Foreign Vulnerabilities, Domestic Risks: The Global Drivers of GDP-at-Risk

Simon Lloyd
Bank of England

Ed Manuel
Bank of England

Konstantin Panchev
University of Oxford

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The Global Drivers of GDP-at-Risk*

Simon Lloyd† Ed Manuel‡ Konstantin Panchev§
April 26, 2022

Abstract
We study how foreign financial developments influence the conditional distribution of domestic GDP growth. Within a quantile regression setup, we propose a method to parsimoniously account for foreign vulnerabilities using bilateral-exposure weights when assessing downside macroeconomic risks. Using a panel dataset of advanced economies, we show that tighter foreign financial conditions and faster foreign credit-to-GDP growth are associated with a more severe left tail of domestic GDP growth, even when controlling for domestic indicators. The inclusion of foreign indicators significantly improves estimates of ‘GDP-at-Risk’, a summary measure of downside risks. In turn, this yields time-varying estimates of higher moments of GDP growth that demonstrate interpretable moves over the cycle. Decomposing historical estimates of GDP-at-Risk into domestic and foreign sources, we show that foreign shocks are a key driver of domestic macroeconomic tail risks.

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†Bank of England. Email Address: simon.lloyd@bankofengland.co.uk (Corresponding Author).
‡Bank of England. Email Address: edward.manuel@bankofengland.co.uk.
§University of Oxford. Email Address: konstantin.panchev@spc.ox.ac.uk.

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1 Introduction

It is well established that domestic financial developments can generate downside risks to domestic economic growth (Adrian, Boyarchenko, and Giannone, 2019; Aikman, Bridges, Hacioglu Hoke, O’Neill, and Raja, 2019) and, in turn, can influence the probability of crises (Schularick and Taylor, 2012). But not all crises have domestic origins. In a highly interconnected and increasingly synchronised global economy, international vulnerabilities can spill over to the domestic risk environment. But how, and to what extent, does this occur?

In this paper, we document the crucial role of foreign vulnerabilities in determining downside risks to domestic economic growth. Tighter foreign financial conditions and faster foreign credit-to-GDP growth can generate significant macroeconomic tail risks, even when controlling for domestic indicators. In particular, we show that they weigh heavily on ‘GDP-at-Risk’—the 5th percentile of the GDP-growth distribution. A summary measure of downside macroeconomic risks, GDP-at-Risk is a now widely used concept in financial stability monitoring and cost-benefit analysis informing macroprudential policy (Carney, 2020).

Foreign financial developments can influence domestic GDP-at-Risk, and the conditional distribution of domestic GDP growth more generally, through a number of channels. First, consistent with evidence of a global financial cycle (Rey, 2013; Miranda-Agrippino and Rey, 2020), characterised by strong cross-country comovement in asset prices, a substantial portion of variation in domestic financial conditions can arise from common global sources. Tighter global financial conditions can impact domestic funding costs and risky asset prices, and, in turn, the conditional distribution of future GDP growth outturns. Second, with financial institutions increasingly holding foreign claims, excessive credit growth and risk taking abroad can generate losses for domestic financial institutions and cause spillovers to the wider economy. Third, a build-up in foreign vulnerabilities that triggers a downturn abroad can spill over to the domestic economy through broader macroeconomic channels—for instance by lowering demand for domestic exports. While the influence of foreign factors on the mean of domestic GDP growth is widely studied in the international business-cycle literature (see Corsetti, 2008, and the references within), their influence on the tails of the domestic GDP growth distribution is the subject of this paper.

In our key methodological contribution, we propose a general and parsimonious approach to account for the influence of foreign vulnerabilities on the conditional distribution of domestic GDP growth. We do so within a quantile regression setup (Koenker and Bassett, 1978) that allows us to estimate the relationship between a range of indicators and the GDP-growth distribution over time and across countries. We account for foreign vulnerabilities by defining a weighted average of indicators in the rest of the world using bilateral-exposure weights. This

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1See, for example, Cesa-Bianchi, Dickinson, Kösem, Lloyd, and Manuel (2021) for a summary of the channels through which these cross-border spillovers can occur.
approach has the advantage of capturing country-specific exposures to foreign vulnerabilities, while also limiting the number of additional regressors—a particular computational challenge for quantile regression.

We then apply this methodology to a cross-country panel dataset of advanced economies. Doing so provides novel empirical evidence demonstrating the link between foreign vulnerabilities and domestic GDP-at-Risk, as well as the conditional distribution of GDP growth. We emphasise four main findings.

First, we show that foreign vulnerabilities significantly and robustly influence the conditional distribution of future domestic GDP growth, even when controlling for domestic indicators. Higher foreign equity volatility is associated with significant reductions in the left tail of domestic GDP in the near term—i.e. less than 1 year. Faster foreign credit-to-GDP growth weighs on the 5th percentile of domestic GDP growth out to longer horizons—i.e. up to 5 years. Moreover, the influence of foreign credit-to-GDP on the distribution of domestic GDP growth is significantly larger at the 5th percentile than at the median, indicating that global credit conditions can have non-linear impacts on domestic GDP.

Second, we demonstrate that foreign indicators provide information relevant for estimating domestic GDP-at-Risk, over and above domestic ones, both in and out of sample. The inclusion of foreign vulnerabilities significantly improves estimates of domestic GDP-at-Risk. The in-sample goodness-of-fit for estimates of the 5th percentile of domestic GDP are materially higher when foreign-weighted variables are included in the quantile regression specification, even when excluding the period containing the 2007-2008 Global Financial Crisis (GFC).

Third, we break down the predictive power of foreign indicators in terms of their ability to estimate different moments of the GDP-growth distribution. We show that a model which includes foreign-weighted indicators can yield estimates of higher moments of GDP growth that are interpretable over the business cycle, especially when done in sample. In addition, we demonstrate that the inclusion of foreign variables tends to generate GDP-growth distributions that exhibit higher variance and more negative skew than those from a model with domestic covariates only in the run-up to the GFC. By capturing vulnerabilities relevant for the tails of the GDP-growth distribution, foreign indicators can help to improve the narrative around higher-order moments estimated within a quantile regression framework.

Finally, we move towards a structural decomposition of historical estimates of GDP-at-Risk by orthogonalising domestic and foreign variables to identify the contribution of foreign shocks. We show that foreign vulnerabilities are a key driver of domestic macroeconomic tail risks. On average, foreign shocks explain up to around 60% of variation in the estimated 5th percentile of advanced-economy GDP growth at the 3-year horizon, more than the comparable figure for the median.

Our results have important implications for financial stability policy. By highlighting the additional explanatory power of foreign variables to domestic GDP-at-Risk, we show the im-
importance of accounting for foreign indicators when monitoring risks to domestic financial stability. In addition, by demonstrating the substantial contribution of foreign shocks to domestic tail risks, our results suggest that international macroprudential policy frameworks that foster cooperation between national authorities when forming regulatory responses to global shocks can be beneficial. More broadly, our general methodology can be applied more widely, for instance to inform analyses of GDP-at-Risk within emerging-market economies, where assessments of tail risks have been more limited in spite of their substantial exposures to foreign events. Such analyses could shed further light on the role of macroprudential policy in guarding against tail risks in the face of foreign shocks (e.g. Coman and Lloyd, 2022).

Related Literature Our paper is related to five main strands of literature. First, and most directly, our work builds on studies applying quantile regression techniques to assess the drivers of macroeconomic tail risks (see, e.g., Adrian, Boyarchenko, and Giannone, 2019; Adrian, Grinberg, Liang, Malik, and Yu, 2022; Aikman, Bridges, Hacioglu Hoke, O’Neill, and Raja, 2019). Using data on advanced economies, these papers identify a strong relationship between domestic vulnerabilities, such as financial conditions and credit growth, and the tails of the conditional GDP growth distribution. But they do not explicitly account for the influence of foreign vulnerabilities. These will only be implicitly captured insofar as foreign vulnerabilities are reflected within domestic indicators. We contribute to this body of work by exploring the independent influence of foreign vulnerabilities, and propose a novel methodological framework for doing so.

Second, our study relates to a literature on financial crisis warning indicators. Building on Schularick and Taylor (2012), who find credit-to-GDP to be a robust predictor of financial crises, others have shown that foreign variables can have significant predictive power. For instance, Cesa-Bianchi, Eguren-Martin, and Thwaites (2019a) and Bluwstein, Buckmann, Joseph, Kang, Kapadia, and Simsek (2020) find that global financial developments influence the probability of domestic crises, over and above domestic indicators. Our analysis extends this literature by documenting the influence of foreign factors on the whole conditional distribution of GDP growth—not just crisis events.

Third, our work contributes to a growing literature assessing the ability of quantile regression models to estimate higher moments of the GDP distribution. Recently, Plagborg-Møller,

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2In part motivated by these papers, there have been a number of other studies of GDP tail risks using quantile regressions. For example: Giglio, Kelly, and Pruitt (2016) for the United States (US) and Europe, Aikman, Bridges, Burgess, Galletly, Levine, O’Neill, and Varadi (2018) for the United Kingdom (UK), Loria, Matthes, and Zhang (2019) for the US, Chavleishvili and Manganelli (2019) and Lhussier (2022) for the euro area, Duprey and Ueberfeldt (2020) for Canada, and Busetti, Caivano, Delle Monache, and Pacella (2021) for Italy. Others have proposed the use of quantile regression tools for high-frequency GDP-at-Risk monitoring (e.g. Ferrara, Mogliani, and Sahuc, 2022).

3Busetti et al. (2021) find a significant association between Italian GDP-at-Risk and US financial conditions, as well as a global purchasing managers’ index. While this demonstrates some role for global factors in the determination of macroeconomic tail risks, the method we propose is more general and—as we go onto explain—has a number advantages over simply adding US variables, or global aggregates, to the explanatory-variable set.
Reichlin, Ricco, and Hasenzagl (2020) argue that conditional moments of US GDP growth other than the mean are poorly estimated by domestic financial conditions within a quantile regression framework. These findings have been challenged by Adrian et al. (2022), who note the importance of distinguishing financial price variables and credit quantity varies due to their different cyclical behaviour. We contribute to this debate by assessing the role of foreign price and quantity variables in driving interpretable moves in higher moments over the business cycle. We also assess alternative approaches to fitting distributions to conditional quantile forecasts from a quantile regression, as recently proposed by Mitchell, Poon, and Zhu (2021). We find that, relative to a parametric approach most regularly applied in the literature to date (e.g. Adrian et al., 2019), a non-parametric alternative can yield additional insights—for example around the contribution of foreign factors in driving higher-order moments and the emergence of multi-modalities in GDP growth.

Fourth, and relatedly, our findings contribute to a broader empirical literature—beyond that which focuses on quantile regression techniques—which documents important higher-order dynamics in GDP growth. Our results support evidence of counter-cyclical GDP-growth variance (Adrian et al., 2019) and pro-cyclical skewness (Delle Monache, De Polis, and Petrella, 2021; Iseringhausen, Petrella, and Theodoridis, 2021), as well as the existence of multimodalities in the predictive distribution of GDP when conditioned on financial conditions—which Adrian, Boyarchenko, and Giannone (2021) emphasise as an important feature of macro-financial dynamics. Our findings can be used to inform the calibration of international macroeconomic models that incorporate such dynamics (see, e.g., Adrian and Duarte, 2018; Corsetti, Lipinska, and Lombardo, 2021).

Finally, our paper has links with the broad literature on disaster risks and economic growth (see, e.g., Barro, 2009; Barro and Ursúa, 2012; Gabaix, 2012; Gourio, 2012; Wachter, 2013). In particular, our evidence emphasising the importance of foreign vulnerabilities for domestic downside risks contributes to recent work highlighting the cross-border transmission of macroeconomic disasters (Gourio, Siemer, and Verdelhan, 2013; Farhi and Gabaix, 2016).

The remainder of this paper is structured as follows. Section 2 presents our general methodology. Section 3 describes the results from a specific application, emphasising the additional information foreign variables provide over and above domestic ones in and out of sample. Section 4 demonstrates the contribution of foreign indicators to estimates of time-varying GDP moments, in and out of sample. Section 5 moves towards a structural assessment, decomposing GDP-at-Risk estimates into domestic and foreign shocks. Section 6 concludes.

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4See also comments by Gertler (2020) and Liang (2020).
2 Methodology to Account for Global Drivers

In this section, we outline our general methodology to account for global drivers of GDP-at-Risk and the conditional distribution of GDP growth. As in previous work, we employ a quantile regression framework (Koenker and Bassett, 1978) to study how changes in a set of conditioning variables are associated with the distribution of future GDP growth. We present our approach within a panel setting, where time is denoted by $t = 1, ..., T$ and the countries for whom we estimate the conditional distribution of GDP are labelled with $i = 1, ..., N$.\(^5\)

We specify the following local-projection model (Jordà, 2005) for the conditional quantile function $Q$ of $h$-period-ahead GDP growth $\Delta^h y_{i,t+h}$:

$$Q_{\Delta^h y_{i,t+h}}(\tau | X_{i,t}, X_{i,t}^*) = \alpha^h_i(\tau) + \beta^h(\tau) X_{i,t} + \vartheta^h(\tau) X_{i,t}^* \quad (1)$$

where $Q$ computes quantiles $\tau$ of the distribution of $\Delta^h y_{i,t+h}$ given covariates: $X_{i,t}$ and $X_{i,t}^*$. \(^6\)

$\alpha^h_i(\tau)$ represents a country- and quantile-specific fixed effect to control for time-invariant unobserved heterogeneity. Estimation of the panel quantile regressions with quantile-specific country fixed effects is feasible when the panel structure has $T$ much larger than $N$ (Galvao and Montes-Rojas, 2015), as is the case in our application.\(^6\) In this $T \gg N$ case, Kato, Galvao, and Montes-Rojas (2012) demonstrate that this panel fixed-effects estimator is consistent and asymptotically normal, a finding verified using a different approach by Galvao, Gu, and Volgushev (2020).\(^7\)

The domestic covariates in equation (1) are denoted by $X_{i,t}$. They include domestic indicators that may influence the conditional distribution of domestic GDP, such as credit growth or proxies for financial conditions. The coefficients $\beta^h(\tau)$ denote the average association between domestic covariates and quantiles $\tau$ of the GDP-growth distribution.

