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A Revisit to Sovereign Risk Contagion in Eurozone with Mutual Exciting Regime-Switching Model*

Shuyi Ge †

November 24, 2020

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This paper proposes a new mutual exciting regime-switching model where crises can spread contagiously across countries. Each country has its own hidden stochastic process that determines whether the country is in a normal or crisis regime. Contagion is defined as a rise in the transition probability to the crisis regime when other countries are in crisis in the past state. Using this new approach, I revisit the sovereign risk contagion in the euro area. I find that there are striking shifts in market pricing functions for the sovereign bond spreads. Multi-country contagion plays a dominant role in driving such shifts, while common risk factors and country-specific fundamentals are much less important.

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1 Introduction

The unfolding of the European Sovereign Debt Crisis shows that extreme events in the financial markets appear in clusters instead of in isolation. And this triggers a surge of interest in the role of contagion in the risk clustering. Testing for the existence of contagion and quantifying the size of it is important for economists and policymakers. This paper proposes a new mutual exciting regime-switching model where crises can spread contagiously across countries.

The first challenge to study contagion regards its definition. Asset returns exhibit co-movement due to exposure to common factors and spillovers. Is contagion just the normal time inter-dependence? Or does it reflect an increase in the co-movement during periods of crisis? This paper does not aim to contribute to this theoretical debate. Rather, I adopt the theoretical framework from the seminal paper of Forbes and Rigobon (2002), which defines contagion as the significant increase in the co-movement beyond what can be explained by normal time interactions. In particular, inter-dependence and contagion are distinguished. While inter-dependence is a result of normal market linkages, contagion is a breakdown of the normal time transmission regime.

Our second challenge is the specification of the crisis regime. Many studies implicitly assume that the crisis regime can be known *a priori*. For example, the correlation-based test for contagion (Boyer et al. (1997), Rigobon (2003), Corsetti et al. (2005)), the factor-model-based test for contagion (Dungey et al. (2002), Dungey et al. (2006), Dungey and Martin (2004), Bekaert and Harvey (2003)), and the structural break test for contagion (Bekaert et al. (2014), Beirne and Fratzscher (2013)) all rely on *a priori* identification of the crisis regime. These methods, while being very popular, given they are simple to implement and interpret, suffer from several problems. First of all, they implicitly assume that the crisis regime is continuous, which is not true according to the empirical findings in this paper. In addition, the *ex-post* nature of these methods makes them not particularly useful for detecting early-warning signals. Another strand of literature assumes the crisis regime is associated with some extreme values of the observed dependent variable. For example, Pesaran and Pick (2007) and Metiu (2012) assumes a country is in crisis regime when its endogenous performance variable is above a pre-specified threshold value. Caporin et al. (2018), in a similar vein, associates crisis regime with some high quantiles of the observed dependent variable. However, crises, in many cases, are more complicated than extreme values of a performance indicator. And they tend to be unobserved processes governed by several mechanisms.

The third challenge we face is the dynamics of the regime-switching process. There could be many reasons for a switch from the normal to crisis regime. In particular, we are interested in the role of contagion in the regime-switching process. That is, for one country, whether the transition to the crisis regime is more likely when other countries are in the crisis regime in the last period (i.e., crises spread contagiously across countries)? Moreover, how to quantify the strength of contagion, if it exists?

This paper proposes a new mutual exciting regime-switching model where crises can spread contagiously across countries. Each country has its own hidden stochastic process that determines whether the country is in a normal or crisis state. The country-specific process is governed by some common macroeconomic factors, country-specific fundamentals, and other countries' past states. Contagion is defined as a rise in the transition probability to the crisis regime when other countries are in crisis in the past state, after controlling for other mechanisms that drive the switching process. Inter-dependence and contagion are distinguished. There are three avenues for inter-dependence in the model. Firstly, asset returns are exposed to common factors and spillovers from others. Secondly, innovations to asset returns are allowed to be cross-sectionally correlated. Lastly, countries' hidden stochastic processes are subject to common risk factors.

The key element in the model that captures the multi-country contagion is the mutual exciting regime-switching process. The idea is closely related to some recent papers by Ait-Sahalia et al. (2014) and Aït-Sahalia et al. (2015). To capture the propagation of jumps across markets, those authors model the jump intensity using the mutual exciting jump process, also known as Hawkes process (Hawkes (1971b), Hawkes (1971a)) where the jump in one market could increase the probability of future jumps elsewhere. In their spirits, I model multi-country contagion using a mutual exciting regime-switching process. Under the framework, one country being in the crisis regime could increase the transition probability to the crisis regime for other countries. As in Ait-Sahalia et al. (2014) and Aït-Sahalia et al. (2015), the cross country contagion is probabilistic rather than certain. And the model goes beyond their work by allowing for a richer contagion pattern. In addition to a break in the mean equation, the model can also accommodate a break in the variance. The dynamics in the variance could be especially important for crisis episodes where increases in volatility are the major symptomatic.

Using the new approach, I revisit the sovereign risk contagion in the euro area. I use daily 10-year sovereign bond spreads of six euro area countries, including Greece, Ireland, Portugal, Spain, Italy (GIPSI countries), and France from 12/02/2008 to 01/12/2011. German government bond yields of the same maturity are used as the benchmark. I deliberately end the sample before the European Central Bank (ECB) announced the Long-Term Refinancing Operations (LTRO) to avoid the clustering of switchings due to the intervention. Some interesting empirical results are found. Firstly, sovereign bond spread pricing functions are highly regime-dependent as there are striking shifts in market pricing behaviors. In the crisis regime, most sample countries experience a significantly positive jump in the intercept. There is a break of exposures to common risk factors. And the directions of the shifts are opposite to the sign of exposures in the normal regime. This might because the risk aversion and uncertainty both start falling since Spring 2009 while the euro area sovereign spreads begin to skyrocket after 2010. It suggests that the factor cannot help to interpret the sharp increases in euro area sovereign bond spreads during the European debt crisis. On the other hand, regional risk spillovers explain more variations in the sovereign bond spreads during periods of crisis as the vector auto-regressive coefficients are much larger in magnitude in the crisis regime. Surprisingly, Greece plays a less important role in directly propagating shocks to others. This is because investors start to isolate Greek bonds from other countries when the Greek default is inevitable. As a result, other countries' bond spreads decouple from the Greek bond spread. Secondly, although Greece is not propagating a lot of shocks to others in a linear way, it is the key player in terms of non-linear contagion. The break in Greece' bond spread pricing function comes earlier, which makes other countries more likely to switch to the crisis regime. All other sample countries, except Greece itself, are subject to considerable contagion effect (i.e., their transition probabilities to the crisis regime all significantly increase when their neighbors are in crisis in the past state). For those countries, multi-country contagion plays a more important role than common risk factors and even country-specific fundamentals in determining their transition probabilities to the crisis regime.

The rest of this paper is organized as follows: Section 2 introduces the mutual exciting regime-switching model. Section 3 discusses the Bayesian estimation procedure and inference. Section 4 presents the empirical application. Section 5 concludes.

2 Mutual Exciting Regime-Switching Model and Contagion

Consider the following regime-switching model:

$$\begin{aligned} y_{1t} &= \alpha_1(s_{1t}) + \boldsymbol{x}_{1t}'\boldsymbol{\beta}_1(s_{1t}) + h_{1t}^{1/2}(s_{1t})\epsilon_{1t} \\ &\vdots \\ y_{nt} &= \alpha_n(s_{nt}) + \boldsymbol{x}_{nt}'\boldsymbol{\beta}_n(s_{nt}) + h_{nt}^{1/2}(s_{nt})\epsilon_{nt} \end{aligned} \tag{1}$$

where y_{it} is a performance indicator for country i at time t for $i = 1, \ldots, n, t = 1, \ldots, T$. x_{it} is a $k \times 1$ vector of explanatory variables for country i at time t, which includes exogenous observed common factors and country specific variables. The country-specific state variable s_{it} is a hidden discrete stochastic process. And the process $\{s_{it}\}$ is assumed to be irreducible and aperiodic first order Markov chain with finite state space $\{0, \ldots, K-1\}$. Different realizations of s_{it} admit different dynamics in the mean and in the variance. It is assumed that $\boldsymbol{\epsilon_t} \sim \mathcal{N}(\mathbf{0}, \Sigma)$, where $\boldsymbol{\epsilon_t} = (\epsilon_{1t}, \ldots, \epsilon_{nt})' \in \mathbb{R}^n$ for $t = 1, \ldots, T$ and Σ is a n-dimensional positive definite matrix with potentially non-zero off-diagonal entries. The variance of the random disturbance term $h_i(s_{it})\epsilon_{it}$ depends on the realization of states via the volatility multiplier $h_{it}(s_{it})$. As for the parameters in the mean equations, the intercepts $\alpha_i(s_{it})$ and slopes $\boldsymbol{\beta}_i(s_{it})$ are also allowed to vary over the realizations of $\{s_{it}\}$.

Inter-dependence and contagion are distinguished in the model. While the inter-dependence is captured by the exposure to common factors, normal time interactions, and the non-zero off-diagonal elements in Σ, contagion is introduced by the mutual excitement component in the country-specific hidden stochastic process $\{s_{it}\}$. The evolution of $\{s_{it}\}$ is sufficiently described by the $K \times K$ time-varying transition matrix, which are governed by some exogenous variables $\boldsymbol{z}_{it}$ and all countries' past states $\boldsymbol{s}_{t-1} = (s_{1t-1}, \ldots, s_{nt-1})$. We say there is contagion from j to i if the regime transition probability to crisis for i increases when j is in crisis in the last period, after controlling for other mechanisms that drive the switching process. For simplicity, in this paper, I only discuss the two-regime [1] ($K = 2$) case so that we can have a natural crisis regime and normal regime distinction. But generalization to more than two regimes is straightforward. I let only the first lagged states to enter the transition equation so that the vector of states $\boldsymbol{s_{it}} = (s_{1t}, \ldots, s_{nt})$ still has the Markov property.[2] The transition probability from state k to state l ($l, k = 0, 1$ where $s = 0$ denotes the normal state and $s = 1$

[1] Allowing for more regimes imposes no theoretical difficulties. But it could be computationally challenging when the number of countries n is big.

