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# Debt crises, fast and slow*

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September 12, 2021

**Abstract**

We build a dynamic model to study how shifts in investors' beliefs can drive either slow-moving debt crises or rollover crises. We show that the threat of slow-moving crises does not necessarily motivate deleveraging: in a recession, unless debt is close enough to the threshold at which the economy becomes vulnerable to such crises, optimizing governments keep borrowing, gambling on economic recovery. The incentive to deleverage is instead strong when the economy is vulnerable to rollover crises at low levels of debt. We show that equilibrium multiplicity remains pervasive independently of bond maturity. In general, short maturities induce more deleveraging.

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# 1 Introduction

After the global financial crisis, the average public debt to GDP ratio in advanced countries rose from below 80 percent to well above 100 percent at the end of 2008. After 2020, the global distress of the COVID-19 pandemic is sparking a further hike in this ratio, expected to end up substantially higher—and raise a host of issues in financial and macroeconomic stability. The academic and policy literature has long reflected on the possibility that countries with relatively high debt face disruptive belief-driven turmoil in the sovereign bond market. This may take the form of hikes in the borrowing costs that raise deficits and feed unsustainable debt dynamics—with a (slow) build-up of liabilities eventually leading to default. It may also coincide with a sudden stop of market financing, where default is immediate in the face of a (fast) rollover crisis. The exercise pursued by this literature is far from a theoretical *curiosum*. The turmoil in the euro area after 2010 provides a vivid and striking example of the widespread disruption caused by this type of crises even among advanced countries.[1]

In this paper, we reconsider the logic of debt crises, specifying a stylised model suitable to address two key open issues in the literature. First, sovereign risk (slow) and rollover (fast) crises appear to be pervasive in the data: under what conditions sovereigns may face hikes in borrowing costs, as opposed to losing market access, due to market beliefs coordinating on a "bad equilibrium"? In particular, is lengthening the debt maturity an effective way to shield countries from these adverse scenarios? Second, and most crucially, is the threat of belief-driven crises enough to motivate optimal deleveraging even when the economy is in a downturn—as opposed to borrowing more, gambling on a future economic recovery?

We address these questions by specifying a model with the same setting of Conesa and Kehoe (2017), except that the timing of investors and fiscal decisions follows Calvo (1988)[2]—hence the model does not feature the bond auction mechanism specified by Cole and Kehoe (2000). Using this framework, we reconsider the mechanisms through which market beliefs may cause a variety of sovereign crises, eliciting different policy responses by optimizing government.

As in Calvo (1988), investors set the bond price based on their expectations of current deficits and future default. Conditional on this price, a discretionary government optimally adjusts its fiscal surplus and takes debt repayment/issuance decisions. Market beliefs about future default thus drive the equilibrium fiscal policy and debt dynamics—beliefs determine the debt tolerance thresholds based on which investors price bonds, and the government

[1] In the words of the ECB president Mario Draghi: "The assessment of the Governing Council is that we are in [...] a 'bad equilibrium', namely an equilibrium where you may have self-fulfilling expectations that feed upon themselves and generate very adverse scenarios." ECB Press Conference, Transcript from the Q&A, September 6 2012.

[2] See the discussion in Lorenzoni and Werning (2019), Corsetti and Dedola (2016) and Ayres et al. (2018) in the Calvo tradition.

takes its default decisions. Calvo (1988) allows for "optimistic" and "pessimistic" beliefs, leading to multiple equilibria in which bonds are traded at either the riskless or the risky price.[3] To enhance comparison with the literature after Cole and Kehoe (2000), we extend the model introducing a regime of "extreme" beliefs, in which investors are willing to finance the government only at the risk-free rate, i.e., provided the government is not expected to default in any circumstances. The distinction between "pessimistic" and "extreme" beliefs allows us to study loss of market access at high and low levels of debt, respectively.

Our main results are as follows. First, it is well understood that, in a dynamic setting after Calvo (1988), beliefs drive sovereign risk hikes and debt crises are "slow moving", as characterized by Lorenzoni and Werning (2019). As a contribution to the debate in the literature, we underscore that rollover crises are also possible in the same setting, when the regime of investors' expectations turns from optimistic to pessimistic. Conditional on this switch, sudden stops occur if investors realize that the outstanding stock of debt is too high for both the equilibrium pricing conditions and the optimal government financing need to be simultaneously satisfied at a positive risky bond price. When this is the case, the government loses market access. For comparison, we also study rollover crises conditional on investors coordinating on extreme beliefs, in which case our model comes close to reproduce the dynamics of fast crises in Cole and Kehoe (2000).

Rollover crises under pessimistic and extreme beliefs have different quantitative implications. In a numerical example using our baseline model, we show that, under pessimistic beliefs, rollover (fast) crises are possible at high levels of debt—between 122% and 206% of GDP—, as opposed to slow-moving crises, which are possible at intermediate levels of initial debt—between 72% and 122% of GDP—. Under extreme beliefs, the debt threshold at which rollover crises can occur is instead very low—in our numerical example, well below 40% of GDP. If and when investors coordinate their beliefs on these regimes, the market for government bonds (prior to the crisis traded at the riskless price) may suddenly disappear at either low or high levels of debt, while at intermediate levels shifts in expectations may raise borrowing costs and ignite high rates of debt accumulation.

Second, lengthening the maturity of government debt per se does not rule out equilibrium multiplicity leading to slow-moving debt crises—these remain pervasive for *all debt maturities*. We derive this result using both our baseline and a version of our model replicating the debt-limit framework adopted by Lorenzoni and Werning (2019). However, longer maturities may rule out fast rollover crises. In a debt-limit framework, this is the case when, in addition to a long debt maturity, the probability of a recovery is non-negligible. In our baseline model, the parameter restrictions for ruling out fast crises are much more stringent.

[3] In an optimistic regime, when the equilibrium is not unique, investors always coordinate on the equilibrium with the highest bond price. In a pessimistic regime, when an equilibrium where government bonds trade at a premium exists, investors coordinate on that equilibrium.

Third, in a sunspot equilibrium where switching from the good to a bad equilibrium is assigned a positive, arbitrarily small probability, the government may have an incentive to deleverage, even during recessions, to bring and keep debt below the crisis threshold. This incentive is strong when the relevant switch is from optimistic to extreme beliefs. As is the case in Cole and Kehoe (2000), the prospect of a rollover crisis leads the government to practice precautionary austerity, and reduce the debt level running pro-cyclical deficit policies. Different from Cole and Kehoe (2000), however, we show that the model predictions are more nuanced when investors and policymakers are concerned with a regime switch from optimistic to pessimistic beliefs. In this case, policymakers find it optimal to deleverage only when the debt level is close enough to the debt threshold below which belief-driven slow-moving debt crises can no longer occur. In a recession, for debt levels sufficiently higher than such thresholds, the consumption smoothing motive dominates fiscal policy, causing deficits and debt accumulation—i.e., gambling on the recovery.

Fourth, we show that short debt maturities induce more deleveraging—a result that resonates with the analysis of debt dilution by Aguiar and Amador (2020). When the maturity of debt is short, the government fully appropriates the benefits from saving to reduce borrowing costs. With long-term debt, instead, the gains from lower borrowing costs are shared between bond holders (as a capital gain) and the government (higher price for new issuance). In line with these considerations, our model suggests that the government finds deleveraging less attractive when debt is long-term.

From a policy perspective, our analysis has key implications for debt sustainability analysis and the design of policies to enhance sustainability. First, estimates of debt tolerance thresholds are a crucial input in assessing the extent to which a country can steer away from default. Our results reiterate that these thresholds are not only contingent on the current and future states of the economy and/or preferences of the policymakers. They can also be quite sensitive to a variety of investors' beliefs. This consideration is a challenge to debt sustainability analysis, motivating an investment in sharpening the analytical toolkit employed in the assessment exercises.

Second, our result that a long debt maturity is not a cure-all solution to the problem of multiple equilibria warns against betting solely on debt management strategies rebalancing the debt maturity structure. By the same token, our result that bond prices may not be falling or move with sunspot probability stresses that market prices may not be reliable signals to detect prospective contingent crises.

Last but not least, our results show that pervasive crisis risk may not provide enough of a welfare incentive for implementing (optimally smoothed) debt reduction strategies. Indeed, our model provides a key benchmark against which to assess political economy factors, e.g., the role of short-sighted or self-interested policymaking. In our framework, even a forward-looking benevolent government generally finds it optimal to raise debt in a recession, smoothing consumption at the cost of keeping the country in a state of vulnerability to self-fullling crises. Specifically, deleveraging is optimal only at relatively low levels of debt—once the stock of liabilities is large enough, the smoothing motive dominates. In light of the evidence on cross-border financial contagion (see, e.g., Brutti and Saure (2015)), this result strengthens the case for an international compact, where the challenge is how to design official assistance combining liquidity and official loans, to favor economically and politically acceptable policies of debt reduction—essentially reducing the costs of (while creating incentives to implement) adjustment policies.

**The literature.** This paper draws on the seminal contributions by Calvo (1988) and Conesa and Kehoe (2017), in turn related to Cole and Kehoe (2000). Calvo (1988) focuses on a two-period model, where the government financing need is taken as given, and the price and quantity of bonds are jointly determined in equilibrium. Self-fulfilling expectations of default generate market "runs" that manifest themselves in a surge in the interest rate charged by investors to the government—but no rollover crisis is modelled in the same context. Conversely, Conesa and Kehoe (2017) focus on liquidity crises whereby the market may suddenly become unwilling to roll over government debt in anticipation of a default. In our paper we aim to reconsider the nature and dynamics of rollover crises—we do so by specifying a model in the style of Calvo (1988) model, but adopting a dynamic setting using the same environment as Conesa and Kehoe (2017) except for the specification of auctions underlying their view of rollover crises.

It is virtually impossible to provide a fair account of the rich literature on debt crises that has contributed to these two paradigms, directly and indirectly.[4] Lorenzoni and Werning (2019) reconsider Calvo (1988) in a dynamic setting, stressing that the increase in the sovereign's borrowing costs driven by self-fulfilling expectations of default leads a country to accumulate debt slowly but relentlessly over time. As the debt stock rises, at some point default occurs unless the conditions of the economy improve sufficiently. Ayres et al. (2018) adopt a framework similar to Arellano (2008) but for the timing assumption, to investigate the likelihood that a country becomes vulnerable to belief-driven crises. Also drawing on Calvo (1988), Corsetti and Dedola (2016) and Bacchetta et al. (2018) write monetary models and discuss how the central bank can backstop government debt, i.e., eliminate self-fulfilling crises by using, respectively, either unconventional (balance sheet) policy, or conventional

[4]A partial list includes Alesina et al. (1989), Aguiar and Gopinath (2006), Chamon (2007), Yue (2010), Grauwe (2011), Chatterjee and Eyigungor (2012), Mendoza and Yue (2012), Arellano and Ramanarayanan (2012), Araujo et al. (2013), Cruces and Trebesch (2013), Ghosh et al. (2013), Lizarazo (2013), the Handbook chapter by Aguiar and Amador (2014) and Aguiar et al. (2016), Aguiar et al. (2015), Collard et al. (2015), Tirole (2015), Chatterjee et al. (2016), Bianchi et al. (2018), Ayres et al. (2019), Bocola et al. (2019), Chatterjee and Eyigungor (2019), Galli (2019), Paluszynski (2019), and Stangebye (2020).

(inflation) policy.[5]

Several papers have developed the model with rollover crises of Cole and Kehoe (2000), in new directions. By way of example, Bocola and Dovis (2019) characterize how the maturity of sovereign debt can be structured to respond to rollover risk and fundamental risk. Rollover crises are also modelled and discussed by Giavazzi and Pagano (1989), Alesina et al. (1992) and Cole and Kehoe (1996). The importance of liquidity lending for a currency union is emphasized by Aguiar et al. (2015), suggesting that the co-existence of high debt and low debt countries in a currency area may create incentive for liquidity provisions that benefit also relatively virtuous countries—and the sustainability of the area overall.

The goal of our paper is very similar to the goal pursued by Aguiar et al. (2020), who also address the need to develop a unified framework to account for the variety of crises that we observe in the data. Aguiar et al. (2020) enrich Cole and Kehoe (2000) allowing for uncertainty about the default decision by the government once the debt auction is closed. Their model specification creates the possibility of belief-driven hikes in borrowing rates conceptually similar to the one stressed by Calvo (1988), however occurring in what these authors dub a "static" dimension (although they do have inter-temporal implications). Our work is clearly complementary to theirs.

This paper is organized as follows. Section 2 lays out the model. Sections 3 analyzes equilibrium multiplicity with optimistic and pessimistic beliefs with short-term debt. Section 4 carries out a quantitative exercise with long-term debt. Section 5 focuses on the question of whether the perceived threat of a belief-driven crisis can prevent a government from running deficits during recessions. Section 6 extends the analysis introducing "extreme" beliefs. Section 7 carries out sensitivity analysis in both our baseline model and in a debt-limit model extension, focusing on debt maturities and probabilities of recovery. Section 8 concludes.

[5] See also Aguiar et al. (2013).

# 2 Model

In this section we specify our baseline model of debt sustainability and default.[6] The environment draws on Conesa and Kehoe (2017), except that we move away from the model of auctions by Cole and Kehoe (2000), and model fiscal and investors' decisions as in Calvo (1988). Two extensions will be discussed in Section 6 and 7 below.

## 2.1 Environment

We consider a small open economy populated by a continuum of identical households, a government, and a continuum of risk-neutral competitive investors with measure one. Time is discrete and indexed by $t = 0, 1, 2, \cdots$. To focus our attention to the sovereign's behaviour, we assume that every period the representative household consumes all its income after paying tax.

The country's output is exogenous and random, given by $y(a, z) = A^{1-a}Z^{1-z}\bar{y}$, with $A < 1$ and $Z < 1$. The parameter $a$ indicates whether the economy is in a recession $a = 0$ or not $a = 1$; $z$ denotes the government decision to default $z = 0$ or repay $z = 1$. If the government defaults, $z = 0$ forever and productivity permanently drops by the factor $Z$. The economy starts out with $a_0 = 0$ and $z_0 = 1$. From period 1, the economy recovers with probability $p$ and once recovered, never falls back into a recession again.[7]

The government issues non-contingent bonds to a continuum of risk-neutral investors. As is customary after Hatchondo and Martinez (2009), we model the maturity of government bonds assuming geometrically decreasing coupons: a bond issued at $t$ pays the sequence of coupons

$$\kappa, (1-\delta)\kappa, (1-\delta)^2\kappa...$$

where $\delta \in [0, 1]$. Hence, under risk neutrality of investors with discount factor $\beta$, the price of a default-risk-free bond is

$$q = \frac{\beta\kappa}{1 - \beta(1-\delta)}$$

To normalize bond prices, it is convenient to set $\kappa = 1 - \beta + \beta\delta$ so that the price

[6] In writing this paper we draw extensively on previous work on debt bailout, especially on Corsetti et al. (2017), which introduces official lending in a Conesa and Kehoe (2017) framework, but also on Corsetti et al. (2020), Conesa and Kehoe (2014), Roch and Uhlig (2018) and Marin (2017).

[7] The model can be extended to adopt the bimodal income process used by Ayres et al. (2018), Chatterjee and Eyigungor (2019), Ayres et al. (2019) and Paluszynski (2019). In such a setting, there are equilibria located at the "wrong" side of the debt Laffer curve, in which a small increase in the bond price creates extra demand of bonds. For the sake of clarity and simplicity, we adopt the discrete distribution such that the debt Laffer curve is locally increasing at all points to filter unstable equilibria without the need to impose an equilibrium selection criterion. Equilibria presented in our model are in line with stable equilibria from a bimodal income process.

of a default-free bond is $\beta$. The parameter $\delta$ indexes the maturity of debt, where $\delta = 0$ corresponds to the case of "consols" (or perpetuities) and $\delta = 1$ corresponds to the case of short-term bonds. Since a bond issued at $t - m$ is equivalent to $(1 - \delta)^m$ bonds issued at $t$, the stock of outstanding bonds can be summarized by a single state variable $B$.

Crucial to equilibrium multiplicity is the assumption that investors can coordinate on different regimes of beliefs that, conditional on a sufficiently high initial level of debt, determine the price of bonds at which market participants are willing to finance the government. Following the literature, coordination is driven by an exogenous state $\rho$—which, when the equilibrium is not unique, determines which one is selected. The aggregate state variable of the economy is then summarized by $s = \{(B, z_{-1}, a, \rho)\}$.

## 2.2 Time-line of Fiscal and Investors' Decisions

The sequence of fiscal and investors' decisions draws on Calvo (1988), essentially following the 'timing convention' as in, e.g., Corsetti and Dedola (2016) and Lorenzoni and Werning (2019) among others. It is worth stressing from the start that, consistent with the time-line below, the government is *discretionary*. In particular, it cannot commit to a particular fiscal or issuance policy, so to coordinate investors' expectations on own preferred equilibrium. Assuming that no default has occurred in the previous period ($z_{-1} = 1$), in each period $t$:

1. The business cycle shock $a$ and the exogenous shock determining the beliefs regime $\rho$ are realized. The aggregate state $s = \{(B, z_{-1}, a, \rho)\}$ is therefore known at the beginning of the period. Conditional on the realized regime of beliefs $\rho$ and the government initial debt stock $B$, a continuum with measure one of investors forms rational expectations of the gross financing need and issuance policy of the government, and coordinates expectations on a planned purchase $b'$ at the bond price $q(b', s)$. Investors offer to buy bonds at that price (subject to an upper bound on aggregate issuance discussed below).

2. Taking $q(b', s)$ as given, fiscal decisions take place.

   (a) If the government decides to repay existing obligations, it optimally adjusts its primary deficit and thus its bond issuance $B'$ consistent with the current market-clearing price of bond. The following period starts again from 1 above.

   (b) If the government opts for a default: it loses market access and suffers an output drop by the factor $Z$, both on a permanent basis. It adjusts its spending to the new, permanently lower, tax revenue. The economy becomes stationary at the new low level of output.

A comment is in order concerning the sequence of decisions detailed above. In the literature on debt crises after Eaton and Gersovitz (1981) and Cole and Kehoe (2000), the government sets the total issuance of discount bonds—hence it chooses the amount of debt it owns in the future—before investors set bond prices.[8] In Calvo, the amount of debt a discretionary government owns in future periods depends on the bond price that investors are willing to offer in the current period. It is precisely the two-way endogeneity of the government future debt burden and the current bond price that can give rise to Calvo-type equilibrium multiplicity—see Lorenzoni and Werning (2019) for a recent discussion of the logic and microfoundations of the mechanism. Under the maintained assumption that the government has the ability to set its future debt burden, multiple equilibria may still be possible, but they emerge from a different type of two-way feedback, e.g., encompassed in the auction mechanism specified by Cole and Kehoe (2000). We further expand on a comparison of our model with this seminal contribution in Section 6 below.

In the logic of Calvo (1988), investors select an equilibrium based on their beliefs, and offer the government a bond price that clears the market and satisfies the first-order conditions of the government (hence its primary surplus and issuance policy) at *the selected equilibrium*. Discretionary fiscal authorities have no commitment power nor instrument to move investors' beliefs from a bad to a better equilibrium—a move that, incidentally, would imply a *lower* issuance $B'$ but a *higher* deficit (spending optimally rises when borrowing conditions improve).[9]

## 2.3 The Investors' Problem

For tractability, investors are assumed to have "deep pockets", such that corner solutions in each investor's problem are ruled out in equilibrium. The equilibrium condition for risk-neutral investors must satisfy the break-even condition, equating the expected return on sovereign debt to the risk-free rate:

$$q(b', s) = \begin{cases} \beta \mathbb{E}\Big[z(s')\Big(\kappa + (1-\delta)q(b'(s'), s')\Big)\Big] & \text{if } z_{-1} = 1 \\ 0 & \text{if } z_{-1} = 0 \end{cases} \tag{1}$$

If the government has no history of default in the past,[10] investors coordinate expectations on a price $q(b', s)$ and form rational expectations of issuance policy $B'$ of the sovereign, that in turn determines the probability of defaulting in the current period or in the future.