The key novelty in equation (1) is the inclusion of foreign covariates $X_{i,t}^*$. The coefficients $\vartheta^h(\tau)$ represent the average association between foreign indicators and the conditional distribution of domestic GDP growth. These foreign indicators can reflect both foreign country-specific factors and common global events. However, as we explain in the next sub-section, the construction of these foreign variables is not trivial.

2.1 Constructing Foreign Covariates

To appreciate these challenges, consider a country $i \in [1, N]$ for whom we estimate the conditional distribution of GDP growth using equation (1). The $\tau$-th quantile of GDP growth in

\(^5\)Our general approach to accounting for global factors can also be applied to country-specific regressions—as we explain in robustness analysis in Section 3.4.

\(^6\)See Lamarche (2021) for a recent survey of panel quantile regression estimators.

\(^7\)Our approach to account for foreign vulnerabilities does not depend on specific assumptions about the constant term. Our main results are robust to using an alternative country fixed-effects structure, in which the fixed effect is the same across quantiles for a given country, i.e. $\alpha^h_i$ for all $\tau$, alongside a quantile-specific intercept (Canay, 2011).
country $i$ can depend on domestic covariates $X_{i,t}$, but also a set of indicators $X_{j,t}$ in a range of other countries $j = 1, ..., N^*$. In order to account for the influence of a single foreign indicator (e.g. credit-to-GDP) on the conditional distribution of domestic (country-$i$) GDP, one approach could be to individually add this indicator for each foreign country $j = 1, ..., N^*$, where $j \neq i$, to the foreign-covariate set $X_{i,t}$. However, this would lead to a proliferation of regressors, adding an extra $N^* - 1$ explanatory variables. This could pose computational challenges for the quantile regression—especially if scaled up to more than one foreign indicator—and, in the limit, would exhaust available degrees of freedom.

To circumvent this ‘curse of dimensionality’, for each indicator (e.g. credit-to-GDP) we define a single foreign covariate $x_{i,t}^* \subseteq X_{i,t}$ as the weighted sum of the indicator $x_{j,t}$ in all other countries $j = 1, ..., N^*$. Defining $\omega_{i,j,t}$ as a time-varying weight capturing the ‘bilateral exposure’ of country $i$ to country $j$ at time $t$, we construct the foreign-weighted sum for each indicator using:

$$ x_{i,t}^* = \sum_{j=1}^{N^*} \omega_{i,j,t} x_{j,t} \tag{2} $$

where $\sum_{j=1}^{N^*} \omega_{i,j,t} = 1$ and $\omega_{i,i,t} = 0$, for all $i, t$. With this definition, each additional foreign indicator (e.g. credit-to-GDP) adds a single regressor (e.g. foreign-weighted credit-to-GDP) to equation (1), offering a parsimonious solution to the curse of dimensionality. Furthermore, by constructing the foreign covariates in this way, we can extend the number of foreign countries $N^*$ that we account for, without increasing dimensionality. There is also no restriction that the number of foreign countries $N^*$ needs to be the same as the number of domestic ones $N$.\(^8\)

Moreover, by using weights $\omega_{i,j,t}$ that capture country-specific bilateral exposures to the rest of the world, we account for heterogeneity in countries’ cross-border links. For instance, we can ensure that countries with stronger ties to country $i$ through trade or financial linkages (i.e. larger $\omega_{i,j,t}$) comprise a larger share of the foreign-weighted covariate and therefore can have a stronger association with the conditional distribution of country-$i$ GDP growth. This desirable economic intuition would be lost were we to specify each $x_{i,t}^*$ as a simple global aggregate (e.g. global credit-to-GDP), i.e. the sum (or unweighted average) of country-$j$ indicators.

In addition, our proposal nests an approach in which only US variables (e.g. US VIX) are used to capture global events (i.e. $\omega_{i,US,t} = 1$ and $\omega_{i,j,t} = 0 \ \forall \ t, j \neq US$). While such a US-specific setup can capture elements of the global financial cycle emanating from the US, our proposal allows for a broader set of cross-border transmission channels and shocks, including the build-up of regional risks (e.g. within the euro area). Moreover, relative to a US-only foreign variable, which is homogeneous for all countries within the panel, our foreign-weighted

\(^8\)For instance, we may estimate GDP-at-Risk for a set of $N$ similar advanced economies, but want to account for spillover channels from a broader set of countries $N^* > N$, which may include major emerging markets in addition to the $N$ advanced economies.
variable is heterogeneous across countries. So, in more general settings, equation (2) can be used alongside fixed effects that are homogeneous with respect to \( i \) (e.g. time fixed effects).

Overall, our approach is parsimonious, while also maintaining a meaningful economic narrative around cross-country links. As in the global vector autoregression (GVAR) literature, where similar weighting schemes are applied to account for the influence of foreign factors at the mean (Pesaran, Schuermann, and Weiner, 2004; Eickmeier and Ng, 2015), there are a number of candidate weighting schemes that can be used too. For example, weights can be constructed based on bilateral trade or financial linkages (or combinations thereof) depending on practitioners’ focus.

3 Documenting the Global Drivers

In this section, we estimate the global drivers of the conditional distribution of GDP growth, emphasising the additional information provided by foreign indicators, in and out of sample.

3.1 Specific Empirical Model

We illustrate our general methodology with a specific empirical model. This model is similar to the specification in Adrian et al. (2022) and is deliberately pared back, in order to highlight the influence of the key global drivers of the conditional distribution of GDP growth. However, as we emphasise in Section 3.4, our key findings are robust to a range of alternative model specifications, reflecting the generality of our approach.

We estimate the conditional distribution of GDP growth for 13 advanced economies: Australia, Canada, Denmark, Finland, France, Germany, Italy, Norway, Spain, Sweden, Switzerland, UK and US. The dataset spans the period 1981Q1 to 2018Q4. Our dependent variable is formally defined as annual average real GDP growth over \( h \) quarters, i.e. \( \Delta^h y_{i,t+h} = (y_{i,t+h} - y_{i,t})/(h/4) \).

Domestic Covariates We include three domestic indicators in the variable set \( X_{i,t} \): the 1-quarter realised volatility of equity prices; the 3-year percentage point change in the aggregate private non-financial credit-to-GDP ratio; and the 1-quarter growth of real GDP. This variable choice is motivated by existing studies focusing on domestic GDP-at-Risk (see, e.g., Aikman et al., 2019; Adrian et al., 2022). Like Aikman et al. (2019), we use equity price volatility as a proxy for financial-market conditions because it is available for a wider set of countries than the financial conditions index (FCI) used by Adrian et al. (2022). We favour the 3-year change.

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9 See Appendix A for a full description of data sources.
10 We discuss the robustness of our findings to different specifications of domestic risk factors in Section 3.4.
11 We show that our results are robust to the use of the FCI in Section 3.4.
in credit-to-GDP to capture persistent changes in credit, which are thought to pose risks to financial stability and are leading indicators of macroeconomic crises (Schularick and Taylor, 2012). Moreover, we choose to separate these two ‘vulnerability’ indicators—rather than use a single aggregated indicator of price- and quantity-based vulnerabilities, as in Plagborg-Møller et al. (2020) for example—to capture the differing influence of risk factors across horizons. As Aikman et al. (2019) and Adrian et al. (2022) show, financial market volatility tends to have a negative near-term influence on the left-tail of GDP growth, while growth in the quantity of credit relative to GDP is associated with a medium-term deterioration in GDP-at-Risk. Quarterly real GDP growth is included as a control for the prevailing state of the macroeconomy.

**Foreign Covariates**  We include the foreign-weighted counterparts of each of the indicators in the foreign variable set $X_{i,t}^*$. This variable choice is, in part, motivated by evidence that global financial market indicators and credit quantities tend to predict domestic financial crises (Cesa-Bianchi et al., 2019a; Bluwstein et al., 2020). For our baseline results, we construct foreign-weighted variables using data on bilateral trade linkages. Using data from IMF Direction of Trade Statistics, we define the weights $\omega_{i,j,t}$ as the fraction of country $i$’s exports to country $j$ at time $t$. This scheme will place higher weight on foreign regions that country $i$ exports more extensively to, reflecting the fact that a downturn in one country $j$ may spill over to another $i$ through reduced demand for country-$i$ exports. Compared to the bilateral financial weights from BIS International Banking Statistics we use in robustness analyses in Section 3.4, these trade weights have the advantage of running back to 1980, enabling us to use time-varying weights in the baseline specification. However, as we discuss there, our key results are robust to different combinations of country weights. Moreover, owing to constraints on data availability, we focus on the same set of foreign countries used in the domestic variable set, i.e. $N = N^* = 13$.$^{12}$

**Interpretation and Inference** For presentational purposes, we standardise all regressors by the country-level mean and standard deviation. So, all coefficients can be interpreted as the association between a one standard deviation change in an indicator and the $\tau$-th quantile of GDP growth. We estimate the local projection regression (1) for $h = 1, 2, \ldots, 20$ quarters. For inference, we follow the block bootstrap procedure of Kapetanios (2008), resampling the data over blocks of different time series dimensions to generate coefficient standard errors for respective quantiles. As in Aikman et al. (2019), we resample time series observations using 8 blocks, replicating the bootstrap 5000 times.

$^{12}$We discuss the robustness of our findings to a broader number of foreign countries in Section 3.4, i.e. $N^* > N$. 

9
3.2 Coefficient Estimates

We present coefficient estimates in two ways. First, we show the relationship between indicators and GDP-at-Risk—i.e. the $\tau = 0.05$th quantile of GDP—across horizons $h$. Second, we present the relationship between indicators across GDP quantiles $\tau$ at a given horizon $h$.

3.2.1 Coefficients Across Horizons

Figure 1 presents coefficient estimates at the 5th percentile of GDP across horizons $h$ for equity volatility and the 3-year change in credit-to-GDP—both domestic and foreign-weighted—from our specific model. These results highlight the differing association between indicators and GDP-at-Risk across horizons.

The upper panels demonstrate the association between domestic indicators and domestic GDP-at-Risk. Here, for comparison, we also present coefficient estimates from a domestic-only specification, which excludes the foreign-weighted variables from the regressor set. As in other studies (Aikman et al., 2019; Adrian et al., 2022), in the domestic-only specification heightened domestic equity volatility (i.e. tighter financial conditions) weighs negatively on GDP-at-Risk in the near term—with the effect peaking in the first quarter, then waning over time. Higher domestic credit-to-GDP also has detrimental effects on the left-tail of GDP in the near-to-medium term—the effect peaks around year 1 and persists out to year 5.

Equity Volatility  The addition of foreign-weighted variables significantly alters the coefficient on domestic equity volatility. Its magnitude is much reduced and generally insignificant across horizons. At the 1-quarter horizon, a one standard deviation increase in domestic volatility is associated with a statistically insignificant 0.1pp deterioration in the 5th percentile of GDP growth, compared to a 1.1pp reduction in the domestic-only specification. In contrast, we find that higher foreign equity volatility is associated with a near-term reduction in the left-tail of annual average domestic GDP growth that is significant at the 32% level. The 1-quarter coefficient indicates that a one standard deviation increase in foreign-weighted equity volatility is linked with a 0.7pp fall in the 5th percentile of GDP growth.

Moreover, there is some evidence that the association between foreign-weighted financial market volatility and the 5th percentile of GDP growth changes sign over horizons. This indicates that financial markets pose an inter-temporal trade-off for GDP-at-Risk: with tight financial conditions weighing negatively on the tails of growth near-term, but supporting growth-at-risk in the medium term by limiting potentially harmful risk taking. Adrian et al. (2022)

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13 Coefficient estimates for the macroeconomic control variables—domestic and foreign-weighted quarterly GDP growth—are presented in Appendix B.1.

14 These coefficient estimates should not be strictly interpreted as causal given potential correlations between domestic and foreign-weighted covariates. We return to the issue of causality in Section 5, where we move towards a structural decomposition of the drivers of GDP-at-Risk.
Figure 1: Association between indicators and the 5th percentile of GDP growth across horizons

**Note:** Estimated association between one standard deviation change in each indicator at time $t$ with 5th percentile of annual average real GDP growth at each quarterly horizon. Red dashed lines denote coefficient estimates from model that excludes foreign covariates. Solid blue lines denote coefficient estimates from model that includes foreign covariates. Light (dark) blue-shaded areas represent 95% (68%) confidence bands from block bootstrap procedure. Additional macroeconomic controls: domestic and foreign-weighted lagged quarterly real GDP growth.

note this trade-off in the context of their study. However, unlike them, our results suggest that the trade-off emanates from global financial conditions, not domestic ones.

**Credit-to-GDP** The domestic credit growth coefficient is negative at all horizons and, while its magnitude falls in the specification with foreign-weighted indicators, it remains significantly negative across horizons. At its peak, a one standard deviation increase in domestic credit-to-GDP is associated with a 0.6pp reduction in GDP-at-Risk in the foreign-augmented specification, compared to a reduction of around 0.9pp in the domestic-only specification. We also find that foreign-weighted credit-to-GDP weighs significantly on GDP-at-Risk in the near-to-medium term. The coefficient is significantly negative from the quarter 3 onward.\(^\text{15}\) At its peak, a one standard deviation increase in foreign-weighted credit-to-GDP is associated with\(^\text{15}\)These point estimates are statistically significant at the 5% level for $h = 6$ to $h = 11$, inclusive.
Note: Estimated association between one standard deviation change in domestic or foreign-weighted realised equity volatility and 3-year change in credit-to-GDP at time $t$ with each quantile $\tau$ of average annual real GDP growth at horizon $h = 1$ and $h = 8$, respectively. Red dashed lines denote coefficient estimates from model that excludes foreign covariates. Solid blue lines denote coefficient estimates from model that includes foreign covariates. Light (dark) blue-shaded areas represent 95% (68%) confidence bands from block bootstrap procedure.

a 0.7pp reduction in the 5th percentile of GDP growth. Foreign credit-to-GDP growth appears to have similar effects on the domestic macroeconomic risk outlook as domestic credit-to-GDP.

