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Jun Chen
Renmin University of China

Matthew Elliott
University of Cambridge

Andrew Koh
Massachusetts Institute of Technology

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CAPABILITY ACCUMULATION AND CONGLOMERATIZATION IN THE INFORMATION AGE*

Jun Chen†, Matthew Elliott‡ and Andrew Koh§

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Abstract

The past twenty years have witnessed the emergence of internet conglomerates fueled by acquisitions. We build a simple model of network formation to study this. We endow firms with scarce capabilities which drive their competitiveness across markets. Firms can merge to combine their capabilities, spin-off new firms by partitioning their capabilities, or procure unassigned capabilities. We study stable industry structures in which no such deviations are profitable. We find an upper and lower bound on the size of the largest firm, and show that as markets value more of the same capabilities abrupt increases in these bounds occur.

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†Renmin University of China.

‡Cambridge University.

§Massachusetts Institute of Technology.

1 Introduction

Someone in 1980s America might have interacted with dozens of different companies in a typical day. In the near future it is not unthinkable for someone to wake up in an Amazon-sourced apartment, check the news on her Facebook Feed and then hail a Google-operated self-driving car on her Apple iPhone to pick up groceries at an Amazon Supermarket paid for with Apple Pay. She might then work from home on her Apple Macbook, collaborating with her co-workers via Google Sheets or on a server hosted on Amazon Web Services. In the evening she might order-in dinner via Amazon Fresh, chat with her parents over Microsoft's Skype and then unwind over in-house content on Amazon Prime, or with a good book on her Amazon Kindle.

In this paper we build a parsimonious theory of industry structure—which firms emerge from merger activity and in what markets they compete—to better understand the rapid emergence of internet conglomerates and their potential longevity. Our theory is based on the resource-based view of competitive advantage in the management literature, pioneered by Wernerfelt (1984), Prahalad and Hamel (1990) and Barney (1991).[1] This is a hugely influential literature that forms a core part of MBA and executive education syllabi, and hence is salient to many managers and executives when contemplating merger and acquisition possibilities. The fundamental idea is that different firms have different immutable and scarce resources or capabilities, and it is these capabilities that deliver competitive advantage and profits. Such capabilities might include unique forms of human capital, production know-how, patents, a strong brand value, a customer base (especially in the context of network externalities or switching costs), and so on.

We endow each firm with a set of capabilities. Not all markets will value all capabilities so we also associate each market with a specific set of capabilities—those valued by that market. For example, a team of molecular biologists might provide a firm with a competitive advantage in biotech markets, but they are unlikely to be a source of competitive advantage for the firm in markets for financial services. We let the competitiveness of a firm in a given market depend on its *relevant capabilities*—the set of capabilities which it both possesses, and are valued by the market. We model this using a pair of hypergraphs to represent (i) which firms have which capabilities; and (ii) which markets value which capabilities. To the best of our knowledge, we are the first to use hypergraphs in this way.[2]

We let firms reorganize their capabilities through *mergers*, *demergers*, *procurements*, and *entries*. We are interested in *stable industry structures* for which there are no profitable mergers, demergers, procurements or entries.[3] Firms pay fixed costs to maintain their capabilities, and

[1] At the time of writing the combined Google Scholar citation count for the aforementioned papers stood at over 150,000 citations. These ideas have received relatively little attention in the economics literature. Exceptions include Goyal et al. (2008), Sutton (2012), Nocke and Yeaple (2007, 2014).

[2] There are only a few papers in the economics literature which use hypergraphs. The closest are probably Malamud and Rostek (2017) and Rostek and Yoon (2020). They use hypergraphs to model financial exchanges, and competition across them.

[3] This approach follows much of the network formation literature. For example, stable financial networks are studied in Cabrales et al. (2017); Farboodi (2017); Erol and Vohra (2018); Elliott et al. (2021) and stable production networks in Carvalho and Voigtländer (2015); Oberfield (2018); Acemoglu and Azar (2020); Elliott

we assume these are increasing and convex in the number of capabilities maintained. Combining capabilities can enable synergies to be realized yielding a more competitive firm across multiple markets, but combining unrelated capabilities incurs additional fixed costs without delivering any benefits.

Our contention is that markets have become more connected in terms of the capabilities they value. First, there are new capabilities that are becoming valued by multiple markets. For example, large data are now valued by healthcare providers who use the data to aid in diagnosis and treatment recommendations, while they are also valuable in advertising markets for targeting adverts. Second, existing capabilities have become valued by new markets. Capabilities—patents for example—associated with implementing payments via tap-and-go technology have linked the market for smart phones to that for payment systems. Third, new markets have arisen which link otherwise unconnected markets. For example, the new self-driving car market values capabilities associated with image recognition as well as those associated with the traditional production of automobiles.

First, we show that this increased connectivity of markets in terms of the capabilities they value can help explain the emergence of internet conglomerates. To take a specific example, consider the acquisition of Whole Foods in 2017 by Amazon for \$13.7B. Arguably, this would have made little sense twenty years ago—Amazon and Whole Foods would, at that time, have had little opportunity to gain competitive advantage by combining their capabilities. This is no longer the case. The merger has allowed Amazon to combine its digital and e-commerce capabilities with the Whole Foods brand, sourcing, and network of stores. Amazon's proprietary data on the shopping habits and interests of Whole Foods customers can help adverts and offers be better targeted (as is now standard), while its logistical and distribution network are valuable for offering online grocery shopping.

Second, we show that the emergence of large conglomerates can be abrupt and brought about by small changes in market connectedness. To do this we take a random-graph approach to modeling the changes in the capabilities that markets value. The size of firms that can be supported in stable industry structures and the number of markets these firms compete in undergo a phase transition. This can help explain the rapid expansion of internet conglomerates. Moreover, as these changes in industry structure are driven by fundamental changes to markets, the theory suggests the changes are likely to persist absent government intervention, at least in the medium run.

Although our focus is different, our approach is related to, and to some extent builds on, the networked markets literature (e.g., Kranton and Minehart, 2001; Elliott, 2015; Nava, 2015; Condorelli et al., 2016; Bimpikis et al., 2019; Goyal, 2017). A recent literature also studies the rise of large firms with market power. For example, Crouzet and Eberly (2019) and De Ridder (2019) attribute rising industry concentration to intangible capital such as intellectual property, branding, and software, while Bessen (2017) and Lashkari et al. (2018) both find that proprietary information technology can explain much of the observed rise in market concen-

et al. (2020); Acemoglu and Tahbaz-Salehi (2020).

