake. In terms of magnitude, the point estimates for the impact of the shock on the downstream and upstream firms at various distances are lower than our baseline specification. For example, in the year following the earthquake, downstream and upstream distance 1 firms experienced 2.3 and 1.9 percentage point lower growth rates compared to control group firms, respectively (compared to 3.6 and 2.9 percentage points in the baseline specification).

Electricity and Power Shortages The triple shocks of earthquake, tsunami, and nuclear disaster resulted in severe disruptions in the supply of electricity well into the summer of 2011, as several nuclear and conventional power plants in northeast Japan went offline, affecting the production capacity of two regional electricity providers, Tohoku Electric Power and Tokyo Electric Power Company (TEPCO). The reduction in generation capacity resulted in rolling (controlled) blackouts throughout March, followed by a power saving edict for the summer months of 2011 (when demand was higher), requiring large-lot users in Kanto and Tohoku regions to reduce power use by 15%.

Given the above, one may wonder the extent to which propagation patterns documented above are mainly driven by disruptions to the supply of electricity.¹⁸ In particular, our estimates may misattribute the disruption in this single—and arguably universal—input to the propagation of the shocks over general input-output linkages if firms close to the disaster area were simultaneously (i) more likely to be affected by the power outages and (ii) more likely to transact with nearby firms inside the disaster area.

To address these concerns, we rely on an unusual feature of Japan's power grid: while Tokyo and the rest of eastern Japan run at 50Hz frequency, the electricity in the western half of the country has a 60Hz frequency. This frequency difference partitions Japan's national grid into two halves of roughly equal sizes, with limited capacity for power transmission across the 50Hz-60Hz divide.¹⁹ This idiosyncrasy of Japanese electric power system means that the adverse effects directly tied to power outages in the aftermath of the earthquake were confined to the eastern half of the country (depicted as the shaded region in Figure 8), with little impact on the supply of electricity in western Japan.

This can be seen from Figure 9, which plots the seasonally-adjusted index of energy production for the two regions. As the figure illustrates, while the eastern half of the country (corresponding to

¹⁸For example, Schnell and Weinstein (2012) argue that the large negative impact on industrial production in the wake of the earthquake was linked to the substantial and persistent drop in energy output. Also see Allcott, Collard-Wexler, and O'Connell (2016), who document a significant effect of electricity shortages on Indian manufacturing plants.

¹⁹At the time of the earthquake, the frequency conversion capacity across the east-west frontier was 1.2 gigawatts (U.S. Energy Information Administration, 2015). According to estimates by Ministry of Economy, Trade and Industry (2011), the earthquake and its aftermaths resulted in a shortfall in generation capacity more than 6 times as large (7.3 gigawatts). The total installed generation capacity of electric utility companies in Japan was 234 gigawatts (Japan Electric Power Information Center, 2015).

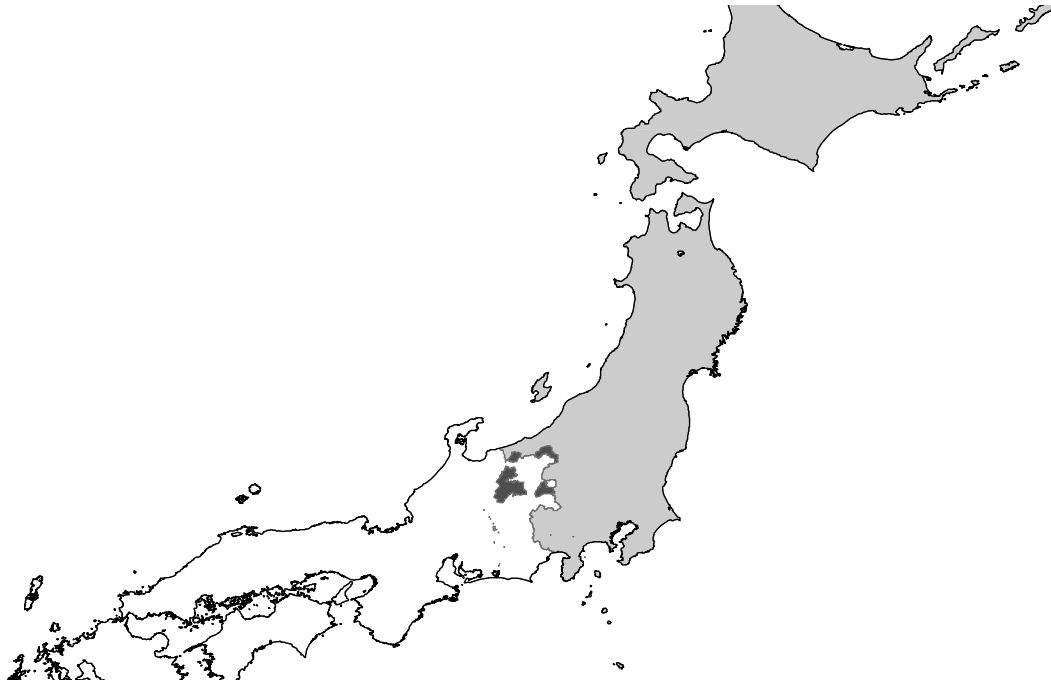


Figure 8. The Frequency Frontier

Note: The map depicts the partition of Japan's electric power system into the 50Hz (lightly shaded area) and 60Hz (unshaded area) regions. The darker areas near the boundary depict regions in which users of both frequencies exist.

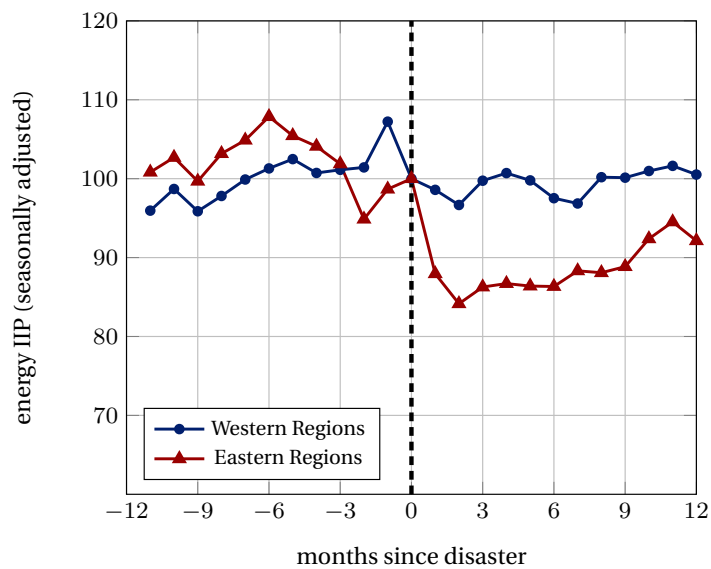


Figure 9. Energy Production

Notes: The figure plots the seasonally adjusted index of energy production for March 2010–February 2012. The index is normalized to 100 for February 2011. The horizontal axis refers to months since the disaster, with 0 corresponding to February 2011. “Eastern Regions” covers Hokkaido, Tohoku, and Kanto, all of which run on 50Hz electricity. “Western Regions” covers Chubu, Kinki, Chugoku, and Kyushu, all of which run on 60Hz electricity. The two regions together cover the entire country, except for the Shikoku region in the west for which METI does not construct an energy IIP index. The two time series are obtained by aggregating regional energy IIPs weighted by the corresponding regional shares of energy production in 2009. IIP data is obtained from Ministry of Economy, Trade and Industry of Japan.

Tohoku, Hokkaido, and Kanto regions, all running on 50Hz electricity) experienced a sharp decline in energy production, production levels in the regions to the west of the 50Hz-60Hz frontier (that is, Chubu, Kinki, Chugoku, and Kyushu) remained roughly at the pre-earthquake levels. Together with the divided nature of Japan’s power grid, this observation illustrates that the western half of the country did not experience any meaningful decline in either production or supply of electricity in the aftermath of the earthquake.

Thus, as our next robustness check, we rerun regression (1) on the subsample of firms located to the west of the 50Hz-60Hz “frequency frontier”. The results are reported in panel (c) of Figure 7, which illustrates that the estimated coefficients (and in particular their pattern) remain largely unchanged, indicating that the shock to the disaster-area firms propagated both upstream and downstream to firms located in the 60Hz region, with a smaller effect on firms with higher network distances to the disaster-area firms.

Placebo Production Network As a final robustness check, we conduct a falsification exercise by rerunning our baseline regression on a randomly-generated production network. To this end, we take the actual production network constructed using TSR data as our starting point, draw a random production network uniformly at random while preserving the identity and the number of customers of all firms, and use the resulting production network to recalculate all firms’ upstream and downstream network distances to the disaster-area firms. We then rerun our baseline specification (1), with the results reported in column 5 of Table A.1. As the table indicates, we find no evidence of propagation of the disaster shock over input-output linkages in this placebo network, as almost all coefficients corresponding to firms at various upstream and downstream distances are small and statistically insignificant.

5 Theoretical Framework

Our results in the previous section provide evidence for significant propagation of the shocks caused by the earthquake and its aftermaths from disaster-area firms to their direct and indirect suppliers and customers throughout Japan. Such a reduced-form analysis, however, is inadequate for quantifying the macroeconomic impact of the disaster on the Japanese economy. In this section, we outline a general equilibrium model of production networks in the spirit of Long and Plosser (1983) and Acemoglu et al. (2012) to capture the role of inter-firm linkages as a shock propagation mechanism. The model serves three purposes: (i) it provides us with a framework to theoretically capture the propagation of firm-level capital destruction shocks over the economy’s production network; (ii) it allows us to quantify the macroeconomic impact of the disaster on the Japanese economy; and (iii) it serves as an essential ingredient for counterfactual analyses.

We outline the model in this section and provide conditions under which its predictions are broadly consistent with our reduced-form empirical findings in Section 4. We then estimate the model’s key parameters in the subsequent section and use the estimated model as the basis of our aggregation and counterfactual analyses.

5.1 Model

Consider a static economy consisting of n competitive firms denoted by $\{1, 2, \dots, n\}$, each of which producing a distinct product. Each product can be either consumed by the households or used as an intermediate input for production of other goods. Firms employ nested constant-elasticity-of-substitution (CES) production technologies with constant returns to transform labor, firm-specific capital, and intermediate goods into output. In particular, the output of firm i is given by

$$y_i = \left[\chi(1 - \mu)^{1/\sigma} \left((z_i k_i)^\alpha l_i^{1-\alpha} \right)^{(\sigma-1)/\sigma} + \mu^{1/\sigma} M_i^{(\sigma-1)/\sigma} \right]^{\sigma/(\sigma-1)}, \quad (2)$$

where l_i is the amount of labor hired by the firm, k_i denotes firm-specific capital, and $z_i \leq 1$ is a capital-augmenting shock to firm i with a steady-state value that is normalized to 1.²⁰ In the above expression, α captures the share of capital in the bundle of primary factors (i.e., labor and capital), μ parametrizes the share of material inputs in firms' production technology, σ represents the elasticity of substitution between primary factors and the intermediate input bundle, and $\chi = (\alpha^\alpha (1 - \alpha)^{1-\alpha})^{1/\sigma-1}$ is a normalization constant whose sole purpose is to simplify the analytical expressions. Finally, the intermediate input bundle, M_i , is itself a CES aggregate of inputs purchased from other firms:

$$M_i = \left[\sum_{j=1}^n a_{ij}^{1/\xi} x_{ij}^{(\xi-1)/\xi} \right]^{\xi/(\xi-1)},$$

where x_{ij} is the amount of good j used in the production of good i and ξ is the elasticity of substitution between different intermediate goods. The coefficient $a_{ij} \geq 0$ designates the importance of good j as an intermediate input for the production of good i : a larger a_{ij} means that good j is a more important input in the production technology of firm i , whereas $a_{ij} = 0$ if firm i does not rely on good j as an intermediate input for production. Throughout, we normalize these coefficients by assuming that $\sum_{j=1}^n a_{ij} = 1$ for all i .