### 3.2.2 Coefficients Across Quantiles

Figure 2 shows complementary coefficient estimates to Figure 1. Focusing on the horizons at which the foreign-weighted indicators have their (near) peak effects on the left tail of GDP growth, we present coefficient estimates across quantiles, $\tau = 0.05, 0.10, 0.15, ..., 0.95$, to assess the extent to which there is a non-linear association between indicators and the conditional distribution of GDP growth.
Equity Volatility  The top left-hand plot demonstrates that the addition of foreign-weighted variables has a significant impact on coefficient estimates for domestic equity volatility across quantiles at $h = 1$. At all quantiles, the magnitude of estimated coefficients is smaller in the foreign-augmented model. Although the coefficient is still most negative at the 5th percentile, coefficient estimates are not significantly different across quantiles.

The top right-hand panel illustrates that foreign-weighted equity volatility has its strongest association with tails of the GDP growth distribution. Coefficient estimates are significantly different from zero at the 32% level at all quantiles except the median. But they are not significantly different across quantiles.

Credit-to-GDP  Coefficient estimates for the domestic and foreign-weighted 3-year change in credit-to-GDP across quantiles at $h = 8$ are presented in bottom half of Figure 2. They highlight a strong non-linear association between the GDP-growth distribution and credit-to-GDP growth. Higher credit growth, relative to GDP, is associated with a significantly more left-skewed distribution. This is most apparent for foreign-weighted credit-to-GDP, whose coefficient at the 5th percentile—of 0.7pp—compares to a near-0 estimate at the median. While domestic credit-to-GDP growth is also associated with a shift in the left tail of the GDP-growth distribution, the addition of foreign indicators reduces the extent to which this is the case.

3.2.3 Discussion of Coefficient Estimates

These results indicate that, holding domestic factors fixed, financial developments abroad can significantly influence the conditional distribution of future GDP growth, with particularly large effects at the left tail. This points to an important role for cross-border spillovers in driving downside macroeconomic tail risk. It also suggests there is important information in foreign variables relevant for estimating GDP tail risk, over and above the information in domestic variables—a point we return to in the following sub-sections.

Our estimates for the impact of heightened global credit-to-GDP growth have some parallels with the early-warning literature, where faster global credit growth has been found to be a significant predictor of crises (Lo Duca and Peltonen, 2013; Cesa-Bianchi et al., 2019a; Bluwstein et al., 2020). This literature finds that domestic and global credit growth have similar effects on the probability of a domestic banking crisis, similar to our findings in Figure 1. However, our results are more general, suggesting that this predictability arises specifically from the association between foreign credit growth and the left tail of the domestic GDP growth distribution, as shown in Figure 2.

There are a number of channels through which heightened credit growth abroad could affect downside tail risks to domestic GDP growth—even holding domestic credit growth fixed. Rapid credit growth abroad may increase the probability and severity of downturns in other countries, which in turn can influence the domestic macroeconomy via crisis contagion. But
there may also be other channels at play. Rapid credit growth abroad may partially reflect additional foreign lending by domestic financial institutions, increasing the exposure of the domestic financial system to developments across borders. In addition, changes in global credit growth may reflect shifts in global sentiment and risk aversion, which could, in turn, affect the sentiment of domestic agents.\footnote{Given that we control for domestic credit-to-GDP growth, we likely partial out some of these spillover effects via changes in sentiment. We consider a specification that accounts for contemporaneous spillovers from global to domestic credit growth in Section 5.}

Our estimates for the association between foreign financial market volatility and the conditional GDP-growth distribution also mirrors findings in previous work, suggesting that financial-market uncertainty is more important for the business cycle when the shocks are global in nature (see e.g. Eguren-Martin and Sokol, 2019; Cesa-Bianchi, Pesaran, and Rebucci, 2019b).\footnote{We find the effect of domestic financial market volatility on GDP-at-Risk is also negative. Although this is insignificantly different from zero, this likely partially reflects difficulties of accurately estimating coefficients when regressors are collinear (the average correlation between domestic and foreign-weighted financial market volatility is around 0.8).} This may reflect the fact that a shock that affects all countries at once can have particularly significant effects via global amplification mechanisms and non-linearities, which may not arise for a shock affecting the domestic economy only.

\section{3.3 Additional Information from Foreign Variables: In Sample}

Our coefficient estimates highlight an important role for cross-border spillovers in driving downside macroeconomic tail risk. We now turn to a natural and complementary question: whether the inclusion of foreign covariates in equation (1) significantly improves estimates of the predicted conditional distribution of GDP growth. This question is of particular relevance when viewing the model as a tool for monitoring financial stability risks. Even if developments abroad are a key driver of downside risks to domestic GDP growth, there may be little gain—from a monitoring and forecasting perspective—to including them in the model if the information contained in foreign variables is already sufficiently captured in domestic indicators. In such a case, estimates of GDP-at-Risk would be little changed when accounting for foreign factors.

To formally test the additional explanatory power provided by foreign-weighted variables, we compute a quantile-specific $R^1(\tau)$ statistic (or quantile score)—a ‘goodness-of-fit’ measure for quantile regression analogous to the conventional $R^2$ statistic for OLS regression (Koenker and Machado, 1999). While the $R^2$ quantifies the success of one model relative to another—typically a constant-only model—at the conditional mean, the $R^1(\tau)$ provides information on the relative performance of models at the $\tau$-th quantile. We carry out this analysis using in-sample estimates, but discuss out-of-sample findings in Section 3.5.

To focus on the additional information from foreign variables, we compare the full (‘unrestricted’) foreign-augmented model to the (‘restricted’) domestic-only model. Defining $\hat{V}^h(\tau)$
Table 1: $R^1_h(\tau)$ across horizons and quantiles

<table>
<thead>
<tr>
<th>Horizons</th>
<th>Quantiles</th>
<th>$\tau = 0.05$</th>
<th>$\tau = 0.25$</th>
<th>$\tau = 0.5$</th>
<th>$\tau = 0.75$</th>
<th>$\tau = 0.95$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$h = 1$</td>
<td></td>
<td>0.060***</td>
<td>0.025***</td>
<td>0.026***</td>
<td>0.027*</td>
<td>0.036***</td>
</tr>
<tr>
<td>$h = 4$</td>
<td></td>
<td>0.069***</td>
<td>0.024**</td>
<td>0.020**</td>
<td>0.028*</td>
<td>0.034***</td>
</tr>
<tr>
<td>$h = 8$</td>
<td></td>
<td>0.056***</td>
<td>0.017**</td>
<td>0.008</td>
<td>0.011</td>
<td>0.018***</td>
</tr>
<tr>
<td>$h = 12$</td>
<td></td>
<td>0.053***</td>
<td>0.015*</td>
<td>0.003</td>
<td>0.006</td>
<td>0.007***</td>
</tr>
<tr>
<td>$h = 20$</td>
<td></td>
<td>0.043***</td>
<td>0.025**</td>
<td>0.020**</td>
<td>0.004</td>
<td>0.026***</td>
</tr>
</tbody>
</table>

Note: $R^1_h(\tau)$ statistics comparing the foreign-augmented (‘unrestricted’) model to the domestic-only (‘restricted’) across horizons and quantiles. Significance at 10%, 5% and 1% levels denoted by *, ** and *** respectively. Statistical significance assessed using likelihood-ratio test from Koenker and Machado (1999).

as the sum of weighted absolute residuals from the unrestricted model at the $\tau$-th quantile and $h$-th horizon, and $\tilde{V}^h(\tau)$ equivalently for the restricted model, we calculate the horizon- and quantile-specific $R^1_h(\tau)$ statistic as: $R^1_h(\tau) = 1 - \frac{\tilde{V}^h(\tau)}{\hat{V}^h(\tau)}$, where $R^1_h(\tau) \in [0, 1]$. We can then interpret $R^1_h(\tau)$ as a measure of how much foreign augmentation alters the goodness-of-fit of the estimated $\tau$-th quantile of $h$-quarter-ahead real GDP growth relative to the domestic-only model. A higher $R^1_h(\tau)$ denotes a larger increase in goodness-of-fit arising from the addition of foreign variables.

Table 1 details $R^1_h(\tau)$ statistics across a range of quantiles and horizons, alongside their statistical significance—assessed using the likelihood-ratio test described in Koenker and Machado (1999). Three observations are noteworthy. First, at all horizons, the $R^1_h(\tau)$ is highest for the 5th percentile of real GDP growth, relative to both the median and the 95th percentile. So, the inclusion of foreign variables in equation (1) improves estimates of the left tail of the conditional GDP-growth distribution most materially. This is consistent with the coefficient estimates across quantiles presented in Figure 2.

Second, $R^1_h(0.05)$ peaks at the 3-quarter-ahead horizon, but remains at around 5% at longer horizons. So, the inclusion of foreign variables has significant explanatory power for the left tail of the real GDP-growth distribution across horizons. This is consistent with the coefficient estimates across quantiles presented in Figure 1.

Third, when carrying out a likelihood-ratio test, we find the $R^1_h(\tau)$ statistics for the 5th percentile of GDP growth to be statistically significant at the 1% level (at least) for all 20 horizons. As such, the inclusion of foreign variables has a significant impact on estimates of the left tail of the conditional GDP distribution in particular, improving the goodness-of-fit over and above a model with only domestic covariates. Together, these observations highlight the importance of accounting for foreign variables when monitoring tail risks to domestic GDP growth.

Excluding GFC Period To assess the extent to which these results are driven by the period around the 2007-2008 GFC, we also compute $R^1_h(\tau)$ statistics by excluding the GFC from the sample—specifically the period from 2006Q1 to 2009Q2. The headline results from Table 1 are
robust to this. $R^1_h(\tau)$ statistics remain positive and significant at a range of horizons—at the 5th percentile especially. For instance, the ex-GFC $R^1_{12}(0.05)$ statistic is 0.050, significant at the 1% level. This indicates that the additional in-sample explanatory power attributable to foreign variables in our model, over and above domestic ones, is not solely driven by the GFC.

### 3.4 Robustness

As explained in Section 3.1, we have so far used a pared back model to focus on the key insights gained from using our general methodology to account for the global drivers of GDP-at-Risk, and the distribution of GDP growth more generally. Using this specific model, we have shown that: (i) foreign-weighted variables exert a significant influence on domestic GDP-at-Risk, even when accounting for domestic variables; (ii) foreign-weighted variables, in particular changes in credit-to-GDP, have a larger influence on the left tail of the conditional GDP-growth distribution than at the median; and (iii) foreign-augmentation can significantly improve the goodness-of-fit of estimates of conditional quantiles of GDP growth within sample, especially at the left tail of the distribution.

As Table 2 summarises, these headline results are robust to a range of alternative model specifications. Across all specifications, the estimated coefficient on foreign-weighted credit-to-GDP at the 5th percentile for medium-term horizons is significantly negative at the 32% level at least. These coefficients are substantially more negative than coefficients estimates for the median, indicating the influence of these foreign variables on the left tail of the conditional GDP-growth distribution in particular. The estimated coefficient on foreign equity price volatility (or alternative measures of financial conditions) at the 5th percentile is significantly negative at near-term horizons across most specifications too.

We discuss each of the robustness exercises summarised in Table 2 in more detail below.