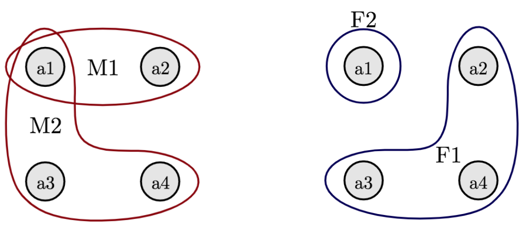
tration in US and French firms respectively. These findings are consistent with our theoretical results.

2 Model

2.1 Capabilities, Firms, and Markets. There is a finite set of capabilities $\mathcal{A}$. These are the hard-to-imitate drivers of competitive advantage. There is a finite set of firms $\mathcal{N} = \{1...n\}$. Each firm i is endowed with some set of capabilities which we denote $F_i \subseteq \mathcal{A}$. We assume $\mathcal{A}$ can be partitioned into $(\{F_i\}_{i=1}^n, S)$ where S is the set of unassigned capabilities not held by any firm. This implies that firms hold disjoint capabilities, i.e., for any two firms i, i', $F_i \cap F_k = \emptyset$. There is also a finite set of markets $\mathcal{M} = \{1...m\}$. Each market j is also associated with some set of capabilities $M_j \subseteq \mathcal{A}$. This represents the capabilities which market j values, and are thus a source of competitive advantage in j. We allow markets to value overlapping capabilities, i.e., we could have $M_j \cap M_{j'} \neq \emptyset$ for $j \neq j'$. It will be helpful to represent this information with a pair of hypergraphs. Call $H_F := \{\mathcal{A}, \{F_1, F_2, ..., F_n\}\}$ the firm hypergraph and $H_M := \{\mathcal{A}, \{M_1, M_2, ...M_m\}\}$ the market hypergraph. Without loss of generality we let each capability be valued by at least one market. Fixing the capabilities of all firms, let $\theta_{ij} := F_i \cap M_j$ be the capabilities firm i possesses which are relevant to market j. Figure 1 illustrates our model.

Figure 1: Representation of firm and market hypergraphs

Notes: This figure illustrates our model. The set of capabilities is $\mathcal{A} = \{a_1, a_2, a_3, a_4\}$. Panel (a) shows the market hypergraph with two markets: $M_1 = \{a_1, a_2\}$ and $M_2 = \{a_1, a_3, a_4\}$. Panel (b) shows the firm hypergraph with two disjoint firms: $F_1 = \{a_2, a_3, a_4\}$ and $F_2 = \{a_1\}$. For illustration, consider competition in market 2: firm 1 has the relevant capabilities $\theta_{12} = F_1 \cap M_2 = \{a_3, a_4\}$ while firm 2 has the relevant capability $\theta_{22} = \{a_1\}$.



(a) Market hypergraph. (b) Firm hypergraph.

We will use $|F_{max}|$ to denote the number of capabilities held by the largest firm. It will be helpful to explicitly define components for the market hypergraph H_M. Define a path between any two nodes $a_1, a_n \in \mathcal{A}$ on H_M as a tuple $(a_1, M_1, a_2, \ldots, a_{n-1}, M_{n-1}, a_n)$, such that for $1 \leq i \leq n-1$, $a_i, a_{i+1} \in M_i$. Then given a market hypergraph H_M, a component of H_M is a subset of the nodes $A \subseteq \mathcal{A}$ such that (i) A and $\mathcal{A} \setminus A$ are disconnected (i.e., there is no path between any pair of nodes $a_1 \in A$ and $a_k \in \mathcal{A} \setminus A$); and (ii) A is self-connected (i.e., for any pair $a_1, a_k \in A$, there exists a path through edges restricted to A). Let $\{C_1, \ldots, C_p\}$ be the set of all components and let $\mathcal{P} = \{1 \ldots p\}$. Since there is a finite set of capabilities, there exists a component with weakly more capabilities than any other. We denote the number of capabilities in a largest component of a hypergraph by $|C_{max}|$.

2.2 Timing. We consider a two stage model. In the first stage firms rearrange their capabilities through merging, demerging, procurements and entries. In the second stage, firms' capabilities are assumed fixed and they compete across the different markets.

2.3 Second Stage Competition. We assume firms compete in each market based on a profit function that is defined on the joint distribution of firms' capabilities relevant for that market. Specifically, let $\Theta_j = 2^{M_j}$ denote the power set of capabilities valued by market j. Firms' profits in market j are given by the vector-valued function $\pi_j : \Theta_j^n \to \mathbb{R}_{\geq 0}^n$ which maps the vector of each firm's relevant capabilities $\boldsymbol{\theta}_j := (\theta_{ij})_{i=1}^n \in \Theta_j^n$ to a vector of profits. We define $\boldsymbol{\theta}_{Sj} := (\theta_{ij})_{i \in S}$ and $-i := \{k \in \mathcal{N} : k \neq i\}$, and use $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})$ to denote the i-th entry of the profit function. Noting that the hypergraph pair (H_M, H_F) pins down $\boldsymbol{\theta}_j$, we will write $\pi_{ij}(H_M, H_F) := \pi_{ij}(\boldsymbol{\theta})$ to denote the *gross* profits i makes in market j given the capabilities that firms hold, and the capabilities that markets value. Then firm i's total *gross* profits $\sum_{j=1}^m \pi_{ij}(H_M, H_F)$ is simply the sum of its gross profits across each individual market. Finally, it will also be convenient to define $J_{i,\pi>0}(H_M, H_F) := \{j \in \mathcal{M} : \pi_{ij}(H_M, H_F) > 0\}$ as the markets firm i operates in, and $I_{j,\pi>0}(H_M, H_F) := \{i \in \mathcal{N} : \pi_{ij}(H_M, H_F) > 0\}$ as the firms operating in market j.

We impose the following assumptions on profits.

Assumption (Primitives on Profits)**.** *The profit functions $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})$ satisfy the following conditions:*

(i) ***Firms with no capabilities make 0 gross profits.***
$\pi_{ij}(\emptyset, \boldsymbol{\theta}_{-ij}) = 0$ *for all* $\boldsymbol{\theta}_{-ij} \in \Theta_j^{n-1}$.

