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Tom Auld

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We present a model linking prices of political binary options to financial assets that applies in the very particular circumstance of the overnight session following an election. Contrary to most of the existing literature, the model is derived from economic first principles and applies in a general setting. We find that under suitable assumptions, election and financial markets will be cointegrated. Deviations from risk neutrality lead to the presence of a non-linear term relating to risk in the cointegrating relationship. The model is tested on three recent political events: The 2014 Scottish independence referendum, the 2016 Brexit referendum and the 2016 US presidential election. Strong support is found for two events (the Brexit referendum and the 2016 Trump win). We find that weak market efficiency broadly holds although there are violations of the order of minutes to tens of minutes. This is apparently caused by betting markets leading financial markets, a phenomena that is observed for all three events. This finding is consistent with the conclusion of the existing literature that prediction markets have superior forecasting ability to other methods. A realistic ex-ante trading strategy is presented for Brexit that profits from these inefficiencies. However, the success is not repeated for the 2016 presidential election. This is due to an apparent deviation from risk neutrality that is not observed on the night of Brexit.

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1 Introduction

How do financial and political betting markets behave in the hours after an election? Does one market outperform the other in terms of reflecting information about an election? Also, are there any violations of market efficiency around political events and can they lead to any realistic profit opportunities? This paper attempts to answer these and other questions.

It can be argued that the overnight hours following many elections are very special times. During these typically overnight hours there is an absence of economic information that may usually inform financial prices. It is possible that financial assets are uniquely determined by the outcome of those elections and nothing else. For many elections the results are drip fed throughout the overnight session as results are announced from different voting areas and constituencies. For example, for US presidential elections there are 51 simultaneous vote counts in 50 states plus the special district of Washington DC.[1] These counts are typically broadcast live on TV and on the Internet. For the 2016 UK Brexit referendum there were 382 different areas that announced local results at various times starting from around midnight. Studying the effects of this unique flow of information on prediction and financial markets is the subject of this paper.

That political risk affects asset prices is an established concept in the literature. Similarly, the information content and superior forecasting ability of political prediction markets is a settled matter. It would naturally follow that political and financial market prices should be generally statistically related. There are many event studies in the literature that study this question. However, there are very few examples of models that apply to either multiple events or a general setting, or are based on economic assumptions. We seek to fill that gap in the particular case of the generic 'election night'. This is achieved by creating a model that links financial and political markets that is based on the idea that financial prices are affected by the election outcome and *nothing else*. This idea is encoded by an assumption that the only information having a persistent effect on financial asset prices is the *probability* of a binary political outcome. Adding further common conditions from the asset pricing literature leads to the finding that political and financial markets will have a non-linear cointegrating relationship. If risk neutral pricing is present in financial markets then the relationship is standard linear cointegration. The model is tested on three real world events. We find strong evidence for two events, whereas for the third, the information contained in the vote is limited and statistical significance is not generated. An exploration of the short run dynamics of the model in the vector error correction form demonstrates that betting markets significantly lead financial markets in the short run, but not vice-versa. This reveals the fact that

[1] The 2020 presidential election is a unique case where, given various legal challenges and recounts, results took days to conclude.

political markets are more efficient at discounting the political information contained in election results on election night.

The contributions of this paper are follows. A model is presented based on a pricing assumption wedded with common asset pricing restrictions that applies in a general setting. This is a first in the literature. Strong support is also found for the model. Further, it is found that political betting markets lead financial markets in the hours after an election. This is an important finding and adds to the weight of evidence in the literature that prediction markets are without peer in their forecasting ability.

The remainder of the paper is organised as follows. Section 2 reviews relevant existing literature. Section 3 presents the pricing model that implies political and financial markets are cointegrated on election night. Section 4 briefly outlines the two statistical tests used in the results section. The bulk of the paper is Section 5, the results section, where the implications of our theory are studied for some real world elections. We end the paper in Section 6 by concluding our results. All tables of results are found in the Appendix.

2 Literature review

This paper is primarily interested in the relationships between, and behaviour of, prediction and financial markets, around political events.

The financial market implications of elections and political risk are well documented. Changes in the composition of government naturally brings about changes in policy. There are many studies that demonstrate either the effects on financial markets of election campaign periods or results. Three large multi-country studies demonstrate changes of asset price volatility around elections. Białkowski et al. (2008) studies 134 elections in 27 OECD countries from 1980 to 2004. They find that national stock exchange index volatility can easily double during the week after the election. Apparently 'investors are surprised by the election outcome'. Kelly et al. (2016), using data from 1990-2012 for a sample of 20 countries, find that this uncertainty is priced in the options market. Prices and implied volatility are higher for options that span elections. The authors also document spillover effects from the election country to other international markets. Pantzalis et al. (2000) is another large multi country study. The paper finds significantly positive returns two weeks prior to polling day for elections in 33 countries between 1974 and 1995. The conclusion is that as election uncertainty is resolved, prices respond positively. Two later papers come to the opposite conclusion for US presidential elections. Goodell and Bodey (2012) study changes of the Graham price to earnings (P/E) and consumer sentiment during the campaign periods of US presidential elections. They find the measures *worsen* as the winner becomes clearer. They conclude that 'during presidential election seasons, the market discovers its distaste for the economic policies of the likely winner'. The analysis is extended in Goodell and Vähämaa (2013) for presidential elections from 1992 to 1998. They find the VIX is positively associated with positive changes in the probability of the winner. This 'indicates that the presidential election process engenders market anxiety as investors form and revise their expectations regarding future macroeconomic policy.'

There are also studies in the literature that demonstrate association between election polls and asset prices. Gemmill (1992) derives the probability of a Conservative party win from polling data for the 1987 UK general election, concluding that the 'FTSE100 index was very closely related to the probability of a Conservative win'. Brander (1991) and Bernhard and Leblang (2006) study the 1988 Canadian election. This was shortly after the implementation of the Canada-US FTA, which the opposition Liberal party opposed. Prices on the Toronto Stock Exchange (TSE) were found to be significantly positively related to polling numbers for the Conservative party during the campaign period.

Studies, many of which are based on the Iowa and UBC markets[2], have demonstrated the remarkable accuracy of forecasts from election markets. There is a consensus in the literature that political markets outperform other methods including polling and expert predictions. For a recent review of political markets see Graefe (2016). Two early papers by Forsythe et al. demonstrate the outperformance of prediction markets when comparing final prices with final polling numbers for vote shares. Forsythe et al. (1992) finds that 'the [Iowa] market worked extremely well, dominating opinion polls' for the 1988 US presidential election. By looking at the positions of different constituent groups they find that traders tended to place bets on their preferred candidates (indicating judgement bias). However, they conclude that prices are set by the marginal trader. Oliven (2004) also find that market-making traders are more rational than price takers in the Iowa market, the implication being that arbitrageurs are indeed making prices efficient. The analysis and conclusions are repeated for the 1993 Canadian federal election in Forsythe et al. (1995). Both these studies focus on shorter-term market predictions. A latter study, Joyce E. Berg and Rietz (2008), extends the analysis to longer-term forecasts. This paper compares vote share prices from the Iowa market with 964 polls for US presidential elections from 1988 to 2004. They find that the prediction market is closer than the polls 74% of the time. Further 'the market significantly outperforms the polls in every election when forecasting more than 100 days in advance', providing

[2]Examples of electronic election markets include University of Iowa's Iowa Electronic Markets, introduced for the 1988 US presidential election, the University of British Columbia's UBC Election Stock Market (now superseded by the Sauder School of Business Prediction Markets), and the Betfair Exchange, prices for which are used in this paper.

evidence that election markets are not only accurate at times close to a vote but have superior explanatory power months from an election.

The literature is clear that both election markets are powerful tools to forecast elections and that the outcome of elections have effects on asset prices. It naturally follows that prediction and financial markets should be related in some way. There are many event studies in the literature that consider elections, but they typically only consider either financial markets, or political markets and rarely both, and when they do they typically only uncover an empirical relationship and are not based on economic theory. The few studies we find in the literature that consider both types of market are discussed below.

There are two studies in the literature that directly relate political prediction market prices to the residuals of CAPM models. Both consider only the 2014 Scottish independence referendum. Acker and Duck (2015) find that the residuals of an estimated CAPM model are significantly positively related to several proxies for the odds of a vote to remain as part of the UK, including a weighted sum of the Betfair exchange odds for 'No'. Similarly Darby et al. (2019) study equities listed on the LSE that were headquartered in Scotland. They find that uncertainty betas help predict cross-sectional returns. Hanna et al. (2021) also use Betfair data to analysis how changes in the betting odds for 'Leave' influence financial markets for the Brexit referendum. Using high frequency data for trades on the Betfair exchange, they regress short-term returns of GBPUSD and major UK and European stock indices on changes in betting prices during stock market opening hours. They find that changes in the odds for leave cause prices of UK equities and the pound to fall in the following 5 minutes. They also find some spillover effects to EU equity prices. Finally, we find two examples in the literature of economically derived relationships between prediction markets and asset prices. These are, firstly Auld and Linton (2019), which examines the behaviour of the pound and the betting markets on the night of Brexit, and secondly, Manasse et al. (2020) which considers the period leading up to the Brexit referendum. Auld and Linton (2019), and the associated corrigendum Auld and Linton (2023), build a similar model of cointegration for the pound on the night of Brexit as to the model presented here. However, the model in this paper is an improvement as it uses common pricing restrictions in its derivation and applies to a general financial asset in an overnight results session following any political event. Separately, the authors of Manasse et al. (2020) build a simple portfolio model for currencies. This implies that currencies are cointegrated with betting prices for the period running up the to 2016 UK Brexit referendum. Under risk neutrality they find a linear cointegrating relationship but that risk aversion leads to the presence of a risk factor related to uncertainty. This leads to a non-linear term of the betting market appearing in the cointegrating relationship. We build a similar model in this paper. However our model only applies to the overnight session following an election. Manasse et al. (2020) find that currency and Betfair data are consistent with their model. However, we do not believe the assumptions behind their model are valid. For it to be so, one would have to believe that the only determinant of the GBPUSD price in the weeks and months preceding the Brexit vote is the result of that vote. We believe this is not plausible. News and information beyond that relating to the referendum, including US economic releases, are likely to affect the British Pound and United States Dollar exchange rate in the period under study.

Contrary to the existing literature, this paper presents a pricing model of political and financial markets that is based on economic principles. It is for a particular setting, the session following an election, but applies to a general election. We believe the fact that there are no similar studies or theories presented in the literature speaks to the contribution of this work.

3 Pricing model

In this section we present a pricing model that links the prices of political and financial markets during the overnight results session following an election.

Notation

E	Binary political event indicator $\in \{0, 1\}$
T	Time at which event is realised
p_t	Price of financial asset at time $t < T$
PB_t	Price of betting contract paying out £1 when $E = 1$ at time $t < T$
$\mathbb{P}_t^B(\cdot)$	Belief of a representative investor in the betting market at time $t < T$
$\mathbb{P}_t^f(\cdot)$	Belief of a representative investor in the financial market at time $t < T$
p_1	Expected value of financial asset conditional on $E = 1$
p_0	Expected value of financial asset conditional on $E = 0$

Scenario

First we outline an overnight scenario. We consider a time period overnight after the vote has closed, but up to and including the result becoming known. Suppose that the outcome of the election can be represented in a *binary* fashion (for example a yes/no referendum). Further suppose there is a healthy liquid prediction market for this event where binary options trade that payout on the outcome of the given event. Such contracts exist and examples include bets that paid £1 in the event of the UK voting to leave the EU in 2016 as well as Donald Trump winning the Presidential election. These bets are effectively contingent claims. Say $E = 1$ if the event occurs and $E = 0$ otherwise. Write PB_t as the price of the contract that pays out £1 when $E = 1$. The outcome of the event is realised at time T. Further, suppose there is a financial asset with price p_t at time $t < T$.

No Shocks condition

We build our model beginning with a key assumption. This is that there are no persistent shocks to the price of financial assets beyond those related to the probability of the outcome of the political event, during the overnight interval. We discuss this assumption in the case of the 2016 Brexit referendum and the pound dollar exchange rate GBPUSD. Simply put, this condition states that the only persistent determinants of the GBP price on the night of the vote were the results of the vote, and that those results only affect the price through their effect on the probability of Brexit, $\mathbb{P}(E = 1)$. Any changes in price that are not related to $\mathbb{P}(E = 1)$ do not persist. Over a longer period of time there are certainly other determinants of the GBP price. For example unanticipated economic information relating to the health or otherwise of the US or UK economies would normally affect the exchange rate. Other examples of such information may even be related to the UK leaving the EU. If after the vote, new information arises regarding the nature of the trading relationship between the EU and the UK, then the exchange rate could be affected. However, on the night of the vote our assumption is that none of this matters. Beyond the very short term, the only thing affecting financial prices is whether or not the UK leaves.

The statement that the only news affecting prices that night was related to the referendum is not controversial. Indeed, there were no major economic releases or other significant news events. The advance Econoday economic calendar, Econoday (2016), listed the final market related news releases on the 23 June 2016 as the US New Home Sales Report at 10:00 am Eastern Time (ET) and the first one for 24 June (beyond the referendum) as Durable Goods Orders at 10 am ET. The calendar wrote regarding the 24th: "In a rare and potentially powerful wildcard, the markets will react to the Brexit outcome". The authors of Wu et al. (2017) come to a similar conclusion and describe the circumstances as "a natural experiment" with "near perfect conditions" to study such a situation where there is a single determinant of prices.

The condition that only the *probability* of Brexit affects the GBPUSD price is more subtle. For instance, it could be believed that the vote share for leaving the EU has an affect on financial prices, over and above the decision to leave the EU. This perhaps could be due to a "harder" Brexit. However, the NS condition means that only the binary result of the vote, and not any particular form of trading relationship following a vote, was on the minds of investors on results night. Whether there is a 50.1% or 99.9% vote share for Brexit, the exchange rate will be the same. Despite there being intense scrutiny of the negotiations between the EU and the UK on the terms of withdrawal in the years following the vote, the term "hard Brexit" only first appeared several months after the referendum at the Conservative party conference in October 2016. The condition that the only thing affecting prices on the night of an election is the binary result of that election, is one we rely on in general to formulate our pricing model.

We operationalise the assumption as follows: For $t < T$ the non-stationary asset price p_t is a function only of $\mathbb{P}_t(E = 1)$ plus some non-persistent term ε_t, representing contributions to the price unrelated to the political event. Then

$$p_t = F(\mathbb{P}_t^f(E = 1)) + \varepsilon_t \tag{3.1}$$

where ε_t is mean-zero and weakly dependent[3] and $\mathbb{P}_t^f(\cdot)$ represents the belief of a representative investor in the financial market. At the resolution of the election, $t = T$, E becomes either zero or one with certainty. Thus $\mathbb{E}_t(p_T) = \mathbb{E}_t(F(E)) + \mathbb{E}_t(\varepsilon_T)$. However, ε_t represents the part of the price beyond that related to the election, which is fleeting. During the night the expected contribution of ε_t to the price at time T ("in the morning") is negligible. This in ensured by the weak dependence assumption of ε_T. $\mathbb{E}_t(\varepsilon_T)$ vanishes quickly as $T - t$ increases and

$$\mathbb{E}_t(p_T) \approx \mathbb{E}_t(F(E)) \qquad t < T.$$

Going forward we assume that $\mathbb{E}_t(\varepsilon_T)$ is in fact zero for all $t < T$. The power of the NS assumption is that the expectations of p_T conditional on the outcome of the election E do not vary. This as

$$\begin{aligned} &\mathbb{E}_t(p_T|E = 0) = F(0) \\ &\mathbb{E}_t(p_T|E = 1) = F(1) \qquad\qquad \forall\, t < T. \end{aligned} \tag{3.2}$$

Applying the total law of expectation to $\mathbb{E}_t(p_T)$ yields

[3] Weak dependence ensures that any deviations away from zero are transient and do not persist.

$$
\begin{aligned}
\mathbb{E}_t(p_T) &= \mathbb{P}_t^f(E=0)F(0) + \mathbb{P}_t^f(E=1)F(1) \\
&= F(0) + \mathbb{P}_t^f(E=1) \times [F(1) - F(0)] \\
&= p_0 + \mathbb{P}_t^f(E=1) \times \triangle p \qquad (3.3)
\end{aligned}
$$

where

$$
\begin{aligned}
p_0 &= F(0) = \mathbb{E}(p_T|E=0) \\
p_1 &= F(1) = \mathbb{E}(p_T|E=1) \\
\triangle p &= p_1 - p_0.
\end{aligned}
$$

Thus the NS assumption implies that the asset price at time T is expected to be p_0 with probability $\mathbb{P}_t^f(E=0)$ and p_1 with probability $\mathbb{P}_t^f(E=1)$.