[8]This assumption ensures a unique equilibrium in Eaton and Gersovitz (1981). In this model, provided investors are willing to finance the government, they offer the best equilibrium price for the bonds. Auclert and Rognlie (2016) expand on Eaton and Gersovitz (1981) and Cole and Kehoe (2000) and discuss conditions such that a unique equilibrium exists.

[9]See Section 3.

[10]Recall that, once the government defaults ($z = 0$), $z$ stays at 0 forever. This assumption implies that the equilibrium bond price is zero in any history with past default.

### 2.4 The Government's Problem

In our economy, in each period the government can access the bond market only conditional on a good credit history, and takes its fiscal decision subject to the bond price $q$ offered by investors. Given this bond price, and assuming for simplicity that the tax rate $\tau$ is constant (so that the sovereign tax revenue is exogenous and equal to $\tau y(a, z)$), the government's problem can be reduced to choose $B', z$ to solve

$$\begin{aligned} V(s) = \max_{B', z} \quad & \mathcal{U}(c, g) + \beta \mathbb{E}[V(s')] \\ \text{s.t.} \quad & g + z\kappa = \tau y(a, z) + zq(B' - (1-\delta)B), \\ & c = (1-\tau) y(a, z), \\ & g \geq \bar{g}, \\ & B' \leq \bar{B} < \infty, \\ & z = 0 \text{ if } z_{-1} = 0 \end{aligned} \tag{2}$$

where $c$ represents the consumption of the representative household who spends all its income after paying tax every period. We denote the endogenous government spending as $g$, and we stipulate that this cannot fall below some critical expenditure level $\bar{g}$. We also stipulate that aggregate debt issuance is kept within a finite boundary. This assumption rules out the possibility for the government to arbitrage by issuing an infinite amount of debt taking the price offered by investors as given, and then defaulting one period ahead.[11]

The government defaults if and only if the utility of repaying debt $V^R(s)$ is smaller than the utility of defaulting $V^D(s)$:

$$V^R(s) < V^D(s)$$

where $V^D(s)$ is pinned down by the simplifying assumption that, in case of default, the country loses market access *and* experiences a discrete but permanent contraction in output by $Z$—output stays at either $AZ\bar{y}$ or $Z\bar{y}$ forever.

As in Conesa and Kehoe (2017), we posit that, for any feasible $B$ such that $\tau A\bar{y} - B$ is an element of the feasible set of government spending $g$, the following condition holds

$$\mathcal{U}_g((1-\tau)A\bar{y}, \tau A\bar{y} - B) > \mathcal{U}_g((1-\tau)\bar{y}, \tau\bar{y} - B)$$

This ensures that, in a recession, the government always has an incentive to raise debt due to higher marginal benefit of government spending in a recession than in normal times.

[11] Implicit in Calvo's approach is the idea that, when investors coordinate their expectations of a given equilibrium price and offer to buy government bonds at that price, their offer is valid only provided that new bond issuance is finite and remains close to the rational expectation equilibrium level.

## 2.5 Equilibrium

An equilibrium is a value function for the government $V(s)$, the policy functions $z(s)$, $g(s)$ and $B'(s)$, and an equilibrium bond price $q$ such that

1. given the policy functions $z(s)$, $g(s)$, and $B'(s)$, $q(b', s)$ is such that investors make zero expected profit (the break-even condition (1) holds);
2. $V(s)$, $B'(s)$, $z(s)$ and $g(s)$ solve the government's optimization problem in (2);
3. the bond market clears. $b' = B'$.

The root of equilibrium multiplicity lies in the fact that the government, facing the bond price $q(b', s)$, optimally adjusts its primary surplus and issuance policy $B'$ and this creates the possibility that multiple pairs of $(q(b', s), B')$ satisfy (1) in equilibrium. The role of the (exogenous) state variable $\rho$ is to *pick* one of the possible regimes of expectations prevailing in the market, when there are more than one available. For tractability, the notion of equilibrium we consider follows a simple Markov structure.

# 3 Investors' Beliefs and Equilibrium Selection

In this section, we illustrate analytically and graphically the logic of belief-driven crises in the model. To do so, we specialize the model assuming that, first, debt is short-term only. Second, all agents in the economy consider the current regime of investors' beliefs as constant over time. The equilibrium thus corresponds to what Aguiar et al. (2020) dubbed "static", in the sense that, once an exogenous beliefs state $\rho$ determines which regime of expectations prevails in the market, agents do not expect to switch across regimes. A switch, if it occurs, is completely unanticipated. We relax both assumptions later on in the text.

Below we discuss the set of beliefs in $\rho$ and their role in selecting the equilibrium. Next, we derive sovereign's debt tolerance thresholds conditional on optimistic and pessimistic beliefs. Finally, we characterize the equilibrium graphically using Gross Financing Need functions and Laffer Curves.

## 3.1 Beliefs: Optimistic vs. Pessimistic

When multiple pairs of $q$ and $B'$ satisfy (1), selection is driven by an exogenous sunspot. As in Calvo (1988), hereafter we let investors beliefs be either "Optimistic" (*opt*) or "Pessimistic" (*pes*), hence $\rho \in \{opt, pes\}$.[12] Heuristically, in an optimistic regime, one can envision investors approaching the debt market with the belief that government bonds are safe and

[12] We will introduce "Extreme beliefs" in Section 6.

the total debt is on a sustainable, moderate growth path. When more than one equilibrium price satisfies (1), optimistic investors naturally coordinate their expectations on the best (riskless) one. In a regime of pessimistic beliefs, instead, investors systematically consider the possibility that debt and deficits may be on an unsustainable expansion path, with two possible consequences. Vis-à-vis the anticipated fiscal path, either the government will be unable to honor its liabilities in the future unless the economy recovers (i.e., it returns to a good state in the following period), or the outstanding stock of debt is already unsustainable. Hence, if multiple prices satisfy (1), they naturally coordinate their expectations on the worse equilibrium—which is either the equilibrium where the government bonds are traded at the default-risky price,[13] or the equilibrium in which the bond price is zero.

## 3.2 The Debt Tolerance Thresholds

Debt tolerance thresholds are contingent on the state of the economy—in particular, on the economic cycle $a$ and the investors' beliefs $\rho$. We denote them as $\bar{B}(a)_\rho$, e.g., in a recession ($a = 0$) under pessimistic beliefs ($\rho = pes$), the maximum sustainable debt level will be $\bar{B}(0)_{pes}$.

### 3.2.1 The threshold in normal times $\bar{B}(1)$.

By assumption, once out of a recession, our economy remains in the high output state permanently and no default takes place. The government's optimization problem conditional on $a = 1$ is deterministic and independent of whether investors' beliefs are optimistic or pessimistic. Conditional on deciding to honor its debt, the government pays $(1 - \beta)B$ in each period to satisfy the no-Ponzi condition. Given our simplifying assumption, the maximum sustainable debt level in normal times will simply be $\bar{B}(1)_{opt} = \bar{B}(1)_{pes} = \bar{B}(1)$. Let $V^R(B, a, \rho)$ and $V^D(a)$ denote the government utility of, respectively, repaying debt, and defaulting:

$$V^R(B, 1, \rho = opt \text{ or } pes) = \frac{\mathcal{U}((1-\tau)\bar{y}, \tau\bar{y} - (1-\beta)B)}{1-\beta}$$

The utility of defaulting when the economy is not in a recession is

$$V^D(1) = \frac{\mathcal{U}((1-\tau)Z\bar{y}, \tau Z\bar{y})}{1-\beta}$$

[13]We should note that, under our simplifying assumptions, once the economy recovers, the default-risky price is no longer an equilibrium price, in that the output uncertainty is resolved. Without loss of generality, we posit that investors offer the riskless bond price after the recovery under pessimistic beliefs regime.

It follows that $\bar{B}(1)$ can be characterized by solving

$$V^R(\bar{B}(1), 1, \rho = opt \text{ or } pes) = V^D(1)$$

and that the utility of the government $V(B, a, \rho)$ in normal times is summarized by

$$V(B, 1, \rho = opt \text{ or } pes) = \max\left\{V^R(B, 1, \rho = opt \text{ or } pes), V^D(1)\right\}$$

The debt tolerance threshold $\bar{B}(1)$ is independent of beliefs, because investors hold a unique consistent view that the economy will never fall back into a recession—output uncertainty is resolved.

### 3.2.2 The threshold in a recession under optimistic beliefs $\bar{B}(0)_{opt}$

As discussed above, when investors hold optimistic beliefs, they presume that the government may remain willing and able to service its debt in full even if the economy remains in a recession in the next period. The question is whether this presumption is self-validating in equilibrium—if not, they could still lend to the government at the default-risky price $\beta p$. While the utility of defaulting in a recession is independent of beliefs:

$$V^D(0) = \frac{\mathcal{U}((1-\tau)AZ\bar{y}, \tau AZ\bar{y})}{1-\beta(1-p)} + \beta \frac{p\mathcal{U}((1-\tau)Z\bar{y}, \tau Z\bar{y})}{(1-\beta)(1-\beta+\beta p)}$$

the utility of repaying debt is belief-dependent. Contingent on optimistic beliefs, the utility of repaying is $V^R_{opt}(B, 0) = \max\{V^R_{opt,1}(B, 0), V^R_{opt,2}(B, 0)\}$, where we allow for the possibility that the government rolls over its liabilities at either the riskless or the risky price:

$$\begin{aligned} V^R_{opt,1}(B, 0) = \max_{0 \le B' \le \bar{B}(0)_{opt}} \quad & \mathcal{U}(c, g) + \beta\Big(pV(B', 1, \rho = opt) + (1-p)V(B', 0, \rho = opt)\Big) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta B', \\ & c = (1-\tau)A\bar{y} \end{aligned} \tag{3}$$

$$\begin{aligned} V^R_{opt,2}(B, 0) = \max_{\bar{B}(0)_{opt} < B' \le \bar{B}(1)} \quad & \mathcal{U}(c, g) + \beta\Big(pV(B', 1, \rho = opt) + (1-p)V^D(0)\Big) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\ & c = (1-\tau)A\bar{y} \end{aligned} \tag{4}$$

The debt threshold $\bar{B}(0)_{opt}$ is the solution of the equation below.

$$V^R_{opt}(\bar{B}(0)_{opt}, 0) = V^D(0)$$

Denote with $\mathbb{B}_{opt}$ the set of debt levels that validate "optimistic belief":

$$\mathbb{B}_{opt} \equiv \{B \mid V^R_{opt}(B,0) \geq V^D(0)\}$$

On this domain, $V(B, 0, \rho = opt) = V^R_{opt}(B, 0)$. For $B > \sup \mathbb{B}_{opt} = \bar{B}(0)_{opt}$, the government defaults.

### 3.2.3 The threshold in a recession under pessimistic beliefs $\bar{B}(0)_{pes}$

Given the equilibrium financing need of the government, investors who hold pessimistic beliefs are concerned with the possibility of future default if the current recession persists. To derive the debt threshold consistent with these beliefs, we again write the utility of repaying debt $V^R_{pes}(B, 0)$ in terms of (3) and (4) above, but replacing the default-free bond price $\beta$ in (3) with the default-risky bond price $\beta p$. To save space, we write up a full description in Appendix A.

The debt threshold $\bar{B}(0)_{pes}$ at which the risky bond price is an equilibrium solves the equation below:

$$V^R_{pes}(\bar{B}(0)_{pes}, 0) = V^D(0)$$

A necessary condition of self-validated pessimistic beliefs is $B' > \bar{B}(0)_{pes}$ in $V^R_{pes}(B, 0)$, so to ensure that the government defaults if the recession persists.

We denote with $\mathbb{B}_{pes}$ the set of initial debt levels that validate "pessimistic belief", defined as follows:

$$\mathbb{B}_{pes} \equiv \{B \mid B' \text{ that solves } V^R_{pes}(B,0) \text{ satisfies } B' > \bar{B}(0)_{pes} \text{ and } V^R_{pes}(B,0) \geq V^D(0)\}$$

The following proposition establishes that $\mathbb{B}_{pes}$ is non-empty.

**Proposition 1.** *For a strictly concave utility function $\mathcal{U}$, a sufficient condition for $\mathbb{B}_{pes} \neq \varnothing$ is a high enough critical expenditure $\bar{g}$, such that under the pessimistic beliefs regime, there exists some debt level at which the government is unable to sustain $\bar{g}$ unless it issues debt above $\bar{B}(0)_{pes}$.*

*Proof.* See Appendix C.1. □

Proposition 1 proves the existence of equilibria given pessimistic beliefs. We should note here that, different from the original analysis in Calvo (1988), the equilibrium when $\rho = pes$ in our model is "stable" in the sense discussed by Lorenzoni and Werning (2019). In our graphical analysis below, Figure 1-3, the slope of the Laffer curve is positive around the pessimistic equilibrium: a small deviation from the equilibrium does not create the kind of instability discussed by these authors, whereby the government can raise more revenue by reducing issuance at the margin.[14]

For future reference, we should also note that, for $B \in \mathbb{B}_{pes}$, the utility of the government in a recession is $V(B, 0, \rho = pes) = V^R_{pes}(B, 0)$. If the initial outstanding debt level is to the left of the set, $B < \inf \mathbb{B}_{pes}$, even if investors hold the pessimistic beliefs, the only equilibrium that can justify investors' break-even condition (1) is the risk-free price $\beta$. When $B > \sup \mathbb{B}_{pes} = \bar{B}(0)_{pes}$, instead, there would be no positive equilibrium price that clears the market. We elaborate these two cases in detail in Section 3.3.

The following proposition establishes that investors' debt tolerance under pessimistic beliefs cannot be above the debt tolerance threshold under optimistic beliefs.

**Proposition 2.** $\bar{B}(0)_{opt} \geq \bar{B}(0)_{pes}$. *Equality holds if and only if, under optimistic beliefs regime, the government borrows into a default-risky level when $B$ is equal to $\bar{B}(0)_{opt}$, i.e., $B'$ is above $\bar{B}(0)_{opt}$. Otherwise inequality is strict.*

*Proof.* See Appendix C.2. □

We conclude our analysis by stressing that the debt thresholds are independent of the initial debt level. By this property, there is a well-defined Laffer curve under each belief scenario, as shown below.

### 3.3 An Intuitive Graphical Analysis using Laffer Curves

In this section, we discuss multiplicity using the graphical apparatus of Laffer Curves—we detail the algorithm for computing an equilibrium analytically in Appendix B. Our goal is to clarify how, once the market coordinates on a bond price, this drives fiscal decisions on debt issuance and deficits in such a way that, depending on the initial debt, the equilibrium in a given period may not be unique. Figure 1-3 are drawn for three different levels of initial debt—low, intermediate and high. To highlight the role of beliefs, each figure includes two panels—the left panel depicting the Laffer Curve under optimistic beliefs ($\rho = opt$), the right-hand panel depicting the curve under pessimistic beliefs ($\rho = pes$). Note that $\bar{B}(0)_{opt}$ in the left-hand panel is higher than $\bar{B}(0)_{pes}$ in the right-hand panel.[15] Each panel also includes the Government Financing Need GFN, as a function of the initial debt and the bond price.

[14] The point is also discussed in Corsetti and Dedola (2016). The endowment process in our model can be viewed as drawn from a bimodal distribution as in Ayres et al. (2018), Ayres et al. (2019), Chatterjee and Eyigungor (2019) and Paluszynski (2019).

[15] For expositional clarity, we restrict our attention to equilibria in which, under optimistic beliefs, the government never borrows up to default-risky levels, i.e., in which the condition $\bar{B}(0)_{opt} \geq \bar{B}(0)_{pes}$, established in Proposition 2, holds as a strict inequality.

As we draw Laffer Curves, the graphs have the amount of discount bonds the government issues during a period, $B'$, on the x axis, and the resources that the government can obtain by issuing debt, $qB'$, on the y axis. Vertical dashed lines denote the debt tolerance thresholds derived above. From the origin to the threshold $\bar{B}(0)_\rho$, the Laffer curve has slope $\beta$, the risk-free bond price. Beyond this threshold, the price of debt falls discretely, as investors anticipate contingent default one period ahead. Due to default risk, any issuance bringing the debt stock in the range between $\bar{B}(0)_\rho$ and $\bar{B}(1)$ is priced $\beta p$: the Laffer curve has a flatter slope but, importantly, still upward sloping.

In each period, the Gross Financing Need of the government (GFN):

$$qB' = \underbrace{g + B - \tau A\bar{y}}_{GFN(q;B)}$$

is endogenous, since (for a given initial debt and conditional on not defaulting) the government sets spending as a function of output and the price of bonds. This endogeneity shapes the two-way feedback effect between the GFN and market beliefs. It can be shown that, for debt levels sufficiently away from zero, the GFN draws a function monotonically decreasing in the price of bonds. Intuitively, facing higher borrowing costs (a lower bond price), the government optimally cuts its spending and deficit, but not enough to reduce its bond issuance $B'$. Graphically, moving down the GFN schedule, think of each point along this schedule as crossing a ray from the origin with slope $q$ (not shown). The GFN depicts the government optimal deficit response (which is a cut) to progressively higher borrowing costs. We should note that the lower bound on government spending is key to the result that the optimal adjustment in the primary surplus falls short of fully compensating the effect of lower bond prices on bond issuance.

As regards the position of the GFN function in the graph, other things equal, a higher stock of liabilities inherited by the government, $B$, raises the GFN, thus shifts the GFN line out to the right. The possibility of equilibrium multiplicity emerges for intermediate and high levels of the initial debt. The equilibrium is instead unique when the GFN function is close enough to the origin (for a low initial debt).

### 3.3.1 Unique Equilibrium with Risk-free Debt

Figure 1 illustrates a case in which the equilibrium is unique and bonds are traded at the default-free price. This case corresponds to a low initial level of liabilities. In the left-hand panel, the GFN intersects the debt Laffer curve at $L_{opt}$, to the left of the debt issuance threshold $\bar{B}(0)_{opt}$ so that debt is issued at the risk-free rate. In the right-hand panel, the GFN also intersects the Laffer curve to the left of the threshold $\bar{B}(0)_{pes}$ at $L_{opt}$.

