The Risk-Premium Channel of Uncertainty: Implications for Unemployment and Inflation

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This paper studies the role of macroeconomic uncertainty in a search-and-matching framework with risk-averse households. Heightened uncertainty about future productivity reduces current economic activity even in the absence of nominal rigidities. A risk-premium mechanism accounts for this result. As future asset prices become more volatile and covary more positively with aggregate consumption, the risk premium rises in the present. The associated downward pressure on current asset values lowers firm entry, making it harder for workers to find jobs and reduces supply. With nominal rigidities the recession is exacerbated, as a more uncertain future reinforces households’ precautionary behavior, which causes demand to contract. Counterfactual analyses using a calibrated model imply that unemployment would rise by less than half as much absent the risk-premium channel. The presence of this mechanism implies that uncertainty shocks are less deflationary than regular demand shocks, nor can they be fully neutralized by monetary policy.

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1 Introduction

Elevated macroeconomic uncertainty contracts economics activity and raises unemployment.\footnote{For empirical evidence see, among many others, Jurado et al. (2015), Baker et al. (2016), Bloom et al. (2018) and Coibion et al. (2021). Baker et al. (2020) and Leduc and Liu (2020) provide evidence of the effect of rising uncertainty triggered by the COVID-19 pandemic.} To account for this empirical relationship, the theoretical literature emphasizes an important interaction of precautionary saving motives and nominal rigidities (Basu and Bundick, 2017). Heightened uncertainty induces prudent households to engage in more precautionary saving. In a standard flexible-price real business cycle model this leads to an investment boom that expands, rather than contracts, economic activity. When prices are rigid as in a standard New Keynesian setting, however, the increase in desired savings is instead diverted into a fall in goods demand that, in equilibrium, contracts economic activity and lowers employment. Motivated by this theoretical mechanism, spikes in uncertainty are sometimes seen as affecting the economy analogously to falls in aggregate demand (e.g., Leduc and Liu, 2016).

This paper uncovers and quantifies a \textit{risk-premium channel}, which can play a pivotal role in shaping the macroeconomics effects of uncertainty, both under flexible and rigid prices. More precisely, we study a canonical search-and-matching (SaM) model with risk averse households. In this framework, firm entry – and thereby unemployment – is governed by the value of firms, i.e., by their associated asset price. These asset prices summarize the equilibrium long-run valuation of firms, and fluctuate with both present and future shocks to operating profits.\footnote{This long-horizon valuation of firms contrasts markedly with a standard real business cycle model in which returns, ceteris paribus, are unaffected by productivity beyond the immediate future (cf. Barro and King, 1984).} Thus, an increase in aggregate uncertainty raises the expected volatility of asset prices. Since asset prices are procyclical, they positively covary with aggregate consumption and negatively with the marginal utility of consumption. Greater uncertainty thus raises the risk premium on equity, leading to a decline in current asset valuations. As a consequence, there is less entry of firms, higher unemployment, and lower economic activity. This mechanism does not hinge on nominal rigidities. Indeed, with flexible prices and for standard parameter values it is sufficiently powerful to eclipse the expansionary forces stemming from a rise in the precautionary motive. Sticky prices introduce additional contractionary forces, operating through the aforementioned conventional demand channels induced by the precautionary motive to save. Finally, the risk-premium channel is quantitatively important: In a calibrated version of the model we show that the effect of an uncertainty shock on unemployment would be only half as large in its absence.

Our first contribution is to tightly characterize the channels through which the mere anticipation of heightened macroeconomic volatility — modeled as a perceived rise in the volatility of future aggregate labor productivity — can contract economic activity in the present. To aid exposition, we introduce a stylized two-period version of the model that facilitates a transparent analysis of the key mechanisms. The most important qualitative result is that a rise in uncertainty can lower economic activity even when prices are flexible. This result emerges due to the risk-premium channel. Uncertainty leads to a
more pronounced negative comovement between the future marginal utility of consumption and the asset value of the firms. The associated rise in the required risk premium for equity depresses hiring incentives and thereby increases unemployment.\(^3\)

But this risk-premium mechanism does not operate in isolation. Under flexible prices there are two countervailing forces. First, in anticipation of greater future volatility, prudent agents value all assets— including risky claims to future cash flows— more highly. The result is upward pressure on asset prices that encourages job creation. In addition, SaM models exhibit an asymmetry in the labor market—simply put, recessions are worse than booms are good—such that larger fluctuations in economic activity lead to higher (expected) unemployment (e.g., Hairault et al., 2010). With preferences that exhibit a desire to smooth consumption over time, this expectation reinforces the precautionary behavior associated with prudence. As both these forces operate virtually indistinguishably—yielding a rise in the expected marginal utility of consumption—we refer to them jointly as the precautionary-motive channel.

Price rigidities add several twists to this story. First, the precautionary-motive channel now amplifies any decline in economic activity caused by the risk-premium channel. The reason is familiar and follows the New Keynesian narrative referred to at the onset: With an increased desire to save, the willingness to purchase consumption goods falls, and firms operating under monopolistic competition reduce their prices in an attempt to restore demand. With nominal rigidities, these price adjustments are insufficient to maintain the original equilibrium, however, and there is a decline in output alongside a fall in inflation. Second, the foregoing demand mechanism amplifies the risk-premium channel in general equilibrium. To see why, suppose that the anticipation of greater volatility in the distant future causes a rise in the risk premium and a concomitant decline in job creation in the near future. The implied fall in expected consumption from the viewpoint of the present induces agents to reduce their demand for consumption goods right now.

The second contribution of the paper is to quantify the relative importance of risk premium and demand mechanisms in accounting for the rise in unemployment. To this end, we first calibrate a quantitative version of the model to jointly match a set of real- and financial moments. We then implement a simple, but novel, decomposition—exploiting a key asset pricing relationship within the model—to construct impulse response functions for a counterfactual economy in which the risk-premium channel is inoperative. This analysis reveals that the risk-premium channel accounts for more than half of the rise in unemployment caused by an uncertainty shock.

In a third contribution, we highlight two implications of the risk-premium channel which show uncertainty shocks to be distinct from other disturbances to the economy. First, from an observational

\(^3\)Appendix A provides some empirical support for this mechanism. Structural vector autoregressions suggest that an increase in uncertainty raises unemployment, reduces vacancy posting, increases the risk premium and puts downward pressure on risk-free returns. In addition, using data on US online job vacancy postings and company valuations, Liu (2021) finds that fluctuations in the employee valuation ratio—the labor market counterpart of the dividend-price ratio of the stock market—are mainly driven by variations of risk premia instead of news about future cash flows.
point of view, uncertainty shocks are less deflationary than regular (negative) demand shocks. This result obtains because uncertainty shocks contain a supply component – the risk-premium channel – in addition to the demand effect. To see the implications for the relationship between unemployment and inflation, envisage the consequences of a regular negative demand shock. In response to a contraction in demand, firms reduce their prices to restore the original equilibrium. With nominal rigidities, however, this adjustment is incomplete, and the result is a contraction in both demand and supply, with lower inflation. But if the risk premium simultaneously increases, asset prices fall, fewer firms enter, and supply contracts even in the absence of any changes in goods prices. Thus, with supply already partly contracted, the reduction in inflation required to restore the equilibrium is smaller. As a consequence, an increase in uncertainty leads to a decline in output akin to a demand shock, but with a less pronounced fall in inflation. This theoretical perspective helps rationalize why, even when the effect of uncertainty on real activity is empirically well-established, findings regarding the impact on inflation are mixed.\textsuperscript{4} Second, and in difference from conventional supply shocks, a sufficiently aggressive response from the monetary authority that stabilizes demand can mitigate the adverse labor market consequences of uncertainty shocks. Unlike in the case of demand shocks, however, mimicking the flexible-price outcome still results in a fall in economic activity and a rise in unemployment, as the risk-premium channel remains operative.

Related literature

Our contribution connects research on uncertainty shocks with a recent literature that highlights the role of movements in risk premia in driving business cycle fluctuations. In relation to the former, the uncertainty literature is too vast to comprehensively discuss here but a few remarks are warranted to contextualize our analysis.\textsuperscript{5}

We build on the important contribution of Leduc and Liu (2016). They document that nominal rigidities are not necessary for uncertainty to worsen unemployment once the labor market is characterized by matching frictions. They conjecture that this is because heightened uncertainty renders firms apprehensive to hire workers due to an increased value of waiting; an option value. However, Den Haan et al. (2021) demonstrate that such option-value considerations play no role in the type of model that Leduc and Liu (2016), as well as this paper, use, because the free-entry condition implies that the expected value of vacancy posting is equal to zero in any state of the world. To develop the

\textsuperscript{4}See, among others, Fernández-Villaverde et al. (2015) and Alessandri and Mumtaz (2019). Castelnuovo’s (2019) survey concludes that more work is needed to resolve this issue.

\textsuperscript{5}Bloom (2014) and Fernández-Villaverde and Guerrón-Quintana (2020) provide excellent surveys. We stress that while focusing our analysis on a relatively simple theoretical model is integral to making our intended contribution, there are many other potential channels through which uncertainty may impact economic activity that we omit. For instance, Bloom (2009) and Schaal (2017) consider the role of real option effects; Born and Pfeifer (2014a), Fernández-Villaverde et al. (2015) and Oh (2020) focus on precautionary pricing effects; and Cacciatore and Ravenna (2018) foreground the consequences of occasionally binding constraints on downward wage adjustment in generating state-dependent amplification of uncertainty shocks.
argument, Den Haan et al. (2021) impose both risk neutrality and flexible prices, thereby leaving open the question what accounts for the effects of uncertainty in a setting with risk aversion and sticky prices. Our paper instead focuses on the consequences of precisely these model features, revealing a novel transmission mechanism in form of the risk-premium channel and examining its quantitative importance and implications.

As regards the effects of precautionary savings, Basu and Bundick (2017) demonstrate that even with recursive preferences and a large coefficient of risk aversion, an otherwise standard real business cycle model predicts an economic expansion in response to an increase in uncertainty. The reason is that absent a quantitatively meaningful risk-premium channel, the precautionary motive is sufficiently strong to generate an investment boom. Indeed, Basu and Bundick (2017) conclude that nominal rigidities are necessary to divert the associated decline in consumption demand into a fall in output through a countercyclical rise in markups. Relatedly, Born and Pfeifer (2020) empirically study this conditionally countercyclical comovement between markups and uncertainty. They find that the extensive labor margin is the only margin along which price-markups increase following an uncertainty shock, indicating the potential relevance of SaM models with nominal rigidities for analyzing the effects of uncertainty.

Our analysis also relates to several recent papers that emphasize the role of financial discounts and risk premia in shaping labor market fluctuations. Hall (2017) argues that as the SaM framework suggests that job-creation incentives are low when discount rates are high, variations in the discount rates in financial markets are likely culprits for volatility in unemployment rates. We build on this idea, with at least three key differences between our argument and that of Hall (2017). First, whereas Hall (2017) explicitly abstracts from the structural shocks accounting for movements in financial discounts, we put forth one particular candidate driver; namely uncertainty shocks. This narrower focus facilitates a precise analysis of the several channels through which uncertainty shocks propagate. Second, Hall (2017) primarily focuses on variation in the mean of the stochastic discount factor, i.e., the risk-free rate (cf. Borovičková and Borovička, 2018). By contrast, we underscore that uncertainty simultaneous increases the valuation of future cash flows, but also leads to a stronger negative covariance between the stochastic discount factor and future asset prices, thereby introducing an explicit role for risk premium effects. Finally, we consider model versions with and without nominal rigidities. Such a joint analysis turns out to be important since the effects of changes in desired savings differ sharply depending on whether the model features nominal rigidities or not.

