UNIVERSITY OF CAMBRIDGE
Faculty of Economics

CAMBRIDGE WORKING PAPERS IN ECONOMICS

Union Debt Management

Juan Equiza-Goni

Elisa Faraglia

Rigas Oikonomou

Abstract

We study the role of government debt maturity in currency unions to identify whether debt management can help governments to hedge their budgets against spending shocks. We first use a detailed dataset of debt portfolios of five Euro Area countries to run a battery of VARs, estimating the responses of holding period returns to fiscal shocks. We find that government portfolios, which in our sample comprise mainly of nominal assets, have not been effective in absorbing idiosyncratic fiscal risks, whereas they have been very effective in absorbing aggregate risks. We then setup a formal model of optimal debt management with two countries, distortionary taxes and aggregate and idiosyncratic shocks. The theoretical model concludes that nominal bonds are not optimal to insure against idiosyncratic fiscal shocks in a currency area. In contrast, we find that long term inflation indexed debt allows governments to take full advantage of fiscal hedging.

Reference Details

CWPE	1890
Published	1 July 2018
Updated	13 October 2022
Key Words	Debt Management, Fiscal Policy, Government Debt, Maturity Structure, Tax Smoothing
JEL Codes	E43, E62, H63
Website	www.econ.cam.ac.uk/cwpe

Union Debt Management

Juan Equiza-Goñi[*] Elisa Faraglia[†] Rigas Oikonomou[‡]

October 13, 2022

Abstract

We study the role of government debt maturity in currency unions to identify whether debt management can help governments to hedge their budgets against spending shocks. We first use a detailed dataset of debt portfolios of five Euro Area countries to run a battery of VARs, estimating the responses of holding period returns to fiscal shocks. We find that government portfolios, which in our sample comprise mainly of nominal assets, have not been effective in absorbing idiosyncratic fiscal risks, whereas they have been very effective in absorbing aggregate risks. We then setup a formal model of optimal debt management with two countries, distortionary taxes and aggregate and idiosyncratic shocks. The theoretical model concludes that nominal bonds are not optimal to insure against idiosyncratic fiscal shocks in a currency area. In contrast, we find that long term *inflation indexed* debt allows governments to take full advantage of *fiscal hedging*.

Debt Management, Fiscal Policy, Government Debt, Maturity Structure, Tax Smoothing
JEL Classification: E43, E62, H63

1 Introduction

An established literature on optimal debt management (DM) in macroeconomic models with distortionary taxes concludes that governments should focus on issuing *long term nominal bonds*. These assets, in fact, allow governments to benefit from two channels to manage and stabilize their debt without increasing tax volatility: the inflation channel and the fiscal insurance channel. Chari and Kehoe (1999), Lustig et al (2008), and Faraglia et al (2013) study the optimality of debt devaluations through inflation when debt is high. Angeletos (2002) and Buera and Nicolini (2004) (hereafter ABN) and Faraglia et al (2010) conclude that governments that issue long term debt can exploit better the negative covariance between long bond prices and government deficits, the so called *fiscal insurance/hedging channel* of DM and the focus of this paper.

This literature has studied optimal DM mainly in a closed economy setting. It remains, however, unexplored whether we can draw the same conclusions when we consider the optimal structure of government debt in an open economy and in particular in a currency union. In this setting the power of the fiscal insurance channel may be diminished because the union members can experience asymmetric shocks. Moreover, the fact that long term interest rates may follow closely future short term rates, which are equal across countries due to the common monetary policy, or even the fact that returns on long term debt are equalised in almost frictionless financial markets in the union, may cause the covariance between long term bond prices and government deficits to be close to zero, conditional on country specific shocks. Fiscal insurance would be impaired.

This paper studies DM in a currency union, first exploring empirically whether debt portfolios have helped some EU governments to hedge against fiscal shocks. Serving a normative purpose, it

[*]Facultad de Economicas y Empresariales, Universidad de Navarra, Edificio Amigos,Pamplona, 31009, Spain
[†]Faculty of Economics, University of Cambridge and CEPR, Sidgwick avenue, Cambridge, CB3 9DD, United Kingdom, Corresponding author: ef307@cam.ac.uk
[‡]Universite Catholique de Louvain and University of Surrey, 3 Place Montesquieu, Louvain la Neuve, 1348, Belgium

then turns to a theoretical investigation to determine which bonds can be optimal to hedge against government spending shocks within a tractable framework adopting the complete market approach to DM of ABN and Faraglia et al (2010), extending the benchmark model to a monetary union.

In Section 2 we lay out our empirical analysis which uses a detailed dataset on bond prices and quantities of all types of debt issued by five Euro Area countries, since the introduction of the euro, in order to test for the presence of fiscal hedging in the data. We estimate a series of panel VARs to identify fiscal shocks and then to study their effects on the holding returns of government portfolios. When shocks lower bond returns, or equivalently when returns covary negatively with the shocks, there is evidence of hedging.

Our identification of fiscal shocks is standard and based on the Blanchard and Perotti (2002) decomposition (Burriel et al (2010), Beetsma and Giuliodori (2011) in the context of the Euro Area). In addition, being interested in decomposing the identified fiscal shocks into aggregate and idiosyncratic components, we augment the VARs with the average spending level of the five countries, letting the aggregate shock but not the idiosyncratic shock affect the average. In this way, we impose that idiosyncratic shocks only redistribute the fiscal burden across countries, without impacting the overall spending level in the Euro area.

Our empirical findings suggest that bond returns responded strongly to spending shocks over the sample period considered and in particular that the real value of debt dropped considerably following a spending shock. This effect is stronger the longer the maturity of debt. This is evidence of fiscal insurance. However, when we decompose the fiscal shock in aggregate and idiosyncratic components, we find that there is evidence of fiscal insurance only when shocks are aggregate but not when they are idiosyncratic.

These results are robust to several model specifications. Most notably our findings do not depend on whether we identify spending shocks from a pooled panel VAR, or we allow coefficients to differ across countries. Moreover the results hold when we consider the period from the introduction of the euro to the financial crisis (1999-2008), when the common currency was widely seen as a stable arrangement, as well as when we extend our sample to include the period of the financial crisis.[1]

Our empirical exercise is complementary to related empirical papers on fiscal hedging, for example Faraglia et al (2008) and Berndt et al (2012). Faraglia et al (2008) provide some evidence of the presence of fiscal insurance in a panel of OECD countries studying the covariance between deficits and bond returns over the period 1970-2000. Berndt et al (2012) use post World War II US data, identifying fiscal shocks as innovations to defense spending, and find a significant impact on holding period returns. Using a log-linear intertemporal budget constraint they propose a decomposition which measures the fraction of the shock that has been absorbed by government portfolios and the fraction absorbed by adjustments in the present value of the surplus. Due to the short time span of our data we do not estimate the intertemporal surplus. However, we exploit the panel dimension of our dataset to document separately the fiscal hedging channel of DM for aggregate and for idiosyncratic shocks in the Euro Area. This is new in the literature.

In Section 3 and 4 we turn to a theoretical model extending ABN to study optimal portfolios in a currency area. The goal of our theoretical analysis is normative and not intended to construct a quantitative framework suitable to match data moments. Rather our aim is to setup a simple model that can allow us to think about policies going forward, in particular the properties of nominal bonds, the most used instrument by the countries in our dataset, and inflation indexed debt, used only by a subset of the countries examined. Like ABN, we setup a policy program where a benevolent planner maximizes welfare under full commitment; governments (implicitly) desire to smooth taxes across

[1] The post 2008-9 turmoil in bond markets and the sovereign debt crisis that followed raised numerous concerns regarding the ability of some of the country-members of the Euro Area to repay their debts. Numerous papers have investigated this empirically, particularly focusing on fiscal divergence and default as driving forces behind interest rate dynamics. We examine separately the two samples (excluding or including the financial crisis years) because we view fiscal hedging and government default as two separate channels. This is in line with the DM literature which has considered creditworthy countries. Though fiscal hedging is a form of default, it does not trigger a credit event, or entail a punishment, such as exclusion from credit, as is the case with outright default. Default however does lead to negative comovement between returns and spending.

time. In the context of our currency union model this implies that fiscal policies are coordinated across countries and also coordinate with monetary policy.[2] In the same spirit, for tractability to solve government portfolios, we assume complete markets, thus assuming the absence of cross border transaction costs (nonetheless a reasonable approximation for the Euro Area) but also of portfolio adjustment costs, liquidity premia, debt constraints etc.[3] These simplifications can be seen as a starting point of a broader agenda on optimal DM in currency areas and before concluding the paper we briefly discuss how giving up the assumption that markets are complete can be an interesting extension of our analysis.

Our model consists of two countries, members of a currency area, that face fiscal shocks and finance them optimally through distortionary taxes and through debt portfolios. Key features are monopolistic competition and sticky prices (e.g. Siu (2004) and Lustig et al (2008)), private sector preferences which exhibit a mild bias towards home goods, and government consumption that is allocated in home goods (e.g. Gali and Monacelli (2008)).

As ABN and others in the debt management literature mentioned previously, our exercise consists of finding portfolios of long and short bonds that can decentralize the optimal policy outcome under complete markets. We first investigate whether with nominal bonds governments can complete the markets. We find that they cannot. Though nominal bonds are useful to hedge against aggregate shocks, in the presence of idiosyncratic/redistributive shocks the complete market outcome generally cannot be attained. Nominal bond prices do not covary negatively with idiosyncratic fiscal shocks, or they vary too little to enable a government to use DM to fully insure against these types of shocks.

Furthermore, we find that not only the fiscal hedging channel is impaired but also the inflation channel cannot help DM when shocks are idiosyncratic. We show that this holds in the case of sticky prices, because there is not enough variation in inflation that DM could exploit to ensure debt solvency. Moreover, the result extends to the limiting case where prices are flexible because large swings in prices should be engineered to complete the markets that are unfeasible under a common monetary policy.

We then consider inflation indexed bonds and explore their ability to complete the markets. We find that indexed bonds do hold promise, since in our model the covariance between real bond prices and government deficits is negative irrespective of the shock, aggregate or idiosyncratic. Moreover, our theory concludes that governments should issue long term bonds to benefit from fiscal hedging.

The success of inflation index bonds and the failure of nominal bonds to provide fiscal insurance simply hinge on the fact that in our model, under complete financial markets, nominal interest rates must be equal across countries, due to the standard risk sharing condition which equates the real exchange rate to the relative consumption levels of the countries. When nominal rates are lined up, they cannot absorb idiosyncratic shocks. However, real rates are not equal across countries, and governments can exploit the ex post variability of holding period returns on bonds that are indexed to inflation, in order to complete the markets.

Finally, this paper is complementary to a large body of work devoted to investigating policies that can promote risk sharing in currency areas. A recent stream of papers (e.g. Farhi and Werning (2017), Dmitriev and Hoddenbagh (2015), Auclert and Ronglie (2014)) studies fiscal transfers in models of currency unions (essentially fiscal unions). Other papers (e.g. Blanchard et al (2016), Acalin (2018), Shiller et al (2018)) have investigated whether introducing new types of debt (primarily GDP growth

[2] These assumptions can be also found in numerous recent papers in international macro. A large number of studies considered optimal coordinated policies, also contrasting their properties with uncoordinated fiscal policies. Gali and Monacelli (2008), Cook and Devereux (2011), Evers (2015), Blanchard et al (2016), Farhi and Werning (2017), Hettig and Mueller (2018) are a few important examples of this work. As a first extension of optimal DM to a currency union in the literature, we choose to focus only on a cooperative optimal policy outcome, leaving to future work to study DM under uncoordinated policies.

[3] Similar assumptions have been employed by numerous papers in the literature on country portfolios in DSGE models under complete or incomplete markets. See for example Baxter and Jermann (1997), Devereux and Sutherland (2007, 2011), Engel and Matsumoto (2009), Coeurdacier et al (2010), Coeurdacier and Rey (2013) and Coeurdacier and Gourinchas (2016). Whilst in these models households choose bond and equity portfolios to hedge consumption against real exchange rate and non tradable income risks, here we focus on the hedging of governments against fiscal shocks. Since governments are the main supplier of debt, our work should be relevant for this literature.

linked debt) can enable risk sharing. We prove explicitly that conventional types of debt (e.g. nominal coupon bearing bonds) do not have desirable properties in the presence of idiosyncratic fiscal risks, even when governments engage in optimal DM.[4] Moreover, we show that when the government issues inflation indexed bonds then the complete market outcome, or complete risk sharing, is attainable. This offers an alternative to GDP linked debt.

2 Fiscal Insurance in the Euro Area

In this section we investigate the presence of fiscal hedging in the Euro Area. Our sample includes five Euro Area countries: Belgium, France, Germany, Italy and Spain which account for roughly 80 percent of aggregate output and spending in the Euro Area. We employ panel SVARs estimating the responses of bond returns to government spending shocks. In Section 2.1 we briefly describe the idea behind fiscal insurance, which motivates the empirical exercise, and in Section 2.2 we present the data and variables used in our empirical analysis. Section 2.3 outlines our identification strategy to separately identify the effects of aggregate and idiosyncratic spending shocks and Sections 2.4 and 2.5 our empirical results.

2.1 Definition of Fiscal Insurance

Before getting to the empirical analysis, we first revisit the definition of fiscal insurance/hedging provided by the DM literature: fiscal hedging results from the negative covariance between the holding period returns on government debt and exogenous spending shocks that governments can exploit to smooth taxes.

Consider the intertemporal budget constraint of a government that issues N zero coupon bonds, B^j, of maturities $j = 1, 2, ...N$:

$$\underbrace{\frac{B^1_{t-1}}{\widetilde{\pi}_t} + \sum_{j\geq 2}^{N} q^{j-1}_t \frac{B^j_{t-1}}{\widetilde{\pi}_t}}_{\text{Market Value of Debt}} = \underbrace{E_t \sum_{k=0}^{\infty} \frac{1}{r_{t,t+k}} S_{t+k}}_{\text{p.d.v. of surpluses in } t} \equiv \widetilde{S}_t. \tag{1}$$

The left hand side is the (real) market value of outstanding debt and q^j_t is the price of a (nominal) bond of maturity j. $\widetilde{\pi}_t$ is the gross inflation rate between $t-1$ and t. The right hand side represents the (real) present value of fiscal surpluses S. $r_{t,t+k}$ is the stochastic discount factor between t and $t+k$. $r_{t,t}$ equals 1.

Note that (1) must hold at every period t if the government is creditworthy. Let $S_t = T_t - G_t$ (real tax revenues minus spending) and consider an exogenous rise in spending levels in t that is not fully compensated by higher current and/or future taxes. According to (1) this extra spending can be financed either through an increase in inflation or through a decrease in bond prices, q, both of which cause a drop in the market value of long term debt when $B^j_{t-1} > 0, j > 1$. The negative comovement between spending levels and (long) bond prices is exactly what the optimal DM literature has defined as 'fiscal insurance'.