**Financial Conditions Index**  Panel B presents results from a specification using a financial conditions index (FCI), as in Adrian et al. (2022), in place of equity price volatility. The FCI, constructed per the method of Koop and Korobilis (2014), is a summary measure that extracts common variation across a range of asset prices. The FCI data is available for a smaller set of countries (10) than our baseline financial conditions indicator (13). We find that a tightening in weighted foreign FCIs has a particularly strong negative effect on the left tail of domestic GDP growth at near-term horizons that is significant at the 1% level, which switches sign in the medium term. The change in coefficient sign for the domestic FCI is less pronounced indicating that the inter-temporal trade-off for GDP-at-Risk from asset prices, identified by Adrian et al. (2022), appears to be driven by global conditions. In addition, as reported in Appendix B.2, $R^1_h(0.05)$ is significant at the 5% level, at least, and is larger than $R^1_h(0.5)$ at most horizons. This indicates a robust and significant improvement in the goodness-of-fit for estimates of the left tail of GDP growth from the inclusion of foreign variables.
<table>
<thead>
<tr>
<th>Foreign variables</th>
<th>$h = 1$</th>
<th>$h = 4$</th>
<th>$h = 8$</th>
<th>$h = 12$</th>
<th>$h = 20$</th>
<th>$h = 1$</th>
<th>$h = 4$</th>
<th>$h = 8$</th>
<th>$h = 12$</th>
<th>$h = 20$</th>
</tr>
</thead>
<tbody>
<tr>
<td>For. Credit-to-GDP</td>
<td>-0.243*</td>
<td>-0.597*</td>
<td>-0.655**</td>
<td>-0.429*</td>
<td>-0.341*</td>
<td>-0.013</td>
<td>-0.451*</td>
<td>-0.699*</td>
<td>-0.414*</td>
<td>-0.679***</td>
</tr>
<tr>
<td>[0.066]</td>
<td>[0.188]</td>
<td>[0.357]</td>
<td>[0.371]</td>
<td>[0.363]</td>
<td>[0.104]</td>
<td>[0.018]</td>
<td>[0.182]</td>
<td>[0.349]</td>
<td>[0.510***]</td>
<td></td>
</tr>
<tr>
<td>For. Fin. Mkt. Vol.</td>
<td>-0.722**</td>
<td>-0.251*</td>
<td>0.135</td>
<td>0.160*</td>
<td>0.017</td>
<td>-1.400***</td>
<td>-0.781*</td>
<td>0.096</td>
<td>0.303*</td>
<td>0.949</td>
</tr>
<tr>
<td>[0.440]</td>
<td>[0.138]</td>
<td>[0.029]</td>
<td>[0.074]</td>
<td>[0.023]</td>
<td>[0.056]</td>
<td>[0.078]</td>
<td>[0.157]</td>
<td>[0.215]</td>
<td>[0.274]</td>
<td></td>
</tr>
<tr>
<td>For. FCI</td>
<td>-0.140***</td>
<td>-0.781*</td>
<td>0.096</td>
<td>0.303*</td>
<td>0.949</td>
<td>1.171***</td>
<td>0.720**</td>
<td>0.201*</td>
<td>0.026</td>
<td>0.073*</td>
</tr>
<tr>
<td>[0.056]</td>
<td>[0.111]</td>
<td>[0.083]</td>
<td>[0.111]</td>
<td>[0.083]</td>
<td>[0.958***]</td>
<td>[0.635***]</td>
<td>[0.304]</td>
<td>[0.304]</td>
<td>[0.111]</td>
<td>[0.083]</td>
</tr>
<tr>
<td>For. GDP gr.</td>
<td>1.171***</td>
<td>0.720**</td>
<td>0.201*</td>
<td>0.026</td>
<td>0.073*</td>
<td>1.171***</td>
<td>0.720**</td>
<td>0.201*</td>
<td>0.026</td>
<td>0.073*</td>
</tr>
<tr>
<td>[0.958***]</td>
<td>[0.635***]</td>
<td>[0.304]</td>
<td>[0.304]</td>
<td>[0.111]</td>
<td>[0.083]</td>
<td>[0.958***]</td>
<td>[0.635***]</td>
<td>[0.304]</td>
<td>[0.304]</td>
<td>[0.111]</td>
</tr>
<tr>
<td>Domestic variables</td>
<td>$h = 1$</td>
<td>$h = 4$</td>
<td>$h = 8$</td>
<td>$h = 12$</td>
<td>$h = 20$</td>
<td>$h = 1$</td>
<td>$h = 4$</td>
<td>$h = 8$</td>
<td>$h = 12$</td>
<td>$h = 20$</td>
</tr>
<tr>
<td>Credit-to-GDP</td>
<td>-0.166*</td>
<td>-0.499***</td>
<td>-0.490***</td>
<td>-0.389***</td>
<td>-0.336***</td>
<td>-0.118</td>
<td>-0.407*</td>
<td>-0.355*</td>
<td>-0.360*</td>
<td>-0.407*</td>
</tr>
<tr>
<td>[0.252]</td>
<td>[0.454***]</td>
<td>[0.518***]</td>
<td>[0.469***]</td>
<td>[0.404***]</td>
<td>[0.159]</td>
<td>[0.189]</td>
<td>[0.312]</td>
<td>[0.384]</td>
<td>[0.416]</td>
<td></td>
</tr>
<tr>
<td>Fin. Mkt. Vol.</td>
<td>-0.096</td>
<td>-0.103</td>
<td>0.104</td>
<td>0.140*</td>
<td>0.025</td>
<td>-0.543*</td>
<td>-0.176</td>
<td>0.117</td>
<td>0.176*</td>
<td>-0.073</td>
</tr>
<tr>
<td>[0.124]</td>
<td>[0.178]</td>
<td>[0.008]</td>
<td>[0.003]</td>
<td>[0.041]</td>
<td>[0.382]</td>
<td>[0.312]</td>
<td>[0.049]</td>
<td>[0.037]</td>
<td>[0.085]</td>
<td></td>
</tr>
<tr>
<td>For. FCI</td>
<td>0.188*</td>
<td>0.258*</td>
<td>0.123*</td>
<td>0.088*</td>
<td>-0.033*</td>
<td>0.258*</td>
<td>0.318*</td>
<td>0.211*</td>
<td>0.107*</td>
<td>-0.011</td>
</tr>
<tr>
<td>[0.274]</td>
<td>[0.266***]</td>
<td>[0.131]</td>
<td>[0.072]</td>
<td>[0.013]</td>
<td>[0.514]</td>
<td>[0.316***]</td>
<td>[0.159]</td>
<td>[0.044]</td>
<td>[0.001]</td>
<td></td>
</tr>
<tr>
<td>N (N*)</td>
<td>13 (13)</td>
<td>10 (10)</td>
<td>13 (13)</td>
<td>11 (11)</td>
<td>11 (11)</td>
<td>13 (13)</td>
<td>13 (13)</td>
<td>17 (32)</td>
<td>17 (32)</td>
<td>17 (32)</td>
</tr>
</tbody>
</table>

### Notes
- Coefficient estimates for 5th percentile and median significance, form block bootstrap, at 32%, 10%, 5%, and 1% levels denoted by *, **, *** and ****. 
- Coefficient estimates for benchmark model and robustness exercises.
**Domestic Covariates**  Panel C presents results from a specification with additional domestic covariates. Here, we include domestic 3-year house price growth, the capital ratio (a measure of overall banking system resilience), the 1-year change in headline central bank policy rates and 1-year inflation in our domestic covariate set—as in Aikman et al. (2019). This allows us to test whether foreign variables provide additional explanatory power, even after accounting for a much wider range of potential domestic covariates. In the this specification, despite the addition on more domestic indicators, the foreign variables we construct continue to have explanatory power, as shown by the significant coefficient estimates in Table 2 and $R^2_T(\tau)$ statistics, shown in Appendix B.2.

**Foreign-Weighting Scheme**  In our baseline specification, we use trade weights to capture countries’ bilateral exposures. These weights have the advantage of running back to 1980, enabling us to use time-varying weights. However, we find similar results when we use bilateral financial weights using BIS International Banking Statistics that capture banks’ exposures to the rest of the world (Panel D).

**Foreign Country Set**  In Panel E, we present results from a specification where we extend the set of countries used to define foreign-weighted covariates. We increase our foreign country set ($N^*$) to 19, by including 6 emerging market economies (China, Korea, Indonesia, Mexico, Turkey and Hong Kong) in addition to the 13 advanced economies used in our baseline specification. We maintain our domestic variable set ($N$) at 13. We shorten the sample for this specification due to limited data availability in some emerging market economies. The results from this model are very similar to our baseline results, although we find slightly larger effects of foreign variables on domestic GDP-at-Risk when we extend the foreign country set.

**Pre-GFC Sample**  To assess the extent to which the GFC drives our results, Panel F reports coefficients from a sample estimated on pre-GFC data, from 1981Q1 to 2005Q4. As in the full sample, we find that foreign-weighted vulnerabilities exert a significant influence on the left tail of GDP growth, with foreign credit weighing more on the 5th percentile in the medium term than the median. In addition, pre-GFC coefficient estimates for foreign-weighted credit-to-GDP, in particular, are similar in magnitude to full-sample estimates, suggesting that the GFC period is not driving our results.

In addition to the robustness exercises presented in Table 2, we carry out two further checks—the results of which are reported in Appendix B.2.

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18 Owing to data limitations, we construct time-invariant bilateral financial weights using average values from 2005 to 2018.
Pooled Country-Specific Results  In our baseline specification, we estimate equation (1) as a panel, accounting for time-invariant country-specific unobservables through the use of country-specific fixed effects $\alpha_{hi}(\tau)$, but assuming that the $\beta_{hi}(\tau)$ and $\varphi_{hi}(\tau)$ coefficients are homogeneous across countries. As an alternative, we estimate equation (1) for single country at a time. This yields 13 sets of estimated coefficients, which we use to compute the mean (and median) coefficient across countries—a quantile regression equivalent of the pooled mean-group estimator for linear regression (Pesaran, Shin, and Smith, 1999). The results indicate that the estimated pooled mean (and median) estimates are similar to those from the panel model.

US-Only Foreign Variables  In Section 2 we noted that our proposal nests one in which only US variables are used in the foreign variable set. When estimating this, we continue to find similar results: US financial market volatility weighs on domestic GDP-at-Risk in the near term, while US credit-to-GDP growth has a significant association with medium-term tails risks. However, the magnitude of estimated coefficients is somewhat smaller. For example, at $h = 8$, the coefficient on US credit-to-GDP growth is $-0.303$ (significant at the 32% level), versus $-0.655$ (significant at the 5% level) in the baseline. While this indicates that the US plays an important role in driving domestic tail risks, there are advantages to using a wider set of countries when constructing the foreign-weighted variables to account for a broader set of cross-border transmission channels and shocks—including the build-up of regional risks.

3.5 Additional Information from Foreign Variables: Out of Sample

In this sub-section, we consider the real-time performance of our model and the extent to which foreign variables also provide additional explanatory power out of sample. To do so, we backtest the model by estimating it in real time from 1995Q1 onwards, with the caveat that we use final revised data only. We estimate regression (1) at four quantiles, $\tau = 0.05, 0.25, 0.75, 0.95$, extending the sample one quarter at a time from 1995Q1. This yields a 24-year quarterly time series of estimated coefficients and out-of-sample forecasts.

We describe two features of the out-of-sample results here. First, we explain that the coefficient estimates presented in Section 3.2 are broadly stable across sub-samples. Second, we evaluate out-of-sample accuracy by analysing the implied probability integral transform (PIT) from the models.\footnote{We additionally discuss and report predictive scores from the models in Appendix B.6.}

Coefficient Sub-Sample Stability  We present real-time coefficient estimates for domestic foreign-weighted financial market volatility and credit-to-GDP at their (near) peak horizons—$h = 1$ and $h = 8$, respectively—at the 5th and 50th percentiles in Appendix B.4. Two observations are particularly noteworthy. First, while the magnitudes of coefficients at the 5th
Figure 3: Out-of-sample PITs for the USA, Canada and France

Note: Empirical cumulative distribution of the probability integral transform (PIT) of out-of-sample estimates for the US, Canada and France at the 1- and 4-quarter-ahead horizons. Blue line shows the estimates from the foreign-augmented model, while red line shows the estimates from the restricted domestic-only model. Dashed lines denote 95% confidence intervals, obtained using the method of Rossi and Sekhposyan (2019). PITs for other countries presented in Appendix B.5.

percentile vary somewhat over time, they are robustly negative for foreign-weighted variables at each period. Second, for foreign-weighted credit-to-GDP especially, the coefficient estimate for the 5th percentile is consistently more negative than the median estimate, supporting our conclusion that foreign factors weigh on the left tail of domestic GDP growth in particular.

Out-of-Sample Accuracy To assess the out-of-sample accuracy of the model in this section, we use our real-time quantile regression estimates at $\tau = 0.05, 0.25, 0.75, 0.95$ to recover estimates of the full forecast distribution at each point in time. To do this, we follow the approach of Adrian et al. (2019) and use the four estimated conditional quantiles of GDP growth to fit a skewed-$t$ distribution (Azzalini and Capitanio, 2003) at each point in time.\footnote{In Section 4, we carry out a fuller assessment of post-quantile regression density fitting, comparing parametric and non-parametric approaches in the context of the narrative around estimates of higher-order GDP moments. We focus on the parametric skewed-$t$ distribution here, for comparison purposes with Adrian et al. (2019), although our results are robust to using the alternative non-parametric approach.}

Using these fitted distributions, we analyse the calibration of the predictive density by computing the empirical cumulative distribution of PITs. This measures the percentage of observations that are below any given quantile. The closer the empirical cumulative distribution of the PITs is to the 45-degree line, the better calibrated the model is.\footnote{In a perfectly calibrated model, the cumulative distribution of the PITs is a 45-degree line, so that the fraction of realisations below any given quantile $Q_{\Delta h_{i,t+h}} (\tau | X_{i,t}, X_{i,t}^*)$ of the predictive distribution is exactly equal to $\tau$.}
Results for the US, Canada and France—three exemplar countries that we use consistently in the remainder of the main body of this paper—are presented in Figure 3 for the 1- and 4-quarter-ahead horizons. Confidence bands around the 45-degree line are constructed per the method of Rossi and Sekhposyan (2019).

The figures illustrate that the foreign-augmented quantile regression generates robust predictive distributions. For all three countries, at all three horizons, the empirical distribution of PITs is within the confidence bands at all quantiles for both the domestic-only and foreign-augmented models—with the upper quantiles of the USA at the 1-quarter horizon the only exception. Moreover, for the 4-quarter-ahead horizon especially, the PITs from the foreign-augmented model are closer to the 45-degree line than the domestic-only model. Comparable PIT distributions for other countries, presented in Appendix B.5, yield similar results.

Overall, the results in this section have demonstrated that foreign-weighted variables exert a significant and robust influence on domestic GDP-at-Risk, even when accounting for domestic variables. Foreign-augmentation can significantly improve estimates of the conditional distribution of GDP growth both in and out of sample, and is especially relevant for pinning down the left tail of the distribution.

4 Estimating Higher Moments of the GDP-Growth Distribution

Building on these findings, we now investigate which features of the GDP-growth distribution the model can be informative about. In particular, we assess the extent to which incorporating foreign variables into our baseline model aids the narrative around changes in higher moments of the GDP-growth distribution over the business cycle.

This analysis is closely related to a recent debate in the literature. Using a different model to ours, Plagborg-Møller et al. (2020) suggest that quantile regression techniques are only able to estimate interpretable changes in the conditional mean of the GDP-growth distribution, offering a negative assessment of the value of such methods for measuring higher-order moments and downside risks. The generality of these findings have been challenged by Adrian et al. (2022), who note that it is important to distinguish financial price variables and credit quantity variables, due to their different cyclical behaviour, within a quantile regression. We contribute to this debate by assessing the ability of our foreign-augmented model to yield interpretable moves in higher-order moments over the business cycle, and in particular by considering the additional value in accounting for foreign factors.

Our analysis is also related to a debate around the appropriate method for fitting distributions to conditional quantile forecasts estimated within a quantile regression. Following Adrian et al. (2019), a common approach is to fit a parametric skewed-$t$ density to estimated quantiles. But Mitchell et al. (2021) note that alternative non-parametric approaches to fit-
ting densities to quantile-regression outputs can yield additional insights about changes in higher-order moments (e.g. around the emergence of multi-modalities in the GDP growth). We contribute to this by comparing the insights from both the parametric and non-parametric approaches—again with a particular focus on how the addition of foreign variables contributes to the narrative around higher-order moments under each approach.

We structure the remainder of this section as follows. First, we describe the parametric and non-parametric approaches to density fitting that we apply. Second, we present in-sample findings for higher-order moments. A key message here is that, regardless of the approach to calculating moments, the foreign-augmented model yields higher moments that are interpretable over the business cycle. Third, we present comparable out-of-sample findings. Here, we find that foreign-augmentation aids interpretation when moments are calculated using non-parametric methods. But, in line with Plagborg-Møller et al. (2020), it is hard to attain meaningful variation in moments using the parametric methods—regardless of whether you account for foreign factors or not. Finally, we discuss the implications of our findings for the wider literature on growth-at-risk and global disaster risk.