(ii) ***Firms which are not operating in market j do not influence profits in j.*** *For all $\boldsymbol{\theta}_j$ and $\boldsymbol{\theta}'_j$ such that (i) $I_{j,\pi>0}(\boldsymbol{\theta}_j) = I_{j,\pi>0}(\boldsymbol{\theta}'_j)$ and (ii) $\boldsymbol{\theta}_{I_{j,\pi>0}j} = \boldsymbol{\theta}'_{I_{j,\pi>0}j}$, we have that for all firms i, $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij}) = \pi_{ij}(\theta'_{ij}, \boldsymbol{\theta}'_{-ij})$.*

(iii) ***Labels do not matter.*** *For any bijection $b : \{1, \ldots, n\} \to \{1, \ldots, n\}$ and all firms i, $\pi_{ij}(\boldsymbol{\theta}_j) = \pi_{b(i)j}(\boldsymbol{\theta}'_j)$ where $\boldsymbol{\theta}'_j := (\theta_{b(i)j})_{i=1}^n$.*

(iv) ***More capabilities or weaker competitors increase gross profits.***

- *If $\theta'_{ij} \supseteq \theta_{ij}$, then $\pi_{ij}(\theta'_{ij}, \boldsymbol{\theta}_{-ij}) \geq \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})$ for all $\boldsymbol{\theta}_{-ij} \in \Theta_j^{n-1}$.*
- *If $\theta'_{kj} \subseteq \theta_{kj}$ for all $k \neq i$, then $\pi_{ij}(\theta_{ij}, \boldsymbol{\theta}'_{-ij}) \geq \pi_{ij}(\theta_{ij}, \boldsymbol{\theta}_{-ij})$ for all $\theta_{ij} \in \Theta_j$.*

(v) ***Strongest monopoly achieves maximum gross profits.*** *For any non-empty $A \subseteq M_j$, $\pi_{ij}(A, \emptyset^{n-1}) > \sum_{k \in \mathcal{N}} \pi_{kj}(\boldsymbol{\theta_j})$ for all $\boldsymbol{\theta}_j \in \Theta_j^n$ such that*

- *$\bigcup_{k\in\mathcal{N}} \theta_{kj} \subseteq A$; and*
- *there does not exist $k \in \mathcal{N}$ such that $\theta_{kj} = A$.*

Assumption (v) states that a single monopolist wielding the capabilities A must generate more total profits in market j than any other configuration of firms who collectively hold at most A. This arises from two forces acting in the same direction: first, the monopolist is the strongest possible firm which can arise from configuring the capabilities A; second, the monopolist does not face any competition which might otherwise erode its profits. We view (i)-(v) as a basic desiderata of any model of competition and maintain them throughout as regularity assumptions; they are consistent with Cournot competition (see Online Appendix A).

We also make the following assumption to capture a conglomeratization cost associated with managing many capabilities.

Assumption 1 (Capability Maintenance Costs)**.** *Firm i bears cost $\kappa(|F_i|)$ where*

(i) $\kappa(0) = 0$;
(ii) *κ is strictly increasing; and*
(iii) *κ is convex, i.e. $\kappa(x) - \kappa(x-1) > \kappa(x-1) - \kappa(x-2)$ for all $x \geq 2$.*

Condition (iii) states that κ exhibits increasing differences, i.e., the cost of maintaining an extra capability is increasing in the number of existing capabilities. This may reflect a wide range of factors including the scarcity of management time or ability of the firm to tailor its corporate culture towards maintaining specific capabilities. Taking firm i's capability maintenance costs into account, we call $\sum_{j=1}^{m} \pi_{ij}(H_M, H_F) - \kappa(|F_i|)$ firm i's *net* profits.

2.4 First Stage Competition. We now endogenize the industry structure by allowing firms to undertake the following deviations:

Definition (Firm Actions)**.** *Firms can reorganize their capabilities through*

(i) ***Procurements.*** *A procurement by firm i lets it procure capabilities $A \subseteq S$. Firm i then has capabilities $F_i \cup A$ and the set of unassigned capabilities shrinks to $S' = S \setminus A$.*
(ii) ***Demergers.*** *A demerger by firm l lets it partition its capabilities among one or more new firms $\mathcal{F}$, while simultaneously disposing of unwanted capabilities denoted by D. As such, $F_l = D \cup \bigcup_{i\in\mathcal{F}} F_i$. The set of unassigned capabilities expands to $S' = S \cup D$. If $|\mathcal{F}| \leq 1$, we call this a **disposal**.*
(iii) ***Mergers.*** *A merger between firms i and k combines their capabilities and creates a new firm l where $F_l = F_i \cup F_k$.*
(iv) ***Entries.*** *An entry creates a new firm l endowed with capabilities $F_l \subseteq S$. The set of unassigned capabilities shrinks to $S' = S \setminus F_l$.*

We exclude trivial firm actions by requiring a firm action to change the firm hypergraph.

Definition (Stability). *We say an industry structure is stable if there is no strictly net profitable procurement, demerger, merger, or entry. An industry structure is unstable if it is not stable.*

In Section 3.2 we show that our main results continue to hold when coalitional deviations are permitted.

2.5 Modeling Choices. Both a strength and weakness of the resource-based view of competitive advantage is the broad interpretation of capabilities. Our modelling inherits this. We intend capabilities to capture a broad range of things rather than trying to provide a more descriptively accurate model for a partial set of capabilities. For example, in practice it might be possible to license some capabilities, like patents, but not others like production know-how. Human capital can decide to move to a new firm, while technological know-how might not be transferable. And so on. Likewise, the strength of the case for convex capability maintenance costs depends on the set of capabilities being considered. For very similar capabilities, there may instead be some economies of scale. Nevertheless, overall, we expect a range of different types of capabilities to often matter in markets and view convexity as a reasonable approximation.

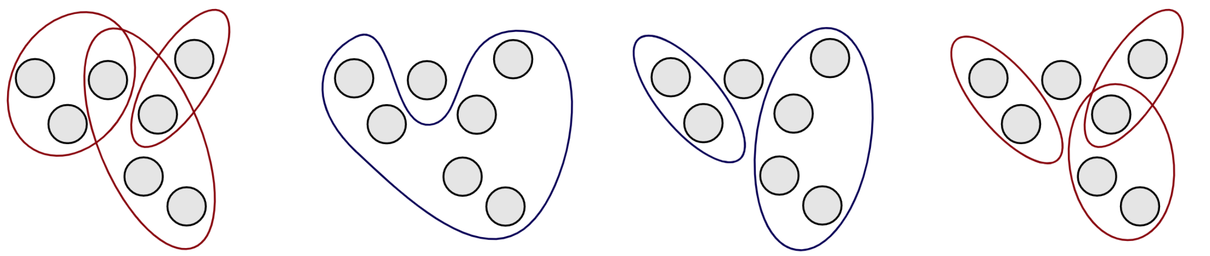
For all capabilities, immutability is key. Capabilities drive persistent competitive advantages by definition, and so it must not be easy for another firm to develop the same capability. We capture this in a simple but extreme way by precluding the development of new capabilities.