Using equation (1) and letting $D_{t-1} = \sum_{j\geq 1}^{N} q^j_{t-1} B^j_{t-1}$ be the market value of debt issued in period $t-1$, we can rewrite the intertemporal constraint of the government as:

$$\sum_{j\geq 1}^{N} \frac{q^{j-1}_t}{\widetilde{\pi}_t q^j_{t-1}} \frac{q^j_{t-1} B^j_{t-1}}{D_{t-1}} = \sum_{j\geq 1}^{N} \underbrace{R^j_{t-1,t}}_{return} \underbrace{w^j_{t-1}}_{share} = \frac{\widetilde{S}_t}{D_{t-1}}, \tag{2}$$

[4] The relevance of this finding extends to the current situation. Though COVID 19 is a shock that impacts all countries simultaneously, the effects are asymmetric, in some countries a much larger rise in spending will be required to stabilize the macroeconomy. There are thus redistributive effects that cannot be warded off through DM or through monetary policy.

where $R^j_{t-1,t} \equiv \frac{q_t^{j-1}}{\tilde{\pi}_t q^j_{t-1}}$ is the real holding period return of buying a bond of maturity j in $t-1$ and selling it in period t at price q_t^{j-1}, adjusted by the realized inflation rate. $w^j_{t-1} \equiv \frac{q^j_{t-1}B^j_{t-1}}{D_{t-1}}$ is the share of bonds of maturity j in the government's portfolio. It is useful, also in order to connect with our empirical exercise below, to use a log-linear approximation of equation (2). Consider an expansion of this equation around a constant portfolio $\overline{w}$, returns $\overline{R}$, discount factors and surpluses. In the online appendix we show that

$$\sum_{j\geq 1}^{N} \overline{R}^j \overline{w}^j \left(\hat{w}^j_{t-1} + \hat{R}^j_{t-1,t}\right) = \frac{\left(E_t - E_{t-1}\right) \sum_{k=0}^{\infty} \frac{1}{\overline{r}_k} \overline{S} \left(\hat{S}_{t+k} - \hat{r}_{t,t+k}\right)}{\frac{1}{\overline{r}_1} \sum_{k=0}^{\infty} \frac{1}{\overline{r}_k} \overline{S}} + \overline{r}_1 E_{t-1} \hat{r}_{t-1,t}, \tag{3}$$

where $(E_t - E_{t-1})x = E_t x - E_{t-1}x$ and $\overline{r}_k$ is the constant k period discount factor. The two terms on the LHS of (3) capture the deviations of the portfolio weights and the holding period returns from their average value. Changes in expectations between $t-1$ and t of the deviations of current and future surpluses and of discount factors make the RHS of (3). Consider now an unexpected rise in spending in period t that is not compensated by an analogous rise in taxes (or a drop in $\left(E_t - E_{t-1}\right) \sum_{k=0}^{\infty} \frac{1}{\overline{r}_k} \overline{S} \hat{r}_{t,t+k}$) so that the term $\left(E_t - E_{t-1}\right) \sum_{k=0}^{\infty} \frac{1}{\overline{r}_k} \overline{S} \hat{S}_{t+k}$ decreases. For the equality to hold, the only term that can absorb this change is $\sum_{j\geq 1}^{N} \overline{R}^j \overline{w}^j \hat{R}^j_{t-1,t}$ since the portfolio weights $\hat{w}^j_{t-1}$ are predetermined.[5] Equation (3) then suggests that

$$cov_t\left(\sum_{j\geq 1}^{N} \hat{R}^j_{t-1,t} \overline{w}^j \overline{R}^j, (E_t - E_{t-1}) \sum_{k\geq 0} \frac{1}{\overline{r}_k} \overline{S} \hat{S}_{t+k}\right) > 0 \tag{4}$$

is necessary for governments to finance the spending shock through the portfolio. Now let $\epsilon_{t,G}$ denote the exogenous shock to government spending. In equilibrium every term in (3) that involves a date t expectation, can be written as function of $\epsilon_{t,G}$:

$$\frac{\left(E_t - E_{t-1}\right) \sum_{k=0}^{\infty} \frac{1}{\overline{r}_k} \overline{S} \hat{S}_{t+k}}{\frac{1}{\overline{r}_1} \sum_{k=0}^{\infty} \frac{1}{\overline{r}_k} \overline{S}} = (\phi_T - \psi_G)\epsilon_{t,G},$$

$$\frac{\left(E_t - E_{t-1}\right) \sum_{k=0}^{\infty} \frac{1}{\overline{r}_k} \overline{S} \hat{r}_{t,t+k}}{\frac{1}{\overline{r}_1} \sum_{k=0}^{\infty} \frac{1}{\overline{r}_k} \overline{S}} = \phi_r \epsilon_{t,G},$$

$$\hat{R}^j_{t-1,t} = \phi^j_R \epsilon_{t,G},$$

where ϕ_T, ϕ_r and ϕ_R control the endogenous responses to the exogenous spending shock of the present value of taxes, of the discount rates and of the holding period return of the maturity j bonds respectively.[6] ψ_G is the exogenous response of spending. We can then rewrite equation (3) as

$$\sum_{j\geq 1}^{N} \overline{R}^j \overline{w}^j \hat{w}^j_{t-1} + \sum_{j\geq 1}^{N} \phi^j_R \epsilon_{t,G} \overline{R}^j \overline{w}^j = (\phi_T + \phi_r - \psi_G)\epsilon_{t,G} + E_{t-1}\overline{r}_1 \hat{r}_{t-1,t}. \tag{5}$$

[5] Since we rely here on a log linear approximation, it is the steady state (zero order) weights that matter (see e.g. Coeurdacier and Rey (2013)). Alternatively, we could expand around the date $t-1$ portfolio. We will use this approach in the empirical model in the next section.

[6] Notice that our argument here is quite general. Since ϕ_T, ϕ_r and ϕ_R measure the responses of the present value of taxes, discount factors and returns, they could be consistent with many different paths of taxes and returns in response to a spending shock. Thus, we are not focusing here on a particular tax policy or a particular endogenous response of $\hat{r}_{t,t+k}$ and $\hat{R}^j_{t-1,t}$. Analogously, even though we use for simplicity a linear approximation of the intertemporal constraints, our argument here applies also to nonlinear models, in which for example ϕ_T can be a function of the debt level, or any other relevant state variable.

Let's consider again the case when the surplus drops because the increase taxes is not large enough to compensate the rise in spending, $(\phi_T + \phi_r - \psi_G) < 0$. Then, given ϕ_R^j, the government has to find a portfolio $(\overline{w}^j)_{j=1}^N$ to satisfy (5). A necessary condition for this is that:

$$cov_t(\hat{R}^j_{t-1,t}, \epsilon_{t,G}) < 0 \tag{6}$$

for some j. If there is a maturity j that satisfies condition (6) then the government will tilt its issuance policy towards this maturity to exploit the negative covariance. In the optimal DM literature (see e.g. ABN and Chari and Kehoe (1999), among others), condition (4) is satisfied. In these models, the government exploits the positive covariance between bond returns and the surplus in order to smooth taxes through time. ABN study the case where government bonds are real. In their model $\phi_R^1 = 0$ and $\phi_R^j < 0$ for $j \geq 2$. Thus, condition (6) holds for long term bonds that become the government's preferred assets to satisfy (5). In contrast, Chari and Kehoe (1999) assume nominal debt and let the government freely choose inflation[7] to satisfy the intertemporal budget. Coefficient ϕ_R^1 can be made sufficiently negative by the government policy so that (5) is satisfied even when $\overline{w}^1 = 1$.

What happens if a portfolio cannot be found, $\phi_R^j \approx 0$, so that condition (6) fails? In this case equation (5) still has to hold and the government will then set $(\phi_T + \phi_r - \psi_G) = 0$ such that current and future taxes will fully absorb the shock.

Consider now a decomposition of the government spending shock, $\epsilon_{t,G}$, into an aggregate, $\epsilon^A_{t,G}$, and an idiosyncratic, $\epsilon^I_{t,G}$, component such that $\epsilon_{t,G} = \epsilon^A_{t,G} + \epsilon^I_{t,G}$, where the aggregate shock affects the spending level of all countries and the idiosyncratic shock is country specific. Then (5) can be written as

$$\begin{aligned} \sum_{j\geq 1}^{N} \overline{R}^j \overline{w}^j \hat{w}^j_{t-1} \quad &+ \quad \sum_{j\geq 1}^{N} (\phi_R^{A,j} \epsilon^A_{t,G} + \phi_R^{I,j} \epsilon^I_{t,G}) \overline{w}^j \overline{R}^j \\ &= \quad (\phi^A_T + \phi^A_r - \psi^A_G)\epsilon^A_{t,G} + (\phi^I_T + \phi^I_r - \psi^I_G)\epsilon^I_{t,G} + \overline{r}_1 E_{t-1} \hat{r}_{t-1,t}, \end{aligned} \tag{7}$$

where coefficients ϕ_R^j, ϕ_T, ϕ_r and ψ_G are allowed to differ across the shocks because policy responses may vary. This decomposition allows us to study if the government manages to hedge both components of the shock. Continuing to follow the previous example, we can find fiscal hedging if

$$cov_t(\hat{R}^j_{t-1,t}, \epsilon^A_{t,G}) < 0 \quad \text{and} \quad cov_t(\hat{R}^j_{t-1,t}, \epsilon^I_{t,G}) < 0. \tag{8}$$

If both of these conditions hold, then governments can, in principle, hedge against both aggregate and idiosyncratic shocks through the portfolio. In contrast, if one of these conditions fails, then governments cannot hedge against one of the components of spending.

Our empirical analysis below first provides a test for (6). Using a structural VAR we identify an exogenous spending shock and verify whether holding period returns covary negatively with the shock. When the covariance is negative, we claim that governments can benefit from fiscal hedging. Then, governments can find a portfolio so that spending shocks are (at least partly) absorbed by holding period returns, thus alleviating the requirement to finance spending through taxes. Moreover, using an identification strategy to decompose the aggregate and idiosyncratic component of the government spending shocks, we study whether (8) holds fully or only against one type of shock. This identification gives us further evidence of how much DM can be successful in hedging the two types of shocks.

It is important to clarify that in our empirical analysis we will not claim that governments in practice follow perfectly the optimal DM strategy choosing optimally the weights, $\overline{w}^j$, that enable them to take full advantage of fiscal hedging. To characterize the optimal portfolios requires a structural model which determines jointly $\overline{w}^j$ and the policy coefficients, ϕ_T, ϕ_r and ϕ_R^j. This will be our task in section 3 of the paper.

[7] Inflation is not distortionary in their model.

Finally, the previous derivations were made under the assumption that governments repurchase in every period the entire outstanding debt. This assumption is common the DM literature (see for example Faraglia et al. (2019)), but in practice buybacks are rare. We can show that our previous derivations continue to hold when we rule out repurchases. We leave the details of this extension to the online appendix.

2.2 Data and Variables

2.2.1 Description of the Data

Bond Quantities and Prices. Our dataset contains information on every outstanding security issued by the central governments of the countries considered (Belgium, France, Germany, Italy and Spain) over the period 1998 to 2013. For each asset we can identify the type, nominal or real, the maturity, from 1 quarter to 30 years, as well as the principal and the coupon payments. These observations have been gathered separately for each country. The uniqueness of this dataset lies on the fact that it determines the maturity structure of each country's government debt taking into account each bond issued not relying on any of the aggregated official time series publicly available. Equiza-Goñi (2016) documents the data sources and for brevity we refer the reader to this paper for details.

All countries issued short term zero coupon bonds maturing in less than a year after their issuance. In general, their share fluctuates around 10 percent of all outstanding debt for each country, except for Germany where they always represent less than 5 percent of total sovereign debt. The rest of debt is long and several types of assets have been used by different countries. There are medium-term coupon bonds, typically maturing 5 years after issuance, and also many types of long-term bonds. In general, these instruments pay fixed coupons. An exception is Italy that issues CTZs that do not pay coupons, and CCTs which are floating rate bonds.[8]

In order to construct the shares of debt for different maturities, w^j, we convert all non-zero coupon bonds in our data set into zero-coupon bonds taking into account both principal repayment and coupons of each bond and their residual maturity; this procedure provides us the face value of debt for every maturity j. We price the zero coupon bond quantities with the zero-coupon yields, to obtain market values of debt. Nominal bond yield estimates for zero coupon bonds from the BIS database are used to price all payment obligations.

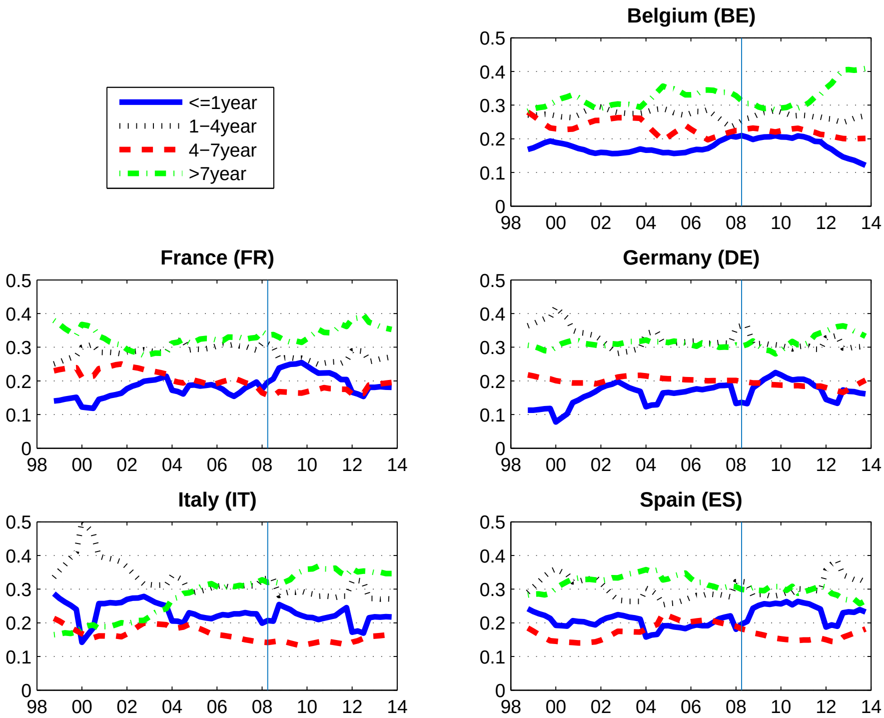
Figure 1 plots the obtained shares of outstanding payment obligations of maturities up to a year (solid lines), between 1 and 4 years (dashed lines) between 4 and 7 years (dotted lines) and above 7 years (dashed-dotted lines). The behavior of the shares reveals several noteworthy features of DM policy in the Euro Area. The shares of short and long term bonds are positive for all countries and are generally stable over time. Governments tend to increase slightly the share of longer maturities in periods where debt accumulates faster (e.g. towards the end of our sample).[9] Moreover, it is interesting to note that Italy had a higher proportion of short and medium term debt than other European countries before entering the Euro. At that time Italy faced a more steeply upward sloping yield curve than Germany, France and Belgium.[10] In relative terms, issuing long bonds was therefore

[8] Short term bonds are called Treasury Certificates or Bills in Belgium, Bubills in Germany, Letras in Spain, BOTs in Italy, or BTFs in France, medium terms bonds Bobls in Germany, BTAN in France, Bonos in Spain and long term bonds OLOs in Belgium, OATs in France, Bunds in Germany.

[9] Issuing more long term debt when overall debt rises is a common feature of DM in Europe and in the US. See Greenwood et al. (2015) for US evidence. When overall debt rises the refinancing risk increases and debt managers face a trade off between issuing more expensive and less risky debt, long term, or cheaper and riskier debt, short term. In general they prefer to issue long term debt to reduce overall refinancing risks of government portfolios.

[10] A plausible explanation for higher yields is inflation risk premia. In the early 90s markets feared that the Italian government could resort to inflation to finance its high debt. In the late 90s Italian monetary policy gained credibility because the exchange rate was locked to the ECU. High long term yields were also the case in Spain, however, this country begun increasing its long term debt issuances even before 1999. By the start of the sample the overall the duration of Spanish debt is similar to the duration in Germany and Belgium. For more details refer to the online appendix.