The analysis of Borovičková and Borovička (2018), while again not concerned with the implications of uncertainty, also exploits the close connection between the valuation of risky financial assets and those of profit flows earned by hiring workers in the labor market. In similarity to our paper, and unlike

\[^6\text{When labor supply and demand clears on a spot market, the equilibrium hours worked increase, since greater uncertainty strengthens a general desire to self-insure, including through an expansion in precautionary labor supply.}\]

\[^7\text{In addition to the papers discussed in the text, also see Kilic and Wachter (2018), Caballero and Simsek (2020) and Basu et al. (2021).}\]
Hall (2017), they explicitly account for fluctuations in the risk-premium component. Borovičková and Borovička (2018) demonstrate that introducing a properly modeled stochastic discount factor at least partially mitigates the need for counterfactually high profit flows to generate an empirically plausible degree of unemployment volatility. Methodologically, their analysis is more general in some dimensions – for instance in their treatment of profit flows and models of the stochastic discount factor – and more narrow in others; e.g. general equilibrium amplification as well as nominal rigidities are absent from their analysis. Hence, and in light of the difference in research questions, we consider the two papers to be complementary.

Lastly, Kehoe et al. (2022) contribute a novel resolution of the well-known unemployment volatility puzzle that hinges on the idea that hiring a worker is akin to investing in a financial asset with risky, long-duration cash flows. On this account, job creation declines when the price of risk rises. The long-lived nature of employment relationships also underpins the quantitatively significant effects of uncertainty in our model, as the current firm value is a function of – and hence hiring incentives fluctuate following shocks to – the entire sequence of expected future net benefits from a hire. We also follow the approach of Kehoe et al. (2022) in incorporating Campbell and Cochrane (1999) type habits into the quantitative version of the model, giving rise to time-varying risk aversion. Kehoe et al. (2022) deviate from standard models by assuming that variations in aggregate productivity alter not only workers’ productivity when employed, but also shift home production and vacancy posting costs. Moreover, they let differential growth of human capital on and off the job play a prominent role. We are instead interested in the transmission of uncertainty shocks in a more standard setting in which productivity shocks cause changes in the profitability of hiring workers, quantifying the contribution of volatility-induced movements in risk premia, and consider the implications of nominal rigidities.

2 Theoretical framework

The economy is populated by a unit measure of households; a competitive sector of intermediate goods firms producing a homogeneous input good; a monopolistically competitive sector of retail good producers, whose outputs are aggregated in a competitive final goods sector; and a monetary authority which sets the policy interest rate.\(^8\) Real quantities are defined in terms of the final good, and are – unless otherwise stated – denoted by lower case letters. Time is discrete and denoted \(t = 0, 1, 2, \ldots\)

The following sections describe this model in some detail. Readers familiar with this class of models may wish to directly move to section 2.6, which summarizes the key equilibrium relationships of interest.

\(^8\)In the regular New Keynesian literature retail firms are normally instead referred to as intermediate goods firms (see, for instance, Chapter 2 in Gali (2015)). However, as models with an underlying SaM structure use an additional layer of firms/goods, we follow the terminology of Ravenna and Walsh (2008), Leduc and Liu (2016), and many others, and refer to these firms as retail firms.
2.1 Households

In a given period \( t \), a household can either be employed, \( n_t \), or unemployed, \( u_t \). The market for idiosyncratic employment risk is complete and the representative household – or simply the household – is comprised of a measure of \( n_t \) members that are working, and \( u_t \) members that are not. Non-employed members of the household may find a job even within the period they get displaced. Thus, the measure of the household’s members that are searching for a job in the beginning of a period is 

\[
us_t = ut - 1 + \delta nt - 1,
\]

where \( \delta \) denotes an exogenous separation rate. The measure of employed individuals working in period \( t \) is therefore given by 

\[
n_t = ftus_t + (1 - \delta)nt - 1,
\]

where \( f_t \) denotes an endogenously determined job finding rate. The real wage is denoted \( wt \) and, as there is no home production, total labor income is given by \( wtn_t \).

In addition to labor income, the household enters the period with nominal bonds, \( B_{t-1} \), and equity \( a_{t-1} \). Equity is valued at the cum-dividend price \( J_t \). However, as a fraction, \( \delta \), of firms goes out of business in each period, the total value of the household’s equity position is 

\[
J_t a_{t-1}(1 - \delta).
\]

The household also receives profits from several other sources. Since none of these can affect – nor be affected by – the household’s decisions, we summarize their total profits in the variable \( \tilde{d}_t \), which is, for the moment, treated as given (see section 2.5 and appendix B.1.3 for a more detailed description).

The household may use the resources available in period \( t \) – i.e. labor income, bond and equity holdings, and the additional profits – to either consume the final good, \( c_t \); purchase new equity, \( a_t \), at the ex-dividend price \( J_t - d_t \); or purchase nominal bonds, \( B_t \), at the price \( 1/(PtR_t) \), where \( P_t \) denotes the aggregate price level, and \( R_t \) the gross nominal interest rate.

Thus, the budget constraint of the household is 

\[
c_t + a_t(J_t - d_t) + \frac{B_t}{P_tR_t} = wtn_t + \tilde{d}_t + \frac{B_{t-1}}{P_t} + a_{t-1}(1 - \delta)J_t, \quad t = 0, 1, 2, \ldots, \tag{1}
\]

where \( a_{t-1} \) and \( B_{t-1} \) are given.

Subject to the above budget constraint, the household decides on a process, \( \{c_t, a_t, B_t\}_{t=0}^\infty \), to maximize the expected present discounted value of lifetime household utility

\[
E_0 \sum_{t=0}^\infty \beta^t u(c_t), \tag{2}
\]

where \( E_0 \) denotes the mathematical expectation operator conditional on time \( t = 0 \) information; the parameter \( \beta \in (0, 1) \) represents the subjective discount factor, and the period utility function, \( u(\cdot) \), satisfies \( u'(\cdot) > 0 \) and \( u''(\cdot) < 0 \).

The first order conditions associated with the household’s problem are given by a bond Euler
equation

\[ u'(c_t) = \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} u'(c_{t+1}) \right], \tag{3} \]
as well as an Euler equation for equity

\[ u'(c_t) = \beta E_t \left[ \frac{J_{t+1}(1-\delta)}{J_t - d_t} u'(c_{t+1}) \right]. \tag{4} \]

Rearranging the latter and defining \( \Lambda_{t,t+1} = \beta u'(c_{t+1})/u'(c_t) \) as the stochastic discount factor gives the asset pricing equation

\[ J_t = d_t + E_t [\Lambda_{t,t+1}J_{t+1}(1-\delta)]. \tag{5} \]

This asset pricing equation for equity will play an integral part of the equilibrium outcome. In particular, intermediate goods producing firms generate dividends \( d_t = x_t z_t - w_t \), where \( z_t \) denotes the marginal product of a worker, and \( x_t \) the relative price of intermediate goods. Thus, if intermediate goods producers generate a dividend process \( \{d_t\}_{t=0}^{\infty} \), their asset price, or firm value, is determined by equation (5). This asset price will, in turn, determine firm entry and unemployment, as discussed in the next sections.

### 2.2 Firms

#### 2.2.1 Intermediate goods producers

There is a large number of potential intermediate goods producing firms, but a finite measure of operating (or active) firms. The firms use labor as the only input to production in a constant returns to scale technology, producing a homogenous good. Thus, without any loss of generality we assume that each active firm employs precisely one worker. As a consequence, the measure of active intermediate firms equals the employment rate, \( n_t \).

An active firm produces \( z_t \) units of intermediate goods, where \( z_t \) represents a workers marginal product. These goods are sold to final goods firms at price \( x_t \), and the firms pay workers the wage \( w_t \). Hence, each intermediate good firms generate (real) profits of \( x_t z_t - w_t \). As a consequence, the value of an intermediate good producing firm is given by

\[ J_t = x_t z_t - w_t + E_t [\Lambda_{t,t+1}J_{t+1}(1-\delta)]. \tag{6} \]

Potential intermediate goods firms may enter the market by posting a vacancy. The (marginal) cost of posting a vacancy is denoted \( \kappa \), which result in the firm meeting a searching household with
probability $h_t$. Thus, the free-entry condition is given by

$$\kappa = h_t J_t. \quad (7)$$

We assume that the aggregate resources devoted to vacancy-posting—i.e., $\kappa v_t$, where $v_t$ denotes the aggregate amount of vacancies posted in period $t$—is rebated back to the households. That is, the households are assumed to own the “vacancy-posting agencies.”

Lastly, there are exogenous stochastic processes for both labor productivity, $z_t$, and the standard deviation of labor productivity shocks, $\sigma_{z,t}$. Both are modeled as AR(1) processes:

$$z_t = (1 - \rho_z)z + \rho_z z_{t-1} + \sigma_{z,t-1} \epsilon_{z,t}, \quad (8)$$

$$\sigma_{z,t} = (1 - \rho_\sigma)\sigma_z + \rho_\sigma \sigma_{z,t-1} + \sigma_\sigma \epsilon_{\sigma,t}. \quad (9)$$

Importantly, the standard deviation of the innovation to productivity, $\epsilon_{z,t}$, is time-varying. The parameters $\rho_z \in (-1, 1)$ and $\rho_\sigma \in (-1, 1)$ measure the persistence of the first- and second-moment shocks, respectively. Additionally, $\sigma_z$ is the steady-state value of the standard deviation of the innovation to productivity. Both shocks $\epsilon_{z,t}$ and $\epsilon_{\sigma,t}$ are normally distributed with $\sigma_{\epsilon_z}$ and $\sigma_{\epsilon_\sigma}$ set to unity; $\sigma_\sigma$ will be calibrated.  

### 2.2.2 Final and retail goods producers

Final goods firms are perfectly competitive and use retail goods as the only input. However, as retailers operate under monopolistic competition, they take into account the demand schedule implied by the final goods firms’ optimal production decisions. We therefore discuss both sectors under the same section, starting with the final goods producers.

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9We make this assumption to maximize transparency when studying the transmission of uncertainty shocks. In particular, resource-draining activities like vacancy posting give rise to a rather mechanical, and economically rather uninteresting, transmission channel, as we discuss in appendix section C.4.

10Two remarks are in place. First, the process for $z_t$ is in levels rather than logs to prevent the expected value of productivity to be different from the deterministic steady-state value through a Jensen’s inequality effect. Second, under the timing assumption in equation (8), which is common in the uncertainty shock literature (e.g., Bloom (2009) or Schaal (2017)), volatility shocks have a delayed impact on the distribution of labor productivity shocks. We observe that the level process (9) (which is common, see e.g., Fernández-Villaverde et al. (2011)) does not restrict $\sigma_{z,t}$ from taking on negative values; this is strictly speaking inconsistent with the definition of a standard deviation. In practical applications, this is not of material concern, however, because the policy functions are locally approximated around the positive deterministic steady-state value $\bar{\sigma}_z$. Accordingly, using a log process for $\sigma_{z,t}$ instead, turns out to produce no noticeably different results.
**Final goods producers** The final consumption good, $y_t$, is produced using a constant elasticity of substitution (CES) production function according to

\[ y_t = \left( \int_0^1 y_t(i)^{\frac{\eta-1}{\eta}} \, di \right)^{\frac{\eta}{\eta-1}}, \]

where $y_t(i)$ denotes the retail good produced by firm $i$, with $i \in [0, 1]$. The parameter $\eta$ denotes the elasticity of substitution between the differentiated retail goods.

Let $p_t(i)$ denote the relative price associated with retail good $i$. The optimization problem facing the final goods producers is then given by

\[ \max_{y_t(i) \in [0, 1]} \left\{ P_t y_t(i) - \int_0^1 p_t(i) y_t(i) \right\}, \]

where $P_t$ denotes the aggregate price level/index.