[2] More recent methodologies like forward-filtering backward sampling (FFBS) algorithm (Frühwirth-Schnatter (2006)) could be applied to allow for richer interaction pattern in different chains. This could be an interesting extension since letting the transition probability to depend on the whole path of the chain could be used to accommodate more interesting phenomena. For example, different duration of past bad states might change the transition probability by a different extent. Maybe some smoothing functions could be applied to summarise the information contained in the past state.

denotes the crisis state) is specified as follows:

$$P(s_{it} = l \mid s_{it-1} = k, \boldsymbol{z}_{it}, \boldsymbol{s}_{-it-1}) = P^i_{kl,t}(\boldsymbol{z}_{it}, \boldsymbol{s}_{-it-1}), \text{ where } l, k = 0, 1 \tag{2}$$

where $\boldsymbol{s}_{-it-1}$ is the vector of states for countries other than i at $t-1$. For the two-regime case, the unit-specific unobserved state variables follow a probit specification as in equation (3). This can be generalized using a logit model if there are more than two regimes. Directed contagion effect from j to i is characterized by a positive λ_{ij}.

$$s_{it} = \begin{cases} 0 \text{ if } u_{it} < \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j \neq i} \lambda_{ij} s_{jt-1} \\ 1 \text{ if } u_{it} \geq \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j \neq i} \lambda_{ij} s_{jt-1} \text{ where } u_{it} \sim \mathcal{N}(0,1) \end{cases} \tag{3}$$

For the identification of a regime-switching model, one needs to deal with the label switching problem. A common way to achieve identification is to impose constraints on the parameters. This is used a lot in macroeconomic literature, and different regimes can have natural interpretations. In the empirical literature on contagion, the normal and the crisis regimes are often identified by different levels of asset returns' volatilities (Corsetti et al. (2005), Dungey* et al. (2005)). Given that line of reasoning, one reasonable identification restriction for the above contagion model is

$$h_{it}(s_{it} = 0) = 1 \text{ and } h_{it}(s_{it} = 1) > 1 \tag{4}$$

where $h_{it}(s_{it} = 1)$ is the volatility multiplier parameter.[3] This identification restriction does not impose an increase in the exposures to factors or a jump in the intercept. Whether a crisis state is associated with a significant break in the pricing function is left to be found out. Of course, this is not the only plausible identification restriction. Different restrictions could be applied, depending on the problem at hand.

3 Bayesian Inference by Gibbs Sampling

Putting everything together, for a two-regime case we have:

$$\begin{aligned} y_{it} &= \alpha_i(s_{it}) + \boldsymbol{x}'_{it}\boldsymbol{\beta}_i(s_{it}) + h^{1/2}_{it}(s_{it})\epsilon_{it} \text{ for } i = 1, \ldots, n \\ s_{it} &= \begin{cases} 0 \text{ if } -u_{it} \geq \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j \neq i} \lambda_{ij} s_{jt-1} \\ 1 \text{ if } -u_{it} < \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j \neq i} \lambda_{ij} s_{jt-1} \text{ for } i = 1, \ldots, n \end{cases} \\ \boldsymbol{\epsilon_t} &= \begin{bmatrix} \epsilon_{1t} \\ \vdots \\ \epsilon_{nt} \end{bmatrix} \sim \mathcal{N}(\boldsymbol{0}, \Sigma) \text{ and } u_{it} \sim \mathcal{N}(0,1) \text{ for } i = 1, \ldots, n \end{aligned} \tag{5}$$

The density function of observed performance variables conditional on states and all the parameters in the model, can be factorized as

$$f(Y_T \mid S_T, X, \boldsymbol{\theta}) = f(Y_1 \mid S_1, X, \boldsymbol{\theta}) \prod_{t=2}^{T} f(\boldsymbol{y}_t \mid Y_{t-1}, S_t, X, \boldsymbol{\theta}) \tag{6}$$

[3]For a two-regime case, another popular way to parameterize the regime-dependent volatility is to use $(1 + v_i * S_{it})\epsilon_{it}$. And v_i can be interpreted as the proportional increase in volatility in the crisis state. Am equivalent identification restriction for such parameterization is $v_i > 0$.

where $Y_t = (\boldsymbol{y}_1, \dots \boldsymbol{y}_t)$ is the history of $\boldsymbol{y}_t = (y_{1t}, \dots, y_{nt})$ up to time t, $X = (\boldsymbol{x_{11}}', \dots, \boldsymbol{x_{1t}}', \dots, \boldsymbol{x_{n1}}', \dots, \boldsymbol{x_{nt}}')'$ is the matrix of exogenous regressors, $Z = (\boldsymbol{z_{11}}', \dots, \boldsymbol{z_{1t}}', \dots, \boldsymbol{z_{n1}}', \dots, \boldsymbol{z_{nt}}')'$ is the matrix of exogenous drivers of the regime switching process, $S_t = (\boldsymbol{s}_1, \dots, \boldsymbol{s}_t)$ is the history of the states $\boldsymbol{s}_t = (s_{1t}, \dots, s_{nt})$ up to time t, and $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$ is the collection of parameters in the model. We collect all the parameters in the main equation in $\boldsymbol{\theta}_1 = (\boldsymbol{\alpha}, \boldsymbol{\beta}, \boldsymbol{h}, \Sigma)$ and all the parameters in the auxiliary regime-switching equation in $\boldsymbol{\theta}_2 = (\boldsymbol{\gamma}, \boldsymbol{\lambda})$.[4] In full, the joint density of the observations and states, is simply the product of the conditional density given states and the density of the states,

$$f(Y_T, S_T \mid X, \boldsymbol{\theta}) = f(Y_T \mid S_T, X, \boldsymbol{\theta}_1) \prod_{t=2}^{T} P(\boldsymbol{s}_t \mid \boldsymbol{s}_{t-1}, Z, \boldsymbol{\theta}_2) \times P(\boldsymbol{s}_1) \tag{7}$$

Direct calculations of the joint likelihood function are messy since brute force marginalization of equation (7) involves 2^{n*T} summations over all possible state sequences $\{\boldsymbol{s}_t\}_{t=1}^{T}$. In this paper, Bayesian inference by Gibbs sampling as described in Albert and Chib (1993) and Kaufmann (2015) is applied to avoid the messy calculations involved in the direct evaluation of the joint likelihood function. For the simulation-based Bayesian procedure, unobserved states are treated as unknown parameters. And they can be simulated given other parameters in the model by Gibbs sampling.

We define the vector of latent indexes governing the transition process as $\boldsymbol{s}_t^* = (s_{1t}^*, \dots, s_{nt}^*)$, where $s_{it}^* = \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j \neq i} \lambda_{ij} s_{jt-1} + u_{it}$. A key step in the procedure is to augment the data by $\boldsymbol{s}_t^*$ (i.e., the latent indexes $\boldsymbol{s}_t^*$ are also treated as unknown parameters). As a result of the data augmentation, the full parameters needed to be estimated are $\boldsymbol{\psi} = \{\boldsymbol{\theta}, S_T, S_T^*\}$, where $S_T = \{\boldsymbol{s}_t\}_{t=1}^{T}$ and $S_T^* = \{\boldsymbol{s}_t^*\}_{t=1}^{T}$ are the history of all states and the history of all latent indexes respectively. Our objective is to derive a Markov chain such that its limiting distribution is the joint distribution of interest. Let us divide the parameter set as $\boldsymbol{\psi} = (\boldsymbol{\psi}_1, \boldsymbol{\psi}_2, \boldsymbol{\psi}_3, \boldsymbol{\psi}_4, \boldsymbol{\psi}_5, \boldsymbol{\psi}_6)$ where

$$\begin{aligned}
\boldsymbol{\psi}_1 &= \{\boldsymbol{\alpha}, \boldsymbol{\beta}\} \\
\boldsymbol{\psi}_2 &= \{\Sigma\} \\
\boldsymbol{\psi}_3 &= \{\boldsymbol{h}\} \\
\boldsymbol{\psi}_4 &= \{S_T\} \\
\boldsymbol{\psi}_5 &= \{S_T^*\} \\
\boldsymbol{\psi}_6 &= \{\boldsymbol{\gamma}, \boldsymbol{\lambda}\}
\end{aligned} \tag{8}$$

Let $[. \mid .]$ denotes the conditional distribution. The joint posterior distribution of $\boldsymbol{\psi}$ leads to very tractable conditional structure. And to sample from the posterior distribution, we iterate over the following steps:

1. Specifying arbitrary initial values $\boldsymbol{\psi}^0$ and set $i = 1$.

2. Cycle through the full conditionals by drawing

   - $\boldsymbol{\psi}_1^i$ from $[\boldsymbol{\psi}_1 \mid \boldsymbol{\psi}_2^{i-1}, \boldsymbol{\psi}_3^{i-1}, \boldsymbol{\psi}_4^{i-1}, \boldsymbol{\psi}_5^{i-1}, \boldsymbol{\psi}_6^{i-1}]$
   - $\boldsymbol{\psi}_2^i$ from $[\boldsymbol{\psi}_2 \mid \boldsymbol{\psi}_1^{i-1}, \boldsymbol{\psi}_3^{i-1}, \boldsymbol{\psi}_4^{i-1}, \boldsymbol{\psi}_5^{i-1}, \boldsymbol{\psi}_6^{i-1}]$
   - $\boldsymbol{\psi}_3^i$ from $[\boldsymbol{\psi}_3 \mid \boldsymbol{\psi}_1^{i-1}, \boldsymbol{\psi}_2^{i-1}, \boldsymbol{\psi}_4^{i-1}, \boldsymbol{\psi}_5^{i-1}, \boldsymbol{\psi}_6^{i-1}]$

[4] $\boldsymbol{\alpha} = (\alpha_{1,0}, \alpha_{1,1}, \dots, \alpha_{n,0}, \alpha_{n,1}), \boldsymbol{\beta} = (\boldsymbol{\beta}_{1,0}, \boldsymbol{\beta}_{1,1}, \dots, \boldsymbol{\beta}_{n,0}, \boldsymbol{\beta}_{n,1})$. For a two-regime model, s_{it} is a dummy variable so that $\alpha_{i,1}$ and $\boldsymbol{\beta}_{i,1}$ correspond to the level shift and slope shift respectively.