Figure 1: **Portfolio Shares: Various Maturity Segments**



Notes: The figure shows the shares w^j of short term (maturity up to one year), medium term (maturity between 1 and 4 years), long term (4-7 years) and very long term debt ($\geq$7 years) debt in government portfolios. To compute these shares we count both principal repayments and coupons using the remaining life of each bond to assign maturity. The vertical lines mark the beginning of the financial crisis at the second quarter of 2008.

more expensive. When long term yields fell, the Italian government begun substituting some of its short and medium term maturity debt with long bonds.

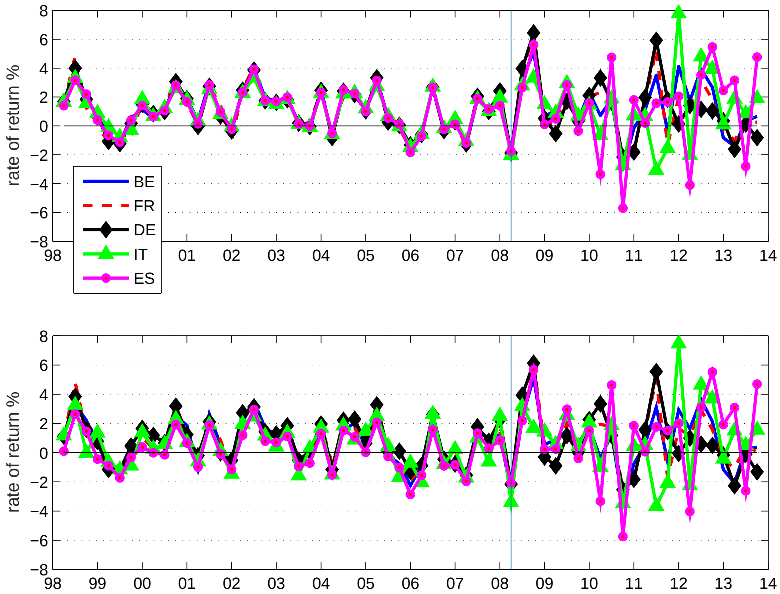
Though most of the debt issued by Euro Area governments is nominal, some medium- and long-term bonds are indexed to inflation. In our sample France seems to have been a pioneer: it was the only country venturing into issuing this type of debt in 1998. Italy followed in 2002. Both countries have increased the use of this instrument throughout our sample reaching a share of 12% in 2013. Then in 2007 Germany joined, but the share of real bonds remained below 5 percent of overall debt throughout the period considered. Belgium and Spain did not use this instrument in our sample period. The bonds issued have maturity ranging from 5 years to 32 years. Italy and France not only issued bonds indexed to the national CPI but also some bonds indexed to the Euro Area HICP.

Due to the fact that issuances of inflation-indexed bonds were virtually zero for most of the sample period for many countries, our empirical analysis will focus only on nominal debt.

GDP, Prices and Spending. Our SVARs will include real GDP, price indices (GDP deflator) and government spending retrieved from the OECD database. Government spending corresponds to consumption of final goods and services by the general government and thus excludes transfers and public investment. Over the period considered mean spending levels range between 21 percent of GDP, in the case of France, to 17 percent, in the case of Spain. Spending varies considerably over time; for example, in Belgium expenditures account for 17 percent of the variability of GDP (i.e. in terms of the ratio of standard deviations), and in Germany for 10 percent.

Sample Selection. Our main empirical exercise uses a sample which covers the period 1998 (Q2) to 2008 (Q2). We start the sample 3 quarters before January 1st 1999, the date of the official introduction of the Euro, to allow our VARs to have the sufficiently rich lag structure suggested by the usual criteria. We truncate our sample in the second quarter of 2008 just before Lehman-Brothers filed for bankruptcy in September 2008. We exclude the years of the financial and sovereign debt crises because we wish to focus on years when the markets perceived government default as extremely

Figure 2: **Holding Period Returns: Aggregate Portfolios**



Notes: The figure shows the holding period return series. The top panel shows returns not adjusted by realized inflation. The bottom panel plots the return series $\mathscr{R}$ on the aggregate portfolio, setting $(\underline{j} = 1, \overline{j} = 120)$.

unlikely for all countries in our sample, as assumed in the optimal DM literature. Later on, in our robustness checks, we will expand our sample and include observations up to 2013.

2.2.2 Bond Return Variables

The previous subsection has explained how we have created the shares of outstanding obligations using bond quantities and prices. We now turn to the construction of the holding period returns used in the SVARs.

For maturity $j = 1, 2, ..., 120$, where j denotes quarters, we have calculated the shares of debt for every maturity, $w^{i,j}_{t-1}$, and the holding period returns, $R^{i,j}_{t-1,t}$ using the zero coupon prices, $q^{i,j}_t$, and the GDP deflator to measure inflation, for every country i in each period t.

We then construct the following return variables:

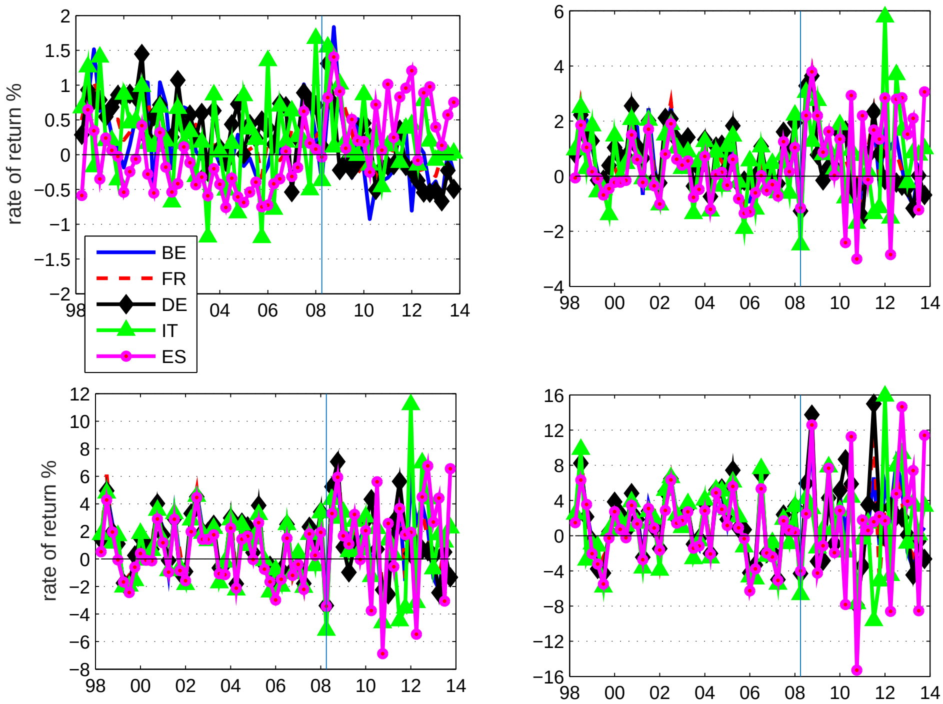
$$\mathscr{R}^{i,\underline{j},\overline{j}}_{t-1,t} \equiv \frac{\sum_{j=\underline{j}}^{j=\overline{j}} (R^{i,j}_{t-1,t} - 1) w^{i,j}_{t-1}}{\sum_{j=\underline{j}}^{j=\overline{j}} w^{i,j}_{t-1}}$$

where $w^{i,j}_{t-1}$ is the share of the market value of debt of maturity j over the total market value of debt issued by government i in $t-1$. $\underline{j}$ and $\overline{j}$ are the lowest and upper bounds of maturities included in the composite return $\mathscr{R}^{i,\underline{j},\overline{j}}_{t-1,t}$.

In our empirical analysis we estimate separate VARs assuming different values for $\underline{j}$ and $\overline{j}$. Our baseline model sets $\underline{j} = 1$ and $\overline{j} = 120$ and $\mathscr{R}^{i,1,120}_{t-1,t}$ denotes the return on the overall portfolio of government i (as in the LHS of equation (2)). In order to identify the effects of spending shocks on different maturity segments we also use values for $\{\underline{j}, \overline{j}\}$: $\{1, 4\}$, $\{5, 16\}$, $\{17, 28\}$, and $\{29, 120\}$ corresponding to short, medium, long and very long maturities respectively.

Figures 2 and 3 plot the resulting holding period return series. Figure 2 shows the series of the aggregate portfolio $(\underline{j} = 1, \overline{j} = 120)$ for each of the five countries in our sample. The bottom panel displays the inflation adjusted returns and the top panel the unadjusted nominal returns. For completeness, we extend the sample period to include observations up to 2013 as in the previous section.

Figure 3: **Holding Period Returns: Various Maturity Segments**



Notes: The figure shows the holding period return series. The top left panel plots the return series for short term debt, ($\underline{j} = 1, \overline{j} = 4$). The top right panel sets ($\underline{j} = 5, \overline{j} = 16$) (medium term debt), the bottom left ($\underline{j} = 17, \overline{j} = 28$) (long term debt) and the bottom right sets ($\underline{j} = 29, \overline{j} = 120$) (very long term debt).

Figure 3 shows the behavior of the inflation adjusted returns for each of the different maturity segments considered. The top left panel displays the returns of short term bonds, the top right medium term, the bottom left long term (maturity 4-7 years) and the bottom right very long maturity debt (>7 years).

From the figures it is evident that prior to the financial crisis holding period returns were very strongly correlated across countries. The lowest correlation coefficient of the inflation adjusted returns on the aggregate portfolio is 0.85 (between Italy and Belgium). The unadjusted return series are slightly more correlated. However, after 2008 Q3, the cross-sectional correlations became considerably weaker; most notably, the returns on Italian and Spanish debt followed a different pattern than in France and Germany: the correlation coefficient between Italy's return series and the returns on German debt is -0.06; the analogous correlation between Spain and Germany is 0.22. As it is well known, during the crisis, Italy and Spain faced high credit spreads as financial markets revised upwards their expectations of the two countries to withdraw from the Euro and default on their debt. In contrast, before the crisis (1999-2008), the Euro was considered a stable arrangement and returns were basically aligned across countries.

Figure 3 shows that these patterns are driven mainly by the behavior of the returns on longer maturity bonds. During the financial crisis, it is evident that returns of long term bonds exhibited higher volatility and weaker correlations. Before the crisis returns on long term bonds were more volatile than returns on short maturities, but they were strongly correlated across countries.

Finally, comparing the behavior of the shares shown in Figure 1 and the pattern followed by the return series in Figures 2 and 3, it is evident that changes in the real payout of debt $\mathscr{R}_{t-1,t}^{i,1,120}$ does not reflect changes in the shares w.

2.3 Identification of Aggregate and Idiosyncratic Spending Shocks

Our goal is to quantify the response of holding returns to government spending shocks and simultaneously to identify the impact of aggregate and idiosyncratic shocks. We thus make two identification assumptions. The first serves to identify spending shocks in general and the second to separate the effects of aggregate shocks from the effects of idiosyncratic shocks. To identify spending shocks, we follow Blanchard and Perotti (2002), Burriel et al (2010), Beetsma and Giuliodori (2011) (among others) and assume that government consumption reacts with lags to innovations in output.[11] To separately estimate the impact of aggregate and idiosyncratic shocks we assume that country specific spending does not impact instantaneously the average spending level of the five countries in our sample.

Formally, let $\mathscr{X}_t^i \equiv \left(\widetilde{g}_t^a, \widetilde{g}_t^i, \widetilde{Y}_t^i, \widetilde{P}_t^i, \mathscr{R}_{t-1,t}^{i,\underline{j},\overline{j}}\right)$ denote the vector of variables that enter in the VAR, where $\widetilde{g}_t^a$ denotes the log of the weighted average of real government spending in t across the five countries in our sample (with weights corresponding to relative GDPs), $\widetilde{g}_t^i$ is the log of real government spending in country i[12], $\widetilde{Y}_t^i$ the log of real GDP, $\widetilde{P}_t^i$ the log of the GDP deflator. We detrended each of these variables using a one-sided Hodrick-Prescott filter with a smoothing parameter of 8330, following Berndt et al. (2012) that shows evidence of fiscal insurance in the U.S. also in a VAR setting.[13]

Our baseline specification is the following pooled panel VAR:

$$\mathscr{X}_t = \mathscr{B}(L)\mathscr{X}_{t-1} + \nu_t, \quad \nu_t = \mathscr{M}\epsilon_t \tag{9}$$

where $\mathscr{X}$ stacks the $\mathscr{X}^i$ matrices, ν_t represents the reduced form innovations, ϵ_t are the structural shocks which have diagonal variance-covariance matrix $\mathscr{D}$. $\mathscr{B}(L)$ is the lag polynomial in the reduced form VAR. Identification of the structural shocks is achieved through the choice of the square matrix $\mathscr{M}$ that maps the errors ϵ_t to the reduced form errors ν_t. We include country fixed effects in the estimation. To condense notation we omit them from (9).

We use the Choleski decomposition and assume that $\mathscr{M} = \mathscr{L} \cdot \mathscr{D}^{1/2}$ where $\mathscr{L} \cdot \mathscr{D}^{1/2}$ is a lower diagonal matrix. Given the ordering of the variables in the VAR, we identify the aggregate spending shock as the shock that impacts all variables instantaneously and the idiosyncratic spending shock as the shock that does not impact (within the same period) average spending. Both spending shocks are identified as in Blanchard and Perotti (2002) assuming that spending responds with lags to innovations in prices and output. The return series enters last in $\mathscr{X}$ since both output shocks and price shocks impact returns instantaneously.

Our identification strategy is similar to Bernardini et al (2020) who estimate spending multipliers in US states using Blanchard and Perotti's scheme. The first variable in their VAR is aggregate spending, the second variable is a redistribution index which captures state specific shocks that leave total spending levels unchanged. Employing the same strategy in our setting, idiosyncratic shocks capture precisely the component of spending that redistributes the fiscal burden across countries: a positive shock in one country is compensated by the opposite shock in another country. Note that this does not require shocks to occur simultaneously and cancel each other out. Suppose for example that there is a rise in spending in Germany, but no change in spending elsewhere. Then, since Germany is a large country the shock increases the average spending level in the Euro Area, hence all countries experience an aggregate shock. Simultaneously, there are also several idiosyncratic shocks. Germany experiences a positive idiosyncratic shock, and all the other countries experience negative idiosyncratic shocks despite keeping their spending levels constant.

This obviously is a simplistic example, since in practice all countries simultaneously experience changes in their spending levels. Changes in the average spending level thus do not originate in one

[11]Because our data is quarterly assuming even two lags is reasonable. More lags would probably be less sensible as the use of Blanchard and Perotti's scheme in low frequency (annual) data is seen as problematic (e.g. Ravn et al (2012)). Born and Mueller (2012) have however shown that the scheme works well even with annual data.

[12]Note that as in Perotti (2005) and Burriel et al (2010) we adjusted government spending by adding deviations of the deflator of spending from its trend and assuming an elasticity of 0.5. The trend is obtained through a standard HP filter. In this way we cleaned the government consumption series from cyclical variations in prices.