The first-order conditions to this optimization problem give rise to the demand schedule

\[ y_t(i) = \left( \frac{p_t(i)}{P_t} \right)^{-\frac{1}{\eta}} y_t, \tag{10} \]

with the associated price index

\[ P_t = \left( \int_0^1 p_t(i)^{\frac{1}{1-\eta}} \, di \right)^{1-\eta}. \]

**Retail goods producers** Differentiated retail goods are produced using the homogeneous intermediate good as the single input. The technology is such that one unit of the intermediate good produces one unit of the retail good. As the relative price of the intermediate good *in terms of the final good* is given by $x_t$, retailers make per-period profits\(^\text{11}\)

\[ \frac{p_t(i)}{P_t} y_t(i) - x_t y_t(i). \tag{11} \]

Since we at times will consider a situation in which retailers cannot adjust prices frictionlessly, but only may do so by incurring a cost, a more general formulation for the retailers profits is given by

\[ \hat{d}_t = \frac{p_t(i)}{P_t} y_t(i) - x_t y_t(i) - \frac{\Omega_p}{2} \left( \frac{p_t(i)}{p_{t-1}(i)} \Pi - 1 \right)^2 y_t, \tag{12} \]

where $\Pi_t = P_t / P_{t-1}$ denotes the gross inflation rate. Thus, the period profits $\hat{d}_t$ nest equation (11) in the special case of $\Omega_p = 0$.

\(^{11}\text{We can equivalently think of } x_t \text{ as the real marginal cost facing the retailer.}\)
Using a pricing relation analogous to equation (5), but denoting the asset price of retailers as $\hat{V}_t(p_t(i))$ yields

$$\hat{V}_t(p_t(i)) = \max_{y_t(i)} \{ \hat{d}_t + E_t [ \Lambda_{t+1} \hat{V}_{t+1}(p_{t+1}(i)) ] \}.$$ 

Taking into account the demand schedule in equation (10), as well as the definition of the per-period profits in equation (12), the first order condition associated with optimizing the firm value above is given by the New Keynesian Phillips curve

$$x_t = \frac{\eta - 1}{\eta} + \frac{\Omega p_t}{\eta} \left\{ \frac{\Pi_t}{\Pi_t \left( \frac{\Pi_t}{\Pi_t - 1} \right)} - E_t \left[ \Lambda_{t+1} \frac{y_{t+1}}{y_t} \frac{\Pi_{t+1}}{\Pi_t} \left( \frac{\Pi_{t+1}}{\Pi_t - 1} \right) \right] \right\},$$

in which we have assumed symmetry, such that $y_t(i) = y_t$ and $p_t(i) = P_t$. As previously mentioned, we assume that the aggregate resources devoted to price changes – the last term in equation (12) – are rebated back to the households. That is, the households are assumed to own the “price-adjusting agency.”

2.3 Labor market

As already discussed in section 2.1 the measure of unemployed workers searching for a job in period $t$ is given by $u_t^s = u_{t-1} + \delta n_{t-1}$. And as discussed in section 2.2.1 there is a measure $v_t$ of aggregate vacancies posted by intermediate goods firms. Matches in the labor market, $M_t$, are then determined according to a standard Cobb-Douglas function,

$$M_t = \psi(u_t^s)^\alpha(v_t)^{1-\alpha},$$

where $\alpha \in (0, 1)$ denotes the elasticity of matches with respect to job seekers $u_t^s$ and $\psi$ scales the matching efficiency. The implied hiring rate, $h_t$, is therefore

$$h_t = \frac{M_t}{v_t} = h(\theta_t) = \psi \theta_t^{-\alpha},$$

where $\theta$ indicates labor market tightness which is given by

$$\theta_t = \frac{v_t}{u_t^s} = \frac{v_t}{1 - (1 - \delta) n_{t-1}}.$$  

Analogously, the job finding probability for a searching worker is given by

$$f_t = \frac{M_t}{u_t^s} = f(\theta_t) = \psi \theta_t^{1-\alpha}.$$
Notice that $h(\theta)$ is strictly decreasing in $\theta$ while $f(\theta)$ is strictly increasing.

We can accordingly formulate the law of motion for employment as

\begin{align}
n_t &= f_t u_t + (1 - \delta)n_{t-1}, \quad (18) \\
&= h_t v_t + (1 - \delta)n_{t-1}, \quad (19) \\
&= M_t + (1 - \delta)n_{t-1}. 
\end{align}

Together with the law of motion for employment, the equilibrium aggregate measure of vacancies posted in any given period, $v_t$, is endogenously determined as the solution to the free-entry condition in equation (7), which is here repeated to explicitly account for the relationship between the asset price, $J_t$, and labor market tightness, $\theta_t$,

\[ \kappa = h(\theta_t)J_t. \quad (20) \]

### 2.3.1 Wage setting

Search frictions in the labor market imply that a matched firm and worker generate a joint surplus, giving rise to a situation of bilateral monopoly. This latter aspect leaves wages, without any further theory, undetermined. To this end, we adopt a wage-setting protocol in the spirit of Hall and Milgrom (2008), which yields the following, linear wage rule

\[ w_t = \omega x_t z_t + (1 - \omega)\chi. \quad (21) \]

Here, $\omega$ denotes worker bargaining power and $\chi$ represents the flow consumption-value the worker receives by delaying agreement during a process of alternating-offer bargaining.\(^{12}\)

This wage protocol contrasts with the more common practice of wage-setting through Nash bargaining. Our approach is motivated by the observation in Den Haan et al. (2021) that even when the mutually agreed-upon wages according to the two alternative schemes coincide in deterministic steady state, they can give to dynamics in response to uncertainty shocks that differ not only quantitatively but also in important, qualitative ways. The reason is that the Nash-bargained wage carries a forward-looking component that has some peculiar implications when analyzing uncertainty shocks. We refer to Den Haan et al. (2021) for details and confine ourself to emphasizing the following point. Holding $x_t$ constant (i.e. under flexible prices), the alternating offers formulation in equation (21) allows us to focus on those nonlinearities that are intrinsic to the matching process, without confounding the results from those arising from any other nonlinearities that are specific to the wage bargain, nor imposing

\(^{12}\)This wage coincides exactly with the protocol proposed by Jung and Kuester (2011), which sets wages by maximizing the Nash product $(w_t - \chi)^{\omega}(x_t z_t - w_t)^{1-\omega}$. Hall and Milgrom (2008) proposes a bargaining specification that partially insulates wages from variations in labor market tightness. Equation (21) is a special case insofar as this isolation is complete.
that wages are completely rigid.\footnote{Additionally, it leaves the wage unresponsive to movements in the marginal utility of consumption. Appendix OB of Den Haan et al. (2021) provides details.}

### 2.4 Policy

The monetary authority sets the nominal interest rate, $R_t$, according to a simple Taylor-type rule

$$\log\left(\frac{R_t}{R}\right) = \phi \log\left(\frac{\Pi_t}{\Pi}\right).$$

\begin{equation}
(22)
\end{equation}

In the presence of nominal rigidities, the monetary policy transmission mechanism to the real economy operates as follows. A lower interest rate, $R_t$, increases demand for the final good through the bond Euler equation in (3). Increased demand for final goods leads retail firms to set higher prices and to increase demand for intermediate goods, putting upward pressure on the relative price of intermediate goods, $x_t$. To the extent that the increase in marginal revenues, $x_tz_t$, is not entirely offset by an increase in wages, $w_t$, the intermediate firms post additional vacancies until the free-entry condition (20) is satisfied, that is, until the probability of filling a vacancy, $h(\theta_t)$, has decreased sufficiently to restore the free-entry condition.

In the case of flexible prices the above chain is broken. In particular, retail firms then adjust prices sufficiently to render the real interest rate unaffected (as inflation expectations change), which entirely offsets the initial increase in demand. Indeed, under flexible prices, i.e. when $\Omega p = 0$, it is trivial to see from equation (13) that the relative intermediate goods price, $x_t$, is constant at $x = (\eta - 1)/\eta$, which implies that there is also no additional entry after a monetary policy intervention (that is, monetary policy is neutral). Nominal rigidities are necessary to prevent these price movements from operating fully.

### 2.5 Market clearing and equilibrium

Since all firms use a constant returns to scale technology – alongside with the fact that intermediate goods use only labor as an input, retail firms use only intermediate goods, and final goods firms use only retail goods – aggregate output is given by $y_t = z_t n_t$. Moreover, taking into account that the household receives dividend income not only from the ownership of intermediate-goods firms but also from retailers, vacancy-posting agencies, and price adjusting firms, it can be shown that in equilibrium, consumption is equal to output, $c_t = y_t$ (cf. appendix B.1.3).\footnote{Aggregate consumption is, thus, not affected by the amount of vacancies created, nor by the costs associated with price adjustments. This is indeed intentional; as we are exploring the role of uncertainty on behavior, any resource draining activity, such as price adjustments, may, somewhat mechanically, alter the marginal utility of consumption. We explore the role of such activities in appendices C.2 and C.4.}

A summarizing definition of equilibrium follows.
Definition 1. A competitive equilibrium is a process of prices \( \{J_t, R_t, \Pi_t, x_t, w_t\}_{t=0}^{\infty} \) and quantities \( \{c_t, B_t, \theta_t, n_t, a_t\}_{t=0}^{\infty} \) such that,

(i) \( \{c_t, B_t, a_t\}_{t=0}^{\infty} \) solves the household’s problem.

(ii) Asset prices \( \{J_t\}_{t=0}^{\infty} \) satisfy the asset pricing equation in (5).

(iii) Labor market tightness, \( \{\theta_t\}_{t=0}^{\infty} \), satisfies the free-entry condition \( \kappa = h(\theta_t)J_t \).

(iv) Employment, \( \{n_t\}_{t=0}^{\infty} \), satisfies the law of motion

\[
n_t = [(1 - n_{t-1}) + \delta n_{t-1}] f(\theta_t) + (1 - \delta) n_{t-1}.
\]

(v) Wages, \( \{w_t\}_{t=0}^{\infty} \), satisfy equation (21).

(vi) The gross nominal interest rate, \( \{R_t\}_{t=0}^{\infty} \), satisfies the Taylor rule in equation (22).

(vii) Relative prices for intermediate goods and inflation, \( \{x_t, \Pi_t\}_{t=0}^{\infty} \), satisfy the Phillips curve in equation (13).

(viii) Bond markets clear, \( B_t = 0 \).

(ix) Equity market clear, \( a_t = n_t \).

(x) Intermediate goods markets clear \( y_t(i) = z_t n_t \).

2.6 Summary of equilibrium relationships

In summary, the key endogenous equilibrium relationships are as follows.

\[
\begin{align*}
&u'(c_t) = \beta E_t \left[ \frac{J_{t+1}(1 - \delta)}{J_t - \delta} u'(c_{t+1}) \right], \\
&\kappa = h(\theta_t)J_t, \\
&n_t = f(\theta_t)u^*_t + (1 - \delta) n_{t-1}, \\
&c_t = z_t n_t, \\
&u'(c_t) = \beta E_t \left[ \frac{R_t}{\Pi_{t+1}} u'(c_{t+1}) \right], \\
&x_t = \frac{\eta - 1}{\eta} + \Omega_p \left\{ \frac{\Pi_t}{\Pi} \left( \frac{\Pi_t}{\Pi} - 1 \right) - E_t \left[ \Lambda_{t,t+1} \frac{y_{t+1}}{y_t} \frac{\Pi_{t+1}}{\Pi} \left( \frac{\Pi_{t+1}}{\Pi} - 1 \right) \right] \right\}, \\
&\log \left( \frac{R_t}{R} \right) = \phi \pi \log \left( \frac{\Pi_t}{\Pi} \right).
\end{align*}
\]

Notice that to understand the dynamics of real macroeconomic variables under flexible prices, only the first four are required (alongside the stochastic processes (8)-(9), of course).
3 Transmission mechanisms: a stylized approach

How does an increase in uncertainty affect unemployment in this economy? To provide a first, qualitative answer to this question, this section studies a stylized two-period version of the model. It transparently reveals the key forces at work. This exercise makes clear that uncertainty sets in motion two forces that affect asset prices in distinct ways – the precautionary motive, and the risk premium channel. It also points to a natural approach to decomposing the relative magnitude of these effects, which is implemented in a quantitative context in the following section.