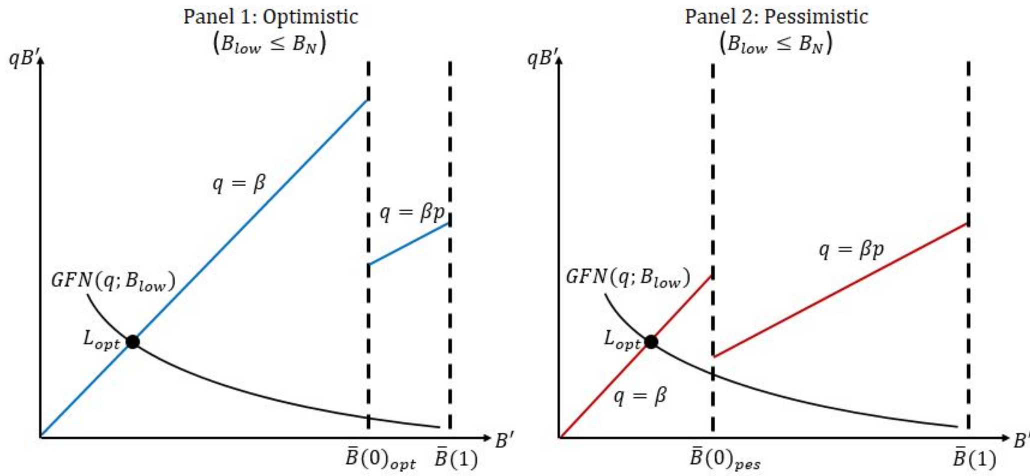


**Figure 1:** Unique equilibrium at a low initial debt

When initial debt level is low, the default-risky price cannot be an equilibrium price since it does not satisfy investors' break-even condition (1), as is clear in the right-hand panel of Figure 1. Even if, under pessimistic beliefs, investors offered a low bond price, the government would still issue debt below the tolerance threshold $\bar{B}(0)_{pes}$, running a moderate (optimal) GFN—investors' pessimistic view would not be validated ex-post. In other words, in the right-hand panel, $q = \beta p$ is not the solution to investors' first order condition (1). It must be the case that:

$$q(b', s) \quad = \quad \beta \overbrace{\mathbb{E}\Big[z\big(\underbrace{B'}_{<\bar{B}(0)}, a', z'_{-1}, \underbrace{\rho = pes}_{\bar{B}(0)=\bar{B}(0)_{pes}}\big)\Big]}^{=1} \neq \beta p \tag{5}$$

Hence, the only self-fulfilling equilibrium is therefore at the point $L_{opt}$, with riskless pricing in equilibrium. Note that, using equation (5), one can address the important question of identifying the level of outstanding debt (determining the position of the GFN function in the graph) above which the equilibrium is no longer guaranteed to be unique—–the economy becomes vulnerable to crises driven by pessimistic beliefs. Define a new debt threshold, labelled $B_N \equiv \inf \mathbb{B}_{pes}$, as the maximum amount of the *initial* debt level in a recession below which the country is "immune to pessimistic beliefs". $B_N$ can be found by solving:

$$B_N \equiv \inf \mathbb{B}_{pes} = \sup_{B} \left\{ \beta p \quad \neq \quad \beta \overbrace{\mathbb{E}\Big[z\big(\underbrace{B'(s)}_{<\bar{B}(0)}, a', z'_{-1}, \underbrace{\rho = pes}_{\bar{B}(0)=\bar{B}(0)_{pes}}\big)\Big]}^{=1} \right\} \tag{6}$$

As we will see in Section 5, $B_N$ will be crucial in shaping a government incentive to deleverage in a recession.

### 3.3.2 Multiplicity with Slow-moving Debt Crises

In Figure 2, the government initial debt is larger than $B_N$ but smaller than $\bar{B}(0)_{pes}$—in the graph, this initial debt level is referred to as $B_{mid}$ (mid for intermediate). The relevant GFN line intersects the Laffer curves either to the left or to the right of the debt tolerance threshold $\bar{B}(0)_\rho$. The two cases are depicted in the left-hand and right-hand panel, respectively. Following Calvo's approach, the government chooses its GFN and debt issuance after investors set the bond price. Hence, either intersection point can be an equilibrium: the left-hand panel depicts the equilibrim in an optimistic world, the right-hand panel in a pessimistic world.

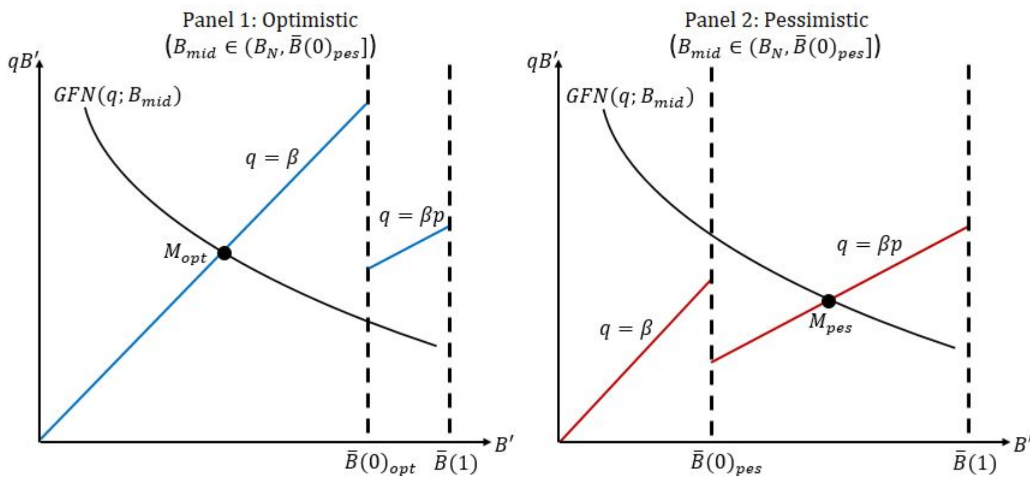


**Figure 2:** Multiple equilibria when $B \in (B_N, \bar{B}(0)_{pes}]$

In the panel to the left, investors offer to buy newly issued sovereign debt at the riskless price $\beta$. Overall borrowing remains below the relevant debt tolerance threshold $\bar{B}(0)_{opt}$, validating ex-post the investors' optimistic beliefs. By contrast, in the panel to the right, investors offer to buy at the lower risky price. Despite the optimal fiscal response (the GFN is lower), new issuance goes beyond the tolerance threshold $\bar{B}(0)_{pes}$ (but remains below $\bar{B}(1)$). One period ahead, unless the economy recovers, the government defaults, validating investors' pessimistic beliefs. For both $q = \beta$ and $q = \beta p$ to be equilibrium prices, the initial stock of debt must be such that these price simultaneously satisfy the equations (7) and (8) from the investors' first order condition (1):

$$q(b', s) = \beta \overbrace{\mathbb{E}\Big[z\big(\underbrace{B'}_{<\bar{B}(0)}, a', z'_{-1}, \underbrace{\rho = opt}_{\bar{B}(0)=\bar{B}(0)_{opt}}\big)\Big]}^{=1} = \beta \tag{7}$$

$$q(b', s) = \beta \overbrace{\mathbb{E}\Big[z\big(\underbrace{B'}_{>\bar{B}(0)}, a', z'_{-1}, \underbrace{\rho = pes}_{\bar{B}(0)=\bar{B}(0)_{pes}}\big)\Big]}^{=p} = \beta p \tag{8}$$

This is the main message in Calvo (1988): for intermediate levels of debt, once investors coordinate their expectations on the equilibrium in which sovereign bonds trade at the default-risky price $\beta p$, trade occurs at this price. The government optimal response is a cut in spending and deficit, but to a degree that is insufficient to prevent an increase in $B'$ above the threshold. Even though the risk-free price $\beta$ is another possible equilibrium price, the government has neither power nor instrument to correct the regime of expectations prevailing in the market.

The type of equilibrium with belief-driven default shown in Figure 2 corresponds to a scenario that Lorenzoni and Werning (2019) dubs a 'slow-moving' debt crisis. Interest rates are high because investors expect the government to default if a recession persists. Because of high borrowing costs, the stock of government debt rises markedly prior to default. But default only occurs if and only if the country remains in a recession in the future.

### 3.3.3 Multiplicity with Fast Debt Crises

By the logic of the model, however, there is also another type of equilibrium with belief-driven default. This is shown in Figure 3, drawn assuming a relatively high initial debt level, higher than $\bar{B}(0)_{pes}$. In the left-hand side panel, the GFN line intersects the Laffer curve at the point $H_{opt}$. If investors buy government bonds at the riskless price $\beta$, despite the high stock of initial liabilities, new debt issuance remains below the threshold under optimistic beliefs, $\bar{B}(0)_{opt}$. However, if investors turn pessimistic—right-hand side panel—the hike in borrowing rates causes the government to become 'intolerant' of the adjustment required to service the (high) outstanding debt. Investors anticipate that, at the default-risky bond price, the government will not be willing to adjust its primary needs enough to keep new issuance of debt below $\bar{B}(1)$: the GFN line does not cross any relevant portion of the debt Laffer curve. A "fast" debt crisis occurs: default occurs the moment markets become pessimistic.

One can look at this fast debt crisis from two angles. From the vantage point of the government, when the market expects a default in a recession, new bonds can only be issued at risky rates. But at these rates, the cut in primary spending required to keep

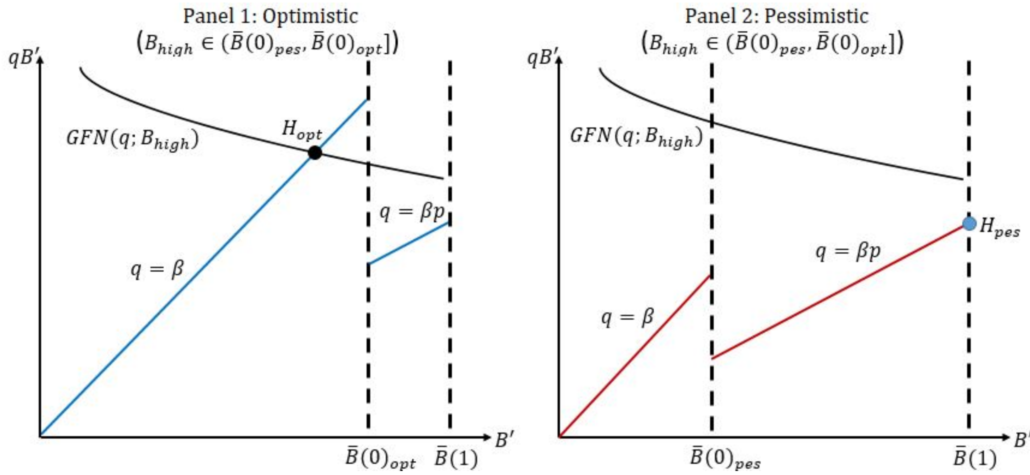


**Figure 3:** Fast crisis when $B \in (\bar{B}(0)_{pes}, \bar{B}(0)_{opt}]$

new issuance below the maximum debt capacity in normal times is suboptimally large. Even if, counterfactually, investors were willing to finance the deficit at finite interest rates (presumably expecting appropriate further cuts in spending), immediate default would be the government's preferred option. Anticipating this, from the investors' vantage point, it is rational not to finance the government at all: the country instantly loses market access.

It is important to clarify why the point of intersection of the Laffer curve with the vertical line above the debt threshold $\bar{B}(1)$, denoted $H_{pes}$ in Figure 3, is not an equilibrium. We have seen that, in Figure 3, once investors turn pessimistic, the government is willing to cut its deficit need moving down *along* the GFN line. But because of the high initial level of liabilities, the optimal primary surplus adjustment conditional on no-default would not be enough to guarantee sustainability. To put it another way, the required deeper deficit cuts to keep issuance below $\bar{B}(1)$ would not be optimal given that spending and the utility of the government remain relatively high after default, i.e., given that post default output remains sufficiently high relative to the critical expenditure level $\bar{g}$.

There is a subtle but important difference between the fast debt crisis in Figure 3 and the rollover crisis in Cole and Kehoe (2000). In Cole and Kehoe (2000), if (enough) investors participated in the bond auction, there would be no default. This is because it is precisely the loss of market access that makes the surplus adjustment (required to repay existing obligations) so large and harsh, that servicing the debt is welfare-dominated by default. In Figure 3, instead, the government would default even if investors were willing to finance the deficit at the (off-equilibrium) risky price—market access is lost because debt is too high to be sustainable when investors charge a premium.

# 4 Multiplicity with Long-Term Debt

For any given stock of debt, longer maturities may help sustainability, by reducing the exposure to rollover risk and the pass-through of hikes on interest rates onto the total cost of servicing the outstanding debt. An important question is whether and under what circumstances maturity can rule out multiplicity leading to either slow-moving or to fast debt crises.

In this and the next sections, we present a numerical example generalizing the economies depicted in Figure 1-3 allowing for long-term debt and calibrating our model with standard parameter values from the literature. We keep assuming that beliefs are "static"—that is, agents in the economy do not attribute a positive probability to a switch in the regime of expectations. We relax this assumption in the next section. Overall, we find that multiplicity of equilibria remains pervasive.

## 4.1 Calibration

In solving the model with long-term debt, we posit the following functional form for the utility function:

$$\mathcal{U}(c, g) = \log(c) + \gamma \log(g - \bar{g}); \tag{9}$$

In our calibration, we set benchmark parameters following Conesa and Kehoe (2017). The parameter values are shown in Table 1.

**Table 1:** Parameter values, baseline

| | | |
|---|---|---|
| $\bar{y}$ | Output | 100 |
| $Z$ | Cost of Default | 0.95 |
| $\beta$ | Discount factor | 0.98 |
| $\gamma$ | Relative weight of $c$ and $g$ in the utility function | 0.20 |
| $\tau$ | Government revenue as a share of output | 0.36 |
| $\bar{g}$ | Level of the critial government expenditure | 25 |
| $A$ | Fraction of output during recession | 0.9 |
| $p$ | Probability of leaving the recession | 0.2 |
| $\delta$ | Amortization rate of market debt | 0.2 |

As shown in the table, we normalize output $\bar{y}$ to 100 so that the units in the model can be interpreted as percentage of GDP: e.g. $B = 50$ means that debt to GDP ratio is 50% in normal times. We set cost of default as $5\% = 1 - Z$. Our default cost is lower relative to the literature (e.g. Alesina et al. (1992)), on the grounds that we assume this cost to be permanent.[16] We assume the relative weight of government utility is 0.2; sensitivity analysis shows that this parameter is unimportant for our result.

The severity of recession $A$ is set to 0.9 so that a recession results in a decrease in output by 10% for the benchmark scenario. This parameter is crucial to generating gambling for recovery in an optimistic world. A more severe recession leads to a stronger smoothing motive for the government, which may induce the government to choose a high-debt risky-debt strategy—we report results for different $A$ in Appendix I.

We set the critical government expenditure $\bar{g}$ at 25% of GDP in normal times: the higher this value, the smaller the room for discretionary spending. Government revenue as a fraction of output is determined by the constant tax rate $\tau$. In normal times, the government income is 36, but in a recession, it drops to 32. We posit $\delta = 0.2$ to match average maturity from 2000-2009 for Greece, Italy and Spain, which is about 5 years. We set $p = 0.2$ so that the expected waiting time for recovery is 5 years.

## 4.2 Pervasive Multiplicity

The key novel result from our analysis is that, as the debt tolerance threshold in a recession moves with investors' expectations, a switch to pessimistic beliefs might result in "fast" debt crises—a result that, as shown by our numerical example, holds also when debt has long maturity. Figure 4 plots the policy functions conditional on a recession, together with the debt tolerance thresholds (in a recession and in normal times), in the optimistic world (left panel) and the pessimistic world (right panel).

A striking feature of the optimistic world—on the left panel of Figure 4—is the high value of the debt tolerance threshold in a recession, about 206% of GDP (or 186% as a ratio of GDP in normal times). As further discussed in Section 7, in our exercises we find that $\bar{B}(0)_{opt}$ is generally not sensitive to the probability of recovery $p$ or debt maturity $\delta$. In a recession, the government smooths consumption by borrowing at the risk-free rate until the stock of outstanding debt reaches $\bar{B}(0)_{opt}$: the figure suggests that the dynamics of debt are mildly increasing.

The right panel of Figure 4 depicts a situation in which investors unexpectedly change their view on government solvency, from optimistic to pessimistic. While $\bar{B}(1)$ is not affected (because of our assumption that, after recovering, the economy never falls back into a recession again), the consequences of such a change on the debt tolerance threshold in a recession are stark. There is a large drop from $\bar{B}(0)_{opt}$ to $\bar{B}(0)_{pes}$.

Now, the country is barely affected by the switch in the regime of beliefs as long as the initial debt is in the region between 0 and $B_N$ (the maximum initial debt level below

[16] Upon a default, in our baseline scenario $Z$ cannot be too small in that the government spending cannot fall below $\bar{g}$. In other words, the conditions $\tau A Z \bar{y} > \bar{g}$ and $\tau Z \bar{y} > \bar{g}$ must be satisfied.

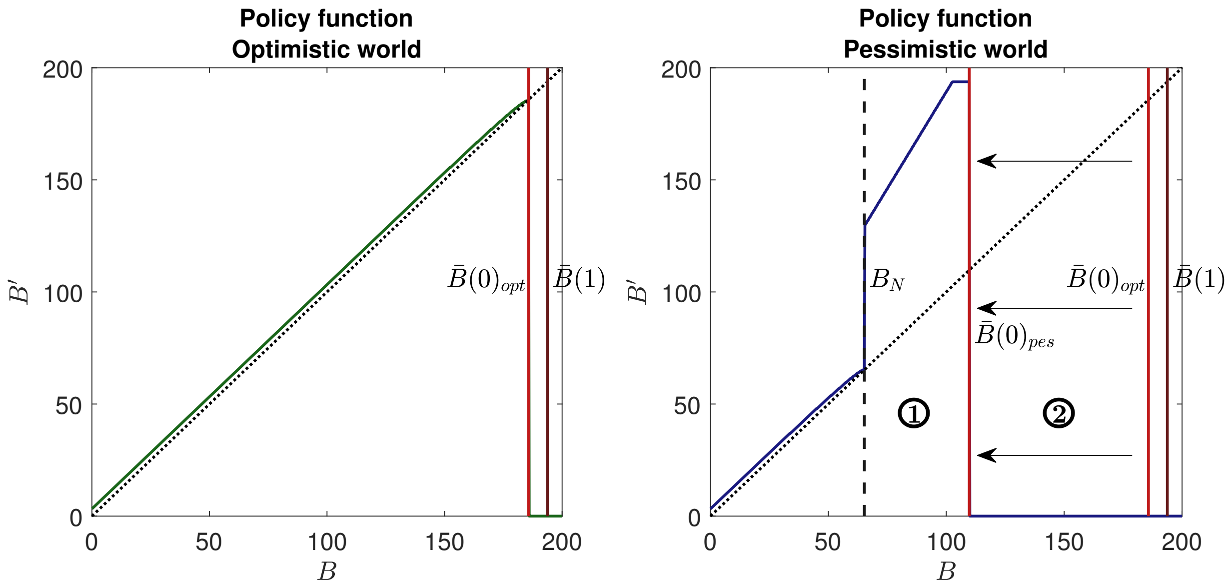


**Figure 4:** Policy functions in the optimistic world and the pessimistic world

which the country is "immune to pessimism"). The government is still able to borrow at the risk-free rate. As a result, in a recession, the government keeps borrowing at this rate for smoothing purposes until the stock of debt reaches $B_N$. At this level, the government cuts its deficits, so to keep the debt stock stationary until the recovery materializes.

The switch in the regime of expectations is instead consequential if the initial debt is anywhere above $B_N$ but below $\bar{B}(0)_{pes}$, i.e., in the region labelled ① in our figure. In this region, the government also keeps borrowing, despite the higher interest rates. But because of these rates, debt accumulates at a much faster pace. In each period, investors anticipate that default may occur depending on whether the economy recovers one period ahead. This is a scenario of a "slow-moving" crisis, in the sense that default is preceded by debt accumulation driven by the hike in interest rates. Note that, under our parameterization, a slow-moving crisis can arise for a debt to GDP ratio as low as 72% in a recession (65% if GDP is measured in normal times).

Furthermore, if debt is in the region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$—the region labelled ② in the figure—the crisis is of the type that we dub "fast": it occurs simultaneously with the (unanticipated) shift in beliefs. It should be stressed that in this fast crisis region, as long as investors are optimistic, the government can actually issue debt at the risk-free rate. But once investors change their view, they understand that the sovereign will be unwilling to reduce its financing need enough to keep new issuance below $\bar{B}(1)$. At the switch, the bond market dries out: there is no "slow-moving" accumulation of debt. Facing a collapse of the bond market, the government defaults immediately. In our calibration, fast crises can occur with a debt to GDP ratio between about 122% and 206% (at the GDP measured in the recessionary state).