[13]Notice that the value 8330 is equivalent to removing frequencies of around 15 year. Also, the fact that the HP filter only uses past values assures that the temporal ordering of the data is preserved. Given our short time span, we use a pre-sample from 1981(Q1) to detrend the series. In the online appendix we show that our results are robust towards alternative values of the smoothing parameter, towards not using the pre-sample and/or removing instead simple linear or quadratic trends. Finally, the series of debt returns is not detrended.

country. However, this example helps to clarify that the opposite is also not true: it is not necessary that fiscal policies in all countries coordinate and increase spending levels, in order to produce an aggregate shock. Idiosyncratic and aggregate shocks will occur simultaneously in our empirical model and they will not load in different periods. We leave to the online appendix further details of the identified shocks and of the forecast error variance decomposition.[14]

Finally, a few recent papers have drawn caution on the ability of the Blanchard and Perotti identification scheme to identify exogenous shocks spending in structural VAR models. Ramey (2011) and Leeper et al (2013) have highlighted the limitations of these models in accounting for 'fiscal foresight', i.e. when fiscal measures not observable by the econometrician are known in advance by private agents. Two solutions have been proposed by the literature to tackle this problem. Sims (2012) suggests to augment the VAR with forward looking variables that may react to news about spending, for example, bond returns, stock prices, spreads etc. In our model we use returns on government debt that obviously react to news about spending. For this reason we are confident that our VAR can provide reliable estimates of the structural shocks to spending. Alternatively, Auerbach and Gorodnichenko (2012) suggest to augment the VAR with forecasts errors of spending. In our case the only database which contains forecasts of spending for the countries in our sample is the OECD database, however these are not available at quarterly frequency. Using biannual forecasts, we find that there is no significant correlation between the additional information of OECD forecasts and the residuals resulting from our VAR.[15] We then conclude that fiscal foresight is not an issue in our analysis.

2.4 Baseline Estimates

Before considering separately idiosyncratic and aggregate shocks, we first study the effects of total spending on holding period returns testing for (6). For this exercise we run model (9) removing average spending from the state vector, letting $\mathscr{X}_t^i \equiv \left(\widetilde{g}_t^i, \widetilde{Y}_t^i, \widetilde{P}_t^i, \mathscr{R}_{t-1,t}^{i,\underline{j},\bar{j}}\right)$.[16] Our estimates then pull together both aggregate and idiosyncratic shock components, since each of the series $\widetilde{g}_t^i$ is influenced by both types of shocks. Applying the usual information criteria, our reduced form estimates indicate that including two lags in the polynomial $\mathscr{B}(L)$ is appropriate.

Figure 4 reports the impulse responses of each variable to one standard deviation shock in 'total' spending when we include the return on the entire portfolio, $\mathscr{R}_{t-1,t}^{i,1,120}$. Starting from the top left panel and going clockwise the figure reports the response of spending, output, returns and finally the price level. The dashed lines indicate the one standard error confidence intervals (equivalent to the 16th and 84th percentiles as typical in the literature).[17]

[14]See online appendix for graphs showing the average and country series of real government consumption that are included in our benchmark VAR (in natural logarithms, detrended, etc), as well as our estimated aggregate and idiosyncratic shocks series in 1998–2008.

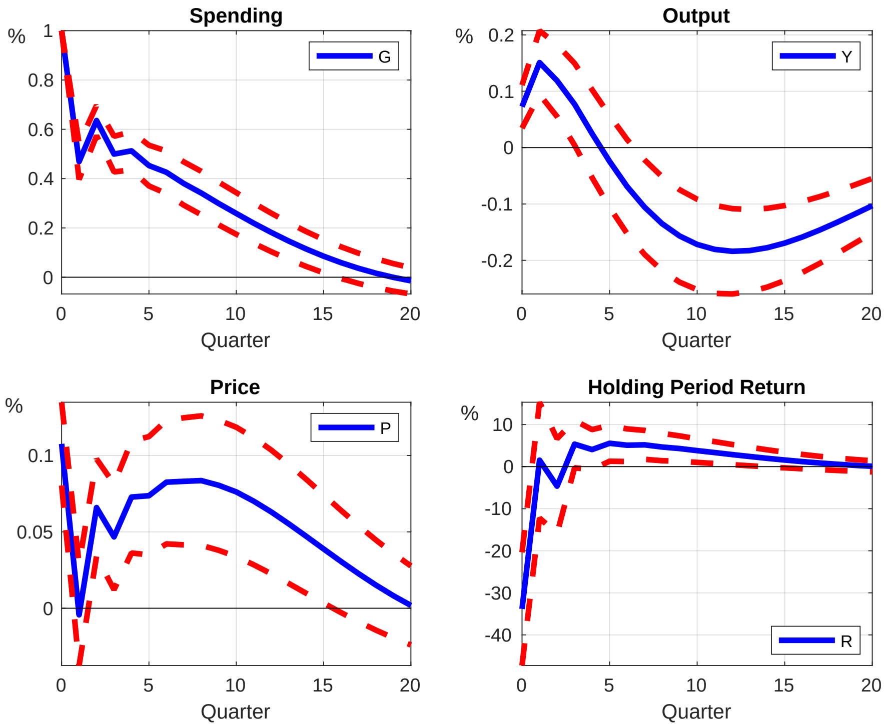
We provide also results of the forecast error variance decomposition. We find that 27% of country-specific government spending forecast errors are caused on impact by aggregate shocks, whereas the other 73% are caused by unforeseen idiosyncratic shocks. Obviously, 100% of the variability of average government spending forecasts is produced, on impact, by aggregate shocks. At longer horizons aggregate and idiosyncratic spending shocks continue to explain most of the variability of spending. For example after 2 quarters 12% of the variability is attributed to aggregate shocks and 87% to idiosyncratic shocks. After 15 quarters these numbers are 36% and 45% respectively.

[15]Following Auerbach and Gorodnichenko (2012), we tested for the significance of the correlation between the components of actual and forecasted growth rates of real government consumption that are orthogonal to the information in our panel VAR. We used in each quarter the closest preceding forecast provided by the OECD in June and December (thus, biannually). The correlation coefficient is 0.1 (with p-value equal to 0.18) for 1998-2008. Similarly, the correlation between our panel VAR innovations to government spending and the residuals obtained from regressing the spending growth forecasts on our VAR lagged values is 0.08 (with p-value of 0.24).

[16]A similar exercise has been performed by Beetsma and Giuliodori (2011) with Euro Area data to identify the effects of spending on output. Their state vector includes spending, output, long term rates and real exchange rates. Their identification assumption is the same as the one employed here.

[17]See for example Blanchard and Perotti (2002) or Beetsma and Giuliodori (2011). The confidence intervals are computed assuming normality of errors and using Monte Carlo simulations (1000 replications). We apply the same procedure to all estimates reported in this section.

Figure 4: **Impulse Responses One Standard Deviation in Spending**



Notes: The figure plots the impulse responses of government spending (top left), output (top right), prices (bottom left) and holding returns (bottom right) to a one standard deviation shock in 'total' spending. The estimates derive from the panel SVAR in Section 2.3. See text for further details.

Following a shock to government spending, prices increase and output increases albeit not substantially. Notice, however, that the estimated response of real output falls within the range of previous estimates in the literature, which typically give a multiplier close to one for the Euro Area.[18] Most importantly the return variable drops significantly (i.e. zero is out of the one-standard deviation confidence bands).

The first column of Table 1 reports the point estimates. The total return on the portfolio drops by 33.8 basis points (bps) on impact. This constitutes a large impact effect of spending on returns[19] and provides *strong evidence of fiscal insurance* in the Euro Area: when spending levels rise, the payout of government debt is lowered and governments enjoy capital gains.

Rows 2 to 5 of Table 1 report the effect on our estimates when we vary the maturity of debt. We consider four maturities, ranging from less than a year to more than seven years. In column 1 all responses are large and statistically significant and the impact effect is larger the longer the maturity. This finding is in line with the predictions of theoretical models of optimal DM theory which suggests that governments should use long maturities to exploit better the fiscal insurance channel.[20]

[18] See for example Burriel et al (2010). Since spending is about 20 percent output in our sample, a spending shock equivalent to 1 percent of GDP increases output by 0.75 percentage points. Thus, despite our short time span (only 10 years compared to other studies), we obtain a multiplier not far from 1.

[19] Though $\mathscr{R}$ continues to respond to the shock even after the initial period, our focus here is on the initial impact. Given that we have constructed the returns $\mathscr{R}$ using time varying portfolio weights, $w_t^{i,j}$, at impact the weights are predetermined (see previous derivations) but in the subsequent periods they may adjust to the shock. This could be due for example to the fact that debt management offices adjust the maturity of new issuances in response to the slope of the yield curve, i.e. issue more of a cheaper security. Focusing on initial period responses guarantees that our estimates do not confound the responses of returns with possible portfolio rebalancing effects.

[20] It is not surprising that even assets of maturity less than or equal to 1 year give a strong and statistically significant impact. Since our horizon is quarterly the prices of six month and one year nominal debt respond to the shock. Changes in P also exert an influence since we adjusted returns by inflation.

Table 1: **Responses of Real Returns to Spending Shocks**

	Pooled-Panel VAR			**Average VAR**		
	All shocks	**Aggr.**	**Idiosync.**	**All shocks**	**Aggr.**	**Idiosync.**
All maturities	$-33.8^{\star}$	$-86.3^{\star}$	-13.4	-16.5	$-84.8^{\star}$	19.1
	(0.01)	(0.00)	(0.40)	(0.76)	(0.12)	(0.79)
≤ 1 year	$-15.2^{\star}$	$-9.3^{\star}$	$-17.3^{\star}$	-1.0	-11.3	2.6
	(0.00)	(0.18)	(0.00)	(0.92)	(0.42)	(0.85)
1-4 years	$-28.6^{\star}$	$-57.9^{\star}$	$-17.0^{\star}$	-7.6	$-61.7^{\star}$	15.6
	(0.00)	(0.00)	(0.08)	(0.83)	(0.09)	(0.72)
4-7 years	$-49.0^{\star}$	$-127.2^{\star}$	-18.1	-29.5	$-128.6^{\star}$	19.9
	(0.01)	(0.00)	(0.38)	(0.67)	(0.08)	(0.82)
>7 years	$-50.2^{\star}$	$-156.9^{\star}$	-8.4	-25.3	$-147.9^{\star}$	43.4
	(0.10)	(0.01)	(0.81)	(0.82)	(0.21)	(0.77)

Notes: The table reports initial period responses (in terms of basis points) of bond returns to 1% increase in government consumption in pooled panel VARs and average VARs. Sample: 1998(Q2)-2008(Q2). All variables are detrended using a one-side HP filter with smoothing parameter of 8330. P-values are reported in parentheses while * denotes that zero is outside the one-standard-deviation confidence interval obtained with Monte Carlo simulations (1000 replications).

We now turn our attention to the main focus of our exercise: to test for (8) identifying separately the effects of aggregate and idiosyncratic shocks on holding period returns. We then run our VAR using $\mathscr{X}_t^i \equiv \left(\widetilde{g}_t^a, \widetilde{g}_t^i, \widetilde{Y}_t^i, \widetilde{P}_t^i, \mathscr{R}_{t-1,t}^{i,\underline{j},\bar{j}}\right)$, as defined in Section 2.3. The second and third columns of Table 1 report the impact effect on the returns of an aggregate shock and of an idiosyncratic shock respectively. We find that holding period returns drop significantly in response to an aggregate shock. The estimated impact is roughly equal to 86 bps, more than twice the one estimated when we pulled both shocks. On the other hand, when the shock is idiosyncratic, the effect is roughly 13 bps and not statistically significant. This suggests that fiscal hedging is powerful, but only in the case of aggregate shocks.

When we consider different maturities (rows 2-5, Table 1) these findings continue to hold. Across all maturity segments aggregate spending shocks have strong negative and significant effects on returns. When shocks are idiosyncratic, the impact effects are small and are significant only when we consider short and medium term bonds.

2.5 Robustness

The evidence presented so far has two important implications for debt management in the Euro Area. First, fiscal hedging exerts a powerful influence on the behavior of returns on government debt and second, this influence is mainly accounted for by shifts in aggregate spending. We now turn to alternative specifications of our empirical model to test the robustness of our findings.

We first put aside the efficiency gains of the pooled panel and allow for heterogeneity in the estimated coefficients, $\mathscr{B}^i(L)$, running separate VARs for each country applying the same identification strategy as in our baseline specification. The impact effects now correspond to the weighted average responses from the five VARs, using the same weights applied to construct $\widetilde{g}^a$. Column 4 to 7 of Table 1 report the results. The estimates suggest that aggregate shocks continue to yield considerable impacts on returns. When we consider idiosyncratic shocks, now all coefficients are insignificant and moreover have the 'wrong sign': returns increase in response to spending shocks so that governments experience capital losses. This holds for the overall portfolio as well as for the different maturity segments. Our previous findings are therefore confirmed when we allow for heterogeneity in the estimated parameters. Aggregate shocks are the key driving force behind fiscal hedging in the Euro Area.

Secondly, in order to isolate the impact of spending shocks on nominal bond prices we now run

Table 2: Robustness Analysis: Different Model Specifications

	Pooled-Panel VAR			Average VAR		
	All shocks	**Aggr.**	**Idiosync.**	**All shocks**	**Aggr.**	**Idiosync.**
Nominal Returns						
All maturities	$-20.8^\star$	$-74.0^\star$	1.0	-14.4	$-73.9^\star$	16.5
	(0.13)	(0.00)	(0.95)	(0.79)	(0.17)	(0.81)
≤ 1 year	$-3.8^\star$	$-51.7^\star$	13.5	0.45	-4.38	0.06
	(0.05)	(0.13)	(0.76)	(0.94)	(0.56)	(0.99)
1-4 years	$-16.1^\star$	$-47.3^\star$	-2.8	-6.77	$-51.74^\star$	13.46
	(0.05)	(0.01)	(0.77)	(0.84)	(0.13)	(0.76)
4-7 years	$-35.9^\star$	$-115.2^\star$	-3.7	-27.99	$-117.4^\star$	17.67
	(0.05)	(0.00)	(0.86)	(0.69)	(0.11)	(0.85)
> 7 years	$-37.1^\star$	$-144.3^\star$	6.0	-22.85	$-136.2^\star$	40.85
	(0.30)	(0.03)	(0.86)	(0.84)	(0.26)	(0.78)
Sample 1998-2013						
All maturities	$-36.1^\star$	$-90.9^\star$	-10.8	-41.6	$-99.1^\star$	3.8
	(0.02)	(0.00)	(0.53)	(0.47)	(0.10)	(0.96)
≤ 1 year	$-18.5^\star$	-15.1	$-18.5^\star$	-7.9	-12.2	-3.6
	(0.00)	(0.02)	(0.00)	(0.42)	(0.37)	(0.78)
1-4 years	$-30.0^\star$	$-46.2^\star$	$-19.6^\star$	$-33.6^\star$	$-60.5^\star$	-9.1
	(0.00)	(0.01)	(0.06)	(0.32)	(0.09)	(0.82)
4-7 years	$-51.8^\star$	$-126.8^\star$	-17.7	-67.4	$-139.9^\star$	10.4
	(0.01)	(0.00)	(0.41)	(0.34)	(0.07)	(0.90)
> 7 years	$-66.3^\star$	$-195.3^\star$	-5.9	-62.1	$-191.8^\star$	36.5
	(0.04)	(0.00)	(0.88)	(0.61)	(0.16)	(0.81)

Notes: The table reports initial period responses (in terms of basis points) of bond returns to 1% increase in government consumption in pooled panel VARs and average VARs. Sample: 1998(Q2)-2008(Q2) except for the last section where the sample is extended to 2013(Q4). All variables are detrended using a one-side HP filter with smoothing parameter of 8330. P-values are reported in parentheses while * denotes that zero is outside the one-standard-deviation confidence interval obtained with Monte Carlo simulations (1000 replications).

VAR (9) replacing $\mathscr{R}$ with holding period returns not adjusted by inflation. The top part of Table 2 reports the results. Not surprisingly, now the estimated coefficients are smaller in absolute value, as these returns do not reflect unexpected higher (lower) current inflation due to positive (negative) spending shocks. However, they remain strongly significant in the case of aggregate shocks for maturities longer than one year and not significant in the case of idiosyncratic shocks. Therefore, part of the response of bond returns $\mathscr{R}$ to spending shocks is accounted for by inflation but aggregate shocks have substantial impacts on nominal bond returns.