In addition to the firms described above, the economy is populated by a unit mass of identical households, who supply L units of labor and firm-specific capital (k_1, \dots, k_n) inelastically to the firms.²¹ Households have logarithmic preferences over the n goods given by

$$u(c_1, \dots, c_n) = \sum_{i=1}^n \beta_i \log(c_i / \beta_i),$$

where c_i denotes the amount of good i consumed. The constants $\beta_i \geq 0$ measure various goods' shares in the household's utility function, normalized such that $\sum_{i=1}^n \beta_i = 1$.

²⁰Note that while we assume a fixed endowment of firm-specific capital k_i that is subject to capital-augmenting shock $z_i \leq 1$, this formulation is equivalent to assuming that the shock results in the partial destruction of the firm's capital stock, resulting in a post-shock endowment of $z_i k_i$. This is our preferred interpretation of the shock in the context of our empirical analysis.

²¹The ownership of the primary factors of production is immaterial. For example, instead of assuming that households own all primary factors, we can assume that capital k_i is owned by firm i , whose shares are owned by the representative household.

The competitive equilibrium of this economy is defined in the usual way: it consists of a collection of prices and quantities such that (i) the representative consumer maximizes her utility; (ii) all firms maximize their profits while taking all prices as given; and (iii) all markets clear.

Before characterizing the equilibrium, we define a few standard but key concepts that are central to the analysis. First, note that we can summarize the inter-firm input-output linkages by matrix $\mathbf{A} = [a_{ij}]$, which also coincides with the economy's steady-state input-output matrix. We also define the economy's *Leontief inverse* as $\mathbf{L} = (\mathbf{I} - \mu\mathbf{A})^{-1}$, whose (i, j) element measures the importance of firm j as a direct and indirect input supplier to firm i . Thus, for example, $\ell_{ij} = 0$ if and only if firm j is not a direct or indirect supplier of i . Finally, we denote firm i 's sales as a share of GDP (which is also known as its Domar weight) by $\lambda_i = p_i y_i / \text{GDP}$, where p_i is the price of good i and y_i is firm i 's output.

5.2 Propagation of Shocks over the Production Network

Our first result provides a characterization of how shocks propagate over the economy's production network. Since the equilibrium in this economy does not have a closed-form representation in general, we consider a first-order approximation of equilibrium quantities and prices around the point where elasticity parameters σ and ξ are close to 1. Besides providing us with a closed-form representation, this approximation leads to a relationship that is linear in σ and ξ , thus enabling us to estimate these parameters using linear regression.²²

Proposition 1. *The impact of a shock to firm j on the sales share of firm i is given by*

$$\begin{aligned} \frac{d \log \lambda_i}{d \log z_j} = & (\sigma - 1) \sum_{h=1}^n \alpha \mu (1 - \mu) (\lambda_h / \lambda_i) \left[\left(\sum_{r=1}^n a_{hr} \ell_{rj} \right) \left(\sum_{s=1}^n a_{hs} \ell_{si} \right) - \ell_{hj} \sum_{r=1}^n a_{hr} \ell_{ri} \right] \\ & + (\xi - 1) \sum_{h=1}^n \alpha \mu (1 - \mu) (\lambda_h / \lambda_i) \left[\sum_{r=1}^n a_{hr} \ell_{ri} \ell_{rj} - \left(\sum_{r=1}^n a_{hr} \ell_{rj} \right) \left(\sum_{s=1}^n a_{ks} \ell_{si} \right) \right], \end{aligned} \quad (3)$$

where $\mathbf{L} = [\ell_{ij}]$ denotes the economy's *Leontief inverse*.

The above result, which will serve as the basis of our aggregation and counterfactual analyses in the next section, highlights the role of the economy's production network (captured by matrices \mathbf{A} and \mathbf{L}) in the propagation of shocks: shocks to firm j induce j 's direct and indirect customers to change the composition of their expenditures on various inputs, thus resulting in a change in demand expenditures on the good produced by firm i . More specifically, the first sets of summands on the right-hand side of (3) captures how substitution between primary and intermediate inputs by any given firm h shapes i 's sales share in response to shocks to j , while the second sets of summands captures the corresponding effect of substitution between various intermediate inputs.²³

²²This is the approach we pursue in Section 6. Reassuringly, our estimates for σ and ξ are consistent with this approximation.

²³See an earlier draft of this paper (Carvalho et al., 2017) as well as Baqaee and Farhi (2019) for related results on the propagation of TFP shocks. Also note that an immediate consequence of Proposition 1 is that in a Cobb-Douglas economy

Before exploring the role of the network structure and the elasticity parameters σ and ξ in further detail, a few remarks are in order. First, we note that since

$$\frac{d \log(p_i y_i)}{d \log z_j} = \frac{d \log \lambda_i}{d \log z_j} + \frac{d \log \text{GDP}}{d \log z_j}, \quad (4)$$

we can use equation (3) to obtain an expression for how propagation of the shock over the production network impacts firms' sales. Second, since the second term on the right-hand side of (4) is common to all firms, the expression in (3) can also be interpreted as the model-implied counterpart to our difference-in-differences specification in Section 4, capturing the differential impact of the shock on firms' sales.

To clarify the role of the economy's production network in the propagation of shocks, in the remainder of this section, we focus our attention to firm pairs that are purely upstream or downstream to one another in the sense formalized below. This enables us to distill the implications of Proposition 1 for upstream and downstream propagation of shocks in a transparent manner.

Definition 1. Firm i is *purely upstream* to firm j and firm j is *purely downstream* to firm i , if (i) $\ell_{ji} > 0$, (ii) $\ell_{ij} = 0$, and (iii) if $\ell_{hj} > 0$ for some firm h , then $\ell_{hi} = \ell_{hj}\ell_{ji}$.

The notions of pure upstreamness and downstreamness defined above have intuitive interpretations. Recall from the definition of the Leontief inverse that the restriction that $\ell_{ji} > 0$ implies that firm i is a direct or indirect supplier of firm j , while the restriction that $\ell_{ij} = 0$ means that j is neither a direct nor indirect supplier of i . Finally, the third restriction in Definition 1 ensures that all directed paths from firm i to any direct or indirect customer of j pass through j , thus ruling out the possibility of a “horizontal” relationship between i and j as co-suppliers of a common set of customers.²⁴ Taken together, these restrictions ensure that the only relationship of j vis-à-vis firm i is as a direct or indirect customer.

With the above definition in hand (and anticipating our estimation results in the following section), we next provide sufficient conditions on the elasticities of substitution under which the model's predictions are broadly consistent with our reduced-form empirical findings in Section 4. We start with upstream propagation:

Proposition 2. Suppose firm i is purely upstream to firm j in the sense of Definition 1. Also suppose $\sigma < \min\{1, \xi\}$. Then,

- (a) a negative shock to firm j reduces the sales share of firm i , i.e., $d \log \lambda_i / d \log z_j > 0$.
- (b) Let i and s be two firms of equal sizes that are purely upstream to firm j . Then $d \log \lambda_i / d \log z_j \geq d \log \lambda_s / d \log z_j$ if and only if $\ell_{ji} \geq \ell_{js}$.

The above result, which is a consequence of Proposition 1, provides sufficient conditions for upstream propagation of shocks. In particular, statement (a) establishes that a negative shock to

(i.e., when $\sigma = \xi = 1$) such as those analyzed in Long and Plosser (1983) and Acemoglu et al. (2012), $d \log \lambda_i / d \log z_j = 0$. This is of course to be expected: when all production functions are Cobb-Douglas, firms' expenditure shares on various inputs do not change in response to shocks.

²⁴We prove this statement in Lemma B.1 in the appendix.



Figure 10. Upstream and downstream propagation of shocks

Note: Each vertex corresponds to an industry, with a directed edge present from one vertex to another if the former is an input-supplier to the latter. In panel (a) firm i is purely upstream to firm j in the sense of Definition 1. In panel (b) firm i is purely downstream to firm j . In addition to intermediate inputs, all firms use labor and capital as inputs for production (not depicted in the figure).

firm j propagates to all its direct and indirect suppliers, resulting in a reduction in their sales shares. Statement (b) then establishes that the shock's impact is greater on firms that are more important (direct and indirect) suppliers of j , consequently implying that the shock's impact is diminished as it travels upstream over the production network.

To see the intuition behind Proposition 2 in a simple setting, consider the production network depicted in Figure 10(a), in which firms i is purely upstream to firm j . If firm j is hit with a negative shock, two effects ensure that the shock results in a reduction in i 's sales shares. First, the assumption that $\sigma < 1$ implies that primary and intermediate inputs are gross complements. Therefore, in response to a capital-augmenting shock, firm j reduces its expenditure share on intermediate inputs, consequently resulting in a reduction in i 's sales share. Second, the assumption that $\sigma < \xi$ implies that, in response to the shock, firm m increases its expenditure share on input h at the expense of firm j and all its direct and indirect suppliers, thus creating a second channel for upstream propagation of the shock to firm i . Note that, in view of equation (4), the predictions of Proposition 2 also apply to firms' sales: the same forces ensure that the negative shock to j reduces the sales of the upstream firm i .

Our next result provides the counterpart to Proposition 2 for downstream propagation.

Proposition 3. *Suppose firm i is purely downstream to firm j . Also, suppose there exists a firm m such that $a_{mi} \in (0, 1)$. Then, for any $\xi > 1$, there exists $\bar{\sigma} < 1$ such that if $\sigma > \bar{\sigma}$, then*

- (a) *a negative shock to firm j reduces the sales share of firm i , i.e., $d \log \lambda_i / d \log z_j > 0$.*
- (b) *Let i and s be two firms of equal size that are purely downstream to firm j . Then $d \log \lambda_i / d \log z_j \geq d \log \lambda_s / d \log z_j$ if and only if $\ell_{ij} \geq \ell_{sj}$.*

This result provides a set of sufficient conditions on the elasticities of substitution under which a

negative shock to firm j reduces the sales shares of its direct and indirect customers (statement (a)), with the impact of the shock decreasing the less such firms rely on firm j as a supplier (statement (b)). The latter statement implies that the shock's impact is diminished as it travels downstream over the production network. Also once again, equation (4) guarantees that the predictions of Proposition 3 apply not just to downstream firms' sales shares but also their sales.

To see the intuition underlying Proposition 3, consider the simple production network in Figure 10(b), in which firm i is purely downstream to firm j . First, note that a negative shock to firm j increases the price of good i used as an input by firm m . Since various intermediate inputs are gross substitutes ($\xi > 1$)—and as long as primary and intermediate inputs are not strong complements—such an increase in i 's price induces firm m to reduce its expenditure share on i 's input, thus effectively creating a downstream propagation channel from firm j to firm i .²⁵

6 Aggregation and Counterfactual Analyses

In this section, we use the TSR firm-level data to estimate the key parameters of the model presented in Section 5. We then use our estimated model to (i) quantify the macroeconomic impact of the disaster on the Japanese economy and (ii) perform various counterfactual analyses.

6.1 Estimation

Recall that equation (3) expresses the change in firms' sales shares in response to shocks as a function of the economy's production network and model parameters. As a result, in conjunction with the TSR data on supplier-customer relations and firm-level sales, it provides us with the natural starting point for estimating the model. Another advantage of using (3) is that it provides us with an expression that is jointly linear in σ and ξ , thus enabling us to estimate these parameters using linear regression. Given the large number of firms in our dataset, this linearity is crucial for making the estimation procedure computationally feasible.²⁶

We start with the following implication of equation (3): in response to a vector of shocks $\Delta \log z$, the log sales of firm i is given by

$$\log(p_i y_i) = \log(p_i^{\text{ss}} y_i^{\text{ss}}) + (\log \text{GDP} - \log \text{GDP}^{\text{ss}}) + (\sigma - 1)\Sigma_i + (\xi - 1)\Xi_i,$$

where GDP^{ss} and $p_i^{\text{ss}} y_i^{\text{ss}}$ are the steady-state (i.e., pre-shock) levels of aggregate output and sales of firm i , respectively, Σ_i and Ξ_i denote the i -th elements of vectors

$$\Sigma = \alpha\mu(1 - \mu)\Lambda^{-1}\mathbf{L}'(\mathbf{A}'\Lambda\mathbf{A} - \mathbf{A}'\Lambda)\mathbf{L} \Delta \log z \quad (5)$$

$$\Xi = \alpha\mu(1 - \mu)\Lambda^{-1}\mathbf{L}'(\text{diag}(\mathbf{A}'\Lambda\mathbf{1}) - \mathbf{A}'\Lambda\mathbf{A})\mathbf{L} \Delta \log z, \quad (6)$$

²⁵This discussion also highlights the role of assumption $a_{mi} \in (0, 1)$ for some firm m in the statement of Proposition 3. It guarantees that firm i has a customer m that can substitute away to other intermediate inputs in the face of an increase in i 's output price.