Weak Market Efficiency (EWMH)

The betting and financial markets are clear examples of segmented markets, having very different participants. However we add the conditions that investors in both sets of markets have identical information sets and very similar beliefs. The assumptions are consistent with those that lead to weak form market efficiency (EWMH). If we write the belief of the probability of $E=1$ of a representative participant in the prediction market as $\mathbb{P}_t^B(E=1)$ then a strict application of this condition means that $\mathbb{P}_t^B(E=1) \equiv \mathbb{P}_t^f(E=1)$. It is unrealistic though that assessments of the probability of the political event (or indeed market efficiency) will hold exactly at all times instantaneously. We thus allows errors in the relationship,

$$
\mathbb{P}_t^f(E=1) = \mathbb{P}_t^B(E=1) + \eta_t \qquad (3.4)
$$

where η_t is the difference between the assessments of the probability of $E=1$ by the representative investors in each market.

Results about the election arrive throughout the night of the election. Each result permanently affects the probabilities $\mathbb{P}_t^f(E=1)$ and $\mathbb{P}_t^b(E=1)$. These are non-stationary. They are also bounded. We model them as integrated $I(1)$ series with absorbing boundaries at zero and unity. Once the boundary is hit, the election has concluded. A strict application of equivalence of beliefs $\Rightarrow \eta_t \equiv 0$. This is unlikely. We distinguish between equivalence of beliefs holding in the long-term (long-term being a few hours here) and not holding at all. If deviations η_t quickly decay, efficiency broadly holds. Again, this occurs when $\mathbb{E}(\eta_t) = 0$ and η_t is weakly dependent. If errors persist and η_t is non-stationary, then the condition does not hold at all. This introduction of the error, η_t, will allow for testing of our model later on.

Substituting equation 3.4 into equation 3.3 results in an expression for expected financial prices conditional on the error in beliefs η_t and dependent on beliefs of the representative investor in the betting market:

$$\mathbb{E}_t(p_T|\epsilon_t) = p_0 + \mathbb{P}_t^B(E=1) \times \triangle p + \epsilon_t \qquad \mathbb{E}(\epsilon_t) = 0 \tag{3.5}$$

where $\epsilon_t = \triangle p \times \eta_t$.

Risk neutral pricing in betting markets

Investors in financial assets are very likely to have correlated exposure to shock election results, leading to the presence of additional election related risk-premia. It is likely that a risk-premia existed for assets that have exposure to say, Scotland voting to leave the UK, the UK voting to leave the EU, or Donald Trump being elected in 2016. However, it is not clear that this is the case at all for bettors in election markets. There is indeed evidence that in the case of Brexit there were very many smaller bettors who favoured leaving the EU, trading against larger pro-EU participants, Economist (2016).

We assume risk neutral pricing in betting markets. This condition formalises the idea that in these markets there are investors present who will take on election risk for ever decreasing edge. This implies that the risk-neutral probability of $E = 1$ is indeed the assessed probability $\mathbb{P}_t^B(E=1)$. As the time for payout of any bets made on the night of the election is very short, any discount factor is effectively unity. Thus the condition reduces to $\mathbb{P}_t^B(E=1) = PB_t$. Then equation 3.5 can be updated with $\mathbb{P}_t^B(E=1)$ replaced with PB_t. The expectation of financial asset prices can now be related to *prices* in the betting markets:

$$\mathbb{E}_t(p_T|\epsilon_t) = p_0 + \triangle p \times PB_t + \epsilon_t \qquad \mathbb{E}(\epsilon_t) = 0. \tag{3.6}$$

The overnight session is 'short'

Our model applies only to the hours overnight following an election vote. The no shocks condition necessarily implies a short time period, as the model only holds whilst the political event dominates the news cycle. Our model so far has resulted in an expression for the expectation of the morning price of a financial asset, given the price of a binary option in the betting market. We need a way to relate financial prices during the night at time t, p_t, to this expectation. We assume a short period so that no interest can be earned and no equity dividends or bond coupons are paid. The expected appreciation of the financial asset overnight is $\mathbb{E}_t(p_T) - p_t$, as there no other cashflows. Since no interest can be earned prices must adjust so that

$$\textit{Risk premium} = \mathbb{E}_t(p_T) - p_t. \tag{3.7}$$

Although we assume risk neutral pricing in betting markets we do not do so in financial markets. The above risk premium is the excess expected return risk-averse investors require to hold the financial asset. During the night the 'morning' price p_T is expected to be p_0 with probability $1 - PB_t$ and p_1 with probability PB_t. Nothing beyond political event odds effects prices (an implication of the no shocks assumption). Another way we think of the overnight session as being 'short' is that any risk premium present is entirely due to the political event and nothing else. Risk factors beyond this particular election will not contribute to the LHS of equation 3.7, as the overnight period is too short for them to be earned. For example, investors cannot earn an additional return by holding, say, high beta stocks versus low beta stocks. Premia can only be earned due to the overnight volatility of the asset, and that volatility if solely caused by the uncertainty of the election result. This is not to say that risk premiums cannot change due to the result of the election (for example Mexican exporters would likely require higher expected returns if Trump were elected versus Clinton in 2016), just that this is already factored into the RHS of equation 3.7 via $\mathbb{E}_t(p_T)$ through the expected morning price under Trump[4].

Turning now to deriving an expression for the risk premium, we note that it must vanish at $E = 0$ or 1, or equivalently $PB_t = 0$ or 1, as at that point the price is certain. Any second order approximation to the premia must then be of the form $\lambda.PB_t.(1 - PB_t)$ where λ is a constant[5]. This closed form can also be recovered by assuming that the risk premium is proportional to the variance of p_T, which is also proportional to $PB_t.(1 - PB_t)$. (The variance is $(\triangle p)^2 \times PB_t.(1 - PB_t)$). Equation 3.7 now reduces to

$$PB_t.(1 - PB_t) \propto \mathbb{E}_t(p_T) - p_t.$$

Rearranging and combining the above with the expression in equation 3.6 yields our pricing model

[4] $\mathbb{E}_t(p_T) = \mathbb{P}_t(CLINTON)\mathbb{E}(p_T|CLINTON) + \mathbb{P}_t(TRUMP)\mathbb{E}(p_T|TRUMP)$.

[5] Any quadratic form which vanishes at the endpoints $PB_t = 0$ and $PB_t = 1$ can be written in this way.

$$p_t \approx p_0 + \triangle p.PB_t + \lambda.R_t + \epsilon_t \qquad \mathbb{E}(\epsilon_t) = 0 \tag{3.8}$$

where

$$R_t = PB_t.(1 - PB_t).$$

We impose one final condition on the parameters of the model. In the absence of risk neutral pricing in financial markets, the relationship, as written in 3.8, could have a non-monotonic form. This occurs when the term related to risk is sufficiently large. For example when $\lambda < 0$ and $p_1 < p_0$ there is an expected depreciation under $E = 1$. If λ has sufficiently large magnitude then a minima can occur in $PB_t \in (0, 1)$. It does not seem plausible that p_t would be lower than the worst case scenario p_1 for intermediate odds, when there is still a chance of the night ending at p_0. Monotonicity is ensured by the condition $|\lambda| \leq |\triangle p|$. [6]

Our pricing model is then summarised as

$$p_t = p_0 + \triangle p.PB_t + \lambda.R_t + \epsilon_t \qquad \mathbb{E}(\epsilon_t) = 0, \;\; |\lambda| < |\triangle p|. \tag{3.9}$$

Note that:

1. ϵ_t represents errors in beliefs between the betting and currency market that are inconsistent with typical conditions for weak market efficiency. If efficiency broadly holds then ϵ_t is mean zero, contains no unit root and 3.9 describes a non-linear cointegrating relationship between financial asset prices and betting market prices. This can be tested.

2. When there is risk neutral pricing in financial markets $\lambda = 0$ and equation 3.8 describes standard linear cointegration.

3. The $\lambda.R_t$ term is an adjustment to the price due to political risk. It results in a political risk premium (either positive or negative depending on the sign of λ). It is greatest when uncertainty is highest ($PB_t = 0.5$) and vanishes when the election outcome becomes known ($PB_t = 0, 1$).

4. As noted in Manasse et al. (2020), changing odds of the political event affect financial prices in two ways. The first is via the direct effect on the probability of yielding p_0 versus p_1. The second is via the effect on the risk premium.

[6] The gradient of the cointegration relationship is $\triangle p - \lambda + 2\lambda.PB_t$ so is $\triangle p \pm \lambda$ at the endpoints. The condition $|\lambda| \leq |\triangle p|$ ensures the gradient does not change sign.

Alternative derivation: Mean-variance preferences

An alternative approach to imposing a 'short' overnight session to derive an explicit expression for the risk permium is outlined in Manasse et al. (2020). This assumes an explicit set of preferences for investors in the financial markets. The conditions that there are no cashflows during the session as well as the quadratic form of the risk premia are effectively baked into the utility function.

Proceed by considering a representative investor who chooses between holding a proportion ω of her wealth in the financial asset and the rest in a risk free asset. Standard mean-variance preferences are assumed so that the investor maximises:

$$U(w) = \omega . \left[\mathbb{E}_t(p_T) - p_t\right] - \frac{r}{2}\omega^2\sigma^2,$$

where r is the coefficient of absolute risk-aversion and σ^2 is the portfolio variance. The first term is the expected appreciation of the asset from a time t overnight before the full results of the event are apparent, and time T, the time at which E is realised. The second term is a penalty (under risk aversion) for holding the risky financial asset and is proportionate to the risk aversion coefficient r and the portfolio variance $\omega^2\sigma^2$.

Firstly we note that the expected appreciation $\mathbb{E}_t(p_T) - p_t$ must be positive for a risk averse investor to hold any of the risky financial assets. When this is the case the first order condition is

$$\mathbb{E}_t(p_T) - p_t = \omega r \sigma^2. \tag{3.10}$$

The portfolio share reduces with increased risk aversion r and asset variance σ^2. Taking the supply of the financial assets S as fixed. Clearing of the financial market implies that $\omega W = S$, where W is the total available wealth of investors (assumed to be greater than S and also fixed). Then $\omega = s$ where $s = S/W$. σ^2 can be evaluated and is the variance of $p_0 + X \times \triangle p$ where X is a Bernoulli random variable with probability $\mathbb{P}_t^f(E = 1)$. This is $(\triangle p)^2 . \mathbb{P}_t^f(E = 1) . (1 - \mathbb{P}_t^f(E = 1))$. Thus the time varying risk premium can be written as

$$\begin{aligned} \lambda_t &= -\lambda \pi_t (1 - \pi_t) \\ \lambda &= -rs\,(\triangle p)^2 \\ \pi_t &= \mathbb{P}_t^f(E = 1). \end{aligned}$$

Substituting the above into equation 3.7 along with the approximation $\pi_t = \mathbb{P}_t^f(E=1) \approx PB_t$ yields an expression for the expected appreciation of the financial asset overnight,

$$\mathbb{E}_t(p_T) - p_t = -\lambda R_t.$$

This can be combined with the previous expression for $\mathbb{E}_t(p_T|\epsilon_t)$, equation 3.6, to recover the cointegration model.

Discussion

Our pricing model relies on a number of restrictions in its derivation. The model is encapsulated in equation 3.9. This describes a cointegrating relationship between a combination of the financial asset price with a non-linear function of the betting market binary option price. Any tests of the model are necessarily joint tests of the model's restrictions. Nonetheless, we can make some statements about how different failures of the model relate to the various restrictions imposed.

The fact that ϵ_t is mean zero and contains no unit root comes from two restrictions. The first is the NS assumption. This states that only changes to the odds of the event, $\mathbb{P}(E=1)$, persistently affects financial prices. This may not hold due to two reasons. The first is that non-election related news is changing asset prices. For instance if aliens invaded the planet during the night of the Brexit referendum we would expect stocks to sell off regardless of $\mathbb{P}(E=1)$! The second is that there is information contained in the election results beyond $\mathbb{P}(E=1)$ (and not perfectly correlated with $\mathbb{P}(E=1)$) that informs the financial price. This could be, for example, due to a higher vote share for one outcome, or the geographical distribution of votes. Either way then ε_t in equation 3.1 is not mean zero. $\mathbb{E}_t(\varepsilon_T|E)$ will not vanish in the conditional expectations in equation 3.2 and contributes to the RHS of equation 3.9. The second restriction which imposes the conditions that lead to cointegration is similarity of beliefs of investors in the two markets, equation 3.4. Testing the absence of a unit root in ϵ_t is thus a joint test of the NS assumption and weak market efficiency (although equivalent beliefs of investors in both markets is not the same as market efficiency, it is one of the conditions that lead to it). However, in our empirical work we will interpret deviations from $\epsilon_t = 0$ as deviations from equivalence of beliefs and thus weak market efficiency (EWMH). This is as ex-post, we know that aliens did not invade the planet (or war broke out or something else) and we can argue that the election really did dominate prices on the nights in question.

Equation 3.9 also specifies the shape of the contemporaneous function of prediction market binary options and financial asset prices that are stationarity. This shape derives from the restrictions we place on preferences of market participants. These are, one, risk neutral pricing in prediction markets, and two, mean-variance preferences of financial investors (when the latter is not assumed we show that the shape of the relationship must be approximately that of equation 3.9 through a second order approximation). Relaxations of these conditions lead to changes in the shape of the contemporaneous relationship, but not the time series properties of ϵ_t. We do not provide a formal proof, but if, for example risk averse pricing is present in the betting markets then the market price PB_t will be a contemporaneous monotonic function of the belief of a representative investor $g(\mathbb{P}_t^b(E=1))$, rather than equal to it. The result will be that the betting price PB_t will occur in a non-linear function in the RHS of equation 3.6 and thus in the pricing relationship. The conclusion that we make (again without formal proof) is that tests on the *shape* of the relationship in equation 3.9 are really tests on the restrictions we make about the preferences of investors.

Finally we comment on the condition that the overnight session is short. We claim that this is a weak condition. Firstly, it implies there are no cash flows. This is easy enough to verify for any particular asset and setting but if there were a cash flow then a performance price series that adjusts for it could be trivially constructed. Secondly, this condition means that no interest or risk premium (beyond that related to election uncertainty) can be earned. If interest, or an additional risk premium is present then the magnitude earned overnight would be proportional to the time left to the morning, $T-t$. There would be an additional term, say $\delta(T-t)$ cropping up in the difference of p_t and $\mathbb{E}_t(p_T)$ and thus on the RHS of the pricing equation 3.9. However, the magnitude of this term would be tiny. For example, if a relatively large annualised premium of 3.65% were being earned throughout the night this term would vary by a fraction of a basis point (there are 365 days in a year and the overnight session is a fraction of a day!). We thus ignore this possibility and assume this condition holds exactly.

4 Statistical tests

This paper will make use of two statistical tests to test our model. They are set out as follows.

Phillips and Ouliaris test for the presence of cointegration

We rely on the Engle–Granger methodology via univariate regressions of financial asset prices against that of the betting market contract

$$p_t = p_0 + PB_t \times \triangle p + \epsilon_t.$$

The presence of a unit root in ϵ_t is tested via a residual regression. However, we use the Philips–Perron test statistics, Z_α and Z_t rather than Dickey–Fuller. These include a non-parametric adjustment to the long run variance, which is robust to misspecified serial correlation and heteroskedasticity. If the statistic is below the critical value we reject the null of a unit root in the residual and conclude that cointegration is present between the asset price and the betting contract.