We now turn our attention to the sample period. As discussed previously, our baseline sample does not include the years of the financial crisis because we do not want our estimates to confound the standard fiscal hedging impact of shocks on returns with the default impact, which we view as a separate channel. During the crisis Italy and Spain experienced large spreads because investors considered more likely that these countries would default and exit the Euro. To the extent that a sequence of high spending shocks drove the rise in spreads, our VARs would identify negative effects of spending on returns, but this would not be due to fiscal hedging.[21]

[21] Note that this could also mean that idiosyncratic shocks would have a significant impact. If a rise in spending increases the spread of one country, then since part of the shock is idiosyncratic, returns would comove negatively with the idiosyncratic shock.

It is, however interesting to extend our estimates to the crisis period, and for this reason we now run VARs including observations up to 2013. The results are shown in the bottom part of Table 2. The estimates confirm our previous results. Aggregate shocks continue to affect returns, whereas idiosyncratic shocks do not. Moreover, the estimates are very similar to the analogous objects reported in Table 1 which suggests that during the turbulent years 2009-2013 there is no evidence of a drastic change in the magnitude of the responses of returns to the shocks. Default and hedging are separate channels, however we do not claim that they cannot co-exist. We are not aware of any model that brings these two channels together. On one hand fiscal hedging presupposes that countries are credit worthy and satisfy their budgets at all times, on the other hand government default implies that governments will be unable to redeem their debts in some states. However, there is no reason why governments could not be able to benefit from hedging despite the fact that investors believe more likely that they will be unable to repay their debts in the future under some contingencies. We thus cannot exclude that in the estimates reported in Table 2 both channels are at work.

Finally, the online appendix presents additional robustness checks of the main results reported in this section. In particular, we have used alternative detrending methods (including linear and quadratic trends), a richer lag structure or different inflation measures. Moreover we have experimented with using the first principal component ($\mathscr{P}$) of the spending series of the five countries in our sample to replace $\widetilde{g}_t^a$ despite our main identification strategy so far has relied on the assumption that idiosyncratic disturbances do not influence the average spending series and this will be consistent to our theoretical approach later. Our baseline results continue to be robust to these different specifications.

3 Optimal Portfolios in a Currency Union: a Theoretical Analysis

With a pure normative purpose and aiming to preserve tractability, we now turn to a theoretical analysis to understand which types of bonds could be optimal in a currency union to profit completely from fiscal hedging. The empirical analysis can only tell us whether governments can (potentially) benefit from the covariance between ex post returns and shocks (conditions (6) and (8)). We do need a structural model to investigate how governments can optimise and take advantage of fiscal hedging. Are there specific types of bonds that are apt to exploit this channel? If nominal bonds cannot be used to hedge against idiosyncratic shocks, can inflation indexed debt prove useful?

The optimal DM literature has provided a framework that can help us answer these questions. This literature assumes a production economy where a benevolent government with full commitment wishes to smooth taxes over time. In every period the government sets the level of distortionary taxes and issues debt to finance spending, assumed to be exogenous and stochastic.

For the time being, assume that there are two possible exogenous spending states, s_1 and s_2, and that the government can issue only two nominal bonds, a one period, B^1, and a N period bond, B^N. Let q_t^j denote the equilibrium nominal bond price of maturity j. In equilibrium, in standard models where bonds are priced by a representative household and the asset pricing kernel is the growth of the marginal utility of consumption, we have $q_t^j = \beta^j E_t\left(\frac{U_{C,t+j}}{U_{C,t}}\frac{P_t}{P_{t+j}}\right)$ where P_t is the price level, $U_{C,t}$ is the marginal utility of consumption of the household and βdenotes the discount rate. The theoretical analog of equation (1) can be written as:

$$\frac{1}{1+\pi_t}\frac{B_{t-1}^1}{P_{t-1}} + \frac{q_t^{N-1}}{1+\pi_t}\frac{B_{t-1}^N}{P_{t-1}} = E_t\sum_{k=0}^{\infty}\frac{S_{t+k}}{U_{C,t}} \equiv \Upsilon_t, \tag{10}$$

where π and S are net inflation and the surplus of the government (the latter is scaled by marginal utility) respectively. Equation (10) needs to be satisfied in every period t, given the optimal tax, inflation and debt issuance policies which endogenously determine S_t, P_t, $U_{C,t}$, q_t^{N-1}.

The literature has studied two different environments. On one hand, ABN and Faraglia et al (2010) consider the case where markets are complete. On the other hand, Lustig et al (2008), Debortoli et al (2017) and Faraglia et al (2019) analyse the same problem assuming incomplete markets, essentially constraining the quantity of debt that governments can issue.

In complete markets, the sequences S, $P\ U_C$ and q^{N-1} can be recovered from the solution to an optimal policy problem in which equation (10) can be satisfied as a residual. Then, given the optimal sequences of taxes, consumption, inflation etc, we can find the optimal bond quantities that make (10) hold. When the number of available non-contingent maturities equals the number of states of the economy, s_1 and s_2, (10) can be expressed as:

$$\underbrace{\begin{bmatrix} \frac{1}{1+\pi_t(s^{t-1},s_1)} & \frac{q_t^{N-1}(s^{t-1},s_1)}{1+\pi_t(s^{t-1},s_1)} \\ \\ \frac{1}{1+\pi_t(s^{t-1},s_2)} & \frac{q_t^{N-1}(s^{t-1},s_2)}{1+\pi_t(s^{t-1},s_2)} \end{bmatrix}}_{\mathscr{Q}_t} \cdot \underbrace{\begin{bmatrix} \frac{B_{t-1}^1(s^{t-1})}{P_{t-1}(s^{t-1})} \\ \\ \frac{B_{t-1}^N(s^{t-1})}{P_{t-1}(s^{t-1})} \end{bmatrix}}_{\mathscr{B}_{t-1}} = \underbrace{\begin{bmatrix} \Upsilon_t(s^{t-1}, s_1) \\ \Upsilon_t(s^{t-1}, s_2) \end{bmatrix}}_{\Upsilon_t}, \tag{11}$$

where s^{t-1} is the history of the stochastic process of spending until $t-1$. The solution of system (11) can pin down the optimal portfolio, B_{t-1}^1 and B_{t-1}^N, given the optimal allocation that provides values for Υ_t, P_t, q_t^{N-1} and π_t. If $\mathscr{Q}_t$ is invertible, the (unique) portfolio satisfies $\mathscr{B}_{t-1} = (\mathscr{Q}_t)^{-1}\Upsilon_t$. If this solution exists then the government can generate the required state contingent payoffs in t through issuing bond quantities in $t-1$ and exploiting ex-post variation in bond prices in t to complete the markets.

When markets are incomplete, the government is assumed to face additional constraints in the amount of bonds that it can issue/hold, $\underline{B}^j \leqslant B_{t-1}^j \leqslant \overline{B}^j$.[22] Then solving (11) is not sufficient to recover the optimal bond quantities and the portfolio problem needs to be solved along with the tax and inflation policies.

The aim of our theoretical section is to determine which types of bonds can be optimal to hedge against idiosyncratic shocks within a tractable framework and we adopt the complete market approach to DM of ABN and Faraglia et al (2010), extending that benchmark model to a monetary union. Our purpose is mainly normative and, as in these papers, we assume that optimal policy in the currency area is designed by a benevolent planner that maximizes the joint welfare of two countries A and B, thus implicitly assuming that fiscal policies are coordinated across countries and also coordinate with monetary policy.[23]

We first characterise the uncertainty faced by the economy (section 3.1) and then we briefly describe our model which close to Faia and Monacelli (2004) (section (3.2)). We provide only a narrative description of the model highlighting the essential building blocks, leaving its formal description to the online appendix since we do not depart from a canonical international macro two country model. We will quickly get to the contribution of the paper in sections (3.3) and (3.4) where we try to decentralize the complete market allocation using two types of non state contingent bonds: nominal bonds and inflation indexed bonds, assessing numerically their optimality to provide fiscal insurance against spending shocks. Lastly, before concluding the paper, we provide a brief discussion of how our results might change in a model with incomplete financial markets.

[22]This could mean ruling out completely issuances of maturity j when $\underline{B}^j = \overline{B}^j = 0$.

[23]As discussed previously, the assumption of optimal coordinated fiscal/monetary policies is common in the international macro literature (see e.g. Gali and Monacelli (2008), Farhi and Werning (2017) among others). Moreover, coordinated fiscal/monetary policies is assumed in the considerable literature of Ramsey models in the closed economy (e.g. Chari and Kehoe (1999), Siu (2004), Lustig et al (2008) and Faraglia et al (2013) among numerous others). As in these papers, monetary policy is not designed independently of fiscal variables and government debt. Inflation can be allowed to respond to debt and deficits fluctuations. However, since differently from these papers we assume a full set of maturities is available to governments and markets are effectively complete, debt acts as a shock absorber, inflation will not ultimately bear the burden of fiscal adjustment.

3.1 Uncertainty

In our model uncertainty stems from fluctuations in the spending levels of the two countries. Let G_t^i denote the level of exogenous government spending in country $i = A, B$. We assume that spending has two components: a common component denoted by g_t^c, which influences the spending levels of the two countries symmetrically, and a country specific component g_t^i, which is i.i.d across countries. We assume $G_t^i = g_t^c + g_t^i$.

G_t^i, g_t^c and g_t^i are first order Markov processes. Let N be the total number of possible realizations of the vector $G_t = [G_t^A, G_t^B]$ of joint spending. This joint process evolves according to the transition matrix $\overline{\mu}$. Finally, let $s_t \in \{s_1, ..., s_N\}$ denote the state in t and s^t represent the history of shocks from dates 0 to t.

The above parameterization of uncertainty is typical in multi-country business cycle models[24] and it is useful to make our theoretical analysis easily comparable to the empirical analysis of the previous section. The spending process assumed has both an idiosyncratic and an aggregate component; as in the data, we will identify idiosyncratic shocks taking deviations of G_t^i from the average spending level and aggregate shocks as shocks to average spending.

To clarify this, consider the following example. Suppose we have $N = 4$, g_t^c constant and $g_t^i \in \left\{\underline{g}, \overline{g}\right\}$. The vector of possible outcomes is:

$$\left(G_t^A - g^c, G_t^B - g^c\right) \in \left\{\left(\underline{g}, \underline{g}\right), \left(\overline{g}, \underline{g}\right), \left(\underline{g}, \overline{g}\right), \left(\overline{g}, \overline{g}\right)\right\}$$

and average spending is

$$\frac{\sum_i G_t^i}{2} \in \left\{g^c + \underline{g}, g^c + \frac{\overline{g} + \underline{g}}{2}, g^c + \frac{\overline{g} + \underline{g}}{2}, g^c + \overline{g}\right\}.$$

In this example, a shock which shifts G_t from state 1 to state 4 is an aggregate shock; the difference between the spending in country i and the average spending, $G_t^i - \frac{\sum_i G_t^i}{2}$, equals 0. A shock which shifts spending from state 2 to state 3 is a pure idiosyncratic shock, since there is no shift in the average.

3.2 A Brief Description of the Two Country Model

Each country, A and B, is populated by a representative household. Households are identical across countries and enjoy utility from leisure and the consumption of a composite bundle of goods produced both domestically and abroad. Preferences exhibit bias towards home goods. Let α denote the parameter that measures the degree of home bias.[25] When $\alpha < \frac{1}{2}$, the household prefers home goods.

Home and foreign goods are imperfect substitutes in utility and η symbolizes the elasticity of substitution. Moreover, home and foreign goods are aggregates of many varieties of differentiated products where ϵ measures the degree of substituability across these goods.

Product varieties are produced by monopolistically competitive firms that operate linear technologies and labor is the sole input in production. Following Siu (2004) we introduce sticky prices assuming that a fraction ν of the intermediate goods firms, in both countries, have to set their prices one period in advance and the remaining firms are flexible price firms and their price is set within the period. Since labour is the only input, flexible prices are a markup over wages (marginal costs) whereas sticky prices are set according to expected marginal costs in the next period.

Households maximize utility choosing consumption and hours worked subject to the budget constraint. As previously explained, we retrieve the complete market allocation first solving the model when agents can trade intertemporally with state contingent assets. From the household utility maximization problem, we can derive Euler equations to price these assets. The corresponding prices then give rise to the standard risk sharing condition which equates the ratio of marginal utilities to

[24] See for example, Krueger et al (2011) for an analogous parameterization of TFP shocks in a multicountry setup.

[25] Introducing notation from the model is useful to later on discuss our calibration of the model.

the consumption based real exchange rate under complete markets (see Backus and Smith (1993) and Kollmann (1995)). In terms of the model's notation this is:

$$\frac{U^B_{C,t}(s^t)}{U^A_{C,t}(s^t)} = \frac{P^B_{C,t}(s^t)}{P^A_{C,t}(s^t)}, \tag{12}$$

where $U^i_{C,t}(s^t)$ and $P^i_{C,t}(s^t)$ are the marginal utility of consumption and the price of consumption respectively in country $i = A, B$.[26]

The competitive equilibrium is found when households and firms optimize, governments and households satisfy their budget constraints and markets clear. The aggregate resource constraint equates aggregate output in country A, B, to the total demand for the home good which is the sum of the consumption of domestic and foreign households and government spending, which as in Gali and Monacelli (2008) is allocated exclusively to domestic goods.

Ramsey (optimal) policy chooses from the set of competitive equilibria the one that maximises the sum of welfare of the two countries equally weighted. In the online appendix we derive sufficient implementability conditions and state formally the Ramsey program. The online appendix presents also the optimality conditions and the methodology used to solve for the optimal policy allocation.

3.3 Decentralizing the Equilibrium with Non-State Contingent Bonds

3.3.1 Nominal Bonds

Let us first assume that governments can issue nominal bonds in different maturities and that the private sector in each country can buy (or sell) any security. Let $\mathscr{J}$ denote the set of N maturities issued. The number of assets in the market is then equal to the number of the possible realizations of G_t and in theory enough to complete the markets. Let $\mathscr{J}_k$ denote the k-th element of $\mathscr{J}$.

All bonds are nominal. $q^{i,j}_{n,t}$ is the price of a nominal asset, n, which promises one unit of income in period $t + j$ and is issued by the government in country i.

A bond of maturity j issued by the government in country A is priced $q^{A,j}_{n,t} = \beta^j E_t\left(\frac{U^A_{C,t+j}}{U^A_{C,t}}\frac{P^A_{C,t}}{P^A_{C,t+j}}\right)$ by the home household according to his first order condition. The same asset is priced $q^{A,j}_{n,t} = \beta^j E_t\left(\frac{U^B_{C,t+j}}{U^B_{C,t}}\frac{P^B_{C,t}}{P^B_{C,t+j}}\right)$ by the household in country B. Then under no arbitrage this price needs to satisfy:

$$q^{A,j}_{n,t} = \beta^j E_t\left(\frac{U^A_{C,t+j}}{U^A_{C,t}}\frac{P^A_{C,t}}{P^A_{C,t+j}}\right) = \beta^j E_t\left(\frac{U^B_{C,t+j}}{U^B_{C,t}}\frac{P^B_{C,t}}{P^B_{C,t+j}}\right). \tag{13}$$

The same holds for a bond of the same maturity issued by the government in country B. The above conditions then imply that the prices of nominal bonds of maturity j are equal across countries, and $q^j_{n,t} = q^{A,j}_{n,t} = q^{B,j}_{n,t}$.