4.1 Constructing Densities From Quantile Regressions

To recover an estimate of the full conditional density of GDP growth, we first estimate quantiles of GDP growth \( Q_{\Delta h y_{i,t+h}}(\tau | X_{i,t}, X^*_i) \) for country \( i \) at horizon \( h \) and time \( t \) using the quantile regression model set out in Section 3.1. We then fit a density to the estimated conditional quantiles and calculate associated moments in order to assess which features of the distribution the model can be informative about. We consider two approaches to fitting a density to the quantiles estimated within the quantile regression.

We follow the parametric approach applied by Adrian et al. (2019) and Plagborg-Møller et al. (2020), amongst others, by fitting a skewed-\( t \) distribution (Azzalini and Capitanio, 2003) to the estimated quantiles at \( \tau = 0.05, 0.25, 0.75, 0.95 \). As Mitchell et al. (2021) note, this approach contrasts with the non-parametric nature of the quantile regressions by assuming a specific functional form for the predictive density and, by construction, rules out certain features of the GDP-growth distribution that may be present in the data (e.g. multi-modality).

So we also consider the non-parametric approach of Mitchell et al. (2021). This method does not superimpose a specific functional form on the estimated quantiles to recover a predictive density. Instead, the conditional quantile estimates are mapped directly to a conditional density, assuming only local uniformity between the quantile forecasts. To construct these

\[\text{For this approach, we follow the methodology of Adrian et al. (2019) and Plagborg-Møller et al. (2020) exactly, with one small exception. We note that the replication codes for these papers estimate moments for the fitted skewed-\( t \) distributions by approximating integrals with discrete sums from the probability distribution function estimated over a finite grid. Instead we use the exact analytical solution for the moments, which can be derived directly from the skewed-\( t \) parameters (see Azzalini and Capitanio, 2003). We find this difference has no effect on the overall direction of results discussed below, although using the exact analytical solution tends to deliver more extreme estimates of the higher moments. This point has also been noted by Mitchell et al. (2021).} \]
densities, we estimate the quantile regression model at \( \tau = 0.05, 0.10, 0.15, \ldots, 0.95 \) and then smooth/interpolate across adjacent quantiles to approximate the true predictive density.\(^{23}\)

### 4.2 Estimated Moments and Distributions: In Sample

In this sub-section, we compare in-sample estimates of GDP-growth moments from our foreign-augmented and domestic-only models to assess the additional information contained in foreign variables. Figures 4 and 5 plot the cross-country average of these moments, from the skewed-t distribution, at each point in time for the 1- and 4-quarter-ahead forecast horizons.\(^{24}\)

The plots demonstrate that our foreign-augmented model picks up interpretable moves in higher-order in-sample moments. At the 1-quarter horizon, the foreign-augmented model yields clear moves in higher moments around crises. For example, it highlights a sharp rise in variance, fall in skew and increase in kurtosis in 2008Q3 around the peak of the GFC, as well as a notable fall in skew and rise in kurtosis in 1987Q4 around the Black Monday stock-market crash. Notably, these moves in higher moments across countries, particularly around the GFC, are more pronounced in the foreign-augmented model than in the restricted domestic-only model. This complements our findings in the previous section, suggesting the additional information contained in foreign variables relates to information about higher-order moments of the GDP-growth distribution.

The conditional mean of the GDP-growth distribution is also highly correlated with higher-order moments at near-term horizons. This has been been established in a range of other empirical literature (see, e.g., Adrian et al., 2019; Delle Monache et al., 2021; Iseringhausen et al., 2021), and can be used to inform the calibration of macroeconomic models with higher-order dynamics (see, e.g., Adrian and Duarte, 2018; Corsetti et al., 2021). In our foreign-augmented model, we find a correlation of around \(-0.8\) between the 1-quarter-ahead mean and variance (i.e. counter-cyclical variance), and a correlation of around \(0.4\) between the 1-quarter-ahead mean and skewness (i.e. pro-cyclical skewness) on average across countries.\(^{25}\)

The foreign-augmented model also yields interpretable moves in higher-order moments at medium-term horizons. At 4 and 8 quarters ahead, both the run-up to the early 1990s downturn—a recessionary period for the majority of countries in our panel—and the years preceding the GFC are characterised by the model as periods of high variance, low and negative skew, and high kurtosis in GDP growth across countries. In contrast, in relatively more...
Figure 4: Estimated in-sample GDP-growth moments: 1-quarter horizon, average across panel

Figure 5: Estimated in-sample GDP-growth moments: 4-quarter horizon, average across panel

Note: Estimates of average time-varying in-sample skewed-$t$ moments of the GDP distribution at the 1- (top) and 4-quarter-ahead (bottom) horizons. Blue line shows the estimates from foreign-augmented model, while red line shows estimates from restricted domestic-only model.
benign periods such as the mid to late 1990s—during which time advanced economies in our panel experienced few recessions and generally above-trend growth—GDP-growth is characterised as having low variance, non-negative skew and relatively low kurtosis. There are some notable differences between the foreign-augmented and domestic-only model estimates at medium-term horizons too. For example, at the 4-quarter horizon specifically, on average across the panel, the foreign-augmented model picks-up a greater rise in variance and kurtosis in the years proceeding downturns in the early 1990s and GFC. But the overall informational advantage from the foreign-augmented model at the 4- and 8-quarter-ahead horizons is less clear than at the 1-quarter forecasting horizon.

We reach similar conclusions for in-sample moments when using the non-parametric approach to density fitting, as Appendix B.8 demonstrates. Like Mitchell et al. (2021), we find that the first two moments from parametric and non-parametric approaches are similar, and that, while the third and (especially) fourth moments differ in levels, they exhibit similar changes over time. And hence the higher moments from the non-parametric approach similarly demonstrate interpretable moves over the business cycle.26

To delve deeper into these results, and turning now to estimates for individual countries, Figure 6 presents in-sample predictive distributions for three countries—the US, Canada and France—for two points in time—1999Q1 and 2008Q4—fitted to the parametric skewed-t density. We choose these dates because of the stark differences in macro-financial conditions at these times. The late 1990s were characterised by a period of relatively low credit growth (relative to GDP) across our panel, as well as low volatility in financial markets, while the period preceding the GFC was characterised by high and rising credit growth, and by a sharp increase in equity volatility at the height of the crisis.

At the 1-quarter-ahead horizon, we compare densities formed in 2008Q3 (i.e. estimates of the GDP-growth distribution for 2008Q4), the quarter in which Lehman Brothers failed, relative to those formed in 1998Q4 (i.e. estimates of the 1999Q1 distribution). Predictive densities from the foreign-augmented model for all three countries in 2008Q3 are not only further to the left (i.e. lower estimated mean), but are also flatter (i.e. higher estimated variance), more left-skewed, and fatter-tailed than densities formed in 1998Q4. We see similar interpretable moves in the estimated distributions at medium-term horizons too. In particular, the estimated GDP-growth distributions formed in 2007Q4 (i.e. 4 quarters prior to 2008Q4) and 2006Q4 (8 quarters) are flatter and more-left skewed than corresponding estimated distributions in the late 1990s.

The differences between the estimated distributions from foreign-augmented and domestic-only models are also notable. At the 1-quarter horizon, the estimated predictive densities from

26In particular, as in Mitchell et al. (2021), we estimate much more extreme estimates for kurtosis from the skewed-t approach (kurtosis ranging between around 3 and 15) than the non-parametric approach (kurtosis ranging between around 2 and 5).
the foreign-augmented model for Canada and France are flatter and more left-skewed ahead of the GFC than for the domestic-only model. This highlights that periods of heightened financial market volatility abroad are associated with a worsening in domestic growth-at-risk, due to changes in higher moments of the domestic GDP-growth distribution. At medium-term 4- and 8-quarter horizons, the moves in higher moments of the growth distribution, and associated worsening in downside tail risk, in the run-up to the GFC are much more stark in the foreign-augmented model than the domestic-only model for all three countries. Again this is particularly true for Canada and France where the foreign-augmented model yields a longer left tail than the domestic-only model. This is consistent with the fact that the rate of credit growth (relative to GDP) was not significantly above its historical average in Canada and France in the years preceding the GFC, and so it is only with the addition of foreign variables that the model picks up a pronounced worsening in downside tail risk driven by changes in higher moments of the GDP-growth distribution.

Figure 7 presents results from the same exercise, but now following the non-parametric approach to fitting densities to the estimated quantiles. The overall headline is similar to Figure 6. Across horizons and all three countries, the model appears to capture interpretable moves in higher moments of the GDP-growth distribution in the run-up to the GFC, and the inclusion of foreign variables tends to lead to estimates of a more extreme left-tail in particular. One interesting feature of the non-parametric distributions is the tendency for sharp kinks in the left tail specifically in the run-up to the GFC—a feature ruled out by the skewed-$t$ distribution—highlighting the heightened possibility of a severe ‘bad’ outcome. Such sharp kinks are particularly noticeable in the estimates from the foreign-augmented model, suggesting a role for global factors in driving disaster risk.

Taken together, these results indicate that the foreign-augmented model can generate in-sample estimates of time-varying moments of the conditional GDP-growth distribution that are interpretable over the business cycle. Moreover, in the run-up to the GFC in particular, the inclusion of foreign variables leads to estimates of the GDP-growth distribution that are flatter, more left-skewed and fatter tailed, particularly in countries where domestic vulnerabilities were relatively muted. These findings complement the results in the previous section around the additional information from foreign variables, highlighting that the inclusion of foreign variables in the model provides information around the higher moments of GDP growth.

4.3 Estimated Moments and Distributions: Out of Sample

We now turn to out-of-sample estimates, recreating predictive densities for 2000Q1 and 2008Q4 for the USA, Canada and France with real-time data. Using these estimates, we find weaker evidence of interpretable moves in higher moments of the GDP-growth distribution in the run-up to the GFC across the three countries, particularly when using the parametric skewed-$t$
Figure 6: Estimated in-sample skewed-t predictive densities

US

Canada

France

Note: Fitted probability density functions for GDP growth in US, Canada and France at the 1-, 4- and 8-quarter horizons. Densities constructed by fitting skewed-t density to quantile regression output in sample. They represent predictions of GDP growth outturns in 1999Q1 and 2008Q4, i.e. formed in 1998Q4 and 2008Q3 (1-quarter-ahead), 1998Q1 and 2007Q4 (4-quarter) and 1997Q1 and 2006Q4 (8-quarter). Blue line shows estimates from the foreign-augmented model, while red line shows estimates from the restricted domestic-only model, as described in Section 3.1.
Note: Fitted probability density functions for GDP growth in US, Canada and France at the 1-, 4- and 8-quarter horizons. Densities constructed by fitting non-parametric density to quantile regression output in sample, following method of Mitchell et al. (2021). They represent predictions of GDP growth outturns in 1999Q1 and 2008Q4, i.e. formed in 1998Q4 and 2008Q3 (1-quarter-ahead), 1998Q1 and 2007Q4 (4-quarter) and 1997Q1 and 2006Q4 (8-quarter). Blue line shows estimates from the foreign–augmented model, while red line shows estimates from the restricted domestic-only model, as described in Section 3.1.
distribution—similar to the findings in Plagborg-Møller et al. (2020). However, we find that non-parametric approach can yield more interpretable results, and in particular highlights the role of global factors in generating multi-modality in the run-up to the GFC.

Figure 8 presents out-of-sample estimates of predictive distributions using the parametric skewed-\(t\) approach. In this case, changes in the shapes of the densities are not as clearly interpretable as those from the in-sample exercise in Figure 6. For example, at the 1-quarter ahead horizon, although the foreign-augmented model estimates more severe downside tail risk (i.e. worse GDP-at-Risk), both relative to the domestic-only model and predictive distributions formed in 1998Q4, this is predominantly driven by a worsening in the mean (i.e. more left-shifted distribution) and not higher moments. In contrast to the in-sample results, the pre-GFC distributions are generally narrower and more right-skewed than the one formed in the late 1990s. So this suggests that the out-of-sample informational content in foreign variables is limited to the location, rather than the shape, of the GDP-growth distribution.

At medium-term horizons, the changes in the shapes of the out-of-sample skewed-\(t\) distributions are somewhat more intuitive. They become flatter and more left-skewed across countries pre-GFC, both relative to the domestic-only model and the densities formed in the late 1990s, particularly at the 8-quarter horizon. However, the differences here are small and much less pronounced than in the in-sample skewed-\(t\) results.

Interestingly, the non-parametric approach seems to capture important features of out-of-sample distributions missed by the parametric approach, thereby yielding more interpretable moves in the shapes of the predictive densities. As shown in Figure 9, and following Mitchell et al. (2021), in this out-of-sample exercise, we find evidence of multi-modality emerging ahead of the GFC—a feature of the distribution that is by construction ruled out by the fitted skewed-\(t\) densities in Figure 8.

At the 1-quarter horizon, the predictive densities for all countries in 2008Q3 exhibit multiple modes, with a modal prediction with positive growth (generally the global mode) and an additional ‘bad’ modal outcome (generally the secondary mode) pointing to negative growth. For France and the US, multi-modality, and the existence of a ‘bad’ (recessionary) modal prediction, appear in the 2008Q3 density—not the 1998Q4 density—and only for the foreign-augmented model. Multi-modality also characterises the pre-GFC predictive distributions for all countries at the 8-quarter horizon. Again, in contrast to the densities estimated in the late 1990s, one mode is centred around negative (average annual) GDP growth. Moreover, the secondary modes in the left-tail of pre-GFC predictive distributions are generally more pronounced (i.e. higher probability) and positioned further to the left (i.e. more extreme) in the foreign-augmented model than in the domestic-only model.