²⁶Note that the expression in (3) is not jointly linear in all model parameters, $(\alpha, \mu, \sigma, \xi)$. However, as we discuss in subsequent paragraphs, α and μ can be calibrated directly using aggregate data.

and $\mathbf{\Lambda} = \text{diag}(\lambda_1^{\text{ss}}, \dots, \lambda_n^{\text{ss}})$ is the diagonal matrix of firms' pre-shock Domar weights. Therefore, provided we can measure the vectors in (5) and (6), the elasticities of substitution σ and ξ can be estimated using the following specification:

$$\log(p_{it}y_{it}) = \gamma_i + \gamma_t + \beta_1(\Sigma_i \times \text{year}_{2012}) + \beta_2(\Xi_i \times \text{year}_{2012}) + \varepsilon_{it}, \quad (7)$$

in which $\sigma = \beta_1 + 1$ and $\xi = \beta_2 + 1$. In the above specification, year_{2012} is a time dummy for the post-earthquake year and γ_i and γ_t denote firm and time fixed effects, respectively.

To construct the covariates Σ_i and Ξ_i on the right-hand side of (7), first note that the matrix of sales shares $\mathbf{\Lambda}$ can be readily constructed from data on firms' pre-earthquake sales. Next, consider parameters μ and α . It is straightforward to verify that, in the model's steady state, these parameters coincide with aggregate intermediate input and capital income shares, respectively. In particular, $\mu = 1 - \text{GDP} / \sum_{i=1}^n p_i y_i$ and $\alpha = \sum_{i=1}^n r_i k_i / \text{GDP}$. We therefore set $\mu = 0.5$ to match Japan's aggregate intermediate input share (Ministry of Economy, Trade and Industry, 2012), while setting parameter $\alpha = 0.36$ to match Japan's capital income share (Hayashi and Prescott, 2002).

Next, we need to construct matrices \mathbf{A} and \mathbf{L} , which capture the economy's production network. Recall that while we can use the TSR data to identify each firm's transaction partners (and thus obtain a binary measure of inter-firm supplier-customer relations), the dataset does not provide us with firms' pairwise transaction volumes. We circumvent this issue by combining our binary production network data with the 2011 industry-level input-output tables constructed by the Ministry of Internal Affairs and Communications to proxy for expenditure shares at the firm level. More specifically, for any given customer-supplier pair of firms (i, j) in our data, we set a_{ij} to be proportional to the entry of the input-output table corresponding to industries that i and j belong to, normalized such that matrix \mathbf{A} has row sums equal to one.²⁷

With matrix \mathbf{A} in hand, we next calculate the Leontief inverse $\mathbf{L} = (\mathbf{I} - \mu\mathbf{A})^{-1}$. Given the large number of firms in our dataset, obtaining \mathbf{L} using a matrix inversion operation is computationally infeasible. As an alternative, we approximate the Leontief inverse by the first 51 terms of its so called Neumann series, given by $\mathbf{L} \approx \sum_{r=0}^{50} (\mu\mathbf{A})^r$.^{28,29}

Finally, to construct the expressions in (5) and (6), we need an estimate for the rate of capital destruction $\Delta \log z$. Since we do not have firm-specific information on the disaster's impact on firms'

²⁷To implement this step, we first use Japan Industrial Productivity database (available at <https://www.rieti.go.jp/en/database/JIP2018/>) to obtain a concordance between sector classifications in input-output tables and Japanese Standard Industry Codes used in the TSR dataset. This procedure results in 315 industries. Since input-output tables do not report transaction volumes for non-manufacturing industries that consign production (wholesale and retail trade, including trading companies and department stores), firm-pairs in such industries cannot be included in this procedure. For these firms, we assign uniform weights across their suppliers. We also use uniform weights if the entry in the input-output table corresponding to a pair of firms is equal to zero. The 2011 input-output tables are available at https://www.soumu.go.jp/english/dgpp_ss/data/io/io11.htm.

²⁸The convergence of the (infinite) Neumann series $\sum_{r=0}^{\infty} (\mu\mathbf{A})^r$ to $\mathbf{L} = (\mathbf{I} - \mu\mathbf{A})^{-1}$ is guaranteed by the facts that $\mu < 1$ and the spectral radius of \mathbf{A} is equal to 1 (Stewart, 1998, p. 55).

²⁹Note that, given the scale of the problem, computing and storing the various powers \mathbf{A}^r in the Neumann series is also computationally prohibitive for large values of r due to memory constraints. Therefore, we forego the explicit computation of \mathbf{A}^r —and directly approximating \mathbf{L} —and instead rely on the observation that to construct the expressions in (5) and (6), it is sufficient to compute vectors of the form $\mathbf{A}^r v$ for some pre-specified vector v . Vectors of the form $\mathbf{A}^r v$ can be computed and stored efficiently using the recursion $\mathbf{A}^r v = \mathbf{A}(\mathbf{A}^{r-1} v)$, which only requires storing matrix \mathbf{A} and a single vector at each step.

Table 3. Estimation of Elasticities of Substitution

	baseline (1)	capital destruction rate		intermediate input share		uniform-weighted network (6)
		low (2)	high (3)	low (4)	high (5)	
σ	0.593*** (0.062)	0.780*** (0.066)	0.417*** (0.090)	0.554*** (0.076)	0.638*** (0.051)	0.569*** (0.062)
ξ	1.183*** (0.034)	1.204*** (0.037)	1.099** (0.047)	1.287*** (0.049)	1.111*** (0.024)	1.155*** (0.035)

Notes: The table reports estimates for the elasticities of substitution implied by regression specification (7), with robust standard errors reported in parentheses. *, **, and *** denote significance for the null hypothesis of the estimate being equal to 1 at the 10%, 5%, and 1% levels, respectively.

capital stock, we rely on two sets of estimates produced by the government of Japan on the extent of capital destruction in disaster-hit municipalities. Using these estimates, we obtain a lower bound of 21.9% and an upper bound of 43.2%.³⁰ We take the mid-point of this range, 32.5%, as our baseline estimate for the capital destruction rate in the disaster area and then evaluate the sensitivity of our results to this choice.

The first column in Table 3 reports the estimates for elasticities of substitution σ and ξ implied by regression (7) under our baseline specification of $\mu = 0.5$ and a capital destruction rate of 32.5%. As the table indicates, we find that (i) firm-level intermediate inputs are gross substitutes with an elasticity of substitution $\xi = 1.183$ (s.e. = 0.034), while (ii) primary and intermediate inputs are gross complements with an elasticity of substitution $\sigma = 0.593$ (s.e. = 0.062). Both of these estimates are statistically different from 1, thus ruling out the benchmark of Cobb-Douglas technologies.

These qualitative findings remain unchanged as we vary the specification. Columns 2 and 3 of Table 3 report the implied estimates for σ and ξ under the lower and upper bound scenarios of 21.9% and 43.2% for capital destruction rates constructed from government estimates. Columns 4 and 5 report the estimates for when the intermediate input share is set to $\mu = 0.4$ and $\mu = 0.6$, respectively. Finally, the last column of Table 3 reports the estimates for σ and ξ when we use a uniform-weighted matrix \mathbf{A} , according to which, if firm j is a supplier of firm i , then a_{ij} is set equal to the reciprocal of firm i 's number of suppliers. As the table indicates, the qualitative conclusions drawn under the baseline case remain unchanged: in all cases, intermediate inputs are estimated to be gross substitutes, while primary and intermediate inputs are estimated to be gross complements.³¹

³⁰These numbers are based on a government study released on June 24, 2011, which estimated a capital loss rate of 15.3% for municipalities affected by the earthquake but not the tsunami and a loss rate of 30.6% for tsunami-hit regions (Cabinet Office, Director General for Disaster Management, 2011a). This was followed by a worst-case scenario estimate of 80% for the tsunami-hit municipalities (Cabinet Office, 2011). We convert these numbers to a capital loss rate for firms in the disaster region by relying on Iwaki et al. (2011), who estimate the value of existing structures in the earthquake-hit and tsunami-hit municipalities as 25 and 19 trillion yen, respectively. These estimates result in a lower bound estimate of $(15.3 \times 25 + 30.6 \times 19)/(25 + 19) = 21.9\%$ and an upper bound estimate of $(15.3 \times 25 + 80 \times 19)/(25 + 19) = 43.2\%$.

³¹We also note that, in view of Propositions 2 and 3, these estimates indicate that capital-destruction shocks in the model generate propagation patterns that are consistent with our reduced-form empirical findings in Section 4.

We conclude with a brief discussion on how our micro-level estimates for the elasticities of substitution compare to other recent estimates in the literature. Our estimates for σ are in line with other recent studies, which also find evidence for complementarity between intermediates and value added. [Oberfield and Raval \(2019\)](#) use U.S. census data to obtain estimates for plant-level elasticity of substitution between primary and intermediate inputs in the manufacturing sector. Depending on the year, they estimate this elasticity to be between 0.57 and 1.03, with an average of 0.78. Our baseline estimate of $\sigma = 0.59$ agrees qualitatively with their findings, falling on the lower range of their reported interval. These estimates are larger than those obtained by [Boehm, Flaaen, and Pandalai-Nayar \(2019\)](#), who study the impact of the 2011 Great East Japan Earthquake on U.S. affiliates of Japanese multinationals. While they also find evidence for complementarity between primary and intermediate inputs, they obtain point estimates around 0.03, though with wide confidence intervals ranging from 0.03 to 0.67. In a more recent study, [Peter and Ruane \(2020\)](#) use plant-level data from Indian manufacturing plants and estimate the elasticity of substitution between the capital-labor bundle and intermediate inputs to be 0.6.

The literature reports a wider range of estimates for micro-level elasticity of substitution between various intermediate inputs: while [Boehm, Flaaen, and Pandalai-Nayar \(2019\)](#) find evidence of complementarities over quarterly horizons (reporting values ranging between 0.20 and 0.62), [Peter and Ruane \(2020\)](#) find evidence in favor of substitutability between broad categories of material inputs over a seven-year horizon (with estimates ranging from 4.7 to 8). Our baseline estimate of $\xi = 1.18$ falls between these two studies' estimates and suggests modest amounts of substitutability over a one-year horizon.³²

6.2 Aggregation

With estimates for the elasticities of substitution σ and ξ in hand, we next use the model to quantify the macroeconomic impact of the earthquake and its aftermaths on the Japanese economy.

We start with the observation that, according to the model, the disaster's impact on the economy's aggregate output is given by

$$\Delta \log \text{GDP} = \alpha(1 - \mu)\mathbf{1}' \left(\frac{\mathbf{\Lambda} + \mathbf{\Lambda}^*}{2} \right) \Delta \log z + \frac{1}{2} \alpha^2 \mu(1 - \mu)(\sigma - 1) \Delta \log z' \mathbf{\Lambda}(\mathbf{I} - \mathbf{A})\mathbf{L} \Delta \log z \quad (8)$$

up to a second-order approximation in the size of the shock, where $\mathbf{\Lambda}$ and $\mathbf{\Lambda}^*$ are diagonal matrices of firms' pre- and post-disaster sales shares, respectively. We rely on a second-order approximation because it allows us to capture the nonlinearities induced by non-unit elasticities, while at the same time providing us with a tractable expression for the disaster's aggregate impact.³³

³²See [Ruhl \(2008\)](#), who emphasizes that the size of the appropriate elasticity is tied to the time horizon considered. Arguably, it is natural to expect that the elasticity of substitution between various intermediate inputs also depends on the level of aggregation, with a lower degree of substitutability between more highly aggregated intermediate input categories. Indeed, using industry-level data from the U.S., [Atalay \(2017\)](#) finds evidence for complementarity between intermediate inputs at the sectoral level.