Denote by $B^{i,j}_{n,t}$ the face value of nominal debt, n, of maturity j issued by the government in i in period t. The period budget constraint of the government in country i is given by:

$$\sum_{j\in\mathscr{J}} q^{j-1}_{n,t} B^{i,j}_{n,t-1} = \sum_{j\in\mathscr{J}} q^j_{n,t} B^{i,j}_{n,t} + P^i_{C,t}\frac{S^i_t}{U^i_{C,t}}. \tag{14}$$

According to (14) the government finances its fiscal deficit, $-P^i_{C,t}\frac{S^i_t}{U^i_{C,t}}$, and the outstanding debt level, $\sum_{j\in\mathscr{J}} q^{j-1}_{n,t} B^{i,j}_{n,t-1}$, with a portfolio of newly issued bonds.[27] Iterating forward (14), we can retrieve

[26] Note that we assume that the two countries have the same initial wealth and focus for simplicity on symmetric equilibria.

[27] Following ABN, Faraglia et al (2010) and Lustig et al (2008) we assume that all debt is bought back one period after issuance.

the intertemporal budget constraint of the government:

$$\sum_{j\in\mathscr{J}} \frac{q_{n,t}^{j-1}}{(1+\pi_t^i)} \frac{B_{n,t-1}^{i,j}}{P_{C,t-1}^i} = E_t \sum_{k=0}^{\infty} \beta^k \frac{S_{t+k}^i}{U_{C,t}^i} \equiv \Upsilon_t^i. \tag{15}$$

that needs to be satisfied for every history s^t. π_t^i denotes the CPI inflation in country i. As ABN and Faraglia et al (2010), we use (15) to characterise the optimal government portfolios. Using the optimal allocation chosen by the Ramsey planner, we can determine the bond prices and the discounted sum of future surpluses for every state of the economy. The bond quantities become then the only unknowns and will need to solve the following system of equations:

$$\underbrace{\begin{bmatrix} \frac{q_{n,t}^{\mathscr{J}_1-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} & \frac{q_{n,t}^{\mathscr{J}_2-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} & \dots & \frac{q_{n,t}^{\mathscr{J}_N-1}(s^{t-1},s_1)}{1+\pi_t^i(s^{t-1},s_1)} \\ \vdots & \dots & & \\ \frac{q_{n,t}^{\mathscr{J}_1-1}(s^{t-1},s_N)}{1+\pi_t^i(s^{t-1},s_N)} & \frac{q_{n,t}^{\mathscr{J}_2-1}(s^{t-1},s_N)}{1+\pi_t^i(s^{t-1},s_N)} & \dots & \frac{q_{n,t}^{\mathscr{J}_N-1}(s^{t-1},s_N)}{1+\pi_t^i(s^{t-1},s_N)} \end{bmatrix}}_{\mathscr{Q}_{n,t}} \cdot \underbrace{\begin{bmatrix} \frac{B_{n,t-1}^{i,\mathscr{J}_1}(s^{t-1})}{P_{C,t-1}^i(s^{t-1})} \\ \vdots \\ \frac{B_{n,t-1}^{i,\mathscr{J}_N}(s^{t-1})}{P_{C,t-1}^i(s^{t-1})} \end{bmatrix}}_{\mathscr{B}_{n,t}} = \underbrace{\begin{bmatrix} \Upsilon_t^i(s^{t-1},s_1) \\ \vdots \\ \Upsilon_t^i(s^{t-1},s_N) \end{bmatrix}}_{\Upsilon_t}.$$

The above system gives a unique solution $\mathscr{B}_{n,t} = (\mathscr{Q}_{n,t})^{-1}\Upsilon_t$, if $\mathscr{Q}_{n,t}$ is invertible. If the solution exists then governments can generate the required state contingent payoffs in t through issuing bonds quantities in $t-1$ and exploiting ex post variation in bond returns in t to complete the markets.

3.3.2 Inflation Indexed Debt

Assume now that the government can use only inflation indexed bonds of different maturities. A real bond, r, of residual maturity j, issued by country A in period t, is a bond that pays out $(1+\pi_{t+1}^A)(1+\pi_{t+2}^A)...(1+\pi_{t+j}^A)$ at $t+j$. From the household's optimization we can show that the price of this bond is $q_{r,t}^{A,j} = \beta^j E_t\left(\frac{U_{C,t+j}^A}{U_{C,t}^A}\right)$ when priced by the household in A. The same bond can be priced also by the household in B and the household in B sets $q_{r,t}^{A,j} = \beta^j E_t\left(\frac{U_{C,t+j}^B}{U_{C,t}^B}\frac{P_{C,t}^B}{P_{C,t+j}^B}\frac{P_{C,t+j}^A}{P_{C,t}^A}\right)$. No arbitrage imposes that prices of the same asset need to be equalised, therefore we have

$$q_{r,t}^{A,j} = \beta^j E_t\left(\frac{U_{C,t+j}^A}{U_{C,t}^A}\right) = \beta^j E_t\left(\frac{U_{C,t+j}^B}{U_{C,t}^B}\frac{P_{C,t}^B}{P_{C,t+j}^B}\frac{P_{C,t+j}^A}{P_{C,t}^A}\right) \tag{16}$$

for $j \in \mathscr{J}$. It is easy to show that the risk sharing condition (12) is sufficient to satisfy (16). It is important to note that differently than the case of nominal bonds, $q_{r,t}^{A,j} \neq q_{r,t}^{B,j}$ as these bonds have different payoffs as they compensate investors for domestic inflation. In the case of inflation indexed bonds the government's intertemporal constraint of country i is:

$$\sum_{j\in\mathscr{J}} q_{r,t}^{i,j-1} \frac{B_{r,t-1}^{i,j}}{P_{C,t-1}^i} = E_t \sum_{k=0}^{\infty} \beta^k \frac{S_{t+k}^i}{U_{C,t}^i} \equiv \Upsilon_t^i, \tag{17}$$

where $B_{r,t-1}^{i,j}$ is the quantity of real bonds, r, of maturity j issued by the government in country i in $t-1$. As in the case of nominal bonds we can now find the optimal portfolio through solving the following system of equations:

$$\underbrace{\begin{bmatrix} q_{r,t}^{i,\mathscr{J}_1-1}(s^{t-1},s_1) & q_{r,t}^{i,\mathscr{J}_2-1}(s^{t-1},s_2) & \dots & q_{r,t}^{i,\mathscr{J}_N-1}(s^{t-1},s_1) \\ \vdots & \dots & & \\ q_{r,t}^{i,\mathscr{J}_1-1}(s^{t-1},s_N) & q_{r,t}^{i,\mathscr{J}_2-1}(s^{t-1},s_N) & \dots & q_{r,t}^{i,\mathscr{J}_N-1}(s^{t-1},s_N) \end{bmatrix}}_{\mathscr{Q}_{r,t}} \cdot \underbrace{\begin{bmatrix} \frac{B_{r,t-1}^{i,\mathscr{J}_1}(s^{t-1})}{P_{C,t-1}^i(s^{t-1})} \\ \vdots \\ \frac{B_{r,t-1}^{i,\mathscr{J}_N}(s^{t-1})}{P_{C,t-1}^i(s^{t-1})} \end{bmatrix}}_{\mathscr{B}_{r,t}} = \begin{bmatrix} \Upsilon_t^i(s^{t-1},s_1) \\ \vdots \\ \Upsilon_t^i(s^{t-1},s_N) \end{bmatrix}. \tag{18}$$

Table 3: **Model Parameters**

		Value	**Target/Reference**
Preferences and Technology			
Discount Rate	β	0.98	Faraglia et al (2010)
Labor Disutility:			
- curvature	γ	2	Elasticity of 0.5
- level	χ	Steady state tax	
Home Bias	α	0.4	Faia and Monacelli (2004)
Substituability:			
- Varieties	ϵ	6	Faia and Monacelli (2004)
- Home vs. Foreign goods	η	2	Faia and Monacelli (2004)
Fraction of Sticky Price Firms	ν	0.05	Siu (2004)
Spending Process			
High Spending State	$\overline{g}$	$(1+0.07)g_s$	Faraglia et al (2010)
Low Spending State	$\underline{g}$	$(1-0.07)g_s$	
Persistence	ρ	0.95	

Note: g_s is used to denote the steady state level of spending (20 percent of GDP).

As in the case of nominal bonds, if $\mathcal{Q}_{r,t}$ is invertible, then markets are effectively complete.

3.4 Simulations

In the next subsections we use a global numerical approximation of the equilibrium to solve the optimal policy problem and obtain portfolios. Details on the numerical algorithm can be found in the online appendix.

3.4.1 Calibration

To solve the model we need calibrate parameter values and assign functional forms. In this paragraph we present our choices which are taken mainly from two sources: Faia and Monacelli (2004), to calibrate the international dimension of the model, and the DM literature (e.g. Buera and Nicolini (2004), Faraglia et al (2010), Siu (2004) and Lustig et al (2008)), to calibrate the rest of the model. Table 3 summarizes the parameter values.

Using standard notation we let β symbolize the discount factor that represents the relative weight of future utility flows in the households' objective functions. Since each model period corresponds to one quarter, we set $\beta = 0.98$. Household preferences are:

$$\log(C_t^i) - \chi \frac{{N_t^i}^{1+\gamma}}{1+\gamma},$$

Setting $\gamma = 2$ gives us a Frisch elasticity of 0.5. Moreover, we normalize hours to 1/3 of the unitary time endowment in the deterministic steady state and χ is calibrated to match this target.

We set the home bias parameter $\alpha = 0.4$, the demand elasticity parameter $\eta = 2$ and the elasticity of substitution across varieties parameter $\epsilon = 6$. These values are taken from Faia and Monacelli (2004) and we treat them as our benchmark. Moreover we experiment with a range of values for parameter ν that governs the degree of price stickiness in our model. [28]

[28] Lustig et al (2008) and Siu (2004) assume the fraction of sticky price firms, ν, equals to 0.08 and 0.05 respectively. These values are somewhat low and in those studies they have been chosen to show that even mild degrees of price stickiness can have dramatic effects on optimal policy. We consider a wider range of values of ν, letting $\nu = 0.05$ be our lower bound, in order to study the impact of varying the degree of price stickiness on optimal portfolios.

We assume for simplicity that the government can issue either two or four maturities (following ABN and Faraglia et al (2010)). In the first case we choose a maturity structure $\mathscr{J} = \{1, 40\}$, with a short bond of one quarter and a long bond of forty quarters. In the second case we choose $\mathscr{J} = \{1, 4, 20, 40\}$, two short and two long bonds.[29] The government can complete the markets with non state contingent assets only if the number of shock realizations in each period, N, is equal to the number of assets available. If we allow only two maturities then $N = 2$. In this case we obviously have to consider separately the case where shocks are idiosyncratic and the case where they are aggregate. When we allow for four maturities then $N = 4$ and we can consider aggregate and idiosyncratic shocks together.[30] The following summarizes the spending process we assume:

$$(G_t^A, G_t^B) \in \begin{cases} \{(\overline{g}, \underline{g}), (\underline{g}, \overline{g})\} & \text{Idiosyncratic shocks } (N = 2) \\ \{(\underline{g}, \underline{g}), (\overline{g}, \overline{g})\} & \text{Aggregate shocks } (N = 2) \\ \{(\underline{g}, \underline{g}), (\overline{g}, \underline{g}), (\underline{g}, \overline{g}), (\overline{g}, \overline{g})\} & \text{Idiosyncratic and aggregate shocks } (N = 4). \end{cases}$$

Finally, the Markov transition matrices across (joint) spending states for the case where $N = 2$ are given by:

$$\Pi_\rho = \begin{bmatrix} \rho & 1-\rho \\ 1-\rho & \rho \end{bmatrix}.$$

For the case $N = 4$ we create the joint transition matrix $\Pi_\rho \otimes \Pi_\rho$. We set $\rho = 0.95$ as Faraglia et al (2010).[31]

In the following pages we present simulations results, assuming that governments have zero debt initially.

3.4.2 Fiscal hedging when shocks are idiosyncratic

Consider first the case when $N = 2$, and the government issues nominal bonds. The top panel of Table 4 reports the optimal portfolios of country A and the market price of the long bond in the two spending states. The portfolios for country B are symmetric and we omit them. The numbers reported are averages of face values ($B^{A,1}$,$B^{A,40}$) and prices ($q^{A,39}$) over a simulation of 1000 periods. The columns report the optimal portfolios when we vary the degree of price stickiness.

Bond prices are virtually identical across the high and low spending states. This implies that matrix $\mathcal{Q}_{n,t}$ is nearly non-invertible and explains the huge values we obtain for ($B^{A,1}$,$B^{A,40}$). Note that even though we can invert the ill-conditioned matrix $\mathcal{Q}_{n,t}$ this is probably due to the numerical approximation of the model equilibrium rather than a feature of the equilibrium itself. We can conclude that effectively markets cannot be completed when governments issue nominal debt and shocks are idiosyncratic.[32]

The bottom panel of Table 4 reports the numerical results for the case where the government issues only inflation indexed bonds. The difference with the case of nominal bonds is stark. Bond prices are now moving across states, they covary negatively with spending, and the government exploits these movements through issuing long term debt financing its position through savings in the short term asset. Bond positions are large (several times GDP which is on average equal to 1/3 in the model) but this is a standard property of optimal DM models (see ABN and Faraglia et al

[29] The results reported below are robust towards assuming other maturity structures.

[30] When $N = 4$ we assume that spending is not correlated across countries. In other words we set g^c constant. We would need at least $N = 8$ to consider a process of the form $G_t^i = g_t^c + g_t^i$ including a common component of spending. This would add computational burden without changing the results.

[31] We set $\overline{g}$ and $\underline{g}$ to be 7% above and below the steady state value which is normalized to 20% of GDP.

[32] To further add to this conclusion, let us note that the linear system that we have to solve by inverting $\mathcal{Q}_{n,t}$ is badly scaled in every period t. In fact we obtain positions that are an order of magnitude (or even two) larger than what is reported in the table, but the signs can alternate over time. Since we add 1000 periods of data, the absolute average positions are smaller, than the absolute positions we obtain in each period. For brevity, we do not show here these simulations.

Table 4: **Optimal Portfolios and Bond Prices - Idiosyncratic Shocks**

	ν	0.05	0.10	0.25	0.50	0.75
Nominal Bonds						
Portfolios						
	$B_n^{A,1}$	38917	19721	7269	9369	5227
	$B_n^{A,40}$	-85569	-43363	-15984	-20618	-11494
Bond Prices						
	$q_n^{A,39}(\overline{g})$	0.4548	0.4548	0.4548	0.4548	0.4548
	$q_n^{A,39}(\underline{g})$	0.4548	0.4548	0.4548	0.4548	0.4547
Inflation Indexed Bonds						
Portfolios						
	$B_r^{A,1}$	-40.7	-41.2	-42.9	-48.2	-64.1
	$B_r^{A,40}$	89.6	90.6	94.5	106.1	140.9
Bond Prices						
	$q_r^{A,39}(\overline{g})$	0.453	0.453	0.453	0.453	0.453
	$q_r^{A,39}(\underline{g})$	0.457	0.457	0.457	0.457	0.456

Notes: The table reports optimal portfolios and prices of the long maturity when $N = 2$ (two spending states). It reports the average face values of a short - 1 period - debt ($B_n^{A,1}$) and long - 40 quarters- debt ($B_n^{A,40}$) and the average repurchases price of the long bond in the high and low spending state ($q_n^{A,39}(\overline{g})$ ($q_n^{A,39}(\underline{g})$)) over 1000 simulations. The repurchase price of the short bond is 1, irrespective of the state of the economy and the type of bond. ν is the fraction of sticky price firms.