3 Upper Bound on Size of the Largest Firm

Our goal is to understand how industry structure will evolve in response to markets valuing more of the same capabilities. The starting point for our analysis is a simple but powerful observation: firms never find it optimal to hold a combination of capabilities if there is a way to partition them without destroying synergies. When such a partition is possible the corresponding demerger generates firms that will obtain exactly the same gross profits in all markets, while capability maintenance costs are reduced. This idea is illustrated in Figure 2 where it is net profitable for firm i, shown in Panel (b), to demerge into firms k and l, as shown in Panel (c). In no markets do both firm k and firm l have relevant capabilities, so they never compete against each other. Moreover, for every market that firm i competed in, either firm k or firm l has exactly the same set of relevant capabilities, and hence generates the same gross profits.

What are the general conditions under which there exist demergers like this? To address this question we introduce the concept of subhypergraphs. We say a hypergraph $\hat{H} = \{\hat{A}, \{\hat{E}_i\}_i\}$ is a *subhypergraph* of $H = \{A, \{E_i\}_i\}$ if (i) $\hat{A} \subseteq A$; and (ii) $\hat{E}_i \subseteq \{E_i \cap \hat{A}\}$ for all edges $\hat{E}_i \in \{\hat{E}_i\}_i$. We say that a subhypergraph $\hat{H} = \{\hat{A}, \{\hat{E}_i\}_i\}$ of $H = \{A, \{E_i\}_i\}$ is *induced by* the nodes $\hat{A} \subseteq A$ if $\hat{E}_i = E_i \cap \hat{A}$ for all i. We will use the notation $\hat{H}_M(A)$ to denote the market subhypergraph induced by the set of capabilities A. The market subhypergraph of H_M induced by the nodes F_i is illustrated in Panel (d) of Figure 2.

Figure 2: A demerger along the component boundaries of the subhypergraph $\hat{H}_M(F_i)$



(a) H_M (b) F_i (c) F_k, F_l (d) $\hat{H}_M(F_i)$

Consider again the demerger of firm i into firm k and l illustrated in Figure 2. The demerger occurs along the component boundaries of the induced subhypergraph $\hat{H}_M(F_i)$ shown in Panel (d). The following lemma generalizes the ideas illustrated in Figure 2.

Lemma 1. *Let $\hat{A}_i := \bigcup_{j \in J_{i,\pi>0}} M_j$ denote the set of capabilities valued by markets firm i is operating in. There exists a demerger that firm i can undertake that weakly increases its gross profits (i.e., $\sum_j \pi_{ij}$ weakly increases) if either of the following two conditions hold*

(i) $F_i \supset \hat{A}_i$,

(ii) the market subhypergraph $\hat{H}_M(F_i)$, with all markets (edges) M_j such that $j \notin J_{i,\pi>0}$ then removed, contains more than one component.

We defer the proof of Lemma 1 to Appendix A.1. Lemma 1 provides sufficient conditions for the existence of a profitable demerger because demergers which weakly increase gross profits strictly increase net profits due to the convexity of capability maintenance costs. It implies that in any stable firm hypergraph, firms cannot hold redundant capabilities—those which are not valued by any market it competes in (condition (i)); furthermore, as discussed earlier, demergers which do not destroy any synergies are always profitable. This resonates with the received wisdom from the finance and management literature—firms should focus on activities aligned with their core capabilities.

It will be helpful to sometimes impose a little more structure on the relationship between capabilities and profits. The capabilities valued by a market are intended to represent the key drivers of competitive advantage in that market. Consistent with this we ensure that an additional relevant capability generates a minimum amount of additional value. Similarly, we require that a monopolist with one capability that is relevant for a market will be able to make sufficient gross profits to cover the cost of maintaining that capability.

Definition (Valued Capabilities)**.** *If firm i is already operating in market j ($\pi_{ij}(\theta_{ij}, \boldsymbol{\theta_{-ij}}) > 0$) or no other firms are operating in market j ($\pi_{kj}(\theta_{kj}, \boldsymbol{\theta_{-kj}}) = 0$ for all $k \in \mathcal{N} \setminus \{i\}$), then*

$$\pi_{ij}(\theta'_{ij}, \boldsymbol{\theta_{-ij}}) - \pi_{ij}(\theta_{ij}, \boldsymbol{\theta_{-ij}}) > \kappa(1)$$

for all $\theta'_{ij} \supset \theta_{ij}$ *such that* $|\theta'_{ij}| - |\theta_{ij}| = 1$.

The next restriction we consider requires capabilities to be complementary in the sense that firms can increase their *gross* profits in a market by combining their capabilities.

Definition (Complementary Capabilities)**.** *For any* $i, k \in \mathcal{N}$ *merging them into a single firm* l *increases gross profits. So, for all markets* j

$$\pi_{lj}(H_M, H'_F) \geq \pi_{ij}(H_M, H_F) + \pi_{kj}(H_M, H_F),$$

where $\theta_{lj} = \theta_{ij} \cup \theta_{kj}$*, the pre-merger industry structure is* (H_M, H_F) *and the post-merger industry structure is* (H_M, H'_F)*. Further, the inequality is strict if* $\theta_{ij} \neq \emptyset$*,* $\theta_{kj} \neq \emptyset$*, and* $\max\{\pi_{ij}(H_M, H_F), \pi_{kj}(H_M, H_F)\} > 0$*.*

This assumption states that synergies have to be sufficiently strong such that even in the presence of competition from other firms in market j, firms i and k merging must deliver enough synergies such that the new firm does at least as well as firms i and k initially did.

The assumptions of valued capabilities and complementary capabilities are not maintained—when results depend on these assumptions we state so.

Denote the markets comprising component r with M_r, and let $|M_{max}| = \max_r |M_r|$ be the maximum number of markets comprising any component.