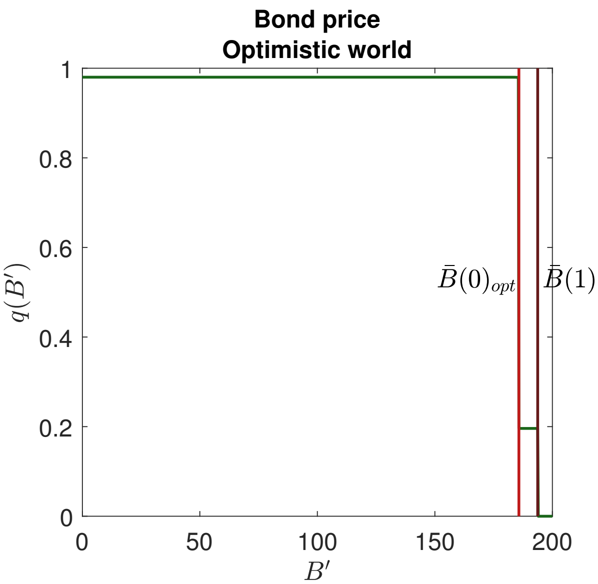


**Figure 5:** Bond prices in the optimistic and the pessimistic world

In Figure 5, we plot the price of new issuance of government bonds in both the optimistic and the pessimistic worlds, contingent on the economy being in a recession. The left panel in the figure shows that the price that optimistic investors offer in equilibrium remains high for a wide range of debt-to-GDP ratio. The bond price they offer, however, drops very markedly in the narrow region between $\bar{B}(0)_{opt}$ and $\bar{B}(1)$. The right panel of Figure 5 illustrates the impact of a change in investors' expectations, from optimistic to pessimistic. If the amount of new issuance lies between $B_N$ and $\bar{B}(0)_{pes}$, the government is exposed to the possibility of slow-moving debt crises next period—a crisis will occur if the economy remains in a recessionary state. Thus, the bond price drops to 0.48.[17] If new issuance is above $\bar{B}(0)_{pes}$ but below $\bar{B}(1)$, instead, the government is at the risk of "fast" debt crises, which decreases the bond price much further.[18]

We conclude by stressing that, while a long debt maturity can substantially increase the debt thresholds in a pessimistic world (improving government welfare), it may not rule out self-fulfilling crises. Our baseline model suggests that the threat of both "slow-moving" and

[17]Different from long-term bonds, in Figure 1-3, the price of one-period bonds remains risk-free for all levels of new issuance below $\bar{B}(0)_{pes}$. When the amount of new issuance lies in between $B_N$ and $\bar{B}(0)_{pes}$, the government may be exposed to slow-moving crisis in the future periods. Yet, one-period debt is still risk-free because, even if the economy enters a slow-moving debt crisis in the following period, the outstanding stock of debt will still be fully rolled over (at higher costs).

[18]We leave to the Appendix D a numerical example assuming one-period bonds ($\delta = 1.0$), using the same calibration as in this section. Comparing the result for one-period bonds with Figure 4 shows that, as $\delta$ converges to unity, $\bar{B}(0)_{pes}$ and $B_N$ are much lower, while $\bar{B}(0)_{opt}$ is not affected at all. We elaborate on these results in Appendix D and Section 7.

"fast" debt crises are pervasive with long-term debt. We will elaborate on this conclusion at the end of Section 7, where we repeat our analysis for a debt-limit version of our model.

# 5 Deleveraging and Debt Dilution

So far we have carried out our analysis under the assumption that, when in an optimistic mode, investors and the government attribute zero probability to a switch to pessimism. In this section, we relax this assumption and construct sunspot equilibria in which all agents in the economy anticipate the possibility of a change in beliefs regime, heavily drawing on the approach by Conesa and Kehoe (2017).

We consider an exercise where the beliefs state $\rho$ transits between optimistic and pessimistic ($\rho \in \{opt, pes\}$). Specifically, we assume that investors are initially optimistic on government solvency, but all agents in the economy are aware that market views may turn pessimistic with probability $\pi$. Conditional on the realization of the sunspot, however, a switch in equilibrium pricing and fiscal policy occurs if and only if the GFN is such that pessimistic beliefs are self-validating. This is the case if the government, when a switch occurs, either borrows more than $\bar{B}(0)_{pes}$ (slow-moving crises), or defaults ("fast" crises). For simplicity, when pessimistic beliefs are self-fulfilling, we assume that investors stick to this expectations regime *forever* afterwards. We posit a small sunspot probability, equal to $\pi = 0.04$. The debt tolerance threshold in a recession is now denoted with $\bar{B}(0)_{\pi}$. To save space, in the following we focus on policy functions and bond price schedules in a recession.

In a sunspot equilibrium, the government may take advantage of the low interest rates in an optimistic regime and choose to decrease debt to safe levels even if the economy is in a recession. The motivation for doing so lies in the potentially large gains in expected utility from either eliminating the possibility of belief-driven crises altogether, or benefiting from a drop in borrowing costs associated to reduced vulnerability, or both. We stress that eliminating belief-driven crises allows the government to steer away from a deep drop in welfare at the crisis threshold—in what follows, we refer to this drop as the welfare 'cliff effect' of self-validating crises, as opposed to their 'price effect'. Our key result is that, different from analyses after Cole and Kehoe (2000), deleveraging is preferred over debt accumulation only for a small range of debt above the threshold at which slow-moving crises become a possibility (around the cliff effect). For a very wide range of debt levels, the government prefers to accumulate liabilities and smooth consumption, gambling on the prospective recovery. The incentive to deleverage is however stronger, the shorter the maturity of debt.

## 5.1 Baseline

Figure 6 displays the policy function (left) and the bond price function (right) in sunspot equilibrium with long-term debt, where investors assign a small probability to a switch to pessimistic beliefs, with $\rho \in \{opt, pes\}$. For debt levels in the region between 0 and $B_N$, the debt dynamics are the same as in the right panel of Figure 4, and the government is able to issue safe debt.

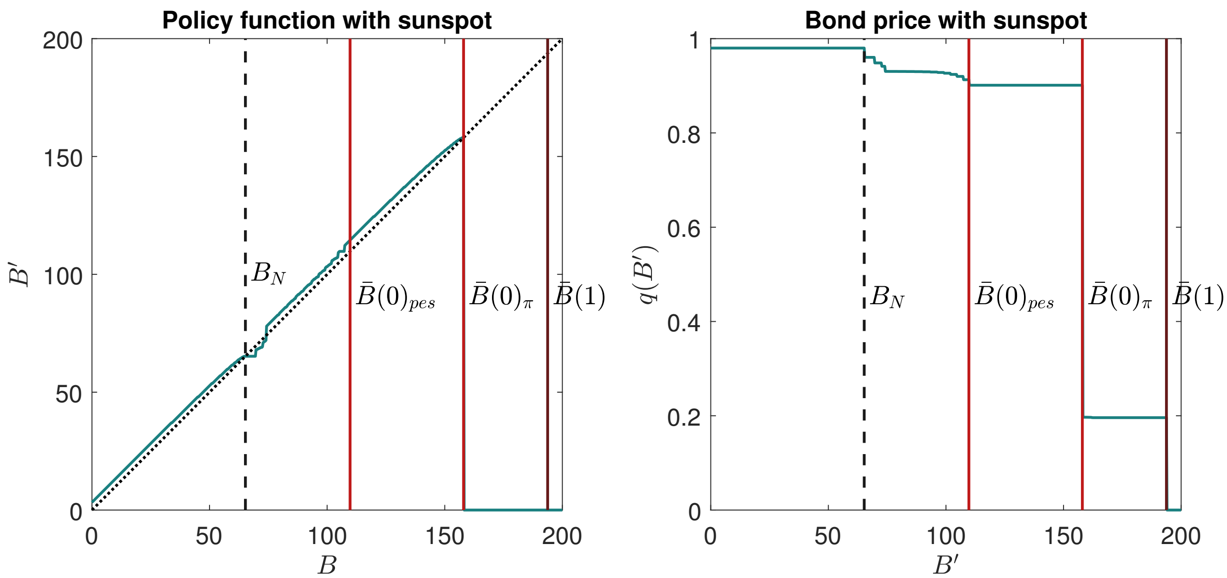


**Figure 6:** $\delta = 0.2$ and $\rho \in \{opt, pes\}$

In the region between $B_N$ and $\bar{B}(0)_\pi$, where the economy is vulnerable to sunspot crises, the debt dynamics are different from what we have seen so far—it is no longer uniform. This region can indeed be split into two subregions. For an initial debt level close to $B_N$, the government chooses to run surpluses and reduce its borrowing. This allows the government to avoid high and increasing borrowing costs, as well as a large utility loss, if self-fulfilling pessimistic expectations materialize. However, for large enough initial debt, the government prefers to keep borrowing. It will do so until its debt level reaches $\bar{B}(0)_\pi$, even for debt levels above (but close to) $\bar{B}(0)_{pes}$, where self-fulfilling crises, if they occur, are fast.[19]

Why is deleveraging optimal for debt levels just above $B_N$, but not so for debt levels just above $\bar{B}(0)_{pes}$? The key insight is that keeping debt below $\bar{B}(0)_{pes}$ shields the country from fast crises, but not from slow-moving ones. Hence, while the government may still have some advantage not to let debt trespass $\bar{B}(0)_{pes}$, this advantage is exclusively in terms of lower borrowing costs (as shown on the right panel of Figure 6), not in terms of eliminating

[19] We find that a government exposed to the risk of fast rollover crises ($B > \bar{B}(0)_{pes}$) with long-term bonds accumulates liabilities faster. This is shown in Appendix E.

the possibility of crises 'tout-court' (the welfare 'cliff effect' is less relevant here).[20] The borrowing costs advantage (about 1.4 percentage point)[21] is not enough to offset the benefits from smoothing consumption in a recession via borrowing. Crucial to this result is the low pass-through of higher interest rates into borrowing costs when the outstanding stock of debt has long maturity—an observation that resonates with the analysis of debt dilution in Aguiar and Amador (2020).[22] The gains in terms of lower borrowing cost from deleveraging are shared between the investors (as a capital gain) and the government (the 'price effect'). With long bond maturities, the former component, which does not provide any incentive to deleverage, has a large weight and therefore weakens the 'price effect' substantially. This is not the case for short-term debt—absent debt dilution, the benefits from lower borrowing costs provide a much stronger incentive to deleverage.

## 5.2 Short-term Debt

For comparison, we show the case of one-period bonds, with $\delta = 1.0$, in Figure 7. Relative to the long-term maturity case, three differences are apparent. First, levels and shifts in thresholds are now substantially different from Figure 6. Second, the bond price remains risk-free in the region between $B_N$ and $\bar{B}(0)_{pes}$, in which a switch in expectations may end up igniting a slow-moving debt crisis. Recall that debt is fully rolled over in the period where a slow-moving crisis materializes. Hence, the risk of such crises does not have any impact on (short-term) bond prices.[23]

Last but not least, the 'price effect' and welfare 'cliff effect' of belief-driven crises play a somewhat different role in shaping government decisions when government debt is short-term. As for the case of long-term debt, deleveraging is optimal for a limited range of debt above $B_N$. But when the maturity is short deleveraging is also optimal for a small range of debt just above $\bar{B}(0)_{pes}$. The right panel of the graph confirms the insight already discussed above. Around the threshold $B_N$, the government deleveraging decision reflects the prospective loss of welfare, which is substantial because of the 'cliff effect'. Although there is no cliff effect around $\bar{B}(0)_{pes}$, the price effect at this threshold is now much higher—the pass-through of the change in market interest rates onto government borrowing costs is full in just one period (about 3.4 percentage points). That is to say, when the maturity of the outstanding debt is short, the government internalizes the gains from reducing the borrowing

[20] Discontinuity in value function, like a 'cliff' in a pessimistic world, motivates the government to deleverage. See Appendix F for details.

[21] We calculate the cost advantage using $\frac{1-q(\bar{B}(0)_{pes}+1)}{q(\bar{B}(0)_{pes}+1)} - \frac{1-q(\bar{B}(0)_{pes})}{q(\bar{B}(0)_{pes})}$. We use the same formula to derive the yield difference of one-period bonds in our simulation below—equal to 3.4%.

[22] See also Aguiar et al. (2019).

[23] This result suggests that bond prices alone may not contain enough information to infer sunspot probability/market beliefs—a beliefs state $\rho$ pins down the equilibrium bond price, but not vice-versa.

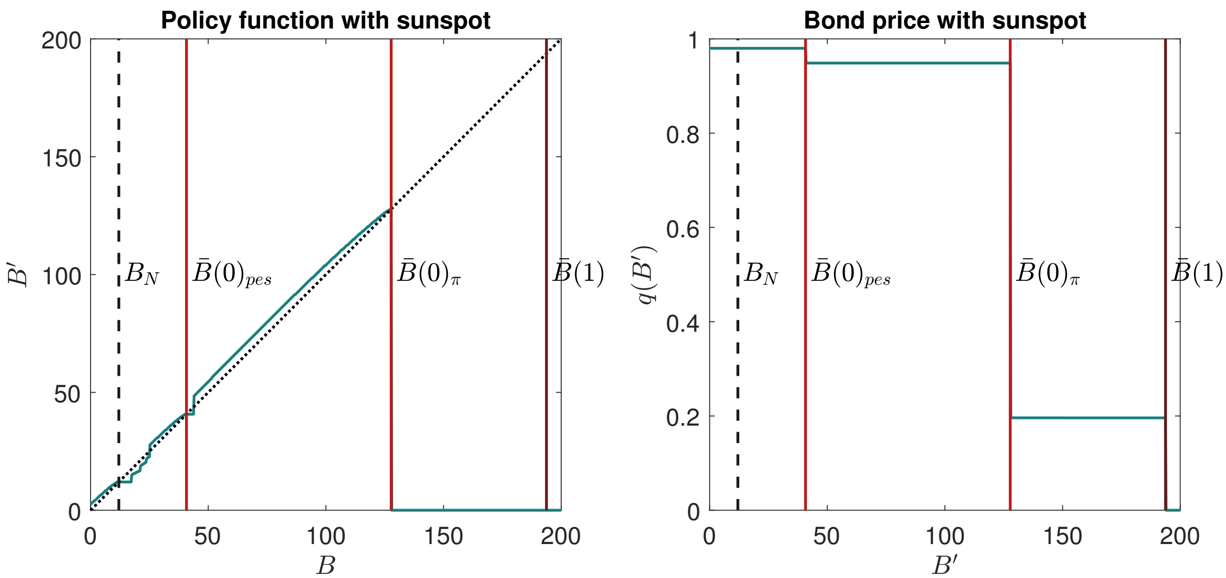


**Figure 7:** $\delta = 1.0$ and $\rho \in \{opt, pes\}$

cost fully, and optimally chooses to deleverage *also* for debt levels close to $\bar{B}(0)_{pes}$. Longer debt maturity would not provide enough of an incentive to do so (as the gains are shared with investors, see again Aguiar and Amador (2020)).

# 6 Default Driven by "Extreme" Beliefs

As a reference benchmark, we extend our model introducing the possibility that investors hold what we dub "extreme beliefs" (denoted $\rho = EX$). Intuitively, in an extreme regime, investors are only willing to finance the government in circumstances under which there is no default in equilibrium. For instance, in our economy with short-term debt, investors with extreme beliefs would always entertain the possibility of a government default. This can only be ruled out under the strict condition that the government remains able and willing to honour its outstanding liabilities in the *current* and *next* period *independent of market financing*. We will see that, while the precise mechanism by which our extreme beliefs may lead to rollover crises is different from Cole and Kehoe (2000), introducing beliefs regime will allow us to compare our model with their seminal work in a meaningful way.

## 6.1 Rollover Crises with Short-term Debt

For clarity, we start, as in Section 3, by studying extreme beliefs in an economy with short-term debt only. Under extreme beliefs, the utility of repaying debt $V_{EX}^{R}$ is

$$V_{EX}^{R}(s) = \max_{0\leq B'} \mathcal{U}((1-\tau)y(a,z), \tau y(a,z) - B + 0 \times B') + \beta\mathbb{E}[V(s')|s] \tag{10}$$

where, given Calvo timing, the loss of market access for the government coincides with markets coordinating on bond market price equal to 0. When $V_{EX}^{R}(s) \geq V^{D}(s)$, the government does not default *today* and it will not default *next period*. As a result, $q = 0$ is not an equilibrium price and "extreme beliefs" are not validated. In contrast, when $V_{EX}^{R}(s) < V^{D}(s)$, the government defaults immediately and, by the assumption that defaulting governments are punished by investors via market exclusion, the loss of market access implied by the investors' extreme beliefs is validated in equilibrium. In other words, in the model, extreme beliefs are self-validating subject to the primitive assumption that the government *permanently* stays at the state of default and market exclusion in any history of default. As mentioned above, such scenario may be rationalized as a strict solvency requirement. For our purpose, however, the key advantage is analytical: the utility off the equilibrium path during a rollover crisis (10) is exactly the same as in Cole and Kehoe (2000).

Analyzing the model under extreme beliefs is particularly useful to appreciate the features of the fast debt crisis in Figure 3. The debt threshold under extreme beliefs (conditional on $\rho = EX$) is always lower than the threshold at which fast crisis can materialize under pessimistic beliefs ($\rho = pes$):

**Proposition 3.** *Strict concavity of the utility function implies* $\bar{B}(0)_{pes} > \bar{B}(0)_{EX}$

*Proof.* See Appendix C.3. □

Under $\rho = pes$, new bonds can be issued at the default-risky rate up to $\bar{B}(1)$—the government is resilient to rollover crises over a wide range of debt. Under $\rho = EX$, instead, default risk translates immediately into a loss of market access. This limits greatly the range of sustainable debt levels.

The fast crises driven by extreme and pessimistic beliefs are different in one, crucial, respect. When $\rho = pes$, fast debt crises occur when investors anticipate that the government is not able and/or willing to adjust enough to keep the stock of debt below $\bar{B}(1)$ at the default-risky rate. When $\rho = EX$, instead, these crises occur when investors find that the government will choose to default immediately unless they are willing to roll over sovereign debt.

## 6.2 The Baseline Revisited

Under extreme beliefs, our baseline calibration of the model yields significantly different results in a number of key dimensions. To start with, we find that long maturities have a significant marginal impact on debt sustainability under extreme beliefs, since they reduce the size of the period-by-period financing need. Other things equal, at the margin lengthening debt maturity raises resilience in a significant way.

However, the debt level at which rollover crises can occur becomes remarkably low. The debt tolerance threshold above which extreme beliefs can cause a rollover crisis is 38 percent in our baseline calibration (with $\delta = 0.2$). It is a mere 8 percent if bond maturity is set to one period (with $\delta = 1$).

## 6.3 Incentives to Deleverage are Stronger

Extreme beliefs impact significantly on the incentive for the government to deleverage. To show this, we extend our analysis to a sunspot equilibrium where agents attach a small positive probability to a switch from optimistic to extreme beliefs, i.e., $\rho \in \{opt, EX\}$.[24] Relative to our previous results contrasting optimistic and pessimistic beliefs, when $\rho \in \{opt, EX\}$, the government deleverages over a much wider range of debt, and yield rates on long-maturity debt are much higher—with bond prices responding more strongly to the sunspot probability.