(2010)).[33] There is no suspicion that solving optimal portfolios under inflation indexed debt requires to invert an ill-conditioned matrix. Thus markets can be completed with inflation indexed bonds.

Notice that this holds regardless of the degree of price stickiness we assume. A higher fraction of sticky prices leads to a fanning out of the positions but matrix $\mathcal{Q}_{r,t}$ continues being invertible. A higher degree of price stickiness leads to a smaller impact of spending shocks on the prices of inflation indexed bonds because consumption now responds less to the shocks.[34] A government which seeks to take full advantage of fiscal hedging must then fan out its position.

3.4.3 Why do inflation indexed bonds work and nominal bonds do not?

The theoretical analysis concludes that governments cannot complete the markets by issuing only nominal debt, or equivalently that nominal bond prices do not respond to idiosyncratic spending shocks. Inspecting matrix $\mathcal{Q}_{n,t}$ reveals the key forces behind this outcome. Assume again that $\left(G_t^A, G_t^B\right) \in \left\{\left(\overline{g}, \underline{g}\right), \left(\underline{g}, \overline{g}\right)\right\}$ and that $\mathcal{J} = \left\{1, 40\right\}$, matrix $\mathcal{Q}_{n,t}$ for country A is:

$$\mathcal{Q}_{n,t} = \begin{bmatrix} \frac{1}{1+\pi_t^A(s^{t-1},s_1)} & \frac{q_{n,t}^{A,39}(s^{t-1},s_1)}{1+\pi_t^A(s^{t-1},s_1)} \\ \\ \frac{1}{1+\pi_t^A(s^{t-1},s_2)} & \frac{q_{n,t}^{A,39}(s^{t-1},s_2)}{1+\pi_t^A(s^{t-1},s_2)} \end{bmatrix},$$

where $q_{n,t}^{A,39} = \beta^{39} E_t\left(\frac{U_{C,t+39}^A}{U_{C,t}^A}\frac{P_{C,t}^A}{P_{C,t+39}^A}\right)$.

[33] Because the variability of prices in standard macro models and given the preferences we assumed, is not large, the government needs to take large positions to fully take advantage of movements in the yield curve. The absolute bond positions could be reduced if we assume preferences that produce large volatility in bond prices (e.g. assuming habit formation or Epstein-Zin preferences).

[34] This property follows clearly from (12). A higher fraction of sticky prices implies less variability in the real exchange rate and less variability of consumption. Moreover, since after 39 quarters consumption in expectation reverts back to its mean value, most of the variability of real long bond prices is explained by current consumption.

The first important condition behind the non-responsiveness of bond prices $q_{n,t}^{A,39}$ is (12). Due to the lack of frictions in financial markets, nominal bond prices are equal across countries. This implies that nominal yields cannot respond to shocks that redistribute the fiscal burden across countries. In our simulations, the ratio $\frac{U^i_{C,t}(s^t)}{P^i_{C,t}(s^t)}$, as well as the expectations $E_t\left(\frac{U^i_{C,t+39}}{P^i_{C,t+39}}\right)$, effectively does not vary across the two states s_1 and s_2. Therefore, nominal bond prices satisfy:

$$q_{n,t}^{39}(s^{t-1}, s_1) \approx q_{n,t}^{39}(s^{t-1}, s_2). \tag{19}$$

Given that bond prices are (nearly) equal across states s_1 and s_2 we can write the second row of $\mathcal{Q}_{n,t}$ as $\frac{1+\pi_t^A(s^{t-1},s_1)}{1+\pi_t^A(s^{t-1},s_2)}$ times the first row. $\mathcal{Q}_{n,t}$ has two (nearly) linearly dependent rows and is therefore ill-conditioned.[35]

The second reason why markets cannot be completed is due to inflation. Issuing nominal bonds and relying on the endogenous response of inflation to create the required state contingent payoffs in $\mathcal{Q}_{n,t}$ does not work for two reasons. First prices are sticky; using inflation to reduce debt when spending is high, distorts production across firms and across countries.[36] Second, even in the case of fully flexible prices $\nu = 0$, it is not possible to set inflation rates in both countries so that the intertemporal budget constraints of both governments are satisfied (which must hold under complete markets) because of (12).[37]

Consider now the case of inflation indexed debt. Matrix $\mathcal{Q}_{r,t}$ is

$$\mathcal{Q}_{r,t} = \begin{bmatrix} 1 & q_{r,t}^{A,39}(s^{t-1}, s_1) \\ \\ 1 & q_{r,t}^{A,39}(s^{t-1}, s_2) \end{bmatrix},$$

where $q_{r,t}^{A,39} = \beta^{39} E_t\left(\frac{U^A_{C,t+39}}{U^A_{C,t}}\right)$.

Notice first that whereas according to (12) nominal bond prices are equal across countries, inflation indexed bond prices do not have to be equal. Since real interest rates can differ, they can respond to idiosyncratic shocks and indeed our simulations show that real rates increase when the spending level increases and fall when spending drops.

Differences in real rates obviously translate into differences in inflation rates. Prices increase on impact when spending rises, but then are expected to move back towards the initial level, when spending returns to steady state.[38] The increase in real interest rates is compensated by a drop in inflation which keeps the nominal interest rate constant. More formally, if an increase in spending leads to a drop in $\beta^{39} E_t\left(\frac{U^A_{C,t+39}}{U^A_{C,t}}\right)$ but $\beta^{39} E_t\left(\frac{U^A_{C,t+39}}{U^A_{C,t}} \frac{P^A_{C,t}}{P^A_{C,t+39}}\right)$ is (roughly) constant then either $E_t\left(\frac{P^A_{C,t}}{P^A_{C,t+39}}\right)$ rises implying mean reversion in the price level, or the covariance between marginal utility growth and inflation decreases so that the inflation risk premium drops. Since in standard

[35] This outcome hinges on the assumption of fixed exchange rates. Outside currency areas exchange rates can respond to redistributive shocks. Nominal bond prices could also respond to these types of shocks even under complete markets.

[36] Note that even in the case where prices are fully flexible in one country, but sticky in the other country, using inflation to reduce debt in the flexible price country may be suboptimal as it would imply inefficient fluctuations in the terms of trade.

[37] In the online appendix we consider the fully flexible price model and show that the well known result of Chari and Kehoe (1999) which states that governments can complete the markets with a single nominal bond by targeting a path of prices that satisfy government budget solvency cannot hold in the currency union.

[38] As we explain in the online appendix, the optimal allocation features history dependence. This is true because of sticky prices, that give rise to constraints involving forward expectations in the policy problem and which will bind in the optimal policy program. The Lagrange multipliers on these constraints become state variables. The presence of these states implies that shocks can persistently drive the economy away from the steady state, but these effects are only mild. According to our simulations, following an idiosyncratic spending shock, consumption tends to revert towards its pre-shock value.

macro models such as ours, risk premia are not significant, it is mainly inflation that compensates for the differences in the real interest rates.[39]

Finally, the policy that issuing long debt is optimal can now be better understood given the response of inflation. Since inflation rises on impact following an increase in spending but then prices move back towards the initial level, the payout of inflation indexed short term bonds is higher than the payout of long bonds. The government leverages on this difference, and investing in the short term asset provides insurance against the fiscal shock.

3.4.4 Fiscal hedging when shocks are aggregate

We now turn to the case in which we allow only aggregate shocks, assuming again $N = 2$. Now shocks are perfectly correlated across countries and the model becomes essentially isomorphic to a closed economy model. Following our previous example, when bonds are nominal, the government can now complete the markets. In our simulations $\frac{U^i_{C,t}(s^t)}{P^i_{C,t}(s^t)}$ varies across the two states due to the aggregate nature of the shocks despite the fact that (12) continues to hold:

$$\frac{U^A_{C,t}(s^{t-1}, s_1)}{P^A_{C,t}(s^{t-1}, s_1)} = \frac{U^B_{C,t}(s^{t-1}, s_1)}{P^B_{C,t}(s^{t-1}, s_1)} \neq \frac{U^A_{C,t}(s^{t-1}, s_2)}{P^A_{C,t}(s^{t-1}, s_2)} = \frac{U^B_{C,t}(s^{t-1}, s_2)}{P^B_{C,t}(s^{t-1}, s_2)}$$

and

$$q^{39}_{n,t}(s^{t-1}, s_1) \neq q^{39}_{n,t}(s^{t-1}, s_2).$$

The optimal allocation features zero inflation and the degree of price stickiness exerts no influence. Under aggregate shocks the planner does not find optimal to use surprise inflation to ward off fiscal shocks, since inflation distorts the allocation of labor across flexible and sticky price goods and since it is possible to exploit variations in the price of real debt to smooth taxes across time. For this reason nominal bonds and inflation indexed bonds are equivalent and the optimal bond portfolio is the same across the two model versions. Markets are then complete. For our parametrization the optimal portfolio is constant and equal to $(B^{A,1}_r, B^{A,40}_r) = (B^{A,1}_n, B^{A,40}_n) = (-13.6, 30.1)$ where again governments issue long bonds and finance this position through short term savings as in ABN. This theoretical conclusion as well is clearly consistent with the empirical evidence of section 2.

3.4.5 Policies with inflation indexed debt

We have now shown that governments should make more use of inflation indexed bonds, and not nominal debt since the latter does not always provide fiscal insurance. This result will hold irrespective of the parameterization of the model and for the remainder of the paper we focus on debt management strategies that exploit the desirable hedging properties of inflation indexed debt, in order to show how changes in the calibration of the model affect the optimal policies.

Consider first the case when $N = 4$. Now our economy features both aggregate and idiosyncratic shocks, and the government in theory needs four assets to complete the markets. We consider two alternative cases: first the governments can issue four inflation indexed bonds of different maturities and then the case where they issue two nominal bonds and two real instruments.

The top panel of Table 5 reports the results of the first case. The model continues to suggest that the government should issue long term debt in order to exploit fiscal insurance. The face value of the short term assets position, $B^{A,1}_r + B^{A,4}_r$, is negative and the value of the long position, $B^{A,20}_r + B^{A,40}_r$, remains positive across all values of ν.[40] Governments desire to issue long bonds, with most of the debt being concentrated in either the 20 quarter maturity (when ν is low) or the 40 quarter bond (for high values of ν).[41]

[39] The expectations hypothesis approximately holds in our model. An interesting extension would be to introduce features that can 'break' the close link between long rates and future short rates. As the literature has shown, this may require to assume Epstein-Zin preferences and demand shocks and/or segmented bond markets (for example by

Table 5: **Optimal Portfolios - Idiosyncratic and Aggregate Shocks**

	ν	0.05	0.10	0.25	0.50	0.75
Inflation Indexed Bonds						
	$B_r^{A,1}$	0.5	0.5	0.5	0.5	0.6
	$B_r^{A,4}$	-30.4	-30.4	-30.3	-30.1	-29.1
	$B_r^{A,20}$	55.1	53.4	47.1	28.5	-21.2
	$B_r^{A,40}$	-20.6	-18.2	-8.9	18.2	90.6
Nominal and Inflation Indexed Bonds						
	$B_n^{A,1}$	-0.2	-0.0	0.6	2.4	7.7
	$B_n^{A,4}$	-38.3	-38.5	-39.6	-42.7	-53.3
	$B_r^{A,20}$	-70.6	-71.6	-75.3	-84.9	-99.4
	$B_r^{A,40}$	185.5	187.2	193.5	210.2	242.4

Notes: The table reports optimal portfolios when $N = 4$ (four spending states). The top panel reports the average face value of debt when the government issues only inflation indexed debt, the bottom panel considers the case when short and medium term debt is nominal and the long maturities are real. The averages are calculated over 1000 simulations.

The bottom panel of Table 5 considers the case where governments issue short nominal bonds (1 and 4 quarters) and real long term bonds (20 and 40 quarters).[42] Notice that issuing a mix of nominal and real assets, allows the government to complete the markets. While nominal debt is not effective in dealing with redistributive shocks, it is effective in dealing with aggregate shocks and, as before, inflation indexed debt can be used to absorb idiosyncratic shocks. As the table shows, government debt is again concentrated in the long term assets, $B_r^{A,20} + B_r^{A,40} > 0$. The optimal portfolio features larger positions in absolute terms than those reported in the top panel of Table 5. This is due to the fact nominal bond prices do not covary with deficits in all states, whereas real price do, and thus fanning out the positions is necessary to complete the markets when nominal bonds are issued.

We now study optimal portfolios when the calibration of the model deviates from the benchmark. In particular we study the influence of degree of home bias and the elasticity of substitution between the home and the foreign good. We assume for simplicity $N = 2$. Table 6 reports the results and compares them to the ones of our benchmark calibration.

When we consider the case of idiosyncratic shocks and decrease α from 0.4 to 0.2, assuming now a stronger home bias, we find that the bond positions are slightly smaller compared to the benchmark case. A lower α leads to a larger impact of an idiosyncratic shock on consumption and therefore to a larger variability in real long bond prices. Governments then can issue fewer long bonds to hedge against the shock. Analogously, a lower degree of home bias, which can be seen as a larger degree of integration of product markets in the currency area, will increase the size of the positions. When $\alpha = \frac{1}{2}$, markets become fully integrated and then it is no longer possible to complete the market via inflation indexed bonds.[43]

When we consider the substitutability between of home and foreign good, η, most papers in the

introducing transaction costs to long term bonds). We leave these extensions to future work.

[40] This is also true when we calculate the market value of the bonds.

[41] It shouldn't be surprising that governments issue small amounts of very short bonds and, in some calibrations, buy the longest maturity available. Faraglia et al (2010) find a similar property in some of the 4 state models they consider, the exact values of the bond quantities that emerge from these models are sensitive to the calibration. However, the principle that optimal debt is of long maturity, continues to hold.

[42] As we have seen, most of the inflation indexed debt issue in Europe is long term. For this reason we let the shortest maturities to be nominal debt, though we could complete the market with two nominal bonds of any maturity.

[43] When $\alpha = \frac{1}{2}$, the CPIs are equal across countries and an efficient allocation sets the ratio of marginal utilities constant. In this case, consumption does not respond to idiosyncratic shocks and so real debt prices do not vary across spending states. Without a mild degree of home bias governments cannot complete markets even when they issue inflation indexed debt.

Table 6: **Optimal Portfolios - Robustness**

	Benchmark $\alpha = 0.4$ $\eta = 2$		$\alpha = 0.2$ $\eta = 2$		$\alpha = 0.4$ $\eta = 1.5$	
ν	0.05	0.75	0.05	0.75	0.05	0.75
Idiosyncratic Shocks						
$B_r^{A,1}$	-30.7	-64.1	-32.3	-63.2	-28.5	-59.8
$B_r^{A,40}$	89.6	140.9	71.0	139.1	62.6	131.6

Notes: The table reports optimal portfolios when $N = 2$ (two spending states). It reports the average face values of a short - 1 period- debt ($B_n^{A,1}$) and long - 40 quarters- debt ($B_n^{A,40}$) over 1000 simulations. ν is the fraction of sticky price firms.

open economy DSGE literature assume values in the range $[1, 2]$. Recently, a stream of studies[44], which estimate structural DSGEs with Bayesian methods, narrow this range further to $[1.5, 2]$. We choose the lower bound of these estimates and set $\eta = 1.5$ to check the impact of varying this parameter of our findings. Assuming a lower η increases the variability of bond prices and again governments slightly reduce the size of bond positions. The qualitative prediction that governments prefer to issue long term debt remains.