Proposition 1. *The following results tie the size of the largest firm,* $\max_i |F_i|$*, and the number of markets any firm operates in,* $\max_i |J_{i,\pi>0}|$*, to the size of the largest component on the market hypergraph,* $|C_{max}|$ *and the maximum number of markets comprising any component on the market hypergraph,* $|M_{max}|$*:*

(i) ***(Upper Bound)*** *In all stable industry structures,* $\max_i |F_i| \leq |C_{max}|$ *and* $\max_i |J_{i,\pi>0}| \leq |M_{max}|$*.*

(ii) ***(Tightness of Bound)*** *If capabilities are valued then for all* κ *not too convex there exists a stable industry structure in which* $\max_i |F_i| = |C_{max}|$ *and* $\max_i |J_{i,\pi>0}| = |M_{max}|$*.*

(iii) ***(Lower Bound)*** *If capabilities are valued and complementary then for all* κ *not too convex, there is a unique stable industry structure in which* $\max_i |F_i| = |C_{max}|$ *and* $\max_i |J_{i,\pi>0}| = |M_{max}|$*.*

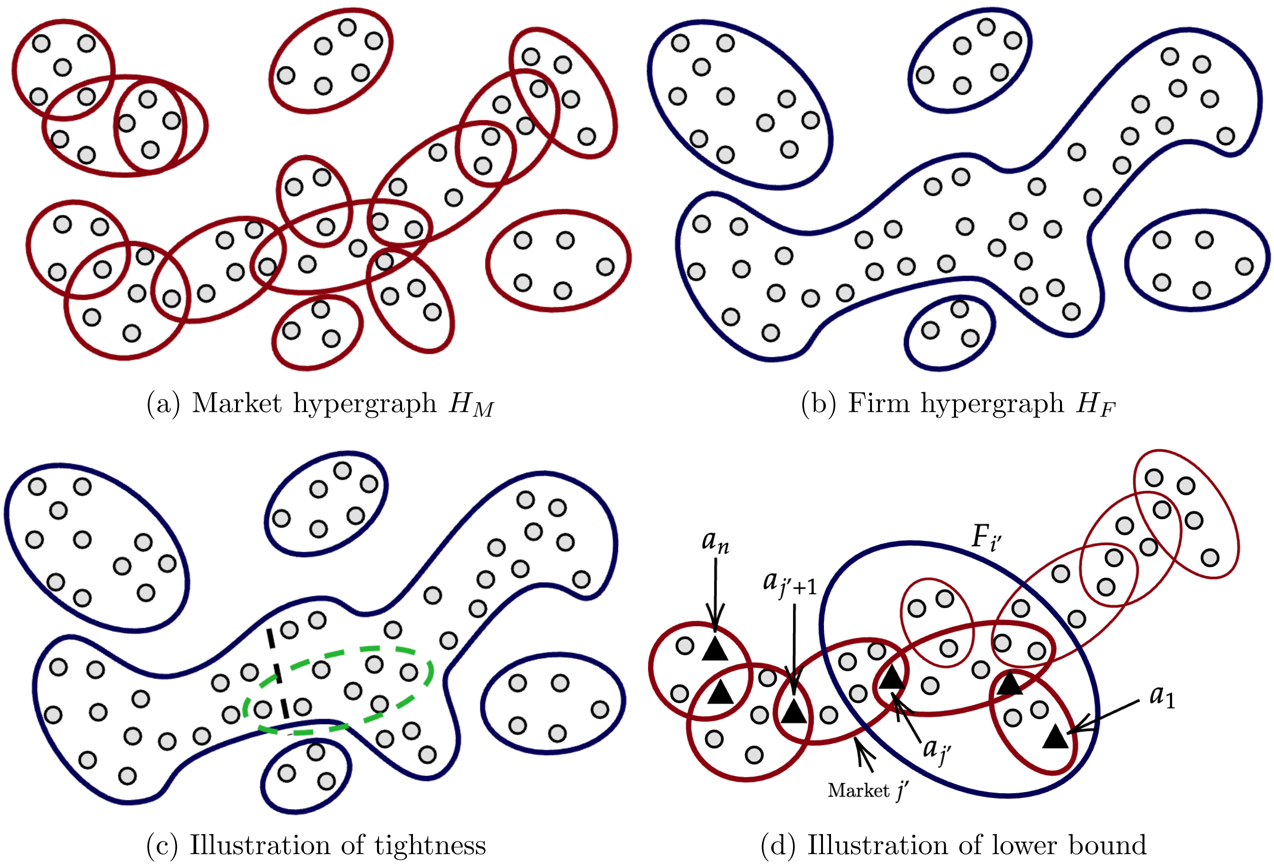
Although the proof of Proposition 1 is fairly rudimentary, it is nevertheless instructive. We defer it until Section 3.1.

Figure 3 provides an illustration of Proposition 1. Panel (a) shows a possible market hypergraph and Panel (b) shows the corresponding firm hypergraph in which each firm holds the same capabilities as the components of the market hypergraph. Part (i) of Proposition 1 implies that there cannot be a firm with more capabilities than the largest firm in the firm hypergraph shown in Panel (b) and provides a corresponding limit on the number of markets any firm operates in. Part (ii) of Proposition 1 implies that if the costs of maintaining capabilities are not too convex, then there will exist a stable firm hypergraph like the one in Panel (b) where the upper bounds from part (i) of Proposition 1 are achieved. Part (iii) of Proposition 1 implies that if capabilities are also complementary, then all stable firm hypergraphs will have a firm that achieves the upper bounds from part (i) as shown in Panel (b).

When the size of the largest component in the market hypergraph increases,[4] the upper-bound on firm sizes given in part (i) of Proposition 1 is relaxed and larger firms can exist in stable industry structures. Indeed, Part (ii) of Proposition 1 shows that this increase in firm size is not just hypothetical, but can occur as the stable industry structure evolves in response to changes in capabilities valued by markets. Part (iii) of Proposition 1 gives conditions under which the increase in firm size is inevitable.

Figure 3: Market hypergraph and implied upper bound on firm size

Notes: Panel (a) illustrates a market hypergraph. Panel (b) shows a firm hypergraph that achieves the upper bound placed on firm size by Proposition 1 (i). Panel (c) shows how demergers breaks synergies in some market. Panel (d) shows the existence of some profitable procurement or merger by firm i'.



(a) Market hypergraph H_M (b) Firm hypergraph H_F

(c) Illustration of tightness (d) Illustration of lower bound

[4] For example, this will happen if previously unconnected markets start to value some of the same capabilities.

