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Tom Auld

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†E-mail address: tja20@cam.ac.uk. Address: Faculty of Economics, University Of Cambridge, The Austin Robinson Building, Sidgwick Ave, Cambridge, CB3 9DD, UK
1 Introduction

How do financial and political betting markets behave in the hours after an election? Does one market outperform the other in terms of reflecting information about an election? Also, are there any violations of market efficiency around political events and can they lead to any realistic profit opportunities? This paper attempts to answer these and other questions.

It can be argued that the overnight hours following many elections are very special times. During these typically overnight hours there is an absence of economic information that may usually inform financial prices. It is possible that financial assets are uniquely determined by the outcome of those elections and nothing else. For many elections the results are drip fed throughout the overnight session as results are announced from different voting areas and constituencies. For example, for US presidential elections there are 51 simultaneous vote counts in 50 states plus the special district of Washington DC. These counts are typically broadcast live on TV and on the Internet. For the 2016 UK Brexit referendum there were 382 different areas that announced local results at various times starting from around midnight. Studying the effects of this unique flow of information on prediction and financial markets is the subject of this paper.

That political risk affects asset prices is an established concept in the literature. Similarly, the information content and superior forecasting ability of political prediction markets is a settled matter. It would naturally follow that political and financial market prices should be generally statistically related. There are many event studies in the literature that study this question. However, there are very few examples of models that apply to either multiple events or a general setting, or are based on economic assumptions. We seek to fill that gap in the particular case of the generic ‘election night’. This is achieved by creating a model that links financial and political markets that is based on the idea that financial prices are affected by the election outcome and nothing else. This idea is encoded by an assumption that the only information having a persistent effect on financial asset prices is the likelihood of a binary political outcome. Adding the assumptions of weak market efficiency and risk neutrality imply that political and financial markets will be cointegrated, whereas relaxation of risk neutrality leads to a non-linear cointegrating relationship. The model is tested on three real world events. We find strong evidence for two events, whereas for the third the information contained in the vote is limited and statistical significance is not generated. An exploration of the short run dynamics of the model in the vector error correction form demonstrates that betting markets Granger cause financial markets, but not vice-versa. This reveals the fact that

1The 2020 presidential election is a unique case where, given various legal challenges and recounts, results took days to conclude.
political markets are more efficient at discounting the political information contained in election results on election night.

The contributions of this paper are follows. A model is presented based on economic first principles that applies in a general setting. This is a first in the literature. Strong support is also found for the model. Further, it is found that political betting markers lead financial markets in the hours after an election. This is an important finding and adds to the weight of evidence in the literature that prediction markets are without peer in their forecasting ability.

The remainder of the paper is organised as follows. Section 2 reviews relevant existing literature. Section 3 presents the theoretical model that implies political and financial markets are cointegrated on election night. Section 4 briefly outlines the cointegration frameworks and associated tests used in the results section. The bulk of the paper is Section 5, the results section, where the implications of our theory are studied for some real world elections. We end the paper in Section 6 by concluding our results.

2 Literature review

This paper is primarily interested in the relationships between, and behaviour of, prediction and financial markets, around political events. Relevant areas of literature include both the financial market effects of political events as well as election prediction markets in general.

The financial market implications of elections and political risk are well documented. Changes in the composition of government naturally brings about changes in policy. There are many studies that demonstrate either the effects on financial markets of election campaign periods or results. Three large multi-country studies demonstrate changes of asset price volatility around elections. Białkowski et al. (2008) studies 134 elections in 27 OECD countries from 1980 to 2004. Using a GARCH methodology they find that the relevant national stock exchange index volatility can easily double during the week after the election. Apparently ‘investors are surprised by the election outcome’. Kelly et al. (2016) find that this uncertainty is priced in the options market. They analyse data from 1990-2012 for options prices on either the national index, or an ETF tracking that index, for a sample of 20 countries. They find that prices and implied volatility are higher for options that span elections. They also document spillover effects from the election country to other international markets. Pantzalis et al. (2000) is another large multi country study. This paper finds significantly positive returns two weeks prior to election dates for elections in 33 countries between 1974 and 1995. The conclusion is that as election uncertainty is resolved, prices respond positively. Two later papers come to the opposite conclusion for US presidential elections. Goodell and Bodey (2012) consider
how the Graham price to earnings (P/E) of the S&P500 index stocks, a valuation metric as well as a measure of consumer sentiment, changes during the campaign periods of US presidential elections. They find the measure *worsens* as the winner becomes clearer, according to the likelihood seen on the Iowa market (that is, as uncertainty reduces). They conclude that for the US, ‘during presidential election seasons, the market discovers its distaste for the economic policies of the likely winner’. The analysis is extended in Goodell and Vähämäa (2013) for the five presidential elections from 1992 to 1998. They consider the effects on the VIX, a measure of implied volatility of options on the S&P500 expiring in under one month. They find the VIX is positively associated with positive changes in the likelihood of the winner. This ‘indicates that the presidential election process engenders market anxiety as investors form and revise their expectations regarding future macroeconomic policy.’

There are also studies in the literature that demonstrate association between elections polls and asset prices, in the run up to an election. Gemmill (1992) considers the campaign period of the 1987 UK general election. In the paper the author derives the probability of a Conservative party win from polling data. They find that the ‘FTSE100 index was very closely related to the probability of a Conservative win’. Further, in the final two weeks before the voteshare options prices showed large increases in implied volatility. This was particularly the case for two nationalisation targets (of the opposition Labour party). Brander (1991) and Bernhard and Leblang (2006) study the 1988 Canadian election. This was shortly after the implementation of the Canada-US FTA. The FTA was widely expected to increase trade between the two countries, being positive for the stock market. However, the opposition Liberal party was opposed to the agreement. Prices on the Toronto Stock Exchange (TSE) were found to be significantly positively related to polling numbers for the Conservative party during the campaign period.

We now briefly review the literature on political prediction markets. Prediction markets are exchange traded financial markets for the purpose of trading on the outcome of events. Election, or political, markets are prediction markets that are based on the outcome of elections. Modern examples of electronic election markets include University of Iowa’s Iowa Electronic Markets, introduced for the 1988 US presidential election, the University of British Columbia’s UBC Election Stock Market (now superseded by the Sauder School of Business Prediction Markets) and the Betfair Exchange, prices for which are used in this paper. There is a plethora of research on the accuracy of prediction, and election markets. For a recent review of prediction markets see Horn et al. (2014) and for political markets see Graefe (2016).

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2Following the inclusion of Mexico in 1994 the CUSFTA became the North American Free Trade Association in 1994.
Studies, many of which are based on the Iowa and UBC markets, have demonstrated the remarkable accuracy of forecasts from election markets. There is a consensus in the literature that political markets outperform other methods including polling and expert predictions. Two early papers by Forsythe et al. demonstrate the outperformance of prediction markets when comparing final prices with final polling numbers for vote shares (despite the presence of judgement bias amongst traders). Forsythe et al. (1992) studies the Iowa market for the 1988 US presidential election. They find that ‘the market worked extremely well, dominating opinion polls’. By looking at the positions of different constituent groups they find that traders tended to place bets on their preferred candidates (indicating judgement bias). However, they conclude that prices are indeed set by the marginal trader. The implication being that even if a majority of participants are irrational and have misspecified beliefs, the existence of a small group of unbiased traders, perhaps arbitrageurs, keep prices in line. Oliven (2004) considers whether the Iowa market is skewed by biased participants. They conclude that market-making traders are more rational than price takers, the implication being that arbitrageurs are indeed making prices efficient. The analysis and conclusions are repeated for the 1993 Canadian federal election in Forsythe et al. (1995). Both these studies focus on shorter-term market predictions. A latter study, Joyce E. Berg and Rietz (2008), extends the analysis to longer-term forecasts. This paper compares vote share prices from the Iowa market with 964 polls over the five US presidential elections from 1988 to 2004. They find that the prediction market is closer than the polls 74% of the time. The average error in vote share for presidential candidates in the final five days of polling is 1.2% versus 1.64% for polls. Further ‘the market significantly outperforms the polls in every election when forecasting more than 100 days in advance’. This provides evidence that election markets are not only accurate at times close to a vote but have superior explanatory power months from an election.

The literature is clear that both election markets are powerful tools to forecast elections and that the outcome of elections have effects on asset prices. It naturally follows that prediction and financial markets should be related in some way. There are many event studies in the literature that consider elections, but they typically only consider either financial markets, or political markets and rarely both, and when they do they typically only uncover an empirical relationship and are not based on economic theory. The few studies in the literature that consider both types of market are discussed below.

There are two studies in the literature that directly relate political prediction market prices to the residuals of CAPM models. Both consider only the 2014 Scottish independence referendum. Acker and Duck (2015) find that the residuals of an estimated CAPM model are significantly positively related to several proxies for the likelihood of a vote to remain as part of the UK, one of which is a weighted sum of the Betfair exchange
odds for ‘No’. Similarly Darby et al. (2019) study equities listed on the LSE that were headquartered in Scotland. They find that uncertainty betas help predict cross-sectional returns. Hanna et al. (2021) also use Betfair data to analysis how changes in the betting odds for ‘Leave’ influence financial markets for the Brexit referendum. They consider the period from January 2016 to the date the referendum was resolved (the early hours of June 24). Using high frequency data for trades on the Betfair exchange they regress short-term returns of GBPUSD and major UK and European stock indices on changes in betting prices during stock market opening hours. They find that changes in the odds for leave cause prices of UK equities and the pound to fall in the following 5 minutes. They also find some spillover effects to EU equity prices. However, the slopes of the regression appear somewhat small when compared to the sensitivities estimated in other studies, including in this paper. Finally we find only a single example of an economically derived relationship between prediction markets and asset prices in the literature which is Manasse et al. (2020). The authors build a simple portfolio model for currencies. This implies that currencies are cointegrated with betting prices for the period running up the to 2016 UK Brexit referendum. Under risk neutrality they find a linear cointegrating relationship but risk aversion leads to the presence of a risk factor related to uncertainty. This leads to a non-linear term of the betting market appearing in the cointegrating relationship. We build a similar model in this paper. However our model only applies to the overnight session following an election. Manasse et al. (2020) find that currency and Betfair data are consistent with their model. However, we do not believe the assumptions behind their model are valid. For it to be so, one would have to believe that the only determinant of the GBPUSD price in the weeks and months preceding the Brexit vote is the result of that vote. We believe this is not plausible. News and information beyond that relating to the referendum, including US economic releases, are likely to affect the British Pound and United States Dollar exchange rate in the period under study.

Contrary to the existing literature, this paper presents an asset pricing model of political and financial markets that is based on rigorous economic principles. It is for a particular setting, the session following an election, but applies to a general election. We believe the fact that there are no similar studies or theories presented in the literature speaks to the contribution of this work.

3 Theoretical framework

In this section we present a pricing framework to link the prices of political and financial markets during the overnight results session following an election.
3.1 Assumptions

First we outline an overnight scenario. Suppose that the outcome of the election can be represented in a binary fashion (for example a yes/no referendum, or whether or not the Democratic candidate in a US presidential election is elected). Further suppose there is a healthy liquid prediction market for this event where bets trade which payout conditional on whether a particular binary outcome occurs. Such contracts exist and examples include bets that paid out £1 in the event of the UK voting to leave the EU in 2016 as well as, say, Matteo Salvini’s party winning the Italian general election in 2018. These bets are effectively binary contracts. Write \( PB_t \) as the price of the contract that pays out £1 in the event of said political risk. The outcome of the event is realised at time \( T \). Further, suppose there is a financial asset with price \( p_t \) at time \( t < T \). Say \( E = 1 \) if the event occurs and \( E = 0 \) otherwise. Further, write \( \Omega \) as the space of all possible outcomes of the election. Note that this is bigger than just the binary outcome \( E \). For example \( \Omega \) will include turnout and vote-shares in different voting regions along with whether or not a particular party or individual is the ultimate winner of a vote.

We now discuss the various economic assumptions of our model which we list in order of strength:

- **NS** No persistent shocks to \( p_t \) beyond those that affect \( P_t(E = 1) \)
- **EMHW** The Efficient Market Hypothesis holds in the weak form
- **RN** Risk neutrality

Not all these assumptions are regarded as holding exactly but they are presented as an approximation. We discuss each one in detail below using the example of pound sterling and the Brexit referendum. In this case \( E = 1 \) refers to a vote to leave the EU, \( p_t = GBP_t \) the price of the pound in dollars at time \( t \) in the overnight session and \( \Omega \) includes such information as the total vote-share for leaving the EU, as well as the vote shares and turnouts in each of the 382 areas that announced overnight. \( PB_t \) will now be the price of the contract that pays out £1 in the event of a vote to leave the EU. Such contracts were listed on the Betfair exchange where around £50m traded during the hours following the close of the vote.

**NS – No Shocks**

The assumption states that the only persistent affects on the financial asset price \( GBP_t \) are due to the probability of the political event occurring. Simply put, the only fundamental determinants of the GBP price on the night were the results of the vote, and that those results only affect the price through their effect on the probability of Brexit,
\( P(E = 1) \). Any changes in price that are not related to \( P_t(E = 1) \) are stationary and will disappear quickly. This is formulated in the next section in equation 3.1.

First we discuss the validity that the only determinant of GBP price during the hours under study were the referendum results. There are certainly other determinants of the GBP price over longer periods of time, for example information regarding the nature of future trading relationships that may became apparent after a positive vote. However, on the night itself, there were no major economic releases, or other significant news events. In advance, the Econoday Economic Calendar (Econoday (2016)) listed the final market related news releases on the 23 June as the US New Home Sales Report at 10:00 am Eastern Time (ET) and the first one for 24 June (beyond the referendum) as Durable Goods Orders at 10 am ET. They predicted that the following would be the market focus for the 24th: “In a rare and potentially powerful wildcard, the markets will react to the Brexit outcome”. This demonstrates that in advance there were beliefs that the main determinant of prices would be the outcome of the referendum. Indeed, the authors of Wu et al. (2017) describe the circumstances as “a natural experiment” with “near perfect conditions” to study such a situation. Next we discuss the assumption that only the likelihood of Brexit affects the GBP price. Before the vote there were several predictions that the pound would sell off significantly in the event of Brexit but rally a little otherwise (see Wu et al. (2017)). Thus, an assumption that GBP decreases monotonically with \( P(E = 1) \) is reasonable. This does not lead directly to the assumption that only \( P(E = 1) \) affects the GBP price. For instance, it could be believed that the vote share \( v_b \) has an affect on GBP over and above the decision to leave the EU through the “hardness” of such a Brexit. This would still be consistent with the NS assumption as unless the support of the distribution of the vote-share for Brexit, \( v_b \), were entirely above 50% (\( P(E = 1) = 1 \)) any move in \( v_b \) would result in a, ceteris paribus, change of \( P(E = 1) \). This is because, as the expectation of \( v_b \) moves further from the 50% cutoff, the probability mass below 50% decreases and \( P(\text{Brexit}) \) decreases, at least for continuous distributions of \( v_b \). Also, we postulate that only the binary result of the vote, and not any particular form of trading relationship following a vote, was on the minds of investors on the night of the vote. Despite there being intense scrutiny of the negotiations between the EU and the UK on the terms of withdrawal in the years following the vote, the term “hard” Brexit only first appeared several months after the referendum at the Conservative party conference in October 2016.

**EMHW – Weak Market Efficiency**

When the EMH holds there can be an interpretation of a market probability for a prediction market, as all information has been aggregated into the price. The market price of the Betfair contract is the risk neutral probability of a vote to leave the EU. An
implication of weak form market efficiency is that different markets will have identical beliefs. For the purposes of this study this means that, although the two markets may have beliefs about the outcome of the referendum which do not agree with fundamental information contained in vote results (violating semi-strong efficiency), they must have the same beliefs. In what follows we make a distinction between weak market efficiency failing on short term timescales where deviations are quickly corrected as opposed to longer term inefficiencies that persist.

**RN – Risk Neutrality**
This is a strong assumption which is not believed to hold in general, but it is a useful approximation that is likely to be roughly valid. We do though only apply this assumption in a very limited way relating to the political risk surrounding the event $E$. To be specific, this relates to risks of price changes that are due to the election and will disappear once $E$ has been resolved. This will be discussed further in the next section. We do note, however, that any deviations from RN may have larger effects than they otherwise might have due to the increased risk of holding the pound during the period under study.\(^3\)

**RA – Risk Aversion**
In the case where we relax risk neutrality we make the assumption that investors, at least in financial markets, are risk averse. This is a very reasonable assumption and is common in the asset pricing literature. Investors will need to be paid a premium to hold assets that have exposure to the political risk $E$ until at least a time when the results have been fully announced. For the purposes of this study this means that there will be a discount applied to the financial asset that is greater when political uncertainty is greater. This is the case when $\mathbb{P}(E = 1)$ is close to 50%, whereas uncertainty is least when the probability is close to 0% or 100%.

### 3.2 Implications
In this section we explore the implications of the assumptions of the model. They are explored below and summarised in Table 1. Notation is outlined below.