3.1 Two-period setup

The asset price in the present is determined by (cf. equation (5))

\[ J = zx - \chi + \frac{\beta(1 - \delta)}{u'(c)} \left[ pu'(c_g) J_g + (1 - p) u'(c_b) J_b \right], \tag{23} \]

where variables without a subscript refer to the current-period values. The variables \( J_g \) and \( J_b \) denote future asset prices in a good and a bad state, respectively, and the notation for future consumption follows a similar taxonomy. Attention is confined to the case in which \( J_g = \bar{J} + \Delta \) and \( J_b = \bar{J} - \Delta \), where \( \bar{J} \) indicates the steady state asset price, with \( \Delta > 0 \), and \( p = 1/2 \). The analysis is conducted with respect to a marginal increase in the value of \( \Delta \).

If the elasticity of matches with respect to job-seekers, \( \alpha \), is 1/2, free entry in any period or state implies \( \theta = (\psi J \kappa)^2 \). Thus, per equation (18), employment is given by

\[ n = (1 - n_{-1} + \delta n_{-1}) \psi^2 J \kappa + (1 - \delta) n_{-1}, \]

where \( n_{-1} \) refers to an initial value of employment.

Current employment, \( n \), and consumption, \( c \), are treated as fixed, and we focus on the immediate effect of uncertainty on asset prices, abstracting from the equilibrium feedback to current period quantities. Of course, insofar asset prices are altered by uncertainty, this will also affect entry, employment, and consumption also in the present. This general-equilibrium effect is discussed in section 3.5 below.

---

15We confine attention to a situation in which there is a direct increase in the uncertainty of future asset prices. The reason is two-fold. First, this focus avoids unnecessary notation that obfuscates the analysis. And second, a shock to productivity, \( z \), affects consumption, \( c = zn \), in two ways: via the direct effect on \( z \), and indirectly via asset prices and employment, \( n \). Focussing on asset prices directly avoids the interaction of these two effects.
3.2 Flexible prices: introducing the risk-premium channel

Inspecting equation (23) reveals that asset prices change if and only if the term

\[
[pu'(c_g)J_g + (1 - p)u'(c_b)J_b],
\]

changes. It is instructive to rewrite this term as

\[
[pu'(c_g)J_g + (1 - p)u'(c_b)J_b] = \frac{1}{2}(u'(c_g) + u'(c_b))\bar{J} + \frac{1}{2}\Delta(u'(c_g) - u'(c_b)).
\]

(24)

This formulation reveals that there are two mechanisms that may affect the asset price: In addition to a precautionary motive arising from a change in the expected marginal utility of consumption, there is a risk-premium effect arising from a change in the covariance between future asset prices and the marginal utility of consumption. Notice that the covariance is negative as long as \(c_g > c_b\), implying that the risk premium is positive, and is increasing in the distance of consumption across states.

Differentiating the above expression with respect to \(\Delta\) gives

\[
\frac{\partial}{\partial \Delta} \left[(u'(c_g) + u'(c_b))\bar{J} + \Delta(u'(c_g) - u'(c_b))\right] = \gamma \left(u'(c_b)e_{cb}J_b - u'(c_g)e_{cg}J_g\right) - \Delta \gamma \left(u'(c_g)e_{cg}J_g + u'(c_b)e_{cb}J_b\right) - (u'(c_b) - u'(c_g)),
\]

where \(e_c > 0\) denotes the elasticity of consumption with respect to asset prices, and \(\gamma\) is the coefficient of relative risk aversion. The last two terms captures the effect of the covariance term, which is always strictly negative for \(\gamma > 0\), as long as \(c_g > c_b\).

**Proposition 1.** For any value of \(\gamma > 0\), an incremental increase in \(\Delta\) leads to a positive precautionary-motive effect, and a negative risk-premium effect. Moreover, if \(\gamma \leq 1\), the overall effect is negative.

**Proof.** See appendix section B.

Thus, in general, a rise in uncertainty increases the precautionary motive which exerts upwards pressure on asset prices. At the same time, the covariance between asset prices and the marginal utility of consumption is aggravated, leading to a higher required risk premium on equity, which exerts negative pressure on asset prices. Moreover, \(\gamma \leq 1\) is a sufficient condition for the risk premium effect to dominate the precautionary-motive effect, such that there is an overall decline in current asset prices.

---

16 The covariance term is intimately linked to the notion of a risk-premium; see equation (27) in section 4.2. In particular, the risk-premium is an affine function of the covariance term, and if the covariance term is zero, so is the risk premium.
3.3 Sticky prices: precautionary motives induce contractionary demand effects

When prices are sticky, the relative price, $x$, moves, and adds an additional force affecting asset prices. To analyze this, the model is augmented with a bond Euler equation, a monetary policy rule, and a forward-looking Phillips curve. The bond Euler equation together with the monetary policy rule is summarized by

$$u'(c) = \beta R \Pi \phi^{-1} \left[ \frac{1}{2} (u'(c_g) + u'(c_b)) \right] E[u(c)],$$

and the Phillips curve can be written as

$$x = \frac{\eta - 1}{\eta} + \frac{\Omega_p}{\eta} \Pi (\Pi - 1).$$

Here, $\Pi$ refers to current inflation, and we assume that future (gross) inflation is equal to one. It should be noticed that the precautionary motive now also appears in the Euler equation for bonds. Hence, a rise in uncertainty will increase the precautionary motive which will lead to a fall in inflation, and to a decline in the relative price $x$. This decline in $x$ will exert negative pressure on the asset price. The next proposition reveals that the effect of the precautionary motive, which under flexible prices exert upward pressure on the asset price, may flip sign, and lead to an overall decline in asset prices.

**Proposition 2.** When prices are sticky, the effect of the precautionary motive is negative for a sufficiently high value of $\Omega_p$.

**Proof.** See appendix section B.

Of course, the main takeaway from Proposition 1 – that the risk premium channel exert negative pressure on asset prices – remains intact. As a consequence, sticky prices may give rise to a situation in which both forces operate in the same direction.

3.4 Precautionary-motive channel: prudence and employment asymmetries

The analysis above has shown that even under flexible prices a rise in uncertainty can lead to a contraction in asset prices through the risk-premium channel. Highlighting this mechanism is a key contribution of this paper. However, we also saw that the consequences of the precautionary motive can be expansionary or contractionary, depending on the degree of nominal rigidities. This result is entirely in line with the insights in Basu and Bundick (2017), who argue that sticky prices can divert an uncertainty-induced rise in the desire to save away from investments towards a decline in current consumption demand, thereby contracting economic activity.
What underpins this rise in the desire to save in the current framework? Partly, of course, it is the rise in future consumption volatility in conjunction with marginal utility being convex. But partly it is also driven by a decline in expected future consumption. To see this, recall the dynamics of employment dynamics in section 3.1, but assuming that $n_{-1} \approx n$,

$$n(J) = \frac{\psi^2 J}{\delta + (1 - \delta) \frac{\psi^2}{\kappa} J}.$$ 

Thus, employment is a concave function of the asset price. For any mean-preserving spread to $J$, the expected employment rate therefore falls short of the employment rate at the expected asset price; $E[n(J)] < n(E[J])$. Hence, a mean-preserving spread does not only lead to more volatility in endogenous variables, but it also lowers the expected value employment and consumption. This feature is in line with the analysis in Hairault et al. (2010) and several others.\(^{17}\) As this effect operates on exactly the same margins as increased consumption volatility – by increasing the expected marginal utility of consumption – we refer to their join impact as the precautionary motive channel.

### 3.5 Decomposition approach

Proposition 2 reveals that when prices are sufficiently sticky, the precautionary motive and the risk-premium channel may operate in the same direction, lowering asset prices – as well as economic activity – when uncertainty increases. A natural question then arises: Provided that an increase in uncertainty reduces production and worsens unemployment, to which extent does each channel contribute to the weakening of the economy? This question does not have an obvious answer, even in a quantitative setting: indeed, while the precautionary motive operates in isolation in the bond Euler equation, both channels are intertwined in the asset pricing equation, which contains the expectation of the multiplication of the future marginal utility of consumption and future asset prices.

A transparent decomposition is possible, however. Following equation (24), these channels can be arranged to enter additively in the asset pricing equation. More precisely, we can rewrite the asset pricing equation in (5) as

$$J_t = d_t + E_t [\Lambda_{t+1} J_{t+1} (1 - \delta)]$$

$$= d_t + E_t [\Lambda_{t+1}] E_t [J_{t+1} (1 - \delta)] + \text{cov}(\Lambda_{t+1}, J_{t+1} (1 - \delta)). \quad (25)$$

Thus, to gauge to which extent the risk-premium channel can account for the affect of uncertainty on the economy, we can suppress the covariance term – that is, we set it to zero – and analyze a counterfactual economy in which the risk-premium channel is absent. The difference between the model outcome and the counterfactual economy can then be attributed to movements of the risk premium. Furthermore,

\(^{17}\)See Jung and Kuester (2011), Petrosky-Nadeau et al. (2018), and Dupraz et al. (2019).
the precautionary motive can be attributed as the remaining residual from a *null result*; that is, the contribution of the precautionary motive is the difference between the counterfactual economy and the (stochastic) steady-state. This follows because, with the risk premium suppressed, if there is no change in the precautionary motive either, asset prices are left entirely unaffected by uncertainty, and so is the overall economy.

Importantly, this method also suppress the general-equilibrium effects of each channel. That is, the counterfactual economy discussed above is also void of the general-equilibrium effects *arising from the risk-premium channel*, notably a fall in consumption when lower asset prices reduce firm entry and employment. We consider this a salient and desirable feature, as it precisely ascribes the equilibrium amplification to the relevant mechanisms.

Lastly, it ought to be noticed that the two channels may well interact. For instance, if the economy is exposed to a persistent rise in uncertainty, economic activity will decline not only in the present, but also in the future. With future consumption depressed, the precautionary motive in the present is further strengthened, and current consumption takers yet another drop, and so on. Of course, the decomposition approach outlined above takes care of these interactions, and attribute their propagating effect to the relevant underlying channel.

4  Quantitative analysis

The previous section showed that uncertainty in the search-and-matching model transmits to the economy through a risk-premium mechanism and a precautionary-motive channel. In this section we quantitatively assess the importance of these channels. Two questions are central. First, is the model capable of matching the empirical evidence of rising unemployment following an uncertainty shock even without imposing nominal rigidities? Second, allowing for such rigidities, what is the relative importance of the risk-premium mechanism and the traditional aggregate demand channel associated with the precautionary motive under sticky prices? We tackle them in turn, finding that the answer to the first question is affirmative, and that, with respect to the second question, the two channels are about equally important.

4.1  Numerical implementation

Before doing so, we outline the benchmark parameterization of the model and briefly explain the numerical solution method.