27We similarly find weaker evidence of interpretability over the business cycle when looking at the time series of estimated out-of-sample moments. Given the shorter time series for the out-of-sample moments, it is hard to assess general moves over the cycle. For these reasons, we do not present out-of-sample analogues to Figures 4-5.
Note: Fitted probability density functions for GDP growth in US, Canada and France at the 1-, 4- and 8-quarter horizons. Densities constructed by fitting a skewed-$t$ density to quantile regression output out of sample, following method of Mitchell et al. (2021). They represent predictions of GDP growth outturns in 1999Q1 and 2008Q4, i.e. formed in 1998Q4 and 2008Q3 (1-quarter-ahead), 1998Q1 and 2007Q4 (4-quarter) and 1997Q1 and 2006Q4 (8-quarter). Blue line shows estimates from the foreign-augmented model, while red line shows estimates from the restricted domestic-only model, as described in Section 3.1.
Note: Fitted probability density functions for GDP growth in US, Canada and France at the 1-, 4- and 8-quarter horizons. Densities constructed by fitting non-parametric density to quantile regression output out of sample, following method of Mitchell et al. (2021). They represent predictions of GDP growth outturns in 1999Q1 and 2008Q4, i.e. formed in 1998Q4 and 2007Q4 (1-quarter-ahead), 1998Q1 and 2007Q4 (4-quarter) and 1997Q1 and 2006Q4 (8-quarter). Blue line shows estimates from the foreign-augmented model, while red line shows estimates from the restricted domestic-only model, as described in Section 3.1.
4.4 Discussion of Results

Taken together, our results show that the foreign-augmented model can yield estimates of higher moments of the GDP-growth distribution that are interpretable over the business cycle, especially when done in sample. In addition, in the run-up to the GFC, the inclusion of foreign variables tends to generate GDP-growth distributions that are wider and more left-skewed than those from a model with domestic covariates only.

Our results around the interpretability of higher moments extend to the 4 and 8-quarter-ahead horizons, around the time at which (domestic and foreign-weighted) credit-to-GDP growth have their maximum effect on GDP-at-Risk. These medium-term-horizon findings are particularly relevant for macroprudential policymakers when monitoring risks to financial stability given policy implementation and transmission lags. These findings complement those in Section 3 that suggest the additional information contained in foreign variables reflects a stronger association with the left tail of the GDP distribution than the median, and therefore higher moments of GDP growth—results that we find to be robust and stable over time.

We also find that the interpretability of higher moments depends on the exact methodology used to fit distributions to the quantile regression results. Under a non-parametric approach, the narrative is around the emergence of sharp kinks, or even additional modes, in the left-tail of the domestic GDP growth distribution in the run-up to crisis episodes driven, in part, by heightened global vulnerabilities. This is true for both in- and out-of-sample estimates of predictive densities. In contrast, under the parametric skewed-t approach, the model picks up interpretable moves in the variance and asymmetry of in-sample distributions. But the informational content of foreign variables in out-of-sample densities appears more limited—focused largely on the location, and not the shape, of distributions.

These conclusions offer two main contributions to recent debates in the literature around the use of quantile regression techniques for estimating higher-order moments of GDP growth. First, in contrast to Plagborg-Møller et al. (2020), our results are generally supportive of the ability of quantile regression models to yield estimates of higher moments of the GDP-growth distribution that are interpretable over the business cycle. In particular, we highlight that the inclusion of foreign factors in the quantile regression framework can aid the interpretability of these higher moments.

Second, our conclusions lend support to using non-parametric methods to fitting densities to quantile regression outputs, especially when fitting predictive distributions out of sample. Relative to parametric alternatives, this approach can let the ‘data speak’ by placing fewer

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28In contrast, Plagborg-Møller et al. (2020) focus their main results only on horizons $h = 1$ and $h = 4$.

29Following Adrian et al. (2022), we also separate out financial price and credit quantity variables in both our domestic-only and foreign-augmented specifications. Like Adrian et al. (2022), we find that credit quantities have a stronger effect on the left tail in the medium-term, while asset price variables have a larger effect in the near-term. Given these differences in effects across horizons, as Adrian et al. (2022) note, conflating these variables into a single index (as in Plagborg-Møller et al., 2020) can be problematic and can mask additional information—in particular contained in credit quantities for tail risks in the medium-term.
restrictions on the shape of predictive densities. In turn, this can generate important insights—e.g. around the emergence of multi-modality—missed by the parametric approach.

Our results also corroborate the findings of a broader empirical literature, beyond that which focuses on quantile regression techniques, which documents important higher-order dynamics in the GDP-growth distribution. In particular, our results support evidence of counter-cyclical GDP-growth variance (as in, e.g., Adrian et al., 2019) and pro-cyclical skewness (as in, e.g. Delle Monache et al., 2021; Iseringhausen et al., 2021). Under a non-parametric approach specifically, we also uncover the existence of multi-modalities driven in particular by changes in financial developments (as in, e.g., Adrian et al., 2021; Mitchell et al., 2021).

Considered together, these contributions have broader implications for the modelling of tail risks and macro-financial dynamics too. Specifically, our results highlight that global developments play an important role in generating multi-modality. Predictive densities from our foreign-augmented models tend to have a more pronounced secondary mode in the left tail in run-up to the GFC, indicating that global factors play a key role in driving severe downside risks to growth. This lends support to assessments of macroeconomic disaster risk that account for global developments (e.g. Gourio et al., 2013; Farhi and Gabaix, 2016) and suggests that results from our foreign-augmented model, fitted with non-parametric densities, could yield further insights for that burgeoning literature.

5 Structural Contribution of Foreign Drivers

So far, we have largely focused on the gains from accounting for foreign variables within a quantile regression framework from a financial-stability monitoring perspective. In this section, we consider the importance of foreign developments for changes in the distribution of GDP in a more structural sense. As part of this, we seek to decompose historical estimates of GDP-at-Risk, assessing the contribution of foreign shocks to domestic tail risks. This mirrors attempts to quantify the contribution of foreign shocks to domestic GDP growth at the mean (e.g. Cesa-Bianchi et al., 2019b), and offers a first assessment of how these factors might vary over the GDP distribution.

An immediate challenge to assessing the structural contribution of foreign variables to domestic GDP-at-Risk is the potential correlation of domestic and foreign covariates in equation (1). For example, consistent with evidence of a global financial cycle (Rey, 2013; Miranda-Agrippino and Rey, 2020), tighter financial conditions abroad could spill over to the domestic economy and generate a contemporaneous tightening in domestic financial conditions that, in turn, could drive changes in domestic GDP-at-Risk. The estimated coefficient on foreign financial conditions in equation (1) effectively ‘partials out’ this effect however, by controlling for domestic financial conditions. So, simply decomposing the drivers of GDP-at-Risk using the fitted values from equation (1)—i.e. $\hat{\beta}^h(\tau)X_{i,t}$ representing domestic drivers and $\hat{\varphi}^h(\tau)X^*_{i,t}$
foreign drivers—will likely not yield an accurate estimate for the relative importance of foreign shocks.

5.1 Method: Towards a Structural Decomposition

To move towards identifying the relative contribution of foreign and domestic shocks to domestic GDP-at-Risk, we build a decomposition using a two-step procedure.

In the first step, we orthogonalise the domestic variables with respect to their foreign-weighted counterparts. To do this, we estimate the following OLS regression for each domestic indicator \( x_{i,t} \subset X_{i,t} \) and for each country \( i = 1, \ldots, N \):

\[
x_{i,t} = a_i + b_i X^*_{i,t} + u^\perp_{i,t}
\]

where \( a_i \) and \( b_i \) denote country- and indicator-specific coefficients, and \( u^\perp_{i,t} \) represents the component of a domestic indicator \( x_{i,t} \) that is orthogonal to contemporaneous variation in foreign-weighted indicators \( X^*_{i,t} \). Given coefficient estimates \( \{\hat{a}_i, \hat{b}_i\} \) from equation (3), we define the estimated orthogonal component as the residual:

\[
\hat{u}^\perp_{i,t} = x_{i,t} - \hat{a}_i - \hat{b}_i X_{i,t}.
\]

In the second step, we then estimate a local-projection model for the conditional quantile function of \( h \)-period-ahead GDP growth using the estimated orthogonal component of domestic indicators, the full set of which is denoted by \( u^\perp_{i,t} \), alongside the set of weighted foreign variables \( X^*_{i,t} \):

\[
Q_{\Delta^h y_{i,t+h} | u^\perp_{i,t}, X^*_{i,t}} = \hat{\alpha}^h(\tau) + \hat{\beta}^h(\tau) \hat{u}^\perp_{i,t} + \hat{\varphi}^h(\tau) X^*_{i,t}
\]

where we distinguish coefficients in this equation, relative to equation (1), with tildes. We can then decompose estimates of GDP-at-Risk by labelling \( \hat{\beta}^h(\tau) u^\perp_{i,t} \) as ‘domestic drivers’ and \( \hat{\varphi}^h(\tau) X^*_{i,t} \) as ‘foreign drivers’.

The key assumption in this procedure is that foreign indicators can contemporaneously influence domestic ones, but domestic indicators cannot contemporaneously affect their foreign counterparts. In effect, we treat the domestic country as a small-open economy, by excluding instantaneous feedback from domestic variables to foreign ones. This mirrors the block exogeneity assumption that has been widely used to estimate the transmission of shocks at the mean using structural vector autoregression methods in the empirical international macroeconomics literature (e.g., Dedola, Rivolta, and Stracca, 2017; Cesa-Bianchi and Sokol, 2022).

However, there are caveats to the block exogeneity assumption. As such, we interpret our results with some caution. First, to the extent that the assumption precludes within-quarter transmission from domestic economies to the rest of the world, we view estimates as an upper bound for the contribution of foreign shocks to domestic macroeconomic tail risks.

Second, while this procedure does orthogonalise foreign-weighted variables with respect to their domestic counterparts, it does not enable a structural decomposition of shocks within
countries. So, the approach can isolate the relative importance of foreign or global shocks for domestic GDP-at-Risk. But it cannot distinguish between, for example, different types of structural shock within that (e.g. shocks to financial conditions vs. credit-growth shocks).

5.2 Results: Contribution of Foreign Shocks to GDP-at-Risk

We apply this orthogonalisation procedure to our specific empirical model to estimate the relative importance of foreign drivers of GDP-at-Risk. To do so, we make one change to the baseline model outlined in Section 3.1. To justify the ‘small-open economy’ assumption implicit in the orthogonalisation, we exclude the US from the set of domestic economies when estimating the structural decompositions. Nevertheless, we include the US in the foreign variable set, so we continue to account for its influence in the global economy.

In Appendix B.9, we present coefficient from regression (4) for our baseline specification. These complement our earlier estimates in Figure 1. Unlike Figure 1, these estimates capture contemporaneous spillovers of global factors to domestic covariates—and as such, global variables have a larger impact on domestic tail risk. In Appendix B.10, we also present orthogonalised decompositions for the estimated 5th percentile of 3-year-ahead real GDP growth. These decompositions indicate the differing importance of foreign and domestic shocks for GDP-at-Risk over time.

We also use this alternate model to assess the relative importance of foreign shocks in driving tail risk more systematically. Equation (3) imposes \( \text{cov}_t(X_{i,t}, \hat{u}^\perp_{i,t}) = 0 \), and so the variance of fitted values of the \( \tau \)-th percentile of the GDP-growth distribution can be decomposed as:

\[
\text{var}_t \left( \Delta^h \hat{y}_{i,t+h}(\tau) \right) = \text{var}_t \left( \hat{\theta}^h(\tau) \hat{u}^\perp_{i,t} \right) + \text{var}_t \left( \hat{\theta}^h(\tau) X^*_{i,t} \right)
\]

Therefore, the share of variation in the fitted value of country-\( i \) GDP growth at the \( \tau \)-th percentile at horizon \( h \) attributable to foreign sources can be defined as:

\[
ForShare^h_i(\tau) \equiv 100 \times \frac{\text{var}_t \left( \hat{\theta}^h(\tau) X^*_{i,t} \right)}{\text{var}_t \left( \Delta^h \hat{y}_{i,t+h}(\tau) \right)}
\]  

The estimated shares \( ForShare^h_i(\tau) \) at \( h = 1, 4, 12 \) and \( \tau = 0.05 \) for each country in our baseline regression are presented in Table 3. As discussed, we interpret these quantities as upper-bound estimates for the share of variation attributable to foreign sources due to the stringency of the orthogonalisation assumption imposed by equation (3).

Overall, Table 3 illustrates that a substantial portion of variation in estimates of the GDP-growth distribution can be attributed to foreign sources for all 12 countries in our sample. At the 1-quarter horizon, the average share of variation in GDP-at-Risk (i.e. \( \tau = 0.05 \)) from foreign sources is 92%, around 13pp more than the variation attributed to foreign sources at
Table 3: Share of Variation in Fitted Values (%) Attributes to Foreign Shocks Across Horizons

<table>
<thead>
<tr>
<th>Country</th>
<th>$h = 1$</th>
<th>$h = 4$</th>
<th>$h = 12$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$\tau = 0.05$</td>
<td>$0.5$</td>
<td>$0.05$</td>
</tr>
<tr>
<td>AUS</td>
<td>91.28 (77.45)</td>
<td>81.34 (63.41)</td>
<td>63.62 (46.97)</td>
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<td>CAN</td>
<td>93.33 (81.47)</td>
<td>78.26 (59.70)</td>
<td>53.84 (33.57)</td>
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<tr>
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<td>81.69 (61.87)</td>
<td>64.22 (47.39)</td>
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<td>54.86 (36.06)</td>
</tr>
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<td>50.42 (29.74)</td>
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<td>91.91 (78.60)</td>
<td>81.84 (64.30)</td>
<td>69.17 (49.88)</td>
</tr>
<tr>
<td>NOR</td>
<td>89.92 (73.81)</td>
<td>73.43 (53.04)</td>
<td>50.78 (31.59)</td>
</tr>
<tr>
<td>SPAIN</td>
<td>92.05 (78.67)</td>
<td>80.85 (61.77)</td>
<td>61.42 (42.71)</td>
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<tr>
<td>SWE</td>
<td>91.81 (81.57)</td>
<td>84.74 (69.13)</td>
<td>72.75 (52.65)</td>
</tr>
<tr>
<td>SWI</td>
<td>93.53 (81.99)</td>
<td>82.82 (63.29)</td>
<td>49.56 (33.11)</td>
</tr>
<tr>
<td>UK</td>
<td>91.53 (80.50)</td>
<td>84.27 (69.69)</td>
<td>82.88 (68.39)</td>
</tr>
<tr>
<td>Avg.</td>
<td>91.88 (78.62)</td>
<td>79.92 (61.20)</td>
<td>60.48 (42.11)</td>
</tr>
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</table>

Note: Share of variation at the 5th percentile ($\tau = 0.05$) and median ($\tau = 0.5$ in parentheses) of country-GDP distributions at different horizons: $h = 1$ (1 quarter), $h = 4$ (1 year), and $h = 12$ (3 years). Share definition in equation (5). Shares constructed from baseline model in which domestic indicators are orthogonalised with respect to all foreign indicators, akin to a small-open economy assumption for domestic countries.