- ψ_4^i from $[\psi_4 \mid \psi_1^{i-1}, \psi_2^{i-1}, \psi_3^{i-1}, \psi_5^{i-1}, \psi_6^{i-1}]$
- ψ_5^i from $[\psi_5 \mid \psi_1^{i-1}, \psi_2^{i-1}, \psi_3^{i-1}, \psi_4^{i-1}, \psi_6^{i-1}]$
- ψ_6^i from $[\psi_6 \mid \psi_1^{i-1}, \psi_2^{i-1}, \psi_3^{i-1}, \psi_4^{i-1}, \psi_5^{i-1}]$

where the conditioning on Y_T, X and Z are suppressed.

3. Let $i = i + 1$ and to back to the previous step.

The process generates a Markov chain, which under mild conditions (Tierney (1994)) has the joint distribution of interest as the limiting distribution. The first M draws have to be discarded, which is called "burn-in". After the "burn-in-period", the simulated values $(\psi_1^i, \psi_2^i, \psi_3^i, \psi_4^i, \psi_5^i, \psi_6^i)$ for $i = M + 1, \ldots, M + K$ can be treated as approximated sample from from the joint posterior distribution. To initialize the sampler, one need initial values. I choose initial values with minimal prior information. The full conditionals and the choice of priors are provided in the appendix. Once we have the posteriors, we can then obtain the credible interval, which is analogous to the confidence interval in frequentist inference, for each parameter of interest.

4 Revisit Sovereign Credit Risk Contagion in the Eurozone

The econometric framework is applied to revisit the sovereign credit risk contagion in the Eurozone. We first discuss the data and its properties. We then examine the drivers of the regime-switching process. In particular, we are interested in the testing and quantification of multi-country contagion.

4.1 Data Description

Sovereign credit risk is measured by government bond yield spreads (relative to benchmark country Germany). Daily sovereign bond spreads of six Eurozone countries, including Greece, Ireland, Portugal, Spain, Italy (GIPSI countries), and France, are constructed using the difference between the 10-year sovereign bond yields of these countries and that of Germany. The daily data spans from 12/02/2008 to 01/12/2011 and are downloaded from Thomson Reuters Eikon. In the spirit of Caporin et al. (2018), we deliberately end the sample before the European Central Bank (ECB) announced the Long-Term Refinancing Operations (LTRO) to avoid the clustering of switchings due to the intervention.

Table 7 presents key macroeconomic fundamentals that affect credit conditions for the sample countries. Germany has the highest credit ratings, the best average fiscal position, and the highest GDP growth within the sample period. Sovereign bonds issued by the German government have very low yields and are considered extremely safe. That justifies why using German yield as the benchmark when constructing the spread is the convention in the literature (Bernoth et al. (2012), Metiu (2012), De Santis (2014), etc). France has the second best credit ratings, and its sovereign bond yield remains low during the whole sample period. Spain and Italy follow and have the next worse credit ratings, with Italy having a much higher public debt level but a better fiscal position. Then we have three countries exhibiting high credit risk, Ireland, Portugal, and Greece. Ireland has the worst fiscal position in the sample, and Greece has the highest public debt level among sample countries.

Figure 1 and Figure 2 show the 10-year sovereign bond spreads of the six sample countries during the sample periods in level and in first difference, respectively. Sovereign bond spreads start dropping from spring 2009 as global uncertainty decreases and countries recovering from the global financial crisis. However, the spreads start to skyrocket at the end of 2009, when the Greek problem reveals. The rise is so sharp that it is hard to reconcile with the gradual deterioration of fundamentals, justifying the use of a regime-switching model to accommodate such breaks. Regime switch in the sovereign credit risk pricing equation for euro area peripheral countries is empirically supported (e.g., Favero and Missale (2012), Delatte et al. (2017)). I aim to go beyond them by allowing cross-sectional interaction in the regime-switching process and using the framework to test and quantify the non-linear contagion effects among sovereigns. Another important feature of Figure 1 is that the sovereign bond spreads show high persistency during the sample period. It is necessary to verify that these variables are stationary since the lack of stationarity will lead to deceptive results. Table 8 presents the results of stationarity tests for the sovereign bond yield spreads of all six sample countries. Augmented Dickey-Fuller test and Phillips-Perron test are both applied, showing these series are difference-stationary. Hence we use a first difference specification.

Based on the empirical literature focusing on the factors that determine individual sovereign credit spread (Edwards (1983), Edwards (1986), Duffie et al. (2003), Longstaff et al. (2011)), the factors affecting the sovereign bond yield spreads are associated with (1) common risk factors, (2) spillover effect, (3) country-specific risk factors, and (4) contagion risk. As for common risk factors, it is found that market risk appetite and uncertainty play an important role in the determination of sovereign risk (Baek et al. (2005)). I use two variables to proxy the market appetite and uncertainty in the euro area. The first one is the spread between the 3-month Euro Interbank Offered Rate (Euribor) and the Euro Overnight Index Average (EONIA). The second one is the VSTOXX Index, which is a forward-looking measure designed to reflect the market's expectations of future volatility in the euro area. For both variables, we use the first lag. To allow for the spillover effect, I use the first two lagged sovereign bond spreads from other countries in the sample. Country-specific default risk is determined by some low-frequency macroeconomic fundamental variables, including public debt/GDP ratio, fiscal balance/GDP ratio, GDP growth, and the current account/GDP ratio. These low-frequency variables will drop out after taking the first difference of the daily data. As stated in the introduction section, contagion is defined a breakdown of the normal time transmission regime. In this model, the contagion risk is captured by a rise in the country-specific probability of being in the crisis regime when others are in crisis in the last period. As in Ait-Sahalia et al. (2014) and Aït-Sahalia et al. (2015), the contagion is probabilistic rather than certain.

A country's regime-switching process also reflects exposure to common risk factors and country-specific fundamentals. This paper contributes by explicitly allowing for the role of multi-country contagion in the regime-switching process so that crises can spread contagiously across countries in a probabilistic way. Again, we use the two common factors described above to control for inter-dependence in the switching process. It has been documented that government debt has non-linear effect on sovereign bond spreads (Bernoth et al. (2012), Delatte et al. (2017)). Due to that reason, I include the country-specific Debt-to-GDP ratio in the switching equation. Because governments might adjust their debt-to-GDP ratio endogenously in response to shocks to credit risk, I use the Debt-to-GDP ratio observed a quarter ahead so that it is predetermined with respect to the bond spread.

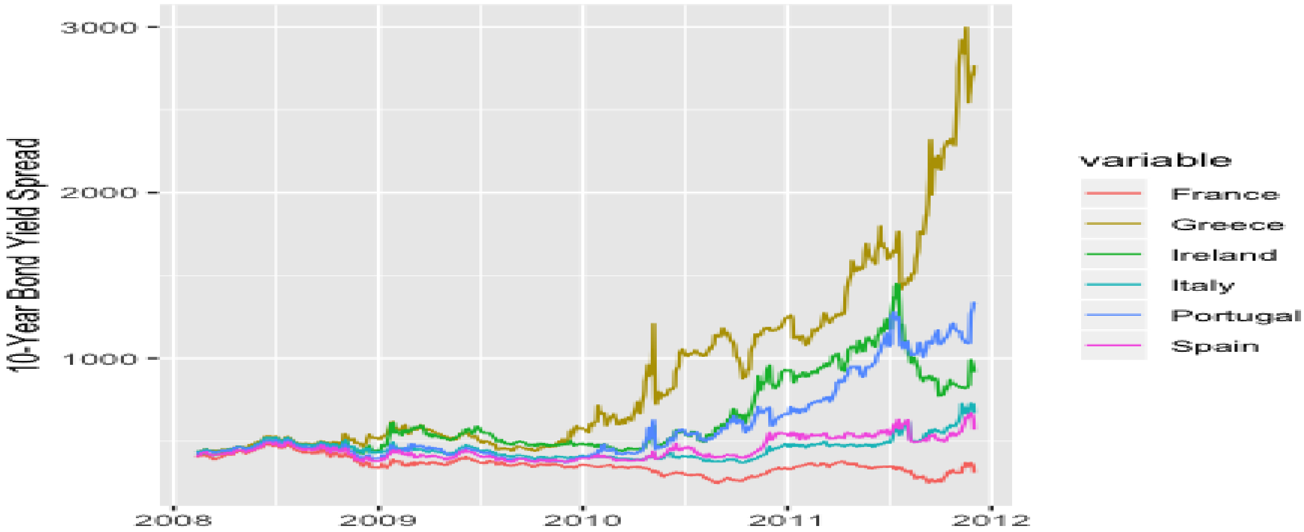


Figure 1: Daily 10-year sovereign bond spreads (in basis points)

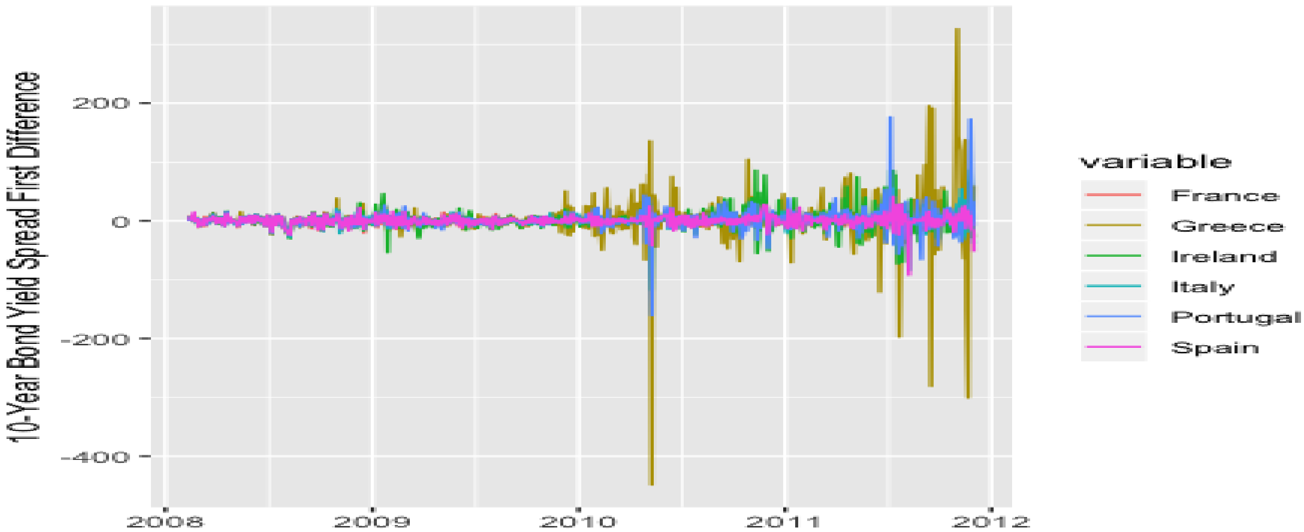


Figure 2: First-differenced daily 10-year sovereign bond spreads (in basis points)