Figure 8 shows the policy function and the bond price schedule for our calibration with long-term bonds. To study the sunspot equilibrium with extreme beliefs, we define a new debt threshold conditional on recovery, denoted $\bar{B}(1)_\pi$. The reason to introduce this new threshold is that, with extreme beliefs, the government remains exposed to the risk of rollover crises even after the economy exits the recession, if its outstanding debt level is large enough—indeed larger than $\bar{B}(1)_{EX}$.[25]

As shown in the figure, when debt is in the region $[0, \bar{B}(0)_{EX}]$, the government borrows in a recession, until the outstanding debt stock reaches $\bar{B}(0)_{EX}$, at which debt is kept stationary.

[24] Extreme beliefs are self-fulfilling if the government, facing a rollover crisis, defaults immediately. Bearing in mind that the utility off the equilibrium path in Conesa and Kehoe (2017) is the same as (10), the policy functions and bond price schedules we derive for this type of sunspot equilibria are consistent with Conesa and Kehoe (2017). The main difference is that we set $\kappa = 1 - \beta + \beta\delta$ in the budget constraint $g + \kappa B = \tau y(s) + q(B' - (1-\delta)B)$ so that the default-free bond price is $\beta$ for all debt maturities. In contrast, in Conesa and Kehoe (2017), the price of default-free long-term bonds is smaller than $\beta$, as $\kappa$ is smaller than $1 - \beta + \beta\delta$.

[25] In accordance with (10), $\bar{B}(1)_{EX}$ is pinned down by solving the following equation: $\mathcal{U}((1-\tau)\bar{y}, \tau\bar{y} - \bar{B}(1)_{EX}) + \beta\frac{\mathcal{U}((1-\tau)\bar{y}, \tau\bar{y})}{1-\beta} = V^D(1)$, unambiguously smaller than $\bar{B}(1)$ where $\rho = opt$ or $\rho = pes$. Even after the recovery, a switch to extreme beliefs may lead to default—these beliefs are self-validating when a collapse of bond market induces an immediate default, regardless of the output states. This added vulnerability deteriorates the utility of repaying conditional on the recovery, hence $\bar{B}(1)_\pi$ in the figure is lower than $\bar{B}(1)$ in Figure 6.

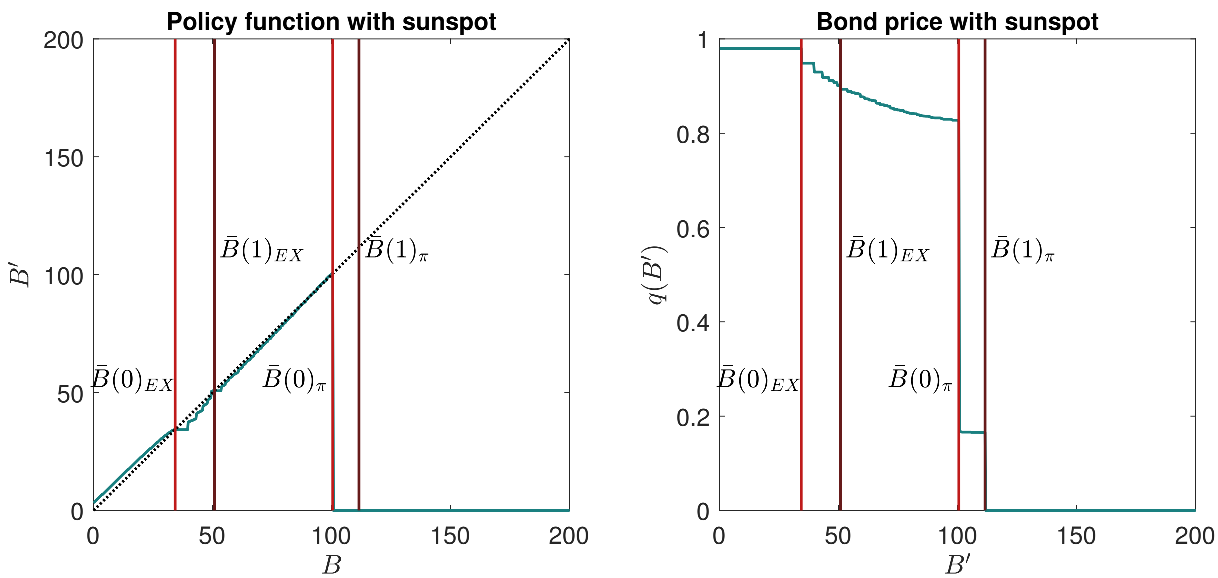


**Figure 8:** $\delta = 0.2$ and $\rho \in \{opt, EX\}$

When debt is in the region $(\bar{B}(0)_{EX}, \bar{B}(0)_\pi]$, the borrowing dynamics are quite complex. The government mostly prefers to deleverage (running surpluses) unless debt is lower than, but close enough to either $\bar{B}(1)_{EX}$ or $\bar{B}(0)_\pi$. Deleveraging is optimal over more than 80% of this debt region. Conversely, the government borrows (gambling on redemption) only over 10% of the region; it prefers to keep the debt level unchanged over about 5% of the region.

The right panel of Figure 8 displays the bond price, which is clearly lower relative to Figure 6. For instance, in terms of bond yields, when $B'$ is 70 (77.8% of GDP in the recessionary state), the bond yield surges to 16.5%, from 5.4% in Figure 6. Although it may not be apparent from the figure, the bond yield increases (the bond price falls) smoothly as debt rises from $\bar{B}(0)_{EX}$ to $\bar{B}(0)_\pi$—with the pace slowing down as $B'$ approaches $\bar{B}(0)_\pi$. The incentive to deleverage is strong when the initial debt level is low enough (far enough from $\bar{B}(0)_\pi$), so that by cutting deficit, the government can not only reduce the borrowing costs to a sizable degree but also steer away from belief-driven crises altogether.

For comparison, Figure 9 illustrates the sunspot equilibrium with short-term bonds. With one-period bonds, debt thresholds are lower, and the deleveraging region wider. Deleveraging is now optimal over more than 85% of the region between $\bar{B}(0)_{EX}$ and $\bar{B}(0)_\pi$; running deficits remains optimal over about 10% of the region. Investors systematically price the rollover crisis risk, with sharp adjustment across each threshold. The price incentive strengthens the incentive to adopt prudent policies and keep debt on a declining path.

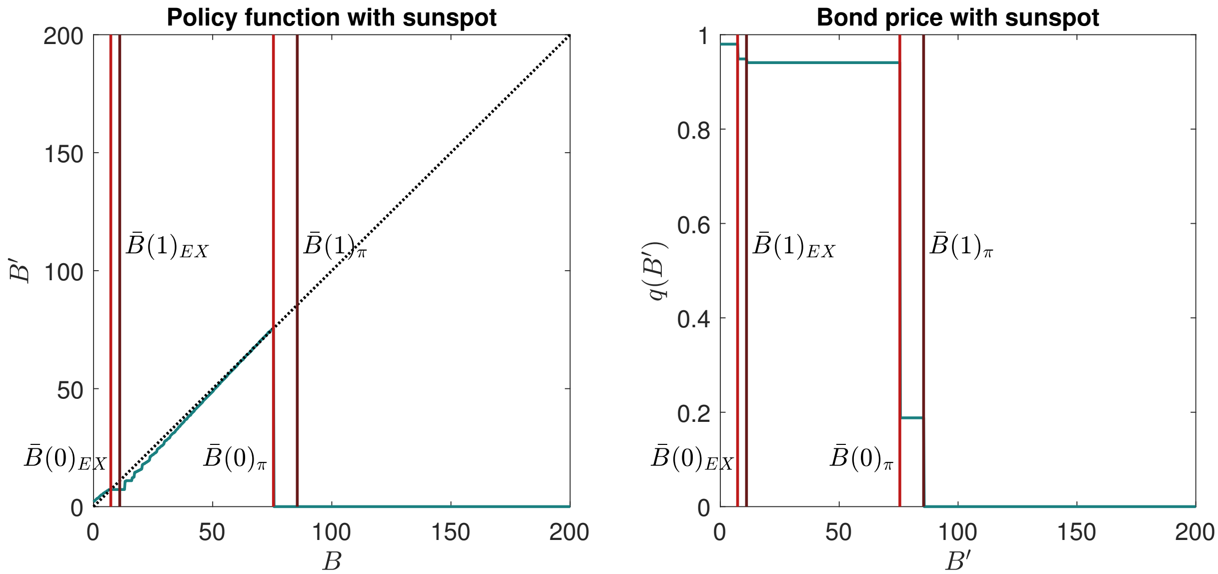


**Figure 9:** $\delta = 1$ and $\rho \in \{opt, EX\}$

# 7 Resilience to Self-Fulfilling Crises

We conclude our study with a close up analysis of a country's vulnerability to self-fulfilling debt crises, depending on the maturity of its public debt and the depth and expected persistence of a downturn. To do so, we carry out extensive sensitivity analysis, as well as by comparing our baseline with another standard model in the literature, the debt-limit model. In the text, we focus on the cases of optimistic and pessimistic beliefs.[26] Our specific goal is to highlight conditions under which self-fulfilling debt crises, especially in the form of rollover crises, may/may not occur.

## 7.1 Baseline Model

For our baseline model, in Figure 10 we plot debt tolerance thresholds in a recession against variable debt maturity (left panel) and probability of recovery (right panel).

Starting from the left panel of Figure 10, we first note that $\bar{B}(0)_{opt}$ (solid line) is largely insensitive to debt maturity, but for extremely small values of $\delta$ ($\delta \to 0$), corresponding to very long-term debt. When investors are optimistic, as long as the government keeps debt below $\bar{B}(0)_{opt}$ (so that it can borrow at the risk-free rate), long-term and short-term debt are basically equivalent. When $\delta \to 0$, however, $\bar{B}(0)_{opt}$ increases somewhat. Intuitively,

[26] To save space, we do not include the case of extreme beliefs in this section. We note that, under these beliefs, a longer maturity and/or a higher probability of recovery have the strongest effect on the threshold $\bar{B}(0)_{EX}$ conditional on a recession. Changing these parameters, everything else equal, raises this threshold much more in percentage terms compared to the shift in $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$.

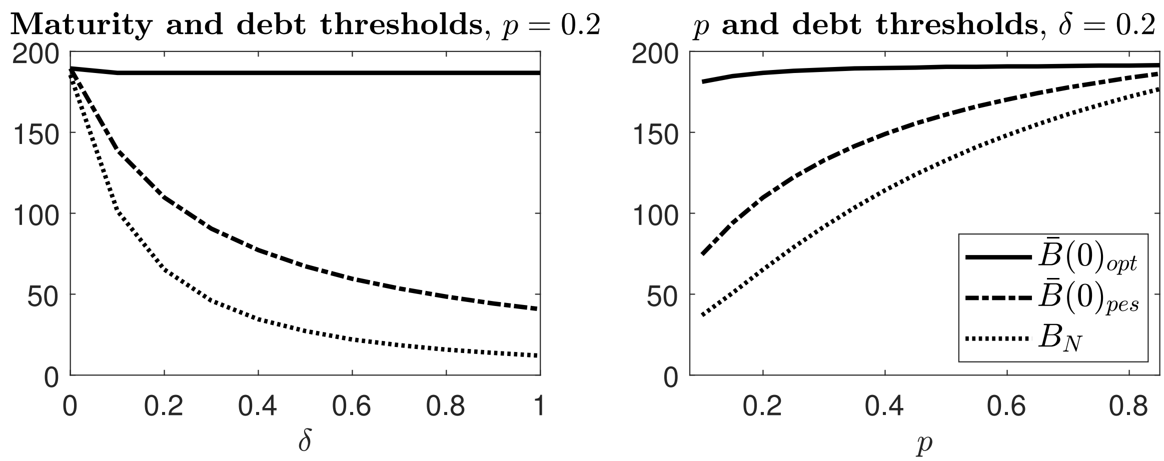


**Figure 10:** Debt thresholds in the baseline model given $A = 0.9$

when debt maturity is very long, even if the interest rate on new issuance is high—raising borrowing costs at the margin—, high rates deteriorate the value of outstanding bonds harming investors. The government benefits from 'diluting' existing debt. When the amount of outstanding debt is large enough, the benefit from debt dilution outweighs the higher borrowing costs: a government has an incentive to issue debt above the threshold $\bar{B}(0)_{opt}$. A high-debt high-risk issuance strategy can yield higher benefit/utility than a low-debt low-risk debt issuance strategy (i.e., $V^R_{opt,1} < V^R_{opt,2}$ in (3) and (4)). In the figure, this is the case for the limit case of consols, where, consistent with Proposition 2, $\bar{B}(0)_{opt}$ equals to $\bar{B}(0)_{pes}$. We will expand on the conditions for, and the meaning of overlapping thresholds in the next subsection.

In contrast, $B_N$ and $\bar{B}(0)_{pes}$ (dotted and dashed lines in the figure) never overlap, and both decrease sharply with $\delta$; that is, they increase with a longer maturity of debt. To see why, consider the net bond revenue in a pessimistic world, $\beta p(B' - (1-\delta)B) - \kappa B$, where $\beta p B'$, $\beta p(1-\delta)B$ and $\kappa B$ denote, respectively, revenue from newly issued bonds, the value of the outstanding stock of bonds, and interest payment to investors. Maturity has opposite effects on these terms. As the maturity of bonds becomes longer ($\delta \downarrow$), the value of the outstanding stock of bonds $\beta p(1-\delta)B$ rises but the interest payments due in the period $\kappa B$ fall. The first effect decreases, while the second effect increases the net bond revenue. Rearranging the net revenue equation as follows $\beta p B' - [1 - \beta(1-p)(1-\delta)]B$ makes it clear that the second effect always dominates the first one: a fall in $\delta$ unambiguously increases net debt revenue—explaining why $\bar{B}(0)_{pes}$ and $B_N$ are larger as the debt maturity becomes longer.[27]

[27] One could note that, when investors have pessimistic beliefs, an official swap of short-term bonds for long-term bonds may improve the debt tolerance threshold of a country (a point discussed is detailed in Corsetti et al. (2017)).

To gauge whether longer debt maturities can eliminate multiplicity, let $\delta \to 0$. In the left-hand side panel, as debt maturity grows, the distance between $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$—the fast crisis region—becomes narrower, up to disappearing in the limit case of consols. On the contrary, the distance between $B_N$ and $\bar{B}(0)_{pes}$—the "slow-moving" crisis zone—remains approximately unchanged. It follows that, while fast crises disappear with very long bond maturities, slow-moving crises are always a possibility, as $B_N$ never coincides with $\bar{B}(0)_{opt}$. Not even maturities approaching "consols" can rule out the multiplicity—slow-moving crises remain pervasive.

The right panel of Figure 10 shows that the probability of recovery $p$ does not have much of an effect on $\bar{B}(0)_{opt}$, while it has a significant impact on both $B_N$ and $\bar{B}(0)_{pes}$. The net bond revenue in an optimistic world, $\beta(B' - (1-\delta)B) - \kappa B$, does not vary with $p$, while the net bond revenue in a pessimistic world, $\beta p(B' - (1-\delta)B) - \kappa B$, is unambiguously increasing in $p$. A higher probability of recovery $p$ significantly narrows the "fast" crisis zone. It also narrows, but to a lesser extent, the "slow-moving" crisis zone.

## 7.2 A Debt-Limit Model Extension

To conclude our sensitivity analysis, we consider an extension of our model encompassing a framework widely discussed in the literature under the headline of "debt-limit" (as opposed to "strategic default") model, see Lorenzoni and Werning (2019) for a recent leading contribution. In this framework, the government is assumed to be willing to exhaust all possibilities of adjustment before repudiating the debt (which implies a permanent fall in the costly low-output financial-autarky state). The default condition is therefore assessed at period-by-period maximum adjustment in the primary surplus, denoted with $\mathcal{PS}^{Max}$, as follows:

$$\mathcal{PS}^{Max} + \max_{B'}\{q(B', s)(B' - (1-\delta)B)\} < \kappa B \tag{11}$$

A detailed analysis of the debt-limit model is given in Appendix G, where we show how it can be derived as a variant of our baseline. Hereafter, we report the results of our sensitivity analysis varying debt maturity and probability of a recovery. Results are shown in Figure 11.

Looking at the left-hand panel of Figure 11, note that $\bar{B}(0)_{opt}$ remains insensitive to debt maturity only for relatively short maturities—for a $\delta$ higher than 0.57. As $\delta$ falls below this value, i.e., for longer maturities, all thresholds are increasing and, most crucially, $\bar{B}(0)_{opt}$ coincides with $\bar{B}(0)_{pes}$—different from the left panel of Figure 10, these two thresholds overlap away from the limit case of consols. We have seen above that, with long-term debt, dilution of existing bond holders strengthens the government incentives to borrow: issuing more debt imparts a capital loss on investors and the pass-through of higher borrowing

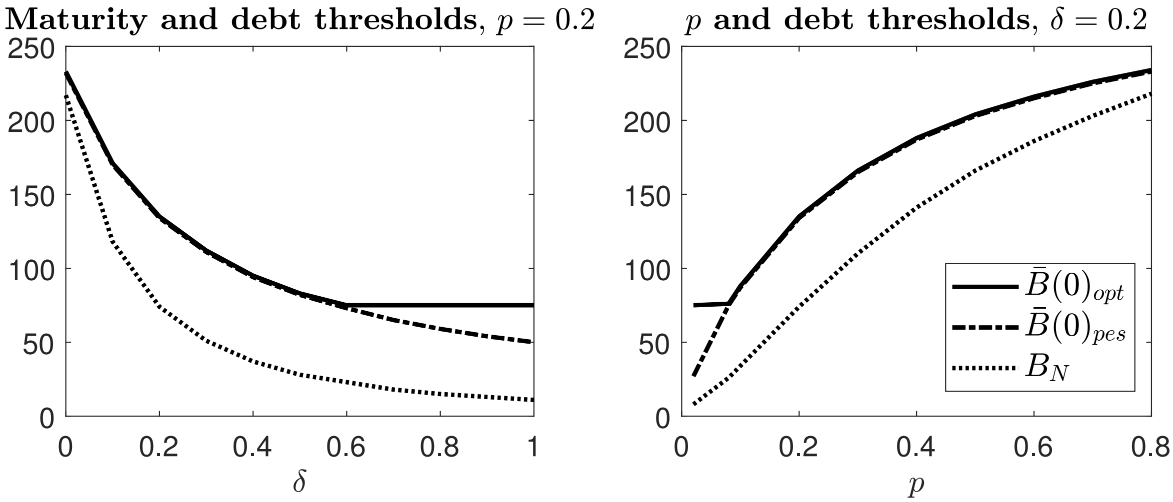


**Figure 11:** Debt thresholds in the debt-limit framework given $A = 0.9$

costs onto the interest bill is slow. Indeed, with long bond maturities and a high stock of outstanding liabilities, issuing (more) debt at the risky price may yield more revenue than issuing (less) debt at the risk-free price, even if beliefs are optimistic. It is therefore possible that, independently of the regime of beliefs prevailing in the market, a government may avoid immediate default only by issuing a large amount of risky debt—when the alternative of keeping issuance below the safe-debt threshold would not generate sufficient revenue to satisfy its gross financing need. In this case, of course, avoiding an immediate crisis comes at the cost of putting debt on a trajectory that foreshadows a crisis in the future if the recession persists. Similar considerations apply in relation to the probability of recovery; sensitivity to this parameter is depicted in the right panel of Figure 11. The two thresholds overlap and the government starts issuing risky debt regardless of beliefs regime at a relatively low level of $p$.