4.1.1  Parameterization

We calibrate the model at monthly frequency, proceeding in three steps. First, we impose functional form restrictions on the utility function. Second, we calibrate a first set of parameters by matching
key steady-state moments, or drawing on external sources. Finally, we set the values of a second set of parameters by matching simulated moments to key asset-pricing and real-economy targets. All parameter values are summarized in Table 1.

Functional forms. Following Kehoe et al. (2022) we combine CRRA preferences with external habits à la Campbell and Cochrane (1999). The period utility function is given by

\[ u(c_t, X_t) = \frac{(c_t - X_t)^{1-\gamma}}{1-\gamma}, \]

where \( X_t \) is the external habit. The law of motion of \( X_t \) is indirectly determined by specifying the law of motion of \( S_t = \left( \frac{(c_t - X_t)}{c_t} \right)^{(1-\gamma)/\gamma} \), which is as follows:

\[ \log(S_{t+1}) = (1 - \rho_s) \log(\bar{S}) + \rho_s \log(S_t) + \lambda_A(\log(S_t))(\Delta z_t + E_t \Delta z_{t+1}) \]

\[ \lambda_A(\log(S_t)) = \frac{1}{\bar{S}} (1 - 2(\log(S_t) - \log(\bar{S})))^{1/2} - 1 \]

where \( \rho_s \) is the persistence of the habit and \( \bar{S} \) is the average habit level. The household treats the evolution of \( S_t \), respectively \( X_t \), as being exogenously given. Notice that the presence of habit leads to the following stochastic discount factor:

\[ \Lambda_{t+1} = \beta \left( \frac{S_{t+1} c_{t+1}}{S_t c_t} \right)^{-\gamma}. \]

Steady-state moments/external sources. We set the elasticity of substitution between differentiated retail products, \( \eta \), to 10, which implies a steady-state markup of 11 percent (Basu and Fernald, 1997). Following Petrongolo and Pissarides (2001), the elasticity of the matching function, \( \alpha \), is set to 0.5. The matching efficiency parameter, \( \psi \), is set to target an unemployment rate of 6.4%. Following Kehoe et al. (2022), the value of \( \delta \) corresponds to a monthly separation rate of 2.8 percent. To calibrate \( \kappa \), we use the law of motion for employment in equation (19) and find the steady-state value of vacancies. Normalizing the steady-state value of labor productivity to unity, we then set \( \kappa \) such that the total cost of vacancy posting is equal to 2 percent of steady-state output (Leduc and Liu, 2016).

For the wage-setting protocol, given a steady-state value of labor market tightness of \( \theta = 0.8777 \) (Leduc and Liu, 2016), the free-entry condition (20), alongside the previously calibrated parameters, pins down the steady-state asset value, \( J \). Together with a steady-state inverse markup equal to \( x = (\eta - 1)/\eta = 0.9 \), and a normalized steady-state value of labor productivity \( z = 1 \), equation (6) implies a steady-state wage equal to 0.8787. The bargaining weight, \( \omega \), is then immediately pinned down by the steady-state version of the wage relationship (21) as \( \omega = (w - \chi)/(xz - \chi) = 0.8062 \).

The parameter governing price stickiness, \( \Omega_p \), is set to 636.26, which gives rise to a slope of the
Table 1: Calibrated parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interpretation</th>
<th>Value</th>
<th>Source/target</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\kappa$</td>
<td>Vacancy-posting cost</td>
<td>0.23643</td>
<td>2 percent of steady-state output</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Efficiency of matching</td>
<td>0.3101</td>
<td>Unemployment rate of 6.4%</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Elasticity of substitution</td>
<td>10</td>
<td>Markup of 11%</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Separation rate</td>
<td>0.028</td>
<td>Kehoe et al. (2022)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Workers bargaining power</td>
<td>0.8062</td>
<td>Steady-state wage relation</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Elasticity of $f(\theta)$</td>
<td>0.5</td>
<td>Petrongolo and Pissarides (2001)</td>
</tr>
<tr>
<td>$\Pi$</td>
<td>Steady-state inflation rate</td>
<td>0.0016</td>
<td>Annual inflation rate of 2 percent</td>
</tr>
<tr>
<td>$\Omega_{\rho}$</td>
<td>Price adjustment cost</td>
<td>636.26</td>
<td>3 quarters price resetting duration</td>
</tr>
<tr>
<td>$\phi_{\pi}$</td>
<td>Taylor rule parameter for inflation</td>
<td>1.5</td>
<td>Taylor principle/convention</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Persistence of productivity</td>
<td>0.983</td>
<td>Leduc and Liu (2016)</td>
</tr>
<tr>
<td>$\rho_\sigma$</td>
<td>Persistence of uncertainty</td>
<td>0.913</td>
<td>Leduc and Liu (2016)</td>
</tr>
<tr>
<td>$\sigma_\sigma$</td>
<td>St. dev. of uncertainty shock</td>
<td>0.00085</td>
<td>Leduc and Liu (2016)</td>
</tr>
<tr>
<td>$\rho_s$</td>
<td>Habit persistence</td>
<td>0.996</td>
<td>Asset pricing moments (Table 2)</td>
</tr>
<tr>
<td>$\bar{S}$</td>
<td>Average habit level</td>
<td>0.09</td>
<td>Asset pricing moments (Table 2)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Coefficient of relative risk aversion</td>
<td>0.5</td>
<td>Asset pricing moments (Table 2)</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount factor</td>
<td>0.9981</td>
<td>Asset pricing moments (Table 2)</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Income while delaying bargaining</td>
<td>0.79</td>
<td>Unemployment volatility (Table 2)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>St. dev. of productivity shock</td>
<td>0.008</td>
<td>Output volatility (Table 2)</td>
</tr>
</tbody>
</table>

Notes. This table lists the parameter values of the model. The calculations and targets are described in the main text. One period in the model corresponds to one month.

Phillips curve that is equal to that of an implied model with Calvo pricing – solved using a first-order approximation – with a price resetting duration of three quarters (cf. Nakamura and Steinsson, 2013).

The coefficient on inflation in the the Taylor rule, $\phi_{\pi}$, is set to the standard value of 1.5.\(^\text{18}\)

The persistence of the productivity (level) process, $\rho_z$, is set to the value of 0.983, which implies a persistence at the quarterly frequency of 0.95, which is standard in the real business cycle literature. The persistence and volatility of the uncertainty shock, $\rho_\sigma$ and $\sigma_\sigma$, are set to match the estimates of Leduc and Liu (2016) at the quarterly frequency. The persistence $\rho_\sigma$ of 0.913 at the monthly frequency is equivalent to the persistence of 0.761 at the quarterly frequency, and the volatility $\sigma_\sigma$ of 0.00085 at the monthly frequency is equivalent to the volatility of 0.392 at the quarterly frequency in logged AR(1) process.\(^\text{19}\)

**Internally calibrated parameters.** Finally, conditional on the parameter values just described, we choose the remaining parameters by moment matching. These are the preference parameters, $\beta$, $\gamma$, $\rho_s$, $\chi$.

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\(^\text{18}\)In section C.2, we study how the effects of uncertainty shocks are mediated by allowing for a central bank that also takes into account output fluctuations.

\(^\text{19}\)Given the persistence of 0.913, the corresponding volatility $\sigma_\sigma$ at the monthly frequency is obtained from the level that aligns the unconditional volatility of our model’s uncertainty shock measured at the quarterly frequency with that of Leduc and Liu (2016).
\( \bar{\delta} \), the bargaining delay value, \( \chi \), and the average TFP volatility, \( \sigma_z \). This set of parameters plays an important role in determining the degree of volatility in the model. Given the connection between labor markets and asset-price movements at the heart of our model, we impose discipline by jointly targeting both real and financial moments (taken from Chen (2017)). Table 1 summarizes the parameter values and Table 2 compares the simulated moments with their data counterpart.

The two habit parameters are informed by key moments highlighted by the the asset pricing literature. In particular, the mean excess asset return increases in the habit persistence \( \rho_s \), and the volatility of the excess asset return decreases in the average habit level \( \bar{\delta} \). Despite there being inseparable interactions between both parameters and both moments, the different sensitivity of the mean and volatility makes it possible to identify \( \rho_s \) and \( \bar{\delta} \). Furthermore, the coefficient of relative risk aversion \( \gamma \) is set at 0.5, which allows the volatility of risk-free return to be similar to the data.\(^{20}\) Finally, the discount factor \( \beta \) pins down the average risk-free return over the business cycle.

The strike value \( \chi \) and the TFP volatility \( \sigma_z \) closely relate to the volatility of unemployment and output, respectively. Notice, in particular, that a higher value of \( \chi \) lowers the fundamental surplus, which in turn generates greater volatility of employment for any given volatility of shocks fed into the model (see Ljungqvist and Sargent (2017)).\(^{21}\)

As an additional validity check, we compare two other asset pricing moments in the simulated data with their empirical counterpart. These are the AR(1) coefficient of the risk-free return and AR(1) coefficient of the excess asset return. Reassuringly, the simulated untargeted moments are at similar levels as their data counterpart.

### 4.1.2 Solution method and IRF construction

We solve the model by third-order pruned perturbation.\(^{22}\) To study the effects of uncertainty shocks, we follow Fernández-Villaverde et al. (2011) and Born and Pfeifer (2014b) and consider impulse response functions (IRFs) that isolate the pure uncertainty effect resulting from higher volatility. That is, we focus on the effect uncertainty has on expectations, and how such changes in expectations trickle through to actual decisions, but ignore materialized shocks to the level of the exogenous processes.

As such, we focus on the effect of uncertainty itself, and not on that of more extreme realizations of productivity shocks. To be more precise, let \( g(\cdot) \) represent the policy function for, say, employment. That is, \( n_t = g(n_{t-1}, z_t, \sigma_z t) \). The pure uncertainty IRF is then given by \( n_{t+s} = g(n_{t+s-1}, z, \sigma_z t+s) \), for \( s = 0, 1, \ldots \) All IRFs are computed around the ergodic mean in the absence of shocks, which is also known as the risky (or stochastic) steady state (e.g., Coeurdacier et al., 2011).

\(^{20}\) The risk-free rate is formally defined in equation (26) in section 4.2.

\(^{21}\) The fundamental surplus is the primary determinant of the steady-state elasticity of labor market tightness with respect to productivity, \( \eta_{\theta z} = \frac{1}{\alpha_{x z}} \frac{\sigma_z}{\bar{z} - \bar{\chi}} \).