4.2 Empirical Results

I implement the estimation methodology outlined above on the first-differenced 10-year sovereign bond yields. For the modeling of multi-country contagion effect, I adopt the formulation in Pesaran and Pick (2007) and aggregate the contagion effect from $N-1$ remaining countries.[5] To be more specific, the regime-switching equation (3) is modified to

$$s_{it} = \begin{cases} 0 \text{ if } u_{it} < \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \lambda_i I(\sum_{j\neq i}^{N} s_{jt-1}) \\ 1 \text{ if } u_{it} \geq \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \lambda_i I(\sum_{j\neq i}^{N} s_{jt-1}) \text{ where } u_{it} \sim \mathcal{N}(0,1) \end{cases} \tag{9}$$

Under this formulation, the crisis indicator $I(\sum_{j\neq i}^{N} s_{jt-1})$ is a dummy variable that takes the value of one as

[5]Estimating directed pairwise contagion as in equation (3) poses no theoretical difficulties. The estimation results are available upon request.

long as any of the $N-1$ remaining countries are in a crisis state at $t-1$. In summary, the empirical specification is as follows:

$$\Delta y_{it} = \alpha_i(s_{it}) + \boldsymbol{x}_{it}'\boldsymbol{\beta}_i(s_{it}) + h_{it}^{1/2}(s_{it})\epsilon_{it} \text{ for } i = 1, \ldots, n$$
$$s_{it} = \begin{cases} 0 \text{ if } u_{it} < \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \lambda_i I(\sum_{j\neq i}^{N} s_{jt-1}) \\ 1 \text{ if } u_{it} \geq \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \lambda_i I(\sum_{j\neq i}^{N} s_{jt-1}) \text{ where } u_{it} \sim \mathcal{N}(0,1) \end{cases} \tag{10}$$
$$\boldsymbol{\epsilon_t} \sim \mathcal{N}(\mathbf{0}, \Sigma) \text{ and } u_{it} \sim \mathcal{N}(0,1) \text{ for } i = 1, \ldots, n$$

where $\boldsymbol{x}_{it} = (\Delta f_{1t-1}, \Delta f_{2t-1}, \Delta y_{1t-1}, \Delta y_{1t-2}, \ldots, \Delta y_{Nt-1}, \Delta y_{Nt-2})'$ are the vector of explanatory variables described in section 4.1 in first difference. f_1 and f_2 correspond to the spread between Euribor and ENOIA, and the VSTOXX Index, respectively. $\Delta y_{it-1}, \Delta y_{it-2}$ for $i = 1, \ldots, N$ are the vector-autoregressive terms with two lags. The mean equation of (10) is essentially a regime-dependent VARX model, and the structural model can be recovered by imposing identification assumptions on Σ. $\boldsymbol{z_{it}} = (1, \Delta f_{1t-1}, \Delta f_{2t-1}, Debt_{i,t_q-1}, S_{i,t-1})'$, together with $I(\sum_{j\neq i}^{N} s_{jt-1})$ gives the drivers of the regime-switching process. Notice that Debt-to-GDP ratio is observed at quarterly frequency, thus the variable has a different time subscript t_q. Prior distributions are provided in Table 9 and Table 10, which are used to initialize the Gibbs Sampler. I run 6000 iterations in total. The first 1000 "burn-in" iterations are discarded, and the 5000 iterations after that are treated as approximate sample from the joint posterior distribution.

Table 1 reports the posterior estimates of coefficients in the normal regime, and Table 2 reports the posterior estimates of the shift parameters (i.e., the changes in parameters when a country switches from the normal to crisis regime). The tables reveal that the sovereign bond spread pricing functions are highly regime-dependent as there there are striking shifts in market pricing behaviors. Our identification scheme is based on a rise in volatility in the crisis regime, and the economic magnitude of that is given by the volatility multiplier parameter h. Greek yield spread experience an almost eight-fold increase in volatility when it switches to the crisis regime. Other countries except for France also show high increases in their volatilities, with the posterior means of their volatility multiplier vary from 2 to 5. In the crisis regimes, all sample countries except France experience a significantly positive jump in the intercept. As the country with the best fundamental in our sample, France has a more "tranquil" crisis regime than others. The break of exposures to common risk factors is also worth noticing since the directions of the shifts are opposite to the sign of exposures in the normal regime. This might because the risk aversion and uncertainty both start falling since Spring 2009 while the euro area sovereign spreads begin to skyrocket after 2010. This suggests that the factor cannot help to interpret the sharp increases in euro area sovereign bond spreads during the European debt crisis. This phenomenon is also documented in De Santis (2014), where the author finds common risk factors stop being important determinants of European bond yields. The spillover pattern among sample countries also changes drastically from one regime to the other. The vector auto-regressive coefficients are much larger in magnitude in the crisis regime, indicating that regional risk spillovers explain more variations in the sovereign bond spreads during periods of crisis. This breakdown is associated with an increase in interconnectedness that is beyond what can be explained by the normal time risk transmission mechanism.

	France(FR)	Spain(ES)	Italy(IT)	Portugal(PT)	Ireland(IE)	Greece(GR)
α	-0.05	0.12	**0.16**	**0.40**	**-0.17**	**0.42**
	[-0.18,0.13]	[-0.03,0.32]	[0.06,0.26]	[0.21,0.60]	[-0.44,-0.02]	[0.15,0.70]
β_1	**0.11**	**0.10**	**0.05**	0.07	**0.18**	**-0.28**
	[0.05,0.17]	[0.05,0.20]	[0.01,0.08]	[-0.01, 0.13]	[0.08,0.30]	[-0.36,-0.20]
β_2	**0.02**	**0.04**	**0.04**	**0.07**	0.01	-0.02
	[0.01,0.03]	[0.03, 0.05]	[0.03, 0.04]	[0.04, 0.08]	[-0.02,0.03]	[-0.03, 0.01]
$\beta_{FR,1}$	-0.01	**-0.07**	**-0.11**	**-0.26**	**-0.30**	-0.00
	[-0.03, 0.03]	[-0.09, -0.05]	[-0.13, -0.08]	[-0.28,-0.23]	[-0.35,-0.25]	[-0.06,0.05]
$\beta_{FR,2}$	0.02	0.03	0.03	**0.04**	**0.23**	**-0.10**
	[-0.02, 0.06]	[-0.02, 0.08]	[-0.01, 0.05]	[0.01, 0.08]	[0.18, 0.27]	[-0.15, -0.03]
$\beta_{ES,1}$	-0.01	**0.07**	0.02	**0.22**	-0.05	0.03
	[-0.04, 0.02]	[0.02,0.15]	[-0.04, 0.07]	[0.19,0.24]	[-0.08,0.00]	[-0.04, 0.08]
$\beta_{ES,2}$	-0.01	-0.00	**0.03**	0.02	**0.18**	**0.10**
	[-0.02, 0.01]	[-0.02,0.02]	[0.01,0.03]	[-0.01,0.04]	[0.15, 0.21]	[0.07, 0.13]
$\beta_{IT,1}$	**0.02**	-0.01	-0.01	**0.05**	**-0.02**	**-0.09**
	[0.01, 0.02]	[-0.03,0.00]	[-0.03,0.01]	[0.03, 0.07]	[-0.04, 0.01]	[-0.12,-0.06]
$\beta_{IT,2}$	**0.01**	0.00	**0.02**	**-0.02**	**0.01**	**0.08**
	[0.00, 0.01]	[-0.00,0.01]	[0.01, 0.02]	[-0.02,-0.01]	[0.00,0.01]	[0.06,0.10]
$\beta_{PT,1}$	**-0.21**	**-0.12**	-0.00	**-0.03**	**-0.16**	**0.06**
	[-0.24, -0.16]	[-0.14, -0.06]	[-0.03, 0.01]	[-0.06, -0.01]	[-0.20, -0.12]	[0.03, 0.12]
$\beta_{PT,2}$	**0.06**	-0.04	**-0.03**	**-0.15**	0.03	**0.09**
	[0.01, 0.08]	[-0.06, 0.00]	[-0.07, -0.01]	[-0.20, -0.12]	[-0.05, 0.06]	[0.06, 0.13]
$\beta_{IE,1}$	**0.13**	**0.18**	**0.07**	**0.14**	**0.15**	**-0.07**
	[0.11, 0.17]	[0.09, 0.23]	[0.05, 0.10]	[0.11, 0.17]	[0.10, 0.17]	[-0.10, -0.03]
$\beta_{IE,2}$	**0.04**	0.03	**0.03**	**0.11**	**-0.06**	**0.06**
	[0.02, 0.05]	[-0.00, 0.05]	[0.01, 0.05]	[0.09, 0.13]	[-0.09,-0.03]	[0.03, 0.08]
$\beta_{GR,1}$	**-0.04**	**-0.06**	**-0.03**	**-0.05**	**0.12**	0.01
	[-0.06, -0.03]	[-0.07, -0.05]	[-0.05, -0.02]	[-0.07, -0.00]	[0.09, 0.14]	[-0.01, 0.03]
$\beta_{GR,2}$	**-0.03**	-0.01	**-0.02**	**-0.02**	**-0.02**	**0.01**
	[-0.03, -0.02]	[-0.02, 0.01]	[-0.02, -0.01]	[-0.03, -0.02]	[-0.02, -0.01]	[0.00,0.01]

Table 1: 95% credible intervals and posterior mean for the normal regime parameters. β_1 is the coefficient on the lagged first-differenced spread between Euribor and ENOIA, β_2 is the coefficient on the lagged first-differenced VSTOXX Index. $\beta_{i,t-k}$ is the coefficient on Δy_{it-k}, for $i = FR, ES, IT, PT, IE, GR$ and $k = 1, 2$. Coefficients significant at the 95 % level are in bold.