**Notation**
- $E$ Binary political event indicator $\in \{0, 1\}$
- $T$ Time at which event is realised

\(^3\)Higher perceptions of risk were evident from, for instance, higher implied volatility from options pricing as well as increased margin requirements from brokers for sterling related products.
\( PB_t \)  
\[ \text{Price of betting contract paying out £1 when } E = 1 \text{ at time } t < T \]

\( p_t \)  
\[ \text{Price of financial asset at time } t < T \]

\( \mathbb{P}_t^B(E = 1) \)  
\[ \text{Aggregate belief of the likelihood of } E = 1 \text{ at time } t \text{ of investors in the betting market} \]

\( \mathbb{P}_t^f(E = 1) \)  
\[ \text{Aggregate belief of the likelihood of } E = 1 \text{ at time } t \text{ of investors in the financial market} \]

\( p_1 \)  
\[ \text{Expected Value of financial asset at time } T \text{ conditional on } E = 1 \]

\( p_0 \)  
\[ \text{Expected Value of financial asset at time } T \text{ conditional on } E = 0 \]

\( u(\cdot) \)  
\[ \text{Bernoulli utility function} \]

\( \epsilon_t \)  
\[ \text{Stationary process} \]

### 3.2.1 No Shocks (NS)

This states that for \( t < T \) the non-stationary asset price \( p_t \) is a function only of \( \mathbb{P}_t(E = 1) \). Write this function as \( F(\cdot) \). Thus

\[ p_t = F(\mathbb{P}_t(E = 1)) \tag{3.1} \]

where \( \epsilon_t \) is stationary. The exogeneity condition follows from the fact that the NS assumption states that \( \epsilon_t \) is unrelated to the likelihood \( \mathbb{P}_t(E = 1) \). Taking expectations at time \( t \) of the prices at the realisations of the event \( t = T \) yields

\[ \mathbb{E}_t(p_T|\Omega) = \mathbb{E}_t(F(\mathbb{P}_T(E = 1))) \]

since \( \mathbb{E}_t(\epsilon_T) = 0 \). At \( t = T, E \) has been realised and is either 0 or 1 and \( \mathbb{P}_T(E = 1) = E \). So

\[ \mathbb{E}_t(p_T|\Omega) = \mathbb{E}_t(F(E)) . \]

\[ = \mathbb{P}_t(E = 1)F(1) + \mathbb{P}_t(E = 0)F(0) \]

Since \( E \subset \Omega \)

\[ \mathbb{E}_t(p_T|\Omega, E) = \mathbb{E}_t(F(E)) = \mathbb{E}_t(p_T|E). \]
Table 1: Assumptions and their implications in the theoretical framework.

<table>
<thead>
<tr>
<th>Assumptions</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>NS+EMHW</td>
<td>$p_t = g(PB_t) + \epsilon_t$  (g(\cdot)) monotonic</td>
</tr>
<tr>
<td>NS+EMHW+RN</td>
<td>single asset: $p_t = p_0 + \Delta p \times PB_t + \epsilon_t$ (\epsilon) cointegration (n) multiple assets: $p_t = (p_{1t}, \ldots, p_{nt}, PB_t)'$ has cointegration rank (n)</td>
</tr>
<tr>
<td>NS+EMHW+RA</td>
<td>$p_t \approx p_0 + \Delta p \times PB_t - \lambda \times R_t + \epsilon_t$ $\lambda \in (0,</td>
</tr>
</tbody>
</table>

This demonstrates that the NS assumption implies conditional mean independence of \(p_t\) from \(\Omega\) given \(E\). This states that the sterling rate is affected by \(\Omega\) only through \(E\), i.e. whether or not Brexit. The GBPUSD price would be expected to be the same if the vote share for Brexit were either 50.01% or 99.99%. This is a strong result, particularly when there is significant probability mass around vote shares close to 50%.

Applying the total law of probability to the expectation yields

$$
\mathbb{E}_t(p_T|\Omega) = \mathbb{P}_t^f(E = 1)F(1) + \mathbb{P}_t^f(E = 0)F(0). \quad (3.2)
$$

We label the probabilities at this stage with the superscript to indicate that they are probabilities of the event as implied by investors in the financial market. As we have not yet made any assumptions about market efficiency it is perfectly possible that other groups including participants in betting markets have other beliefs. Note that

$$
F(1) = \mathbb{E}(p_T|E = 1) \\
F(0) = \mathbb{E}(p_T|E = 0)
$$

the values of the asset price expected under the different outcomes of the binary political event \(E\). These are constant in time (as by NS the asset price only depends on \(P_t(E = 1)\)).

The expectation can further be written as

\(^4\text{In the paper Auld and Linton (2019) NS and Conditional Mean Independence (CMI) were listed as separate assumptions but we show here that CMI follows from NS.}\)
\[ \mathbb{E}_t(p_T|\Omega) = \mathbb{P}_t^f(E = 1)F(1) + (1 - \mathbb{P}_t^f(E = 1))F(0) \]
\[ = F(0) + \mathbb{P}_t^f(E = 1) \times [F(1) - F(0)] \]
\[ = p_0 + \mathbb{P}_t^f(E = 1) \times \triangle p \]

(3.3)

where

\[ p_0 = F(0) = \mathbb{E}(p_T|E = 0) \]
\[ \triangle p = \mathbb{E}(p_T|E = 1) - \mathbb{E}(p_T|E = 0). \]

Thus the NS assumption implies that the distribution of the asset price at time \( T \) is effectively a linear function of a Bernoulli random variable with probability equal to the probability of the political event being realised.

### 3.2.2 Weak Market Efficiency (EWMH)

Turning now to the betting market and using the expected utility of the contract that pays out £1 when \( E = 1 \)

\[ u(PB_t) = \mathbb{P}_t^B(E = 1) \times u(\£1). \]

Normalising the utility function so that \( u(\£1) = 1 \) implies

\[ \mathbb{P}_t^B(E = 1) = u(PB_t). \]

Weak market efficiency implies that the probabilities of the event evaluated by the betting markets are the same as those evaluated by the financial markets and so

\[ \mathbb{P}_t^f(E = 1) = \mathbb{P}_t^B(E = 1) = u(PB_t). \]

It is unrealistic that market efficiency will hold exactly at all times instantaneously. We introduce an error to the relationship \( \eta_t \) so that
If EMH holds strictly then $\eta_t \equiv 0$. Results about the election arrive throughout the night of the election. Each result permanently affects the long-term expectation of the likelihoods $P^f_t(E = 1)$ and $P^b_t(E = 1)$. They are thus non-stationary. Non-stationarity can be shown by the fact that the long-term expectation of these time series is equal to the value at time $t$. Thus no long-term expectation can exist. The time series of the likelihoods are bounded. However these likelihoods can be thought of as an integrated $I(0)$ series up to and until the value of either 0 or 1 is achieved at the conclusion of the election. Turning now to market efficiency we will distinguish between market efficiency holding in the long term and not holding at all. If deviations $\eta_t$ quickly decay, efficiency broadly holds. Then $E_t(\eta_t) = 0$ and $\eta_t$ is stationary. If errors persist and $\eta_t$ is non-stationary, then weak market does not hold at all.

Substituting equation 3.4 into the NS assumption equation 3.1 recovers

$$p_t = F(u(PB_t) + \eta_t) \approx F(u(PB_t)) + \frac{d}{dPB_t}[F(u(PB_t))] \times \eta_t$$

When $\eta_t$ is small and stationary this describes an approximate contemporaneous relationship between the prices in the betting and financial markets. It is intuitively likely that the relationship between the asset price and the probability of the event described by the function $F(\cdot)$ is monotonic. Since the utility function is increasing this would lead to a monotonic relationship between the prices. We can explore this a little more by substituting equation 3.4 into the expectation in equation 3.3 giving

$$E_t(p_T|\Omega) = p_0 + u(PB_t) \times \Delta p.$$
\[ u(p_t) = \mathbb{E}_t(u(p_T)|\Omega) = \mathbb{P}_t^f(E = 1) \times u(p_1) + \mathbb{P}_t^f(E = 0) \times u(p_0) \]
\[ = u(p_0) + \mathbb{P}_t^f(E = 1) \times [u(p_1) - u(p_0)] \]
\[ = u(p_0) + \mathbb{P}_t^f(E = 1) \times \Delta u(p) \]  
(3.5)

where \( p_1 = F(1) = \mathbb{E}(p_T|E = 1) \). Substituting in the Weak Market Efficiency condition equation 3.4 implies

\[ u(p_t) = u(p_0) + [u(PB_t) + \eta_t] \times \Delta u(p) \]
\[ p_t = u^{-1}[u(p_0) + u(PB_t) \times \Delta u(p) + \Delta u(p) \times \eta_t] \]
\[ = u^{-1}[u(p_0) + u(PB_t) \times \Delta u(p)] + \]
\[ \frac{d}{dPB_t} \left( u^{-1}[u(p_0) + u(PB_t) \times \Delta u(p)] \right) \bigg|_{u(p_0) + u(PB_t) \times \Delta u(p)} \times \Delta u(p) \times \eta_t + o(\eta_t^2) \]  
(3.6)

\[ = g(PB_t) + \epsilon_t + o(\eta_t^2) \]  
(3.7)

where

\[ g(\cdot) = u^{-1}[u(p_0) + u(\cdot) \times \Delta u(p)] \]
\[ \epsilon_t = \eta_t \times \Delta u(p) \cdot \frac{d}{dPB_t} \left( u^{-1}[u(p_0) + u(PB_t) \times \Delta u(p)] \right) \bigg|_{u(p_0) + u(PB_t) \times \Delta u(p)} . \]

The Bernoulli utility function \( u(\cdot) \) is monotonic and increasing. Thus \( g(\cdot) \) is monotonic. Further if an appreciation in the asset price is expected when \( E = 1 (\Delta p > 0 \Rightarrow \Delta u(p) > 0) \) then \( g(\cdot) \) is increasing and there is a positive relationship between the prices. This is as expected as the asset price increases with increasing odds of \( E = 1 \) in the betting markets.

Thus equation 3.7 describes a monotonic relationship between the financial asset and the betting market which, when market efficiency holds in the long term, has errors from that relationship which quickly decay. By linking the probabilities of the event occurring in the two markets via market efficiency, we see that any deviation from the relationship \( g(\cdot) \) does not persist. However, when EWMH holds instantaneously and exactly, adding
NS implies that the prices in the two markets should move contemporaneously tick by tick.

3.2.3 Risk Neutrality (RN)

Under the assumption of risk neutrality the Bernoulli Utility function is linear. We can recover a linear relationship between the two markets firstly by linearising equation 3.5 and substituting in the market efficiency condition equation 3.4:

\[
p_t = p_0 + \Delta p \cdot PB_t + \epsilon_t \tag{3.8}
\]

\[
\epsilon_t = \Delta u(p) \times \eta_t \tag{3.9}
\]

Thus the two markets will be cointegrated with cointegrating vector \((\Delta p, -1)\). Taking \(\Delta p\) as given, an arbitrage opportunity may exist if the error term deviates from zero. This would be facilitated by taking a position in the betting market contract and taking an opposing position in the financial asset, in the ratio of the cointegrating vector.

The cointegration relationship can similarly be recovered by applying the Uncovered Interest Parity (UIP) asset pricing relationship which holds under perfect mobility of capital and risk aversion. This usually applies only to foreign exchange where there is an interest rate differential. However, in our situation there is no interest rate differential realised as we are considering only a few hours which do not in general cross from one day to the next for clearing or funding purposes. Just as a currency can be bought or sold over this hypothetical period, so can a share or commodity. There is no cost or benefit of carry. UIP applied to currencies states that

\[
E_t(p_T) - p_t = i - i^* \tag{5}
\]

where \(i - i^*\) is the interest rate differential from \(t\) to \(T\).\(^5\) As this is zero in our case this immediately results in the relationship

\[
p_t = E_t(p_T) = P_t^0(E = 0) \times p_0 + P_t^1(E = 1) \times p_1 \quad \text{using equation 3.2}
\]

\[
= p_0 + \Delta p \cdot PB_t \quad \text{substituting in the market efficiency condition.}
\]

\(^5\)Strictly speaking this relationship holds for logarithm of the asset prices, but we can ignore this fact as there is no interest rate differential realised in our situation and to first order the equation is identical.
Note that this applies to assets beyond foreign exchange. We note that as UIP assumes the asset price always equates to its future expectation exactly, we have lost the stationary error term $\epsilon_t$. We could if pushed interpret $\epsilon_t$ as observed errors from UIP. However this is not as satisfactory as the first derivation as we lose the interpretation of $\epsilon_t$ as deviations from the long term relationship. Nonetheless the application of UIP does serve as another way to derive the linear long term cointegration relationship from economic first principles.

The cointegration relationship resulting from the three assumptions of NS, EWMH and RN has a pleasing interpretation. Rather than there being separate non-stationary asset prices during the hours after the vote, prices across markets share a single common trend. This non-stationary trend is the likelihood that the political event will or will not be realised. The assumptions mean that this is the only factor that persistently affects prices during these unique times.

### 3.2.4 Multiple assets

We now consider the case when there are $n$ assets indexed by $i$. Each asset is separately cointegrated with the prediction market so that

$$p_{it} = p_{i0} + \triangle p_i P B_t + \epsilon_{it}.$$

We can also consider the vector of $n+1$ prices where the final price is that of the political contract $PB_t$. Then our model becomes

$$\begin{pmatrix} p_{1t} \\ \vdots \\ p_{nt} \\ PB_t \end{pmatrix} = \begin{pmatrix} p_{10} \\ \vdots \\ p_{n0} \\ 0 \end{pmatrix} + PB_t \begin{pmatrix} \triangle p_1 \\ \vdots \\ \triangle p_n \\ 1 \end{pmatrix} + \begin{pmatrix} \epsilon_{1t} \\ \vdots \\ \epsilon_{nt} \\ 0 \end{pmatrix}.$$

Note our model says nothing about whether or not idiosyncratic deviations from the cointegrating relationships are correlated or not. In fact we may expect dependence among these deviations among economically related assets. However, EWMH does imply that these deviations will not be predictable and thus must be martingale differences.

Using matrix notation we write $P_t$ as the $n+1$ dimensional vector of prices which include the betting contract price, $P_0$ as the $n$ dimensional vector of expected prices of the financial asset conditional on $E = 1$ and $\epsilon$ the $n$-dimensional vectors of idiosyncratic martingale differences.
\[ P_t = \begin{pmatrix} p_{1t} \\ \vdots \\ p_{nt} \\ PB_t \end{pmatrix}, \quad P_0 = \begin{pmatrix} p_{10} \\ \vdots \\ p_{n0} \end{pmatrix}, \quad \epsilon = \begin{pmatrix} \epsilon_{1t} \\ \vdots \\ \epsilon_{nt} \end{pmatrix}. \]

The system of equations can be written as

\[ \Pi P_t - P_0 = \epsilon \]  

(3.10)

where

\[ \Pi = \begin{pmatrix} 1 & 0 & \cdots & 0 & 0 & -\triangle p_1 \\ 0 & \ddots & 0 & \vdots \\ \vdots & 1 & \ddots & \vdots \\ 0 & \ddots & 0 & \vdots \\ 0 & 0 & \cdots & 0 & 1 & -\triangle p_n \end{pmatrix} \]

an \( n \times (n+1) \) dimensional matrix of \( n \) cointegrating relationships. The \( n \) rows of equation 3.10 are stationary. Thus there is again a single common trend among the asset prices. This is equal to the likelihood of the outcome \( E = 1 \) and is equal to the price of the betting contract \( PB_t \). By assuming that the only persistent determinant of asset prices is the likelihood of the outcome of the political event we see that the cointegrating rank of the \( n + 1 \) asset prices which include the political market must be \( n \) during the results session. This is a proposition which can be tested empirically.

### 3.2.5 Risk Aversion (RA)

We now consider the case where investors are risk averse. We recovered the linear cointegration relationship in the case of risk neutrality with two different approaches. The first was using the implications of the NS assumption that led to the conditional mean independence of \( p_t \) from \( \Omega \) given \( E \). In the case of risk aversion a non-linear monotonic relationship is recovered in terms of the utility function \( u(\cdot) \) and expected appreciation/depreciation of the asset from \( E = 0 \) to \( E = 1 \) (\( \triangle p \)). This is written down in equation 3.7. The second way cointegration was derived was via UIP. The strict form of UIP, that the expected appreciation is equal to the interest rate differential, only holds when assets in different currencies are perfect substitutes. The equation can be modified by the introduction of a time-varying risk premium term \( \lambda_t \):
\[
E_t(p_T) - p_t = i - i^* + \lambda_t = \lambda_t
\]

since no yield can be realised in the short period of time under consideration \((i - i^* = 0)\). When \(\lambda_t\) is positive, spot prices are lower relative to their expectation. This is as investors need to be compensated for the risk of holding the asset. In our model we are not concerned with default risk or lack of monetary integration between different currencies. The risk premium we are concerned with is due to political uncertainty and will be fully resolved when the election concludes. Note that any contribution to the risk premium due to other factors will not contribute to \(\lambda_t\). Such a premium will be present both before and after the election results and discount \(p_T\) and \(E_t(p_T)\) in equal amounts. Thus any effect would be canceled out on the left hand side of the above relationship. It makes sense that when uncertainty about the election is highest the risk premium is highest, and this is the case when the probability of \(E = 1\) is closest to 50%. We follow a similar methodology to that outlined in Manasse et al. (2020) to derive an explicit expression for \(\lambda_t\) in terms of \(\mathbb{P}_t^f(E = 1)\).