\(^{22}\) A perturbation method of at least the third order is necessary to obtain policy functions that contain volatility shocks as independent arguments; that is, a third-order approximation allows the second moments of both exogenous and endogenous variables to affect expectations.
Table 2: Simulated moments and the data counterpart

<table>
<thead>
<tr>
<th>Moments</th>
<th>Data</th>
<th>Model</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Asset pricing moments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean risk-free return (%p.q)*</td>
<td>0.24</td>
<td>0.2378</td>
<td>Chen (2017)</td>
</tr>
<tr>
<td>Volatility of risk-free return (%p.q)*</td>
<td>0.4</td>
<td>0.6915</td>
<td>Chen (2017)</td>
</tr>
<tr>
<td>AR(1) coef. of risk-free return (q)</td>
<td>0.88</td>
<td>0.9378</td>
<td>Chen (2017)</td>
</tr>
<tr>
<td>Mean excess return (%p.q)*</td>
<td>1.47</td>
<td>1.0012</td>
<td>Chen (2017)</td>
</tr>
<tr>
<td>Volatility of excess return (%p.q)*</td>
<td>8.46</td>
<td>12.6435</td>
<td>Chen (2017)</td>
</tr>
<tr>
<td>AR(1) coef. of risk premium (q)</td>
<td>0.08</td>
<td>-0.0231</td>
<td>Chen (2017)</td>
</tr>
<tr>
<td><strong>Real economy moments</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemployment volatility (%p.q)*</td>
<td>12.5</td>
<td>14.46</td>
<td>Hagedorn and Manovskii (2008)</td>
</tr>
<tr>
<td>Output volatility (%p.q)*</td>
<td>2.06</td>
<td>2.95</td>
<td>NIPA</td>
</tr>
</tbody>
</table>

*Notes. The moments with * are the targeted moments. The volatilities of unemployment and output are obtained from applying the HP filter with a smoothing parameter of 1600 to the time series of the (logged) variables.*

4.2 Flexible prices

The solid line in Figure 1 shows the pure-uncertainty effects of a one standard-deviation shock to volatility under flexible prices (i.e., when setting $\Omega_p = 0$). The primary outcome of interest is a decline in economic activity and a rise in the unemployment rate. Thus, our first main quantitative result is that due to the risk-premium channel a rise in uncertainty lowers economic activity even when prices are flexible.\(^{23}\)

As suggested in section 3, and specifically Proposition 1, this outcome arises because the contractionary risk-premium channel dominates the precautionary-motive mechanism, the latter being expansionary under flexible prices. To verify this claim, we can practically implement the decomposition approach set out in theoretical terms in section 3.5. Following this idea, the dashed line in Figure 1 shows how the uncertainty shock propagates through a counterfactual economy in which the risk-premium channel is absent. It is evident that in this counterfactual environment uncertainty leads to an economic boom, as the higher valuation of future cash flows leads firms to increase investment into vacancy creation, leading to a lower unemployment rate.

A complementary explanation rationalizes these macroeconomic effects through the lens of two

\(^{23}\)The working-paper version of this article showed that this result also obtains under a simpler calibration with log utility and no habits.
Figure 1: flexible prices

Notes: The figure illustrates the pure uncertainty IRFs for a one standard-deviation shock to volatility under flexible prices.
key asset-price moments. First, define the risk-free real interest rate as

\[ R_{t}^{rf} = \frac{1}{E_{t}[\Lambda_{t,t+1}]} \]  \quad (26)

Figure 1 shows that \( R_{t}^{rf} \) declines, as the expected stochastic discount factor, \( E_{t}[\Lambda_{t,t+1}] \) increases. As discussed in section 3.4, the rise in the expected marginal utility of consumption originates in two related but distinct forces; the prudent behavior of agents, as well as their expectation that future expected consumption will be lower. When the risk-free interest rate is lower, all future cash flows, including risky ones, are valued more highly. The result is upward pressure on asset prices that encourages job creation. This is the precautionary-motive channel.

On the other hand, the risk premium on equity, defined as

\[ RP_{t} = \frac{E_{t}[J_{t+1}](1 - \delta)}{J_{t} - d_{t}} - R_{t}^{rf} \]

\[ = - \frac{\text{cov}(\Lambda_{t,t+1}, J_{t+1}(1 - \delta))}{J_{t} - d_{t}} \times \frac{1}{E_{t}[\Lambda_{t,t+1}]} \]  \quad (27)

increases following an uncertainty shock. The reason is that heightened uncertainty leads to a more pronounced negative comovement between the future marginal utility of consumption and the asset value of the firms. The second equality above reveals a tight relationship between this covariance and the risk premium. Due to the rise in the required risk premium, equity prices fall. Importantly, since shocks are persistent, this mechanism is expected to repeat itself in the future, and there is a reinforcing effect arising from an additional anticipated decline in future equity prices, which puts additional downward pressure on current prices, and so on.

This story is not without economic appeal. A rise in uncertainty brings about a desire for intertemporal substitution, that is, to save more; both because of prudence, and the nonlinear dynamics of employment. This enhanced motive to save would, in isolation, put upward pressure on equity prices and result in an expansion. However, as consumption and asset prices are positively correlated, there is a negative covariance between future asset prices and the stochastic discount factor, indicating that equity indeed is a poor asset for hedging against this increase in risk. As a consequence of this desired shift in the composition of savings, prices for safe assets increase, while those on risky assets decline. When this channel dominates the former – as in Figure 1 – the unemployment increases alongside a rise in the risk premium.

### 4.3 Sticky prices

The second main quantitative result is that the risk-premium channel keeps playing an important role even when allowing for standard demand effects under sticky prices. Yet, Proposition 2 suggests
that establishing this point requires a more subtle argument compared to the preceding section. For while under flexible prices the risk-premium channel is the unambiguous culprit behind a rise in the unemployment rate, this is not the case under sticky prices. Now the increase in desired savings associated with the precautionary motive may likewise contract economic activity, through a decline in goods demand.

The solid line in Figure 2 indicates the effects of a one standard-deviation uncertainty shock when prices are rigid. Notice that the graph containing the risk premium has been replaced by the relative price of intermediate goods, $x_t$, which is now time-varying. As can be seen, the fall in the price of equity and the rise in unemployment qualitatively line up with those of Figure 1, but they are quantitatively more pronounced. At its peak, the unemployment rate increases by slightly more than 0.04 percentage points, which is due to an almost one-percent fall in the firm value.

A key reason behind this result is that the previously stabilizing force associated with the precautionary motive is now destabilizing. The reason follows a familiar New Keynesian narrative. The rise in uncertainty puts upward pressure on the expected stochastic discount factor and thereby downward pressure on the risk-free real interest rate. However, as the monetary authority is constrained in its reaction by the Taylor rule, the nominal interest rate does not change unless there is visible disinflation. Thus, absent a decline in inflation the real interest rate would be left unchanged, and demand for final goods would fall short of supply. The reduction in demand, however, encourages retail firms to lower their prices. Because of price-adjustment costs their response is muted, which results in a decline in the demand for intermediate goods, and in their relative price, $x_t$. As a consequence, the equity price falls, there is less entry and vacancy posting, less production, and supply approaches the reduced level of demand. Indeed, since the equity price is forward-looking, an expected, persistent, decline in the relative price puts severe negative pressure on the equity price already in the present. At the same time, the reduction in the price level leads to disinflation and thereby a reduction in real and nominal interest rates, which serves to mute the initial fall in demand. This process ends when there is an equal decline in both the demand and supply for goods, and the equilibrium is restored. Thus, the same mechanisms that stabilized the economy under flexible prices – those that put upward pressure on the expected stochastic discount factor – are now, via the aggregate demand channel, destabilizing. Consequently, and in marked contrast to the case with flexible prices, the equilibrium equity price is now lower for two reasons: partly as a result of a decline in the relative price for intermediate goods, which is driven by demand; and partly as a result of an increase in the risk premium.

It is natural to ask what the relative importance is of, respectively, the risk-premium channel and the fall in aggregate-demand associated with the precautionary-motive channel. Figure 3 provides an answer. The left-hand side depicts a decomposition of the cumulative rise in unemployment along the IRF. To conduct this decomposition, we apply the method outlined in section 3.5 and first solve the model using equation (25) yet suppressing the covariance term to be zero. The difference between the baseline result and the outcome of this exercise is due to the dynamics of the risk premium;
Figure 2: benchmark result under sticky prices

Notes: The figure illustrates the pure uncertainty IRFs for a one standard-deviation shock to volatility under sticky prices.
the remainder is due to a rise in the expected marginal utility of consumption associated with the precautionary motive.\footnote{We deliberately abstract from any effects associated with “precautionary pricing” by linearizing the New Keynesian Phillips curve in equation (13). In Appendix C.3 we show such effects to be quantitatively small in the present environment.} This exercise shows that absent the risk-premium mechanism, the rise in the unemployment rate is 38\% of that in the baseline economy and, thus, less than half as large.

To shed further light on the role played by the risk-premium mechanism, the dashed line in Figure 2 shows the effects in a counterfactual economy in which this channel is inoperative (paralleling the dashed line in Figure 1). This exercise reveals that the importance of the risk-premium mechanism in a setting with nominal rigidities partly derives from an interaction with demand forces that play out in general equilibrium, as foreshadowed in section 3.5. We can see, in particular, that in the counterfactual economy the fall in the firm value is muted partly because the relative price also declines less steeply. The reason is that the risk-premium channel leads agents to expect lower income/consumption in the future. The desire to smooth consumption over time then leads to lower demand already in the present, putting downward pressure on both the aggregate price level and the relative price of intermediate goods. These “risk-premium induced negative demand effects” mitigate the direct inflationary effect of the rising risk premium – which we dissect in further detail in the subsequent section 5.1 – and amplify the fall in the asset price and concomitant rise in unemployment. Hence, we conclude that to fully appreciate the propagation and amplification properties of the risk-premium channel it is important to account for nominal rigidities, over and above the implications those carry by themselves.

Notes: The figure illustrates the cumulative effect of the various transmission mechanisms under sticky prices prices on two macroeconomic aggregates: unemployment (left axis) and inflation (right axis). The computations are described in the main text.
5 Implications for inflation and monetary policy

A cursory reading of Figure 2 suggests that uncertainty shocks affect economic activity no differently from regular (aggregate) demand shocks, such as contractionary monetary policy. Indeed, both inflation and the risk-free real interest rate decline, output contracts, and the unemployment rate rises. This section evaluates to which extent uncertainty shocks differ from aggregate demand shocks by comparing the implications of these two types of shocks for the observable relationship between unemployment and inflation. We also briefly sketch implications for monetary policy.

5.1 Uncertainty shocks are not aggregate demand shocks

In a first step, we modify the Taylor rule in equation (22) to include a shock to monetary policy and reverse engineer a persistent rise in the nominal interest rate such that the impulse response function of unemployment exactly coincides with that of Figure 2. The effect on inflation and on the risk premium is documented in Figure 4 which makes clear that uncertainty shocks have a somewhat muted effect on inflation, alongside a much more pronounced effect on the risk premium.

What explains this result? In short, uncertainty shocks contain a supply component – the risk-premium channel – in addition to a demand effect. Consider first the propagation of a contractionary monetary policy shock. A higher interest rate reduces demand for final goods through the bond Euler equation. Facing lower demand, retail firms reduce their prices, leading to an overall decline in the price level. As prices are sticky, however, the resulting price-adjustment is incomplete, and retailers demand fewer intermediate goods. As a consequence, the relative price of intermediate goods, \( x_t \), falls, leading to lower asset values, which then contracts supply. This process continues until the goods market is in equilibrium, at a lower level of economic activity.

An uncertainty shock operates through comparable mechanisms, but with one pronounced difference, viz., the risk-premium channel. On the one hand, and as outlined before, an increase in uncertainty renders a decline in the risk-free real interest rate due to the precautionary motive. While a reduction in the risk-free interest rate can materialize both due to a decline in the nominal rate, or because of a rise in expected inflation, the Taylor rule in equation (22) reveals that the nominal rate will only be lowered if there is a reduction in current inflation. However, as the risk premium rises, equity prices fall, the unemployment rate increases, current private consumption declines, and the marginal utility of consumption rises. As a consequence, the rise in the expected discount factor is less pronounced than it would be in the absence of a variable risk premium, and the fall in both the nominal interest rate and inflation is therefore somewhat muted. Put simply, disinflation materializes to bring supply towards demand. But as an uncertainty shock contracts supply even in the absence of any price movements, less disinflation is needed to bring markets back into equilibrium. Phrased differently, the movement in the covariance term in equation (25) is akin to a negative supply shock,
Notes: The figure illustrates the pure uncertainty effect of a one standard-deviation shock to the volatility of productivity shocks (solid line) and the impact of a shock to the nominal interest rate (dashed line). The sequence of interest rate shocks is such, in terms of magnitude and persistence, that the resulting effect on real economics activity is identical to that resulting from an uncertainty shock.

which are commonly associated with inflationary pressure.