	France(FR)	Spain(ES)	Italy(IT)	Portugal(PT)	Ireland(IE)	Greece(GR)
$\Delta\alpha$	0.27	**2.35**	**2.34**	**4.71**	**3.94**	**8.19**
	[-0.12,0.60]	[1.29,3.24]	[1.73,3.39]	[2.84,6.77]	[2.39,5.18]	[2.38,12.77]
$\Delta\beta_1$	-0.06	**-2.66**	**-1.34**	**-2.24**	**-2.93**	**-7.76**
	[-0.15,0.12]	[-2.87,-2.37]	[-1.77, -1.06]	[-3.03, -1.20]	[-3.67,-2.42]	[-9.31, -6.26]
$\Delta\beta_2$	-0.03	0.06	**-0.17**	0.12	-0.08	-0.10
	[-0.05, -0.00]	[-0.01, 0.22]	[-0.36, -0.05]	[-0.08,0.31]	[-0.25, 0.06]	[-0.41, 0.14]
$\Delta\beta_{FR,1}$	**0.33**	0.07	-0.07	**0.48**	**0.70**	**-3.29**
	[0.26, 0.41]	[-0.03, 0.19]	[-0.23, 0.19]	[0.23, 0.79]	[0.39, 0.91]	[-4.03, -2.63]
$\Delta\beta_{FR,2}$	0.01	**0.83**	**0.46**	**1.39**	**1.04**	**1.11**
	[-0.07, 0.11]	[0.59, 0.94]	[0.28,0.59]	[1.06, 1.62]	[0.47,1.33]	[0.33, 2.13]
$\Delta\beta_{ES,1}$	**0.21**	**-0.52**	**-0.12**	**-1.13**	**-1.33**	**3.07**
	[0.15, 0.25]	[-0.63, -0.37]	[-0.25, -0.01]	[-1.34, -0.79]	[-1.63, -0.65]	[1.75, 3.74]
$\Delta\beta_{ES,2}$	**-0.13**	**-0.17**	**-0.31**	-0.02	**-0.20**	**-0.26**
	[-0.15, -0.11]	[-0.26, -0.07]	[-0.42, -0.20]	[-0.16,0.05]	[-0.30, -0.10]	[-0.47, -0.03]
$\Delta\beta_{IT,1}$	**-0.07**	**0.21**	**0.20**	**0.20**	**0.20**	0.03
	[-0.10, -0.01]	[0.10, 0.32]	[0.12, 0.26]	[0.13, 0.27]	[0.10,0.34]	[-0.17,0.20]
$\Delta\beta_{IT,2}$	**0.03**	-0.03	**0.03**	0.04	0.03	-0.04
	[0.02, 0.05]	[-0.06, -0.00]	[0.01, 0.05]	[0.00, 0.10]	[-0.01,0.05]	[-0.10, 0.03]
$\Delta\beta_{PT,1}$	**0.71**	0.13	**-0.54**	0.23	**-0.42**	**-2.78**
	[0.62, 0.76]	[-0.05, 0.29]	[-0.79, -0.10]	[-0.16, 0.89]	[-0.70,-0.20]	[-3.17, -2.21]
$\Delta\beta_{PT,2}$	**0.13**	0.00	-0.15	**-1.39**	**-1.23**	**-2.17**
	[0.061, 0.23]	[-0.08,0.12]	[-0.47, 0.25]	[-1.64, -1.07]	[-1.47,-0.90]	[-2.68, -1.60]
$\Delta\beta_{IE,1}$	**-0.29**	**-0.31**	-0.20	**0.66**	**0.69**	**1.86**
	[-0.36, -0.25]	[-0.43, -0.23]	[-0.56, 0.00]	[0.31, 0.89]	[0.30, 1.03]	[1.46, 2.23]
$\Delta\beta_{IE,2}$	**-0.12**	0.02	0.06	**-0.21**	0.02	**-0.31**
	[-0.14, -0.11]	[-0.03, 0.08]	[-0.00, 0.12]	[-0.27,-0.12]	[-0.08, 0.10]	[-0.60, -0.07]
$\Delta\beta_{GR,1}$	**0.16**	**0.18**	-0.00	**0.22**	**0.22**	0.18
	[0.13, 0.19]	[0.12,0.23]	[-0.09, 0.08]	[0.16, 0.27]	[0.10,0.32]	[-0.12, 0.52]
$\Delta\beta_{GR,2}$	**0.03**	**-0.05**	**0.03**	0.06	**-0.07**	**-0.14**
	[0.01, 0.05]	[-0.08, -0.01]	[0.02, 0.05]	[0.00, 0.12]	[-0.11, -0.04]	[-0.19, -0.06]
h	**1.11**	**2.71**	**3.65**	**4.89**	**4.46**	**7.80**
	[1.00,1.36]	[2.32,3.21]	[3.10,4.30]	[4.31, 5.54]	[3.93, 5.07]	[6.90, 8.82]

Table 2: 95% credible intervals and posterior mean for the shift parameters in the mean equation. A typical shift parameter $\Delta\beta = \beta(s = 1) - \beta(s = 0)$, is the change of that parameter from normal regime to crisis regime. Coefficients significant at the 95 % level are in bold.

To look closer into the roles played by countries in the regional risk spillover, I calculate the variance decomposition network as in Diebold and Yılmaz (2014) for the two regimes. For the orthogonalization of shocks, instead of using Cholesky decomposition, which requires an ordering of variables, I adopt the generalized impulse response function from Pesaran and Shin (1998). Table 3 and Table 4 report the variance decomposition network when all countries are in normal and crisis regime, respectively. The tables summarize all dependencies up to lag h by means of the forecast error variance decomposition. A typical element on the ith row and jth column gives the percentage of h-step ahead forecast error variance of Δy_{it} that is due to innovations in Δy_{jt}. The row sums are ones as a result of normalization, and the kth column sum gives the from-degree of country k (i.e., total spillover from k). Comparing the variance decomposition networks during normal and crisis regime reveals some interesting features. Spain, Italy, and Portugal are the most important systemically risk contributors. On the other hand, the two countries that are considered as the origin of the "fever", do not propagate a lot of risk to the system. This result is also documented in Caporin et al. (2018) and Dumitru and Holden (2019). [6] On top of their findings, we find that while Italy is the hub of the network in the normal regime, and its role gets replaced by Portugal in the crisis regime. This might because Portugal is the systemically most important debtor based on the network structure of debt among sample countries. When the system is in stress, the credit risk from Portugal quickly spreads to other countries via debt exposure. On the other hand, Greece plays a less important role in directly propagating shocks to others. This might be explained by the fact that investors start to isolate Greek bonds from other countries when the Greek default is inevitable. However, it will be seen later that Greek plays an important role in non-linear contagion as its break comes earlier and makes other countries more likely to switch to the crisis regime. Overall, we confirm that there is strong evidence of parameters instability during our sample period. Regional spillovers gain importance during periods of stress, with Italy, Spain, and Portugal playing important roles in directly spill over risk to others. Common risk factors, on the other hand, fail to explain the sharp increases in euro area sovereign bond spreads.

	France(FR)	Spain(ES)	Italy(IT)	Portugal(PT)	Ireland(IE)	Greece(GR)
France(FR)	14.82	20.56	28.54	15.90	12.81	7.38
Spain(ES)	12.52	21.20	28.75	16.55	13.61	7.36
Italy(IT)	12.38	20.50	29.07	16.56	13.57	7.92
Portugal(PT)	11.48	19.91	27.91	18.25	14.53	7.93
Ireland(IE)	11.43	19.76	27.65	17.42	15.85	7.88
Greece(GR)	10.98	18.40	27.57	16.52	13.60	12.92
From-Degree	73.61	120.33	169.50	101.19	83.97	51.40

Table 3: Variance decomposition network when all countries are in normal regime. The prediction horizon is 5 days.

[6] Caporin et al. (2018) documents Italy's role as the hub of the network of sovereign contagion during the European debt crisis. Spain and Portugal's important roles are also found in Dumitru and Holden (2019). Both studies find other countries' bond spreads decouple from the Greek bond spread.

	France(FR)	Spain(ES)	Italy(IT)	Portugal(PT)	Ireland(IE)	Greece(GR)
France(FR)	17.17	15.45	18.94	32.27	11.57	4.60
Spain(ES)	8.24	34.22	25.23	18.41	11.72	2.19
Italy(IT)	9.20	15.50	22.33	35.23	13.07	4.66
Portugal(PT)	15.00	5.46	12.07	46.24	17.38	3.85
Ireland(IE)	11.41	17.66	16.86	34.19	16.05	3.83
Greece(GR)	8.22	13.18	21.57	29.26	22.36	5.42
From-Degree	69.24	101.47	117.0	195.59	92.15	24.55

Table 4: Variance decomposition network when all countries are in crisis regime. The prediction horizon is 5 days.