³³See Appendix B for the detailed derivations that lead to equation (8). Using an approach similar to [Baqae and Farhi \(2019\)](#), one can also express the second-order approximation to disaster's aggregate impact in terms of the discretized Divisia index of firms' capital expenditures as a share of GDP i.e., $\Delta \log \text{GDP} = (1/2) \sum_{i=1}^n (\eta_i + \eta_i^*) \Delta \log z_i$, where η_i and η_i^*

Thus, in conjunction with the data on the economy’s production network and our estimates for the elasticity parameters, equation (8) provides us with an expression to quantify the aggregate impact of the disaster. Using our baseline estimates from Table 3, we estimate that the disaster resulted in a 0.47 percentage point decline in Japan’s real GDP growth in the year following the disaster. For comparison, Japan’s average growth rate in the decade prior to the disaster was equal to 0.6% (with a standard deviation of 2.4%), while a value-added accounting based on the economic size of the affected prefectures (as in footnote 2) would have only attributed 0.1 percentage point decline in GDP growth to the disaster.

We can also use the model to quantify the contribution of supply chain linkages between firms inside and outside the disaster area to the disaster’s aggregate impact. To this end, we consider a counterfactual economy that is identical to the actual economy, except with no input-output linkages between disaster-area firms and the firms in the rest of the country.³⁴ By construction, input-output linkages in such an economy can no longer propagate the shock to firms outside the disaster area. Applying equation (8) to this counterfactual economy, we find that the disaster would have resulted in a 0.21 decline in GDP growth rate, thus implying that the propagation of the shock to firms outside the disaster area over input-output linkages amplified the disaster’s aggregate impact by more than twofold.

As a final remark, we note that our estimate for the macroeconomic impact of the disaster is comparable to those obtained by Raddatz (2009) and Noy (2009), who measure the aggregate impact of a host of different natural disasters using cross-country reduced-form time-series models. Focusing on a sample of high-income countries, Raddatz (2009) finds that natural disasters have moderate but significant macroeconomic impacts, with a climatic disaster affecting at least half a percent of a country’s population translating into a 0.25% reduction in GDP per capita. Noy (2009), on the other hand, estimates that large natural disasters—defined as those that are one standard deviation above the mean natural disaster in terms of direct damages—reduce subsequent output growth by about 1 percentage point in developed economies.

6.3 Counterfactual Analysis: The Tokai Earthquake

We conclude this section by using our estimated model to obtain general equilibrium forecasts for the macroeconomic impact of what is known as the Tokai Earthquake. The Tokai Earthquake is a large-scale earthquake expected to occur along the Nankai Trough, with an assumed epicenter area extending from Suruga Bay to the inland area of Shizuoka prefecture in central Japan. The

are firm i ’s pre- and post-disaster capital expenditures as a share of GDP, respectively. However, since we do not have access to firm-level information on capital expenditures, quantifying the shock’s aggregate impact using such a sufficient statistic approach is not feasible. As a result, we instead rely on the model-implied equation (8). This equation also illustrates that firms’ pre- and post-disaster sales shares are not sufficient statistics for the disaster’s aggregate impact either, as the second term on the right-hand side of (8) also depends on matrix \mathbf{A} and elasticity σ . This is because, when $\sigma \neq 1$, firms’ capital expenditure shares may not remain constant in response to shocks.

³⁴Formally, such a counterfactual economy is constructed as follows. Let \mathbf{A} and $\tilde{\mathbf{A}}$ denote the input-output matrices of the actual and counterfactual economies, respectively. We set $\tilde{a}_{ij} = \tilde{a}_{ji} = 0$ whenever exactly one firm in the pair (i, j) belongs to the disaster area. We set $\tilde{a}_{ij} \propto a_{ij}$ whenever both firms are either in the disaster area or outside of it. Such a construction ensures that all input-output linkages between the disaster area and the rest of the country are broken, while at the same time preserving the economy’s input-output structure within the two regions.

Table 4. Aggregate Impact of a Potential Earthquake in the Tokai Region

	Exceedance Probability				
	50%	40%	30%	20%	10%
Number of disaster-area firms	27,812	40,258	59,187	74,548	95,640
GDP impact (percentage points)	−0.48	−0.88	−1.25	−1.51	−2.17

Notes: The table reports the number of disaster-area firms and the macroeconomic impact of a potential earthquake for five different scenarios. Each disaster area is defined by intersecting the area designated by Central Disaster Management Council as the “Disaster Prevention Strengthening Area” with areas obtained from probabilistic seismic hazard maps. The exceedance probability is the probability that ground shaking exceeds an intensity level of 6-upper in JMA seismic intensity scale within a 30-year horizon. GDP impact measures the change in GDP growth in the year after the earthquake in percentage points.

government’s figures put the odds of a magnitude 8.0-plus Nankai Trough earthquake at 40–50% in the next 20 years, 60–70% in the next 30 years, and more than 90% in the next half century (Headquarters for Earthquake Research Promotion, 2013).³⁵ Aside from its high odds, the potentially significant economic costs of a major earthquake in the Tokai region has made it one of the main points of focus of disaster management in Japan. The region constitutes the industrial heartland of Japan and serves as one of the central hubs of Japanese and global manufacturing supply chains.

To determine the potential disaster area in the wake of a major earthquake in the Tokai region, we take the area designated by the Central Disaster Management Council as the “Disaster Preparation Strengthening Area” as our starting point (Cabinet Office, Director General for Disaster Management, 2012). This area consists of 157 municipalities spanning seven prefectures of Aichi, Gifu, Kanagawa, Mie, Nagano, Shizuoka, and Yamanashi and three remote islands in Tokyo. We then overlay this area with probabilistic seismic hazard maps—available from Japan Seismic Hazard Information Station—to determine the probability that ground shaking exceeds an intensity level of 6-upper at any given location within a 30-year period.³⁶ We define five nested disaster areas with exceedance probabilities of 50%, 40%, 30%, 20%, and 10% and, using the TSR dataset, identify the firms within each area. Table 4 reports the number of disaster-area firms under each scenario, with the geographic distribution of disaster-area firms depicted in Figure 11.

Having identified the disaster-area firms, we then use equation (8) and our estimates in Table 3 to quantify the macroeconomic impact of the earthquake under each scenario. As in our quantification exercise in the previous subsection, we assume a capital destruction rate of 32.5% for disaster-area firms. Table 4 reports the resulting estimates. We find that, with probability exceeding 50% within a 30-year horizon, a major earthquake in the Tokai region would result in a 0.48 percentage point

³⁵Major Nankai Trough earthquakes have occurred regularly at intervals of approximately 100 to 150 years. The trough is split into three main sections, known as Nankai, Tonankai, and Tokai. While Nankai and Tonankai regions were hit in 1946 and 1944, respectively, the last Tokai earthquake occurred in 1854.

³⁶The earthquake intensity is reported in Japan Meteorological Agency (JMA) seismic intensity units, which measures the intensity of local ground shaking and is different from the moment magnitude scale (which measures the amount of energy released by the earthquake). For more on the JMA intensity scale see <https://www.jma.go.jp/jma/en/Activities/inttable.html>. The probabilistic seismic hazard maps are available at <http://www.j-shis.bosai.go.jp/map>.

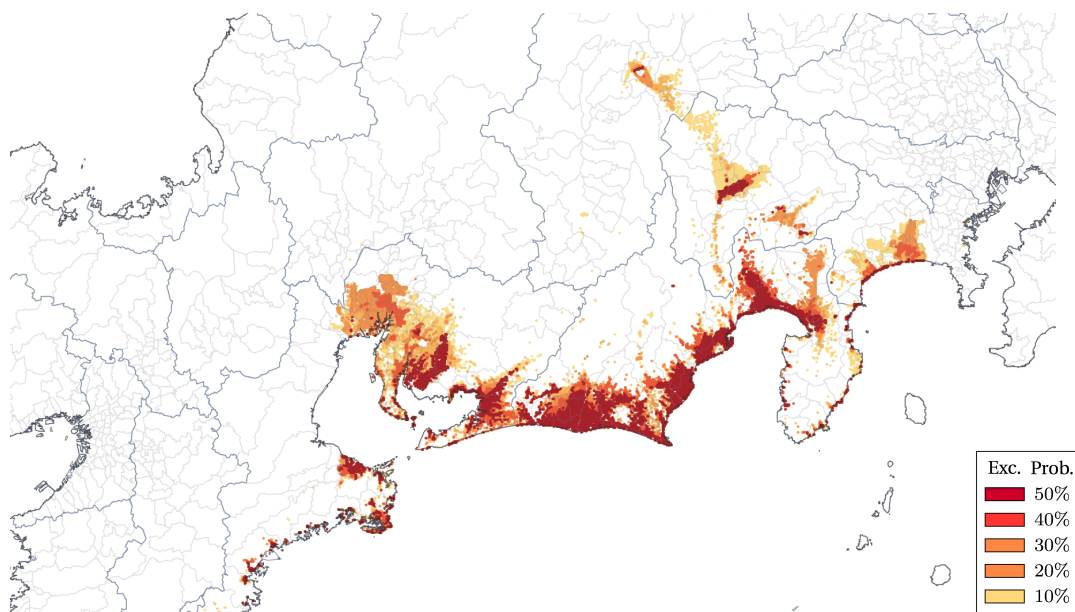


Figure 11. Geographic Distribution of Firms Exposed to the Tokai Earthquake

Note: The figure maps the geographic distribution of headquarters locations of firms in the five counterfactual disaster areas. Each disaster area is obtained by overlying the area designated by Central Disaster Management Council as the “Disaster Preparation Strengthening Area” with probabilistic seismic hazard maps from Japan Seismic Hazard Information Station. The exceedance probability is the probability that ground shaking exceeds an intensity level of 6-upper in JMA seismic intensity scale within a 30-year horizon.

decline in GDP growth in the following year, whereas with probability exceeding 10%, it would result in up to 2.17 percentage point decline in GDP growth.³⁷ The significantly larger estimated impact of such a potential earthquake compared to that of the Great East Japan Earthquake reflects the role of Tokai region as part of Japan’s industrial core and a key link in the country’s most important supply chains. We should note that these figures abstract from economic losses due to fatalities and damage to infrastructure, both of which are also expected to be significant.

7 Conclusions

At the backbone of any modern economy is an entangled web of specialized production units, ensuring the flow of goods and services from suppliers to other producers and final consumers. Precisely due to their vital role in the production process, a growing literature has argued that these input linkages can serve as a channel for the propagation and amplification of shocks throughout the economy.

In this paper, we investigate the nature and extent of these propagation effects by using a large-scale dataset on supply chain linkages among Japanese firms together with information on firm-

³⁷The upper range of these estimates are consistent with the government’s assessment of the maximum economic loss due to the earthquake, evaluated at 11 trillion yen in 2003, consisting of 3 trillion yen damage to production stoppage, 2 trillion yen due to transportation disruption, and 6 trillion yen due for propagation to outside of the disaster area (Cabinet Office, Director General for Disaster Management, 2003). Given Japan’s GDP in 2003 was equal to 502 trillion yen, this translates to $11/502 = 2.19\%$ reduction in GDP.

level exposures to the Great East Japan Earthquake in 2011. Leveraging the exogenous and localized nature of the earthquake and its aftermaths, we find strong evidence for the importance of inter-firm linkages as a shock transmission mechanism, documenting that the earthquake not only resulted in a decline in the growth rates of disaster-area firms' immediate downstream customers and upstream suppliers, but also propagated to firms that were only indirectly linked to disrupted firms. Using a general equilibrium model of production networks, we then obtain an estimate for the overall macroeconomic impact of the disaster by taking these propagation effects into account.