Overall, Figure 11 shows that, in the debt-limit framework, $\bar{B}(0)_{opt}$ coincides with $\bar{B}(0)_{pes}$ over a large region of parameters—both thresholds increase as debt maturity lengthens ($\delta$ falls) and the probability of recovery rises. As detailed in the Appendix G.3, when the two thresholds coincide, multiple equilibria are still possible and the beliefs regime makes a difference at intermediate debt levels: bond prices are lower and debt accumulates faster when inverstors are pessimistic. However, as debt approaches the thresholds, the government eventually adopts the same risky-debt issuance strategy regardless of the beliefs regime prevailing in the market. When this occurs, the equilibrium is unique and fast crises are no longer a possibility. With overlapping thresholds, therefore, the multiplicity region is narrower than the region between $B_N$ and $\bar{B}(0)_{pes} = \bar{B}(0)_{opt}$ and, most crucially, belief-driven crises can only occur in the form of slow-moving crises. We should stress that this result is not specific to the debt-limit model. As shown in Appendix I, $\bar{B}(0)_{opt}$ may coincide with $\bar{B}(0)_{pes}$ also in our baseline specification, but the conditions are more stringent than the debt-limit framework à la Lorenzoni and Werning (2019).[28]

A key conclusion of our sensitivity analysis is that, for intermediate debt maturities and non-negligible probability of recovery, rollover crises are still possible in our baseline, but not in the debt-limit counterpart. Yet, multiplicity remains pervasive for *all debt maturities* in both models—a long debt maturity may eliminate the possibility of fast (rollover) crises at high levels of debt (especially in a debt-limit model), but not the risk of slow-moving ones.[29]

# 8 Conclusion

The literature has long emphasized that, once a country debt is sufficiently high, the equilibrium is no longer unique and the country is vulnerable to disruptive self-fulfilling crises. As the COVID-19 pandemic is causing widespread economic crises across the globe, it is unavoidable that debt stocks rise virtually everywhere, potentially undermining stability in the bond markets in advanced countries and raising issues in which instruments are available to keep these markets in a "good equilibrium".

This paper shows that different types of self-fulfilling crises, one emphasized by Calvo (1988), the other by Cole and Kehoe (2000), may occur in the same dynamic Calvo (1988) setting. In particular, both slow-moving debt crises and rollover crises are possible when investors coordinate on what we dub "pessimistic" beliefs, while rollover crises are a form of self-fulfilling debt crises specific to "extreme" beliefs.

We revisit debt dynamics and the incentive to deleverage when governments operate under the threat of self-fulfilling debt crisis. This is an important issue, that may dominate debates on fiscal policy in the post-COVID, high-debt regime. Under the threat of rollover crises driven by investors coordinating on "extreme" beliefs, in line with the literature, we find that a forward-looking benevolent government would optimally reduce its debt even in a recession. As a contribution to the literature, however, we also show that, if crises are anticipated to be slow-moving—driven by "pessimistic" beliefs—, deleveraging is optimal only over a relatively small range of debt. We stress that this result is obtained independently of political economy considerations, with policymakers modelled as short-sighted or self-interested. Even for forward-looking benevolent governments, the threat of slow-moving

[28] The parameter restrictions that rule out "fast" debt crises in our baseline are: the country is in a deep recession, the probability of recovery is high, and debt maturity is sufficiently long. These results are discussed in Appendix I.

[29] Differences in our results relative to Lorenzoni and Werning (2019) originate from a different assumption concerning output uncertainty. In Lorenzoni and Werning (2019), output is drawn from normal distribution with a single peak—which in their debt-limit framework rules out multiple equilibria for one-period bonds. Our model, instead, features a bimodal distribution for which, as explained in footnote 7, stable equilibria exist at high interest rates for all debt maturities.

debt crises is generally not enough to motivate precautionary fiscal policy of risk reduction. This suggests that, after a crisis causing an abrupt rise in borrowing, debt may remain at high levels for a long time, even if governments are aware that their failure to deleverage keeps their country exposed to the threat of belief-driven crises. In light of cross-border contagion effects undermining stability at global level, there is an argument for international fiscal compacts associated with institutional liquidity provision.

As a direction for future research, theory can be brought to bear on debt sustainability when the government can choose the maturity of its debt and rely on external bailouts or liquidity assistance. The logic of our model suggests that the way debt management and/or official support can impinge on the dynamics of debt and vulnerability to crisis crucially rests on how these policies are able to affect the incentives of a government to gamble on prospective recovery—which, as we have shown, depends on the regime of beliefs. A key question is under what conditions maturity swaps and bailouts may help and speed up deleveraging—as opposed to give an extra incentive to smooth adjustment by borrowing. Bailouts may create a trade-off between resilience to rollover crises and vulnerability to default at high levels of debt. Understanding this trade-off is crucial in the design of an efficient governance of official lending institutions.

# References


Aguiar, M. and M. Amador (2014). Sovereign Debt. In *Handbook of International Economics*, Volume 4, Chapter 11, pp. 647–687. Elsevier B.V.

Aguiar, M. and M. Amador (2020). Self-Fulfilling Debt Dilution: Maturity and Multiplicity in Debt Models. *American Economic Review 110*(9), 2783–2818.

Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2013). Crisis and commitment: Inflation credibility and the vulnerability to sovereign debt crises. *NBER Working Paper No. 19516*.

Aguiar, M., M. Amador, E. Farhi, and G. Gopinath (2015). Coordination and Crisis in Monetary Unions. *Quarterly Journal of Economics 130*(4), 1727–1779.

Aguiar, M., M. Amador, H. Hopenhayn, and I. Werning (2019). Take the Short Route: Equilibrium Default and Debt Maturity. *Econometrica 87*(2), 423–462.

Aguiar, M., S. Chatterjee, H. Cole, and Z. Stangebye (2016). Quantitative Models of Sovereign Debt Crises. In *Handbook of Macroeconomics*, Chapter 21, pp. 1697–1755. Elsevier B.V.

Aguiar, M., S. Chatterjee, H. L. Cole, and Z. Stangebye (2020). Self-Fulfilling Debt Crises, Revisited. *PIER Working Paper 20-003*.

Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics 69*, 64–83.

Alesina, A., M. D. Broeck, A. Prati, G. Tabellini, M. Obstfeld, and S. Rebelo (1992). Default Risk on Government Debt in OECD Countries. *Economic Policy*, 428–463.

Alesina, A., A. Prati, and G. Tabellini (1989). Public confidence and debt management: a model and a case study of Italy. *NBER Working Paper No. 3135*.

Araujo, A., M. Leon, and R. Santos (2013). Welfare Analysis of Currency Regimes with Defaultable Debts. *Journal of International Economics 89*, 143–153.

Arellano, C. (2008). Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review 98*(3), 690–712.

Arellano, C. and A. Ramanarayanan (2012). Default and the Maturity Structure in Sovereign Bonds. *Journal of Political Economy 120*(2).

Auclert, A. and M. Rognlie (2016). Unique equilibrium in the Eaton-Gersovitz model of sovereign debt. *Journal of Monetary Economics 84*, 134–146.

Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2018). Sovereign Default: The Role of Expectations. *Journal of Economic Theory 175*, 803–812.

Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2019). Self-Fulfilling Debt Crises with Long Stagnations. *unpublished manuscript*.

Bacchetta, P., E. Perazzi, and E. V. Wincoop (2018). Self-fulfilling debt crises: What can monetary policy do ? *Journal of International Economics 110*, 119–134.

Bianchi, J., J. C. Hatchondo, and L. Martinez (2018). International Reserves and Rollover Risk. *American Economic Review 108*(9), 2629–2670.

Bocola, L., G. Bornstein, and A. Dovis (2019). Quantitative Sovereign Default Models and the European Debt Crisis. *Journal of International Economics 118*, 20–30.

Bocola, L. and A. Dovis (2019). Self-Fulfilling Debt Crises: A Quantitative Analysis. *American Economic Reiwew 109*(12), 4343–4377.

Brutti, F. and P. Saure (2015). Transmission of sovereign risk in the euro crisis. *Journal of International Economics 97*(2), 231–248.

Calvo, G. A. (1988). Servicing the Public Debt: The Role of Expectations. *American Economic Review 78*(4), 647–661.

Chamon, M. (2007). Can Debt Crises be Self-fulfilling? *Journal of Development Economics 82*(1), 234–244.

Chatterjee, S., D. Corbae, K. Dempsey, and J. Rios-Rull (2016). A Theory of Credit Scoring and the Competitive Pricing of Default Risk. *Working Paper*.

Chatterjee, S. and B. Eyigungor (2012). Maturity, Indebtedness and Default Risk. *American Economic Review 102*(6), 2674–2699.

Chatterjee, S. and B. Eyigungor (2019). Endogenous Political Turnover and Fluctuations in Sovereign Default Risk. *Journal of International Economics 117*, 37–50.

Cole, H. L. and T. J. Kehoe (1996). A Self-Fulfilling Model of Mexico's 1994–1995 Debt Crisis. *Journal of International Economics 41*(3-4), 309–330.

Cole, H. L. and T. J. Kehoe (2000). Self-Fullfiling Debt Crises. *The Review of Economic Studies 67*(1), 91–116.

Collard, F., M. Habib, and J. C. Rochet (2015). Sovereign Debt Sustainability in Advanced Economies. *Journal of the European Economic Association 13*(3), 381–420.

Conesa, J. C. and T. J. Kehoe (2014). Is it too late to Bail Out the Troubled Countries in the Eurozone? *American Economic Review: Papers & Proceedings 104*(5), 88–93.

Conesa, J. C. and T. J. Kehoe (2017). Gambling for Redemption and Self-Fulfilling Debt Crises. *Economic Theory 64*(4), 707–740.

Corsetti, G. and L. Dedola (2016). The Mystery of the Printing Press: Monetary Policy and Self-Fulfilling Debt Crises. *Journal of the European Economic Association 14*(6), 1329–1371.

Corsetti, G., A. Erce, and T. Uy (2017). Debt Sustainability and the Terms of Official Support. *ADEMU Working Paper*.

Corsetti, G., A. Erce, and T. Uy (2020). Official Sector Lending Strategies During the Euro Area Crisis. *The Review of International Organisations 15*, 667–705.

Cruces, J. J. and C. Trebesch (2013). Sovereign Defaults: The Price of Haircuts. *American Economic Journal: Macroeconomics 5*(3), 85–117.

Eaton, J. and M. Gersovitz (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *The Review of Economic Studies 48*(2), 289–309.

Galli, C. (2019). Self-Fulfilling Debt Crises, Fiscal Policy and Investment. *unpublished manuscript*.

Ghosh, A. R., J. I. Kim, E. G. Mendoza, J. D. Ostry, and M. S. Qureshi (2013). Fiscal Fatigue, Fiscal Space and Debt Sustainability in Advanced Economies. *Economic Journal 123*(566), 4–30.

Giavazzi, F. and M. Pagano (1989). Confidence Crises and Public Debt Management. In *Public Debt Management: Theory and History.*

Grauwe, P. D. (2011). The European Central Bank : Lender of Last Resort in the Government Bond Markets? *CESifo Working Paper Series 3569*.

Hatchondo, J. C. and L. Martinez (2009). Long-Duration Bonds and Sovereign Defaults. *Journal of International Economics 79*(1), 117–125.

Lizarazo, S. V. (2013). Default Risk and Risk Averse International Investors. *Journal of International Economics 89*(2), 317–330.

Lorenzoni, G. and I. Werning (2019). Slow Moving Debt Crises. *American Economic Review 109*(9), 3229–3263.

Marin, E. (2017). Self-fulfilling Crises and Country Solidarity. *unpublished manuscript*.

Mendoza, E. G. and V. Z. Yue (2012). A General Equilibrium Model of Sovereign Default and Business Cycles. *Quarterly Journal of Economics 127*(2), 889–946.

Paluszynski, R. (2019). Learning About Debt Crisis. *unpublished manuscript*.

Roch, F. and H. Uhlig (2018). The Dynamics of Sovereign Debt Crises and Bailouts. *Journal of International Economics 114*, 1–13.

Stangebye, Z. (2020). Belief Shocks and Long-Maturity Sovereign Debt. *Journal of International Economics 127*.

Tirole, J. (2015). Country Solidarity in Sovereign Crises. *American Economic Review 105*(8), 2333–2363.

Yue, V. Z. (2010). Sovereign Default and Debt Renegotiation. *Journal of International Economics 80*(2), 176–187.

# Appendices

## A Derivation of $V^R_{pes}(B,0)$

Conditional on pessimistic beliefs, investors offer the default-risky bond price $\beta p$. To derive the relevant debt threshold for these beliefs to be self-validating, as in the text we write $V^R_{pes}(B,0) = \max\{V^R_{pes,1}(B,0), V^R_{pes,2}(B,0)\}$, where:

$$
\begin{aligned}
V^R_{pes,1}(B,0) = \max_{0 \leq B' \leq \bar{B}(0)_{pes}} \quad & \mathcal{U}(c,g) + \beta\Big(pV(B',1,\rho=pes) + (1-p)V(B',0,\rho=pes)\Big) \\
\text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\
& c = (1-\tau)A\bar{y}
\end{aligned} \tag{12}
$$

$$
\begin{aligned}
V^R_{pes,2}(B,0) = \max_{\bar{B}(0)_{pes} < B' \leq \bar{B}(1)} \quad & \mathcal{U}(c,g) + \beta\Big(pV(B',1,\rho=pes) + (1-p)V^D(0)\Big) \\
\text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\
& c = (1-\tau)A\bar{y}
\end{aligned} \tag{13}
$$

Different from (3) and (4), under pessimistic beliefs, the government assesses its value function taking a default-risky price for its bonds, i.e., $\beta p$. Depending on initial debt level, pessimistic beliefs may/may not be self-validating (see the graphical analysis in Section 3.3).

## B The algorithm for computing value functions

### B.1 Baseline

The algorithm computes two debt thresholds in an optimistic world, and three debt thresholds in a pessimistic world.

#### B.1.1 Optimistic beliefs

1. Compute the debt tolerance threshold in normal times $\bar{B}(1)$ by solving:

$$\frac{\mathcal{U}((1-\tau)\bar{y}, \tau\bar{y} - (1-\beta)\bar{B}(1))}{1-\beta} = V^D(1)$$

After the economy recovers, the government's optimization problem is deterministic.

Thus, the value function in normal times can be characterized by

$$V(B, a=1) = \begin{cases} \frac{\mathcal{U}((1-\tau)\bar{y}, \tau\bar{y}-(1-\beta)B)}{1-\beta} & \text{if } 0 \le B \le \bar{B}(1) \\ V^D(1) & \text{if } \bar{B}(1) < B \end{cases}$$

2. Guess initial values for the threshold $\bar{B}(0)_{opt}$ and the bond price function $\tilde{q}_{opt}(B', 0)$ in a recession.

3. Given the bond price function $\tilde{q}_{opt}(B', 0)$ and $\bar{B}(0)_{opt}$, guess the value function $\tilde{V}(B, 0)$ in an optimistic world. Perform value function iteration and update initial guess until it satisfies convergence criterion $\max_B \left|V(B,0) - \tilde{V}(B,0)\right| < \epsilon$.

   $V(B,0) = \max\{V^R_{opt,1}(B,0), V^R_{opt,2}(B,0), V^D(0)\}$, where

$$\begin{aligned} V^R_{opt,1}(B,0) = \quad & \max_{0 \le B' \le \bar{B}(0)_{opt}} \quad \mathcal{U}(c,g) + \beta\Big(pV(B',1) + (1-p)\tilde{V}(B',0)\Big) \\ & \text{s.t.} \quad g + \kappa B = \tau A\bar{y} + \tilde{q}_{opt}(B',0)\big(B' - (1-\delta)B\big), \\ & \qquad\quad c = (1-\tau)A\bar{y} \end{aligned}$$

$$\begin{aligned} V^R_{opt,2}(B,0) = \quad & \max_{\bar{B}(0)_{opt} < B' \le \bar{B}(1)} \quad \mathcal{U}(c,g) + \beta\Big(pV(B',1) + (1-p)V^D(0)\Big) \\ & \text{s.t.} \quad g + \kappa B = \tau A\bar{y} + \beta p(B' - (1-\delta)B), \\ & \qquad\quad c = (1-\tau)A\bar{y} \end{aligned}$$

4. Derive a new value of $\bar{B}(0)_{opt}$ by solving:

$$V(\bar{B}(0)_{opt,new}, 0) = V^D(0)$$

5. Update the bond price function and compute the error. New values of $q_{opt}(B', 0)$ are

$$q_{opt}(B',0) = \begin{cases} \beta\Big(p + (1-p)\big(\kappa + (1-\delta)\tilde{q}_{opt}\big(B'(B',0),0\big)\big)\Big) & \text{if } 0 \le B' \le \bar{B}(0)_{opt} \\ \beta p & \text{if } \bar{B}(0)_{opt} < B' \le \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

6. If $\max_{B'} \left|\tilde{q}_{opt}(B',0) - q_{opt}(B',0)\right| > \epsilon$ or/and $\left|\bar{B}(0)_{opt} - \bar{B}(0)_{opt,new}\right| > \epsilon$, set $\tilde{q}_{opt}(B',0) = q_{opt}(B',0)$ and $\bar{B}(0)_{opt} = \bar{B}(0)_{opt,new}$, and go back to 3. Else, start to solve the equilibrium problem in a pessimistic world.

### B.1.2 Pessimistic beliefs

1. Repeat A.1.1 step 1.

2. Guess initial values for the threshold $\bar{B}(0)_{pes}$ and derive value function $V^R_{pes,2}(B,0)$.

$$\begin{aligned} V^R_{pes,2}(B,0) = \max_{\bar{B}(0)_{pes} < B' \leq \bar{B}(1)} \quad & \mathcal{U}(c,g) + \beta\Big(pV(B',1) + (1-p)V^D(0)\Big) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\ & c = (1-\tau)A\bar{y} \end{aligned}$$

3. According to the proof of Proposition 1 in Appendix C, we have $V^R_{pes,2}(\bar{B}(0)_{pes},0) > V^R_{pes,1}(\bar{B}(0)_{pes},0)$. Hence, derive $\bar{B}(0)_{pes}$ by solving:

$$V^R_{pes,2}(\bar{B}(0)_{pes},0) = V^D(0)$$

4. Guess the value function in a pessimistic world $\tilde{V}(B,0)$ and the threshold $B_N$.

5. Guess the bond price function $\tilde{q}_{pes}(B',0)$ in a recession. Perform value function iteration and update initial guess until it satisfies convergence criterion $\max_B \big|V(B,0) - \tilde{V}(B,0)\big| < \epsilon$.

$$V(B,0) = \begin{cases} V_{safe}(B,0) & \text{if } 0 \leq B \leq B_N \\ V^R_{pes,2}(B,0) & \text{if } B_N < B \leq \bar{B}(0)_{pes} \\ V^D(0) & \text{if } \bar{B}(0)_{pes} < B \end{cases}$$

   where

$$\begin{aligned} V_{safe}(B,0) = \max_{0 \leq B' \leq B_N} \quad & \mathcal{U}(c,g) + \beta\Big(pV(B',1) + (1-p)\tilde{V}(B',0)\Big) \\ \text{s.t.} \quad & g + \kappa B = \tau A\bar{y} + \tilde{q}_{pes}(B',0)\big(B' - (1-\delta)B\big), \\ & c = (1-\tau)A\bar{y} \end{aligned}$$

6. Compute the government utility and policy function given the bond price $q(B',0) = \beta p$, denoted as $V^R_{pes}(B,0) = \max\{V^R_{pes,1}(B,0), V^R_{pes,2}(B,0)\}$ and $B'_{pes}(B,0)$, respectively.

$$\begin{aligned} V^R_{pes,1}(B,0) = \max_{0 \leq B' \leq \bar{B}(0)_{pes}} \quad & \mathcal{U}(c,g) + \beta\Big(pV(B',1) + (1-p)\tilde{V}(B',0)\Big) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\ & c = (1-\tau)A\bar{y} \end{aligned}$$

7. Derive a new value of $B_N$ by solving equation below.

$$B_{N,new} = \sup_{B}\{B'_{pes}(B,0) \leq \bar{B}(0)_{pes}\}$$

8. Update the bond price function and compute the error. New values of $q_{pes}(B',0)$ are

$$q_{pes}(B',0) = \begin{cases} \beta\Big(p + (1-p)\big(\kappa + (1-\delta)\tilde{q}_{pes}\big(B'(B',0),0\big)\big)\Big) & \text{if } 0 \leq B' \leq B_N \\ \beta\Big(p + (1-p)\big(\kappa + (1-\delta)\beta p\big)\Big) & \text{if } B_N < B' \leq \bar{B}(0)_{pes} \\ \beta p & \text{if } \bar{B}(0)_{pes} < B' \leq \bar{B}(1) \\ 0 & \text{if } \bar{B}(1) < B' \end{cases}$$

9. If $\max\limits_{B'}\big|\tilde{q}_{pes}(B',0) - q_{pes}(B',0)\big| > \epsilon$ or/and $\big|B_N - B_{N,new}\big| > \epsilon$, then update values: $\tilde{q}_{pes}(B',0) = q_{pes}(B',0)$, $B_N = B_{N,new}$, and go back to 5. Else, exit.