This account provides one potential explanation for the ambiguous findings in the empirical literature regarding the response of inflation to uncertainty shocks, even when that on real economic activity is well-established.\textsuperscript{25}

### 5.2 Monetary policy can partially offset uncertainty shocks

With flexible prices, the conduct of monetary policy is irrelevant for real variables, and merely serves to separate nominal from real interest rate movements. When prices are sticky, however, monetary policy affects real economic activity and can have profound implications regarding the response of the economy to various shocks.

Figure 5 illustrates the effect of the uncertainty shock, varying the aggressiveness of the monetary authority’s response to inflation.\textsuperscript{26} The solid line shows the benchmark result under sticky prices from Figure 2, and the dashed lines shows the results as $\phi_\pi$ takes on higher values, eventually approaching infinity. The dashed line illustrates the flexible price benchmark result from Figure 1.

As can be seen in the figure, the effect of an uncertainty shock on unemployment approaches that observed under flexible prices as the monetary authority response more aggressively to inflation. That

\textsuperscript{25}Incorporating additional mechanisms that propagate uncertainty shocks through “supply channels” would further reinforce this theoretical point. Two examples of such channels are precautionary pricing (on which see appendix C.3) and real options effects (see Bloom (2009), Schaal (2017) and Den Haan et al. (2021)).

\textsuperscript{26}Appendix C.2 studies the transmission of uncertainty shocks when the central bank also responds to output fluctuations.
is, by stabilizing inflation the monetary authority successfully stabilizes output as well. This finding corresponds to the well-known result of the “divine coincidence” (Blanchard and Galí, 2007), here applied to uncertainty shocks.

Figure 5 also illustrates a second point. Even though the divine coincidence holds, and in contrast to regular demand shocks, the flexible-price outcome nevertheless involves higher unemployment relative to a counterfactual economy that experienced no shock. The explanation follows immediately from the discussion in section 4.2; the rise in the required risk premium on equity induced by greater uncertainty is sufficient to cause a recession.

6 Concluding remarks

Using a canonical search-and-matching (SaM) model with risk-averse households, this paper has shown that a risk-premium channel plays a pivotal role in shaping the macroeconomics effects of uncertainty. Heightened future volatility strengthens the negative comovement between the marginal utility of consumption and the (procyclical) dividends paid out by firms, giving rise to an equity risk premium that weighs on the value of firms in the present. As hiring incentives in the SaM model are determined by this firm value, greater uncertainty can lower vacancy posting and push up unemployment even under flexible prices.

Moreover, in the presence of nominal rigidities, an uncertainty-induced rise in the motive for precautionary saving amplifies the decline in economic activities through a standard demand channel. Using a calibrated model disciplined by both real and asset pricing moments, and applying a simple but novel decomposition method, we found that in a counterfactual economy that features no risk-premium
channel, the effect of uncertainty shocks on unemployment would be more than halved.

Heuristically, the risk-premium mechanism operates akin to a negative supply shock that lowers employment and puts upward pressure on prices. Uncertainty shocks are, therefore, less deflationary than regular demand shocks. Moreover, they cannot be fully neutralized by monetary policy.

A central feature of the model that underpins our results is the long-lasting nature of firms. In particular, a persistent shock – or, indeed, an anticipated shock – affects firms’ profits for an extended period of time, which generates large fluctuations in asset values already in the present, thereby also affecting returns and incentives for job creation. We believe this feature may have implications for a number of issues in macroeconomics beyond those analyzed in this paper, including policies such as forward guidance, and more generally, the management of agents’ expectations.
Appendix A

A.1 Empirical evidence on the effects of uncertainty shocks

To provide context for the theoretical analysis in the main text, here we briefly summarize the empirical effects on key macroeconomic and financial variables of an increase in perceived uncertainty. Methodologically, we follow a vast literature, including Leduc and Liu (2016), whose specification we marginally extend by including evidence on vacancy-posting by firms and a measure of the equity risk premium. Specifically, we estimate a simple vector autoregression (VAR) model that includes a forward-looking measure of economic uncertainty ordered first, and identify the uncertainty shock recursively. We rely on monthly US data spanning 1978:1-2016:12 and include the following six variables in the given order: a measure of perceived economic uncertainty;\(^{A.1}\) the U-3 unemployment rate from the Current Population Survey; the composite help-wanted index constructed in Barnichon (2010); the one-year ahead equity risk premium (ERP) from Duarte and Rosa (2015); the year-on-year CPI inflation rate; and the 3-month Treasury bill rate. The model includes six lags and is estimated using standard Bayesian methods.

Figure A.1 describes the resulting impulse responses to a one standard-deviation uncertainty shock. We make three observations. First, elevated uncertainty leads to a sizeable contraction in economic activity. In particular, the shock causes the unemployment rate to rise in a persistent, hump-shaped manner, with a sizeable peak effect of more than 0.2 percentage points. Moreover, vacancy posting activity as captured by help-wanted advertising is negatively affected. Second, despite these unambiguous adverse consequences for real economic activity, the response of the inflation rate is small and at no point is the zero not included in the 95 percent credible interval. Finally, and turning to financial variables, while the essentially risk-free Treasury bill rate declines, the risk premium increases by up to 30 basis points.

\(^{A.1}\)Specifically, this measure is given by the fraction of respondents in the Michigan Consumer Survey pointing to an “uncertain future” as negatively affecting their spending on durable goods over the coming year. The results are highly comparable when using various alternative uncertainty proxies, as for instance the Equity Market Volatility Tracker of Baker et al. (2019).
Figure A.1: VAR evidence on the effects of uncertainty shocks

Notes: The figure describes the empirical effects of a one standard-deviation uncertainty shock identified from a Cholesky decomposition. The data included are described in the main text. The VAR is estimated using Markov Chain Monte Carlo Methods and employing a normal-diffuse prior for the coefficient matrix and the covariance matrix of the reduced-form innovations, respectively. Solid lines indicate the median posterior density of impulse responses, while the shaded area represents the highest posterior density (HPD) interval.
Appendix B

B.1 Proofs and derivations

B.1.1 Proof of Proposition 1

The main equation is

\[ \gamma \left( u'(c_b)e_{cb} \frac{\bar{J}_b}{J_b} - u'(c_g)e_{cg} \frac{\bar{J}_g}{J_g} \right) - \Delta \gamma \left( u'(c_g) \frac{\epsilon_{cg}}{J_g} + u'(c_b) \frac{\epsilon_{cb}}{J_b} \right) - (u'(c_b) - u'(c_g)). \]

where \( \epsilon_c \) is the elasticity of consumption with respect to the asset price

\[ \epsilon_{c,g} = 1 - \left( 1 - \delta \right) \frac{n}{n_g}, \quad \epsilon_{c,b} = 1 - \left( 1 - \delta \right) \frac{n}{n_b}. \]

Notice that \( \epsilon_{c,g} > \epsilon_{c,b} > 0 \). The precautionary motive effect is captured by

\[ \gamma \left( u'(c_b)e_{cb} \frac{\bar{J}_b}{J_b} - u'(c_g)e_{cg} \frac{\bar{J}_g}{J_g} \right). \]

Using the fact that the elasticities can be written as

\[ \epsilon = \frac{(1 - n_{-1} + \delta n_{-1})}{n} \psi^2 \frac{J}{\kappa}, \]

this simplifies to

\[ \gamma(1 - n_{-1} + \delta n_{-1}) \psi^2 \frac{J}{\kappa} \left( \frac{u'(n_b)}{n_b} - \frac{u'(n_g)}{n_g} \right) > 0, \]

where the inequality obtains if \( n_g > n_b \). Thus, the precautionary saving effect is always positive.

For the total effect, the main equation above simplifies to

\[ \gamma \left[ c_g^{-\gamma}(1 - \gamma \epsilon_{c,g}) - c_b^{-\gamma}(1 - \gamma \epsilon_{c,b}) \right]. \]

Thus, if \( \gamma \leq 1 \) we have

\[ \gamma \left[ c_g^{-\gamma}(1 - \gamma \epsilon_{c,g}) - c_b^{-\gamma}(1 - \gamma \epsilon_{c,b}) \right] < \gamma \left[ c_b^{-\gamma}(1 - \gamma \epsilon_{c,g}) - c_b^{-\gamma}(1 - \gamma \epsilon_{c,b}) \right] = \gamma \left[ c_b^{-\gamma}(\epsilon_{c,b} - \epsilon_{c,g}) \right] < 0. \]
B.1.2 Proof of Proposition 2

From the bond Euler equation we have

\[ \Pi = \left[ \frac{u'(c)}{\beta R^{\frac{1}{2}}(u'(c_g) + u'(c_b))} \right]^{\frac{1}{\phi - 1}}. \]

Thus,

\[ \frac{\partial \Pi}{\partial \Delta} = -\frac{1}{\phi - 1} R^{\phi} \frac{\beta}{u'(c)} \frac{1}{2} \gamma \left[ u'(c_b)e_{cb} f_b - u'(c_g)e_{cg} f_g \right] \]

and using the elasticities from above gives

\[ \frac{\partial \Pi}{\partial \Delta} = -\frac{1}{\phi - 1} R^{\phi} \frac{\beta}{u'(c)} \frac{1}{2} \gamma (1 - n_{-1} + \delta n_{-1}) \frac{\psi^2}{\kappa} \left( \frac{u'(n_b)}{n_b} - \frac{u'(n_g)}{n_g} \right) < 0. \]

Combining with the Phillips curve reveals that the total precautionary motive effect is

\[ \left( \frac{u'(n_b)}{n_b} - \frac{u'(n_g)}{n_g} \right) \gamma (1 - n_{-1} + \delta n_{-1}) \frac{\beta}{u'(c)} \frac{1}{2} \frac{\psi^2}{\kappa} \left( \bar{f}(1 - \delta) - (1 - \omega)2\Omega(2\Pi - 1) \right), \]

which can be arbitrarily negative for \( \Omega \) being sufficiently large.

B.1.3 Derivation of the resource constraint

Since all firms use a constant returns to scale technology – alongside with the fact that intermediate goods use only labor as an input, retail firms use only intermediate goods, and final goods firms use only retail goods – aggregate output is given by \( y_t = z_t n_t \).

As mentioned in section 2.1, the household makes additional profits, \( \tilde{d}_t \). These profits are in excess of the dividends arising from the ownership of intermediate firms, and instead include per-period profits from retailers, vacancy-posting agencies, and price adjusting firms. Aggregate profits arising from vacancy-posting agencies are equal to \( \kappa v_I \). Moreover, the aggregate profit arising from retailers net of price adjustment costs is

\[ \frac{p_t(i)}{P_t} y_t(i) - x_t y_t(i). \]

Using the fact that in a symmetric equilibrium \( p_t(i) = p_t(j) = P_t \), alongside with the demand relation in equation (10) together with \( y_t = z_t n_t \), reveals that these profits amount to \( z_t n_t(1 - x_t) \). Thus,

\[ \tilde{d}_t = \kappa v_I + z_t n_t(1 - x_t). \]  

(B.1)

35
Using the equilibrium relations \( B_t = 0 \) and \( n_t = a_t \), the household’s budget constraint is

\[
c_t + n_t(J_t - d_t) = w_t n_t + \hat{d}_t + n_{t-1} (1 - \delta) J_t.
\]

Rearranging and using that fact that \( d_t = x_t z_t - w_t \) gives

\[
c_t + J_t (n_t - n_{t-1} (1 - \delta)) = n_t x_t z_t + \hat{d}_t.
\]

Using the law of motion for employment in equation (19), and the definition of \( \hat{d}_t \) above reveals that

\[
c_t + \kappa v_t = n_t x_t z_t + \kappa v_t + z_t n_t (1 - x_t),
\]

or simply \( y_t = c_t = z_t n_t \).