At the *micro* level, our findings suggest that, when faced with a supply-chain disruption, individual firms are unable to find suitable alternatives in order to completely insulate themselves from the shock (at least in the short run). This is consistent with an emerging literature (e.g., [Bernard, Moxnes, and Saito \(2019b\)](#), [Eaton, Kortum, and Kramarz \(2018\)](#), and [Barrot and Sauvagnat \(2016\)](#)) that emphasizes the role of search frictions and relation-specific investments along supply chains. However, it should be noted that our results are indicative of relatively small firm-level effects on average—corresponding to roughly a 2 to 3 percentage point decline in firm-level annual sales growth in our baseline specification—specially when compared to average firm-level sales growth volatility (which, as per [Davis, Haltiwanger, Jarmin, and Miranda \(2007\)](#), is arguably one order of magnitude larger).

At the *macro* level, our results point to the structure of production network linkages as a key driver of aggregate fluctuations. In particular, they indicate that, even if average firm-level effects are not necessarily large, the potential propagation of shocks over the economy's production network can propagate to a significant fraction of firms, thus resulting in movements in the aggregates.

We view our paper as a step towards a systematic empirical investigation of the role of input-output linkages as a mechanism for propagation and amplification of shocks. Several important issues, however, remain open to future research. First, while binary information on the presence of firm-level linkages enabled us to obtain estimates for the extent to which shocks propagate, using more detailed information on the value of firm-to-firm transactions would pave the way for a more complete analysis of the macroeconomic implications of production networks. Administrative firm-level data from countries with value-added tax (such as the dataset constructed by [Dhyne et al. \(2015\)](#) and [Magerman et al. \(2016\)](#) for Belgium) would serve as the ideal dataset for such a study.

Second, even though the input-output network in our model is assumed to be exogenous, in reality, firms decide on the set of suppliers and customers that they transact with. It is reasonable to expect that the extent to which firms can form new linkages has first-order implications for the nature and intensity of shock propagation (in particular, in horizons longer than what we considered in the paper). While recent work, such as [Oberfield \(2018\)](#) and [Acemoglu and Azar \(2020\)](#), have focused on how firm-level decisions shape productivity and the organization of production, developing a comprehensive framework for endogenous formation of networks would be crucial for the theoretical and empirical investigation of the role of input-output linkages in the propagation and amplification of shocks.

Last but not least, while our theoretical framework treats all supplier-customer relationships

as homogeneous—in the sense that any specific supplier is as easy to substitute away from as any other—real world supply chains are likely to exhibit important heterogeneity across input-output linkages. Arguably, firm-to-firm linkages are better approximated by the coexistence of easily-substitutable suppliers alongside suppliers producing customized inputs that are not easily substitutable. This, in turn, suggests the existence of key bottleneck firms with no short-run, satisfactory alternatives, whose outright exit may result in the destruction of relation-specific surplus between firms. On the theoretical side, developing a framework to capture the implications of movements in the extensive margin for the relationship-specific surplus that is at the heart of supply chains may require a departure from the competitive analysis that is prevalent in most of the literature. On the empirical side, documenting the relationship between patterns of firm failures and the ensuing rewiring process of the economy's production network would be a natural next step in understanding the role of firm-level production networks in propagating shocks.

A Additional Figures and Tables

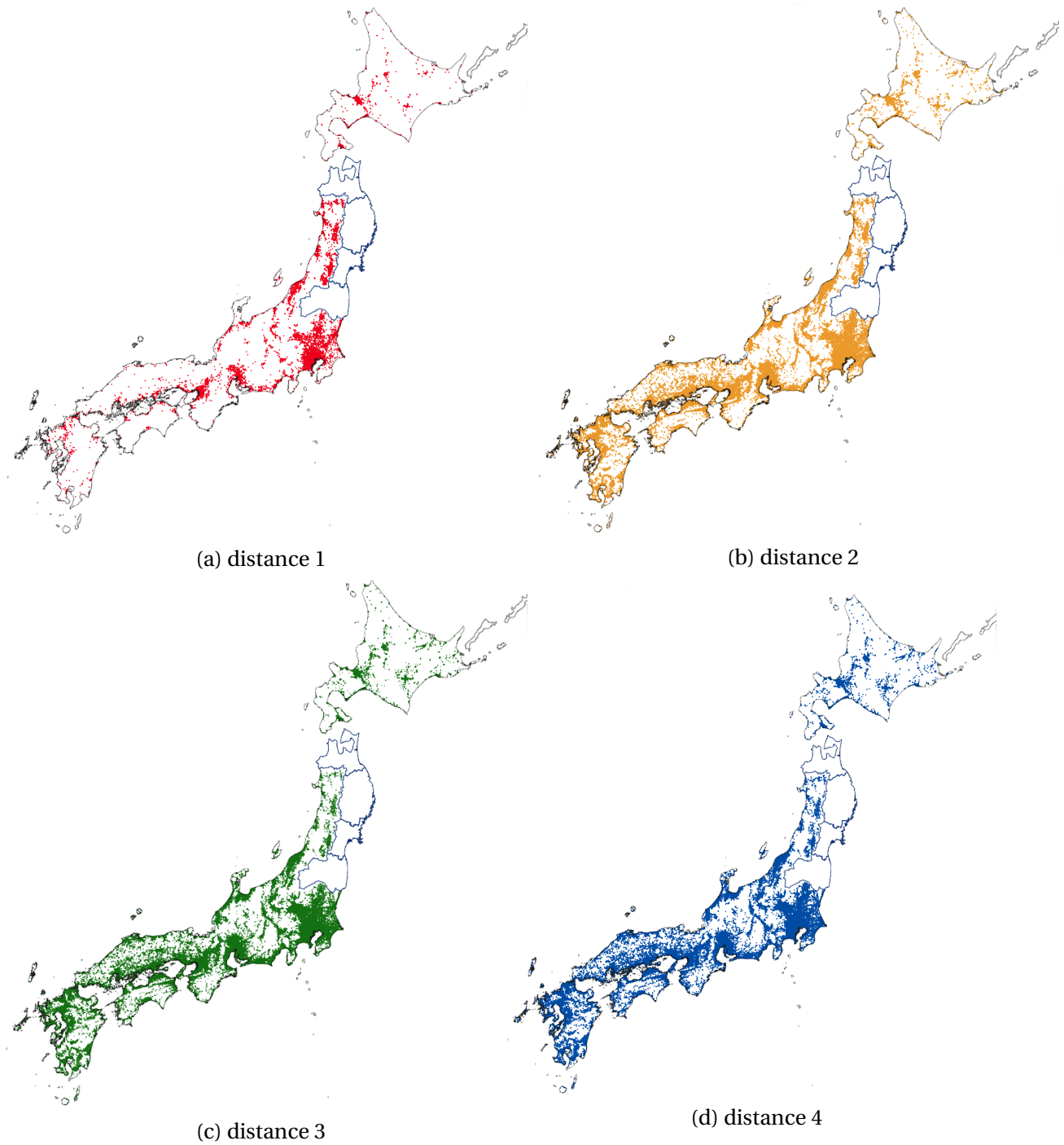


Figure A.1. Geographic Distribution of Firms of Various Network Distances to Disaster-Area Firms

Notes: The figure maps the geographic distribution of headquarters locations of firms of various network distances (upstream and downstream) to firms in the disaster area. Each dot represents a firm. The blue curve represents the boundary of the four disaster-stricken prefectures of Aomori, Fukushima, Iwate, and Miyagi.

Table A.1. Impact of Disaster on Firm Sales: Baseline and Robustness Checks

	baseline (1)	flooded area (2)	60Hz region (3)	single-plant firms (4)	placebo network (5)
Downstream ⁽¹⁾ × year ₂₀₁₂	−0.036*** (0.007)	−0.040*** (0.008)	−0.036*** (0.010)	−0.023*** (0.008)	0.001 (0.002)
Downstream ⁽²⁾ × year ₂₀₁₂	−0.027*** (0.004)	−0.025*** (0.004)	−0.030*** (0.004)	−0.024*** (0.003)	−0.000 (0.002)
Downstream ⁽³⁾ × year ₂₀₁₂	−0.021*** (0.004)	−0.022*** (0.004)	−0.023*** (0.003)	−0.017*** (0.003)	−0.000 (0.001)
Downstream ⁽⁴⁾ × year ₂₀₁₂	−0.020*** (0.004)	−0.018*** (0.004)	−0.015*** (0.004)	−0.018*** (0.006)	−0.002 (0.003)
Upstream ⁽¹⁾ × year ₂₀₁₂	−0.029*** (0.005)	−0.029*** (0.008)	−0.030*** (0.005)	−0.019*** (0.007)	−0.001 (0.003)
Upstream ⁽²⁾ × year ₂₀₁₂	−0.021*** (0.004)	−0.019*** (0.004)	−0.022*** (0.003)	−0.015*** (0.003)	−0.001 (0.002)
Upstream ⁽³⁾ × year ₂₀₁₂	−0.013*** (0.004)	−0.014*** (0.004)	−0.012*** (0.003)	−0.010** (0.004)	−0.001 (0.001)
Upstream ⁽⁴⁾ × year ₂₀₁₂	−0.009** (0.004)	−0.008** (0.003)	−0.009 (0.006)	0.001 (0.004)	0.000 (0.004)
Downstream ⁽¹⁾ × year ₂₀₁₀	−0.012 (0.008)	−0.001 (0.011)	−0.010 (0.009)	−0.014 (0.009)	−0.002 (0.002)
Downstream ⁽²⁾ × year ₂₀₁₀	−0.010 (0.006)	−0.007 (0.006)	−0.008 (0.005)	−0.013* (0.007)	−0.003 (0.002)
Downstream ⁽³⁾ × year ₂₀₁₀	−0.004 (0.006)	−0.006 (0.006)	−0.005 (0.005)	−0.005 (0.006)	−0.002 (0.002)
Downstream ⁽⁴⁾ × year ₂₀₁₀	0.001 (0.006)	−0.003 (0.005)	0.011** (0.004)	0.000 (0.006)	−0.003 (0.003)
Upstream ⁽¹⁾ × year ₂₀₁₀	−0.011 (0.007)	−0.005 (0.009)	−0.013* (0.007)	−0.015* (0.008)	−0.004 (0.003)
Upstream ⁽²⁾ × year ₂₀₁₀	−0.004 (0.006)	−0.005 (0.006)	−0.003 (0.005)	−0.004 (0.006)	−0.004** (0.002)
Upstream ⁽³⁾ × year ₂₀₁₀	−0.003 (0.007)	−0.002 (0.006)	−0.002 (0.005)	−0.002 (0.006)	−0.001 (0.002)
Upstream ⁽⁴⁾ × year ₂₀₁₀	−0.006 (0.009)	−0.012 (0.007)	−0.006 (0.011)	−0.006 (0.011)	−0.006 (0.004)
Firm fixed effects	Yes	Yes	Yes	Yes	Yes
Prefecture × industry × year FE	Yes	Yes	Yes	Yes	Yes
Observations	1,600,019	1,600,019	763,479	755,340	1,554,912
R ²	0.012	0.012	0.014	0.011	0.012

Notes: This table reports estimates from regression specification (1), presented in Figures 5 and 7 in the main body of the paper as well as the placebo network robustness check. The dependent variable in all columns is the logarithm of firms' annual sales. Downstream^(k) and Upstream^(k) are dummy variables indicating network distance to disaster-area firms. year₂₀₁₀ and year₂₀₁₂ are time dummies. All regressions include a set of control variables consisting of the logarithm of the number of transaction partners, age, logarithm of the number of employees, and distance to the disaster area. All control variables are measured for the year 2010 and interacted with 2010 and 2012 time dummies. The data covers 2010, 2011, and 2012. R² is the within-firm R². Standard errors are two-way clustered at the level of the prefecture and the industry. *, **, and *** denote significance at the 10%, 5%, and 1% levels, respectively.

B Proofs and Derivations

We start by stating and proving the following lemma, which relates the notion of pure upstream and downstreamness—defined in Definition 1 in terms of the economy’s Leontief inverse—to the structural properties of the underlying production network.