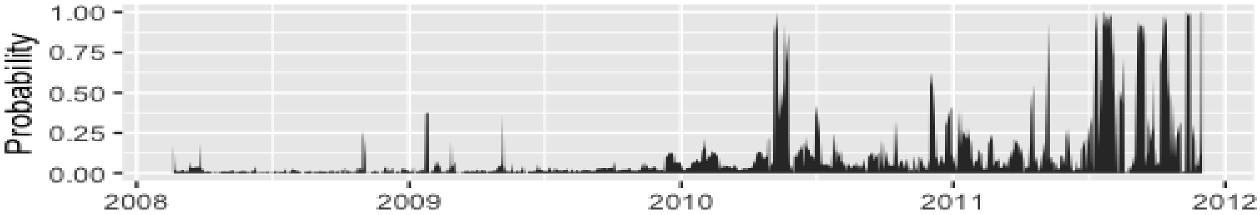
Table 5 reports the posterior estimates of coefficients in the auxiliary regime-switching equation. The first important observation is that common risk factors do not play a role in determining the probabilities of regime-switching. For a country, its lagged Debt-to-GDP ratio, past state, and other countries' past states (the contagion component) determine its transition probability. As documented in Bernoth et al. (2012) and Delatte et al. (2017), government debt level has an non-linear effect on sovereign bond spreads. For all sample countries except Italy, a higher Debt-to-GDP ratio corresponds to a higher probability of entering the crisis regime. A country's own past state largely affects the transition probability. A normal state is much more likely to be followed by a normal state, while a crisis state increases the probability of getting another crisis state in the next period by a large margin. Apart from a country's own past states, other countries' past states also significantly affect the country's transition probability. There is strong evidence of multi-country contagion. Except for Greece, other countries' transition probabilities to crisis regime all significantly increase when at least one of their neighbors are in crisis in the past state. Greece is not affected by multi-country contagion since its break comes earlier than others, and conditional on its own past states, other countries' past states do not have an additional effect on its transition probability. For France and Spain, the contagion effect on transition probability is equivalent to an increase in the Debt-to-GDP ratio of around 20%. Portugal and Ireland are subject to an even large contagion effect, and its effect on transition probability is equivalent to an increase in the Debt-to-GDP ratio of around 40%. To better interpret the intensity of contagion, I conduct a partial effect analysis since the coefficients alone in the non-linear regime-switching process could be less indicative. Table 6 shows the partial effect (PE) of contagion on transition probabilities when countries' other switching variables are at different percentiles. Since a country's own past state is discrete, we separately analyze the partial effect on transition probability from normal to crisis and crisis to crisis. The economic magnitude of the contagion effect is large. It contributes to more than a 10% increase in transition probability from normal to crisis regime for France, Spain, Portugal, and Ireland, given their Debt-to-GDP ratio at any percentiles considered. Especially for Portugal and Ireland, multi-country contagion is associated with more than 25% increases in transition probabilities when their Debt-to-GDP ratio is at the median level. The incremental effect of contagion is smaller when Portugal's debt level is high, as its own fundamentals now contribute a lot to the switching. The contagion effect on transition probability from crisis to crisis regime is also sizeable, although smaller for most countries. The smaller partial effect is because the probability of staying in the crisis regime is already high, making the incremental effect of contagion smaller in magnitude. Overall, I find strong evidence of multi-country contagion during the sample period. It plays a more important role than common risk factors and even country-specific fundamentals in determining the transition probability to crisis regime.

	France(FR)	Spain(ES)	Italy(IT)	Portugal(PT)	Ireland(IE)	Greece(GR)
γ_1	**-1.895**	**-2.071**	**-2.161**	**-1.672**	**-1.841**	**-1.648**
	[-1.939,-1.855]	[-2.094, -2.047]	[-2.214, -2.076]	[-1.738, -1.614]	[-1.897, -1.810]	[-1.717,-1.621]
γ_2	**-0.008**	0.007	0.005	-0.006	-0.000	0.008
	[-0.0168, -0.002]	[-0.004, 0.016]	[-0.002, 0.012]	[-0.015,0.005]	[-0.011, 0.014]	[-0.002, 0.014]
γ_3	0.001	-0.000	0.002	-0.001	-0.001	-0.002
	[-0.001,0.004]	[-0.002, 0.001]	[-0.001, 0.004]	[-0.002, 0.001]	[-0.003, 0.001]	[-0.003,-0.000]
γ_4	**0.028**	**0.026**	-0.004	**0.027**	**0.019**	**0.025**
	[0.025, 0.030]	[0.023, 0.031]	[-0.007, 0.001]	[0.026, 0.028]	[0.018, 0.020]	[0.021, 0.027]
γ_5	**1.586**	**1.896**	**2.407**	**0.827**	**1.353**	**2.052**
	[1.458,1.741]	[1.732,2.133]	[2.312, 2.484]	[0.732, 0.985]	[1.254,1.489]	[1.964, 2.128]
λ	**0.444**	**0.410**	**0.531**	**0.872**	**0.671**	0.041
	[0.337, 0.509]	[0.339,0.455]	[0.469, 0.590]	[0.794,0.922]	[0.582, 0.748]	[-0.046, 0.129]

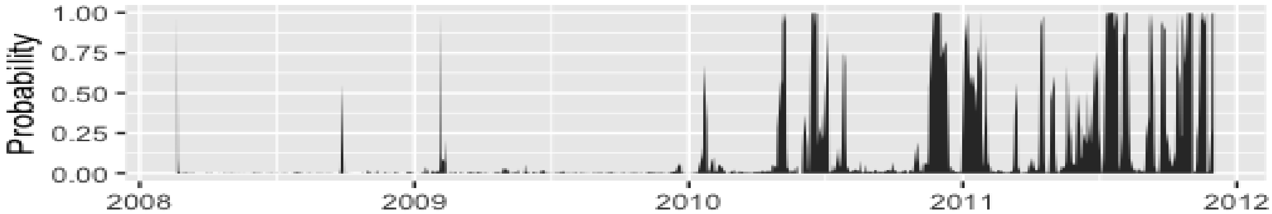
Table 5: 95% credible intervals and posterior mean of the coefficients in the auxiliary regime-switching equation. γ_1 is the constant of the switching equation, γ_2 is the coefficient on the lagged first-differenced spread between Euribor and ENOIA, γ_3 is the coefficient on the lagged first-differenced VSTOXX Index, γ_4 is the coefficient on the lagged Debt-to-GDP ratio and γ_5 is the coefficient on own past state. λ is the coefficient of the contagion effect. Coefficients significant at the 95 % level are in bold.

	Coefficient (λ_i)	PE at 50^{th}	PE at 75^{th}	PE at 90^{th}
(1) Normal to Crisis				
France(FR)	0.444	14.50%	13.60%	13.41%
Spain(ES)	0.410	14.49%	15.59%	16.10%
Italy(IT)	0.531	3.62%	3.62%	3.62%
Portugal(PT)	0.872	20.76%	14.24%	8.57%
Ireland(IE)	0.671	25.02%	26.00%	25.19%
(1) Crisis to Crisis				
France(FR)	0.444	1.59%	1.28%	1.23%
Spain(ES)	0.410	6.05%	4.54%	3.53%
Italy(IT)	0.531	18.42%	18.42%	18.42%
Portugal(PT)	0.872	6.83%	3.60%	1.64%
Ireland(IE)	0.671	15.73%	8.85%	7.07%

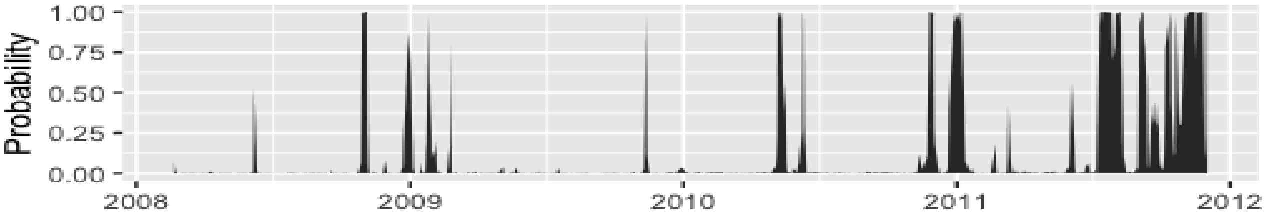
Table 6: Partial effect (PE) of contagion at different percentiles of other regime-driving variables. For each sample country, the effect on transition probability from normal to crisis and crisis to crisis are given the the upper panel (1) and lower panel (2), respectively. I consider the lagged Debt-to-GDP ratio at their 50^{th}, 75^{th} and 90^{th} percentiles.



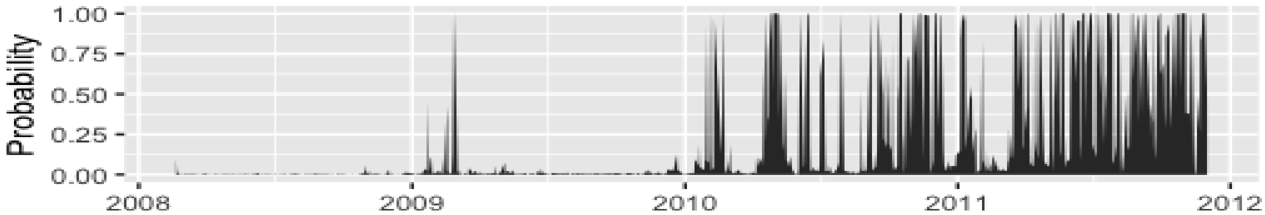
(a) France



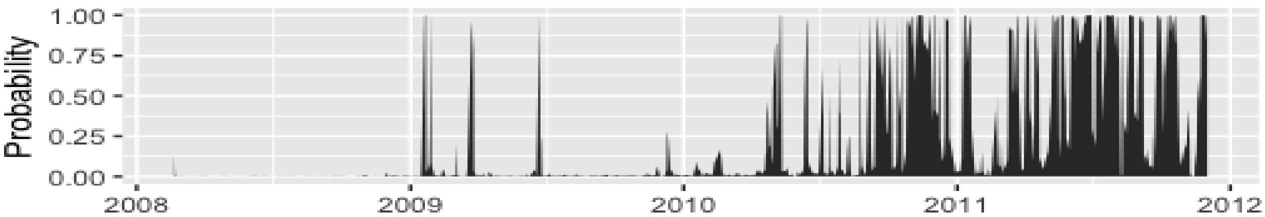
(b) Spain



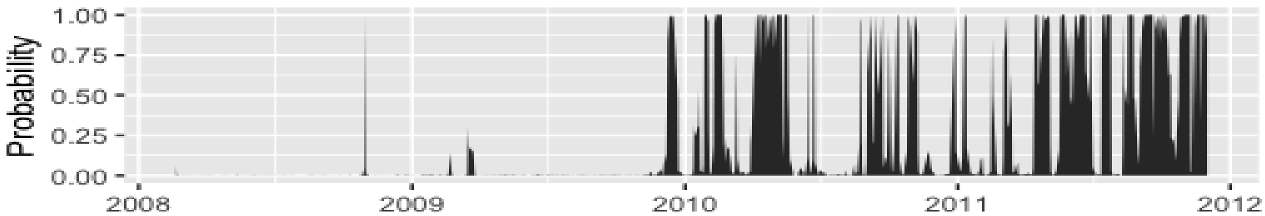
(c) Italy



(d) Portugal



(e) Ireland



(f) Greece

Figure 3: Country-specific smoothed probabilities for crisis regime.