We consider a representative investor who chooses between holding a proportion \(\omega\) of her wealth in the financial asset and the rest in a risk free asset. For simplicity assume standard mean-variance preferences so that the investor maximises:

\[
U(w) = \omega \cdot [E_t(p_T) - p_t] - \frac{r}{2}\omega^2\sigma^2,
\]

where \(r\) is the coefficient of absolute risk-aversion and \(\sigma^2\) is the portfolio variance. The first term is the expected appreciation of the asset from a time \(t\) overnight before the full results of the event are apparent, and time \(T\), the time at which \(E\) is realised. The second term is a penalty for holding the risky financial asset and is proportionate to the risk aversion coefficient \(r\) and the portfolio variance \(\omega^2\sigma^2\). Risk aversion \(\Rightarrow r > 0\).

Firstly we note that the expected appreciation \(E_t(p_T) - p_t\) must be positive for an investor to hold any of the risky financial assets. When this is the case the first order condition is

\[
E_t(p_T) - p_t = \omega r \sigma^2.
\]

The portfolio share reduces with increased risk aversion \(r\) and asset variance \(\sigma^2\). Taking the supply of the financial asset as fixed (exogenous), clearing of the financial market implies that \(\omega = s\). \(\sigma^2\) can be evaluated and is the variance of \(p_0 + X \times \Delta p\) where
$X$ is a Bernoulli random variable with probability $\mathbb{P}_t^f(E = 1)$. This is $(\triangle p)^2 \cdot \mathbb{P}_t^f(E = 1). [1 - \mathbb{P}_t^f(E = 1)]$. Thus the time varying risk premium can be written as

$$
\lambda_t = \lambda \pi_t (1 - \pi_t)
$$

$$
\lambda = rs (\triangle p)^2
$$

$$
\pi_t = \mathbb{P}_t^f(E = 1).
$$

We form a non-linear cointegration relationship by using the approximation $\pi_t = \mathbb{P}_t^f(E = 1) \approx PB_t$ and substituting the expectation of $p_T$ from equation 3.3 into the modified UIP formula

$$
p_t \approx p_0 + \triangle p. PB_t - \lambda. R_t + \epsilon_t.
$$

(3.11)

where

$$
R_t = PB_t(1 - PB_t).
$$

This derivation is not entirely satisfactory. The approximation $\mathbb{P}_t^f(E = 1) \approx PB_t$ implicitly ignores the possibility of risk aversion of gamblers in the betting markets. We could proceed with calculating a discount between the price of a bet that pays out £1 in the event of $E = 1$, using risk aversion and a risk premium as above. However, betting markets will usually list pairs of opposing bets. For instance, just as there is a contract that pays out £1 in the event $E = 1$ there will be one that pays out £1 when $E = 0$. These markets are typically efficient to within transaction costs so that the sum of the price of the two bets is approximately £1. Any meaningful discount on the price of one bet will result in a premium in the price of the opposite bet. We can reconcile the apparent contradiction by assuming that bettors in the political markets are risk neutral and will be correctly evaluating the odds of the event, whereas investors in financial markets are risk averse. Financial investors require a premium to invest in an asset in the overnight period prior to the uncertainty in $E$ being resolved.

Another way to derive the non-linear relationship in equation 3.11 is to form an approximation to the relationship in 3.7. Firstly we know that at the point at which all uncertainty has been removed (ie $PB_t = 0, 1$) any risk premium must be zero and the prices are the same as those from the linear relationship ($p_t = p_0 + PB_t \times \triangle p$). Thus the approximation must pass through the 2 points $(0, p_0)$ and $(1, p_1)$. The simplest way to approximate $g(\cdot)$ is to form a quadratic approximation. We could proceed with a Taylor

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series to second order and attempt to solve for the coefficients. However, it is more simple to note that any quadratic which passes through two points has a single degree of freedom. Noting that the risk term \( R_t = PB_t (1 - PB_t) \) is zero at \( PB_t = 0, 1 \) suggests that we can add any multiple of this term to the linear relationship \( (p_t = p_0 + PB_t \times \Delta p) \) and the resulting quadratic will still be correct at the endpoints. Thus the general quadratic must be of the form

\[
p_t \approx \tilde{g}(PB_t) = p_0 + \Delta p \cdot PB_t - \lambda R_t \quad \lambda \in \mathbb{R}.
\]

Risk aversion means there must be a discount to the price (and expected appreciation) when there is uncertainty which ensures \( \lambda > 0 \). The fact that the relationship is monotonic places further restrictions on \( \lambda \). The first derivative of \( \tilde{g}(PB_t) \) is

\[
\tilde{g}'(PB_t) = \Delta p - \lambda + 2\lambda PB_t.
\]

When \( \Delta p > 0 \) (expected appreciation when \( E = 1 \)) this must be positive for all values of \( PB_t \in [0, 1] \). This implies \( \lambda < \Delta p \). Similarly a negative gradient under an expected depreciation means that \( \lambda < -\Delta p \). Thus \( \lambda < |\Delta p| \).

As noted in Manasse et al. (2020), changing odds of the political event affect financial prices in two ways. The first is via the direct effect on the likelihood of yielding \( p_0 \) versus \( p_1 \) under different outcomes of the vote. The second is via the effect on the risk premium, which will depress prices if the odds move towards 50%. Further testing the hypothesis \( H_0 : \lambda = 0 \) versus \( H_1 : \lambda = 0 \) in the (non-stationary) regression of \( p_t \) on \( PB_t \) and \( R_t \) above will be a test of whether investors are risk averse versus the null hypothesis of risk neutral.

4 Statistical specifications

We aim to test the theoretical pricing model on a selection of political events. In each case we will study the overnight price series of a betting contract with a collection of key actively traded financial assets. The key implication of the theoretical model is one of cointegration. Our assumptions imply that there exist linear combinations of the non-stationary asset prices which are stationary. The statistical phenomena of cointegration was introduced by Granger (1983) who used it to study long-run economic relationships. For our model the “long-run” relationship lasts only a few hours. It exists between political and financial asset prices and arises due to the very special circumstances of the hours directly after an election. There are three statistical frameworks in which
cointegration is studied, each producing various methods by which the theory can be tested. We describe these briefly below.\textsuperscript{6}

4.1 Cointegration frameworks

4.1.1 The regression framework

Given a multivariate process $p_t = (p_{1t}', p_{2t}')'$ where the dimension of $p_{1t}$ and $p_{2t}$ are $n_1$ and $n_2$ respectively the model is

$$p_{1t} = \beta' p_{2t} + \epsilon_{1t}$$
$$\Delta p_{2t} = \epsilon_{2t}$$

where $\epsilon_t = (\epsilon_{1t}', \epsilon_{2t}')'$ is a mean zero, finite variance stationary linear invertible process. $p_t$ is non-stationary, $p_{2t}$ is not cointegrating and $p_t$ is cointegrating of rank $n_1$. The cointegrating vectors are the rows of $\Pi = [I_{n_1}, -\beta']$. Our pricing model naturally fits into this framework with $n_1 = n$ the number of financial assets ($n = 1$ in the single asset case), $n_2 = 1$ and $p_{2t}$ being the price of the political betting contract $PB_t$. $\beta = \Delta p$ the vectors of sensitivities of asset prices to the binary political event. Engle and Granger (1987) showed that, in the presence of cointegration, OLS estimators of $\beta$ are superconsistent. Stock (1987) demonstrated that the asymptotic distributions of $\hat{\beta}$ are mixed normal distributions. Tests for cointegration are based on the residuals of the regression where the null hypothesis is the presence of a unit root in $\epsilon_t$. This null hypothesis is equivalent to no cointegration in $p_t$. Tests of an econometric theory typically proceed with the assumption that the null hypothesis supports the model under consideration and a failure to reject the null is support for the theory. This is not the case here. However, rejection of the null at some significance level indicates a rejection of no cointegration in favour of cointegration. This would indicate stronger evidence of the pricing model than a failure to reject stationarity of $\epsilon_t$ at the same significance level.

Note that deviations from the long term relationships $\epsilon_t$ are not restricted to be i.i.d. Hansen and Phillips (1990), Phillips and Ouliaris (1990) and Phillips (1991) propose efficient corrections for the long run variance of $u_t$ due to serial correlation. This is important in our work as financial time-series typically exhibit heteroskedasticity and short term serial correlation. Despite the natural fit of our model into this framework there are some drawbacks. Models of cointegration rank are not nested and cannot be tested, and we rely on other frameworks for this purpose.

\textsuperscript{6}For simplicity we omit lags of differenced variables in the following exposition.
4.1.2 The autoregressive framework

The model for the $N$ dimensional multivariate process is now

$$\Delta p_t = \alpha(\beta'x_{t-1} - E(\beta'x_{t-1})) + u_t$$

where $u_t$ are mean zero, finite variance i.i.d. errors. $\alpha$ and $\beta$ are $N \times r$ matrices. The cointegration rank is $r \leq N$ and the space of cointegrating vectors is the space spanned by the rows of $\beta'$. The framework allows modeling of the short term dynamics towards the long run relationship ($\beta'x_{t-1} = E(\beta'x_{t-1})$) through adjustment speeds $\alpha$. Statistical inference in this framework is based on the Gaussian likelihood (Johansen, 1995), and likelihood ratios are used when testing various hypotheses. The fact that models of different cointegration rank $r$ are nested enables sequential testing of specific ranks against alternatives of higher rank. Another benefit over the regression framework is that test statistics of cointegration are independent of which variable is chosen as a dependent variable in any regression. This is not the case with the Engle–Graigner framework.

4.1.3 The unobserved component framework

The process $p_t$ is now given by

$$p_t = \xi\eta' \sum_{i=1}^{t} \varepsilon_i + u_t$$

where $u_t$ is a mean zero, finite variance i.i.d process independent of $\varepsilon_t$. The $N-r$ common stochastic trends are the elements of the vector ($\sum_{i=1}^{t} \varepsilon_i$). The parameters are related to the autoregressive framework via the relationships $\xi = \beta_{\perp}$ and $\eta = \alpha_{\perp}$. Again models of different rank (or the numbers of common trends) are nested, via the rank of $\xi$. Whereas in the autoregressive framework models of rank $r$ can be tested against alternatives of higher rank, here specific numbers of common trends are tested against alternatives of higher numbers of trends. The testing of rank proceeds in the “opposite” direction to the Johansen tests. Note that in our model there is a single common trend equivalent to the probability of a particular political outcome implying $N-1$ cointegration vectors. Shin (1994) proposed a residual based test for the bivariate case (or univariate regression) where the null is one of a single common trend (cointegration). Tests of higher numbers of common trends in the general multivariate case have been proposed by Nyblom and Harvey (2000).
4.2 Non-linear cointegration

The theory of linear cointegration is well developed. That of non-linear cointegration less so, with many open questions. In fact it is not entirely clear how non-linear cointegration should be defined. Generalisations of the properties of short memory (I(0)) and long memory (I(1)) are required. See Escanciano and Escribano (2009) and Wang (2015) for surveys of the field.

Much of the work extending cointegration to a non-linear setting involves extending ECMs to non-linear error correcting models (NECs). This in general involves replacing the linear gap function $\alpha(\beta' x_{t-1} - E(\beta' x_{t-1}))$ to some non-linear reaction function $f(\beta' x_{t-1} - E(\beta' x_{t-1}))$. The argument of $f(\cdot)$ is a stationary linear cointegrating vector of non-stationary variables. Of potentially more interest to our application is the study of nonlinear cointegrating regression models. These have been studied in both the parametric and non-parametric setting.

Turning to our particular problem we recall that the most general form of the theoretical model is

$$p_t = g(PB_t) + \epsilon_t$$

where $p_t$ is the non-stationary financial asset price, $PB_t$ is the non-stationary (bounded) price of the betting contract and $\epsilon_t$ is stationary. Our theoretical model uses a Bernoulli utility function with an assumption of increasing marginal utility. This places a further monotonic restriction on $g(\cdot)$. We also recall that assuming risk neutrality implies that $g(\cdot)$ is linear. Karlsen et al. (2007) estimate the function $g(\cdot)$ nonparametrically using the Nadaraya–Watson estimator. The asymptotics have been worked out and the distribution of $\hat{g}(\cdot)$ is Gaussian. This opens the door to specification testing. Unfortunately rates of convergence in the non-stationary case are slower than the stationary case. $\hat{g}(\cdot)$ converges to $g(\cdot)$ at a rate of $T^{1/4}$. Other developments of this theory include Gao et al. (2009).

This paper presents a bootstrap scheme and test of whether $g(\cdot)$ is of a known parametric form $g(\cdot, \beta)$. Wang and Wang (2013) extends the Kernel estimate of $\hat{g}(\cdot)$ to include $\epsilon_t$ as having a nonlinear nonstationary heteroskedastic process. This is of particular relevance for applications with financial time series.

In this paper we are primarily interested in testing whether our theory holds. We seek extensions of the tests with a null of $\epsilon_t$ non-stationary, such as Dickey–Fuller and associated tests, to the non-linear setting. Rejection of such a test in favour of stationary $\epsilon_t$ would provide strong evidence of our theory holding. KPSS-type tests of stationarity in the nonlinear parametric case (see Choi and Saikkonen (2010)) are available. However, we find no tests in the literature with a null of no nonlinear cointegration in the nonparametric case.
Of secondary interest in our model is whether there is significant risk aversion present. This is equivalent to the cointegration relationship deviating significantly from linearity. The methods presented in both Gao et al. (2009) and Wang and Wang (2013) do provide such tests conditional on cointegration holding. However, there are two drawbacks for our problem. Firstly, there is no way to restrict the estimate of the relationship to being monotonic. Secondly, we are studying periods of a few hours in an overnight session of the markets. We sample prices every minute. This leads to a $T$ being of the order of hundreds. As convergence of $\hat{g}(\cdot)$ is of the order of $T^{1/4}$ we would not expect convergence to apply. It does not appear that the application of non-parametric methods is suitable for this paper.

Luckily though the pricing framework developed in our model does provide a straight-forward way to proceed in the linear setting with minimal restrictions. An assumption of standard mean variance risk preferences yields the convenient form for $g(\cdot)$ of

$$g(PB_t) = p_0 + \Delta p.PB_t + \lambda R_t$$

(4.1)

$$R_t = PB_t(1 - PB_t).$$

$R_t$ represents the non-stationary measure of political risk and is proportional to the variance of the Bernoulli variable $E$ representing the political event. $g(\cdot)$ is now linear in $PB_t$ and $R_t$. We can also recover this form of the cointegrating relationship using a second order quadratic approximation to $g(\cdot)$, making use of the fact that once the event has been realised ($PB_t = 0$ or $1$) there is no longer political risk and the contribution to the relationship must vanish. Using this form of the relationship means we can now exploit tests within the well developed field of linear cointegration theory. Testing for significant risk preferences (and non-linearity in $PB_t$) is equivalent to testing whether $\lambda$ is significant.

We note that $R_t$ is a quadratic function of $P_t$. This means that the Granger representation theorem does not apply and so the Johansen framework is not strictly valid. There are methods in the literature that yield specification tests for non-stationary quadratic regression models (see Wagner and Hong, 2016). The application of such a test is beyond the scope of this paper and we will rely on tests solely within the linear setting. Furthermore there is no test that has a null of no-cointegration we can find.

We argued earlier that $P_t$ behaves as an $I(0)$ process until a value of zero or unity is achieved (at the point at which the political event is resolved). $R_t$ will also not be stationary although not necessarily $I(1)$. We do test both $R_t$ and its first difference $\Delta R_t$
with both the Philips–Perron and KPSS tests. For all cases presented in this paper, test results for the risk variable \( R_t \) are consistent with an \( I(1) \) variable.

Using the linear framework also allows us to easily deal with the monotonic restriction, \( |\lambda| \leq |\Delta p| \). When the estimate of 4.1 yields a non-monotonic result with \( |\hat{\lambda}| > |\hat{\Delta p}| \) we can simply re-run the regression with the non-stationary explanatory variables \( PB_t \) and \( R_t \) replaced with a single explanatory variable. This will be either \( PB_t + R_t \) or \( PB_t - R_t \) depending on whether \( PB_t \) and \( R_T \) have the same sign. This variable is monotonic and recovers the estimate at the boundary where \( |\lambda| = |\Delta p| \).

### 4.3 Statistical tests

This paper will make use of several statistical tests derived from the three linear cointegration frameworks. They are set out as follows.

#### 4.3.1 Phillips and Ouliaris test for the presence of cointegration

We apply the Engle–Grainger methodology via univariate regressions of each price against that of the betting market contract

\[
p_t = p_0 + PB_t \times \Delta p + \epsilon_t.
\]

The presence of a unit root in \( \epsilon_t \) is tested via a residual regression. However, we use the Philips–Perron test statistics, \( Z_\alpha \) and \( Z_t \) rather than Dickey–Fuller. These include a non-parametric adjustment to the long run variance which is robust to misspecified serial correlation and heteroskedasticity. If the statistic is below the critical value we reject the null of a unit root in the residual and conclude that cointegration is present between the asset price and the betting contract.