Although there is variation across countries, this finding emphasises the crucial role for foreign vulnerabilities at the left tail of the GDP growth distribution specifically. This suggests that there may be important cross-border contagion effects in the global economy that amplify the macroeconomic consequences of tail events over and above more general interdependence between nations (Forbes, 2012).

While the share of variation attributable to foreign sources tends to decline as the horizon increases, the relative importance of foreign factors remains substantial. At the 3-year horizon, the average share of GDP-at-Risk variation linked with foreign shocks is 60%, around 18pp more than the corresponding estimate at the median.

We also assess the robustness of these results by estimating comparable decompositions for two alternative models. First, we estimate our baseline model, but construct foreign-weighted variables using bilateral financial weights from BIS International Banking Statistics. Second, we estimate a model with more domestic covariates, mirroring the specification in Aikman et al. (2019). We continue to exclude the US from the domestic variable set when constructing these decompositions. The results are discussed in Appendix B.10. Our key finding—that foreign factors play a substantial role in explaining variation in the estimated 5th percentile of GDP growth, more so than for the median—is robust across model specifications.
6 Conclusion

This paper has shown that foreign vulnerabilities matter for domestic macroeconomic tail risks. Faster global credit-to-GDP growth and tighter global financial conditions exert a significant negative influence on the left tail of the GDP-growth distribution. Moreover, these foreign indicators provide information relevant for estimating domestic GDP-at-Risk, over and above domestic ones, both in and out of sample. In turn, these foreign indicators help to generate estimates of higher GDP moments that are interpretable over the cycle. Decomposing historical estimates of GDP-at-Risk into orthogonalised domestic and foreign shocks, we show that foreign vulnerabilities on average explain up to around 60% of variation at the 3-year horizon, more than the comparable figure for the median.

Taken together, our findings have important implications for macroprudential policymakers. By highlighting the relevance of global spillovers, they emphasise the importance of monitoring global variables when assessing risks to domestic financial stability. Moreover, by demonstrating the substantial contribution of foreign shocks to domestic tail risks, they point to the potential benefits of international macroprudential policy cooperation in response to global shocks. More broadly, our general methodology can be applied more widely, for instance to inform analyses of GDP-at-Risk within emerging-market economies, where assessments of tail risks have been more limited in spite of their substantial exposures to foreign events.

References


Appendix

A Data Sources

Table 4 presents a full list of data sources used in this paper—both in the main body and the appendices.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Source</th>
<th>Frequency</th>
<th>Notes</th>
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<tbody>
<tr>
<td><strong>Dependent Variable</strong></td>
<td></td>
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<td></td>
</tr>
<tr>
<td>Real GDP</td>
<td>OECD</td>
<td>Quarterly</td>
<td>Construct annual average growth across quarterly horizons</td>
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<tr>
<td><strong>Covariates</strong></td>
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<td></td>
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<tr>
<td>Equity Volatility</td>
<td>Datastream</td>
<td>Daily</td>
<td>Calculate realised volatility within quarter using standard deviation of daily returns</td>
</tr>
<tr>
<td>FCI</td>
<td>IMF</td>
<td>Quarterly</td>
<td>See (Adrian et al., 2022) and Koop and Korobilis (2014)</td>
</tr>
<tr>
<td>Credit-to-GDP</td>
<td>BIS</td>
<td>Quarterly</td>
<td>Construct 3-year change in ratio</td>
</tr>
<tr>
<td>House Prices</td>
<td>OECD</td>
<td>Quarterly</td>
<td>Construct 3-year growth</td>
</tr>
<tr>
<td>Capital Ratio</td>
<td>Aikman et al. (2019)</td>
<td>Annual</td>
<td>Ratio of tangible common equity to tangible assets</td>
</tr>
<tr>
<td>Inflation</td>
<td>OECD</td>
<td>Quarterly</td>
<td>Annual growth of CPI</td>
</tr>
<tr>
<td>Policy Rates</td>
<td>BIS</td>
<td>Quarterly</td>
<td>Annual change in central bank policy rates</td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Export Weights</td>
<td>IMF DOTS</td>
<td>Quarterly</td>
<td>Construct weights by averaging across each calendar year to smooth seasonal variation</td>
</tr>
<tr>
<td>Financial Weights</td>
<td>BIS, IBS, Tbl. 9D</td>
<td>Quarterly</td>
<td>Construct weights by averaging across each calendar year to smooth seasonal variation</td>
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</table>
B Additional Results

B.1 Baseline Empirical Model

In this Appendix sub-section, we report additional results for our specific model described in Section 3.1.

Coefficient Estimates for Macroeconomic Controls Across Horizons  Figure 10 presents coefficient estimates for the macroeconomic control variables—domestic and foreign-weighted quarterly real GDP growth—at the $\tau = 0.05$th quantile across horizons in our specific model described in Section 3. Both domestic and foreign-weighted real GDP growth are associated with higher estimates of the 5th percentile of real GDP growth.

Figure 10: Association between indicators and the 5th percentile of GDP growth across horizons

Note: Estimated association between one standard deviation change in each indicator at time $t$ with 5th percentile of average annual real GDP growth at each quarterly horizon. Red dashed lines denote coefficient estimates from model that excludes foreign covariates. Solid blue lines denote coefficient estimates from model that includes foreign covariates. Light (dark) blue-shaded areas represent 95% (68%) confidence bands from block bootstrap procedure.

Impulse Responses Across Quantiles  Figure 11 compares impulse responses from our baseline foreign-augmented model at the 5th and 50th percentiles. The coefficient estimates for foreign equity volatility and credit highlight notable differences over the distribution. The near-term impacts of tighter financial conditions are much more negative at the 5th percentile than at the median, while the inter-temporal reversal is also specific to the left tail. For foreign-weighted credit, coefficient estimates are negative at all horizons for the 5th percentile. But faster foreign credit growth is associated with higher median GDP growth in the near-term.
Figure 11: Association between indicators and GDP growth across horizons at the 5th and 50th percentiles

Note: Solid blue lines denote estimated association between one standard deviation change in each indicator at time $t$ with 5th percentile of average annual real GDP growth at each quarterly horizon. Light (dark) blue-shaded areas represent 95% (68%) confidence bands around these estimates from block bootstrap procedure. Black dashed lines denote corresponding coefficient estimates at the 50th percentile (i.e. median).
B.2 Additional Robustness for In-Sample Goodness of Fit

Table 5 reports $R^1_h(\tau)$ statistics for two of the robustness exercises discussed in Section 3.4, namely: the model using an FCI as an alternative to the VIX (as per Adrian et al., 2022), and the model with additional domestic covariates (as per Aikman et al., 2019). In both cases, we find that the addition of foreign-weighted covariates leads to a significant increase in quantile scores, especially at the 5th percentile.

Table 5: $R^1_h(\tau)$ across horizons and quantiles for robustness exercises

<table>
<thead>
<tr>
<th>Horizons</th>
<th>$\tau = 0.05$</th>
<th>$\tau = 0.25$</th>
<th>$\tau = 0.5$</th>
<th>$\tau = 0.75$</th>
<th>$\tau = 0.95$</th>
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<tr>
<td>$h = 1$</td>
<td>0.096***</td>
<td>0.028*</td>
<td>0.019</td>
<td>0.025</td>
<td>0.035**</td>
</tr>
<tr>
<td>$h = 4$</td>
<td>0.057***</td>
<td>0.020</td>
<td>0.014</td>
<td>0.023</td>
<td>0.045***</td>
</tr>
<tr>
<td>$h = 8$</td>
<td>0.050***</td>
<td>0.006</td>
<td>0.005</td>
<td>0.013</td>
<td>0.036***</td>
</tr>
<tr>
<td>$h = 12$</td>
<td>0.036**</td>
<td>0.022*</td>
<td>0.011</td>
<td>0.019</td>
<td>0.011</td>
</tr>
<tr>
<td>$h = 20$</td>
<td>0.021***</td>
<td>0.120***</td>
<td>0.072***</td>
<td>0.034**</td>
<td>0.029***</td>
</tr>
</tbody>
</table>

(A) Alternative Financial Conditions

<table>
<thead>
<tr>
<th>Horizons</th>
<th>$\tau = 0.05$</th>
<th>$\tau = 0.25$</th>
<th>$\tau = 0.5$</th>
<th>$\tau = 0.75$</th>
<th>$\tau = 0.95$</th>
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</thead>
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<tr>
<td>$h = 1$</td>
<td>0.067***</td>
<td>0.040***</td>
<td>0.035***</td>
<td>0.030*</td>
<td>0.026***</td>
</tr>
<tr>
<td>$h = 4$</td>
<td>0.063***</td>
<td>0.032***</td>
<td>0.034**</td>
<td>0.037*</td>
<td>0.056***</td>
</tr>
<tr>
<td>$h = 8$</td>
<td>0.021**</td>
<td>0.025**</td>
<td>0.015</td>
<td>0.014</td>
<td>0.028***</td>
</tr>
<tr>
<td>$h = 12$</td>
<td>0.016*</td>
<td>0.017*</td>
<td>0.006</td>
<td>0.004</td>
<td>0.012**</td>
</tr>
<tr>
<td>$h = 20$</td>
<td>0.006</td>
<td>0.010</td>
<td>0.012</td>
<td>0.005</td>
<td>0.002</td>
</tr>
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</table>

(B) Additional Domestic Covariates

Note: $R^1_h(\tau)$ statistics comparing the foreign-augmented (‘unrestricted’) model to the domestic-only (‘restricted’) across horizons and quantiles. Significance at 10%, 5% and 1% levels denoted by *, ** and ****, respectively. Statistical significance assessed using likelihood-ratio test from Koenker and Machado (1999).
B.3 Pooled Country-Specific Results

Figure 12 plots a comparison of our baseline coefficient estimates for foreign-weighted financial market volatility and credit-to-GDP growth from a panel model with the mean and median of coefficient estimates from individual country regressions. The results indicate that the estimated pooled mean and median estimates are similar to those from the panel model.

Figure 12: Average association between indicators and the 5th percentile of GDP growth across horizons from country-specific regressions

Note: Estimated association between one standard deviation change in each indicator at time $t$ with 5th percentile of annual average real GDP growth at each quarterly horizon. Black line denotes mean coefficient estimate when pooling individual-country estimates. Black crosses represent the median. Blue line denotes point estimates from our baseline panel model. Light (dark) blue-shaded areas represent 95% (68%) confidence bands from block bootstrap procedure for the corresponding coefficient estimate over the full 1981Q1-2018Q4 sample.
B.4 Out-of-Sample Coefficient Stability

Figure 13 plots real-time coefficient estimates for domestic and foreign-weighted variables in the foreign-augmented model.

Figure 13: Real-time estimates of the association between domestic and foreign-weighted indicators and the 5th (50th) percentile of GDP growth

Note: Estimated real-time association between one standard deviation change in each indicator at each forecast date and 5th percentile (blue line) and median (black dashed line) of annual average real GDP growth at 1-quarter horizon for domestic and foreign-weighted financial market volatility and $h = 8$-quarter horizon for domestic and foreign-weighted credit-to-GDP. Light (dark) blue-shaded areas represent 95% (68%) confidence bands from block bootstrap procedure for the corresponding 5th percentile coefficient estimate over the full 1981Q1-2018Q4 sample.
B.5 Out-of-Sample PITs

In this Appendix sub-section, we report out-of-sample PITs for the other countries in our sample. This supports the PITs for the US, Canada and France presented in Figure 3 of Section 3.5. Figures 14 and 15 report the 1- and 4-quarter-ahead PITs for the 12 countries in turn. Figure 16 reports PITs for the 8-quarter-ahead horizon.

Figure 14: Out-of-sample PITs: one quarter ahead

Note: Empirical cumulative distribution of the probability integral transform (PIT) of out-of-sample estimates at the 1-quarter-ahead horizon. Blue line shows the estimates from the foreign-augmented model, while red line shows the estimates from the restricted domestic-only model. Dashed lines denote 95% confidence intervals, obtained using the method of Rossi and Sekhposyan (2019).
Figure 15: Out-of-sample PITs: one year ahead

Note: Empirical cumulative distribution of the probability integral transform (PIT) of out-of-sample estimates at the 1-year-ahead horizon. Blue line shows the estimates from the foreign-augmented model, while red line shows the estimates from the restricted domestic-only model. Dashed lines denote 95% confidence intervals, obtained using the method of Rossi and Sekhposyan (2019).
Figure 16: Out-of-sample PITs: two years ahead

![Graphs showing empirical cumulative distribution of the probability integral transform (PIT) of out-of-sample estimates at the 2-year-ahead horizon. Each graph compares estimates from the foreign-augmented model (blue line) and the restricted domestic-only model (red line). Dashed lines denote 95% confidence intervals, obtained using the method of Rossi and Sekhposyan (2019).]

*Note:* Empirical cumulative distribution of the probability integral transform (PIT) of out-of-sample estimates at the 2-year-ahead horizon. Blue line shows the estimates from the foreign-augmented model, while red line shows the estimates from the restricted domestic-only model. Dashed lines denote 95% confidence intervals, obtained using the method of Rossi and Sekhposyan (2019).
B.6 Out-of-Sample Predictive Scores

In this Appendix sub-section, we assess the reliability of predictive distributions by measuring the accuracy of density forecasts using the predictive score—where higher scores indicate more accurate predictions, i.e. outcomes that the model considers more likely are closer to the *ex post* realisation.\(^{30}\) Table 6 presents the average scores of the predictive distribution for the foreign-augmented and domestic-only models, as well as an indication of whether the former is significantly greater than the latter, for a range of horizons and all countries. Across the panel, the average predictive scores from the foreign-augmented model tend to be slightly larger than those from the domestic-only model. At \(h = 1\) and \(h = 4\), the scores from the foreign-augmented model are significantly larger than those from the domestic-only model for over half of the countries. Thus, the conditional distribution from the foreign-augmented model is broadly accurate, and the information contained in the foreign-weighted variables appears to be a robust and genuine feature of the data.