## B.2 Debt thresholds in the debt-limit version of the model

### B.2.1 Optimistic beliefs

1. Derive the debt threshold in normal times. $\bar{B}(1)$ can be characterized by

$$\bar{B}(1) = \frac{\tau\bar{y} - \bar{g}}{1-\beta}$$

2. Derive the debt threshold in a recession.

$$\bar{B}(0)_{opt} = \max\left\{\frac{\tau A\bar{y} - \bar{g}}{1-\beta}, \frac{\tau A\bar{y} - \bar{g} + \beta p\bar{B}(1)}{1-\beta(1-p)(1-\delta)}\right\}$$

3. The methodology to derive the government policy choice and the bond price schedule is similar to the one followed for our baseline, but in two respects. First, in the debt-limit framework, debt thresholds can be computed directly without any iterative procedure. Second, the utility of repaying on finite discretized space in a simulation must be larger than the utility of defaulting. For instance, when sovereign bond space $[0, 200]$ is discretized into finite grid points, the utility of defaulting must be low enough (e.g. $V_d(0) = -99999999$) so that the utility of repaying in discretized space is always larger than the utility of defaulting. Computationally, defaulting is never an optimal choice in a simulation—this rules out strategic default.

#### B.2.2 Pessimistic beliefs

1. Repeat A.2.1 step 1.

2. Derive the debt threshold in a recession.

$$\bar{B}(0)_{pes} = \frac{\tau A\bar{y} - \bar{g} + \beta p\bar{B}(1)}{1 - \beta(1-p)(1-\delta)}$$

3. Set $V_d(0) = -99999999$ and use the debt thresholds derived in stage 1 and 2 to solve the equilibrium.

# C Proofs

## C.1 Proof of Proposition 1

*Proof.* To prove that $\mathbb{B}_{pes}$ is non-empty, we only need to show that there exists some debt level that simultaneously satisfies $V^R_{pes} = \max\{V^R_{pes,1}, V^R_{pes,2}\} = V^R_{pes,2}$ and $V^R_{pes,2} \geq V^D(0)$, i.e., the government chooses to issue debt above the threshold $\bar{B}(0)_{pes}$ and repay the existing debt at the default-risky rate.

Posit that, for some $\tilde{B} \leq \bar{B}(0)_{pes}$, the critical expenditure $\bar{g}$ is large enough to exceed $\tau A\bar{y} - \tilde{B} + \beta p\bar{B}(0)_{pes}$. $\bar{B}(0)_{pes}$ is derived by solving the following equation (see $V^R_{pes,2}$ in (13)):

$$V^R_{pes,2}(\bar{B}(0)_{pes}, 0) = V^D(0) \tag{14}$$

For $B \in [\tilde{B}, \bar{B}(0)_{pes}]$, since $\bar{g} > \tau A\bar{y} - \tilde{B} + \beta p\bar{B}(0)_{pes}$, if the government issued bonds below $\bar{B}(0)_{pes}$ at the risky rate (see $V^R_{pes,1}$ in (12)), it would have to cut spending below $\bar{g}$.

Hence, it must be the case that $V^R_{pes}(B,0) = V^R_{pes,2}(B,0)$ for all $B \in [\tilde{B}, \bar{B}(0)_{pes}]$. The amount of newly issued bonds $B'$ in this range must be unambiguously larger than $\bar{B}(0)_{pes}$, validating the pessimistic beliefs.

It follows that, when bonds trade at the default-risky price, a sufficient condition for a non-empty set $\mathbb{B}_{pes}$ is a large enough $\bar{g}$. Note that the proof above also implies that $\bar{B}(0)_{pes}$ is pinned down by solving the equation $V^R_{pes,2}(\bar{B}(0)_{pes}, 0) = V^D(0)$, as referred to the computation algorithm. See Appendix B for details.

□

## C.2 Proof of Proposition 2

*Proof.* First, we prove that $\bar{B}(0)_{opt}$ is not smaller than $\bar{B}(0)_{pes}$. By Proposition 1, when $B = \bar{B}(0)_{pes}$, in a pessimistic world the government always borrows into a default-risky level (above $\bar{B}(0)_{pes}$ but below $\bar{B}(1)$). $\bar{B}(0)_{pes}$ is characterized by solving the equation below (see $V^R_{pes,2}$ in (13)):

$$V^R_{pes,2}(\bar{B}(0)_{pes}, 0) = V^D(0)$$

In an optimistic world, instead, when the initial debt level is $\bar{B}(0)_{opt}$, the government can either borrow into a risky level (above $\bar{B}(0)_{opt}$ but below $\bar{B}(1)$), or keep the issuance at safe levels (not larger than $\bar{B}(0)_{opt}$).

If the government adopts the risky issuance strategy, $\bar{B}(0)_{opt}$ is characterized by solving:

$$V^R_{opt,2}(\bar{B}(0)_{opt}, 0) = V^D(0)$$

(see $V^R_{opt,2}$ in (4)). It is easy to verify that $V^R_{opt,2}$ and $V^R_{pes,2}$ represent the same optimization problem and therefore $\bar{B}(0)_{opt} = \bar{B}(0)_{pes}$.

If the government adopts the safe issuance strategy, it must be the case that this enhances debt sustainability relative to the risky one (see $V^R_{opt,1}$ in (3)):

$$V^R_{opt,1}(\bar{B}(0)_{pes}, 0) > V^R_{opt,2}(\bar{B}(0)_{pes}, 0) = V^D(0)$$

$\bar{B}(0)_{opt}$ in this case is unambiguously larger than $\bar{B}(0)_{pes}$. □

## C.3 Proof of Proposition 3

*Proof.* Rewrite $V^R_{pes,1}(B, 0)$ when $B = \bar{B}(0)_{EX}$:

$$\begin{aligned} V^R_{pes,1}(\bar{B}(0)_{EX}, 0) = \max_{0 \leq B' \leq \bar{B}(0)_{pes}} \quad & \mathcal{U}(c, g) + \beta\Big(pV(B', 1, \rho = pes) + (1-p)V(B', 0, \rho = pes)\Big) \\ \text{s.t.} \quad & g + B = \tau A\bar{y} + \beta p B', \\ & c = (1-\tau)A\bar{y} \end{aligned}$$

If we set the choice variable $B'$ to zero, the value of $V^R_{pes,1}(\bar{B}(0)_{EX}, 0)$ is equal to $V^R_{EX}(\bar{B}(0)_{EX}, 0)$ (see $V^R_{EX}$ in (10)). However, the optimal choice of $B'$ is unambiguously positive in $V^R_{pes,1}(\bar{B}(0)_{EX}, 0)$. By strict concavity of $\mathcal{U}$, this implies that $V^R_{pes,1}(\bar{B}(0)_{EX}, 0)$ is larger than $V^R_{EX}(\bar{B}(0)_{EX}, 0)$. Hence, $\bar{B}(0)_{pes} > \bar{B}(0)_{EX}$. □

## C.4 Proposition Characterizing $\bar{B}(0)_{EX}$

Denote with $\mathbb{B}_{EX}$ the set of initial debt level that validates "extreme belief", defined as follows

$$\mathbb{B}_{EX} \equiv \{B \mid V^R_{EX}(B, 0) < V^D(0)\}$$

The following proposition shows that $\bar{B}(0)_{EX}$ is always positive:

**Proposition 4.** *A strictly concave utility function with the property* $\lim_{g \to \bar{g}} \mathcal{U}(c, g) = -\infty$ *ensures* $\bar{B}(0)_{EX}$ *is positive.*

*Proof.* Define $\mathcal{E}(B) \equiv V^R_{EX}(B, 0) - V^D(0)$. Since $V^D(0)$ is a constant, by the properties of the value of repaying debt, $\mathcal{E}(B)$ is a continuous and monotonically decreasing function. Given that $\lim_{g \to \bar{g}} \mathcal{U}(c, g) = -\infty$, by the intermediate value theorem the following inequalities imply existence and the uniqueness of a debt threshold $\bar{B}(0)_{EX}$ in the region $(0, \tau A\bar{y} - \bar{g})$:

$$\begin{aligned} \mathcal{E}(0) &> 0 \\ \lim_{B \to \tau A\bar{y} - \bar{g}} \mathcal{E}(B) &= -\infty \end{aligned}$$

□

# D Policy functions with short-term bonds

To highlight the role of debt maturity, Figure 12 shows the policy functions conditional on a recession for the case of one-period bonds ($\delta = 1.0$). Comparing this with Figure 4, it is apparent that, as $\delta$ converges to unity, $\bar{B}(0)_{pes}$ is much lower, while $\bar{B}(0)_{opt}$ is not affected. The result that $\bar{B}(0)_{opt}$ is not sensitive to debt maturity follows from the fact that, when investors hold an optimistic view on government solvency, they lend the government at the risk-free rate: thus there is little scope for maturity to make a difference. Indeed, the left panel in Figure 12 features exactly the same dynamics as the left panel of Figure 4.

Maturity instead makes a difference for the region of debt in which fast and slow crises are possible. The region between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$ is much wider with short-term debt. The rollover crises might occur for low levels of debt (above 41% of GDP in normal times). Moreover, a larger region of fast crises is not fully compensated by a narrowing of the region of slow crises, as both $B_N$ and $\bar{B}(0)_{pes}$ shrink when maturity is shorter. $B_N$ falls from 53 to 12! When government bonds are all short-term, a country in a recession might suffer a slow-moving crisis even when its outstanding debt is relatively low.

# E Debt evolution in baseline sunspot equilibria

Figure 13 displays the debt path starting from $B = 75$ in sunspot equilibria. The government accumulates debt over time as long as the sunspot event does not occur. Debt accumulation is slow when the debt level is below $\bar{B}(0)_{pes}$, whereas it accelerates when debt enters the fast crises zone, above $\bar{B}(0)_{pes}$.

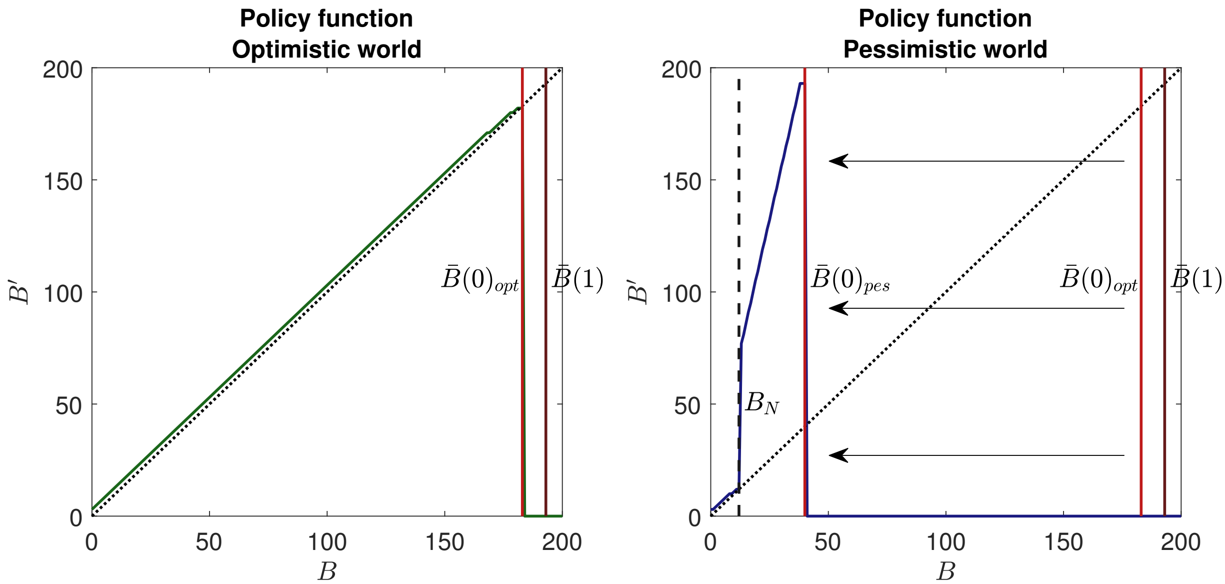


**Figure 12:** Policy functions for one-period bonds, $\delta = 1.0$

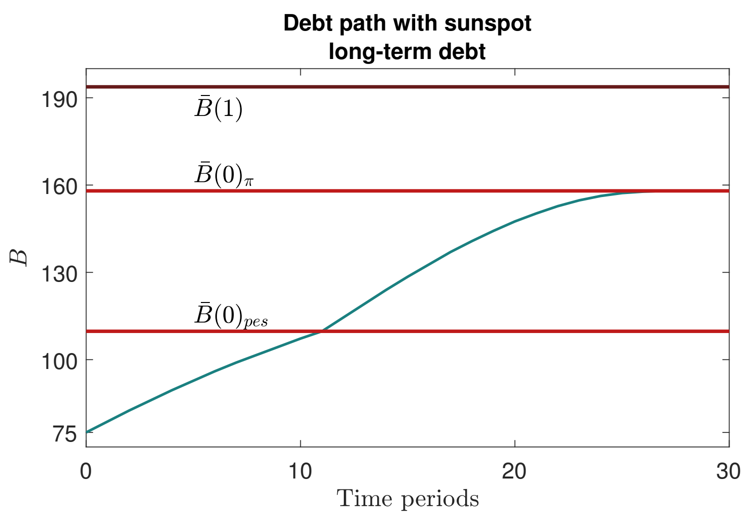


**Figure 13:** Debt path starting from $B = 75$

# F The 'Cliff Effect' in welfare due to self-fulfilling crises

In Figure 14, we show the government value function in a pessimistic world for $\delta = 0.2$. We refer to the discontinuity in the value function at the debt threshold as the 'welfare cliff.' A cliff is apparent at $B_N$. In a sunspot equilibrium, the large loss of utility the economy suffers when it becomes vulnerable to belief-driven crises motivates the government to deleverage and keep debt at safe levels, below $B_N$. Observe, however, that there is no cliff around the other, higher threshold — the welfare incentive for the government to deleverage is significant only around $B_N$.

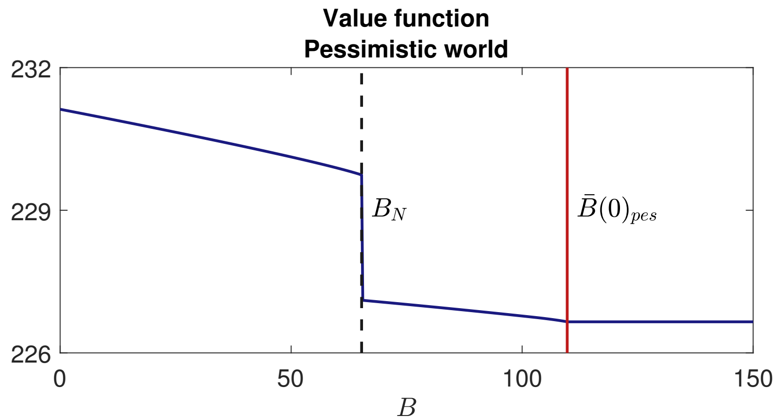


**Figure 14:** Cliff effect ($\delta = 0.2$)

# G Multiplicity in a debt-limit version of our model

In this appendix, we reconsider our main results using a debt-limit version of our model. In the main text, we introduced this new model by rewriting the sustainability condition as (11). Below, we first characterize the debt tolerance thresholds based on this condition, focusing on the case of one-period bonds. Then, we show how to derive the debt-limit model as a variant of our baseline, and carry out numerical analysis for a generic bond maturity.

## G.1 The debt tolerance threshold in the debt-limit framework

In the debt-limit framework, the debt tolerance thresholds are pinned down by the maximum adjustment in primary surpluses the government is willing/able to generate. As for our baseline model, these thresholds may be shifting in response to the regime of investors' expectations. We derive the thresholds conditional on short debt maturity in the following.

### G.1.1 The debt tolerance threshold in normal times $\bar{B}(1)$

In normal times, the government budget constraint is

$$B = \tau\bar{y} - g + q(B', s)B'$$

Since by assumption, once the economy recovers, it never falls back into a recession again, there is no reason to borrow or lend for consumption smoothing purposes. The government's optimization problem is deterministic independently of whether the regime of beliefs is optimistic or pessimistic ($\rho = opt$ or $pes$). If no default has occurred in the past, the government will simply service its existing debt at the risk-free rate, paying $(1-\beta)B$ to investors each period, to satisfy the no-Ponzi condition. Given $\tau$, the government does not default if and only if

$$B \leq \frac{\tau \bar{y} - \bar{g}}{1-\beta} = \bar{B}(1)$$

where $\bar{g}$ is the critical expenditure level.[30]

#### G.1.2 The debt tolerance threshold(s) in a recession $\bar{B}(0)$

In a recession, the debt thresholds depend on the regime of beliefs. Below we consider pessimistic and optimistic beliefs respectively.

**Pessimistic Beliefs**

In a recession, the government budget constraint reflects the decline in tax revenue due to the downturn in activity ($A < 1$):

$$B = \tau A \bar{y} - g + qB'$$

In a pessimistic world, investors are only willing to buy bonds at the low risky price. Given the definition of the debt tolerance threshold, the maximum the government can borrow is capped by the stock of debt that the government can service if the economy recovers, that is, $\max\{q(B', s)B'\} = \beta p \bar{B}(1)$. Hence, to rule out immediate default, the current debt level must be low enough to satisfy:

$$B \leq \tau A \bar{y} - \bar{g} + \beta p \bar{B}(1) = \bar{B}(0)_{pes}, \tag{15}$$

an expression that gives us the current debt tolerance threshold $\bar{B}(0)_{pes}$. Analogously, we can also derive the threshold $B_N$ below which the government is immune to pessimism:

$$B \leq \tau A \bar{y} - \bar{g} + \beta p \bar{B}(0)_{pes} = B_N.$$

If this condition is satisfied, the government will keep the debt level below $\bar{B}(0)_{pes}$, even if bonds trade at the default-risky price.