Lemma B.1. *Suppose firm i is purely upstream to firm j in the sense of Definition 1 and let $h \neq i, j$. Then, $\ell_{hj} > 0$ if and only if all directed paths from i to h over the production network pass through j .*

Proof. First, suppose all directed paths from firm i to firm h pass through firm j . This immediately guarantees that there exists a directed path from firm j to firm h . As a result, the corresponding element of the economy’s Leontief inverse matrix has to be strictly positive, i.e., $\ell_{hj} > 0$.

To prove the converse implication, suppose that $\ell_{hj} > 0$ for some firm h . Since firm i is purely upstream to firm j in the sense of Definition 1, we have $\ell_{hi} = \ell_{hj}\ell_{ji}$. Similarly, since $\ell_{jj} > 0$ and firm i is purely upstream to firm j , we have $\ell_{ji} = \ell_{jj}\ell_{ji}$, which guarantees that $\ell_{jj} = 1$. Hence,

$$\ell_{hi} = \ell_{hj}\ell_{ji}/\ell_{jj}. \quad (\text{B.1})$$

On the other hand, note that the Leontief inverse matrix $\mathbf{L} = (\mathbf{I} - \mu\mathbf{A})^{-1}$ is the inverse of a nonsingular M-matrix.³⁸ Therefore, by Theorem 3.9 of McDonald, Neumann, Schneider, and Tsatsomeros (1995), equation (B.1) implies that there is no directed path connecting firm i to firm h if we remove firm j from the production network. This therefore establishes that all directed paths from firm i to firm h over the production network pass through firm j . \square

Throughout the rest of this appendix, we let $\Omega = [\omega_{ij}]$ denote the matrix with elements $\omega_{ij} = p_j x_{ij} / p_i y_i$ and let $\Psi = (\mathbf{I} - \Omega)^{-1}$. We define the economy’s steady state as the constellation of prices and quantities when $z_i = 1$ for all firms i . In addition, we assume that the endowment of capital specific to each firm is proportional to the firm’s steady-state size. Under such specifications, it is easy to verify that, in steady state, $\Omega = \mu\mathbf{A}$ and $\Psi = \mathbf{L}$. Finally, we choose the price of the household’s consumption basket as the numeraire, thus normalizing the ideal price index to 1.

Proof of Proposition 1

As a first observation, note that market clearing for good i is given by $y_i = c_i + \sum_{s=1}^n x_{si}$. Multiplying both sides of this equation by p_i/GDP leads to $\lambda_i = \beta_i + \sum_{s=1}^n \omega_{si}\lambda_s$, where λ_i denotes the Domar weight of firm i . Differentiating both sides of the above equation with respect to $\log z_j$ implies that

$$\frac{d \log \lambda_i}{d \log z_j} = \frac{1}{\lambda_i} \sum_{h=1}^n \sum_{s=1}^n \lambda_s \omega_{sh} \psi_{hi} \frac{d \log \omega_{sh}}{d \log z_j}, \quad (\text{B.2})$$

³⁸A square matrix \mathbf{U} is called an M-matrix if there exist a nonnegative square matrix \mathbf{V} and a constant $r \geq \rho(\mathbf{V})$ such that $\mathbf{U} = r\mathbf{I} - \mathbf{V}$, where $\rho(\mathbf{V})$ is the spectral radius of \mathbf{V} . If $r > \rho(\mathbf{V})$, then \mathbf{U} is a nonsingular M-matrix. That $\mathbf{I} - \mu\mathbf{A}$ is a nonsingular M-matrix follows from the observations that $\rho(\mathbf{A}) = 1$ and $\mu < 1$.

where $\Psi = (\mathbf{I} - \Omega)^{-1}$. Next, note that first-order condition with respect to intermediate input x_{sh} purchased by firm s implies that

$$\omega_{sh} = \frac{p_h x_{sh}}{p_s y_s} = \mu a_{sh} p_s^{\sigma-1} p_h^{1-\xi} Q_s^{\xi-\sigma}, \quad (\text{B.3})$$

where Q_s is the price index of the intermediate good bundle M_s and is given by

$$Q_s = \left(\sum_{m=1}^n a_{sm} p_m^{1-\xi} \right)^{1/(1-\xi)}. \quad (\text{B.4})$$

Consequently, by (B.3),

$$\begin{aligned} \frac{d \log \omega_{sh}}{d \log z_j} &= (\sigma - 1) \frac{d \log p_s}{d \log z_j} + (1 - \xi) \frac{d \log p_h}{d \log z_j} + (\xi - \sigma) \frac{d \log Q_s}{d \log z_j} \\ &= (\sigma - 1) \frac{d \log p_s}{d \log z_j} + (1 - \xi) \frac{d \log p_h}{d \log z_j} + (\xi - \sigma) \sum_{m=1}^n \left(\frac{\omega_{sm}}{\sum_{b=1}^n \omega_{sb}} \right) \frac{d \log p_m}{d \log z_j}, \end{aligned}$$

where the second equality follows from differentiating (B.4) with respect to $\log z_j$. The juxtaposition of the above equation with equation (B.2) and imposing the steady-state conditions that $\omega_{sh} = \mu a_{sh}$ and $\psi_{sh} = \ell_{hs}$ implies that

$$\begin{aligned} \frac{d \log \lambda_i}{d \log z_j} &= \mu(\sigma - 1) \frac{1}{\lambda_i} \sum_{h=1}^n \sum_{s=1}^n \lambda_s a_{sh} \ell_{hi} \left(\frac{d \log p_s}{d \log z_j} - \sum_{m=1}^n a_{sm} \frac{d \log p_m}{d \log z_j} \right) \\ &\quad + \mu(1 - \xi) \frac{1}{\lambda_i} \sum_{h=1}^n \sum_{s=1}^n \lambda_s a_{sh} \ell_{hi} \left(\frac{d \log p_h}{d \log z_j} - \sum_{m=1}^n a_{sm} \frac{d \log p_m}{d \log z_j} \right). \end{aligned}$$

Writing the above equation in matrix form implies that the derivative of the vector of Domar weights with respect to $\log z_j$, evaluated at the economy's steady state, is given by

$$\begin{aligned} \frac{d \log \lambda}{d \log z_j} &= \mu(\sigma - 1) \mathbf{\Lambda}^{-1} \mathbf{L}' \mathbf{A}' \mathbf{\Lambda} (\mathbf{I} - \mathbf{A}) \frac{d \log p}{d \log z_j} \\ &\quad + \mu(1 - \xi) \mathbf{\Lambda}^{-1} \mathbf{L}' (\text{diag}(\mathbf{A}' \mathbf{\Lambda} \mathbf{1}) - \mathbf{A}' \mathbf{\Lambda} \mathbf{A}) \frac{d \log p}{d \log z_j}, \end{aligned} \quad (\text{B.5})$$

where $\mathbf{\Lambda} = \text{diag}(\lambda)$ and $\text{diag}(q)$ denotes a diagonal matrix with diagonal entries given by vector q . The above equation characterizes the derivatives of Domar weights with respect to $\log z_j$ in terms of the derivatives of prices, the economy's production network, and elasticities of substitution.

As our next step, we obtain a second system of equations that characterizes the derivatives of prices in terms of the derivatives of Domar weights. To this end, note that the price of good i satisfies

$$p_i^{1-\sigma} = (1 - \mu) (z_i^{-\alpha} r_i^{\alpha} w^{1-\alpha})^{1-\sigma} + \mu Q_i^{1-\sigma}, \quad (\text{B.6})$$

where w denotes the wage, r_i is the rental rate of the capital specific to firm i , and Q_i is the price of the intermediate input bundle M_i and is given by (B.4). Differentiating the above with respect to $\log z_j$ implies that, as long as $\sigma \neq 1$,

$$p_i^{1-\sigma} \frac{d \log p_i}{d \log z_j} = (1 - \mu) (z_i^{-\alpha} r_i^{\alpha} w^{1-\alpha})^{1-\sigma} \left(\alpha \frac{d \log r_i}{d \log z_j} - \alpha \frac{d \log z_i}{d \log z_j} + (1 - \alpha) \frac{d \log w}{d \log z_j} \right) + \mu Q_i^{1-\sigma} \frac{d \log Q_i}{d \log z_j}.$$

Consequently,

$$\frac{d \log p_i}{d \log z_j} = (1 - \mu p_i^{\sigma-1} Q_i^{1-\sigma}) \left(\alpha \frac{d \log r_i}{d \log z_j} - \alpha \frac{d \log z_i}{d \log z_j} + (1 - \alpha) \frac{d \log w}{d \log z_j} \right) + \sum_{r=1}^n \omega_{ir} \frac{d \log p_r}{d \log z_j},$$

where we are using equations (B.6) and (B.4) to obtain the first and the second terms on the right-hand side above, respectively. Furthermore, note that summing both sides of (B.3) over all h implies that $\mu p_s^{\sigma-1} Q_s^{1-\sigma} = \sum_{h=1}^n \omega_{sh}$. Therefore, replacing for the first term on the right-hand side above and imposing the steady-state condition $\omega_{ir} = \mu a_{ir}$ implies that

$$\frac{d \log p_i}{d \log z_j} = (1 - \mu) \left(\alpha \frac{d \log r_i}{d \log z_j} - \alpha \frac{d \log z_i}{d \log z_j} + (1 - \alpha) \frac{d \log w}{d \log z_j} \right) + \mu \sum_{r=1}^n a_{ir} \frac{d \log p_r}{d \log z_j}. \quad (\text{B.7})$$

On the other hand, the first-order condition corresponding to firm i 's problem implies that $r_i k_i = \alpha(1 - \mu) y_i p_i^\sigma (z_i^{-\alpha} r_i^\alpha w^{1-\alpha})^{1-\sigma}$. As a result,

$$(1 + \alpha(\sigma - 1)) \frac{d \log r_i}{d \log z_j} = \frac{d \log \lambda_i}{d \log z_j} + \frac{d \log \text{GDP}}{d \log z_j} + (\sigma - 1) \left(\frac{d \log p_i}{d \log z_j} + \alpha \frac{d \log z_i}{d \log z_j} - (1 - \alpha) \frac{d \log w}{d \log z_j} \right). \quad (\text{B.8})$$

Plugging the expression for $d \log r_i / d \log z_j$ from the above equation into (B.7) therefore implies that

$$\begin{aligned} \frac{d \log p_i}{d \log z_j} &= \frac{1 - \mu}{1 + \alpha\mu(\sigma - 1)} \left(\alpha \frac{d \log \lambda_i}{d \log z_j} + \alpha \frac{d \log \text{GDP}}{d \log z_j} - \alpha \frac{d \log z_i}{d \log z_j} + (1 - \alpha) \frac{d \log w}{d \log z_j} \right) \\ &\quad + \mu \frac{1 + \alpha(\sigma - 1)}{1 + \alpha\mu(\sigma - 1)} \sum_{r=1}^n a_{ir} \frac{d \log p_r}{d \log z_j}. \end{aligned}$$

Writing the above equation in matrix form and solving for vector $d \log p / d \log z_j$, we obtain

$$\begin{aligned} \frac{d \log p}{d \log z_j} &= \left(\alpha \frac{d \log \text{GDP}}{d \log z_j} + (1 - \alpha) \frac{d \log w}{d \log z_j} \right) \mathbf{1} \\ &\quad + \frac{\alpha(1 - \mu)}{1 + \alpha\mu(\sigma - 1)} \left(\mathbf{I} - \mu \frac{1 + \alpha(\sigma - 1)}{1 + \alpha\mu(\sigma - 1)} \mathbf{A} \right)^{-1} \left(\frac{d \log \lambda}{d \log z_j} - \frac{d \log z}{d \log z_j} \right). \end{aligned} \quad (\text{B.9})$$

where $\mathbf{1}$ denotes the vector of ones. Consequently, we can use the above equation to substitute for $d \log p / d \log z_j$ in (B.5). Before doing so, however, two remarks are in order. First, note that the right-hand side of (B.5) is linear in $\sigma - 1$ and $\xi - 1$. Therefore, to obtain a first-order approximation of $d \log \lambda / d \log z_j$ in $\sigma - 1$ and $\xi - 1$ around the point $\sigma = \xi = 1$, it is sufficient to plug in the expression for (B.9) evaluated at $\sigma = 1$, i.e.,

$$\frac{d \log p}{d \log z_j} = \left(\alpha \frac{d \log \text{GDP}}{d \log z_j} + (1 - \alpha) \frac{d \log w}{d \log z_j} \right) \mathbf{1} + \alpha(1 - \mu) \mathbf{L} \left(\frac{d \log \lambda}{d \log z_j} - \frac{d \log z}{d \log z_j} \right). \quad (\text{B.10})$$