Table 6: Average predictive scores from real-time forecasts across across countries and horizons

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<th></th>
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<td>AUS</td>
<td>0.109</td>
<td>0.106</td>
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<td>0.175</td>
<td>0.165</td>
<td>***</td>
<td>0.216</td>
<td>0.219</td>
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<tr>
<td>CAN</td>
<td>0.126</td>
<td>0.120</td>
<td>**</td>
<td>0.170</td>
<td>0.158</td>
<td>***</td>
<td>0.195</td>
<td>0.191</td>
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<tr>
<td>DNK</td>
<td>0.079</td>
<td>0.075</td>
<td>***</td>
<td>0.165</td>
<td>0.158</td>
<td>**</td>
<td>0.239</td>
<td>0.236</td>
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<tr>
<td>FIN</td>
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<td>0.065</td>
<td>**</td>
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<td>0.103</td>
<td></td>
<td>0.123</td>
<td>0.120</td>
<td>**</td>
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<tr>
<td>FRA</td>
<td>0.191</td>
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<td>0.257</td>
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<td>0.252</td>
<td>0.304</td>
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<td>GER</td>
<td>0.099</td>
<td>0.09</td>
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<td>0.187</td>
<td>0.166</td>
<td>***</td>
<td>0.198</td>
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<td>ITA</td>
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<td>0.139</td>
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<tr>
<td>SPA</td>
<td>0.144</td>
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<td>0.158</td>
<td>0.171</td>
<td></td>
<td>0.144</td>
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<td>0.164</td>
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<td>0.193</td>
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<td>0.127</td>
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<td>0.165</td>
<td></td>
<td>0.167</td>
<td>0.169</td>
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</table>

Notes: Average predictive scores for foreign-augmented and domestic-only models for each country at \(h = 1, 4, 8\) quarters. Predictive scores constructed by estimating the model in real time from 1995Q1, and extending sample one quarter at a time to 2018Q4. Predictive distribution constructed by fitting a skewed-\(t\) distribution (Azzalini and Capitanio, 2003) each period. ‘For.>Dom.’ column denotes difference in mean significance test, with alternative hypothesis that foreign-augmented model has higher predictive score than domestic-only model. Significance at 10%, 5% and 1% levels denoted by *, ** and *** respectively.

\(^{30}\)Formally, this is computed by evaluating a model’s predictive distribution at the realised value of a time series.
B.7 Parametric GDP Moments

In this Appendix sub-section, we present the cross-country average of 8-quarter-ahead in-sample moments, as well as estimates of in-sample time-varying moments of US, France and Canada GDP growth from our baseline specification. These moments are estimated using a skewed-$t$ distribution. These complement the cross-country time-varying moment plots described in Section 4.2.

Figure 17: Estimated in-sample parametric GDP-growth moments: 8-quarter horizon, average across panel

Note: Estimates of time-varying in-sample skewed-$t$ moments of the GDP distribution at the 8-quarter-ahead horizon. Blue line shows the estimates from foreign-augmented model, while red line shows the estimates from restricted domestic-only model.
Figure 18: Estimated in-sample parametric GDP-growth moments: 1-quarter horizon, US

Note: Estimates of time-varying in-sample skewed-$t$ moments of the US GDP distribution at the 1-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
Figure 19: Estimated in-sample parametric GDP-growth moments: 4-quarter horizon, US

Note: Estimates of time-varying in-sample skewed-t moments of the US GDP distribution at the 4-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
Figure 20: Estimated in-sample parametric GDP-growth moments: 8-quarter horizon, US

Note: Estimates of time-varying in-sample skewed-t moments of the US GDP distribution at the 8-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
Figure 21: Estimated in-sample parametric GDP-growth moments: 1-quarter horizon, Canada

Note: Estimates of time-varying in-sample skewed-$t$ moments of the Canada GDP distribution at the 1-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
Figure 22: Estimated in-sample parametric GDP-growth moments: 4-quarter horizon, Canada

Note: Estimates of time-varying in-sample skewed-$t$ moments of the Canada GDP distribution at the 4-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
Figure 23: Estimated in-sample parametric GDP-growth moments: 8-quarter horizon, Canada

Note: Estimates of time-varying in-sample skewed-t moments of the Canada GDP distribution at the 8-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
Figure 24: Estimated in-sample parametric GDP-growth moments: 1-quarter horizon, France

Note: Estimates of time-varying in-sample skewed-t moments of the France GDP distribution at the 1-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
Figure 25: Estimated in-sample parametric GDP-growth moments: 4-quarter horizon, France

Note: Estimates of time-varying in-sample skewed-$t$ moments of the France GDP distribution at the 4-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
Figure 26: Estimated in-sample parametric GDP-growth moments: 8-quarter horizon, France

Note: Estimates of time-varying in-sample skewed-\(t\) moments of the France GDP distribution at the 8-quarter horizon. Blue line shows estimates from the foreign-augmented model while red line shows estimates from restricted domestic-only model.
B.8 Non-Parametric GDP Moments

In this Appendix sub-section, we present estimates of time-varying moments of GDP growth across countries in our panel using a non-parametric approach to fitting a distribution to estimated quantiles. These complement the discussion in Section 4.2.

Figure 27: Estimated in-sample non-parametric GDP-growth moments: 1-quarter horizon, average across panel

*Note:* Estimates of time-varying in-sample non-parametric moments of the GDP distribution at the 1-quarter-ahead horizon. Blue line shows the estimates from foreign-augmented model, while red line shows the estimates from restricted domestic-only model.
Figure 28: Estimated in-sample non-parametric GDP-growth moments: 4-quarter horizon, average across panel

Note: Estimates of time-varying in-sample non-parametric moments of the GDP distribution at the 4-quarter-ahead horizon. Blue line shows the estimates from foreign-augmented model, while red line shows the estimates from restricted domestic-only model.
Figure 29: Estimated in-sample non-parametric GDP-growth moments: 8-quarter horizon, average across panel

*Note:* Estimates of time-varying in-sample non-parametric moments of the GDP distribution at the 8-quarter-ahead horizon. Blue line shows the estimates from foreign-augmented model, while red line shows the estimates from restricted domestic-only model.
B.9 Structural Coefficient Estimates

In this Appendix sub-section, we report coefficient estimates at the 5th percentile for our alternate ‘structural’ model described in Section 5. The coefficient estimates are similar to those in Figure 1, although the estimates of the impact of foreign credit-to-GDP and foreign volatility on domestic GDP-at-Risk are now larger in magnitude. For example, in this specification, at peak, a one standard deviation increase in foreign-weighted equity volatility is linked with a 1pp fall in the 5th percentile of GDP growth compared to a 0.7pp fall in Figure 1. This is because the first-stage orthogonalisation allows us to capture the contemporaneous impact of foreign variables on domestic covariates—e.g. capturing the fact that a sharp tightening in global financial conditions can spill over contemporaneously to domestic financial conditions (and thereby worsen domestic GDP-at-Risk via this tightening in domestic conditions).

Figure 30: Association between orthogonalised vulnerability indicators and GDP growth across horizons at the 5th percentile

Note: Solid blue lines denote estimated association between one standard deviation change in each indicator at time $t$ with 5th percentile of average annual real GDP growth at each quarterly horizon. Light (dark) blue-shaded areas represent 95% (68%) confidence bands around these estimates from block bootstrap procedure.
B.10 Towards a Structural Decomposition

In this Appendix, we report exemplar decompositions from our orthogonalisation procedure.

Figure 31 shows the orthogonalised decomposition for the estimated 5th percentile of 3-year-ahead UK real GDP growth. The orthogonalised decomposition suggests that the estimated fall in UK 3-year GDP-at-Risk in the run-up to the 1990-1991 recession was predominantly driven by domestic drivers (red bars). Foreign drivers (blue bars) played a limited role. Following this recession, these factors reversed with the estimated rise in the 5th percentile of UK 3-year GDP growth supported by both domestic and foreign factors.

Tail risks built up substantially over the 2000s though—with estimated GDP-at-Risk becoming more negative from 2005 especially, driven almost entirely by a build-up in foreign-weighted credit-to-GDP. This accords with the well established view that the GFC had global origins, driven by worldwide trends in an increasingly interconnected international financial system.

Since the GFC, these drivers of tail risks have again reversed, likely tempered by enhanced macroprudential policy toolkits and global monitoring of the financial system.

Figure 32 presents the comparable decomposition for German 3-year GDP-at-Risk. The relative evolution of domestic and foreign shocks in the run-up to the GFC is particularly notable for Germany. Domestic factors are associated with improvements in the left tail of the GDP growth distribution from 2004 to 2008. In contrast, foreign-weighted indicators are associated with a worsening in tail risk over the same period. In sum, these foreign factors dominate and contribute to an overall fall in fitted GDP-at-Risk over the period, exemplifying the importance of accounting for global influences when monitoring macro-financial risks.

We also present details of the robustness exercises we run to complement the structural decompositions in Section 5. Specifically, we estimate structural decompositions from two model variants, in addition to the baseline model.

First, we re-estimate our baseline model, weighting foreign variables using bilateral financial linkages measured using BIS International Banking Statistics. Compared to our baseline model, which includes 12 countries in the domestic variable set, this financially-weighted model includes 10 domestic economies owing to data availability.

Second, we estimate an extended model, akin to that in Aikman et al. (2019). Here, the domestic variable set includes 3-year house price growth, the current account, bank capital ratios, 1-year CPI inflation and the 1-year change in central bank policy rates, in addition to our baseline domestic indicators (3-year change in credit-to-GDP growth and lagged quarterly real GDP growth).

The estimated share of variation in fitted values attributable to foreign shocks $ForShare^h_i(\tau)$, defined in equation (5), at $h = 1, 4, 12$ and $\tau = 0.05, 0.5$ from these two models, alongside the baseline, are presented in Table 7.
Figure 31: Estimated orthogonalised decomposition of UK GDP-at-Risk at the 3-year horizon

Note: Solid black line denotes estimated 5th percentile of annual average 3-year-ahead GDP growth at each point in time. The bars show the contribution of domestic and foreign-weighted indicators to that total from estimates of equation (4).

Figure 32: Estimated orthogonalised decomposition of German GDP-at-Risk at the 3-year horizon

Note: Solid black line denotes estimated 5th percentile of annual average 3-year-ahead GDP growth at each point in time. The bars show the contribution of domestic and foreign-weighted indicators to that total from estimates of equation (4).
In all three models, a substantial share of variation in estimated percentiles of GDP growth is attributable to foreign shocks. Moreover, foreign factors exert a larger influence on fitted values at the left tail of the GDP distribution, i.e. the 5th percentile, than at the median, corroborating the results in Table 3. Although the foreign share is lowest for the extended model, this is unsurprising given that it includes more domestic covariates than the baseline or its financially-weighted variant. Even so, the results in Table 7 indicate that, across models, between 35 and 52% of variation in the 5th percentile of 3-year GDP growth is attributable to foreign shocks.
Table 7: Share of Variation in Fitted Values (%) Attributes to Foreign Shocks Across Horizons

<table>
<thead>
<tr>
<th>Country</th>
<th>(1) Baseline</th>
<th>(2) Financial Weights</th>
<th>(3) Extended Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>h = 1</td>
<td>h = 4</td>
<td>h = 12</td>
</tr>
<tr>
<td></td>
<td>τ = 0.05</td>
<td>0.5</td>
<td>0.05</td>
</tr>
<tr>
<td>AUS</td>
<td>91.28 (77.45)</td>
<td>81.34 (63.41)</td>
<td>63.62 (46.97)</td>
</tr>
<tr>
<td>CAN</td>
<td>93.33 (81.47)</td>
<td>78.26 (59.70)</td>
<td>53.84 (33.57)</td>
</tr>
<tr>
<td>DNK</td>
<td>91.34 (76.35)</td>
<td>81.69 (61.87)</td>
<td>64.22 (47.39)</td>
</tr>
<tr>
<td>FIN</td>
<td>91.85 (76.27)</td>
<td>77.87 (56.10)</td>
<td>54.86 (36.06)</td>
</tr>
<tr>
<td>FRA</td>
<td>92.17 (78.83)</td>
<td>74.07 (54.92)</td>
<td>50.42 (29.74)</td>
</tr>
<tr>
<td>GER</td>
<td>91.82 (77.94)</td>
<td>77.85 (57.19)</td>
<td>52.22 (33.21)</td>
</tr>
<tr>
<td>ITA</td>
<td>91.91 (78.60)</td>
<td>81.84 (64.30)</td>
<td>69.17 (49.88)</td>
</tr>
<tr>
<td>NOR</td>
<td>89.92 (73.81)</td>
<td>73.43 (53.04)</td>
<td>50.78 (31.59)</td>
</tr>
<tr>
<td>SPAIN</td>
<td>92.05 (78.67)</td>
<td>80.85 (61.77)</td>
<td>61.42 (42.71)</td>
</tr>
<tr>
<td>SWE</td>
<td>91.81 (81.57)</td>
<td>84.74 (69.13)</td>
<td>72.75 (52.65)</td>
</tr>
<tr>
<td>SWI</td>
<td>93.53 (81.99)</td>
<td>82.82 (63.29)</td>
<td>49.56 (33.11)</td>
</tr>
<tr>
<td>UK</td>
<td>91.53 (80.50)</td>
<td>84.27 (69.69)</td>
<td>82.88 (68.39)</td>
</tr>
<tr>
<td>Avg.</td>
<td>91.88 (78.62)</td>
<td>79.92 (61.20)</td>
<td>60.48 (42.11)</td>
</tr>
</tbody>
</table>

Share of variation at the 5th percentile (τ = 0.05) and median (τ = 0.5 in parentheses) of country-GDP distributions at different horizons: h = 1 (1 quarter), h = 4 (1 year), and h = 12 (3 years). Share definition in equation (5). Shares constructed from three models in which domestic indicators are orthogonalised with respect to all foreign indicators, akin to a small-open economy assumption for domestic countries.