#### 4.3.2 Johansen max eigenvalue test

The Johansen methodology considers the error-correcting form for our multivariate price process \( p_t \)

\[
\Delta p_t = \alpha((\beta' p_{t-1} - E(\beta' p_{t-1})) + \sum_{i=1}^{k-1} \Gamma_i \Delta p_{t-i} + u_t.
\]

This specification includes lags of \( \Delta p_t \). The cointegration rank is the rank of \( \beta \) and models of smaller rank are sequentially nested in models of higher rank. Johansen demonstrated that the Gaussian Maximum Likelihood for a given rank \( r \) is a simple expression based on the smallest \( r \) eigenvalues of the canonical correlation matrix relating to a reduced
rank regression of the above equation. The Likelihood Ratio depends on the \( N - r \) largest eigenvalues. There are two forms of rank test, both where the null hypothesis is cointegration of rank \( r \leq r^* \). In the trace test the alternative is that cointegration is full rank. This is immediately discounted in our application as it is equivalent to all prices being stationary. The second form, the max eigenvalue test, has the alternative higher rank \( r = r^* + 1 \). This test is based on \( \lambda_{r+1} \), the \((r + 1)^{th}\) smallest eigenvalue. The asymptotic distribution of the test statistic is non-standard and depends on Brownian motion. It has been tabulated in Johansen and Juselius (1990). Rejection of rank \( r = r^* \) occurs when \( \lambda_{r+1} \) is “large”. Care will be required when choosing the lag length \( p \) as tests will be biased and inconsistent if the model is misspecified.

4.3.3 Johansen constraint test

Likelihood Ratio tests can be formulated for restrictions on model parameters in the Johansen framework. We use this to test whether risk aversion is significant in equation 3.11 (i.e. \( H_0 : \lambda = 0 \)). This is done by considering the trivariate prices

\[
x_t = (p_t, P^t, R_t)'
\]

and checking whether the coefficient in the last dimension (the dimension of risk) are not significantly different from zero.

4.3.4 Nyblom and Harvey common trends test

Nyblom and Harvey (2000) work in the unobserved component framework and consider the multivariate local level model

\[
\begin{align*}
p_t & = \mu_t + \epsilon_t, \quad \epsilon_t \sim IID(0, \Sigma_{\epsilon}) \\
\mu_t & = \mu_{t-1} + \eta_t, \quad \eta_t \sim IID(0, \Sigma_{\eta}).
\end{align*}
\]

\( \Sigma_{\eta} \) is the variance of the multivariate disturbance driving the unobserved random walks. Cointegrating vectors \( \beta \) satisfy \( \Sigma_{\eta}\beta' = 0 \). The rank of \( \Sigma_{\eta} \) is equal to the number of common trends. Tests for a given number of common trends \( k^* \) are proposed with alternatives of higher rank. Non-parametric corrections that are robust to serial correlation and heteroskedasticity are included. Test statistics are now based on the sum of the \( N - K^* \) smallest eigenvalues of a matrix. When this statistic is large we reject \( k = k^* \) in favour of \( k > k^* \). The theoretical model presented in the previous section assumes \( k = 1 \). As before, we discount the situation \( k = 0 \) (stationary prices) but can test \( k = 1 \)
against \( k > 1 \). Rejection of this null would be a rejection of our theory. Failure to reject is consistent with our theory but would give less support than a rejection of cointegration rank, \( r = N - 2 \) in favour of \( r = N - 1 \), under the max eigenvalue test.

### A note on lag specification

The above frameworks require lag length choices to be made when specifying tests. For both the Phillips and Ouliaris residual regressions and the common trends test, nonparametric adjustments are included that are robust to misspecified serial correlation in the disturbances. All that is required for these methods is to choose a lag truncation parameter in the estimator of the long-run variance. We use the Newey and West (1994) plug-in procedure of \( 4 \left( \frac{T}{100} \right)^{2/9} \). The Johansen rank and constraint tests are not robust to model misspecification although we note that there is some evidence that testing constraints are not unduly effected in the presence of serial correlation, Silvapulle and Podivinsky (2000). Schwert (1989) suggests choosing the lag length that minimises the AIC or BIC up to a maximum lag of \( 12 \left( \frac{T}{100} \right)^{1/4} \). The choice of lag involves a trade off between accuracy of specification and statistical power. Models of higher numbers of lags and hence parameters will necessarily be more accurate (as shorter lag lengths are nested) but will result in a loss of statistical power and we note that cointegration tests often do lack power. We start by only adding lags if they result in a statistically significantly different model, as implied by a likelihood ratio test. Lags are added to the base (zero lag) model until the likelihood ratio test fails to reject the restricted model. As the max eigenvalue test is not robust in the presence of serial correlation, we check that the residuals are indeed not correlated (according to a Portmanteau test). If they are, then further lags are added until serial correlation is no longer present.

### 5 Results

In this section we evaluate our theory on real world data from some political events from the last few years. The choice of events is important. For our model to apply, the results of the elections need to come during overnight hours. This rules out, for instance, the 2020 US presidential election, where various legal challenges and recounts in several states took days and even weeks to resolve. Our model also has no applicability if the result is realised instantaneously. We require a meaningful period of time for the information flow to occur. This rules out UK General Elections. This is due to the very high accuracy of the exit poll. These polls measure how people declare they have voted on the day itself, at a selection of particular, secret, polling stations. They are much more accurate than any pre-election polling (Curtice et al. (2011)), due to the fact that there is no measurement error of respondents. The polls are released just after votes closed and are effectively an
announcement of the winner. Instead we chose three elections where we believe our model is likely to apply. In each case we consider an actively traded binary betting contract along with a collection of heavily traded financial assets. We begin our investigation with the first great political shock of 2016. This was the United Kingdom European Union membership referendum, commonly referred to as the Brexit referendum.

5.1 The Brexit referendum

On 23rd June 2016 the UK voted in a country wide referendum to leave the European Union. This was one of only three UK wide referendums. The first was in 1975 and involved a vote to join the European Community. This is known as the common market and is what became the European Union. The second was the United Kingdom Alternative Vote referendum which was rejected by a wide margin. The third was the Brexit plebiscite. Turnout was historically high at 72% and the result was narrow: 51.9% to leave the EU versus 48.1% to stay.

The vote was split up into a large number of voting areas (382) and each area announced at different times throughout the night as their counts were finalised. The result

Figure 1: Betfair contract for ‘Remain’ on the night of the Brexit referendum.
was unexpected. There was widespread polling data that showed a small but consistent lead to stay in the European Union (‘Remain’). For example, a poll was conducted by YouGov which was published shortly after voting closed at 10pm (YouGov (2016)). This showed the vote-share for leave at 48.4% with a standard sampling error of 3%.

There was an actively traded political market that traded both up to the day of voting and overnight as results were announced. Contracts that paid out £1 in the event of both ‘Remain’ and ‘Leave’ were listed on the Betfair Exchange market. This operates like a limit order book. There did not appear arbitrage opportunities in the exchange in that the sum of the prices of the contracts do not deviate sufficiently from £1.\(^7\) Around £130m was wagered in total with £50m changing hands on the night.

There was widespread belief that the country would vote to remain in the EU. Figure 1 shows the price action of the contract price for Remain. Voting closed at 22:00 on 23rd June.\(^8\) The YouGov poll on the day was released shortly afterwards. The first result released was for Gibraltar around 23:36. This was inconsequential. Gibraltar is an overseas territory located at the southern tip of the Iberian Peninsula bordering Spain. As expected, the electorate there voted overwhelmingly in favour of Remain (96%). As can be seen, this did not affect the prices in the betting market. The risk neutral probability of remaining in the EU seen close to 90%. Meaningful results started to be announced from midnight with Newcastle upon Tyne being the first to announce. As can be seen from Figure 1, prices had already started to move a little against Remain from 23:45, possibly due to information leakage, or to private polling conducted by some hedge funds.

There was a large quick move from 73% to 61% for Remain between 00:16 and 00:18 as Sunderland announced. The result there showed a lead for Brexit over Remain of 21% versus an expectation of around 6%. Price action expectations moved against Remain for the next few hours, with a particular collapse around 2am which mostly recovered shortly afterwards. However, by 4am Remain was trading at under 10% and a probability for Brexit of 99% was implied at 05:21. This BBC finally projected Brexit at 5:39am and there was no doubt that the country had voted to leave the European Union.

5.1.1 Financial assets

We consider five financial assets for this event. Two currencies, one stock index future, one commodities future and one fixed income future. All asset classes are covered by this collection and all but GBPUSD was the most liquid leading indicator of those asset classes. GBPUSD is included as it is especially relevant for the UK and a country-specific indicator of the health of the economy. All futures were listed on the Chicago Mercantile Exchange.

\(^7\)Owning both one contract for Remain and one for Leave guarantees a payout of £1.

\(^8\)Times in this section are all quoted in British Summer Time. This was the current timezone in the UK on this date.
Note: Assets that depreciated are shown versus the Remain contract price whereas the asset that appreciated is shown versus the contract price for Brexit.

Figure 2: Rebased financial asset prices versus Betfair contracts.
Exchange. Foreign exchange and futures are studied as these were open for trading during the results session. Cash markets do not generally trade overnight and the UK specific FTSE100 future only opened part way through the night and so is excluded. We consider the period 23:00 BST on 23rd June to 05:30 on 24th June and sample prices every minute. During these hours all contracts are open, the only information released to the market is that contained in the vote results and the betting markets converge to certainty.

Table 2 lists the five assets with a description, their starting and ending prices and the percentage change. As can be seen four assets depreciated whilst the treasury future appreciated. This is as expected as the result was not expected and was considered negative for trade and hence the economy. Generally fixed income assets appreciate in times of economic uncertainty as both expectations of future interest rates fall and money moves out of risky assets. The Japanese yen is considered a safe asset so appreciated against the USD whilst GBP is seen as risky versus the USD and depreciated.

Figure 2 plots assets that depreciated against the betting contract for Remain and the treasury Future is plotted against the contract for Brexit. The financial assets do appear to be moving more or less in lock-step with the betting markets. This is pleasing as this is implied by our theory. However, we now turn to statistical tests of the theory in the following sections.

### 5.1.2 Evidence for cointegration

We first test the cointegration of each asset price with the betting contract for Brexit. Table 3 shows the results of the Phillips–Ouliaris $Z_t$ and $Z_α$ tests for cointegration. Both tests reject the null of a unit root in the residual $ε_t$ at the 99% level in favour of stationarity. The results of the Johansen max eigenvalue test are shown Table 4. The maximum lag length $k − 1$ was chosen using the procedure outlined in subsection 4.3. This test also rejects a cointegration rank of 0 in favour of a single common trend for all assets, albeit at lower significance levels for the S&P500 and crude oil futures prices.

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9Note, the E-mini S&P500 Future hit a trading limit shortly after 05:30 on 24th and was thus put into auction.

10The futures were closed for an hour between 22:00 and 23:00 British Summer Time. This was straight after the vote closed but before results were announced.
Table 3: Results of the Phillips–Ouliaris $Z_t$ and $Z_\alpha$ tests for cointegration.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\hat{p}_0$</th>
<th>$\Delta p$</th>
<th>$p_{Z_t}(a = 0)$</th>
<th>$p_{Z_\alpha}(a = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>2134.0</td>
<td>-115.6</td>
<td>0.008***</td>
<td>0.003***</td>
</tr>
<tr>
<td>ZNU6</td>
<td>130.32</td>
<td>2.97</td>
<td>0.001***</td>
<td>0.001***</td>
</tr>
<tr>
<td>CLU6</td>
<td>51.38</td>
<td>-3.37</td>
<td>0.006***</td>
<td>0.004***</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>1.5199</td>
<td>-0.1751</td>
<td>0.001***</td>
<td>0.001***</td>
</tr>
<tr>
<td>USDJPY</td>
<td>107.28</td>
<td>-6.15</td>
<td>0.001***</td>
<td>0.001***</td>
</tr>
</tbody>
</table>

The first stage regression is $p_t = \hat{p}_0 + \Delta p PB + \epsilon_t$

The residual regression is $\hat{\epsilon}_t = a\hat{\epsilon}_{t-1} + \eta_t$

Table 4: Results of the bivariate Johansen max eigenvalue test.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\hat{p}_0$</th>
<th>$\Delta p$</th>
<th>$k - 1$</th>
<th>$p(r = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>2134.0</td>
<td>-121.9</td>
<td>1</td>
<td>0.024**</td>
</tr>
<tr>
<td>ZNU6</td>
<td>130.32</td>
<td>3.00</td>
<td>1</td>
<td>0.001***</td>
</tr>
<tr>
<td>CLU6</td>
<td>51.34</td>
<td>-3.48</td>
<td>2</td>
<td>0.090*</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>1.5188</td>
<td>-0.1784</td>
<td>2</td>
<td>0.003***</td>
</tr>
<tr>
<td>USDJPY</td>
<td>107.24</td>
<td>-6.22</td>
<td>0</td>
<td>0.001***</td>
</tr>
</tbody>
</table>

The regression is $\Delta p_t = \alpha(\beta' p_{t-1} - c_0) + \sum_{i=1}^{k-1} \Gamma_i \Delta p_{t-i} + u_t$

$r = \text{rank}(\hat{\beta})$, when $r = 1$, $\beta' = (1, -\Delta p)$, $c_0 = -\hat{p}_0$

Table 5: Results of the multivariate Johansen max eigenvalue test.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$h$</th>
<th>stat</th>
<th>cValue</th>
<th>eigVal</th>
<th>pValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>43.32</td>
<td>40.96</td>
<td>0.106</td>
<td>0.027**</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>36.13</td>
<td>34.81</td>
<td>0.089</td>
<td>0.035**</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>31.15</td>
<td>28.59</td>
<td>0.077</td>
<td>0.023**</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>17.77</td>
<td>22.30</td>
<td>0.045</td>
<td>0.191</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>7.92</td>
<td>15.89</td>
<td>0.020</td>
<td>0.593</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>3.73</td>
<td>9.16</td>
<td>0.010</td>
<td>0.523</td>
</tr>
</tbody>
</table>

The regression is $\Delta p_t = \alpha(\beta' p_{t-1} - c_0) + \sum_{i=1}^{k-1} \Gamma_i \Delta p_{t-i} + u_t$

$r = \text{rank}(\hat{\beta})$
During a typical trading day there will be information that would persistently affect the financial assets over and above that which affects the likelihood of the UK voting to leave the European Union. The fact that the null hypothesis of no cointegration is firmly rejected in favour of a single common trend in the hours after the Brexit referendum is strong evidence in favour of our theory.

We consider next the 6-dimensional multivariate price process $p_t$ which includes all five financial assets and the betting contract. Our pairwise cointegration tests show strong evidence of each financial asset being cointegrated with the betting contract. Within the space of time series, cointegration is a transitive relation. That is, if $x_t$ is cointegrated with $y_t$ and $y_t$ is cointegrated with $z_t$, then $x_t$ is cointegrated with $z_t$. If every financial asset is cointegrated with the betting market, then every financial asset is pairwise cointegrated, cointegration is of rank $n - 1$ and there is a single common trend. This is implied by our theory. We test this proposal directly using the Johansen max eigenvalue test. Using the procedure outlined in subsection 4.3 we chose a lag length in the error correction model of 3. Results of the tests for various cointegration ranks are shown in Table 5. Cointegration ranks of 0, 1 and 2 are rejected in favour of higher ranks at the 95% level. This suggests that the rank is at least 3. Our theory predicts a rank of 5. The failure to reject ranks 3 and 4 in favour of rank 5 is a rejection of our theory. It may be the case that our model does not sufficiently describe the behaviour observed after the Brexit referendum. However, it may also be the case that the Johansen’s test may also have insufficient power to reject ranks of 3 and 4. As another check, we compute the test statistic used in the Nyblom and Harvey test for a single common trend. If the statistic is above a critical level the single common trend is rejected in favour of a higher number of common trends. The statistic is 0.0045. This is well below the level of rejection at the 90% level. Thus the null hypothesis, of a single common trend and cointegration rank of $n - 1$, is not rejected at any meaningful significance level. Although it would be preferable for the Johansen test to reject cointegration ranks of 3 and 4 in favour of a single common trend, the results of this test taken together with the pairwise tests and the common trends test is strong evidence that the pricing theory outlined in section 3 held during the night after the Brexit referendum.