The model also produces country-specific probabilities for each regime. Figure 3 reports the crisis probability for each country. These figures provide some interesting results. Firstly, sample countries' regimes are not fully synchronized, and there is a considerable degree of heterogeneity in the cross-sectional regime-switching patterns. Secondly, the crisis regime is not continuous, hindering the usefulness of sample splitting type of contagion test as they rely on *ex-post* identification of the crisis regime, which implicitly assumes that the crisis regime is continuous and homogeneous. These figures also reveal why Greece is not subject to the contagion effect from others. At the end of 2009, while other countries are in the normal regime, Greece is the first country that enters the crisis regime as its trouble reveals in December 2009, when it admits its debts have reached $300bn$ euros, the highest in modern history. Thus, conditional on its own past states, other countries' states do not affect Greece's transition probability anymore.

5 Conclusion

This paper contributes by proposing a new methodological framework of the multi-country contagion problem. It develops a procedure to test and quantify contagion based on a mutual exciting regime-switching model. Contagion is defined as a rise in the transition probability to the crisis regime when other countries are in crisis in the past state. The model has several advantages. Firstly, it does not rely on *ex-post* identification of the crisis regime—this type of identification scheme implicitly assume that the crisis regime is continuous and homogeneous. However, from the empirical results, we can see that the country-specific crisis regime is neither continuous nor homogeneous. Secondly, different from the strand of literature that assumes the crisis regime is associated with some extreme values of the observed dependent variable, we let the crisis state to be an unobserved stochastic process. This is motivated by the fact that crises, in many cases, are more complicated and can not be sufficiently captured by the tail events of one particular value. Thirdly, we explicitly model multi-country contagion as a source of regime-switching. We can quantitatively analyze the roles of different mechanisms, especially multi-country contagion, in determining the transition process. Lastly, this framework accommodates a rich contagion pattern. In addition to a break in the mean equation, which is the parameters instability in the asset pricing equation, the model also accommodates a break in the second moment. The dynamics in the variance could be especially important for crisis episodes where increases in volatility are the major symptomatic.

The project also contributes from an empirical point of view. There is an extensive body of research examining sovereign bond prices in the context of the European debt crisis and whether there is a contagion effect remains the center of the debate. Empirical evidence is very much mixed. The empirical study in this paper provides some new important findings. First of all, there are striking shifts in market pricing behaviors. There is not only a jump in the intercept but also breaks in the exposures to common risk factors and the intensities of the regional spillover effect. It is vitally important to take into account this regime-dependent pricing behavior. Secondly, countries are subject to a strong contagion effect. Actually, contagion plays a more important role than common risk factors and country-specific fundamentals in determining their transition probabilities to the crisis regime.

Appendices

(A) Supplementary figures and tables

Country	Public Debt (%GDP, average 2008-2012)	GDP Growth(%, average 2008-2012)	Fiscal Position(%GDP, average 2008-2012)	Credit Ratings, 2012 (Moody's, Fitch, S&P)
Germany	70.2	0.7	-1.7	Aaa,AAA,AAA
France	75.9	0.6	-5.4	Aaa,AAA,AA+
Italy	108.7	0.6	-3.7	A3,A+,BBB+
Spain	47.5	0.6	-7.9	A3,AA-,A
Ireland	55.0	-0.3	-14.1	Ba1,BBB+,BBB+
Portugal	81.1	0.5	-7.4	Ba3,BBB-,BB
Greece	123.1	0.2	-10.9	Ca,CCC,CC

Table 7: Macroeconomic summary statistics for sample countries (source: ECB)

Variable	Augmented Dickey-Fuller		Phillips-Perron	
	Level	First Difference	Level	First Difference
FR	-2.16	-8.58***	-11.7	-753***
ES	-2.37	-9.41***	-12.5	-616***
IT	-0.09	-9.75***	-2.97	-682 ***
PT	-0.59	-8.98***	-2.44	-600***
IE	-1.83	-9.25***	-8.01	-730***
GR	0.58	-10.22***	-0.31	-611***

Table 8: Stationarity tests of 10-year sovereign bond spreads for six sample countries. ***, **, * denote the rejection of unit root hypothesis at the 1%, 5%, 10% level of significance, respectively.

(B) Prior Distributions

Parameters ($s_{it} = 0$)	Mean	Std.dev	Shift Parameters	Mean	Std.dev
α	0	10	$\Delta\alpha$	0.1	10
β_1	0	10	$\Delta\beta_1$	0	10
β_2	0	10	$\Delta\beta_2$	0	10
$\beta_{FR,1}$	0	10	$\Delta\beta_{FR,1}$	0	10
$\beta_{FR,2}$	0	10	$\Delta\beta_{FR,2}$	0	10
$\beta_{ES,1}$	0	10	$\Delta\beta_{ES,1}$	0	10
$\beta_{ES,2}$	0	10	$\Delta\beta_{ES,2}$	0	10
$\beta_{IT,1}$	0	10	$\Delta\beta_{IT,1}$	0	10
$\beta_{IT,2}$	0	10	$\Delta\beta_{IT,2}$	0	10
$\beta_{PT,1}$	0	10	$\Delta\beta_{PT,1}$	0	10
$\beta_{PT,2}$	0	10	$\Delta\beta_{PT,2}$	0	10
$\beta_{IE,1}$	0	10	$\Delta\beta_{IE,1}$	0	10
$\beta_{IE,2}$	0	10	$\Delta\beta_{IE,2}$	0	10
$\beta_{GR,1}$	0	10	$\Delta\beta_{GR,1}$	0	10
$\beta_{GR,2}$	0	10	$\Delta\beta_{GR,2}$	0	10
$-$			h	1.2	10

Table 9: Mean and standard deviation of priors on the main equation. The table shows the prior distributions for a typical country. We use the same sets of prior distributions for each sample country.

Parameters ($s_{it} = 0$)	Mean	Std.dev
γ_1	2	10
γ_2	0	10
γ_3	0	10
γ_4	0	10
γ_5	-2	10
λ	0	10

Table 10: Mean and standard deviation of priors on the auxiliary regime-switching equation. The table shows the prior distributions for a typical country. We use the same sets of prior distributions for each sample country.

(C) Sampling from Full Conditionals

To sample from the joint posterior distribution of full parameters $\boldsymbol{\psi}$ given data, we sample from the following conditional posteriors iteratively:

(1) $\boldsymbol{\psi}_1 = \{\boldsymbol{\alpha}, \boldsymbol{\beta}\}$

We first rearrange equation (1) as a linear regression model given other parameters,

$$\begin{aligned} &h_1^{-1}(s_{1t})(y_{1t} - \alpha_{1,0} - \alpha_{1,1}s_{1t} - \boldsymbol{x}_{1t}'\boldsymbol{\beta}_{1,0} - (\boldsymbol{x}_{1t} * s_{1t})'\boldsymbol{\beta}_{1,1}) = \epsilon_{1t} \\ &\vdots \\ &h_n^{-1}(s_{nt})(y_{nt} - \alpha_{n,0} - \alpha_{n,1}s_{nt} - \boldsymbol{x}_{nt}'\boldsymbol{\beta}_{n,0} - (\boldsymbol{x}_{nt} * s_{nt})'\boldsymbol{\beta}_{n,1}) = \epsilon_{nt} \end{aligned} \tag{11}$$

Let $\tilde{y}_{it} = h_i^{-1}(s_{it}) * y_{it}$, $\tilde{\boldsymbol{x}}_{it} = (h_i^{-1}(s_{it}), h_i^{-1}(s_{it}) * s_{it}, h_i^{-1}(s_{it})\boldsymbol{x}_{it}', h_i^{-1}(s_{it}) * s_{it} * \boldsymbol{x}_{it}')'$. Let $\tilde{Y}_i = (\tilde{y}_{i1}, \ldots, \tilde{y}_{iT})'$ and $\tilde{X}_i = (\tilde{\boldsymbol{x}}_{i1}, \ldots, \tilde{\boldsymbol{x}}_{iT})'$. Denote $\tilde{\boldsymbol{\beta}}_i = (\alpha_{i,0}, \alpha_{i,1}, \boldsymbol{\beta}_{i,0}', \boldsymbol{\beta}_{i,1}')'$ so that equation (9) can be rewritten as:

$$\begin{aligned} &\tilde{y}_{1t} = \tilde{\boldsymbol{x}}_{1t}'\tilde{\boldsymbol{\beta}}_1 + \epsilon_{1t} \\ &\vdots \\ &\tilde{y}_{nt} = \tilde{\boldsymbol{x}}_{nt}'\tilde{\boldsymbol{\beta}}_n + \epsilon_{nt} \end{aligned} \tag{12}$$

Given ψ_2, ψ_3, ψ_4, with $\tilde{y}_{it}$ and $\tilde{\boldsymbol{x}}_{it}$ being observed, equation (10) is a system of linear regressions with known variance covariance matrix. I use normal prior on $\tilde{\boldsymbol{\beta}}_i$

$$\tilde{\boldsymbol{\beta}}_i \sim \mathcal{N}(\tilde{\boldsymbol{\beta}}_i^0, \sigma_i^2 P_i^0) \text{ for } i = 1, \ldots, n \tag{13}$$

Subscript 0 and 1 represent the parameters for regime 0 and 1, respectively. Superscript 0 and 1 indicate the prior and posterior, respectively. σ_i^2 is the ith diagonal entry of Σ. The posterior distribution of $\tilde{\boldsymbol{\beta}}_i$ is given by:

$$\tilde{\boldsymbol{\beta}}_i \sim \mathcal{N}(\tilde{\boldsymbol{\beta}}_i^1, \sigma_i^2 P_i^1) \text{ for } i = 1, \ldots, n \tag{14}$$

where $P_i^1 = ((P_i^0)^{-1} + \tilde{X}_i'\tilde{X}_i)^{-1}$ and $\tilde{\boldsymbol{\beta}}_i^1 = P_i^1((P_i^0)^{-1}\tilde{\boldsymbol{\beta}}_i^0 + \tilde{X}_i'\tilde{Y}_i)$

(2) $\boldsymbol{\psi}_2 = \{\Sigma\}$

Given ψ_1, ψ_3, ψ_4, $\boldsymbol{y_t} \in \mathbb{R}^n$ for $t = 1, \ldots, T$ follows a multivariate normal distribution with known mean $\boldsymbol{\mu} \in \mathbb{R}^n$.

$$\boldsymbol{y_t} = \begin{bmatrix} y_{1t} \\ \vdots \\ y_{nt} \end{bmatrix} \sim \mathcal{N}(\boldsymbol{\mu}, \Sigma) \tag{15}$$

The natural conjugate prior for a covariance matrix is the inverse Wishart (IW) prior, thus I impose:

$$\Sigma \sim \mathcal{IW}(\nu^0, \Lambda^0) \tag{16}$$

And the posterior of Σ is

$$\Sigma \sim \mathcal{IW}(T + \nu^0, S_\mu + \Lambda^0) \tag{17}$$

where $S_\mu = \sum_{t=1}^T (\boldsymbol{y_t} - \boldsymbol{\mu})(\boldsymbol{y_t} - \boldsymbol{\mu})'$ and $\boldsymbol{\mu} = (\mu_1, \ldots, \mu_n)' \in \mathbb{R}^n$ with $\mu_i = h_i^{-1}(s_{it})(\alpha_{i,0} - \alpha_{i,1}s_{it} - \boldsymbol{x}_{it}'\boldsymbol{\beta}_{i,0} - (\boldsymbol{x}_{it} * s_{it})'\boldsymbol{\beta}_{i,1})$.