In the online appendix we explore in greater detail the behaviour of the net asset positions and of the real exchange rate for different parameterization of the model.[45] We show that net asset positions are not sensitive to the degree of price stickiness as well as to degree of home bias or degree of good substitutability. More interesting are the dynamics of the real exchange rate. Our results show that the more volatile the real exchange rate the smaller are the bond positions and this happens when prices are more flexible, there is a lower degree of home bias or a lower substitutability between the home and foreign good. This maps the response of relative consumption due to the Backus and Smith condition (12).

Finally inspired by our empirical analysis, we looked at a different indexation of the bonds. In section 2 we reported that France and Italy have issued not only bonds linked to home inflation but also to Euro Area inflation. We therefore have found interesting to extend our analysis and allow for bonds indexed to the union-wide inflation. For brevity all details of this version of the model are provided in the online appendix. Our findings suggest that when shocks are idiosyncratic governments continue to issue long term debt, however the optimal portfolio positions are now extremely large compared to the ones obtained with bonds indexed to home inflation.[46] While governments can complete the markets through issuing debt indexed to union-wide inflation, this becomes more difficult because these bonds prices respond less to spending shocks than when the bonds were indexed to the country inflation. These bonds lie in between nominal debt and debt indexed to country inflation. Governments thus have to leverage more on the size of the positions to achieve an optimal fiscal insurance and complete the markets.

To summarise, the optimality of issuing long term inflation indexed debt is a robust prediction of our model. We have shown also that a large degree of product market integration- a low degree of home bias or when home and foreign goods are close substitutes - reduces the hedging value of inflation indexed bonds. Governments will then have to fan out positions to take advantage of the

[44]See for example De Walque et al (2005), Justiniano and Preston (2010) and Rabanal and Tuesta (2010).

[45]It is important to notice that private portfolios cannot be pinned down unless we impose restrictions on the private positions. Assuming that households in country A can invest only in domestic assets, we characterize private portfolios and show that the influence of parameters ν, η and α is essentially the same as for government portfolios. See online appendix for the further discussion.

[46]For example, in our benchmark calibration, where $N = 2$, $\nu = 0.05$ and shocks are idiosyncratic, long debt is 1065 times larger than GDP. When the prices are stickier and $\nu = 0.75$, the long bond position increases to 2085 times of GDP.

covariance between bond prices and deficits. Lastly, debt linked to union-wide inflation has less desirable properties than debt indexed to domestic inflation. These policy relevant findings should be of interest.

3.5 Discussion of Possible Extensions

Before closing the paper, it is worth to briefly discuss a few interesting extensions of this optimal DM exercise. Throughout our theoretical exercise, we have studied optimal policies assuming that a Ramsey planner can issue debt using state contingent assets (the complete market paradigm), that fiscal policies are coordinated across countries and that monetary policy is jointly determined with fiscal policy. Though this may correspond to the best possible policy setting in a currency area, these conditions are unlikely met in practice. The fiscal policies of the union are not necessarily coordinated and the optimal DM policies identified in this paper may be unimplementable due to financial market frictions.

A recent stream of papers in the DM literature has explored the interplay between frictions and optimal maturity of debt in a closed economy (e.g. Lustig et al (2008), Debortoli et al (2017) and Faraglia et al (2019)). These papers assume that optimizing governments are constrained in the number of assets available so that markets become incomplete. In addition, Lustig et al (2009), Faraglia et al (2019) impose constraints in the size of the positions so that governments cannot finance a very large issuance of long term bonds through savings in a short term private asset. In the context of the currency union optimal DM model, such constraints may not only affect trades between the government and the private sector within a single country as in the closed economy paradigm, but may also affect cross border bond transactions. In the presence of cross border transaction costs and constraints the close relation between nominal interest rates across countries may not hold, and thus the ability of governments to hedge through nominal assets may be partly restored.[47]

We can draw an interesting related insight from the recent literature on optimal currency areas: Completing the markets may not always be optimal in the presence of asymmetric shocks and sticky prices. Auray and Eyquem (2014) document a welfare loss under complete markets relative to financial autarky and to incomplete markets with transaction costs. Analogously, Farhi and Werning (2017) show that it may be optimal in a currency area to introduce constraints in capital flows across borders, or use fiscal policy to curb trades in financial markets. A similar result could arise in a model of optimal DM under incomplete markets. If completing the market is not optimal, then perhaps issuing only nominal debt and not fully exploiting fiscal insurance is desirable. Otherwise, the presence of constraints in cross border transactions will also improve welfare by enabling governments to hedge against idiosyncratic shocks through nominal assets.

Besides allowing for relevant financial market frictions, another interesting extension of this work could be to study jointly the optimal portfolios of private agents along with the government optimal DM, bringing together the hedging motives of governments and households. When we have only fiscal shocks and complete markets, our set-up does not generate any incentive for the households to hedge through the international capital market. For this reason, we have assumed complete home bias in equities. Introducing financial frictions and more shocks will crucially allow agents to trade in equity. This extension could then bridge the valuable insights of the vast literature on cross country equity portfolios with optimal DM issues.

Finally, modelling fiscal and monetary policies through empirically grounded tax and interest rate rules seems a relevant alternative to the coordinated Ramsey outcome we considered in this paper. This alternative may allow to meaningfully study realistic asymmetries in product market

[47] Also, our paper showed that the choice between real and nominal bonds is crucial in the currency area: Inflation indexed bonds can be used to hedge the government budgets against idiosyncratic shocks. However, financial market frictions are likely to affect differently nominal and real debt markets. For example, it has been well documented that US inflation indexed bonds are less liquid. This could imply that these types of bonds carry higher transaction costs than the more liquid nominal long term bonds. It would thus be interesting to know whether higher yields associated with lower liquidity, reduce the attractiveness of inflation indexed debt for debt managers in a currency area, when markets are incomplete.

frictions and in the conduct of fiscal policy. It is for example well known that inflation responses even to aggregate shocks could be asymmetrical across countries when monetary policy targets the union wide inflation (e.g. Benigno (2004)). These differences ought to be relevant for DM policy, and translate into asymmetries in optimal portfolios. Analogously, different tax policies generate asymmetries in the response of government budgets to shocks that will then also be reflected in optimal portfolios.

4 Conclusions

In the previous sections we have studied the role of government debt maturity in currency unions to identify whether DM can help governments to hedge their budgets against spending shocks. Using a detailed dataset of debt portfolios of five Euro Area countries to run a battery of VARs, we estimate the responses of holding period returns to fiscal shocks. We find that government portfolios, mainly of nominal assets, have not been effective in absorbing idiosyncratic fiscal risks, whereas they have been very effective in absorbing aggregate risks. These results are robust to several specifications. We have then used a theoretical model of optimal DM that features a two country currency union, distortionary taxes, aggregate and idiosyncratic shocks. This complete market model of DM advises governments in a currency union to issue inflation indexed long term debt. In this way, budgets are insulated from the effects of spending shocks and taxes are smoothed, since portfolios absorb a large part of the fiscal shocks. Our paper builds on the insights of the recent macro DM literature, bringing together its positive and normative elements. Our analysis aims to shed light on the management of government debt in currency areas, which recently has received considerable attention in policy circles.

Acknowledgments: Faraglia and Oikonomou thank the Keynes Fund-Cambridge for financial support. Faraglia gratefully acknowledges support from Cambridge Endowment for Research in Finance (CERF). We are grateful to Mirko Abbritti, Hafedh Bouakez, Charles Brendon, Chryssi Giannitsarou, Robert Kollmann, Luisa Lambertini, Albert Marcet, Jochen Mankart, Antonio Moreno, Gert Peersman and Romanos Priftis and an anonymous referee for useful comments and suggestions.

References

[1] Angeletos, G-M, (2002), "Fiscal policy with non-contingent debt and optimal maturity structure", *Quarterly Journal of Economics*, 27, 1105-1131.

[2] Auerbach, A. J. and Gorodnichenko, Y., (2012), "Measuring the output responses to fiscal policy ", *American Economic Journal: Economic Policy*, 4(2), 1-27.

[3] Auray, S. and Eyquem, A. (2014), "Welfare Reversals in a Monetary Union", *American Economic Journal: Macroeconomics*, 6(4), p. 246-290

[4] Backus, D.K. and Smith, G.W., (1993), "Consumption and Real Exchange Rates in Dynamic Economies with Non-traded Goods", *Journal of International Economics,* 35(3-4), 297-316.

[5] Baxter, M. and Jermann, U., (1997), "The International Diversification Puzzle is Worse Than You Think", *American Economic Review*, 87(1), 170-80.

[6] Beetsma, R. and Giuliodori, M., (2011), "The effects of government purchases shocks: review and estimates for the EU", *The Economic Journal*, 121(550).

[7] Benigno, P., (2004), "Optimal Monetary Policy in a Currency Area", *Journal of International Economics*, Vol. 63, Issue 2, 293-320.

[8] Bernardini, M., De Schryder, S. and Peersman, G., (2020), "Heterogeneous Government Spending Multipliers in the Era Surrounding the Great Recession", *The Review of Economics and Statistics*, 102(2), 304-322.

[9] Berndt, A., Lustig, H. and Yeltekin, S., (2012), "How does the US government finance fiscal shocks?", *American Economic Journal: Macroeconomics*, 4(1), 69-104.

[10] Blanchard, O. and Perotti, R., (2002) "An empirical characterization of the dynamic effects of changes in government spending and taxes on output", *Quarterly Journal of economics,* 117(4), 1329-1368.

[11] Blanchard, O., Mauro, P. and Acalin, J., (2016), "The Case of Growth-Indexed Bonds in Advanced Economies Today", *Policy Brief Peterson Institute for International Economics,* No PB16-2.

[12] Born, B. and Mueller, G.J., (2012), "Government spending shocks in quarterly and annual time series", *Journal of Money, Credit and Banking*, 44 (203), pp.507-517.

[13] Buera F. and Nicolini, J.P., (2004), "Optimal Maturity of Government Debt with Incomplete Markets", *Journal of Monetary Economics*, 51, 531-554.

[14] Burriel, P., De Castro, F., Garrote, D., Gordo, E., Paredes, J. and Perez, J.J., (2010), "Fiscal Policy Shocks in the Euro Area and the US: an Empirical Assessment", *Fiscal Studies*, 31(2), 251-285.

[15] Chari, V.V. and Kehoe, P., (1999), "Optimal Fiscal and Monetary Policy", in *Handbook of Macroeconomics*, Volume 1C, John Taylor and Mike Woodford, Eds. North Holland, Amsterdam.

[16] Coeurdacier,N., Kollmann, R. and Martin, P., (2010),"International Portfolios, Capital Accumulation, and Portfolio Dynamics", *Journal of International Economics*, 80(1), 100-112.

[17] Coeurdacier, N. and Rey, H., (2013), "Home bias in open economy financial macroeconomics", *Journal of Economic Literature*, 51(1), 63-115.

[18] Coeurdacier, N. and Gourinchas, P-O., (2016) "When Bonds Matter Home Bias in Goods and Assets", *Journal of Monetary Economics*, 82, 119-137.

[19] Cook, D. and Devereux, M. B., (2011), "Optimal fiscal policy in a world liquidity trap ", *European Economic Review*, 55(4), 443–462.

[20] Debortoli, D., Nunes, R. and Yared, P., (2017), "Optimal Time-Consistent Government Debt Maturity", *Quarterly Journal of Economics*, 132 (1), 55-102.

[21] Devereux, M. and Sutherland, A., (2007), "Monetary Policy and Portfolio Choice Choice in an Open Economy Macro Model", *Journal of the European Economic Association*, 5(2-3), 491-499.

[22] Devereux, M. and Sutherland, A., (2011), "Country Portfolios in Open Economy Macro-Models" *Journal of the European Economic Association*, 9(2), 337-369.

[23] De Walque, G., Smets, F. and Wouters, R., (2005), "An estimated two-country DSGE model for the Euro area and the US economy", European Central Bank, mimeo.

[24] Engel, C. and Matsumoto, A., (2009), "The International Diversification Puzzle When Goods Prices Are Sticky: It's Really about Exchange-Rate Hedging, Not Equity Portfolios", *American Economic Journal: Macroeconomics*, 1(2), 155-188.

[25] Equiza-Goñi, J., (2016), "Government Debt Maturity and Debt Dynamics in Euro Area Countries", *Journal of Macroeconomics,* 49, 292-311.

[26] Evers, M. P., (2015), "Fiscal federalism and monetary unions: A quantitative assessment ", *Journal of International Economics*, 97(1), 59–75.

[27] Farhi, E. and Werning, I., (2017), "Fiscal Unions", *American Economic Review*, 107(12), 3788-3834.

[28] Faia, E. and Monacelli, T., (2004), "Ramsey monetary policy and international relative prices", ECB working paper 344.

[29] Faraglia, E, Marcet, A and Scott, A., (2008) "Fiscal Insurance and Debt Management in OECD Economies", *The Economic Journal*, 118, 363-386.

[30] Faraglia, E, Marcet, A and Scott, A., (2010), "In Search of a Theory of Debt Management", *Journal of Monetary Economics*, 57, 821-836.

[31] Faraglia, E., Marcet, A., Oikonomou, R., and Scott, A., (2013), "The impact of Debt Levels and Debt Maturity on Inflation", *Economic Journal*, 123(566), 164-192.

[32] Faraglia, E., Marcet, A., Oikonomou, R., and Scott, A., (2019),"Government Debt Management: The Long and the Short of It, "*Review of Economic Studies,* 86 (6), 2554-2604.

[33] Gali, J. and Monacelli, T., (2008),"Optimal monetary and fiscal policy in a currency union ", *Journal of International Economics*, 76 (2008) 116–132.

[34] Hettig, T. & Muller, G. J., (2018), "Fiscal policy coordination in currency unions at the effective lower bound", *Journal of International Economics*, 115(C), 80-98.

[35] Justiniano, A. and Preston, B., (2010), "Can Structural Small Open Economy Models Account for the Influence of Foreign Disturbances?", *Journal of International Economics*, 81(1), 61-74.

[36] Kollmann, R., (1995), "Consumption, Real Exchange Rates and the Structure of International Asset Markets,", *Journal of International Money and Finance*, 14(2), 191-211.

[37] Krueger, D. Kubler, F., and Malin, B., (2011), "Solving the Multi-Country Real Business Cycle Model using a Smolyak-Collocation Method", *Journal of Economic Dynamics and Control* 35, 229-239.

[38] Leeper, E. M., Walker, T. B., and Yang, S. C. S., (2013), "Fiscal foresight and information flows", *Econometrica*, 81(3), 1115-1145.

[39] Lustig, H., Sleet, C., Yeltekin, S., (2008), "Fiscal hedging with nominal assets", *Journal of Monetary Economics* 55, 710-727.

[40] Perotti, R., (2005), "Estimating the effects of fiscal policy in OECD countries", CEPR discussion paper 4842.

[41] Rabanal, P. and Tuesta, V., (2010), "Euro-dollar Real Exchange Rate Dynamics in an Estimated Two-country Model: An Assessment", *Journal of Economic Dynamics and Control*, 34(4), 780-797.

[42] Ramey, V.A., (2011), "Identifying government spending shocks: It's all in the timing", *The Quarterly Journal of Economics*, 126(1), 1-50.

[43] Ravn, M.O., Schmitt-Grohé, S. and Uribe, M., (2012), "Consumption, government spending, and the real exchange rate", *Journal of Monetary Economics*, 59(3), 215-234.

[44] Sims, E. R., (2012), "News, Non-Invertibility, and Structural VARs", in *DSGE Models in Macroeconomics: Estimation, Evaluation, and New Developments*, 81-135. Emerald Group Publishing Limited.

[45] Siu, H., (2004), "Optimal Fiscal and Monetary policy with sticky prices", *Journal of Monetary Economics*, 51, 575-607.