5.1.3 Risk Aversion

Next we turn to the question of risk aversion. Manasse et al. (2020) derive a model of foreign exchange prices and the Betfair contracts for the months before the referendum. This is based on cointegration and includes a component of risk aversion. The results

\footnote{Nyblom and Harvey (2000) only compute critical values for $n \leq 4$. However, we can avoid the simulation of critical values here. We note that critical values increase with $n$. The published value of 0.427 for the 90% when $n = 4$ is a lower bound for higher $n > 4.$}
Table 6: Results of the trivariate Johansen constraint test for risk aversion.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\hat{\lambda}$</th>
<th>$k - 1$</th>
<th>$p(\lambda = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>-2.165</td>
<td>0</td>
<td>0.913</td>
</tr>
<tr>
<td>ZNU6</td>
<td>-0.442</td>
<td>3</td>
<td>0.084*</td>
</tr>
<tr>
<td>CLU6</td>
<td>-0.718</td>
<td>0</td>
<td>0.294</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>0.018</td>
<td>2</td>
<td>0.508</td>
</tr>
<tr>
<td>USDJPY</td>
<td>1.274</td>
<td>1</td>
<td>0.237</td>
</tr>
</tbody>
</table>

The regression is $\Delta p_t = \alpha (\beta' p_{t-1} - c_0) + \sum_{i=1}^{k-1} \Gamma_i \Delta p_{t-i} + u_t$ with $p_t = (p_t, PB_t, R_t)'$. As described in the theoretical model, the third component in $x_t$ relates to risk, and is identical to that considered in Manasse et al. (2020). With or without risk aversion, our theory implies a single cointegrating vector $\beta' = (1, -\Delta p, -\lambda)$ and two common trends $PB_t$ and $R_t$. We test the restriction $\lambda = 0$ in the Johansen framework with $r = 1$ using the constraint test. The results are shown in Table 6. We do not reject the null hypothesis of risk neutrality for any asset at the 95% level and only reject the hypothesis for a single asset, the treasury future at the 90% level. The constraint test p-value for the pound of 0.51 suggests there is no evidence at all for the GBPUSD exhibiting risk aversion behaviour with regards to Brexit. This is in contrast to the conclusions of Manasse et al. (2020). However, this author takes issue with the application of a cointegration theory for longer term periods. The basis for the model is that the only determinant of the GBPUSD price in the weeks and months preceding the Brexit vote is the result of that referendum. Whilst we agree that the likelihood of the referendum result is a large determinant of prices, and is the only determinant in the overnight hours after the vote, we do not believe it is the only information affecting exchange rates for longer periods. Their theory assumes no other news beyond that relating to the referendum affects the British Pound and United States dollar exchange rate in the months up to the vote. For example, this would imply that any and all news about the health of the US economy, be it consumer demand, trade barriers, protectionism etc., would have no effect on GBPUSD. We find this implausible and postulate that including risk aversion is simply an exercise is over-fitting to avoid rejecting a model that should never have been applied on this timescale. We suggest that

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$^{12}$PBt cannot possibly be cointegrated with $R_t = PB_t(1 - PB_t)$ if it is non-stationary.
other models such as vector auto-regressive or factor models would be more economically justified for longer periods of time.

Table 7: Fitted short-run parameters in the error correction model for Brexit, with Granger causality tests.

Short-Run Regression Results

\[
\Gamma_1 = 10^{-3} \times \begin{pmatrix}
-190^{**} & - & -3860^{***} & 13000^{***} & - & -20600^{***} \\
-3.44^{***} & - & 441^{*} & - & - & 185^{*} \\
6.11^{***} & -0.312^{***} & -159^{***} & -2890^{**} & - & -28.9^{***} \\
0.14^{*} & - & - & -105^{*} & - & - \\
11.4^{**} & - & - & 6720^{*} & -116^{**} & -1640^{**} \\
- & - & - & -452^{*} & - & -
\end{pmatrix}
\]

Based on \( \Delta p_t = \alpha(\beta' p_{t-1} - c_0) + \Gamma_1 \Delta p_{t-1} + u_t \)

\( p_t = (\text{ESU6}, \text{ZNU6}, \text{CLU6}, \text{GBPUSD}, \text{USDJPY}, PB_t)' \)

1 Statistically insignificant coefficients omitted

Null Hypothesis | LR-Test | Chi-square | Prob
\hline
(ESU6, ZNU6, CLU6, GBPUSD, USDJPY)' does not Granger cause \( PB_t \) & 4.84 & 0.436
\( PB_t \) does not Granger cause (ESU6, ZNU6, CLU6, GBPUSD, USDJPY)' & 11.4^{**} & 0.0441

5.1.4 Deviations from long term relationships

The strong evidence for cointegration in the hours after the referendum is a pleasing result and agrees with our theoretical model. Our assumptions include that the only persistent affects on asset prices are related to the likelihood of voting to the leave the European Union. This gives rise to the single common trend. Another main assumption is that weak market efficiency holds. However, the presence of significant autoregressive terms in the error correction model of the Johansen framework suggests that efficiency may be less clear cut.

We can study short term deviations from the long term relationship in two ways. The first is via the coefficients of the ECM of the full multivariate system. The second is via the univariate errors of the cointegration regression. The full ECM is

\[
\Delta p_t = \alpha(\beta' p_{t-1} - c_0) + \sum_{i=1}^{p} \Gamma_i \Delta p_{t-i} + u_t
\]

where
It is found that an error correcting model with a single short-run autoregressive term \( \Gamma_1 \) is significantly different to one with no term but that higher numbers of lags are not significant. Table 7 shows the significant fitted values of \( \Gamma_1 \) in the ECM. Of particular interest is whether the betting markets Granger cause the financial markets and/or vice-versa. These hypotheses can be tested. Jointly testing the significance of the first \( n - 1 \) values of the final column of \( \Gamma_1 \) is a test of whether the betting markets Granger cause financial markets. Testing the first \( n - 1 \) values of the last row tests whether financial markets Granger cause betting markets. LR tests for these hypotheses are also shown in table 7. There is strong evidence that betting markets jointly Granger cause financial markets with the null of no Granger causality rejected at the 5% level. However, there is no significant causation in the other direction. This is a very interesting result. Short term deviations from the long term relationship that revert occur as the betting and
financial markets do not move exactly in lock-step. However, during the establishment and convergence of these deviations the betting markets are leading the financial markets. This suggests the betting markets are discounting political information more quickly (and on the scale of minutes) than the financial markets.

We now study inefficiencies via the simpler residuals of the univariate cointegrating regressions. This error is a linear combination of two asset prices (a financial asset and a betting contract) and can be traded. The return of the error can be created by holding the two assets in a ratio equal to the cointegration ratio. It is also largely risk free, at least with respect to political risk. The exposure to the result of the referendum in one asset is hedged with equal and opposite exposure in the other. Serial correlation in the cointegration error implies that deviations from the long term relationship (long term in this context being a few hours) can be predicted. This again demonstrates violations of weak market efficiency on a short term timescale.

The estimated cointegration error is \( \hat{\epsilon}_t = p_t - \hat{p}_0 - PB_t \times \hat{\Delta}p \). We consider the quantity \( \hat{\epsilon}_t/|\hat{\Delta}p| \) which is the cointegration error normalised to units of the betting contract. This can be more readily compared across different assets whose prices have different magnitudes. The normalised cointegration errors with the sample Auto-Correlation Function and robust Bartlett intervals for each asset are plotted in Figure 3.\(^\text{13}\) This shows significantly positive autocorrelations out to around 20 minutes. This provides evidence of deviations from weak market efficiency of the order of minutes to tens of minutes on an ex-post basis. We fit autoregressive models with varying numbers of lags to the estimated error \( \hat{\epsilon}_t \). Note that the constant term is fixed to zero as \( E(\hat{\epsilon}_t) = 0 \) by construction.

Results are shown in Table 8. A single lag provides a good fit to the data. However likelihood ratio tests show that two lags produces a significantly different model to ones with a single lag for the S&P500 future and the pound but that those with more lags are no different. These results contradict EMH. The possibility of profiting from them systematically is explored in the next section.

### Table 8: Estimated autoregressive models for cointegration errors.

<table>
<thead>
<tr>
<th>lag</th>
<th>ESU6</th>
<th>ZNU6</th>
<th>CLU6</th>
<th>GBPUSD</th>
<th>USDJPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \hat{\epsilon}_{t-1} )</td>
<td>0.746***</td>
<td>0.761***</td>
<td>0.897***</td>
<td>0.675***</td>
<td>0.818***</td>
</tr>
<tr>
<td></td>
<td>(0.039)</td>
<td>(0.029)</td>
<td>(0.021)</td>
<td>(0.046)</td>
<td>(0.020)</td>
</tr>
<tr>
<td>( \hat{\epsilon}_{t-2} )</td>
<td>0.166***</td>
<td></td>
<td>0.166***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td></td>
<td>(0.044)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The regression is \( \hat{\epsilon}_t = \psi_1 \hat{\epsilon}_{t-1} + \ldots + \psi_k \hat{\epsilon}_{k-1} + \eta_t \). \( \hat{\epsilon}_t \) is estimated from the first stage regression.

\(^{13}\) The Bartlett intervals are adjusted to allow for serial correlation in the variance of \( \hat{\epsilon}_t \). Such serial conditional heteroskedasticity is expected as this is a financial time series.
We now turn to the possibility of profiting from mean reverting deviations to the long term relationship. Rather than focus on trading assets outright (or “naked” in trading parlance) via predictions from an error correction model, we focus on trading the cointegration error. This is effected by buying or selling the financial asset against an opposing position in the betting contract with sizes equal in ratio to the cointegrating ratio.\textsuperscript{14} This is effectively politically risk free and involves taking positions in pairs of contracts simultaneously rather than larger numbers of assets outright which be would exposing oneself to greater risk.

We use a modification of the common Bollinger Band trading signal. When used to trade non-stationary price series, Bollinger Bands are used to produce directional trading indicators. A corridor around a moving average of the price of the asset is constructed by adding and subtracting twice the sample standard deviation, $\sigma_t$, of the price calculated along the length of the preceding moving average window. For a contrarian strategy the signal will be to sell when above the upper band and to buy when below the lower band.

\textsuperscript{14}To be clear $\Delta p$ notional of the betting contract is traded for every unit notional of financial asset exposure.
All positions are closed out once the betting contract has converged to certainty. As the cointegration error $\epsilon_t$ has zero expectation we do not calculate the corridor around a moving average but about zero. An investor may wish to be more aggressive when applying this strategy to a stationary error $\epsilon_t$ than to an unbounded non-stationary asset price. Also the theoretical model implies that any non-zero value of $\epsilon_t$ is a deviation from market efficiency and should be fleeting. As such, using a multiple of less than $2\sigma_t$ may be more appropriate in this context. For example, if the unconditional distribution of $\epsilon_t$ is normal then using two standard deviations would imply generating a trading signal, and hence an opening position, only around 4% of the time. Given there are only a few hours to trade this is very conservative. Fatter tails in the unconditional distribution are expected but a smaller corridor may still be needed to generate a reasonable amount of trades.

We first try this strategy in the most studied asset with respect to Brexit which is the pound. When $\hat{\epsilon}_t$ is above the upper band we sell and buy when below the lower band. Both long and short positions are closed out when $\hat{\epsilon}_t$ has converged to zero, the expected equilibrium level. Note that selling the error is equivalent to selling the pound and buying the betting contract for Remain (or selling the contract for Leave) in the cointegrating ratio. At time $t$ where $t >$ midnight, the period from 11pm, 23rd June to $t$ is used to calculate the sample standard deviation $\hat{\sigma}_t = \hat{\epsilon}_t/|\Delta p|$, Bollinger Bands and trades for both $2\sigma$ and $1.5\sigma$ strategies are shown in Figure 4.

The Bollinger strategy appears excellent. The $2\sigma$ signals generate seven trades, six of them winning, with a total gross profit of 52p for every £1 Betfair contract traded. This is impressive as the total move in the contract is only around 90p in the whole night. The strategy is able to capture a large amount of the entire overnight move in the betting contract without apparently taking political risk. The $1.5\sigma$ strategy generates 10 trades (9 winning) of lower average profit but a greater total gross profit of 94p for every £1 of Betfair contract. Trading costs for these markets are relatively small. They are well below the order of gross profits and so net profits will still be significant. Figure 5 shows the $1.5\sigma$ and $2\sigma$ strategies applied to all assets. Results are presented in table 9. Again the strategies appear excellent. Across all assets the $1.5\sigma_t$ strategy has 57 trades, 55 winning, with average profit of 7.8p for every contract traded on Betfair. The $2\sigma$ strategy has 36 winning out of 38 total trades with an average profit per trade of 10p. Trading costs and slippage should not be above 1p per contract so these profits appear real.

\[15\] In terms of transaction costs, selling the pound would cost about 2–3 hundredths of a cent at that time, whereas the Betfair cost is 3–5% levied on any bets that pay out. This would slightly change the ratio of the portfolio but not significantly affect profits or these conclusions.
Figure 5: Bollinger Band trading strategies for Brexit.
Table 9: Bollinger Band gross trading profits for Brexit.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>1.5σ</th>
<th>2σ</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>nWin</td>
<td>Profit</td>
</tr>
<tr>
<td></td>
<td>nTrade</td>
<td>Profit</td>
</tr>
<tr>
<td>ESU6</td>
<td>11/12</td>
<td>70.7p</td>
</tr>
<tr>
<td>ZNU6</td>
<td>19/19</td>
<td>104.9p</td>
</tr>
<tr>
<td>CLU6</td>
<td>8/8</td>
<td>74.4p</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>9/10</td>
<td>66.2p</td>
</tr>
<tr>
<td>USDJPY</td>
<td>8/8</td>
<td>127.1p</td>
</tr>
<tr>
<td>Total</td>
<td>55/57</td>
<td>443.3p</td>
</tr>
</tbody>
</table>

*aProfits are shown for every £1 contract traded on Betfair

Unfortunately, being able to apply this strategy ex-ante is not at all realistic. Firstly the trader would need to be confident weak market efficiency would ultimately hold. Secondly, and more importantly, the calculation of $\hat{\epsilon}_t$ uses the cointegrating relationship estimated ex-post using data from the whole night. Ex-ante, a successful investor would have to correctly foresee the cointegrating relationship. This is equivalent to knowing the conditional expectations $GBP_L = E(GBP_T|\text{BREXIT})$ and $GBP_H = E(GBP_T|\text{BREXIT})$, ie where the pound settles given a vote to leave the EU (and the price it would have achieved in the counterfactual remain scenario).

We investigate an application of the same strategy when the cointegrating relationship is informed by forecasts of the conditional expectation from a market commentator. Prior to the referendum in late April the investment bank JPMorgan published a forecast of precisely where the pound would be priced given a vote to leave the EU. This was 1.32 (Peters, 2016). If a trader on the night used this value for $GBP_L$ and assumed that prices were efficient at 11pm then she would evaluate $GBP_H$ at 1.522 and $\hat{\Delta}p = GBP_L - GBP_H = -0.2024$. This is a larger predicted drop than estimated from the ex-post regression. Simply put, JP Morgan’s prediction was too pessimistic. The pound did not fall (in the overnight session) as much as expected given the positive referendum result for Brexit. The cointegration error estimated in this way does not converge to zero as the pound does not fall as much as expected. The misspecified error is shown on the left hand side of figure 6 with Bollinger Band strategies. The error does drift downwards. Remarkably though, and against this author’s expectations, the trading strategy is still profitable, albeit less so. The error is biased downwards and so only 16This is because at 11pm GBPUSD was at 1.5007 and the betting contract implied a 10.7% chance of leaving the EU. If 100% chance of leaving the EU results in $GBP = 1.32$ then this implies that a zero chance implies $GBP_H = (1.5007 - 0.107 \times 1.32)/0.893 = 1.5224$. Note this is similar to $\hat{p}_c$ calculated from the ex-post cointegrating relationship shown in Table 3.
“buys” are executed. It appears that JP Morgan’s conditional estimate for the pound, albeit a little pessimistic, was close enough to allow a profitable strategy. Although biased, there does seem to be some mean reversion of this estimated cointegration error. The conservative natures of the Bollinger Band signals avoid losing money. However, it is not the case that conditional predictions for the other financial assets are available and so this strategy does not appear readily applicable to the other financial assets.

Bollinger Bands strategies for misspecified cointegration error when using $E(GBP_T|BREXIT) = 1.32$ (left) and when using the first part of the night to estimate (right).

Figure 6: Possible ex-ante estimated cointegration errors and Bollinger Band strategies.

There does seem to be one way in which ex-ante profits may have been possible for the other symbols without the foresight of conditional predictions of the asset prices. This is by using the first few hours of the night to estimate the cointegrating relationship and trading in the latter part of the night. This actually does result in profits. An investor has the most chance of estimating the cointegrating relationship if there is sufficient support of the relationship within the training set. However, waiting for too much training data reduces the window of opportunity. We settle on using data up to the point where the betting market predicts a 50% chance of Brexit. This occurs at 2:02am. The strategy is shown on the right hand side of Figure 6. The error does appear to converge to zero in the trading period. There are five trades with all but the final trade profitable. We note that the final trade also lost money when trading using the ex-post estimated relationship. It appears that the long term relationship estimated from the first few hours of the night is stable and persists into the later hours of the night.

Turning now to the other assets, we first check the cointegration tests for the shorter training period. The relevant p-values are shown in table 10. Naturally these tests have less power with less data. Nevertheless, four out of the five assets have results which reject the null of no cointegration in favour of cointegration at high significance levels. The exception is the crude oil future. We note that the significance was less for this
Table 10: Cointegration tests for the smaller training period.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>( p_{Z_t}(a = 0) )(^a)</th>
<th>( p_{Z_t}(a = 0) )(^a)</th>
<th>( p(r = 0) )(^b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>0.026**</td>
<td>0.007***</td>
<td>0.004***</td>
</tr>
<tr>
<td>ZNU6</td>
<td>0.059*</td>
<td>0.014**</td>
<td>0.001***</td>
</tr>
<tr>
<td>CLU6</td>
<td>0.344</td>
<td>0.356</td>
<td>0.423</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>0.115</td>
<td>0.027**</td>
<td>0.057*</td>
</tr>
<tr>
<td>USDJPY</td>
<td>0.011**</td>
<td>0.003***</td>
<td>0.008***</td>
</tr>
</tbody>
</table>

\(^a\)Phillips–Ouliaris tests.
\(^b\)Johansen max eigenvalue test.