(3) $\boldsymbol{\psi}_3 = \{\boldsymbol{h}\}$

$$y_{it} - \alpha_{i,0} - \alpha_{i,1}s_{it} - \boldsymbol{x}_{it}'\boldsymbol{\beta}_{i,0} - (\boldsymbol{x}_{it} * s_{it})'\boldsymbol{\beta}_{i,1} = h_i(s_{it})\epsilon_{it} \tag{18}$$

Denote $\check{x}_i = y_{it} - \alpha_{i,0} - \alpha_{i,1}s_{it} - \boldsymbol{x}'_{it}\boldsymbol{\beta}_{i,0} - (\boldsymbol{x}_{it} * s_{it})'\boldsymbol{\beta}_{i,1}$. Given ψ_1, ψ_3 and ψ_4, and with $\check{x}_i$ being is observed, and a natural way to interpret equation h_i is that it is the ratio of the standard deviation of $\check{x}_{it}$ under high and low volatility regime:

$$h_i = \frac{\sigma(\check{x}_{it}\mathcal{I}(s_{it} = 1))}{\sigma(\check{x}_{it}\mathcal{I}(s_{it} = 0))} \text{ for } i = 1, \ldots, n \tag{19}$$

Given ψ_2, the variance of low volatility regime is known, which is given by the diagonal entries of Σ. Further condition on ψ_1 and ψ_3, the inference on h_i boils down to the inference of the variance of a normally distributed univariate random variable with known mean.

$$\check{x}_{it}\mathcal{I}(s_{it} = 1) \sim \mathcal{N}(\check{\mu}_i, \check{\sigma_i}^2) \text{ for } i = 1, \ldots, n \tag{20}$$

We impose an inverse gamma prior:

$$\check{\sigma}_i^2 \sim g^{-1}(\frac{\nu^0}{2}, \frac{\nu^0(\sigma^0)^2}{2}) \text{ for } i = 1, \ldots, n \tag{21}$$

which leads to an inverse gamma posterior:

$$\check{\sigma}_i^2 \sim g^{-1}(\frac{\nu^1}{2}, \frac{\nu^0(\sigma^1)^2}{2}) \text{ for } i = 1, \ldots, n \tag{22}$$

where $\nu^1 = \nu^0 + T$, and $(\sigma^1)^2 = \frac{1}{\nu^1}(\nu^0(\sigma^0)^2 + \sum_{i \in \mathbb{B}_i}(\check{x}_{it}\mathcal{I}(s_{it} = 1) - \check{\mu}_i)^2$. We denote $\mathbb{B}_i = \{i \mid s_{it} = 1\}$ as the set of high volatility observations for country i. Since our identification restriction is $h_i > 1$, we keep drawing from the posterior until such restriction is satisfied.

(4) $\boldsymbol{\psi}_4 = \{S_T\}$

The key feature of simulation-based Bayesian inference of hidden Markov model is the simulation of the states from the joint conditional distribution of all states given other parameters in the model. The procedure for drawing states is based on Albert and Chib (1993). We avoid the intractable simulation of the whole chain at a time by drawing a single state at each step recursively. The conditional distribution that we hope to simulate from is $P(S_T \mid \boldsymbol{\psi}_{-4})$, which could be written as:

$$\begin{aligned} P(S_T \mid \boldsymbol{\psi}_{-4}, \Omega_T) = {} & P(s_{1T}, \ldots, s_{nT} \mid \boldsymbol{\psi}_{-4}, \Omega_T) \times \ldots \\ & \times P(s_{1t}, \ldots, s_{nt} \mid \boldsymbol{\psi}_{-4}, \Omega_T, S^{t+1}) \times \ldots \\ & \times P(s_{11}, \ldots, s_{n1} \mid \boldsymbol{\psi}_{-4}, \Omega_T, S^2) \end{aligned} \tag{23}$$

where $S_t = (s_{11}, \ldots, s_{n1}, \ldots, s_{1t}, \ldots, s_{nt})$ is the history of states up to time t, as defined earlier. And $S^{t+1} = (s_{1t+1}, \ldots, s_{nt+1}, \ldots, s_{1T}, \ldots, s_{nT})$ is the future of states from $t+1$ to T. $\Omega_t = (Y_t, X_t, Z_t)$, which is the collection of information on dependent, independent variables and exogenous drivers of the regime switching process up to time t. A typical elements in equation (21), excluding the terminal point, is

$$P(s_{1t}, \ldots, s_{nt} \mid \boldsymbol{\psi}_{-4}, \Omega_T, S^{t+1}) \tag{24}$$

By the argument in Albert and Chib (1993),

$$\begin{aligned} & P(s_{1t}, \ldots, s_{nt} \mid \boldsymbol{\psi}_{-4}, \Omega_T, S^{t+1}) \propto \\ & P(s_{1t}, \ldots, s_{nt} \mid \boldsymbol{\psi}_{-4}, \Omega_t) \times P(s_{1t+1}, \ldots, s_{nt+1} \mid s_{1t}, s_{2t}, \Omega_t, \boldsymbol{\psi}_{-4}) \end{aligned} \tag{25}$$

$P(s_{1t}, \ldots, s_{nt} \mid \boldsymbol{\psi}_{-4}, S^{t+1})$ is the product of two terms. The first term is the mass function of $(s_{1t}, \ldots, s_{nt})$ given Ω_t and other parameters in the model. This term can be derived iteratively by a prediction step and an update step. These mass functions $P(s_{1t},\dots,s_{nt} \mid \boldsymbol{\psi}_{-4},\Omega_t)_{t=1}^{T}$ are stored in a $T \times 2^n$ matrix F since there are 2^n possible combinations of $(s_{1t},\dots,s_{nt})$ for each t. The second term is the transition probability, which can be derived given ψ_6. The last state $(s_{1T},\dots,s_{nT})$ is simulated using $P(s_{1T},\dots,s_{nT} \mid \boldsymbol{\psi}_{-4},\Omega_T)$, which is the last row of F. And then the remaining states can be simulated using equation (23).

(5) $\boldsymbol{\theta}_5 = \{S_T^*\}$

Performing direct inference on $\{\boldsymbol{\gamma},\boldsymbol{\lambda}\}$ is complicated since no conjugate prior exists for the parameters of the auxiliary probit regression model. In the spirit of Kaufmann (2015), I overcome this problem by augmenting the original model in the following way:

$$s_{it}^* = \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j\neq i}\lambda_{ij} * s_{j,t-1} + u_{it} \text{ for } i = 1,\dots,n \tag{26}$$

Given ψ_4,ψ_6, $\boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j\neq i}\lambda_{ij} * s_{j,t-1}$ can be calculated. And u_{it} is draw from a standard normal distribution that is consistent with the prediction of the random utility model. That is, a draw u_{it} will be accepted only if

$$\begin{cases} \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j\neq i}\lambda_{ij} * s_{j,t-1} + u_{it} \geq 0 \text{ if } s_{it} = 1 \\ \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j\neq i}\lambda_{ij} * s_{j,t-1} + u_{it} < 0 \text{ if } s_{it} = 0 \text{ for } i = 1,\dots,n \end{cases} \tag{27}$$

When we have our first accepted draw $u_{it}^{(1)}$, the latent index s_{it}^* is computed as $s_{it}^* = \boldsymbol{z_{it}}'\boldsymbol{\gamma_i} + \sum_{j\neq i}\lambda_{ij} * s_{j,t-1} + u_{it}^{(1)}$

(6) $\boldsymbol{\psi}_6 = \{\boldsymbol{\gamma},\boldsymbol{\lambda}\}$

$$s_{it}^* = \tilde{\boldsymbol{z}}_{it}'\tilde{\gamma}_i + u_{it} \text{ for } i = 1,\dots,n \tag{28}$$

Given ψ_4 and ψ_5, we have a linear regression model, where $\tilde{\boldsymbol{z}}_{it} = (\boldsymbol{z_{it}}', \boldsymbol{s_{-i}}')'$ and s_{it}^* are observed. $\tilde{\gamma}_i = (\boldsymbol{\gamma}_i', \boldsymbol{\lambda_i}')'$. Again, I impose a conjugate normal prior:

$$\tilde{\gamma}_i \sim \mathcal{N}(\tilde{\gamma}_i^0, M_i^0) \text{ for } i = 1,\dots,n \tag{29}$$

which leads to normal posterior:

$$\tilde{\gamma}_i \sim \mathcal{N}(\tilde{\gamma}_i^1, M_i^1) \text{ for } i = 1,\dots,n \tag{30}$$

where $M_i^1 = ((M_i^0)^{-1} + \tilde{Z}_i'\tilde{Z}_i)^{-1}$ and $\tilde{\boldsymbol{\gamma}}_i^1 = M_i^1((M_i^0)^{-1}\tilde{\boldsymbol{\gamma}}_i^0 + \tilde{Z}_i'S^*)$

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