3.1 Proof of Proposition 1. ***Part (i):*** Without loss of generality, suppose that firm i is a largest firm (i.e., $|F_i| = |F_{max}|$). In a stable industry structure no firm can have a strictly net profitable demerger. Recall $\hat{A}_i$ is the set of capabilities that are valued by markets i operates in. Hence, by Lemma 1 (i), $F_i \subseteq \hat{A}_i$. But $|\hat{A}_i| \leq |C_{max}|$ otherwise $\hat{H}_M(F_i)$ with markets $j \notin J_{i,\pi>0}$ removed contains more than one component so by Lemma 1 (b) firm i has a strictly profitable demerger. Hence $|F_i| \leq |C_{max}|$.

Fix any component r. By the argument above, if any firm k operates in some market comprising r, $F_k \subseteq C_r$ in all stable industry structures. By regularity assumption (i) this implies $\max_i |J_{i,\pi>0}| \leq |M_{max}|$.

Part (ii): We proceed by construction. Recall we indexed the components of H_M with $\{1...p\}$. For each $i \in \{1...p\}$, let $F_i = C_i$. This generates the firm hypergraph illustrated in Figure 3 (b). We show that the ensuing firm hypergraph is stable. As all capabilities are valued by at least one market, there are no unassigned capabilities and hence no possible procurements or entries. Now consider any two firms i, k and note that any merger between them generating firm l is strictly net unprofitable since, by Lemma 1, there exists a strictly net profitable demerger of firm l along the component boundaries C_i and C_k which exactly undoes this merger.

Finally consider demergers. Pick an arbitrary firm i and note (i) firm i is a monopoly in every market $j \in \{j \in \mathcal{M} : M_j \subseteq C_i\}$; and (ii) capabilities are valued, which implies $J_{i,\pi>0} = \{j \in \mathcal{M} : M_j \subseteq C_i\}$. Consider a demerger generating the set of firms $\mathcal{F}$ and disposing of capabilities $D \subseteq F_i$. Let the new firm hypergraph be H'_F. Partition $J_{i,\pi>0}$ into J, markets j for which there exists $k \in \mathcal{F}$ such that $M_j \subseteq F_k$, and $J' = J_{i,\pi>0} \setminus J$. Note that $J' \neq \emptyset$.

Define

$$
\begin{aligned}
\Delta\pi(i, \mathcal{F}) :&= \sum_j \sum_{i' \in \mathcal{F}} \pi_{i'j}(H_M, H'_F) - \sum_j \pi_{ij}(H_M, H_F) \\
&= \sum_{j \in J'} \Big(\sum_{i' \in \mathcal{F}} \pi_{i'j}(H_M, H'_F) - \pi_{ij}(H_M, H_F) \Big).
\end{aligned}
$$

From firm hypergraph H'_F consider the mergers creating firm $F_l = \bigcup_{i' \in \mathcal{F}} F_{i'}$ and denote the resulting hypergraph by H''_F. Observe, $\Delta\pi(i, l) = \pi_{lj}(H_M, H''_F)) - \sum_{j \in J'} (\pi_{ij}(H_M, H_F) < -|D|\kappa(1)$ as capabilities are valued. Further, $\Delta\pi(l, \mathcal{F}) = \sum_{i' \in \mathcal{F}} \pi_{i'j}(H_M, H'_F)) - \sum_{j \in J'} (\pi_{lj}(H_M, H''_F) < 0$ by regularity condition (v). Combining these inequalities,

$$
\Delta\pi(i, \mathcal{F}) = \underbrace{\Delta\pi(i, l)}_{< -|D|\kappa(1)} + \underbrace{\Delta\pi(l, \mathcal{F})}_{< 0}
$$

Thus, for all κ not too convex $\Delta\pi(i, \mathcal{F}) + \kappa(|F_i|) - \sum_{i' \in \mathcal{F}} \kappa(|F_{i'}|) < 0$, hence the demerger is strictly unprofitable.

Finally, let r be the component comprised of $|M_{max}|$ markets. In the stable industry structure we constructed, there exists some firm which holds capabilities C_r, and hence operates in $|M_{max}|$ markets.

Part (iii): By Lemma 1 in a stable industry structure there does not exist any firm $i \in \mathcal{N}$ which spans multiple components of the market hypergraph. Fix a component $r \in \{1, \dots p\}$ of the market hypergraph and restrict our attention to firms $\mathcal{F} := \{i \in \mathcal{N} : F_i \subseteq C_r\}$. We will show that in all stable industry structures, there must exist a single firm $i \in \mathcal{F}$ such that $F_i = C_r$. If $|\mathcal{F}| = 0$, then $C_r \subseteq S$ and since capabilities are valued, for sufficiently low convexity of κ, there exists some strictly net profitable entry. Thus $|\mathcal{F}| \geq 1$.

Now suppose, towards a contradiction, there does not exist a firm i such that $F_i = C_r$. Pick any firm $i' \in \mathcal{F}$, any $a_1 \in F_{i'}$, and any $a_n \in C_r \setminus F_{i'}$. There exists a path through C_r, starting at a_1 and ending at a_n. Label markets so that this path is $(a_1, M_1, a_2, \dots, a_{n-1}, M_{n-1}, a_n)$. We iterate backwards from a_n until we find a market j' such that $a_{j'} \in F_{i'}$ and $a_{j'+1} \in C_r \setminus F_{i'}$. Panel (d) of Figure 3 shows an example of a path from a capability $a_1 \in F_{i'}$ to a capability $a_n \in C_r \setminus F_{i'}$.

Conclude by noting that either i' operates in market j' or not. If it does, there are two subcases: if $a_{j'+1}$ is unassigned it procures it and increases its net profits by $\kappa(1)+\varepsilon-(\kappa(|F_{i'}|+1)-\kappa(|F_{i'}|))$ for some $\varepsilon > 0$, which is strictly positive for all κ not too convex; if $a_{j'+1}$ is held by another firm k', then merging with k' is strictly profitable for all κ not too convex by complementary capabilities. If i' does not operate in j', then there must exists a firm k' that does operate in market j' (otherwise i would be operating in j' since capabilities are valued). As both firm i' and k' hold at least one capability valued by j' a merger between i' and k' is strictly profitable for all κ not too convex by complementary capabilities. This shows that for the industry structure to be stable there must exist a firm i such that $F_i = C_r$. But since component r was chosen arbitrarily, this, combined with the result from Part (ii), pins down the unique stable industry structure.