Table 11: Ex-ante Bollinger Band gross trading profits for Brexit.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>( n\text{Win} )</th>
<th>( n\text{Trade} )</th>
<th>Profit(^a)</th>
<th>( n\text{Win} )</th>
<th>( n\text{Trade} )</th>
<th>Profit(^a)</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>0/1</td>
<td>-40.4p</td>
<td>-40.4p</td>
<td>0/1</td>
<td>-37.8p</td>
<td>-37.8p</td>
</tr>
<tr>
<td>ZNU6</td>
<td>10/10</td>
<td>58.0p</td>
<td>5.8p</td>
<td>7/7</td>
<td>46.8p</td>
<td>6.7p</td>
</tr>
<tr>
<td>CLU6</td>
<td>7/7</td>
<td>64.5p</td>
<td>9.2p</td>
<td>5/5</td>
<td>62.2p</td>
<td>12.4p</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>9/10</td>
<td>56.8p</td>
<td>5.7p</td>
<td>4/5</td>
<td>29.3p</td>
<td>5.9p</td>
</tr>
<tr>
<td>USDJPY</td>
<td>6/6</td>
<td>106.4p</td>
<td>17.7p</td>
<td>5/5</td>
<td>100.1p</td>
<td>20.0p</td>
</tr>
<tr>
<td>Total</td>
<td>32/34</td>
<td>245.3p</td>
<td>7.2p</td>
<td>21/23</td>
<td>200.6p</td>
<td>8.7p</td>
</tr>
</tbody>
</table>

\(^a\)Profits are shown for every £1 contract traded on Betfair

asset when testing on the whole night and conclude that the smaller data-set generates insufficient power to reject the null. The fact that there is strong evidence of cointegration for four of the assets in this smaller training period is pleasing.

The results of applying the Bollinger Band strategy to all assets is illustrated in Figure 7. Gross profits are shown in Table 11. For four of the assets the strategy is impressive. There are less trades due to the smaller trading period, yet profits per trade are similar to those using the ex-post cointegration relationship. The exception is the S&P500 future. A single trade is executed which unfortunately loses around 40p. This occurs as the estimated error does not converge. The cointegration relationship does not appear stable for this asset. Nevertheless, the losses of this trade are outweighed by the profits in the other contracts.

These trading results are quite remarkable. Starting from three reasonable assumptions we wrote down a theory of asset prices which implied cointegration. We find that there is, for most assets, strong evidence of that theory generated in the first part of the night. Moreover the estimated cointegration relationship is so stable that trading profits
Figure 7: Ex-Ante Bollinger Band trading strategies for Brexit.
can, apparently for most assets, be generated by taking positions against deviations from those long-term relationships. Whether or not they would be realistically crystallised by an intelligent investor is debatable. We have written down a theory after the fact. Evidence does appear to have quickly emerged on the night that the theory holds. However, to execute the trades described in this paper would require a market participant to have confidence that the theory would indeed continue to apply for the remaining hours. It would take a brave soul to do so. It is not the case that there have been large numbers of referendums or political events where our approach has been shown to work\textsuperscript{17}. Whether or not these profits are realistic in practice is a philosophical point. One thing is certain though, and that is that it is far easier for me to write this apparently successful study with hindsight after the fact than actually risk my money upfront on the night!

5.2 The 2016 United States presidential election

We next study the second great political shock of 2016, the US presidential election, held on Tuesday 8th November. The Republican ticket of Donald Trump and Mike Pence, against expectations, beat the Democratic ticket of Hilary Clinton and Tim Kaine.

Under the Electoral College system the winner needs at least 270 of the 538 electors. There are 51 voting areas, 50 states plus the special federal district of Washington D.C., that each award electoral votes. Electors, for the most part, vote for the winners of the popular vote within their respective area\textsuperscript{18}. The democratic candidate led in the vast majority of nationwide and swing-state polls. However the margin decreased as the election was approached. On election day Donald Trump out-performed his polls, winning all of the key battleground states of Florida, North Carolina, Ohio and Iowa. Additionally, and against all expectations, he took the three formerly Democrat “rust-belt” states of Pennsylvania, Michigan and Wisconsin. The Republican ticket’s votes were exceptionally well distributed. Donald Trump won 30 states with 306 electoral votes whereas Hillary Clinton won 20 states with 232 votes.\textsuperscript{19} This is despite Trump garnering 2.87 million less votes than Hilary Clinton.

Similar to the Brexit referendum there were multiple vote counts (51 versus 382) from different regions occurring throughout the night after voting ended. The situation is complicated further by polls closing at different times in different states. However, evolving vote counts were published in real time on all the major news networks as well as the Internet. As with the Brexit referendum the only information affecting the market

\textsuperscript{17}Arguably, if there had been a history of success for this strategy, then the opportunity would have disappeared due to the actions of arbitrageurs.

\textsuperscript{18}Exceptions include Maine and Nebraska where electors are allocated based on a combination of the plurality of votes as well as the popular winner in each of their congressional districts. There are also typically a handful of “faithless electors” in each election who chose to vote against the candidate for whom they had pledged to vote.

\textsuperscript{19}There were seven faithless electors in total; five defections from Clinton and two from Trump.
that night was the vote counts and results. Similar to Brexit there were also various betting markets open and trading. Bets that paid out in the event of either a Trump or a Clinton win were widely traded. Figure 8 shows the price series for the Betfair contract that pays out £1 in the event of a Republican win in Greenwich Mean Time (GMT).\(^{20}\) Between midnight and 1:00am the risk neutral odds of a Trump win varied between 10% and 20%. However, by 1:30am GMT (20:30 EST) the count of the crucial swing state of Florida was almost completed and showed a lead for Trump of 0.7% versus an expectation that Clinton would win by 0.6%. From this point on the odds for a Trump win improved as various other counts showed Trump consistently out-performing his polling. By a little after 4am the betting markets implied a Trump win with 95% likelihood which slowly increased to 98% by 6am. At 7:50am Donald Trump made his victory speech.

5.2.1 Financial assets

We use a similar basket of financial assets as we did with the Brexit referendum. The exception being we swap out the UK specific GBPUSD cross and include the USDMXN

\(^{20}\)GMT is 5 hours ahead of Eastern Standard Time (EST) and is the time in London on the date of the election. All times in this section are quoted in GMT.
Table 12: Financial assets and changes for the 2016 presidential election.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>( p_{t=1} )</th>
<th>( p_{t=T} )</th>
<th>( \Delta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESZ6</td>
<td>E-mini S&amp;P500 Future</td>
<td>2142.25</td>
<td>2038.500</td>
<td>-4.8%</td>
</tr>
<tr>
<td>ZNZ6</td>
<td>10-Year T-Note Future</td>
<td>129.484</td>
<td>130.6094</td>
<td>0.9%</td>
</tr>
<tr>
<td>CLZ6</td>
<td>Crude Oil Future</td>
<td>44.880</td>
<td>43.72</td>
<td>-2.6%</td>
</tr>
<tr>
<td>USDMXN</td>
<td>US Dollar Mexican Peso Cross</td>
<td>18.310</td>
<td>20.684</td>
<td>13.0%</td>
</tr>
<tr>
<td>USDJPY</td>
<td>US Dollar Japanese Yen Cross</td>
<td>104.990</td>
<td>101.789</td>
<td>-3.1%</td>
</tr>
</tbody>
</table>

exchange rate. Trump had proposed that if he won he would renegotiate or exit various trade agreements including those with Mexico. Given the dependence of Mexico’s economy on trade and exports to the US, a Trump win was seen as extremely negative for that country’s economy.

We consider the period midnight to 6:00am BST on 9th November and sample prices every minute. During these hours all contracts are open, the betting market almost converges and the only information released to the market is that contained in the vote results. We do not consider beyond 6am as this is the start of the trading day in London. Other economic news beyond the election may be released which would invalidate our model and, either way, the result had become apparent by then. Table 12 lists the five assets with a description, their starting and ending prices and the percentage change. The election of Donald Trump was a shock. Not only had he pledged to renegotiate various trade deals, reducing world trade and hence the outlook for the economy, he was widely seen as unpredictable and inconsistent. The US dollar depreciated against the safe haven Japanese yen, and the oil and stock market futures depreciated too. The treasury future appreciated as would be expected in a time of increasing risks to the US economy and the US dollar appreciated a large 13% against the Mexican peso. This is as the market re-priced the very significant risks to the Mexican economy following the Trump win.

Figure 9 plots depreciating and appreciating assets versus the Betfair contracts that pay out £1 for a Clinton and Trump win respectively. By and large the financial markets do seem to be moving together with the betting contracts. The relationship does not look quite as established as that for the Brexit referendum with some reversal of the large falls past 5am for the S&;P500 and oil futures as well as the USDJPY exchange rate. We turn to the statistical specifications and tests in the next sub-section to make robust conclusions.
Note: Assets that depreciated are shown versus the “Clinton” contract price whereas assets that appreciated are shown versus the contract price for “Trump”.

Figure 9: Rebased financial asset prices versus US presidential election contracts.
Table 13: Results of the Phillips–Ouliaris $Z_t$ and $Z_\alpha$ tests for cointegration.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\hat{p}_0$</th>
<th>$\hat{\Delta}p$</th>
<th>$p_{Z_t}(a = 0)$</th>
<th>$p_{Z_\alpha}(a = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>2152.59</td>
<td>-126.27</td>
<td>0.109</td>
<td>0.094*</td>
</tr>
<tr>
<td>ZNU6</td>
<td>129.314</td>
<td>1.673</td>
<td>0.709</td>
<td>0.671</td>
</tr>
<tr>
<td>CLU6</td>
<td>44.99</td>
<td>-1.67</td>
<td>0.497</td>
<td>0.467</td>
</tr>
<tr>
<td>USDMXN</td>
<td>18.031</td>
<td>2.751</td>
<td>0.015**</td>
<td>0.014**</td>
</tr>
<tr>
<td>USDJPY</td>
<td>105.298</td>
<td>-4.041</td>
<td>0.308</td>
<td>0.279</td>
</tr>
</tbody>
</table>

The first stage regression is $p_t = \hat{p}_0 + \hat{\Delta}pPB_t + \hat{\epsilon}_t$

The residual regression is $\hat{\epsilon}_t = a\hat{\epsilon}_{t-1} + \eta_t$

Table 14: Results of the bivariate Johansen max eigenvalue test.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\hat{\rho}_0$</th>
<th>$\hat{\Delta}p$</th>
<th>$k - 1$</th>
<th>$p(r = 0)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>2159.49</td>
<td>-126.6967</td>
<td>6</td>
<td>0.006***</td>
</tr>
<tr>
<td>ZNU6</td>
<td>129.174</td>
<td>1.5993</td>
<td>4</td>
<td>0.003***</td>
</tr>
<tr>
<td>CLU6</td>
<td>45.21</td>
<td>-1.64</td>
<td>7</td>
<td>0.028**</td>
</tr>
<tr>
<td>USDMXN</td>
<td>17.908</td>
<td>2.764</td>
<td>5</td>
<td>0.024**</td>
</tr>
<tr>
<td>USDJPY</td>
<td>105.600</td>
<td>-3.981</td>
<td>4</td>
<td>0.009***</td>
</tr>
</tbody>
</table>

The regression is $\Delta p_t = \alpha(\beta'p_{t-1} - c_0) + \sum_{i=1}^{k-1} \Gamma_i \Delta p_{t-i} + u_t$

$r = \text{rank}(\beta)$, when $r = 1$, $\beta' = (1, -\Delta p)$, $c_0 = -p_0$

Table 15: Results of the multivariate Johansen max eigenvalue test.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$h$</th>
<th>stat</th>
<th>cValue</th>
<th>eigVal</th>
<th>pValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>58.75</td>
<td>40.96</td>
<td>0.152</td>
<td>0.001***</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>27.54</td>
<td>34.81</td>
<td>0.074</td>
<td>0.311</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>18.81</td>
<td>28.59</td>
<td>0.051</td>
<td>0.538</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>12.11</td>
<td>22.30</td>
<td>0.033</td>
<td>0.659</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>6.46</td>
<td>15.89</td>
<td>0.018</td>
<td>0.742</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>2.69</td>
<td>9.16</td>
<td>0.008</td>
<td>0.679</td>
</tr>
</tbody>
</table>

The regression is $\Delta p_t = \alpha(\beta'p_{t-1} - c_0) + \sum_{i=1}^{k-1} \Gamma_i \Delta p_{t-i} + u_t$

$r = \text{rank}(\beta)$
5.2.2 Evidence for cointegration

We now consider the evidence for cointegration between the financial assets and the betting markets. Table 13 shows the results of the Phillips–Ouliaris regression tests for cointegration and Table 14 shows the results of the bivariate Johansen max eigenvalue tests. Neither the $Z_\alpha$ or $Z_t$ tests reject the null of no cointegration for assets beyond the Mexican Peso whereas the Johansen test does reject the cointegration rank of 0 for all assets at either the 95% or 99% level. There is weaker evidence for cointegration than in the Brexit case. This is because the Phillips–Ouliaris tests are robust to serial correlation and heteroskedasticity. In applying the Johansen test we do remove serial correlation from the residual through the inclusion of sufficient lags but we cannot remove conditional heteroskedasticity. Thus the Johansen test is not as robust as the Phillips–Ouliaris tests.

We conclude there is strong evidence for our model in the case of USDMXN and weak evidence for the remaining assets. Turning now to the order of cointegration, Table 15 shows the results of the max eigenvalue tests of the 6-dimensional system which includes all financial assets and a betting contract. The evidence here for our model is weak. The tests do reject a cointegration rank of zero in favour of a single cointegration vector but higher orders of cointegration are not rejected. It may be that the tests do not have sufficient power, but compared to the Brexit case where ranks of below 3 were rejected, the data do not appear to fit our model (which implies rank 5) as well for this election. The Nyblom and Harvey test for a single common trend statistic is 0.008. This is well below the level of rejection at the 90% level so the data also do not reject our model’s prediction of cointegration rank 5. We conclude that there is strong evidence that there is at least a cointegrating relationship for the Mexican Peso but that the evidence for cointegration of the other assets and hence higher orders of cointegration is mixed.

5.2.3 Risk Aversion

Our model that assumes risk neutrality implies a linear cointegrating relationship. There is weak evidence for the risk neutral model for four of the five financial assets considered. Could the variant of our model that includes risk aversion provide a better fit to the data for the US presidential election? Again we apply the Johansen constraint test to the trivariate systems $x_t = (p_t, PB_t, R_t)'$ for each financial asset $p_t$ to test this idea. Results are shown in table 16. Unlike in the Brexit case, there does appear to be evidence of non-trivial risk preferences and non-linear relationships. The coefficient relating to risk, $\lambda$, is significant and negative (as expected) for the yen and the oil future. This is interesting. It is not significant for the S&P500 future, the treasury future, or the Peso (although the linear model seems adequate to describe the USDMXN behaviour). To further explore

---

21Over 80% of Mexican exports in 2015 were to the US.
Table 16: Results of the trivariate Johansen constraint test for risk aversion.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>( \hat{\lambda} )</th>
<th>( k - 1 )</th>
<th>( p(\lambda = 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>-15.951</td>
<td>4</td>
<td>0.904</td>
</tr>
<tr>
<td>ZNU6</td>
<td>13.378</td>
<td>3</td>
<td>0.132</td>
</tr>
<tr>
<td>CLU6</td>
<td>-5.74</td>
<td>5</td>
<td>0.001***</td>
</tr>
<tr>
<td>USD MXN</td>
<td>-4.988</td>
<td>4</td>
<td>0.575</td>
</tr>
<tr>
<td>USD JPY</td>
<td>-7.481</td>
<td>4</td>
<td>0.010***</td>
</tr>
</tbody>
</table>

The regression is \( \Delta p_t = \alpha (\beta' p_{t-1} - c_0) + \sum_{i=1}^{k-1} \Gamma_i \Delta p_{t-i} + u_t \)
\( p_t = (p_t, PB_t, R_t)' \)
\( r = \text{rank}(\beta) = 1, \beta' = (1, -\Delta p, -\lambda), c_0 = -p_0 \)