Notice that aggregate consumptions is therefore not affected by the amount of vacancies created, nor the costs associated with price adjustments. This is indeed intentional; as we are exploring the role of uncertainty on behavior, any resource draining activity, such as price adjustments, may, somewhat mechanically, alter the marginal utility of consumption. We explore the role of such activities in appendix section C.4.

**B.1.4 Derivation of the wage-setting rule**

Wage setting based on alternating offers stems from the observation that severing a match is not a credible threat; indeed the worker and the firm will always reach an agreement within the period the meeting occurs. Common knowledge of this feature implies that future variables bear no consequence on the currently agreed wage.

The alternating-offers game takes place in fictional time, in which each time-period is of length \( \Delta \). If the worker has the opportunity of proposing a wage, \( w_t \), she will offer the highest possible value that the firm will accept. That is, the wage will yield the worker a maximum value of \( v_w = w_t \), and the firm a minimum value of \( v_f = x_t z_t - w_t \). However, as the firm can reject the wage proposal and wait to the next (fictional) time-period to make a counteroffer, the minimum value must also satisfy \( v_f = e^{-\Delta \omega} \times v_f \), where \( v_f \) denotes the firm’s maximum value, and \( e^{-\Delta \omega} \) the discount factor.

Conversely, if the firm has the opportunity of proposing a wage, \( w'_t \), it will yield the firm a maximum value of \( v_f = x_t z_t - w'_t \), and the worker a minimum value of \( v_w = w' \). Again, as the worker can reject the wage proposal and wait to the next (fictional) time-period to make a counteroffer, the worker’s minimum value must also satisfy \( v_w = \Delta \hat{\chi} + e^{-\Delta (1-\omega)} \times v_w \), where \( \Delta \hat{\chi} \) represents the flow utility the worker receives by not working.\(^{B.1}\)

Notice that the worker and the firm discounts fictional time differently; a higher value of \( \omega \) renders workers more patient which will play to the worker’s advantage,

\(^{B.1}\)We did not explicitly introduce the parameter \( \chi \) into the utility function but doing so would be straightforward.
Figure C.1: Sensitivity to $\chi$

Notes: The figure illustrates the cumulative response of unemployment to a one standard-deviation uncertainty shock, varying the flow consumption-value the worker receives by delaying agreement during the alternating offers bargaining process, $\chi$.

and vice versa.

The above set-up provides six (linear) equations in six unknowns. Solving this system and letting $\Delta$ approach zero gives rise to a unique (subgame perfect) wage that is agreed upon immediately

$$w_t = \omega x_t z_t + (1 - \omega)\chi,$$

with $\chi = \hat{\chi}/(1 - \omega)$. That is, the wage agreed by alternating offers is a combination of the firm’s revenues and the flow consumption-value the worker receives by delaying agreement.

Appendix C

C.1 Sensitivity analysis: the role of $\chi$

This appendix section shows that the extent to which uncertainty shocks cause mild or severe recessions in the model is shaped by the value of the parameter $\chi$, which in the alternating offer bargaining game over wages represents the worker’s outside option.

To illustrate this property, we solve the model repeatedly for a grid of values for $\chi$, holding the other parameters constant, and, each time, compute the cumulative response of unemployment to the uncertainty shock. The solid line in Figure C.1 depicts the result. It can be seen that the cumulative impact of an uncertainty shock on the unemployment rate monotonically increases in $\chi$.\(^{C.1}\) The reasons

\(^{C.1}\)The same result obtains under flexible prices.
for this result relate to the size of the fundamental surplus $xz - \chi$ (Ljungqvist and Sargent, 2017). Intuitively, if the fundamental surplus is small relative to output, then a given percentage change in productivity induces a greater percentage change in investment into vacancies (and, concomitantly, a larger responses of employment). This same logic carries through to the analysis of uncertainty shocks, with a smaller fundamental surplus translating into more unemployment volatility.\(^{C.2}\)

Lastly, and for completeness, the dashed line in Figure C.1 quantifies the magnitude of the uncertainty effect on the unemployment rate that is due to the risk-premium channel as a function of $\chi$. Unsurprisingly, per the above reasoning, that particular channel is stronger for higher values of $\chi$. The relative contribution of the risk-premium channel declines somewhat for still higher values of $\chi$.

\section*{C.2 A more general Taylor rule}

The baseline analysis considered a Taylor rule according to which the central bank responds to inflation only. This section explores the implications of allowing for a more general specification according to which the monetary authority also responds to output fluctuations as it is quite commonly used in the literature (and is, arguably, is of empirical relevance). The correponding coefficient is denoted $\phi_y$.\(^{C.3}\)

Figure C.2a shows that the recessionary impact of elevated uncertainty on unemployment is much less severe if the monetary authority responds not only to inflation but also to output (with $\phi_y = 0.2$). This is for two reasons. First, $\phi_y > 0$ directly implies a more accommodative policy stance given the contraction in output in the present. Second, expectations for future production are stabilized, which mutes not only precautionary savings due to prudence but also the force of employment asymmetries generating a fall in expected consumption that would incentivize saving in the present. To see the latter feature, notice first that the direct impact of, say, an increase in the productivity of the intermediate goods sector is to push down the intermediate price of inputs, $x_t$. However, the same shock also increases income and stimulates demand, which instead puts upward pressure on the relative price, $x_t$. The reverse holds true for negative productivity shocks. By responding to output, policymakers can limit the second effect, engineer a relatively more countercyclical relative price, and thus dampen business cycle volatility in hiring (see Lepetit (2020) for a detailed discussion). Given the asymmetric employment dynamics of the model, the expected employment gap – caused by the anticipation of greater future volatility – and resulting pessimism about the future, is then smaller in absolute

\(^{C.2}\) An additional nuance is that employment asymmetries become disproportionately more important when employment is more volatile. That is, when $\chi$ is high, then following an increase in perceived future exogenous volatility agents to downgrade their expectations for future average employment by a significant amount. Indeed, higher volatility makes it more likely that the model gives rise to what Petrosky-Nadeau et al. (2018) describe as “endogenous disasters”. The anticipation of such an outcome leads agents to change their behavior in the present, giving rise to strong pure uncertainty effects.

\(^{C.3}\) Indeed, both Leduc and Liu (2016) and Basu and Bundick (2017) consider $\phi_y = 0.2$ in their baseline parameterization. Basu and Bundick (2017) assume that $\phi_y$ applies to output in deviation from its own lag, rather than its deviation from the deterministic steady-state.
The implications of setting $\phi_y > 0$ for inflation are more intricate in two respects. First, as uncertainty shocks tend to push inflation and output in the same direction, responding more aggressively to either variable helps stabilizing the other one as well. Yet in response to a productivity shock, the inflation rate and output move in opposite directions. Accordingly, by stabilizing future output the monetary authority also leaves future inflation relatively more volatile. Thus, the monetary authority faces a tradeoff between stabilizing either future real or nominal dynamics, even when this is not the case with respect to today’s economy in which, by construction, only uncertainty shocks materialize.

Second, when price adjustments are costly in resource terms, then stochastic volatility renders the tradeoffs facing the monetary authority more complex. To see this, suppose we relax the assumption, maintained thus far, that Rotemberg adjustment costs subtracted from retail firms’ profits are rebated to the household as a lump-sum. Then the GDP identity is instead given by $y_t = c_t + ac_t$, where $ac_t$ represents the convex adjustment cost

$$ac_t = \frac{\Omega_p}{2} \left( \frac{p_t(i)}{p_{t-1}(i)\Pi} - 1 \right)^2 y_t.$$

Figure C.2b illustrates that under this assumption the economy may even fall into a deeper recession.

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C.4 Notice that in a standard New Keynesian model without asymmetric employment dynamics, the expectation for more volatile employment would typically not affect the conditional mean.

C.5 This statement implicitly rests on the premise that overall dynamics are more strongly influenced by shocks to levels than by shocks to second moments. In practice, this is the case.
following an uncertainty shock when the monetary authority actively responds to output. This result stands in marked contrast to Figure C.2a. The key insight is that by setting $\phi_y > 0$ the monetary authority may stabilize expected future production, but now tolerating greater future inflation variability no longer guarantees stabilizing future consumption. The convex nature of adjustment costs implies that greater expected future price adjustments translate into lower expected consumption for any given level of production. As before, expectations for lower future consumption then ripple through the economy, leading to a fall in current demand but also the expectation of lower future asset prices. To summarize, with resource draining price-adjustment costs, an increase in anticipated volatility leaves households more pessimistic about future consumption, either because employment is expected to be significantly more volatile – when the monetary authority ignores output fluctuations – or because inflation is expected to be more variable – when the Taylor rule does give weight to output. The monetary authority thus faces an uncomfortable tradeoff between expected future output volatility and inflation volatility; both of which contribute to exacerbating the effects of an uncertainty.

### C.3 Precautionary pricing

When firms are subject to nominal rigidities, an increase in uncertainty about future demand conditions may give rise to precautionary pricing that renders markups more countercyclical.\(^{C.6}\) Intuitively, such behavior arises when firms’ marginal revenue product exhibits convexity; it is more costly for a given firm to set too low a price relative to its competitors (more units need to be sold at a sub-optimally low price) compared to setting it suboptimally high (the higher price per unit partially compensated for fewer units sold).

To avoid confounding the key mechanisms of interest in our analysis with such effects, all results in the main text are derived by imposing a linearized version of the New Keynesian Phillips curve (NKPC) in equation (13). This approach eliminates non-linear terms that could potentially generate an upward pricing bias (cf. Fernández-Villaverde et al. (2015, section VI)). To verify the robustness of our results, Figure C.3 plots the usual pure uncertainty IRFs under the benchmark parameterization (solid line), but illustrating also the outcome when the underlying model is solved such that the nonlinearity of the NKPC is preserved (dashed line).

Figure C.3 illustrates why we consider our baseline implementation not only a useful simplification, in that it strengthens tractability, but also an innocuous one. As can be seen from the figure, nonlinearities in the NKPC play virtually no role when $\phi_\pi = 1.5$ and $\phi_y = 0$. Moreover, even when we allow for a non-zero response to output, which leads to greater volatility in future inflation, the difference in inflation responses is modest. Consistent with the principles of precautionary pricing, to the extent that there do exist nonlinearities in the NKPC, they bias inflation upward and the relative price (i.e., the inverse relative markup) downward; the latter generally exerts a negative effect on job creation.

\(^{C.6}\)For details on precautionary pricing, see Fernández-Villaverde et al. (2015), Oh (2019), and Born and Pfeifer (2020).
Figure C.3: The limited role of precautionary pricing

Notes: The figure illustrates the pure-uncertainty IRFs for a one standard-deviation shock to volatility. The different IRFs vary according to the coefficient on output in the Taylor rule, $\phi_y$, and differ in whether the structural New Keynesian Phillips curve equation is linearized or not when solving the model.

To the extent that nonlinearities in the NKPC do exert upward pressure on inflation, this only goes to reinforce our finding that uncertainty shocks are less deflationary than regular demand shocks.

C.4 Resource-draining vacancy posting costs

In the benchmark model we assume that vacancy posting costs are rebated to the household. If one were to suppose, instead, that these expenditures subtract from consumption in the resource constraint – that is, $c_t + \kappa v_t = z_t n_t$ – then this gives rise to an additional transmission channel that operates under both flexible and sticky prices. As vacancies $v_t$ are convex in productivity, a spike in expected volatility lowers the expected level of output available for consumption. Such an expectation then sets off a greater desire for savings in the present. Quantitatively, this channel turns out to be insignificant (figures are available upon request), so that we abstract from it in order to focus on the key transmission channels of interest.
References