3.2 Coalitional Stability. Proposition 1 (i)-(iii) continues to hold when firms are permitted to undertake a broader set of deviations. Showing this requires a stronger notion of stability.

Definition (Coalitional Stability)**.** *A coalitional deviation by the firms $\mathcal{F} \subseteq \mathcal{N}$ reorganises the capabilities $\bigcup_{i \in \mathcal{F}} F_i$ into a new (possibly empty) set of firms $\mathcal{F}'$ such that $\bigcup_{i' \in \mathcal{F}'} F_{i'} \subseteq \big\{S \cup \{\bigcup_{i \in \mathcal{F}} F_i\}\big\}$. The industry structure (H_M, H_F) is coalitionally stable if there are no strictly profitable coalitional deviations.*

The coalitional deviations we allow are very permissive. Any set of firms can combine their joint capabilities with any unassigned capabilities, and then assign these capabilities in any way they wish among any number of firms, while disposing of any unwanted capabilities.

Proposition 2. *Proposition 1 (i)-(iii) on the upper bound, tightness, and lower bound on the size of the largest firm in stable industry structures continue to obtain under coalitional stability.*

Coalitional stability implies stability so the set of coalitionally stable industry structure must weakly contract relative to the set of stable industry structures. As such, the upper bound (Proposition 1 (i)) continues to obtain under coalitional stability. The existence result (Proposition 1 (ii)) extended to coalitional stability is proved in Appendix A.2. This existence of coalitionally stable industry structures, together with the lower bound (Proposition 1 (iii)), implies the same lower bound on firm size across all coalitionally stable structures.

4 Sensitivity of the Bound on Firm Size

Proposition 1 links the size of the largest firm to the capabilities that markets value and specifically the size of the largest component in the market hypergraph. As markets value more of the same capabilities, both the size of the largest component, as well as the number of markets which comprise it will increase. However, if these changes are only gradual, this mechanism would not offer a satisfactory account of the rapid expansion of internet conglomerates into an ever-increasing array of new markets. Should we expect sudden changes? To explore this question, we need to add some structure to how the market hypergraph evolves. We do this by modelling it as a *random* hypergraph. This provides a natural benchmark and will illustrate how small changes in connectivity can massively relax our upper bound on firm size, as well as the number of markets a firm can enter in equilibrium.

For simplicity, we consider a standard random hypergraph model which yields a neat closed-form characterization though these results hold more broadly. We let each edge (market) of size k in our random market hypergraph occur independently from each other, and independently from edges of other sizes, with probability p_k.[5] We denote the random hypergraph model by $\mathcal{R}(\mathcal{A}, \mathbf{p})$ with $\mathbf{p} := (p_1, p_2, \ldots, p_t)$ where $t < \infty$ is the largest edge size permitted. In a network setting the expected degree provides a key measure of connectivity. Consistent with this, we use a generalized notion of degree for hypergraphs and define the degree of capability a as the number of node-edge pairs (a_i, E_i) such that $\{a, a_i\} \subseteq E_i$. The expected degree[6] of the market hypergraph is increasing in the markets generated (increasing in p_k for any k) and increasing in the size of markets (as $\mathbf{p}$ places more weight on larger markets in place of smaller ones).

It turns out that the expected degree of a random hypergraph is a sufficient statistic for determining whether the random market hypergraph drawn according to $\mathbf{p}$ can support large firms or not.

Proposition 3. *There exists a finite constant $\bar{d}$ such that with high probability (i.e., with probability going to 1 as $|\mathcal{A}|$ goes to infinity),*[7]

(i) [subcritical case] if $\mathbb{E}[d(|\mathcal{A}|, \mathbf{p})] < \bar{d}$, then $|C_{max}| = O(\log |\mathcal{A}|)$ and $|M_{max}| = O(\log |\mathcal{A}|)$;

(ii) [supercritical case] if $\mathbb{E}[d(|\mathcal{A}|, \mathbf{p})] > \bar{d}$, then $|C_{max}| = \Omega(|\mathcal{A}|)$ and $|M_{max}| = \Omega(|\mathcal{A}|)$.

[5] This generalizes Erdős-Rényi graphs.

[6] The expected degree of a random hypergraph H with distribution $\mathcal{R}(\mathcal{A}, \mathbf{p})$ is

$$\mathbb{E}[d(|\mathcal{A}|, \mathbf{p})] = \sum_{k=2}^{t} (k-1) \binom{|\mathcal{A}| - 1}{k-1} p_k.$$

[7] $f(n) = O(g(n))$ if there exists $M > 0$ such that $|f(n)| \leq M|g(n)|$ for all n; $f(n) = \Omega(g(n))$ if there exists $m > 0$ such that $|f(n)| \geq m|g(n)|$ for all n.

Proposition 3 shows there is a critical threshold for the connectivity of markets around which a phase transition in the number of markets a firm can potentially enter occurs. Part (i) of Proposition 3 states that if the expected degree of the random market hypergraph is below this threshold, then all components of the market hypergraph will contain a vanishing proportion of capabilities and markets. The latter imposes a clear upper-bound on the number of markets any single firm can operate in. Part (ii) of Proposition 3 states that if the expected degree of the random market hypergraph is above this threshold, then at least one component of the market hypergraph—the component with the most capabilities—will contain a large number of markets. This implies that if some firm holds $|C_{max}|$ capabilities—all those which comprise the largest component—then it must also operate in a large number of markets. Finally, note that this is a threshold phenomena: the probability that the market hypergraph can sustain giant firms spanning many markets goes from 0 just below the key connectivity threshold $\bar{d}$ to 1 just above it. Even small changes to the connectivity of markets can have a huge impact on the number of markets a firm can enter in a stable industry structure.

The results on the phase transition of capabilities held by the largest component ($|C_{max}|$) was proven by Schmidt-Pruzan and Shamir (1985), and we complete the proof of Proposition 3 in Online Appendix B by employing related techniques to obtain results on the phase transition of the largest number of markets comprising any component ($|M_{max}|$). While the space of capabilities is abstract and intended to capture a wide variety of things, the phrase transition results on the number of markets a firm can enter in equilibrium is more concrete, and provides predictions consistent with how industry structure has evolved in recent years.