Second, note that the fact that $\sum_{s=1}^n a_{is} = 1$ for all i implies that $(\text{diag}(\mathbf{A}' \mathbf{\Lambda} \mathbf{1}) - \mathbf{A}' \mathbf{\Lambda} \mathbf{A}) \mathbf{1} = 0$ and $(\mathbf{I} - \mathbf{A}) \mathbf{1} = 0$. As a result, plugging (B.10) into (B.5) implies that

$$\begin{aligned} \frac{d \log \lambda}{d \log z_j} &= \alpha\mu(1 - \mu)(1 - \sigma) \mathbf{\Lambda}^{-1} \mathbf{L}' (\mathbf{A}' \mathbf{\Lambda} \mathbf{A} - \mathbf{A}' \mathbf{\Lambda}) \mathbf{L} \left(\frac{d \log \lambda}{d \log z_j} - \frac{d \log z}{d \log z_j} \right) \\ &\quad + \alpha\mu(1 - \mu)(1 - \xi) \mathbf{\Lambda}^{-1} \mathbf{L}' (\text{diag}(\mathbf{A}' \mathbf{\Lambda} \mathbf{1}) - \mathbf{A}' \mathbf{\Lambda} \mathbf{A}) \mathbf{L} \left(\frac{d \log \lambda}{d \log z_j} - \frac{d \log z}{d \log z_j} \right) \end{aligned}$$

up to a first-order approximation in $\sigma - 1$ and $\xi - 1$. Solving for the vector $d \log \lambda / d \log z_j$ using the first-order approximation one more time, we obtain

$$\begin{aligned} \frac{d \log \lambda}{d \log z_j} &= \alpha \mu (1 - \mu) (\sigma - 1) \Lambda^{-1} \mathbf{L}' (\mathbf{A}' \Lambda \mathbf{A} - \mathbf{A}' \Lambda) \mathbf{L} \frac{d \log z}{d \log z_j} \\ &\quad + \alpha \mu (1 - \mu) (\xi - 1) \Lambda^{-1} \mathbf{L}' (\text{diag}(\mathbf{A}' \Lambda \mathbf{1}) - \mathbf{A}' \Lambda \mathbf{A}) \mathbf{L} \frac{d \log z}{d \log z_j} \end{aligned} \quad (\text{B.11})$$

up to a first-order approximation in $\sigma - 1$ and $\xi - 1$. Noting that $d \log z / d \log z_j$ is the j -th unit vector, it is then immediate to verify that the above equation coincides with equation (3). \square

Proof of Proposition 2

We first state and prove a simple lemma.

Lemma B.2. *Suppose firm i is purely upstream to firm j in the sense of Definition 1. If $\ell_{hj} > 0$ and $a_{hm} \ell_{mi} > 0$ for a pair of firms h and m , then $\ell_{mi} = \ell_{mj} \ell_{ji}$.*

Proof. The assumption that $a_{hm} \ell_{mi} > 0$ implies that there exists a directed path from i to m to h and that m is a direct supplier of h . On the other hand, since i is purely upstream to j and $\ell_{hj} > 0$, Lemma B.1 implies that all directed paths from i to h have to pass through j , including the path that connects i to m to h . Since m is a direct supplier of h this implies that $\ell_{mj} > 0$. Now using the assumption that i is purely upstream to j one more time guarantees that $\ell_{mi} = \ell_{mj} \ell_{ji}$. \square

With the above lemma in hand, we now prove Proposition 2.

Proof of part (a) Recall from Proposition 1 that the impact of a shock to firm j on the sales share of firm i is given by (3). Rearranging terms, we obtain

$$\frac{d \log \lambda_i}{d \log z_j} = \alpha \mu (1 - \mu) \frac{1}{\lambda_i} \left((\xi - \sigma) \sum_{h=1}^n \lambda_h \Delta_h + (1 - \sigma) \sum_{h=1}^n \sum_{r=1}^n \lambda_h a_{hr} \ell_{ri} (\ell_{hj} - \ell_{rj}) \right), \quad (\text{B.12})$$

where

$$\Delta_h = \sum_{r=1}^n a_{hr} \left(\ell_{ri} - \sum_{s=1}^n a_{hs} \ell_{si} \right) \left(\ell_{rj} - \sum_{s=1}^n a_{hs} \ell_{sj} \right). \quad (\text{B.13})$$

We start by simplifying the first term on the right-hand side of (B.12). Note that the Leontief inverse satisfies $\mathbf{L} = \mu \mathbf{A} \mathbf{L} + \mathbf{I}$, which implies that $\ell_{hj} \geq \mu a_{hr} \ell_{rj}$ for all firms h and r . Therefore, Δ_h in equation (B.13) is equal to zero whenever $\ell_{hj} = 0$. Consequently, $\sum_{h=1}^n \lambda_h \Delta_h = \sum_{h: \ell_{hj} > 0} \lambda_h \Delta_h$, and as a result,

$$\sum_{h=1}^n \lambda_h \Delta_h = \sum_{h: \ell_{hj} > 0} \lambda_h \sum_{r=1}^n a_{hr} \left(\ell_{ri} - \sum_{s=1}^n a_{hs} \ell_{si} \right) \left(\ell_{rj} - \sum_{s=1}^n a_{hs} \ell_{sj} \right).$$

Since firm i is purely upstream to firm j , Lemma B.2 implies that whenever $\ell_{hj} > 0$, then $\ell_{mi} = \ell_{mj} \ell_{ji}$ for any firm m such that $a_{hm} \ell_{mi} > 0$. Therefore,

$$\sum_{h=1}^n \lambda_h \Delta_h = \ell_{ji} \sum_{h: \ell_{hj} > 0} \lambda_h \sum_{r=1}^n a_{hr} \left(\ell_{rj} - \sum_{s=1}^n a_{hs} \ell_{sj} \right)^2, \quad (\text{B.14})$$

which means that we can rewrite equation (B.12) as

$$\frac{d \log \lambda_i}{d \log z_j} = \alpha \mu (1 - \mu) \frac{1}{\lambda_i} \left[\ell_{ji} (\xi - \sigma) \sum_{h=1}^n \sum_{r=1}^n \lambda_h a_{hr} \left(\ell_{rj} - \sum_{s=1}^n a_{hs} \ell_{sj} \right)^2 + (1 - \sigma) \sum_{h,r=1}^n \lambda_h a_{hr} \ell_{ri} (\ell_{hj} - \ell_{rj}) \right]. \quad (\text{B.15})$$

To simplify the above further, note that

$$\begin{aligned} \mu \sum_{h=1}^n \sum_{r=1}^n \lambda_h a_{hr} \ell_{ri} (\ell_{hj} - \ell_{rj}) &= \sum_{h=1}^n \lambda_h \ell_{hj} (\ell_{hi} - \mathbb{I}_{\{h=i\}}) - \sum_{h=1}^n (\lambda_h - \beta_h) \ell_{hi} \ell_{hj} \\ &= \sum_{h=1}^n \beta_h \ell_{hi} \ell_{hj} - \lambda_i \ell_{ij}, \end{aligned} \quad (\text{B.16})$$

where \mathbb{I} denotes the indicator function and we are using the fact that $\mu \mathbf{A} \mathbf{L} = \mathbf{L} - \mathbf{I}$ and $\mu \lambda' \mathbf{A} = \lambda' - \beta'$. Since firm i is purely upstream to firm j in the sense of Definition 1, $\ell_{ij} = 0$ and $\ell_{hi} = \ell_{hj} \ell_{ji}$ whenever $\ell_{hi} > 0$. As a result, the juxtaposition of equations (B.15) and (B.16) implies that

$$\frac{d \log \lambda_i}{d \log z_j} = \alpha \ell_{ji} \lambda_i^{-1} (1 - \mu) \left[\mu (\xi - \sigma) \sum_{h=1}^n \sum_{r=1}^n \lambda_h a_{hr} \left(\ell_{rj} - \sum_{s=1}^n a_{hs} \ell_{sj} \right)^2 + (1 - \sigma) \sum_{h=1}^n \beta_h \ell_{hj}^2 \right]. \quad (\text{B.17})$$

It is now immediate that as long as $\sigma < \min\{1, \xi\}$ the right-hand side of the above expression is always positive. \square

Proof of part (b) Let i and s denote two firms that are purely upstream to firm j . Also suppose i and s have the same size in steady state. Then, equation (B.17) implies that

$$\frac{d \log \lambda_s}{d \log z_j} / \frac{d \log \lambda_i}{d \log z_j} = \ell_{js} / \ell_{ji}.$$

Since, $d \log \lambda_i / d \log z_j > 0$, it is then immediate that $d \log \lambda_i / d \log z_j \geq d \log \lambda_s / d \log z_j$ if and only if $\ell_{ji} \geq \ell_{js}$. \square

Proof of Proposition 3

Proof of part (a) Recall from the proof of Proposition 1 that the impact of a shock to firm j on firms' sales shares is given by (B.11). Using the fact that the Leontief inverse satisfies $\mu \mathbf{L} \mathbf{A} = \mu \mathbf{A} \mathbf{L} = \mathbf{L} - \mathbf{I}$, we can rewrite this equation as

$$\begin{aligned} \frac{d \log \lambda}{d \log z_j} &= \alpha (1 - \mu) (\sigma - 1) \mu^{-1} \mathbf{A}^{-1} (\mathbf{L}' - \mathbf{I}) \mathbf{A} ((1 - \mu) \mathbf{L} - \mathbf{I}) \frac{d \log z}{d \log z_j} \\ &\quad + \alpha \mu (1 - \mu) (\xi - 1) \mathbf{A}^{-1} \mathbf{L}' (\text{diag}(\mathbf{A}' \mathbf{A} \mathbf{1}) - \mathbf{A}' \mathbf{A} \mathbf{A}) \mathbf{L} \frac{d \log z}{d \log z_j}. \end{aligned}$$

Therefore,

$$\begin{aligned} \frac{d \log \lambda_i}{d \log z_j} &= \alpha (1 - \mu) (\sigma - 1) \frac{1}{\mu \lambda_i} \sum_{h=1}^n \lambda_h (\ell_{hi} - \mathbb{I}_{\{h=i\}}) ((1 - \mu) \ell_{hj} - \mathbb{I}_{\{h=j\}}) \\ &\quad + \alpha \mu (1 - \mu) (\xi - 1) \sum_{h=1}^n \lambda_h \sum_{r=1}^n a_{hr} \left(\ell_{ri} - \sum_{s=1}^n a_{hs} \ell_{si} \right) \left(\ell_{rj} - \sum_{s=1}^n a_{hs} \ell_{sj} \right). \end{aligned}$$

Since firm i is, by assumption, purely downstream to j , we can simplify the first term on the right-hand side above by noting that $(\ell_{hi} - \mathbb{I}_{\{h=i\}})((1 - \mu)\ell_{hj} - \mathbb{I}_{\{h=j\}}) = (1 - \mu)\ell_{ij}\ell_{hi}^2\mathbb{I}_{\{h \neq i\}}$. Furthermore, steps identical to the derivation of equation (B.14) in the proof of Proposition 2 allows us to simplify the second term, thus leading to the following expression:

$$\frac{d \log \lambda_i}{d \log z_j} = \alpha(1 - \mu) \frac{\ell_{ij}}{\lambda_i} \left[(1 - \mu)(\sigma - 1) \frac{1}{\mu} \sum_{h \neq i} \lambda_h \ell_{hi}^2 + \mu(\xi - 1) \sum_{h,r=1}^n \lambda_h a_{hr} \left(\ell_{ri} - \sum_{s=1}^n a_{hs} \ell_{si} \right)^2 \right]. \quad (\text{B.18})$$