There are other common cointegrating methodologies available in the literature that we could use. These include the Johansen maximum eigenvalue test, based on the error-correcting form of the bivariate process $(p_t, PB_t)'$. We prefer the Phillips and Ouliaris test due to the convenience of the non-parametric adjustment (a lag length must be chosen in the Johansen framework). We have checked our results using the max eigenvalue test though and they confirm our findings. Results are available by contacting the author directly.

Johansen constraint test

The Johansen methodology considers the error-correcting form for our multivariate price process p_t

$$\triangle \boldsymbol{p_t} = \boldsymbol{\alpha}(\boldsymbol{\beta' p_{t-1}} - E(\boldsymbol{\beta' p_{t-1}})) + \sum_{i=1}^{k-1} \boldsymbol{\Gamma_i} \triangle \boldsymbol{p_{t-i}} + \boldsymbol{u_t}.$$

Likelihood ratio tests can be formulated for restrictions on model parameters in this framework called constraint tests. We use this to test whether risk aversion is significant in equation 3.9 (i.e. $H_o : \lambda = 0$). This is done by considering the trivariate process

$$x_t = (p_t, PB_t, R_t)'$$

and checking whether the coefficient in the last dimension (the dimension of risk) are not significantly different from zero.

A note on lag specification

The above frameworks require lag length choices to be made when specifying tests. For the Phillips and Ouliaris residual regressions non parametric adjustments are included that are robust to misspecified serial correlation in the disturbances. All that is required is to choose a lag truncation parameter in the estimator of the long-run variance. We use the Newey and West (1994) plug-in procedure of $\left[4.\left(\frac{T}{100}\right)^{2/9}\right]$. The Johansen constraint test is not robust to model misspecification, although we note that there is some evidence that testing constraints are not unduly effected in the presence of serial correlation, Silvapulle and Podivinsky (2000). The choice of lag involves a trade off between accuracy of specification and statistical power. Models of higher numbers of lags and hence parameters will necessarily be more accurate (as shorter lag lengths are nested) but will result in a loss of statistical power. We start by only adding lags if they result in a statistically significantly different model, as implied by a likelihood ratio test. Lags are added to the base (zero lag) model until the likelihood ratio test fails to reject the restricted model. As the constraint test is not robust in the presence of serial correlation, we check that the residuals are indeed not correlated (according to a Portmanteau test). If they are, then further lags are added until serial correlation is no longer present.

5 Results

In this section we evaluate our theory on real world data from some political events from the last few years. All results tables are found in the Appendix. The choice of events is important. For our model to apply, the results of the elections need to come during overnight hours. This rules out, for instance, the 2020 US presidential election, where various legal challenges and recounts in several states took days and even weeks to resolve. Our model also has no applicability if the result is realised instantaneously. We require a meaningful period of time for the information flow to occur. We exclude UK general elections. This is due to the very high accuracy of the exit poll. These polls measure how people declare they have voted on the day itself, at a selection of particular, secret, polling stations. They are much more accurate than any pre-election polling, Curtice et al. (2011), due to the fact that there is no measurement error of respondents. The polls are released just after votes closed and are effectively an announcement of the winner. Instead we chose three elections where we believe our model is likely to apply. In each case we consider an actively traded binary betting contract along with a collection of heavily traded financial assets. We begin our investigation with the first great political shock of 2016. This was the United Kingdom European Union membership referendum, commonly referred to as the Brexit referendum.

5.1 The Brexit referendum

On 23rd June 2016 the UK voted in a country wide referendum to leave the European Union. This was one of only three UK wide referendums. The first was in 1975 and involved a vote to join the European Community. This is known as the common market and is what become the European Union. The second was the United Kingdom Alternative Vote referendum which was rejected by a wide margin. The third was the Brexit plebiscite. Turnout was historically high at 72% and the result was narrow: 51.9% to leave the EU versus 48.1% to stay.

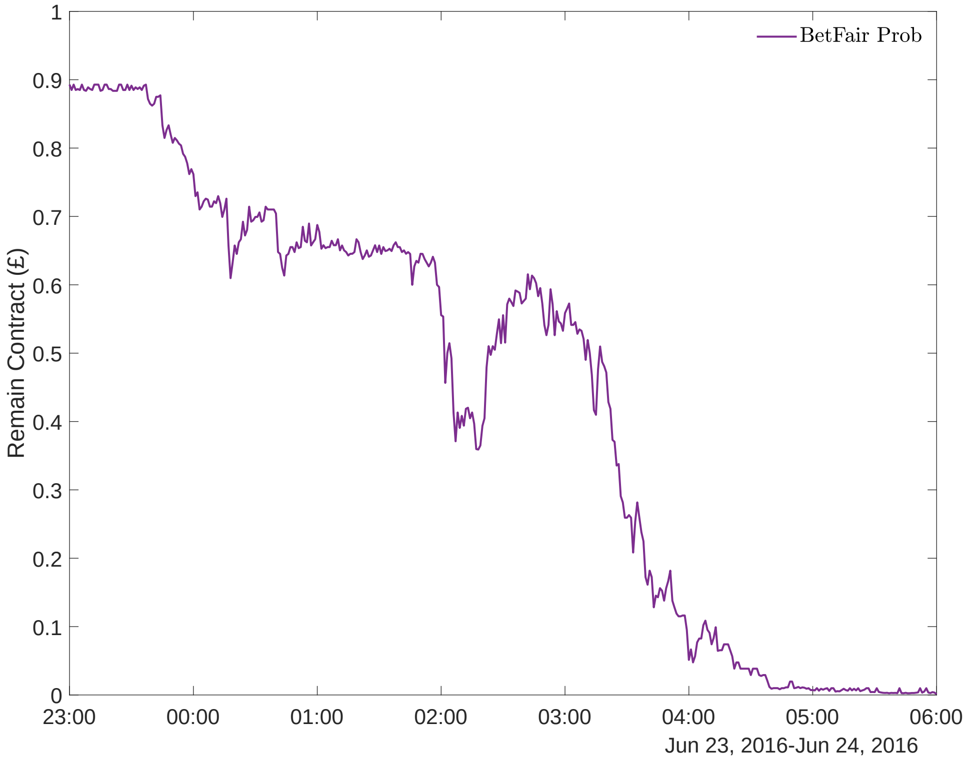


Figure 1: Betfair contract for 'Remain' on the night of the Brexit referendum.

The vote was split up into a large number of voting areas (382) and each area announced at different times throughout the night as their counts were finalised. The result was unexpected. There was widespread polling data that showed a small but consistent lead to stay in the European Union ('Remain'). For example, a poll was conducted by YouGov which was published shortly after voting closed at 10pm, YouGov (2016). This showed the vote-share for leave at 48.4% with a standard sampling error of 3%.

There was an actively traded political market that traded both up to the day of voting and overnight as results were announced. Contracts that paid out £1 in the event of both 'Remain' and 'Leave' were listed on the Betfair Exchange market. This operates like a limit order book. There did not appear arbitrage opportunities in the exchange in that the sum of the prices of the contracts do not deviate sufficiently from £1.[7] Around £130m was wagered in total with £50m changing hands on the night.

There was widespread belief that the country would vote to remain in the EU. Figure 1 shows the price action of the contract price for Remain. Voting closed at 22:00 on

[7]Owning both one contract for Remain and one for Leave guarantees a payout of £1.

23rd June.[8] The YouGov poll on the day was released shortly afterwards. The first result released was for Gibraltor around 23:36. This was inconsequential. Gibraltor is an overseas territory located at the southern tip of the Iberian Peninsula bordering Spain. As expected, the electorate there voted overwhelmingly in favour of Remain (96%). As can be seen, this did not affect the prices in the betting market. The risk neutral probability of remaining in the EU seen close to 90%. Meaningful results started to be announced from midnight with Newcastle upon Tyne being the first to announce. As can be seen from Figure 1, prices had already started to move a little against Remain from 23:45, possibly due to information leakage, or to private polling conducted by some hedge funds. There was a large quick move from 73% to 61% for Remain between 00:16 and 00:18 as Sunderland announced. The result there showed a lead for Brexit over Remain of 21% versus an expectation of around 6%. Prices moved against Remain for the next few hours, with a particular collapse around 2am which mostly recovered shortly afterwards. However, by 4am Remain was trading at under 10% and a probability for Brexit of 99% was implied at 05:21. The BBC finally projected Brexit at 5:39am.

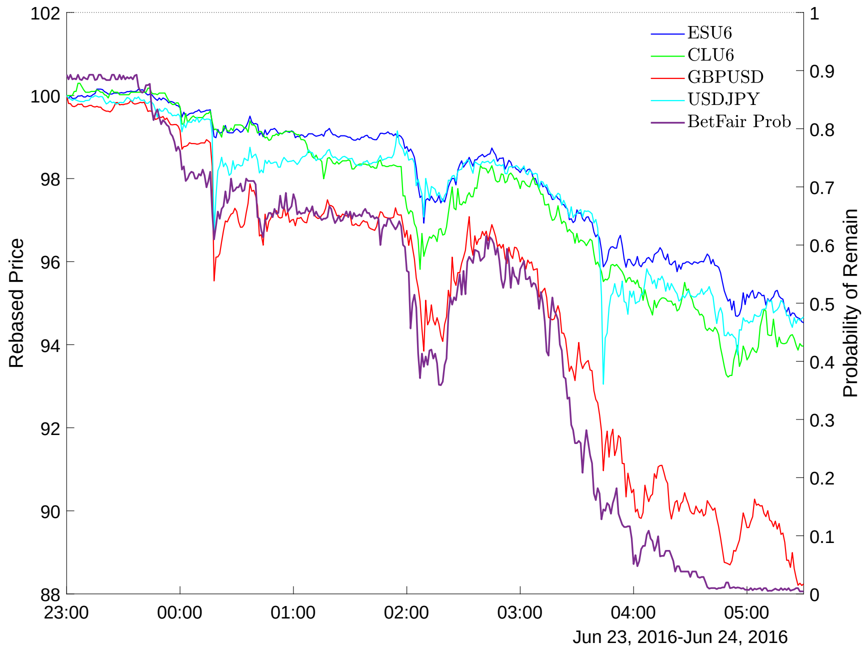
Financial assets

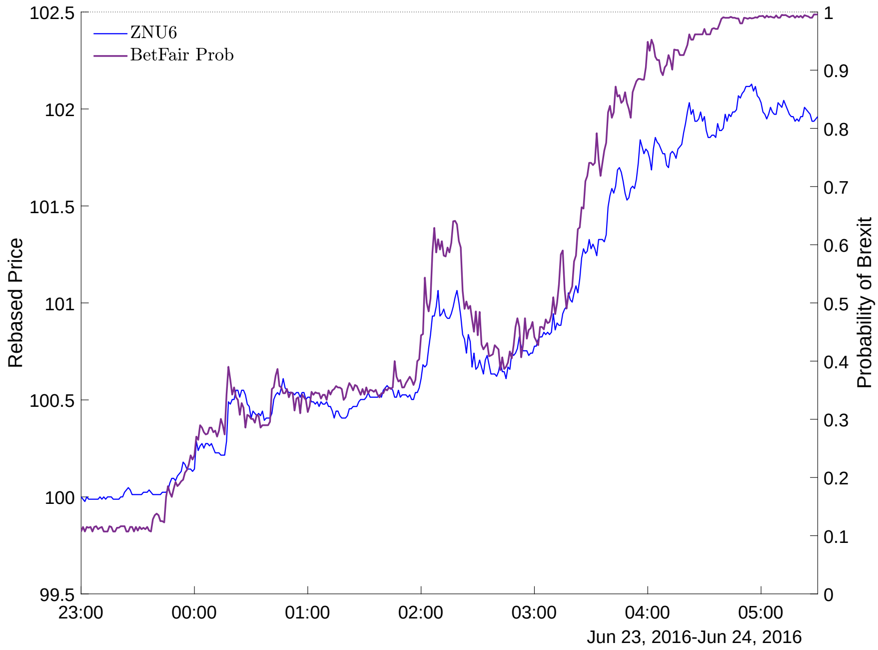
We consider five financial assets for this event. Two currencies, one stock index future, one commodities future and one fixed income future. All asset classes are covered by this collection and all but GBPUSD was the most liquid leading indicator of those asset classes. GBPUSD is included as it is especially relevant for the UK and a country-specific indicator of the health of the economy. All futures were listed on the Chicago Mercantile Exchange. Foreign exchange and futures are studied as these were open for trading during the results session. Cash markets do not generally trade overnight and the UK specific FTSE100 future only opened part way through the night and so is excluded. We consider the period 23:00 BST on 23rd June to 05:30 on 24th June and sample prices every minute. During these hours all contracts are open,[9] the only information released to the market is that contained in the vote results and the betting markets converge to certainty.[10] Table **??** lists the five assets with a description, their starting and ending prices and the percentage change. As can be seen four assets depreciated whilst the treasury future appreciated. This is as expected as the result was not expected and was considered negative for trade and hence the economy. Generally fixed income assets appreciate in times of economic uncertainty as both expectations of future interest rates

[8]Times in this section are all quoted in British Summer Time. This was the current timezone in the UK on this date

[9]Note, the E-mini S&P500 Future hit a trading limit shortly after 05:30 on 24th and was thus put into auction.

[10]The futures were closed for an hour between 22:00 and 23:00 British Summer Time. This was straight after the vote closed but before results were announced.





Note: Assets that depreciated are shown versus the Remain contract price whereas the asset that appreciated is shown versus the contract price for Brexit.

Figure 2: Rebased financial asset prices versus Betfair contracts.

fall and money moves out of risky assets. The Japanese yen is considered a safe asset so appreciated against the USD whilst GBP is seen as risky versus the USD and depreciated.

Figure 2 plots assets that depreciated against the betting contract for Remain and the treasury future is plotted against the contract for Brexit. The financial assets do appear to be moving more or less in lock-step with the betting markets. This is pleasing as this is implied by our theory. However, we now turn to statistical tests of the theory in the following sections.

Cointegrating relationships

Table 2 shows the results of the Phillips–Ouliaris Z_t and Z_α tests for linear cointegration. Both tests reject the null of a unit root in the residual ϵ_t at the 99% level in favour of stationarity. During a typical trading day there will be information that would persistently affect the financial assets over and above that which affects the odds of the UK voting to leave the European Union. The fact that the null hypothesis of no cointegration is firmly rejected in favour of a single common trend in the hours after the Brexit referendum is strong evidence in favour of our theory.

Next we turn to the question of non-standard risk preferences. Manasse et al. (2020) derive a model of foreign exchange prices and the Betfair contracts for the months before the referendum. This is based on cointegration and includes a component related to risk. The results of the previous subsection suggest that we do need to deviate from risk neutrality to explain the high frequency price action observed during the hours when the referendum results were released. However, could a model that includes a non-zero risk aversion parameter provide a better fit? To answer this question we consider the trivariate systems $x_t = (p_t, PB_t, R_t)'$ for each financial asset p_t. As described in the pricing model in Section 3, the third component in x_t relates to risk, and is identical to that considered in Manasse et al. (2020). With or without risk aversion our theory implies a single cointegrating vector $\beta' = (1, -\triangle p, -\lambda)$ and two common trends PB_t and R_T[11]. We test the restriction $\lambda = 0$ in the Johnasen framework with $r = 1$ using the constraint test. The results are shown in table 3. We do not reject the null hypothesis of risk neutrality for any asset at the 95% level and only reject the hypothesis for a single asset, the treasury future at the 90% level. The constraint test p-value for the pound of 0.51 suggests there is no evidence at all for the $GBPUSD$ exhibiting risk aversion behaviour with regards to Brexit. This is in contrast to the conclusions of Manasse et al. (2020). However, this author takes issue with the application of a cointegration theory for longer term periods. The basis for the model is that the only determinant of the $GBPUSD$ price in the weeks and months preceding the Brexit vote is the result of that referendum. Whilst we agree that the probability of the referendum result is a large

[11] PB_t cannot possibly be cointegrated with $R_t = PB_t.(1 - PB_t)$ if it is non-stationary.

determinant of prices, and is the only determinant in the overnight hours after the vote, we do not believe it is the only information affecting exchange rates for longer periods. Their theory assumes no other news beyond that relating to the referendum affects the British Pound and United States dollar exchange rate in the months up to the vote. For example, this would imply that any and all news about the health of the US economy, be it consumer demand, trade barriers, protectionism etc., would have no persistent effect on GBPUSD. We find this implausible and postulate that including risk aversion is simply an exercise in over-fitting to avoid rejecting a model that should never have been applied on this timescale. We suggest that other models such as vector auto-regressive or factor models would be more economically justified for longer periods of time.