5 Conclusions

We have proposed a capability-based explanation for the sudden emergence of what we have termed internet conglomerates. This helps to close a gap between the economics and management literature. Our explanation is that changes in technology have made markets more connected and that has facilitated the rapid emergence of internet conglomerates. While this is not the whole story, we do think it is a useful framework for making sense of recent acquisitions by large tech firms.

We have assumed that each capability can only be wielded by a single firm. In a separate paper (Chen et al., 2022), we relax this assumption by allowing for multiple instances of a capability so that multiple firms can wield the same capability. Versions of our main result continue to hold in this setting, but a richer set of industry structures can be supported. Moreover, we derive a lower bound on the minimum firm size across all stable industry structures, and show that it is related to the scarcity of capabilities.

Our framework is relatively simple and versatile. While here we use it to study the forces underlying recent trends toward conglomeratization, it might also be used to address a range of other theoretical and empirical questions. For example, it could underpin a dynamic analysis of the evolution of industry structure. Such an approach would be able to systematically address issues such as hoarding—firms holding multiple instances of a capability. While hoarding is unprofitable in our framework because extra capabilities have no myopic value and are costly to maintain, it might be dynamically profitable by suppressing future competition in a similar spirit to the "killer acquisitions" documented by Cunningham et al. (2021).[8] This possibility is important for antitrust policy, particularly regarding conglomerate mergers and regulation of anticompetitive behaviour. Questions about hoarding are inevitably about capabilities—our framework offers a tractable means of putting them at the heart of the analysis.

[8] Indeed, in an accompanying paper Chen et al. (2022), we document simulations which show that hoarding can often be profitable, especially as the market hypergraph becomes more connected.

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A Omitted Proofs

A.1 Proof of Lemma 1.

Proof. If $F_i \not\subseteq \hat{A}_i$ then there exists a capability $a \in F_i$ that firm i does not use it in any market. Disposing of such a capability therefore leaves i's relevant capabilities unaffected for all markets $j \in J_{i,\pi>0}$ and hence firm i's gross profit are unaffected.

Now suppose the market subhypergraph $\hat{H}_M(F_i)$, with all markets (edges) M_j such that $j \notin J_{i,\pi>0}$ then removed, contains more than one component. Since we are considering demergers, we can restrict our attention to the market hypergraph H'_M which is obtained from H_M by removing all markets $\mathcal{M} \setminus J_{i,\pi>0}$. This is because if firm i did not initially operate in market j, none of the firms generated by the demerger will do so (regularity assumption (iv)). Next, denote the components of $\hat{H}'_M(F_i)$ by $\{C_r\}_{r=1,\ldots,p}$, $p > 1$. Now consider a demerger of firm i generating firms k and l where $F_k = C_1$, $F_l = \bigcup_{r=2,\ldots,p} C_r$. For each market $j \in J_{i,\pi>0}$, $\theta_{ij} = \theta_{kj}$ or $\theta_{ij} = \theta_{lj}$. But since firm labels do not matter, this implies k or l makes identical gross profits as firm i in j. Hence

$$\begin{aligned}\sum_{j\in\mathcal{M}}\pi_{ij}(H_M,H_F) &= \sum_{j\in J_{i,\pi>0}}\pi_{ij}(H'_M,H_F)\\ &= \sum_{j\in J_{i,\pi>0}}\Big(\pi_{kj}(H'_M,H_F)+\pi_{lj}(H'_M,H_F)\Big)\\ &= \sum_{j\in\mathcal{M}}\Big(\pi_{kj}(H_M,H_F)+\pi_{lj}(H_M,H_F)\Big)\end{aligned}$$

where the first and last equalities are from the definition of H'_M. We have then found a demerger which weakly increases gross profits. $\square$

A.2 Proof of Proposition 2.

Proof. We already argued the upper and lower bounds continue to obtain under coalitional stability since coalitional stability implies stability. For tightness, we proceed as before by construction: for each $i \in \{1...p\}$, let $F_i = C_i$. We claim this firm hypergraph is coalitionally stable.

Consider any deviation which reconfigures $\mathcal{F} \subseteq \mathcal{N}$ into $\mathcal{F}'$ such that $\bigcup_{i'\in\mathcal{F}'} F_{i'} \subseteq \big\{S \cup \bigcup_{i\in\mathcal{F}} F_i\big\}$. If any firm $i \in \mathcal{F}'$ generated by the coalitional deviation spans multiple components, by Lemma 1 it has a strictly net profitable demerger. This demerger reduces the total fixed costs borne by the coalition and leaves the competitive landscape in each market unchanged so is strictly net profitable. Perform all such demergers until none remain, and denote the output by $\mathcal{F}''$. We now show this makes strictly less profits than the original coalition. For each $r \in \{1...p\}$, denote $\mathcal{F}''_r := \{i \in \mathcal{F}'' : F_i \cap C_r \neq \emptyset\}$ and denote $\mathcal{C}_r := \{j \in \mathcal{M} : M_j \subseteq C_r\}$.

Excluding the trivial case $\mathcal{F}'' = \{C_r\}$, note that for all $r \in \{1...p\}$,

$$\sum_{j\in\mathcal{C}_r}\pi_{rj}(H_M,H_F) > \sum_{j\in\mathcal{C}_r}\sum_{i\in\mathcal{F}''_r}\pi_{ij}(H_M,H'_F)$$

where H_F (H'_F) is the hypergraph before (after) the deviation. To see this, note that there must exist some market $j \in \mathcal{C}_r$ for which there does not exist a firm in $\mathcal{F}$ which holds all the capabilities M_j (otherwise, since components are path-connected, we have a contradiction). For such a market j, the inequality is strict i.e. firm r makes strictly more gross profits than the collective gross profits of the firms $\mathcal{F}''$ from regularity assumption (v). Now since this is true for all r, by the independence of profits, we have

$$\sum_{j\in\mathcal{M}}\sum_{i\in\mathcal{F}}\pi_{ij}(H_M,H_F) > \sum_{j\in\mathcal{M}}\sum_{i\in\mathcal{F}''}\pi_{ij}(H_M,H'_F)$$

and for any $\varepsilon > 0$, we can reduce the convexity of κ until $\sum_{i\in\mathcal{F}}\kappa(|F_i|) - \sum_{i\in\mathcal{F}''}\kappa(|F_i|) < \varepsilon$ which implies the deviation generating $\mathcal{F}''$ is strictly dominated by not deviating at all. Then since generating $\mathcal{F}''$ in turn dominates generating $\mathcal{F}'$, this concludes the proof. $\square$