Table 17: Results of the Phillips–Ouliaris \( Z_t \) and \( Z_a \) tests for cointegration. \( R_t \) included.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>( \hat{p}_0 )</th>
<th>( \hat{\Delta p} )</th>
<th>( \hat{\lambda} )</th>
<th>( pZ_t(a = 0) )</th>
<th>( pZ_a(a = 0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>2170.038</td>
<td>-135.277</td>
<td>-101.664</td>
<td>0.001***</td>
<td>0.001***</td>
</tr>
<tr>
<td>ZNU6</td>
<td>128.873</td>
<td>1.900</td>
<td>2.570</td>
<td>0.036**</td>
<td>0.026**</td>
</tr>
<tr>
<td>CLU6</td>
<td>45.50</td>
<td>-1.94</td>
<td>-3.00</td>
<td>0.010***</td>
<td>0.007***</td>
</tr>
<tr>
<td>USD MXN</td>
<td>17.817</td>
<td>2.861</td>
<td>1.252</td>
<td>0.003***</td>
<td>0.003***</td>
</tr>
<tr>
<td>USD JPY</td>
<td>106.032</td>
<td>-4.419</td>
<td>-4.274</td>
<td>0.007***</td>
<td>0.007***</td>
</tr>
<tr>
<td>ZNU6(^a)</td>
<td>128.980</td>
<td>1.863</td>
<td>1.863</td>
<td>0.065*</td>
<td>0.039**</td>
</tr>
<tr>
<td>CLU6(^a)</td>
<td>45.35</td>
<td>-1.88</td>
<td>-1.88</td>
<td>0.036**</td>
<td>0.021**</td>
</tr>
</tbody>
</table>

The first stage regression is \( p_t = \hat{p}_0 + \Delta p PB_t + \lambda R_T + \epsilon_t \)
\(^a\)The first stage regression is \( p_t = \hat{p}_0 + \Delta p [PB_t + R_t] + \epsilon_t \) so \( \hat{\lambda} = \hat{\Delta p} \)

The residual regression is \( \hat{\epsilon}_t = a_0 \epsilon_{t-1} + \eta_t \)

51
Table 18: Results of the multivariate Johansen max eigenvalue test. $R_t$ included.

<table>
<thead>
<tr>
<th>r</th>
<th>h</th>
<th>stat</th>
<th>cValue</th>
<th>eigVal</th>
<th>pValue</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>59.78</td>
<td>47.08</td>
<td>0.155</td>
<td>0.002***</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>32.91</td>
<td>40.96</td>
<td>0.088</td>
<td>0.330</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>27.75</td>
<td>34.81</td>
<td>0.075</td>
<td>0.297</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>23.79</td>
<td>28.59</td>
<td>0.065</td>
<td>0.182</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
<td>10.76</td>
<td>22.30</td>
<td>0.030</td>
<td>0.773</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>8.47</td>
<td>15.89</td>
<td>0.024</td>
<td>0.537</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
<td>3.17</td>
<td>9.16</td>
<td>0.009</td>
<td>0.608</td>
</tr>
</tbody>
</table>

The regression is $\Delta p_t = \alpha(\beta' p_{t-1} - c_0) + \sum_{i=1}^{k-1} \Gamma_i \Delta p_{t-i} + u_t$

$r = \text{rank}(\beta)$

the possibility of non-linear cointegration we re-run the Phillips–Ouliaris tests with the risk factor $PB_t, (1 - PB_t)$ included as an explanatory variable in the regression. Results are shown in Table 17. The results are striking. The null of no cointegration is now rejected at the 99% level for four assets and at the 95% level for the treasury future. We also note that for the treasury and oil futures $|\lambda| > |\Delta p|$ which violate our model. A coefficient of risk with larger magnitude than $\Delta p$ implies a non-monotonic utility function which decreases at an end point. This is likely due to interpolation to the slight pull back in the asset prices observed between 5am and 6am. We re-run the regression using the single non-linear monotonic explanatory variable $PB_t + R_t$. This also results in the null of no cointegration being rejected for both assets.

The signs of the estimated risk parameters are encouraging. For all “risk” assets that depreciated on the shock Trump win the parameters are negative, implying risk aversion. For the asset that appreciated, ZNU6, the sign is positive, indicating risk loving behaviour. This is discussed further below.

According to the Johansen test the coefficient is significant for the oil future and the dollar denominated in yen. The sign is negative implying risk aversion. To be clear, the Japanese yen is considered a “risk-off” asset. The US dollar denominated in yen is “risky” and depreciated 3.1% overnight on the shock result that Donald Trump was elected. An explanation for this behaviour is that speculators will borrow in the lower yielding yen to finance investment in assets of higher yielding currencies when risk appetite increases.\textsuperscript{22} The coefficient in the Phillips–Ouliaris regression for USDMXN is positive. This is consistent with risk aversion, as this is the price of the dollar denominated in the risky asset, the Peso. A positive value of $\lambda$ for USDMXN is equivalent to a

\textsuperscript{22} Japanese interest rates and asset yields have historically, and at the time of the vote, been lower than elsewhere.
negative value in the regression of MXNUSD against $PB_t$ and $PB_t \cdot (1 - PB_t)^{23}$. The coefficient of ESU6 is also negative whereas that for ZNU6 is positive. The positive (albeit not necessarily significant) coefficient for ZNU6 indicates risk loving preferences, which on first impressions may be surprising. However, our model incorporated risk preferences by assuming that the participants in the betting market were risk neutral and those for financial assets were risk averse. This led to a discount on asset prices proportional to the uncertainty in the outcome of the political event. Uncertainty is highest when $PB_t$ is around the middle of its range $[0, 1]$ and furthest from the endpoints. For ZNU6 there instead appears to be a premium on the price when uncertainty is highest. This is not unexpected. US treasuries are considered the ultimate safe haven asset, and are typically bought when there is a flight to quality and risk appetite decreases. Investors sell risky assets such as commodities (CLU6), stocks (ESU6) or emerging market currencies (MXNUSD), recover any funding in lower yielding currencies (USDJPY) and invest the proceeds in treasuries. This behaviour reveals itself on the night not only by the ZNU6 appreciating on the shock result, but as a risk loving preference and a positive value of $\lambda$ in the cointegrating relationship.

Finally we check the implications for the order of cointegration for the 7-dimensional system which includes the five asset prices, the betting contract, and the risk factor $R_t$. When risk is included in the system our model still implies a cointegration rank of five but there are now two common stochastic trends, relating to the likelihood of the political event, and the risk factor. Results when the Johansen test is re-run with risk included are shown in Table 18. Disappointingly the inclusion of the non-linear risk factor does not provide any more evidence of a higher cointegration rank. Again, only the zero rank is rejected in favour of higher ranks. The situation of the NH test is similar. The test statistic for two common trends ($PB_t$ and risk) of 0.0029 again are well below the 90% level of rejection in favour or higher numbers of trends and a lower cointegration rank than 5. The strong evidence in favour of a non-linear cointegrating relationship between the financial and betting markets from the Phillips–Ouliaris tests is not repeated in the rank tests.

### 5.2.4 Deviations from long term relationships

As in the Brexit case, the presence of significant autoregressive terms in both the cointegrating regressions and the ECM suggests deviations from efficiency on a short term scale. There is also evidence for the 2016 presidential election that risk neutrality does

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23Mathematically: negative $\lambda$ is equivalent to a convex cointegrating relationship and positive $\lambda$ implies concavity. If USDMXN is concave, then MXNUSD = USDMXN\(^{-1}\) is convex. Economically: if USD is “safe” relative to the MXN, then MXN is “risky” relative to the USD.
Table 19: Fitted short-run parameters in the error correction model for the Trump election and Granger causality tests.

**Short-Run Regression Results**

\[
\Gamma_1 = \begin{pmatrix}
-0.196^{**} & -9.28^{**} & 4.92^* & - & - & -36.4^{***} & - \\
- & - & -0.0955 & -0.0956 & - & 0.502^{***} & - \\
- & - & - & -0.147^* & - & -0.426^{***} & - \\
- & -2.40 & 0.200^* & - & -0.274^{***} & -1.10^{***} & -0.893* \\
- & - & -0.0555^{**} & -0.0500^{**} & - & -0.329^{***} & - \\
- & - & - & - & - & -0.277^{***} \\
\end{pmatrix}^{1}
\]

Based on \( \Delta p_t = \alpha (\beta p_{t-1} - \epsilon_0) + \Gamma_1 \Delta p_{t-1} + u_t \)

\( p_t = (\text{ESZ6}, \text{ZNZ6}, \text{CLZ6}, \text{USDMXN}, \text{USDJPY}, PB_t, R_t)' \)

\( R_t = PB_t (1 - PB_t) \)

1 Statistically insignificant coefficients omitted

**Null Hypothesis**

<table>
<thead>
<tr>
<th>LR-Test</th>
<th>Chi-square</th>
<th>Prob</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{ESZ6}, \text{ZNZ6}, \text{CLZ6}, \text{USDMXN}, \text{USDJPY})' \text{ does not Granger cause } (PB_t, R_t)</td>
<td>15.7</td>
<td>0.108</td>
</tr>
<tr>
<td>(PB_t, R_t) \text{ does not Granger cause } (\text{ESZ6}, \text{ZNZ6}, \text{CLZ6}, \text{USDMXN}, \text{USDJPY})'</td>
<td>32.8^{***}</td>
<td>&lt; 0.001</td>
</tr>
</tbody>
</table>

not hold. The “long-term” relationship, which lasts a few hours, is now non-linear. The full multivariate system is

\[
p_t = (\text{ESZ6}, \text{ZNZ6}, \text{CLZ6}, \text{USDMXN}, \text{USDJPY}, PB_t, R_t)'
\]

where \( R_t \) is the non-linear political risk measure \( PB_t (1 - PB_t) \). The fitted ECM, as in the case with Brexit, has a single significant autoregressive short-run matrix \( \Gamma_1 \). Table 19 shows the significant values of this matrix. Testing for Granger causality in this system requires considering both the risk measure \( R_t \) in addition to the betting contract price \( P_t \). To test causation from the betting markets to financial markets, consideration of the \((n-2) \times 2\) sub-matrix of \( \Gamma_1 \), which is the first \((n-2) \) values of the last two columns, is required. Testing in the opposite direction requires joint testing of the \( 2 \times (n-2) \) sub-matrix which is the first \((n-2) \) values of the last two rows. Table 19 shows results of these two tests. As was the case with the previous event, there is significant causation from the betting markets to the financial markets (at the 1% level) but no significant causation in the other direction.

The cointegrating error for this election is \( \hat{e}_t = p_t - \hat{p}_0 - PB_t \times \Delta p - \hat{\lambda} \times R_t \). Unlike in the linear case with Brexit, this error cannot be recreated simply by trading a portfolio.
of the betting contract with the financial asset. The non-linearity would require constant adjustment, or delta hedging. This would be difficult and expensive to achieve in practice. However, as the relationship is monotonic, if the error can be predicted, this does imply that the two assets separately can be predicted. Figure 10 plots $\hat{\epsilon}_t / |\Delta p|$, the cointegration error normalised to units of the Betting contract, with the sample Auto-Correlation Function and robust Bartlett intervals. As in the case with Brexit, this shows significant and positive autocorrelations out to around 20–30 minutes. The presence of long term cointegration of the markets implies market efficiency on a longer term timescale. The fact that deviations appear predictable suggests an inefficiency on a shorter timescale. We fit autoregressive models to $\hat{\epsilon}_t$. We find, for all assets, that models with two lags are significantly different to a model with a single lag but that further lags are not significant. Table 20 shows the results of the regressions. This again provides evidence that EMH was violated on a short time scale on the night of the 2016 US presidential election.
Figure 11: Ex-Post Bollinger Band trading strategies for the 2016 presidential election.
Table 20: Estimated autoregressive models for cointegration errors.

<table>
<thead>
<tr>
<th>lag</th>
<th>ESZ6</th>
<th>ZNZ6</th>
<th>CLZ6</th>
<th>USDMXN</th>
<th>USDJPY</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{\epsilon}_t$</td>
<td>0.559***</td>
<td>0.783***</td>
<td>0.771***</td>
<td>0.629***</td>
<td>0.658***</td>
</tr>
<tr>
<td></td>
<td>(0.038)</td>
<td>(0.034)</td>
<td>(0.043)</td>
<td>(0.040)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>$\hat{\epsilon}_{t-2}$</td>
<td>0.398***</td>
<td>0.196***</td>
<td>0.204***</td>
<td>0.302***</td>
<td>0.308***</td>
</tr>
<tr>
<td></td>
<td>(0.036)</td>
<td>(0.036)</td>
<td>(0.044)</td>
<td>(0.041)</td>
<td>(0.044)</td>
</tr>
</tbody>
</table>

The regression is $\hat{\epsilon}_t = \psi_1 \hat{\epsilon}_{t-1} + \ldots + \psi_k \hat{\epsilon}_{k-1} + \eta_t$.

$\hat{\epsilon}_t$ is estimated from the first stage regression.

5.2.5 Profit opportunities

We now turn to the possibility of profiting from the apparently predictable deviations from the long term cointegrating relationship. In practice it is not possible to recreate the return of the non-linear error so we will focus on the linear error. Again, this is reproduced by trading the financial asset against the political bet in a ratio equal to the linear cointegrating ratio. This is estimated from the ex-post regression and the parameters are shown in Table 13. We apply the $1.5\sigma$ and $2\sigma$ sigma trading strategies. Results are shown in Table 21 and the normalised error $\hat{\epsilon}_t/|\Delta p|$ is shown with bands and trades in Figure 11. The strategies are profitable although there were far fewer trades than in the Brexit case. This may be because for four of the five assets linear cointegration does hold. The $1.5\sigma$ strategy generates 17 trades, 15 profitable, compared with 57 trades for Brexit. The $2\sigma$ strategy only generates eight trades, all profitable compared with 38 for Brexit. The errors appear to mean revert with far less frequency than for Brexit. However, each trade is more profitable, indicating a greater volatility on the night of the Trump election. As discussed earlier, recreating these profits in practice is not at all realistic. It would require an investor to correctly evaluate the conditional expectations of each asset price given the two possible outcomes of the election. The results of the ex-post strategies are at best thought experiments, and rely on the conditional expectations that have been revealed after the event. In fact applying the Bollinger strategy in this way ex-post to any two unrelated random walks is always likely to generate apparent profits. There will always be mean reversion to an apparent relationship that is estimated from the (spurious) regression. Given there is not even evidence of linear cointegration for the financial assets (beyond the Peso) the results below are likely pure fiction.

The USDMXN cross showed evidence of linear cointegration. Could profits be generated from this asset using an ex-ante estimated forecast for the conditional expectations of the asset prices? To answer this question we exploit a Reuters poll of several economists that was published on 1st November 2016, a week before the election, Cascione (2016). The prospect of a Republican win was expected to be very grave for the Mexican economy. The Reuters poll asked a question as to where the Mexican Peso would settle in the
Table 21: Ex-Post Bollinger Band gross trading profits for the 2016 presidential election.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>nWin/nTrade</th>
<th>Profit*</th>
<th>Profit/Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>3/3</td>
<td>27.6p</td>
<td>9.2p</td>
</tr>
<tr>
<td>ZNU6</td>
<td>2/3</td>
<td>23.8p</td>
<td>7.9p</td>
</tr>
<tr>
<td>CLU6</td>
<td>3/4</td>
<td>51.8p</td>
<td>12.0p</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>5/5</td>
<td>45.4p</td>
<td>9.1p</td>
</tr>
<tr>
<td>USDJPY</td>
<td>2/2</td>
<td>20.2p</td>
<td>10.1p</td>
</tr>
<tr>
<td></td>
<td>15/17</td>
<td>168.8p</td>
<td>9.9p</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Symbol</th>
<th>nWin/nTrade</th>
<th>Profit*</th>
<th>Profit/Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>USDJPY</td>
<td>1/1</td>
<td>11.7p</td>
<td>11.7p</td>
</tr>
<tr>
<td>ZNU6</td>
<td>1/1</td>
<td>20.7p</td>
<td>20.7p</td>
</tr>
<tr>
<td>CLU6</td>
<td>2/2</td>
<td>41.7p</td>
<td>20.8p</td>
</tr>
<tr>
<td>GBPUSD</td>
<td>3/3</td>
<td>31.1p</td>
<td>10.4p</td>
</tr>
<tr>
<td>USDJPY</td>
<td>1/1</td>
<td>25.4p</td>
<td>25.4p</td>
</tr>
<tr>
<td></td>
<td>8/8</td>
<td>130.6p</td>
<td>16.3p</td>
</tr>
</tbody>
</table>

*Profits are shown for every £1 contract traded on Betfair.

event that Donald Trump were elected. The median forecast for USDMXN was 21.50. In the event this was a little pessimistic. The Peso bottomed out at 5:23am (GMT) with a USDMXN price of 20.77. It slightly appreciated to 20.68 by 6am. Assuming efficiency at midnight and using the Reuters median for \( p_1 = 21.50 \) implies \( p_0 = 17.70 \). Thus \( \Delta p = 3.80 \) and \( \hat{e}_t = GBP_t - 17.70 - 3.80 \times PB_t \). This ex-ante estimated error from the Reuters forecast is plotted with Bollinger Band strategies and trades on the left hand side of Figure 12. The overly pessimistic forecast makes the error diverge from zero, yielding a large loss making trade in the second half of the night. For the \( 2\sigma \) strategy there are two profitable trades made in the first half of the night. These occur before the error diverges. These do more than offset the loss of the final trade but the total gross profits of 3.2p are likely to be negligible after trading costs. The profits of around 1p per trade compare very poorly with that of the (unrealistic) ex-post strategy (16.3p).