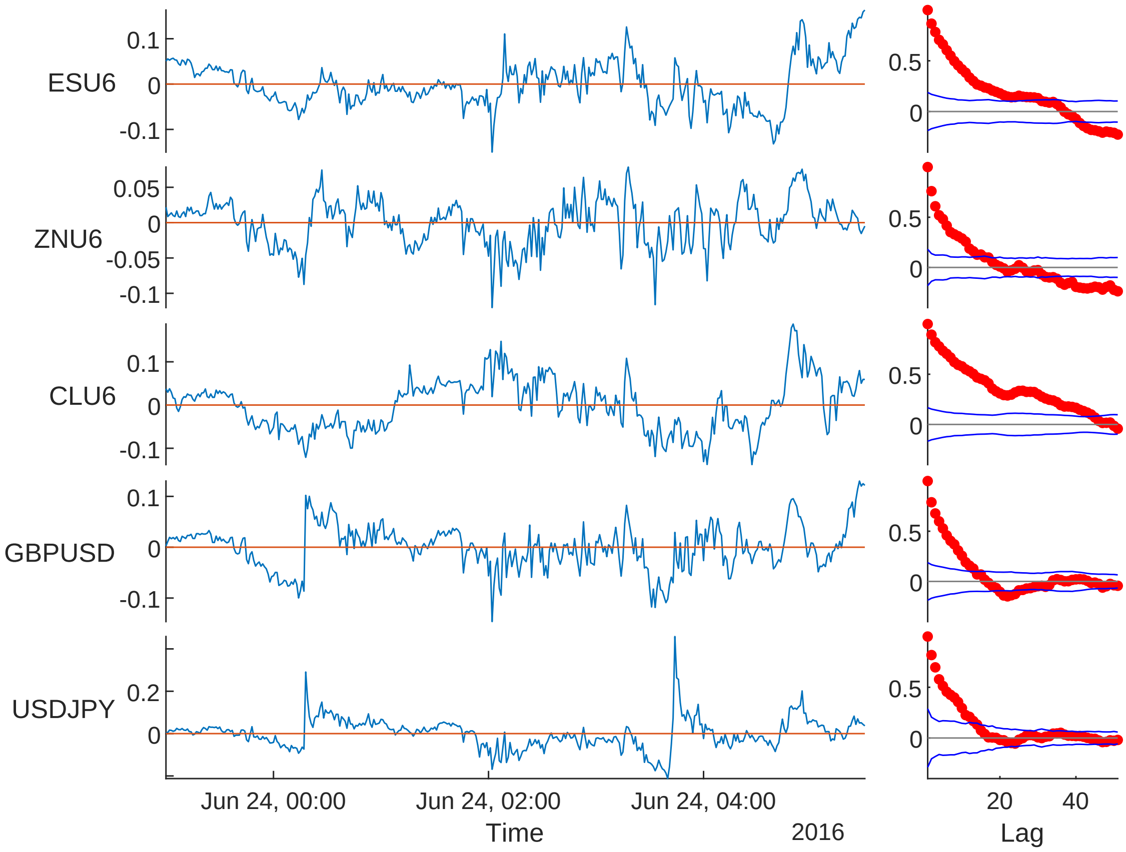


Figure 3: Cointegration errors with autocorrelation functions for Brexit.

Deviations from long term relationships

The strong evidence for cointegration in the hours after the referendum is a pleasing result and agrees with our pricing model. The model holds under conditions including that the only persistent affects on asset prices are related to the probability of voting to the leave the European Union. This gives rise to the single common trend. Other conditions of our model are consistent with those of weak market efficiency holding. However, the presence of significant autoregressive terms in the error correction model of the Johansen framework[12] suggests that market efficiency may be less clear cut.

We can study short term deviations from the long term relationship in two ways. The first is via the coefficients of the VECM of the full multivariate system. The second is via the univariate errors of the cointegration regression. The full VECM is

$$\triangle \boldsymbol{p_t} = \boldsymbol{\alpha}(\boldsymbol{\beta' p_{t-1}} - \boldsymbol{c_0}) + \sum_{\boldsymbol{i=1}}^{\boldsymbol{p}} \boldsymbol{\Gamma_i} \triangle \boldsymbol{p_{t-i}} + \boldsymbol{u_t}$$

where

$$\boldsymbol{p_t} = (\text{ESU6, ZNU6, CLU6, GBPUSD, USDJPY, } PB_t)'.$$

It is found that an error correcting model with a single short run autoregressive term Γ_1 is significantly different to one with no term but that higher numbers of lags are not significant. Table 4 shows the significant fitted values of Γ_1 in the VECM. Deviations from the long-run relationships ($\boldsymbol{\beta' p_{t-1}} - \boldsymbol{c_0} = 0$) cause adjustments in both markets through the first term, $\boldsymbol{\alpha}$. The second term, $\boldsymbol{\Gamma_1} \triangle \boldsymbol{p_{t-1}}$, governs short-run dynamics. Of particular interest is whether the betting markets lead the financial markets and/or vice-versa. For short run dynamics this can be explored by testing the coefficients of the coefficient matrix $\boldsymbol{\Gamma_1}$. Jointly testing the significance of the first $n-1$ values of the final column of $\boldsymbol{\Gamma_1}$ is a test of whether short run changes in the betting markets cause short run changes in financial markets. Testing the first $n-1$ values of the last row tests whether financial markets cause betting markets in the short run. LR tests for these hypotheses are also shown in table 4. There is strong evidence that short run changes in betting markets cause short-run changes in financial markets with the null of no causality rejected at the 5% level. However, there is no significant causation in the other direction. This is a very interesting result. Short term deviations from the long term relationship that revert occur as the betting and financial markets do not move exactly in lock-step. However, during the establishment and convergence of these deviations the betting markets are leading the financial markets. This suggests the betting markets are discounting political information more quickly (and on the scale of minutes) than financial markets.

We can also study inefficiencies via the simpler residuals of the univariate cointegrating regressions. This error is a linear combination of two asset prices (a financial asset and a betting contract) and can be traded. The return of the error can be created by

[12]Significant lags are shown in the third column of Table 3.

holding the two assets in a ratio equal to the cointegration ratio. It is also largely risk free, at least with respect to political risk. The exposure to the result of the referendum in one asset is hedged with equal and opposite exposure in the other. Serial correlation in the cointegration error implies that deviations from the long term relationship (long term in this context being a few hours) can be predicted. This again demonstrates violations of weak market efficiency on a short term timescale.

The estimated cointegration error is $\hat{\epsilon}_t = p_t - \hat{p}_0 - PB_t \times \hat{\triangle p}$. We consider the quantity $\hat{\epsilon}_t / |\hat{\triangle p}|$ which is the cointegration error normalised to units of the betting contract. This can be more readily compared across different assets whose prices have different magnitudes. The normalised cointegration errors with the sample Auto-Correlation Function and robust Bartlett intervals for each asset are plotted in Figure 3.[13] This shows significantly positive autocorrelations out to around 20 minutes. This provides evidence of deviations from weak market efficiency of the order of minutes to tens of minutes on an ex-post basis.

Profit opportunities

We now turn to the possibility of profiting from mean reverting deviations to the long term relationship. Rather than focus on trading assets outright (or "naked" in trading parlance) via predictions from an error correction model, we focus on trading the cointegration error. This is effected by buying or selling the financial asset against an opposing position in the betting contract with sizes equal in ratio to the cointegrating ratio.[14] This is effectively politically risk free and involves taking positions in pairs of contracts simultaneously rather than larger numbers of assets outright which be would exposing oneself to greater risk.

We use a modification of the common Bollinger Band trading signal. When used to trade non-stationary price series, Bollinger Bands are used to produce directional trading indicators. A corridor around a moving average of the price of the asset is constructed by adding and subtracting twice the sample standard deviation, σ_t, of the price calculated along the length of the preceding moving average window. For a contrarian strategy the signal will be to sell when above the upper band and to buy when below the lower band. All positions are closed out once the betting contract has converged to certainty. As the cointegration error ϵ_t has zero expectation we do not calculate the corridor around a moving average but about zero.

Bollinger Bands and trades are shown in Figure 4 for all five assets. When $\hat{\epsilon}_t$ is above the upper band we sell and buy when below the lower band. Both long and short

[13]The Bartlett intervals are adjusted to allow for serial correlation in the variance of ϵ_t. Such serial conditional heteroskedasticity is expected as this is a financial time series.

[14]To be clear $\hat{\triangle p}$ notional of the betting contract is traded for every unit notional of financial asset exposure

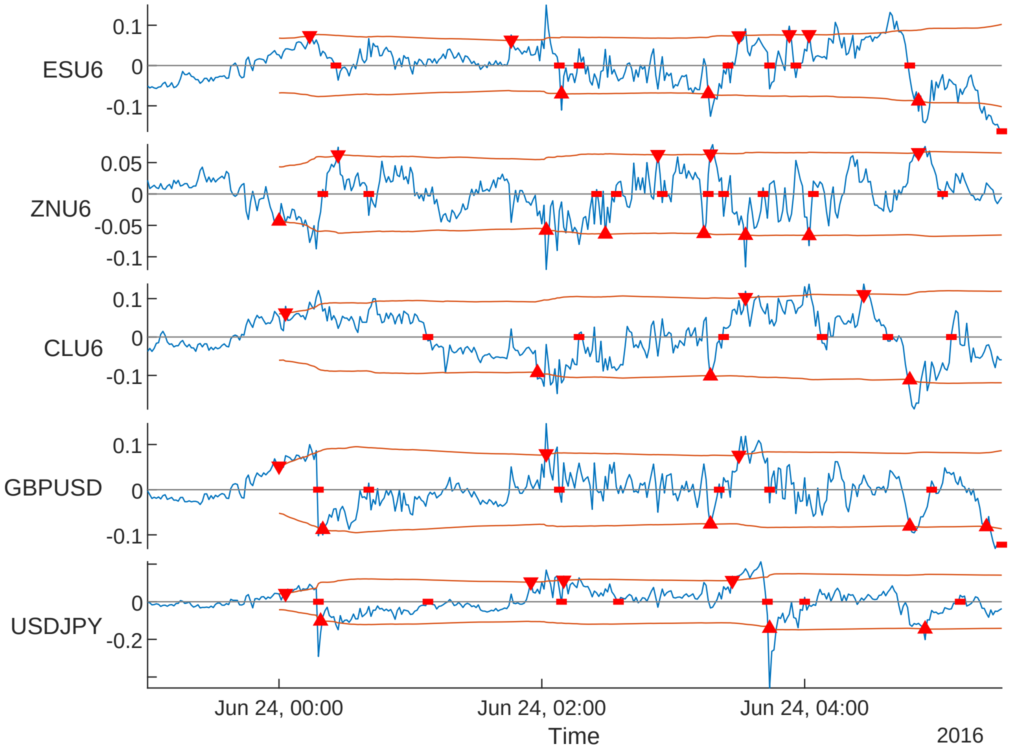


Figure 4: Bollinger Band trading strategies for Brexit.

positions are closed out when $\hat{\epsilon}_t$ has converged to zero, the expected equilibrium level. Note that selling the error is equivalent to selling the financial asset and buying the betting contract for Remain (or selling the contract for Leave) in the cointegrating ratio. At time t where $t >$midnight, the period from 11pm, 23rd June to t is used to calculate the sample standard deviation σ_t. $\hat{\epsilon}_t/|\hat{\triangle p}|$. Table 5 shows the profits and success of the trades the scheme generates. The strategies appear excellent. Across all assets the strategy has 36 winning out of 38 total trades with an average profit per trade of 10p. Trading costs for these markets are relatively small[15]. They are well below the order of gross profits and combined with slippage should not be above 1p per contract. There would be positive net profits from executing these trades.

Unfortunately, being able to apply this strategy ex-ante is not at all realistic. Firstly the trader would need to be confident weak market efficiency would ultimately hold. Secondly, and more importantly, the calculation of $\hat{\epsilon}_t$ uses the cointegrating relationship estimated ex-post using data from the whole night. Ex-ante, a successful investor would have to correctly foresee the cointegrating relationship. In the case of the pound, this is equivalent to knowing the conditional expectations $GBP_L = E(GBP_T|BREXIT)$ and

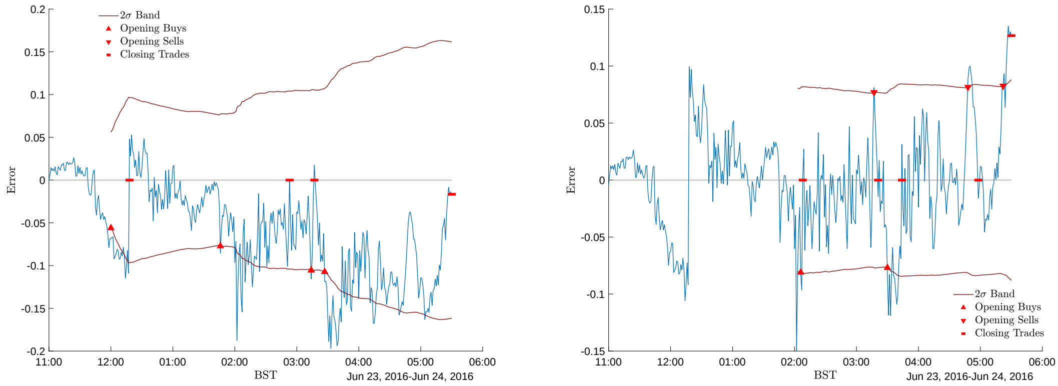
[15] In terms of transaction costs, selling the pound would cost about 2–3 hundredths of a cent at that time, whereas the Betfair cost is 3–5% levied on any bets that pay out. This would slightly change the ratio of the portfolio but not significantly affect profits or these conclusions.

$GBP_H = E(GBP_T|REMAIN)$, ie where the financial asset settles given a vote to leave the EU (and the price it would have achieved in the counterfactual remain scenario).

We investigate an application of the same strategy when the cointegrating relationship is informed by forecasts of the conditional expectation from a market commentator. Prior to the referendum in late April the investment bank JPMorgan published a forecast of precisely where the pound would be priced given a vote to leave the EU. This was 1.32, Peters, 2016. If a trader on the night used this value for GBP_L and assumed that prices were efficient at 11pm then she would evaluate GBP_H at 1.522[16] and $\hat{\triangle p} = GBP_L - GBP_H = -0.2024$. This is a larger predicted drop than estimated from the ex-post regression. Simply put, JP Morgan's prediction was too pessimistic. The pound did not fall (in the overnight session) as much as expected given the positive referendum result for Brexit. The cointegration error estimated in this way does not converge to zero, as the pound does not fall as much as expected. The misspecified error is shown on the left hand side of figure 5 with Bollinger Band strategies. The error does drift downwards. Remarkably though, and against this author's expectations, the trading strategy is still profitable, albeit less so. The error is biased downwards and so only "buys" are executed. It appears that JP Morgan's conditional estimate for the pound, albeit a little pessimistic, was close enough to allow a profitable strategy. Although biased, there does seem to be some mean reversion of this estimated cointegration error. The conservative natures of the Bollinger Band signals avoid losing money. However, it is not the case that conditional predictions for the other financial assets are available and so this strategy does not appear readily applicable to the other financial assets.

There does seem to be one way in which ex-ante profits may have been possible for the other symbols without the foresight of conditional predictions of the asset prices. This is by using the first few hours of the night to estimate the cointegrating relationship and trading in the latter part of the night. This actually does result in profits. An investor has the most chance of estimating the cointegrating relationship if there is sufficient support of the relationship within the training set. However, waiting for too much training data reduces the window of opportunity. We settle on using data up to the point where the betting market predicts a 50% chance of Brexit. This occurs at 2:02am. The strategy is shown on the right hand side of Figure 5. The error does appear to converge to zero in the trading period. There are five trades with all but the final trade profitable. We note that the final trade also lost money when trading using the ex-post estimated relationship. It appears that the long term relationship estimated from the first few hours of the night is stable and persists into the later hours of the night.