Since $\xi > 1$, the left-hand side of the above equation is nonnegative if and only if

$$\frac{1 - \sigma}{\xi - 1} \leq \frac{\mu^2 \sum_{h,r=1}^n \lambda_h a_{hr} \left(\ell_{ri} - \sum_{s=1}^n a_{hs} \ell_{si} \right)^2}{(1 - \mu) \sum_{h \neq i} \lambda_h \ell_{hi}^2}. \quad (\text{B.19})$$

If the expression on the right-hand side of the above inequality is strictly positive, then for any $\xi > 1$, there exists $\bar{\sigma} < 1$ such that for all $\sigma > \bar{\sigma}$, the inequality is satisfied, which in turn implies that $d \log \lambda_i / d \log z_j \geq 0$. To establish this, recall that, by assumption, firm i has at least one customer firm with a direct supplier that is distinct from i , that is, there exists a firms m such that $0 < a_{mi} < 1$. Given such an m , note that

$$a_{mi} \left(\ell_{ii} - \sum_{s=1}^n a_{ms} \ell_{si} \right)^2 = a_{mi} \left(\sum_{s \neq i} a_{ms} (\ell_{ii} - \ell_{si}) \right)^2. \quad (\text{B.20})$$

On the other hand, since $\mathbf{L} = (\mathbf{I} - \mu \mathbf{A})^{-1}$ is the inverse of a row diagonally dominant M-matrix,³⁹ statements (i) and (ii) of Theorem 3.2 of McDonald, Neumann, Schneider, and Tsatsomeros (1995) imply that $\ell_{ii} \geq \ell_{si}$ for all $s \neq i$ and $\ell_{ii} > \ell_{mi}$, respectively. This, coupled with the fact that $0 < a_{mi} < 1$ then guarantees that the right-hand side of (B.20) is strictly positive, thus implying that the right-hand side of (B.19) is also strictly positive. As a result, there exists a $\bar{\sigma} < 1$ such that for all $\sigma > \bar{\sigma}$ the right-hand side of (B.18) is strictly positive. \square

Proof of part (b) Let i and s denote two firms that are purely downstream to firm j in the sense of Definition 1. Also suppose i and s have the same size in steady state, i.e., $\lambda_i = \lambda_s$. Then, equation (B.18) implies that

$$\frac{d \log \lambda_s}{d \log z_j} \Big/ \frac{d \log \lambda_i}{d \log z_j} = \ell_{sj} / \ell_{ij}.$$

Since $d \log \lambda_i / d \log z_j > 0$, it immediately follows that $d \log \lambda_i / d \log z_j \geq d \log \lambda_s / d \log z_j$ if and only if $\ell_{ij} \geq \ell_{sj}$. \square

³⁹An M-matrix \mathbf{U} is said to be row diagonally dominant if $\mathbf{U}\mathbf{1} \geq 0$. The row diagonal dominance of $\mathbf{I} - \mu \mathbf{A}$ follows from the observation that $(\mathbf{I} - \mu \mathbf{A})\mathbf{1} = (1 - \mu)\mathbf{1} > 0$. See footnote 38 for the definition of an M-matrix.

Derivation of Equation (8)

The second-order approximation to the aggregate impact of the disaster around the economy's steady state is given by

$$\Delta \log \text{GDP} = \sum_{j=1}^n \frac{d \log \text{GDP}}{d \log z_j} \Big|_{\text{ss}} (\Delta \log z_j) + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \frac{d^2 \log \text{GDP}}{d \log z_i d \log z_j} \Big|_{\text{ss}} (\Delta \log z_i)(\Delta \log z_j).$$

By Hulten's theorem, $d \log \text{GDP} / d \log z_j = \eta_j$, where $\eta_j = r_j k_j / \text{GDP}$ is firm j 's capital expenditure as a share of GDP. Therefore,

$$\Delta \log \text{GDP} = \alpha(1 - \mu) \sum_{j=1}^n \lambda_j (\Delta \log z_j) + \frac{1}{2} \alpha(1 - \mu) \sum_{i=1}^n \sum_{j=1}^n \lambda_i \frac{d \log \eta_i}{d \log z_j} \Big|_{\text{ss}} (\Delta \log z_i)(\Delta \log z_j), \quad (\text{B.21})$$

where we are using the fact that, in steady state, $\eta_j = \alpha(1 - \mu)\lambda_j$. To simplify the second term on the right-hand side above, note that equations (B.5) and (B.9) in the proof of Proposition 1 can be rewritten as

$$\begin{aligned} \frac{d \log \lambda}{d \log z_j} &= \mathbf{M} \frac{d \log p}{d \log z_j} \\ \frac{d \log p}{d \log z_j} &= \left(\alpha \frac{d \log \text{GDP}}{d \log z_j} + (1 - \alpha) \frac{d \log w}{d \log z_j} \right) \mathbf{1} + \mathbf{B} \left(\frac{d \log \lambda}{d \log z_j} - \mathbf{e}_j \right), \end{aligned}$$

where \mathbf{e}_j is the j -th unit vector and matrices \mathbf{M} and \mathbf{B} are given by

$$\begin{aligned} \mathbf{M} &= \mu(\sigma - 1) \mathbf{\Lambda}^{-1} \mathbf{L}' \mathbf{A}' \mathbf{\Lambda} (\mathbf{I} - \mathbf{A}) + \mu(1 - \xi) \mathbf{\Lambda}^{-1} \mathbf{L}' (\text{diag}(\mathbf{A}' \mathbf{\Lambda} \mathbf{1}) - \mathbf{A}' \mathbf{\Lambda} \mathbf{A}) \\ \mathbf{B} &= \frac{\alpha(1 - \mu)}{1 + \alpha\mu(\sigma - 1)} \left(\mathbf{I} - \mu \frac{1 + \alpha(\sigma - 1)}{1 + \alpha\mu(\sigma - 1)} \mathbf{A} \right)^{-1}, \end{aligned}$$

respectively. Solving the above system of equations for vectors $d \log \lambda / d \log z_j$ and $d \log p / d \log z_j$ and using the observation that $\mathbf{M} \mathbf{1} = 0$, we obtain

$$\begin{aligned} \frac{d \log \lambda}{d \log z_j} &= -(\mathbf{I} - \mathbf{M} \mathbf{B})^{-1} \mathbf{M} \mathbf{B} \mathbf{e}_j \\ \frac{d \log p}{d \log z_j} &= \left(\alpha \frac{d \log \text{GDP}}{d \log z_j} + (1 - \alpha) \frac{d \log w}{d \log z_j} \right) \mathbf{1} - \mathbf{B} (\mathbf{I} - \mathbf{M} \mathbf{B})^{-1} \mathbf{e}_j. \end{aligned}$$

Plugging in the expressions for $d \log \lambda / d \log z_j$ and $d \log p / d \log z_j$ into equation (B.8) in the proof of Proposition 1 then implies that the derivative of the vector of firms' capital expenditures as a fraction of GDP with respect to $\log z_j$ satisfies

$$\frac{d \log \eta}{d \log z_j} = \left(\mathbf{I} - \frac{1}{1 + \alpha(\sigma - 1)} [\mathbf{I} + (\sigma - 1) \mathbf{B}] \right) (\mathbf{I} - \mathbf{M} \mathbf{B})^{-1} \mathbf{e}_j + \frac{d \log \lambda}{d \log z_j}.$$

Consequently,

$$\frac{d \log \eta}{d \log z_j} = \mathbf{T} \mathbf{e}_j + \frac{d \log \lambda}{d \log z_j}$$

up to a first-order approximation in $\sigma - 1$ and $\xi - 1$, where $\mathbf{T} = \alpha\mu(\sigma - 1)(\mathbf{I} - \mathbf{A})\mathbf{L}$. Plugging the above into the second-term on the right-hand side of (B.21) implies that

$$\Delta \log \text{GDP} = \alpha(1 - \mu) \sum_{j=1}^n \lambda_j (\Delta \log z_j) + \frac{1}{2} \alpha(1 - \mu) \sum_{i=1}^n \sum_{j=1}^n \left(\lambda_i t_{ij} + \frac{d\lambda_i}{d \log z_j} \right) (\Delta \log z_i) (\Delta \log z_j).$$

Let λ_i^* denote post-shock sales share of firm i . Since up to a first-order, $\sum_{i=1}^n \frac{d\lambda_i}{d \log z_j} \Delta \log z_j = \lambda_i^* - \lambda_i$, the above equation implies that

$$\Delta \log \text{GDP} = \frac{1}{2} \alpha(1 - \mu) \sum_{j=1}^n (\lambda_j + \lambda_j^*) (\Delta \log z_j) + \frac{1}{2} \alpha(1 - \mu) \sum_{i=1}^n \sum_{j=1}^n \lambda_i t_{ij} (\Delta \log z_i) (\Delta \log z_j),$$

which coincides with equation (8). □

C Disaster's Impact on Infrastructure

This appendix provides a brief summary of the extent of infrastructure reconstruction in the aftermath of the disaster based on information compiled by the Central Disaster Management Council (established by the Cabinet Office) from the materials reported by MLIT (Cabinet Office, Director General for Disaster Management, 2011b). Figure C.1 plots the recovery rate for highways, national routes, ports, Shinkansen, conventional railways, and airports in the region. As the figure indicates, by late March, highways, national routes, ports, and airports were back to their pre-earthquake levels, while Shinkansen and conventional rail lines were almost fully restored by end of April. The spike on April 7 is due to the magnitude 7.0 aftershock off the coast of Miyagi Prefecture.

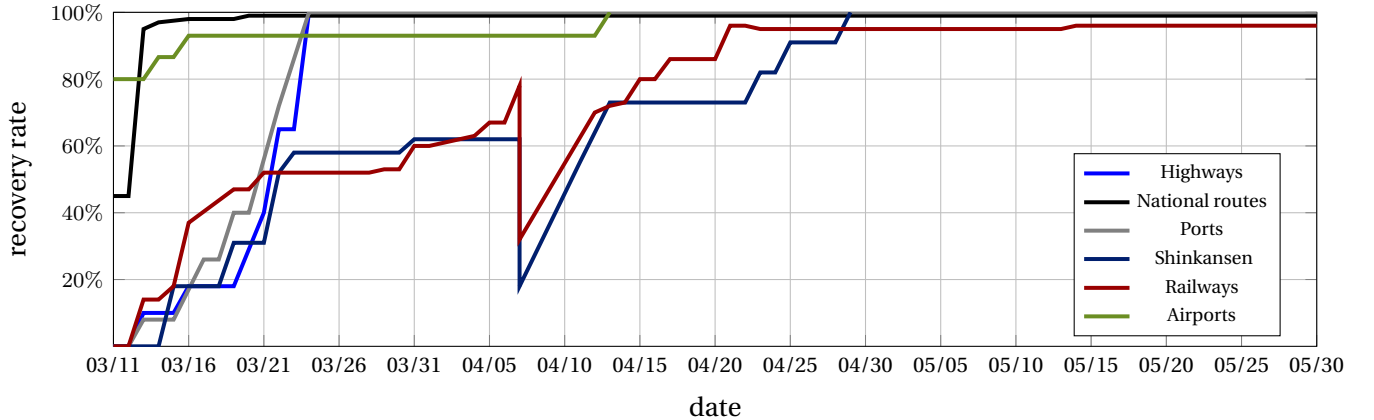


Figure C.1. Infrastructure Recovery Rate

Source: Compiled by Cabinet from the materials reported by MLIT

Notes: This figure reports the timeline of infrastructure recovery rate in the disaster area. Highways (979km): Tohoku highway, Joban highway; National routes (1119km): Routes 4, 45, and 6 (within Iwate, Miyagi, and Fukushima prefectures); Ports (15): Aomori Port–Kashima Port; Shinkansen (990km): Tohoku shinkansen, Akita shinkansen, and Yamagata shinkansen; Railways (1012km): Joban line, Tohoku line (Ueno station–Aomori station); Airports (13): airports in Tohoku region and Ibaraki prefecture and Haneda, Narita, and Niigata airports.

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