**Optimistic Beliefs**

In an optimistic world, we need to examine two possible issuance strategies for the government. One consists of issuing a lot of debt, at a low, risky price—essentially this is the same strategy as described above, and is therefore associated to the same debt threshold in (15). The other one consists of keeping new issuance in check, so to ensure that debt remains

[30] In normal times, the debt threshold may still depend on extreme beliefs. In the debt limit model, this threshold is derived by solving $B$ in the following equation $B = \tau \bar{y} - \bar{g}$.

safe. This can be dubbed as a "low-risk low-debt" issuance strategy. By using the same steps above, we can derive the maximum sustainable debt conditional on the safe-debt strategy as:

$$B \leq \frac{\tau A\bar{y} - \bar{g}}{1-\beta}$$

Thus, $\bar{B}(0)_{opt}$ can be characterized as follows:

$$\bar{B}(0)_{opt} = \max\left\{\frac{\tau A\bar{y} - \bar{g}}{1-\beta}, \bar{B}(0)_{pes}\right\}$$

Which strategy gives the government higher revenue in an optimistic world depends on parameters. If all government debt is short-term, we find that $\bar{B}(0)_{opt} > \bar{B}(0)_{pes}$, and thus a safe-debt strategy makes the government better off.[31]

## G.2 The government welfare function

To study the debt-limit model, we specify a variant of our baseline building on the idea that the government suffers a utility cost $\Gamma$ if it cuts spending below $\bar{g}$. Specifically, we replace (9) with a new objective function:

$$\mathcal{U}(c,g) = \mathbb{1}_{g>\bar{g}}\big(\log(c) + \gamma\log(g - \bar{g} + \epsilon)\big) - (1 - \mathbb{1}_{g>\bar{g}}) \times \Gamma,$$

where $\mathbb{1}_{g>\bar{g}}$ is an indicator function equal to 0 if spending falls below the critical value. We assume an arbitrary small positive $\epsilon$ to ensure that $\mathcal{U}(c,g)$ is bounded below when $g \to \bar{g}$. This is the key implication: if defaulting brings spending below the critical level $\bar{g}$ and a utility penalty $\Gamma$ is cruel enough, the value of repaying will never be below that of defaulting—the government never defaults strategically, and thus (11) holds. Yet, as shown below, crises are still possible, depending on the initial conditions, the persistence of recessions and investors' expectations.

Using this new framework, we now set $Z = 0.8$, $\tau = 0.35$, $\bar{g} = 30$ such that government spending falls below the critical level $\bar{g}$ upon a default.[32] For the other parameters, we adopt the same values as in the baseline of Table 1. We discuss the case of "static" beliefs in Appendix G.3 and a more general analysis of sunspots in Appendix G.4. We will show that, as discussed in Section 7, in this debt-limit framework long-term debt tends to rule out "fast" debt crises more easily, but remains ineffective in ruling out "slow-moving" debt crises.

[31] In the case of extreme beliefs, investors gauge the current debt sustainable if it satisfies the following condition: $B \leq \tau A\bar{y} - \bar{g} = \bar{B}(0)_{EX}$. This means that debt will be repaid even if the government loses market access in a recession.

[32] We observe that the initial recessionary state can be quite adverse—$A$ can be so low that the government cannot finance the critical level of spending $\bar{g}$ without borrowing, i.e., $A\tau\bar{y} < \bar{g}$. We discuss this case in Appendix H.

## G.3 Differences between the debt-limit and the baseline model

The main results from our exercise are shown in the two panels of Figure 15, which depicts the policy functions with long-term bonds (left panel) and one-period bonds (right panel). Each panel illustrates both the optimistic and the pessimistic world.

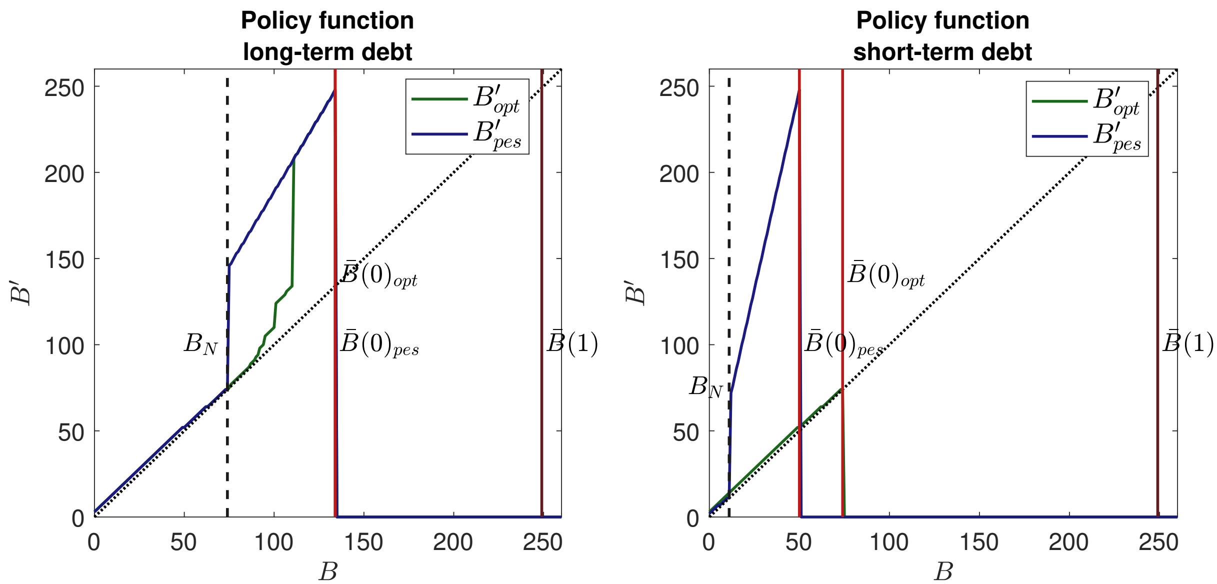


**Figure 15:** Policy functions where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

With one-period bonds—the case shown in the right panel of Figure 15—the debt dynamics are very similar to the ones in our baseline model.[33] In an optimistic world, the government accumulates debt over time to smooth consumption till it reaches $\bar{B}(0)_{opt}$. In a pessimistic world, the government issues safe debt at a slow pace in the region between 0 and $B_N$; it starts to accumulate risky debt at a fast pace in the region between $B_N$ and $\bar{B}(0)_{pes}$; fast, rollover crises can nonetheless occur for debt levels between $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$.

The debt dynamics with long-term bonds shown by the left panel in Figure 15 are instead quite different from our baseline in Figure 4. As in our baseline, the equilibrium is unique at a low level of debt (in the region between 0 and 74), but additionally at a high level of debt; there are multiple equilibria for intermediate levels of debt. However, in this intermediate debt region, as investors price the risk of slow-moving crises in the future regardless of beliefs regimes, the bond price falls in the level of debt driving a fast pace of debt accumulation. The beliefs regime nonetheless matters: from the figure, it is apparent that, when a recession persists for many periods, debt accumulates faster under pessimistic beliefs.

Most notably, different from Figure 4, the two thresholds $\bar{B}(0)_{pes}$ and $\bar{B}(0)_{opt}$ coincide and the multiplicity region is narrower than the region between the thresholds $B_N$ and

[33] Relative to Figure 4, the thresholds in Figure 15 are much lower, reflecting the difference in debt maturity in the two figures.

$\bar{B}(0)_{pes} = \bar{B}(0)_{opt}$. In our simulation for the debt-limit framework, multiple equilibria are possible for a debt-to-GDP ratio comprised between 82 and 122 percent (74 and 110 percent if GDP is measured in normal times), while the overlapping debt thresholds are close to 150 percent. With a long debt maturity, the revenue from issuing debt above the safe-debt threshold, $\beta p(B' - (1-\delta)B) - \kappa B$, is high and may exceed the net bond revenue from keeping issuance below the threshold (pursuing a low-debt safe-debt issuance strategy). For a high enough stock of outstanding debt, keeping issuance below the safe-debt threshold may not yield enough income to avoid immediate default, even if investors hold optimistic beliefs. When this is the case, the government has no alternative but issuing risky debt above the threshold, to avoid immediate debt repudiation, regardless of beliefs regimes. The equilibrium is unique, with the country entering a slow-moving crisis mode. Remarkably, this rules out the possibility of "fast" debt crises. The unique equilibrium region at high debt levels widens with a longer bond maturity, since the net bond revenue from issuing risky bonds rises with maturity.

These results suggest that debt maturity is much more consequential in the debt-limit than in the baseline model. As discussed in Section 7, we find that "fast" debt crises are ruled out in the debt-limit version of our model for any $\delta$ below 0.57, corresponding to a debt maturity of seven quarters. For longer debt maturities, "none" and "slow" are the only possible outcomes in the debt-limit framework.

## G.4 Sunspot equilibria in a debt-limit framework

We study how sunspots can affect government behavior in the debt-limit framework. To keep things simple and for the sake of comparison with Lorenzoni and Werning (2019), in the following we restrict our attention to a scenario in which a switch is possible from optimistic to pessimistic beliefs regime.

In a debt-limit framework, a sunspot equilibrium modifies our previous analysis in two respects. First, when government bonds are long-term, at intermediate levels of debt, there is an acceleration of debt accumulation. Second, when debt is short-term, debt thresholds become sensitive to the probability attributed to the sunspot—they shift at low values of these probabilities.

In Figure 16, we display the bond price functions and debt accumulation in the time domain for two different levels of debt in our economy with long-term bonds. Each panel illustrates both the optimistic equilibrium and the sunspot equilibrium. We do omit the policy function from the graph because this is visually very similar to the optimistic world in Figure 15.

The center and right panels of Figure 16 clarify the main difference between the optimistic and the sunspot equilibria. In both panels, default may occur with positive probability, but

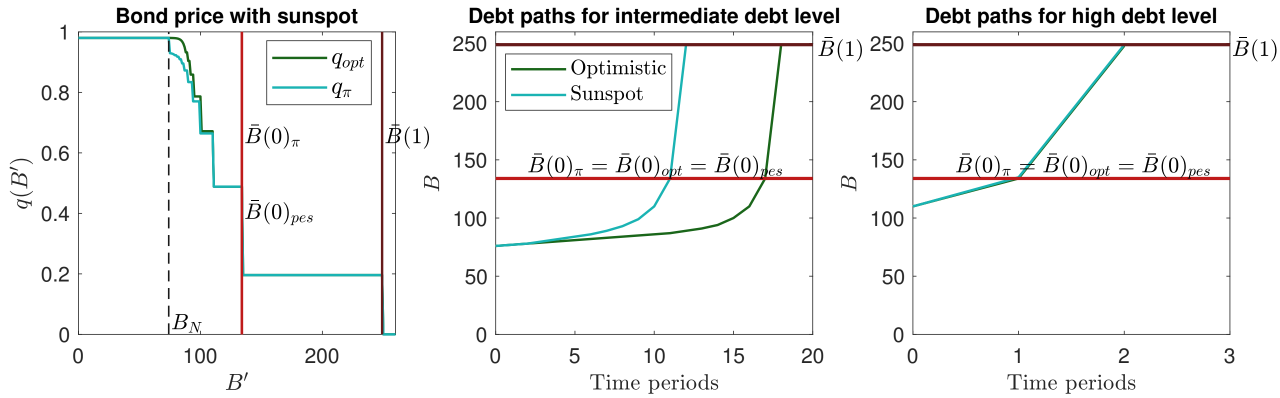


**Figure 16:** Bond price schedules and debt paths ($\delta = 0.2$)

in the center panel default is possible starting from a moderate debt level ($B = 76$), while in the right panel default is possible starting from a high debt level ($B = 110$).

The center panel of Figure 16 clarifies when and how the sunspot equilibrium makes a difference. When the economy is exposed to sunspot crises, the government has to pay a higher spread at intermediate levels of debt. This accelerates debt accumulation: as debt crises arrive earlier, the spread rises even further, larger than $\pi = 4\%$ for $B'$ close to $B_N$ in the left panel of Figure 16.

When debt level is already sufficiently high, pricing in a sunspot equilibrium is less consequential. The right panel of Figure 16 shows that the debt paths are identical in the sunspot and the optimistic equilibrium. Intuitively, investors may turn pessimistic at $T = 1$, but the government always chooses risky-debt high-debt issuance strategy at $T = 1$ regardless of the investors' beliefs (there is no multiplicity at high debt levels, see Figure 15). The sunspot is immaterial for the equilibrium.

The economy with one-period debt features different debt dynamics. Strikingly, $\bar{B}(0)_\pi$ coincides with $\bar{B}(0)_{pes}$: with one-period debt, in the sunspot equilibrium, the debt tolerance threshold shrinks towards $\bar{B}(0)_{pes}$. We find that for any $\pi$ above 1% (consistent with the policy function), the government always issues default-free debt up to $\bar{B}(0)_{pes}$.

# H Deep recessions in a debt-limit framework

In a deep recession, the government is only able to sustain $\bar{g}$ via borrowing ($\bar{g} > \tau A \bar{y}$). A numerical example is shown in Figure 17 where $A = 0.8$. The figure shows the path of optimal debt accumulation over time, contrasting the economy with long-term bonds (left) and one-period bonds (right). The initial debt level is set to 0 in both panels. In either case, the government keeps increasing its debt, beyond safe levels if a recession persists.

Note that, when the recession persists, the government accumulates debt faster and

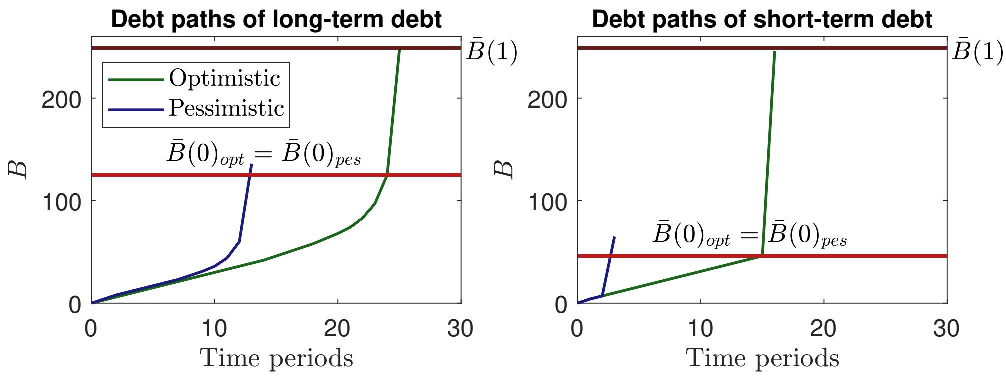


**Figure 17:** Deep recession $A = 0.8$

defaults earlier in a pessimistic world. Comparing these two panels also suggests that, when debt is short-term, debt reaches unsustainable levels faster/earlier due to a lower $\bar{B}(0)$.

# I Ruling out "fast" debt crises in the baseline model

In the sensitivity analysis discussed in the text, we have seen that long debt maturities are effective in eliminating equilibria with fast (rollover) crises in the debt-limit model, while they are not effective in our baseline. In this appendix we extend our sensitivity analysis and characterize three conditions under which the equilibria with fast crises disappear also in our baseline. These are: (i) the country is in a very deep recession ($A$ is very low), (ii) the probability of recovery is quite high, and (iii) debt maturity is sufficiently long—as to mute the pass-through of high interest rates on the total cost of debt servicing. Under these conditions, the government has a strong incentive to pursue high-debt risky-debt issuance strategy even when investors' expectations are optimistic.

In Figure 18 we set $A = 0.8$ and $p = 0.6$: the current recession is exceptionally deep (with a loss of output equal to 20%), but the probability of a recovery one period ahead is higher than 50%. Figure 18 shows the policy functions when debt is long-term ($\delta = 0.2$) in the left panel, and when debt is short-term ($\delta = 1.0$) in the right panel.

When government bonds are short-term—the right panel of Figure 18—the two thresholds $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$ are distinct, and thus fast debt crises are possible. With short debt maturity (high $\delta$), the pass-through of interest rate on borrowing costs is very rapid. In an optimistic world, the government smooths consumption through the recession sticking to a safe-debt low-issuance strategy, and keeps debt stationary at the threshold $\bar{B}(0)_{opt}$. In a pessimistic world, a slow-moving debt crisis materializes at intermediate levels of debt; a fast one occurs at high levels of debt.

In contrast, as shown in the left panel of Figure 18, when debt maturity is sufficiently

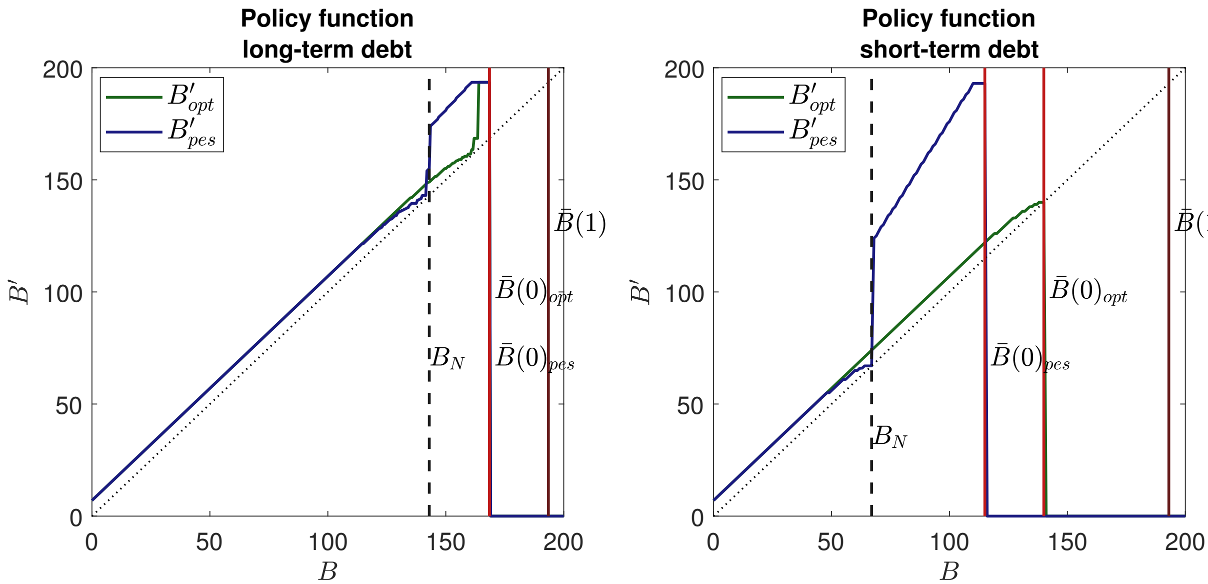


**Figure 18:** Policy functions in a very severe recession and high probability of recovery, $A = 0.8$ and $p = 0.6$, where $\delta = 0.2$ (left) and $\delta = 1.0$ (right)

long, the equilibrium is unique before the outstanding level of debt reaches the risky-debt thresholds. When debt is long-term, facing a deep recession and a high probability of recovery, the government has a stronger incentive to borrow. Below $\bar{B}(0)_{opt}$ already, the government switches to the risky-debt issuance strategy and keeps accumulating debt also in an optimistic world. Borrowing against the future (uncertain) recovery is the preferred option regardless of the prevailing regime of beliefs. Indeed, as is the case in the debt-limit version of the model, the two thresholds $\bar{B}(0)_{opt}$ and $\bar{B}(0)_{pes}$ coincide, implying that no fast debt crises are possible any longer.

A striking feature of this economy is that, in a pessimistic world, a government with an outstanding debt level below $B_N$ prefers to keep borrowing and let debt grow over this threshold, even if this means that the country enters the slow-moving crises zone. In the figure, for any debt level below $B_N$, the government slowly accumulates debt over time, and keeps doing so before debt exceeds $B_N$. Given a high probability of recovery, the consumption smoothing motive drives the optimal government policy. Once debt is in the region between $B_N$ and $\bar{B}(0)_{pes}$, the government effectively starts gambling on the recovery.