[16]This is because at 11pm GBPUSD was at 1.5007 and the betting contract implied a 10.7% chance of leaving the EU. If 100% chance of leaving the EU results in $GBP = 1.32$ then this implies that a zero chance implies $GBP_H = (1.5007 - 0.107 \times 1.32)/0.893 = 1.5224$. Note this is similar to $\hat{p_o}$ calculated from the ex-post cointegrating relationship shown in Table 2.



Bollinger Bands strategies for misspecified cointegration error when using $E(GBP_T|BREXIT) = 1.32$ (left) and when using the first part of the night to estimate (right).

Figure 5: Possible ex-ante estimated cointegration errors and Bollinger Band strategies.

Turning now to the other assets, we first check the cointegration tests for the shorter training period. The relevant p-values are shown in table 6. Naturally these tests have less power with less data. Nevertheless, four out of the five assets have results which reject the null of no cointegration in favour of cointegration at high significance levels. The exception is the crude oil future. We note that the significance was less for this asset when testing on the whole night and conclude that the smaller data-set generates insufficient power to reject the null. The fact that there is strong evidence of cointegration for four of the assets in this smaller training period is pleasing.

The results of applying the ex-ante Bollinger Band strategy to all assets is illustrated in Figure 6. Gross profits are shown in Table 7. For four of the assets the strategy is impressive. There are less trades due to the smaller trading period, yet profits per trade are similar to those using the ex-post cointegration relationship. The exception is the S&P500 future. A single trade is executed which unfortunately loses around 40p. This occurs as the estimated error does not converge. The cointegration relationship does not appear stable for this asset. Nevertheless, the losses of this trade are outweighed by the profits in the other contracts.

These trading results are quite remarkable. Starting from some reasonable conditions we wrote down a model of prices which implied cointegration. We find that there is, for most assets, strong evidence of that theory generated in the first part of the night. Moreover the estimated cointegration relationship is so stable that trading profits can, apparently for most assets, be generated by taking positions against deviations from those long-term relationships. Whether or not they would be realistically crystallised by an intelligent investor is debatable. We have written down a theory after the fact. Evidence does appear to have quickly emerged on the night that the theory holds. However, to

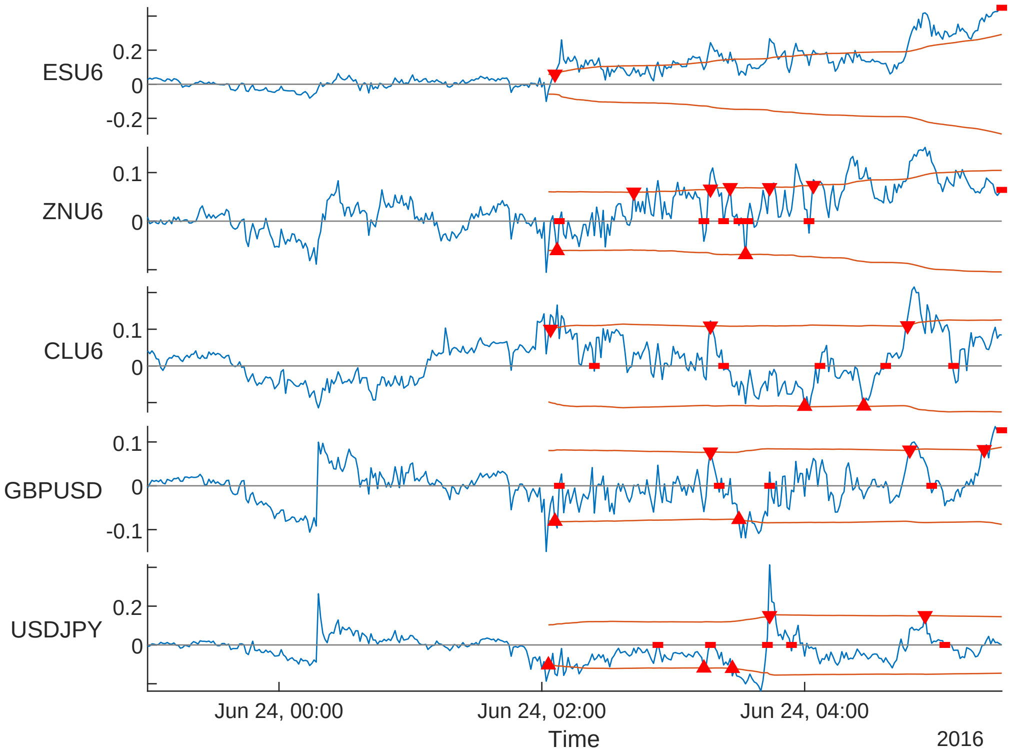


Figure 6: Ex-Ante Bollinger Band trading strategies for Brexit.

execute the trades described in this paper would require a market participant to have confidence that the theory would indeed continue to apply for the remaining hours. It would take a brave soul to do so. It is not the case that there have been large numbers of referendums or political events where our approach has been shown to work[17]. Whether or not these profits are realistic in practice is a philosphical point. One thing is certain though, and that is that it is far easier for me to write this apparently successful study with hindsight after the fact than actually risk my money upfront on the night!

5.2 The 2016 United States presidential election

We next study the second great political shock of 2016, the US presidential election, held on Tuesday 8th November. The Republican ticket of Donald Trump and Mike Pence, against expectations, beat the Democratic ticket of Hilary Clinton and Tim Kaine.
Under the Electoral College system the winner needs at least 270 of the 538 electors. There are 51 voting areas, 50 states plus the special federal district of Washington D.C., that each award electoral votes. Electors, for the most part, vote for the winners of

[17]Arguably, if there had been a history of success for this strategy, then the opportunity would have disappeared due to the actions of arbitrageurs.

the popular vote within their respective area[18]. The democratic candidate led in the vast majority of nationwide and swing-state polls. However the margin decreased as the election was approached. On election day Donald Trump out-performed his polls, winning all of the key battleground states of Florida, North Carolina, Ohio and Iowa. Additionally, and against all expectations, he took the three formerly Democrat "rust-belt" states of Pennsylvania, Michigan and Wisconsin. The Republican ticket's votes were exceptionally well distributed. Donald Trump won 30 states with 306 electoral votes whereas Hillary Clinton won 20 states with 232 votes.[19] This is despite Trump garnering 2.87 million less votes than Hilary Clinton.

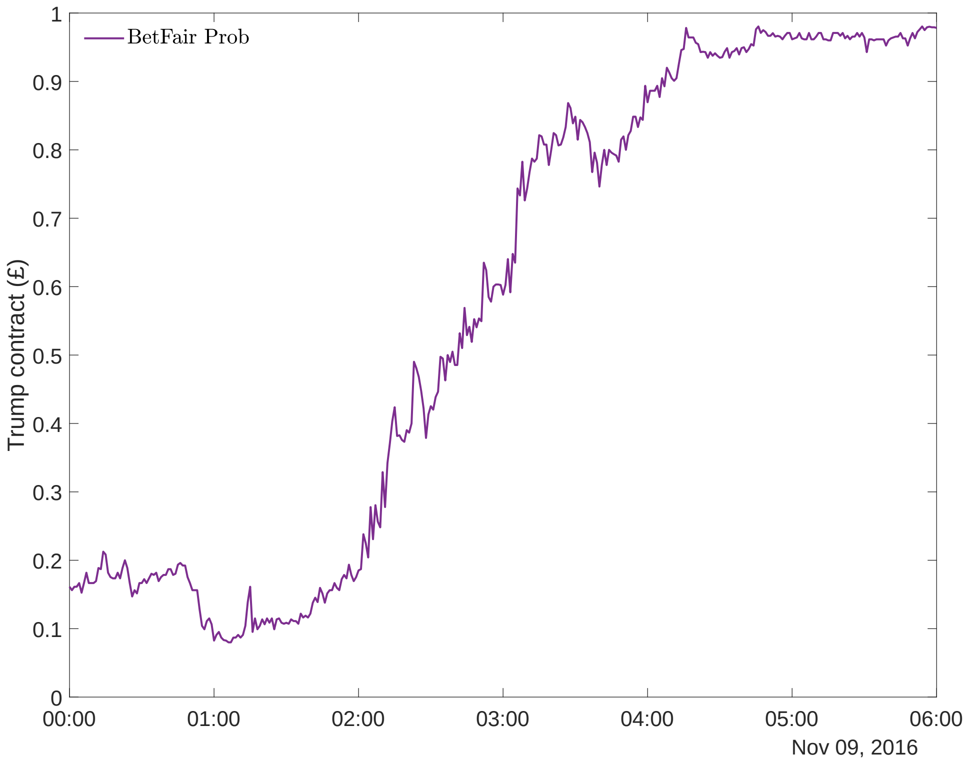


Figure 7: Betfair contract for a Trump win in the 2016 presidential election.

Similar to the Brexit referendum there were multiple vote counts (51 versus 382) from different regions occurring throughout the night after voting ended. The situation is complicated further by polls closing at different times in different states. However, evolving vote counts were published in real time on all the major news networks as well

[18]Exceptions include Maine and Nebraska where electors are allocated based on a combination of the plurality of votes as well as the popular winner in each of their congressional districts. There are also typically a handful of "faithless electors" in each election who chose to vote against the candidate for whom they had pledged to vote.

[19]There were seven faithless electors in total; five defections from Clinton and two from Trump.

as the Internet. As with the Brexit referendum the only information affecting the market that night was the vote counts and results. Similar to Brexit there were also various betting markets open and trading. Bets that paid out in the event of either a Trump or a Clinton win were widely traded. Figure 7 shows the price series for the Betfair contract that pays out £1 in the event of a Republican win in Greenwich Mean Time (GMT).[20] Between midnight and 1:00am the risk neutral odds of a Trump win varied between 10% and 20%. However, by 1:30am GMT (20:30 EST) the count of the crucial swing state of Florida was almost completed and showed a lead for Trump of 0.7% versus an expectation that Clinton would win by 0.6%. From this point on the odds for a Trump win improved as various other counts showed Trump consistently out-performing his polling. By a little after 4am the betting markets implied a Trump win with 95% probability which slowly increased to 98% by 6am. At 7:50am Donald Trump made his victory speech.

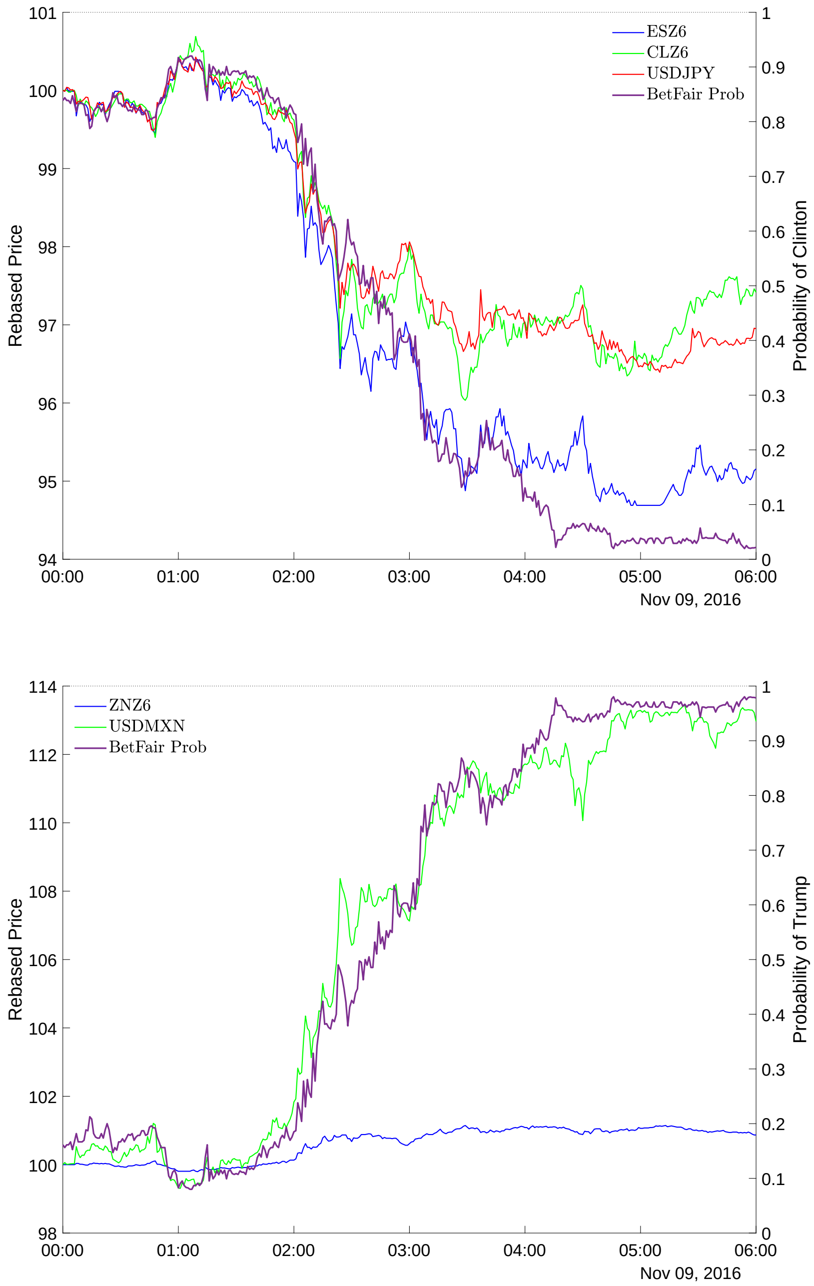
Financial assets

We use a similar basket of financial assets as we did with the Brexit referendum. The exception being we swap out the UK specific GBPUSD cross and include the USDMXN exchange rate. Trump had proposed that if he won he would renegotiate or exit various trade agreements including those with Mexico. Given the dependence of Mexico's economy on trade and exports to the US,[21] a Trump win was seen as extremely negative for that country's economy.

We consider the period midnight to 6:00am BST on 9th November and sample prices every minute. During these hours all contracts are open, the betting market almost converges and the only information released to the market is that contained in the vote results. We do not consider beyond 6am as this is the start of the trading day in London. Other economic news beyond the election may be released which would invalidate our model and, either way, the result had become apparent by then. Table 8 lists the five assets with a description, their starting and ending prices and the percentage change. The election of Donald Trump was a shock. Not only had he pledged to renegotiate various trade deals, reducing world trade and hence the outlook for the economy, he was widely seen as unpredictable and inconsistent. The US dollar depreciated against the safe haven Japanese yen, and the oil and stock market futures depreciated too. The treasury future appreciated as would be expected in a time of increasing risks to the US economy and the US dollar appreciated a large 13% against the Mexican peso. This is as the market re-priced the very significant risks to the Mexican economy following the Trump win.

[20] GMT is 5 hours ahead of Eastern Standard Time (EST) and is the time in London on the date of the election. All times in this section are quoted in GMT

[21] Over 80% of Mexican exports in 2015 were to the US.



Note: Assets that depreciated are shown versus the "Clinton" contract price whereas assets that appreciated are shown versus the contract price for "Trump".

Figure 8: Rebased financial asset prices versus US presidential election contracts.

Figure 8 plots depreciating and appreciating assets versus the Betfair contracts that pay out £1 for a Clinton and Trump win respectively. By and large the financial markets do seem to be moving together with the betting contracts. The relationship does not look quite as established as that for the Brexit referendum with some reversal of the large falls past 5am for the S&P500 and oil futures as well as the USDJPY exchange rate. We turn to the statistical specifications and tests in the next sub-section to make robust conclusions.

Cointegrating relationships

We now consider the evidence for cointegration between the financial assets and the betting markets. Table 9 shows the results of the Phillips–Ouliaris regression tests for linear cointegration. The Z_α or Z_t tests reject the null of no cointegration for only a single asset, USDMXN, at the 95% level. At the 90% level, one more asset (the Mexican Peso) has the null of no cointegration rejected, and only just. There is much less evidence of standard linear cointegration between financial assets and the prediction market than in the case of Brexit.