Next we study whether the cointegrating relationship can be estimated for the USDMXN in the first few hours and successfully exploited later in the night. The betting markets implied a 50% probability of a Trump win by 2:39am. Using the period midnight to 2:39am to estimate a linear cointegrating relationship estimates \( p_0 = 17.81 \) and \( p_1 = 21.69 \). Again the conditional expectation of USDMXN given a Trump win is overly pessimistic. It results in an estimated error that diverges from zero. This yields a terrible trade in the second half of the night as demonstrated by the right hand side of Figure 12 where the misspecified error and Bollinger bands are shown.

The results obtained for GBPUSD for Brexit, where the linear relationship estimated from the first period was stable, yielded a successful strategy. The answer as to why the performance is so different in this instance is the non-linearity and apparent deviation from risk neutrality. Figure 13 illustrates the difference. The GBPUSD and USDMXN are plotted against the relevant betting contract along with both the linear relationship
Bollinger Bands strategies for misspecified cointegration error when using $E(USDMXN|TRUMP) = 21.50$ (left) and when using the first part of the night to estimate (right).

Figure 12: Possible ex-ante estimated cointegration errors and Bollinger Band strategies for USDMXN.

from the first part of the night as well as the ex-post non-linear cointegrated relationship. The non-linear plot clearly diverges from the linear plot for USDMXN whereas there is almost no difference in the two lines for GBPUSD. There is no hope of estimating the long term non-linear cointegrating relationship for USDMXN from the first part of the night and making money from any convergence. Figure 14 shows similar plots for the remaining assets for the 2016 presidential election as well as Brexit. The situation is replicated for these assets. There is a clear difference in behaviour for the Trump election. In the Brexit case the non-linear relationships are virtually identical to the linear one. This is consistent with the earlier Johansen constraint tests where there was no evidence of significant risk parameters $\lambda$. The only Brexit contract where the market does not converge to the linear cointegrated relationship is the ESU6, and the divergence is far smaller than for the presidential election. Note that this was the contract that lost money when applying the Bollinger ex-ante strategy errors. However, this does not appear to be due to the presence of risk aversion but more to a slight misestimation in the linear cointegration ratio. The application of the ex-ante Bollinger strategy on the night of the Trump election would be ruinous. Moreover the data from the first period appear to indicate a linear relationship. Table 22 shows the pValues for the null hypothesis of no (linear) cointegration for the Phillips–Ouliaris $Z_t$ and $Z_\alpha$ tests and the Johansen max eigenvalue test. Cointegration is indicated. This could lead any brave investor that profited from this strategy on the night of the Brexit referendum to trade a linear relationship that does not exist. The apparent divergence from risk neutrality after the presidential election leads to far different behaviour than that observed after
The non-linear relationship is calculated ex-post on all data whereas the initial linear relationship uses data from an initial period when event probabilities were under 50%.

Figure 13: Estimated cointegration relationships for USDMXN (presidential election) and GBPUSD (Brexit referendum).

Table 22: Cointegration tests for the smaller training period for the presidential election.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$p_{Z_a}(a = 0)^a$</th>
<th>$p_{Z_a}(a = 0)^a$</th>
<th>$p(r = 0)^b$</th>
</tr>
</thead>
<tbody>
<tr>
<td>ESU6</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.187</td>
</tr>
<tr>
<td>ZNU6</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.014**</td>
</tr>
<tr>
<td>CLU6</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.032**</td>
</tr>
<tr>
<td>USDMXN</td>
<td>0.002***</td>
<td>0.003***</td>
<td>0.028**</td>
</tr>
<tr>
<td>USDJPY</td>
<td>0.001***</td>
<td>0.001***</td>
<td>0.040**</td>
</tr>
</tbody>
</table>

$^a$Phillips–Ouliaris tests.

$^b$Johansen max eigenvalue test.

Brexit, where risk neutrality appeared to hold. We conclude that there appear to be no realistic opportunities to profit from our model for the 2016 US presidential election.

5.3 The 2014 Scottish independence referendum

The 2014 Scottish independence referendum was held on 18th September 2014. It concerned whether Scotland should remain in the UK. The question asked “Should Scotland become an independent country?” and it required a simple majority to pass. All EU and Commonwealth citizens residing in Scotland were eligible to vote. This was the first time that an election was open to 16 and 17 year olds. Turnout, at 84.6%, was the largest for any UK election since the general election of 1910. The “No” side won with 55.3% of the vote against 44.7% for “Yes”.

Voting took place from 7am to 10pm BST. Voters still queuing at the close of polls were allowed to vote. Each of the 32 local authority areas announced their results at
Figure 14: Estimated cointegration relationships for (a) the 2016 presidential election and (b) the Brexit referendum.

Note: The non-linear relationship (orange line) is calculated ex-post on all data whereas the initial linear relationship (yellow line) uses only data from the first few hours when event probabilities were under 50%.
separate times overnight. This makes this event suitable for our model as the information was drip fed to the public throughout the overnight hours.

During July and August 2014, opinion polls showed a consistent lead for “No” with the average difference being 6%, Curtice (2014). However, polls tightened at the start of September and on the 6th of that month YouGov published a poll that showed the Yes side ahead with a small lead of 2%, Dahlgreen (2014). This was the first time a poll showed the Yes side ahead. It was also the first time many commentators took the possibility of a Yes vote, and the resulting disruption to the UK’s economy, seriously. In response sterling lost around 1% against the euro and dollar and companies with links to Scotland sold off sharply, Wearden (2015). However the 6th September poll was the only poll that showed Yes ahead. The polls subsequently reverted to a Yes trailing No. The final poll released was conducted by YouGov on the day of voting. The results were released just after the close of voting and predicted an 8 point lead for No.

Figure 15 shows the Betfair contract that pays out £1 in the event of a No vote, along with the GBPUSD price. The betting market had previously, following the release of the 6th September poll, implied a probability of Scotland leaving the UK of as high as 35%. However, on the night of the vote the likelihood was much lower. From 11pm to midnight Betfair implied around a 90–92% chance of remaining. This contract started rising after 1am. The first local authority to announce their result was Clackmannanshire at 1:28am. This showed the vote for No nearly 8% ahead of Yes and in line with the poll on the day released by YouGov. The SNP, whose headline policy was independence, had achieved their highest vote share in Clackmannanshire in the 2012 council elections and so this result was seen as very negative for the Yes campaign. The No contract on Betfair continued its ascent and shortly after this result priced a 98–99% likelihood of No winning the referendum. The currency was sensitive to the possibility, albeit small, of a yes vote. It rallied around a cent to the dollar as this possibility was effectively ruled out. From the point of this first result onward the result did not seem in doubt. The betting contract price remained above 98% for the most part with only a brief fall to 96% shortly after 4am which quickly reversed.

5.3.1 Evidence for cointegration

For this event we consider GBPUSD only. The currency was expected to sell off sharply in the event that the Yes camp prevailed, and recover a little if Scotland voted to stay in the UK. The price at 10pm was 1.6395, it peaked at 2:26am at 1.6523 and ended the night at 6am at 1.6476. The appreciation was modest but this has to be considered against the smaller move in the betting market contracts of only around 10p.

In table 23 we show the results of various cointegration tests for GBPUSD and the betting contract. The first line of the table shows the results for linear cointegration.
Figure 15: Betfair price for “No” bet and GBPUSD on the night of the independence referendum.

Both Phillips and Ouliaris tests as well as the Johansen max eigenvalue test do not reject the null of no cointegration at the 95% level. The $Z_\alpha$ test marginally rejects the null at the 90% level. This does not provide sufficient evidence for cointegration. The second line shows the results for the case where the risk factor is included in the cointegration regression (or in the system for the Johansen test). Here, the Phillips and Ouliaris tests do reject the null at the 99% level but the Johansen rank test does not. However, we note that the magnitude of the risk parameter $\lambda$ is significantly larger than the linear parameter $\Delta p$. This violates our model as it results in a non-monotonic cointegrating relationship and utility function. The estimated value for the counterfactual conditional expectation of the pound given a Yes vote of $\hat{p}_0 = 3.18$ is nonsensical. The pound was expected to depreciate given the unlikely scenario of Scotland voting to be independent, not appreciate nearly 100%. The third and final line of the table shows the tests when the non-linear cointegration relationship is constrained to be positive with $|\lambda| = |\Delta p|$. This is effected by regressing $GBP_t$ against the single monotonic factor $[PB_t + R_t]$. The three cointegration tests do not reject the null of no cointegration at the 90% level. Again the value of $\hat{p}_0 = 0.703$ is implausible. Although the pound was expected to
depreciate significantly given a Yes vote, this value implies a huge fall of over 50%. After all, GBPUSD only sold off around 12% on the night of the Brexit referendum. Of all three models considered, the linear model seems the most realistic but there is still no compelling evidence that our model holds.

We investigate the implied cointegration models in a little more depth. Figure 16 shows estimated cointegration relationships and market data for the night. The yellow line is the linear relationship found from regression using the first few hours when the betting probability of No was below 95%\(^{24}\). This is decreasing which makes no sense. In this first period the betting contract only varied between 90 and 95p, and for the most part was no greater than 93p. Similarly the pound moved within a roughly 0.3 cent range. These were small noisy moves of no consequence as no information was released. The non-linear monotonic relationship fitted ex-post on all data is at least increasing. However, it is a poor fit to much of the data and explains the failure to reject the null of no cointegration in the non-linear regression. Deviations from this relationship do not appear stationary nor converge. These insights provide further evidence that our pricing model does not apply on the night.

The truth is that on the night of the Scottish referendum the markets did not move a great deal, and what little movement there was for each market came relatively quickly and not quite in sync. The betting contract moved only 10p in the overnight session. This contrasts with the two events in 2016 where the results were a shock. The betting contracts moved close to their full range of £1. Our model fits well there as these political events dominated the markets. There was also sufficient coverage of price data there to generate statistical power validating our model. In both cases in 2016 relevant information was released over a matter of hours. What is perhaps more interesting is the fact that the betting market appeared to lead the currency during the move up.\(^{24}\)

The probability of a vote for Scotland to remain in the UK rose above 95% at 1:21am, shortly before the Clackmannanshire announcement.

Table 23: Results of cointegration tests for the Scottish independence referendum.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>(\hat{p}_0)</th>
<th>(\hat{\Delta}p)</th>
<th>(\hat{\lambda})</th>
<th>(p_{Z_t}(a=0))</th>
<th>(p_{Z_{a}}(a=0))</th>
<th>(p(r=0)^{d})</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBPUSD(^a)</td>
<td>1.5570</td>
<td>0.0937</td>
<td>–</td>
<td>0.126</td>
<td>0.099*</td>
<td>0.132</td>
</tr>
<tr>
<td>GBPUSD(^b)</td>
<td>3.1833</td>
<td>-1.5309</td>
<td>-2.4388</td>
<td>0.008***</td>
<td>0.007***</td>
<td>0.161</td>
</tr>
<tr>
<td>GBPUSD(^c)</td>
<td>0.7031</td>
<td>0.9464</td>
<td>0.9464</td>
<td>0.182</td>
<td>0.150</td>
<td>0.369</td>
</tr>
</tbody>
</table>

\(^a\)The first stage regression is 
\(p_t = \hat{p}_0 + \hat{\Delta}pPB_t + \hat{\epsilon}_t\)

\(^b\)The first stage regression is 
\(p_t = \hat{p}_0 + \hat{\Delta}pPB_t + \hat{\lambda}PB_t(1 - PB_t) + \hat{\epsilon}_t\)

\(^c\)The first stage regression is 
\(p_t = \hat{p}_0 + \hat{\Delta}p[PB_t + PB_t(1 - PB_t)] + \hat{\epsilon}_t\) so 
\(\hat{\lambda} = \hat{\Delta}p\)

The residual regression is 
\(\hat{\epsilon}_t = a\hat{\Delta}p + \eta_t\)

\(^d\)Result of the Johansen max eigenvalue test for zero cointegration rank
Note: The non-linear relationship (orange line) is calculated ex-post on all data whereas the initial linear relationship (yellow line) uses only data from the first few hours when event probabilities were under 95%.

Figure 16: Supriously estimated cointegrating relationships between the pound and the betting markets for the 2014 independence referendum.

This is similar to the Granger causality from political to financial markets observed in the previous two events. There may in fact be a common trend in the markets relating to the likelihood of a ‘No’ vote; it is just that the currency appears to lag that trend by around 30 minutes. To investigate this a little further we plot the betting price against the GBPUSD price bought forward 30 minutes from the future. This is shown in Figure 17. To the eye the market move is much more synchronized. We check the linear cointegration tests for this adjusted data. Results are shown in Table 24. We now see that there is evidence of a common trend between the two time series with the Phillips and Ouliaris tests both rejecting the null of no cointegration at the 95% level. The reader should note that this is not a rigorous statistical test of cointegration. The author has plotting more than one adjusted data series, choosing the one most pleasing to the eye.\textsuperscript{25} A joint hypothesis test has effectively been performed whilst the pValues in Table 24

\textsuperscript{25}Full disclosure: graphs for GBPUSD bought forward by 20, 30, 35 and 40 minutes were eyeballed.
Figure 17: Betfair price for “No” bet and GBPUSD price bought forward 30 minutes from the future, on the night of the independence referendum.

relate to only single hypothesis tests. Either way this exercise has demonstrated that the lack of statistical significance is likely due to the presence of a deviation of weak market efficiency of similar order in time to that of the move in the common trend. For the most part, and either side of the Clackmannashire result, the markets simply drifted.

Table 24: Results of linear cointegration tests for Scottish independence referendum where GBPUSD is shifted forward 30 minutes.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>$\hat{p}_0$</th>
<th>$\hat{\Delta p}$</th>
<th>$\hat{\lambda}$</th>
<th>$p_{Z_1}(a = 0)$</th>
<th>$p_{Z_1}(a = 0)$</th>
<th>$p(r = 0)^a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>GBPUSD$^a$</td>
<td>1.5580</td>
<td>0.0934</td>
<td>–</td>
<td>0.041**</td>
<td>0.030**</td>
<td>0.601</td>
</tr>
</tbody>
</table>

The first stage regression is $p_t = \hat{p}_0 + \hat{\Delta p}PB_t + \hat{\epsilon}_t$

The residual regression is $\hat{\epsilon}_t = a\hat{\epsilon}_{t-1} + \eta_t$

$^a$Result of the Johansen max eigenvalue test for zero cointegration rank
6 Conclusion

In this paper we attempted to describe the behaviour of financial and political betting markets in the hours after elections. The key assumptions are that: 1) the only information that has a persistent affect on asset prices is that related to the likelihood of a political outcome (and nothing else), and; 2) weak market efficiency. Further assuming risk neutrality implies cointegration between financial prices and the price of political bets whereas deviations from risk neutrality lead to non-linear cointegrating relationships. We test the model on data from three recent elections and find strong support for the theory for two of them.

For the 2016 Brexit referendum our base linear cointegrating model finds excellent support. The long term cointegration relationship is found to be so stable that a brave trader could estimate it in the first part of the night and profit from the convergence of deviations from that relationship in the second. It appears that market efficiency holds on a longer term basis that night (long term meaning hours) whereas there are some deviations from efficiency on a timescale of tens of minutes. Our results are also in contrast to the model in Manasse et al. (2020) where a divergence from risk neutral preferences is required to explain the behaviour of the currency markets in the weeks and months preceding the vote. Of further interest is the fact that with respect to short term deviations, the betting markets appear to lead and Granger cause the financial markets but there is no apparent causation in the other direction. This suggests betting markets were more efficient at discounting the results of the vote in the hours after the referendum.

For the 2016 US presidential election linear cointegration, and hence risk neutrality, cannot explain the observed price action. However, we find strong evidence for our theory when deviations from risk neutrality are incorporated with a non-linear cointegrating relationship. The interpretation of the computed risk parameters are pleasing. Risk aversion is revealed for “risk-on” assets, whereas risk loving preferences are observed for the safe haven treasury future asset. Similar to Brexit we find deviations from the long term (albeit non-linear here) cointegrating relationship are predictable, indicating deviations from weak market efficiency on similar smaller timescales. The result that the betting markets Granger cause the financial markets but that there is no causation in the other direction is also repeated. However, that brave trader who profited from the Brexit deviations would have had a terrible night repeating that strategy for this election. This is explained by the presence of non-trivial risk preferences. It is not possible to infer or estimate the non-linear cointegrating relationship in the first few hours. Foresight of the conditional expectations of the asset prices given election outcomes would be needed to make money.
Finally we do not find evidence for our model on the night of the 2014 Scottish independence referendum. The markets did not move very much at all that night, as the result was largely as expected. The move itself was also short lived. This is in contrast to the other two events where the results were both a huge shock, and large moves in all prices were observed which took several hours. The betting markets do appear to lead the currency market though by around 30 minutes. We explain the failure to demonstrate statistical significance as due to the similarity of timescales of deviations from market efficiency with those of the time taken for market to move upwards that night.

For all three events it appears that betting markets reflected political information more quickly than financial markets, and on a timescale of minutes to tens of minutes. This is consistent with, and an extension of, the conclusion of the existing literature that political prediction markets offer superior forecasting ability. This finding, along with the development of an asset pricing model that holds in a general election night setting that is based on economic principles are the main contributions of this paper. Whether or not political markets lead financial markets at times other than the results session following a vote is an open research question.

References