In our model a linear cointegrating relationship occurs when there is risk neutral pricing in both markets. Could the variant of our model that includes risk aversion provide a better fit to the data for the US presidential election? Again we apply the Johansen constraint test to the trivariate systems $x_t = (p_t, PB_t, R_t)'$ for each financial asset p_t to test this idea. Results are shown in table 10. Unlike in the Brexit case, there does appear to be evidence of non-trivial risk preferences and non-linear relationships. The coefficient relating to risk, λ, is significant and negative for the yen and the oil future. This is interesting.

To further explore the possibility of non-linear cointegration we re-run the Phillips–Ouliaris tests with the risk factor $R_t = PB_t.(1 - PB_t)$ included as an explanatory variable in the regression. Results are shown in Table 11. The results are striking. The null of no cointegration is now rejected at the 99% level for four assets and at the 95% level for the treasury future. We also note that for the treasury and oil futures $|\hat{\lambda}| > |\hat{\triangle p}|$ which violate our model. A coefficient of risk with larger magnitude than $\hat{\triangle p}$ implies a non-monotonic relationship, which decreases at an end point. This is likely due to interpolation to the slight pull back in the asset prices observed between 5am and 6am. We re-run the regression for these two assets, using the single non-linear monotonic explanatory variable $PB_t + R_t$. This also results in the null of no cointegration being rejected for both assets.

The signs of the estimated risk parameters are encouraging. For all "risk" assets that depreciated on the shock Trump win, the parameters are negative, implying risk aversion. For the asset that appreciated, ZNU6, the sign is positive, indicating risk loving behaviour. This warrants further comment.

According to the Johansen constraint test the coefficient is significant for the oil future and the dollar denominated in yen. The sign is negative implying risk aversion. To be clear, the Japanese yen is considered a "risk-off" asset. The US dollar denominated in yen is "risky" and depreciated 3.1% overnight on the shock result. An explanation for this behaviour is that speculators will borrow in the lower yielding yen to finance investment in assets of higher yielding currencies when risk appetite increases.[22] The coefficient in the Phillips–Ouliaris regression for USDMXN is positive. This is consistent with risk aversion, as this is the price of the dollar denominated in the risky asset, the Peso. A positive value of λ for USDMXN is equivalent to a negative value in the regression of MXNUSD against PB_t and $PB_t.(1 - PB_t)$[23]. The coefficient of ESU6 is also negative whereas that for ZNU6 is positive. The positive (albeit not necessarily significant) coefficient for ZNU6 indicates risk loving preferences. Our model incorporated risk preferences by assuming risk neutral pricing in the betting markets and whereas a risk premium was allowed overnight in financial markets. Under risk aversion, this led to a discount on asset prices proportional to the uncertainty in the outcome of the political event. Uncertainty is highest when PB_t is around the middle of its range $[0, 1]$ and furthest from the endpoints. For ZNU6 there instead appears to be a premium on the price when uncertainty is highest. This is not unexpected. US treasuries are considered the ultimate safe haven asset, and are typically bought when there is a flight to quality and risk appetite decreases. Investors sell risky assets such as commodities (CLU6), stocks (ESU6) or emerging market currencies (MXN), recover any funding in lower yielding currencies (JPY) and invest the proceeds in treasuries.

5.2.1 Deviations from long term relationships

As in the Brexit case, the presence of significant autoregressive terms in the VECM suggests deviations from efficiency on a short term scale. There is also evidence for the 2016 presidential election that risk neutrality does not hold. The "long-term" relationship, which lasts a few hours, is now non-linear. The full multivariate system is

$$\boldsymbol{p_t} = (\text{ESZ6, ZNZ6, CLZ6, USDMXN, USDJPY, } PB_t,\, R_t)'$$

where R_t is the non-linear political risk measure $PB_t.(1 - PB_t)$. The fitted VECM, as in

[22]Japenese interest rates and asset yields have historically, and at the time of the vote, been lower than elsewhere.

[23]Mathmatically: negative λ is equivalent to a convex cointegrating relationship and positive λ implies concavity. If USDMXN is concave, then MXNUSD = USDMXN^{-1} is convex. Economically: if USD is "safe" relative to the MXN, then MXN is "risky" relative to the USD.

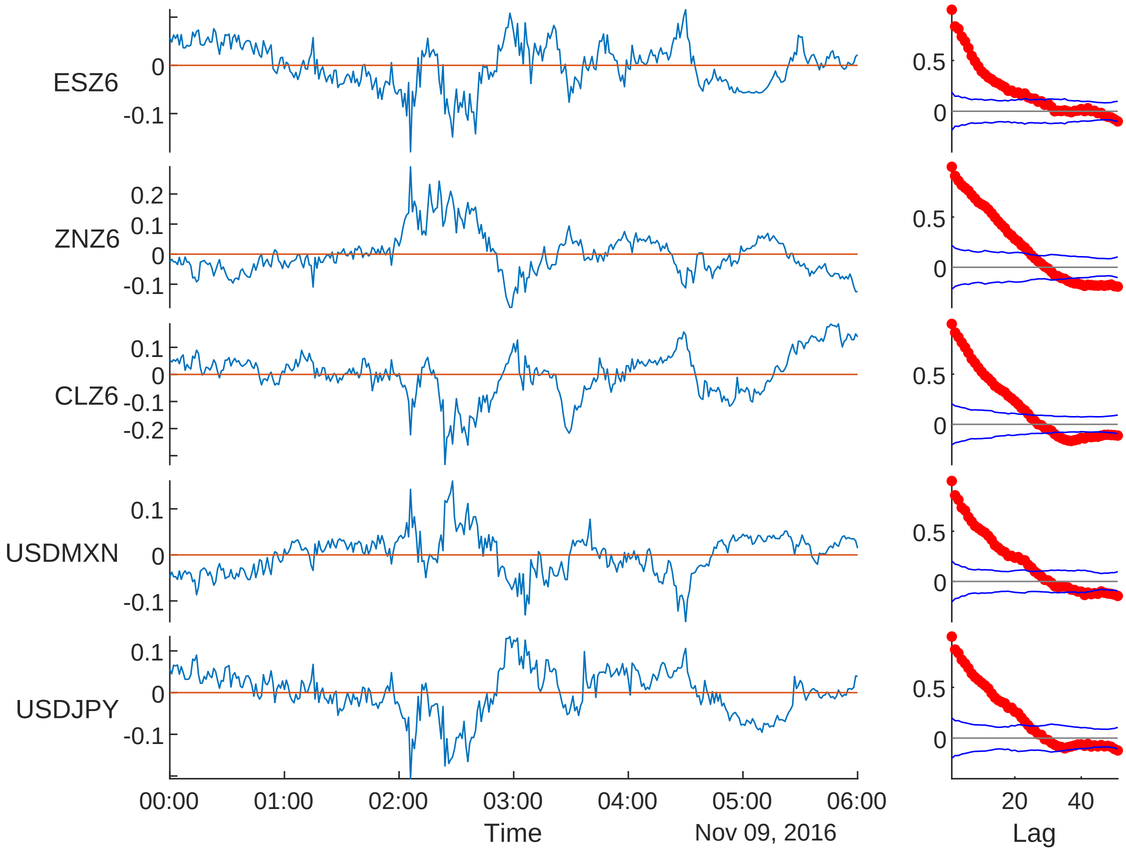


Figure 9: Non-linear cointegration errors with autocorrelation functions.

the case with Brexit, has a single significant autoregressive short run matrix $\mathbf{\Gamma_1}$. Table 12 shows the significant values of this matrix. Testing for short-run causality in this system requires considering both the risk measure R_t in addition to the betting contract price P_t. To test causation from the betting markets to financial markets, consideration of the $(n-2) \times 2$ sub-matrix of $\mathbf{\Gamma_1}$, which is the first $n-2$ values of the last two columns, is required. Testing in the opposite direction requires joint testing of the $2 \times (n-2)$ sub-matrix which is the first $n-2$ values of the last two rows. Table 12 shows results of these two tests. As was the case with the previous event, there is significant causation from the betting markets to the financial markets in the short run (at the 1% level) but no significant causation in the other direction.

The cointegrating error for this election is $\hat{\epsilon}_t = p_t - \hat{p}_0 - PB_t \times \hat{\triangle p} - \hat{\lambda} \times R_t$. Unlike in the linear case with Brexit, this error cannot be recreated simply by trading a portfolio of the betting contract with the financial asset. The non-linearity would require constant adjustment, or delta hedging. This would be difficult and expensive to achieve in practice. However, as the relationship is monotonic, if the error can be predicted, this does imply that the two assets separately can be predicted. Figure 9 plots $\hat{\epsilon}_t/|\hat{\triangle p}|$, the cointegration error normalised to units of the Betting contract, with the sample auto-correlation function and robust Bartlett intervals. As in the case with Brexit, this shows significant and positive autocorrelations out to around 20–30 minutes. The presence of long term cointegration of the markets implies market efficiency on a longer term time timescale. The fact that deviations appear predictable suggests an inefficiency on a shorter timescale.

5.2.2 Profit opportunities

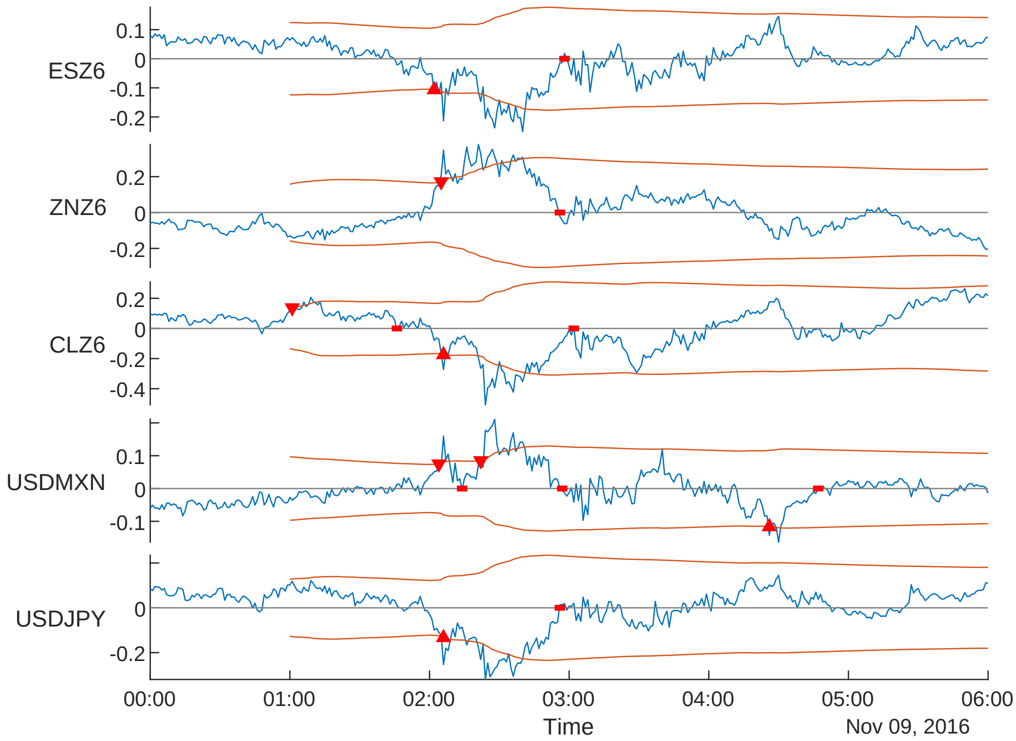
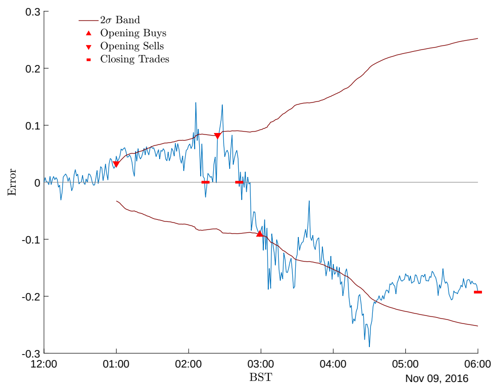
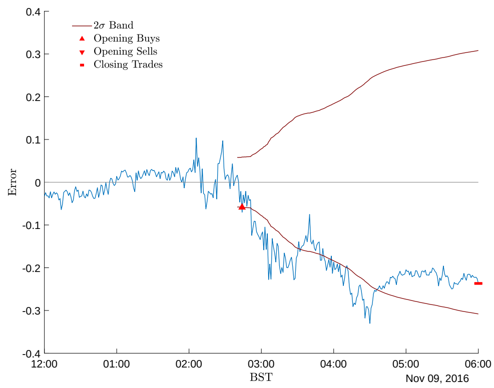


Figure 10: Ex-post Bollinger Band trading strategies for the 2016 presidential election.

We now turn to the possibility of profiting from the apparently predictable deviations from the long term cointegrating relationship. In practice it is not possible to recreate the return of the non-linear error so we will focus on the linear error. Again, this is reproduced by trading the financial asset against the political bet in a ratio equal to the linear cointegrating ratio. This is estimated from the ex-post regression and the parameters are shown in Table 9. We apply the Bollinger Band trading strategy. Results are shown in Table 13 and the normalised error $\hat{\epsilon}_t/|\hat{\triangle p}|$ is shown with bands and trades in Figure 10. The strategies are profitable although there were far fewer trades than in the Brexit case. The strategy only generates eight trades, all profitable, compared with 38 for Brexit. The errors appear to mean revert with far less frequency than for Brexit. However, each trade is more profitable, indicating a greater volatility on the night of the Trump election. As discussed earlier, recreating these profits in practice is not at all realistic. It would require an investor to correctly evaluate the conditional expectations of each asset price given the two possible outcomes of the election. The results of the ex-post strategies are at best thought experiments, relying on conditional expectations that have been revealed after the event. In fact applying the Bollinger strategy in this way ex-post to any two unrelated random walks is always likely to generate apparent profits. There will always be mean reversion to an apparent relationship that is estimated from the ex-post (spurious) regression. Given the lack of evidence for linear cointegration the results below are likely pure fiction.

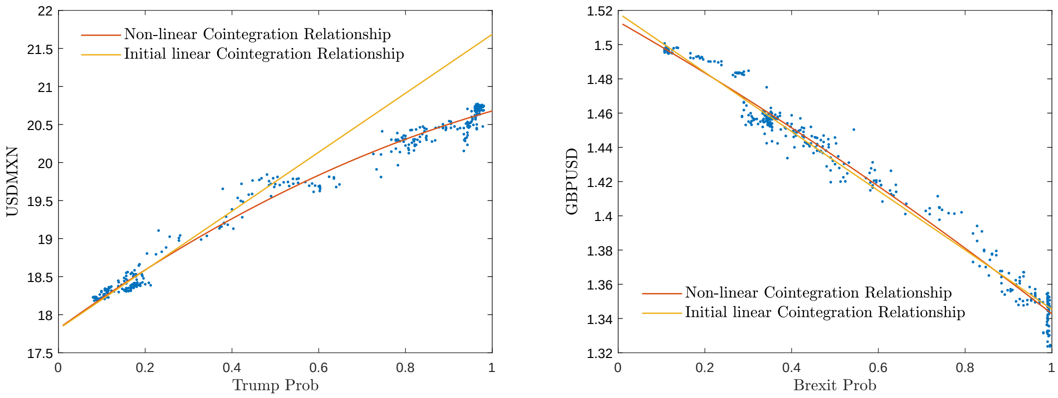




Bollinger Bands strategies for misspecified cointegration error when using $E(USDMXN|TRUMP) = 21.50$ (left) and when using the first part of the night to estimate (right).

Figure 11: Possible ex-ante estimated cointegration errors and Bollinger Band strategies for USDMXN.

USDMXN showed some weak evidence of linear cointegration. Could profits be generated from this asset using an ex-ante estimated forecast for the conditional expectations of the asset prices? To answer this question we exploit a Reuters poll of several economists that was published on 1st November 2016, a week before the election, Cascione (2016). The prospect of a Republican win was expected to be very grave for the Mexican economy. The Reuters poll asked a question as to where the Mexican Peso would settle in the event that Donald Trump were elected. The median forecast for USDMXN was 21.50. In the event this was a little pessimistic. The Peso bottomed out at 5:23am (GMT) with a USDMXN price of 20.77. It slightly appreciated to 20.68 by 6am. Assuming efficiency at midnight and using the Reuters median for $p_1 = 21.50$ implies $p_0 = 17.70$. Thus $\triangle p = 3.80$ and $\hat{\epsilon}_t = GBP_t - 17.70 - 3.80 \times PB_t$. This ex-ante estimated error from the Reuters forecast is plotted with Bollinger Band strategy on the left hand side of Figure 11. The overly pessmistic forecast makes the error diverge from zero, yielding a large loss making trade in the second half of the night. There are two profitable trades made in the first half of the night. These occur before the error diverges. These do more than offset the loss of the final trade but the total gross profits of 3.2p is underwhelming. The profits of around 1p per trade compare very poorly with that of the (unrealistic) ex-post strategy (16.3p).



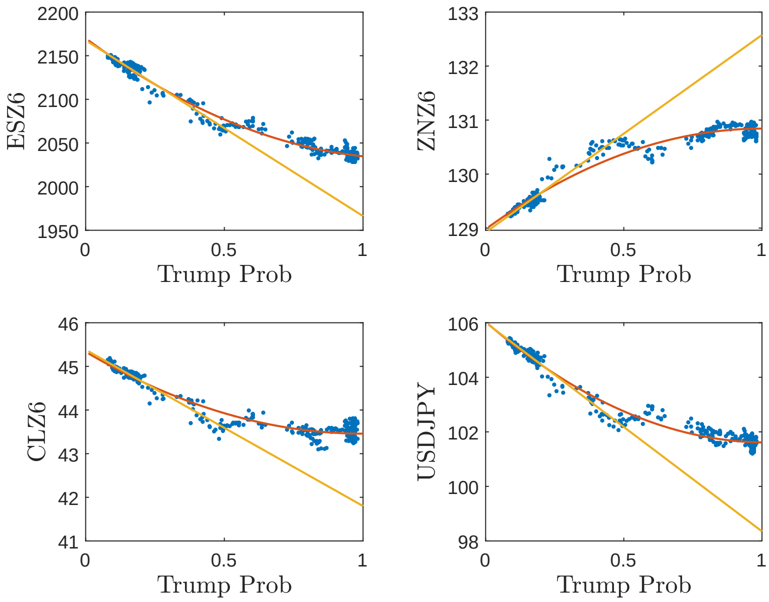
The non-linear relationship is calculated ex-post on all data whereas the initial linear relationship uses data from an initial period prior to event probabilities crossing 50%.

Figure 12: Estimated cointegration relationships for USDMXN (presidential election) and GBPUSD (Brexit referendum).

Next we study whether the cointegrating relationship can be estimated for the USDMXN in the first few hours and successfully exploited later in the night. The betting markets implied a 50% probability of a Trump win by 2:39am. Using the period midnight to 2:39am to estimate the linear cointegrating relationship yields $\hat{p}_0 = 17.81$ and $\hat{p}_1 = 21.69$. Again the conditional expectation of USDMXN given a Trump win is overly pessimistic. It results in an estimated error that diverges from zero. This yields a terrible trade in the second half of the night as demonstrated by the right hand side of Figure 11 where the misspecified error and Bollinger bands are shown.

The results obtained for GBPUSD for Brexit, where the linear relationship estimated from the first period was stable, yielded a successful strategy. The answer as to why the performance is so different in this instance is the non-linearity and apparent deviation from risk neutrality. Figure 12 illustrates the difference. The GBPUSD and USDMXN are plotted against the relevant betting contract along with both the linear relationship from the first part of the night as well as the ex-post non-linear cointegrated relationship. The non-linear plot clearly diverges from the linear plot for USDMXN whereas there is almost no difference in the two lines for GBPUSD. There is no hope of estimating the long term non-linear cointgrating relationship for USDMXN from the first part of the night and making money from any convergence. Figure 13 shows similar plots for the remaining assets for the 2016 presidential election as well as Brexit. The situation is replicated for these assets. There is a clear difference in behaviour for the Trump

a



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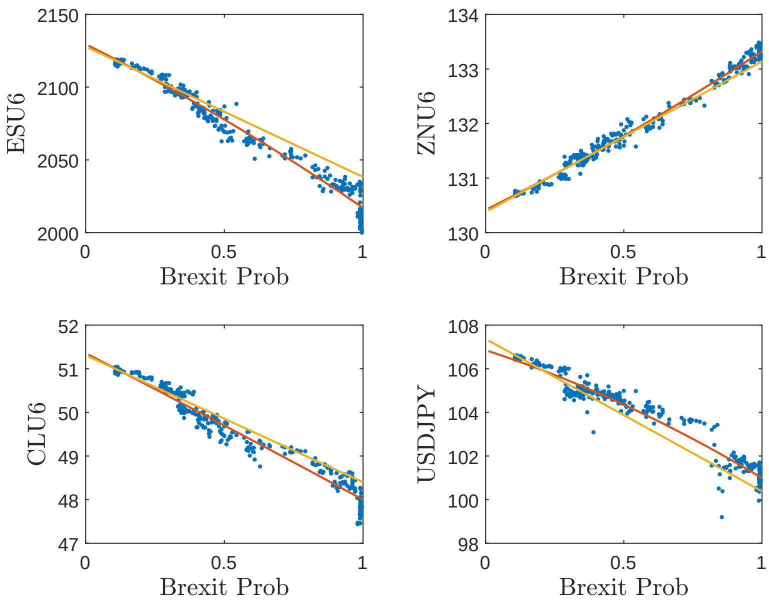

Note: The non-linear relationship (orange line) is calculated ex-post on all data whereas the initial linear relationship (yellow line) uses only data from the first few hours when event probabilities were under 50%.

Figure 13: Estimated cointegration relationships for (a) the 2016 presidential election and (b) the Brexit referendum.

election. In the Brexit case the non-linear relationships are virtually identical to the linear one. This is consistent with the earlier Johansen constraint tests where there was no evidence of significant risk parameters λ. The only Brexit contract where the market does not converge to the linear cointegrated relationship is the ESU6, and the divergence is far smaller than for the presidential election. The application of the ex-ante Bollinger strategy on the night of the Trump election would be ruinous. The apparent divergence from risk neutrality after the presidential election leads to far different behaviour than that observed after Brexit, where risk neutrality appeared to hold. We conclude that there appear to be no realistic opportunities to profit from our model for the 2016 US presidential election.

5.3 The 2014 Scottish independence referendum

The 2014 Scottish independence referendum was held on 18th September 2014. It concerned whether Scotland should remain in the UK. The question asked "Should Scotland become an independent country?" and it required a simple majority to pass. All EU and Commonwealth citizens residing in Scotland were eligible to vote. This was the first time that an election was open to 16 and 17 year olds. Turnout, at 84.6%, was the largest for any UK election since the general election of 1910. The "No" side won with 55.3% of the vote against 44.7% for "Yes".

Voting took place from 7am to 10pm BST. Voters still queuing at the close of polls were allowed to vote. Each of the 32 local authority areas announced their results at separate times overnight. This makes this event suitable for our model as the information was drip fed to the public throughout the overnight hours.

During July and August 2014, opinion polls showed a consistent lead for "No" with the average difference being 6%, Curtice (2014). However, polls tightened at the start of September and on the 6th of that month YouGov published a poll that showed the Yes side ahead with a small lead of 2%, Dahlgreen (2014). This was the first time a poll showed the Yes side ahead. It was also the first time many commentators took the possibility of a Yes vote, and the resulting disruption to the UK's economy, seriously. In response sterling lost around 1% against the euro and dollar and companies with links to Scotland sold off sharply, Wearden (2015). However the 6th September poll was the only poll that showed Yes ahead. The polls subsequently reverted to Yes trailing No. The final poll released was conducted by YouGov on the day of voting. The results were released just after the close of voting and predicted an 8 point lead for No.

Figure 14 shows the Betfair contract that pays out £1 in the event of a No vote, along with the GBPUSD price. The betting market had previously, following the release of the 6th September poll, implied a probability of Scotland leaving the UK of as high as 35%. However, on the night of the vote the implied probability was much lower. From 11pm to midnight Betfair implied around a 90–92% chance of remaining. This contract started rising after 1am. The first local authority to announce their result was Clackmannanshire at 1:28am. This showed the vote for No nearly 8% ahead of Yes and in line with the poll on the day released by YouGov. The SNP, whose headline policy was independence, had achieved their highest vote share in Clackmannanshire in the 2012 council elections and so this result was seen as very negative for the Yes campaign. The No contract on Betfair continued its ascent and shortly after this result priced a 98–99% probability of No winning the referendum. The currency was sensitive to the possibility, albeit small, of a yes vote. It rallied around a cent to the dollar as this possibility was effectively ruled out. From the point of this first result onward the result did not seem in doubt. The betting contract price remained above 98% for the most part with only a brief fall to 96% shortly after 4am which quickly reversed.

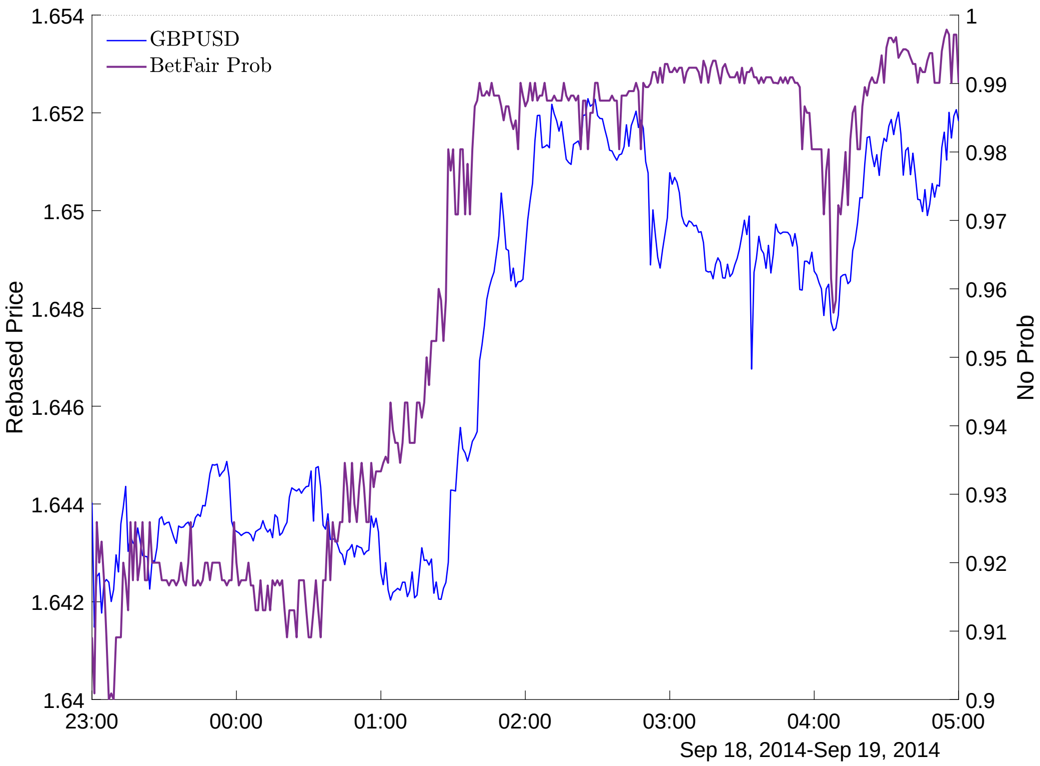

Figure 14: Betfair price for "No" bet and GBPUSD on the night of the independence referendum.

Cointegrating relationships

For this event we consider GBPUSD only. The currency was expected to sell off sharply in the event that the Yes camp prevailed, and recover a little if Scotland voted to stay in the UK. The price at 10pm was 1.6395, it peaked at 2:26am at 1.6523 and ended the night at 6am at 1.6476. The appreciation was modest but this has to be considered against the smaller move in the betting market contracts of only around 10p.

Figure 14 shows the results of cointegration tests for GBPUSD and the betting contract. The first line of the table shows the results for linear cointegration. Both Phillips and Ouliaris tests do not reject the null of no cointegration at the 95% level. The Z_α test marginally rejects the null at the 90% level. This does not provide sufficient evidence for cointegration. The second line shows the results for the case where the risk factor is included in the cointegration regression. Here, tests do reject the null at the 99% level but the Johansen rank test does not. However, we note that the magnitude of the risk parameter λ is significantly larger than the linear parameter $\triangle p$. This violates our model as it results in a non-monotonic cointegrating relationship. The estimated value for the counterfactual conditional expectation of the pound given a 'Yes' vote of $\hat{p_0} = 3.18$ is nonsensical. The pound was expected to depreciate given the unlikely scenario of Scotland voting to be independent, not appreciate nearly 100%. The third and final line of the table shows the tests when the non-linear cointegration relationship is constrained to be positive with $|\lambda| = |\triangle p|$. This is effected by regressing GBP_t against the single monotonic factor $[PB_t + R_t]$. The cointegration tests do not reject the null of no cointegration at the 90% level. Again the value of $\hat{p_0} = 0.703$ is implausible. Of all three models considered, the linear model seems the most realistic but there is still no compelling evidence that our model holds.

The truth is that on the night of the Scottish referendum the markets did not move a great deal, and what little movement there was for each market came relatively quickly and not quite in sync. The betting contract moved only 10p in the overnight session. This contrasts with the two events in 2016 where the results were a shock. The betting contracts moved close to their full range of £1. Our model fits well there as these political events dominated the markets. There was also sufficient coverage of price data there to generate statistical power, validating our model. In both cases in 2016 relevant information was released over a matter of hours. What is perhaps more interesting is the fact that the betting market appeared to lead the currency during the move up. This is similar to the observation that political markets lead financial prices on a short timescale in the previous two events. There may in fact be a common trend in the markets relating to the odds of a 'No' vote; it is just that the currency appears to lag that trend by around 30 minutes. To investigate this a little further we plot the betting price against the GBPUSD price bought forward 30 minutes from the future. This is shown in Figure 15. To the eye the market move is much more synchronized. We check the linear cointegration tests for this adjusted data. Results are shown in Table 15. We now see that there is evidence of a common trend between the two time series with the

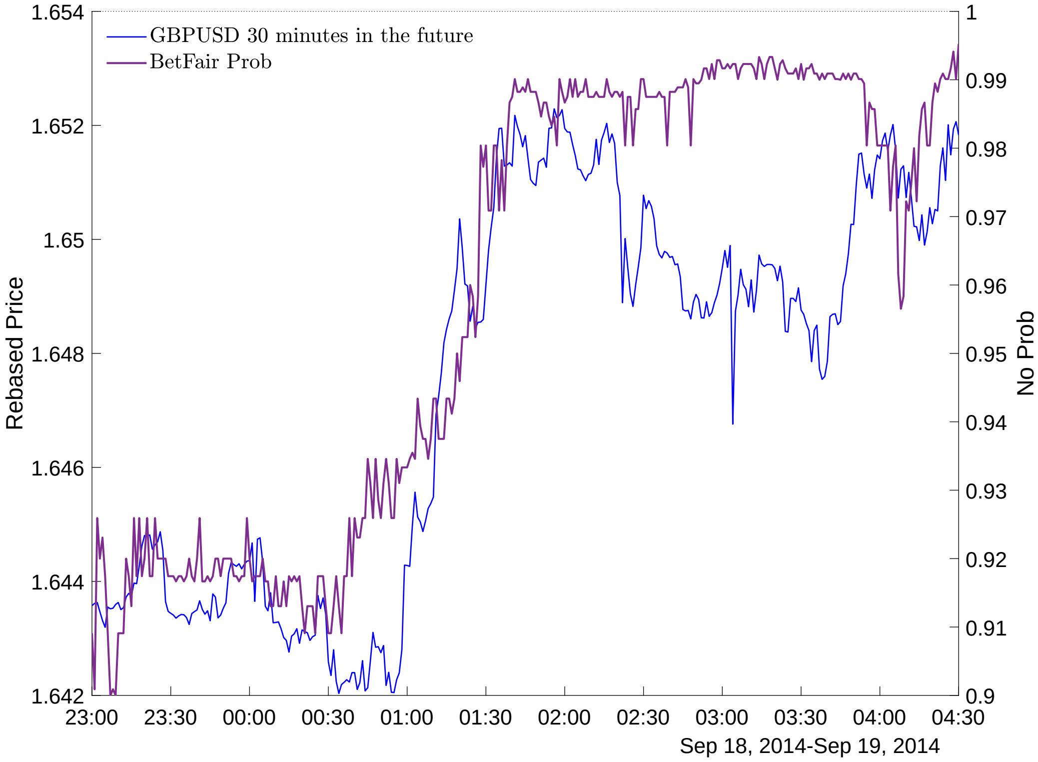

Figure 15: Betfair price for "No" bet and GBPUSD price bought forward 30 minutes from the future, on the night of the independence referendum.

Phillips and Ouliaris tests both rejecting the null of no cointegration at the 95% level. The reader should note that this is not a rigorous statistical test of cointegration. The author has plotted more than one adjusted data series, choosing the one most pleasing to the eye.[24] A joint hypothesis test has effectively been performed whilst the pValues in Table 15 relate to only single hypothesis tests. Either way this exercise has demonstrated that the lack of statistical significance is likely due to the presence of a deviation of weak market efficiency of similar order in time to that of the move in the common trend. For the most part, and either side of the Clackmanshire result, the markets simply drifted.

6 Conclusion

In this paper we attempted to describe the behaviour of financial and political betting markets in the hours after elections. The key assumption is that the only information that has a persistent affect on asset prices is that related to the *probability* of a political outcome (and nothing else). The further restrictions of equivalence of beliefs between

[24]Full disclosure: Graphs for GBPUSD bought forward by 20, 30, 35 and 40 minutes were eyeballed.

the two markets (consistent with weak market efficiency), risk neutral pricing in betting markets and a 'short' overnight session lead to a non-linear cointegrating relationship between financial asset and binary options prices in the betting markets. When there are risk neutral investors in financial markets the relationship between the markets is one of standard linear cointegration. We test the model on data from three recent elections and find strong support for the theory for two of them.

For the 2016 Brexit referendum our base linear cointegrating model finds excellent support. The long term cointegration relationship is found to be so stable that a brave trader could estimate it in the first part of the night and profit from the convergence of deviations from that relationship in the second. It appears that market efficiency holds on a longer term basis that night (long term meaning hours) whereas there are some deviations from efficiency on a timescale of tens of minutes. Our results are also in contrast to the model in Manasse et al. (2020) where a divergence from risk neutral preferences is required to explain the behaviour of the currency markets in the weeks and months preceding the vote. Of further interest is the fact that with respect to short term dynamics, the betting markets appear to lead and cause the financial markets but there is no apparent causation in the other direction. This suggests betting markets were more efficient at discounting the results of the vote in the hours after the referendum.

For the 2016 US presidential election linear cointegration, and hence risk neutrality, cannot explain the observed price action. However, we find strong evidence for our theory when deviations from risk neutrality are incorporated with a non-linear cointegrating relationship. The interpretation of the computed risk parameters are pleasing. Risk aversion is revealed for "risk-on" assets, whereas risk loving preferences are observed for the safe haven treasury future asset. Similar to Brexit we find deviations from the long term (albeit non-linear here) cointegrating relationship are predictable, indicating deviations from weak market efficiency on similar smaller timescales. The result that in the short run betting markets lead financial markets but that there is no causation in the other direction is also repeated. However, that brave trader who profited from the Brexit deviations would have had a terrible night repeating that strategy for this election. This is explained by the presence of non-trivial risk preferences. It is not possible to infer or estimate the non-linear cointegrating relationship in the first few hours. Foresight of the conditional expectations of the asset prices given election outcomes would be needed to make money.

Finally we do not find evidence for our model on the night of the 2014 Scottish independence referendum. The markets did not move very much at all that night, as the result was largely as expected. The move itself was also short lived. This is in contrast to the other two events where the results were both a huge shock, and large moves in all prices were observed which took several hours. The betting markets do appear to lead the currency market though by around 30 minutes. We explain the failure to demonstrate statistical significance as due to the similarity of timescales of deviations from market efficiency with those of the time taken for market to move upwards that night.

For all three events it appears that betting markets reflected political information more quickly than financial markets, and on a timescale of minutes to tens of minutes. This is consistent with, and an extension of, the conclusion of the existing literature that political prediction markets offer superior forecasting ability. This finding, along with the development of a pricing model that holds in a general election overnight setting that is based on economic principles are the main contributions of this paper. Whether or not political markets lead financial markets at times other than the results session following a vote is an open research question.

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Appendix - Results tables

Table 1: Financial assets and changes for the Brexit referendum.

Symbol	**Description**	$p_{t=1}$	$p_{t=T}$	$\triangle$
ESU6	E-mini S&P500 Future	2115.75	2000.00	-5.5%
ZNU6	10-Year T-Note Future	130.703	133.266	2.0%
CLU6	Crude Oil Future	50.89	47.82	-6.0%
GBPUSD	British Pound US Dollar Cross	1.5007	1.3242	-11.8%
USDJPY	US Dollar Japanese Yen Cross	106.61	100.93	-5.3%

Table 2: Results of the Phillips–Ouliaris Z_t and Z_α tests for cointegration for Brexit.

Symbol	$\hat{p_0}$	$\hat{\triangle p}$	$p_{Z_t}(a=0)$	$p_{Z_\alpha}(a=0)$
ESU6	2134.0	-115.6	0.008***	0.003***
ZNU6	130.32	2.97	0.001***	0.001***
CLU6	51.38	-3.37	0.006***	0.004***
GBPUSD	1.5199	-0.1751	0.001***	0.001***
USDJPY	107.28	-6.15	0.001***	0.001***

The first stage regression is $p_t = \hat{p_0} + \hat{\triangle p} PB_t + \hat{\epsilon_t}$

The residual regression is $\hat{\epsilon_t} = a\hat{\epsilon_{t-1}} + \eta_t$

Table 3: Results of the trivariate Johansen constraint test for risk aversion for Brexit.

Symbol	$\hat{\lambda}$	$k-1$	$p(\lambda=0)$
ESU6	-2.165	0	0.913
ZNU6	-0.442	3	0.084*
CLU6	-0.718	0	0.294
GBPUSD	0.018	2	0.508
USDJPY	1.274	1	0.237

The regression is:

$\triangle \boldsymbol{p_t} = \boldsymbol{\alpha}(\boldsymbol{\beta' p_{t-1}} - \boldsymbol{c_0}) + \sum_{i=1}^{k-1} \boldsymbol{\Gamma_i} \triangle \boldsymbol{p_{t-i}} + \boldsymbol{u_t}$

$\boldsymbol{p_t} = (p_t, PB_t, R_t)'$

$r = \text{rank}(\boldsymbol{\beta}) = 1$, $\boldsymbol{\beta}' = (1, -\triangle p, -\lambda)$, $c_0 = -p_0$

Table 4: Fitted VECM for Brexit with tests on short-run dynamics.

$$\boldsymbol{\Gamma_1} = 10^{-3} \times \begin{pmatrix} -190^{**} & - & -3860^{***} & 13000^{***} & - & -20600^{***} \\ -3.44^{***} & - & 44.1^{*} & - & - & 185^{*} \\ 6.11^{***} & -0.312^{***} & -159^{***} & -2890^{**} & - & - \\ 0.14^{*} & - & - & -105^{*} & - & -28.9^{***} \\ 11.4^{**} & - & - & 6720^{*} & -116^{**} & -1640^{**} \\ - & - & - & -452^{*} & - & - \end{pmatrix}^{1}$$

Based on $\triangle \boldsymbol{p_t} = \boldsymbol{\alpha}(\boldsymbol{\beta' p_{t-1}} - \boldsymbol{c_0}) + \boldsymbol{\Gamma_1} \triangle \boldsymbol{p_{t-1}} + \boldsymbol{u_t}$

$\boldsymbol{p_t} =$ (ESU6, ZNU6, CLU6, GBPUSD, USDJPY, $PB_t)'$

[1]Statistically insignificant coefficients omitted

Null Hypothesis	LR-Test	
	Chi-square	Prob
(ESU6, ZNU6, CLU6, GBPUSD, USDJPY)' does not cause PB_t	4.84	0.436
PB_t does not cause (ESU6, ZNU6, CLU6, GBPUSD, USDJPY)'	11.4^{**}	0.0441

Table 5: Bollinger Band gross trading profits for Brexit.

Symbol	$\frac{\textbf{nWin}}{\textbf{nTrade}}$	**Profit**[a]	$\frac{\textbf{Profit}}{\textbf{Trade}}$
ESU6	7/8	54.8p	6.8p
ZNU6	10/10	77.6p	7.8p
CLU6	6/6	69.5p	11.6p
GBPUSD	6/7	52.0p	7.4p
USDJPY	7/7	125.4p	17.9p
Total	36/38	379.3p	10.0p

[a]Profits are shown for every £1 contract traded on Betfair

Table 6: Cointegration tests for the smaller training period for Brexit.

Symbol	$\boldsymbol{p_{Z_t}(a=0)^a}$	$\boldsymbol{p_{Z_\alpha}(a=0)^a}$
ESU6	0.026^{**}	0.007^{***}
ZNU6	0.059^{*}	0.014^{**}
CLU6	0.344	0.356
GBPUSD	0.115	0.027^{**}
USDJPY	0.011^{**}	0.003^{***}

[a]Phillips–Ouliaris tests.

Table 7: Ex-ante Bollinger Band gross trading profits for Brexit.

Symbol	$\frac{\textbf{nWin}}{\textbf{nTrade}}$	**Profit**[a]	$\frac{\textbf{Profit}}{\textbf{Trade}}$
ESU6	0/1	-37.8p	-37.8p
ZNU6	7/7	46.8p	6.7p
CLU6	5/5	62.2p	12.4p
GBPUSD	4/5	29.3p	5.9p
USDJPY	5/5	100.1p	20.0p
Total	21/23	200.6p	8.7p

[a]Profits are shown for every £1 contract traded on Betfair

Table 8: Financial assets and changes for the 2016 presidential election.

Symbol	**Description**	$p_{t=1}$	$p_{t=T}$	$\triangle$
ESZ6	E-mini S&P500 Future	2142.25	2038.500	-4.8%
ZNZ6	10-Year T-Note Future	129.484	130.6094	0.9%
CLZ6	Crude Oil Future	44.880	43.72	-2.6%
USDMXN	US Dollar Mexican Peso Cross	18.310	20.684	13.0%
USDJPY	US Dollar Japanese Yen Cross	104.990	101.789	-3.1%

Table 9: Results of the Phillips–Ouliaris Z_t and Z_α tests for cointegration for the Trump election.

Symbol	$\hat{p_0}$	$\hat{\triangle p}$	$p_{Z_t}(a=0)$	$p_{Z_\alpha}(a=0)$
ESU6	2152.59	-126.27	0.109	0.094*
ZNU6	129.314	1.673	0.709	0.671
CLU6	44.99	-1.67	0.497	0.467
USDMXN	18.031	2.751	0.015**	0.014**
USDJPY	105.298	-4.041	0.308	0.279

The first stage regression is $p_t = \hat{p_0} + \hat{\triangle p} PB_t + \hat{\epsilon_t}$

The residual regression is $\hat{\epsilon_t} = a\hat{\epsilon_{t-1}} + \eta_t$

Table 10: Results of the trivariate Johansen constraint test for risk aversion for the Trump election.

Symbol	$\hat{\boldsymbol{\lambda}}$	$\boldsymbol{k-1}$	$\boldsymbol{p(\lambda=0)}$
ESU6	-15.951	4	0.904
ZNU6	13.378	3	0.132
CLU6	-5.74	5	0.001***
USDMXN	-4.988	4	0.575
USDJPY	-7.481	4	0.010***

The regression is:
$\triangle \boldsymbol{p_t} = \boldsymbol{\alpha}(\boldsymbol{\beta' p_{t-1}} - \boldsymbol{c_0}) + \sum_{i=1}^{k-1} \boldsymbol{\Gamma_i} \triangle \boldsymbol{p_{t-i}} + \boldsymbol{u_t}$
$\boldsymbol{p_t} = (p_t, PB_t, R_t)'$
r =rank$(\boldsymbol{\beta}) = 1$, $\boldsymbol{\beta}' = (1, -\triangle p, -\lambda)$, $c_0 = -p_0$

Table 11: Results of the Phillips–Ouliaris Z_t and Z_α tests for cointegration for the Trump election. R_t included.

Symbol	$\hat{\boldsymbol{p_0}}$	$\hat{\triangle \boldsymbol{p}}$	$\hat{\boldsymbol{\lambda}}$	$\boldsymbol{p_{Z_t}(a=0)}$	$\boldsymbol{p_{Z_\alpha}(a=0)}$
ESU6[a]	2170.038	-135.277	-101.664	0.001***	0.001***
ZNU6[a]	128.873	1.900	2.570	0.036**	0.026**
CLU6[a]	45.50	-1.94	-3.00	0.010***	0.007***
USDMXN[a]	17.817	2.861	1.252	0.003***	0.003***
USDJPY[a]	106.032	-4.419	-4.274	0.007***	0.007***
ZNU6[b]	128.980	1.863	1.863	0.065*	0.039**
CLU6[b]	45.35	-1.88	-1.88	0.036**	0.021**

[a]The first stage regression is $p_t = \hat{p_0} + \hat{\triangle p} PB_t + \hat{\lambda} R_T + \hat{\epsilon_t}$
[b]The first stage regression is $p_t = \hat{p_0} + \hat{\triangle p}\,[PB_t + R_t] + \hat{\epsilon_t}$ so $\hat{\lambda} = \hat{\triangle p}$
The residual regression is $\hat{\epsilon_t} = a\hat{\epsilon_{t-1}} + \eta_t$

Table 12: Fitted VECM for the Trump election with tests on short-run dynamics.

$$\mathbf{\Gamma_1} = \begin{pmatrix} -0.196^{**} & -9.28^{**} & 4.92^{*} & - & - & -36.4^{***} & - \\ - & - & -0.0955^{*} & -0.0956^{*} & - & 0.502^{***} & - \\ - & - & - & -0.147^{*} & - & -0.426^{***} & - \\ - & - & - & - & - & 0.567^{***} & - \\ - & -0.240^{*} & 0.200^{*} & - & -0.274^{***} & -1.10^{***} & -0.893^{*} \\ - & - & -0.0554^{**} & -0.0500^{**} & - & -0.329^{***} & - \\ - & -0.0555^{**} & - & - & - & - & -0.277^{***} \end{pmatrix}^{1}$$

Based on $\triangle \boldsymbol{p_t} = \boldsymbol{\alpha}(\boldsymbol{\beta' p_{t-1}} - \boldsymbol{c_0}) + \boldsymbol{\Gamma_1} \triangle \boldsymbol{p_{t-1}} + \boldsymbol{u_t}$

$\boldsymbol{p_t} = (\text{ESZ6, ZNZ6, CLZ6, USDMXN, USDJPY, } PB_t,\ R_t)'$

$R_t = PB_t.(1 - PB_t)$

[1]Statistically insignificant coefficients omitted

Null Hypothesis	LR-Test	
	Chi-square	Prob
(ESZ6, ZNZ6, CLZ6, USDMXN, USDJPY)' does not cause (PB_t, R_t)	15.7	0.108
(PB_t, R_t) does not cause (ESZ6, ZNZ6, CLZ6, USDMXN, USDJPY)'	32.8***	< 0.001

Table 13: Ex-Post Bollinger Band gross trading profits for the 2016 presidential election.

Symbol	$\frac{\textbf{nWin}}{\textbf{nTrade}}$	**Profit**[a]	$\frac{\textbf{Profit}}{\textbf{Trade}}$
ESU6	1/1	11.7p	11.7p
ZNU6	1/1	20.7p	20.7p
CLU6	2/2	41.7p	20.8p
GBPUSD	3/3	31.1p	10.4p
USDJPY	1/1	25.4p	25.4p
	8/8	130.6p	16.3p

[a]Profits are shown for every £1 contract traded on Betfair

Table 14: Results of cointegration tests for the Scottish independence referendum.

Symbol	$\hat{p_0}$	$\hat{\triangle p}$	$\hat{\lambda}$	$p_{Z_t}(a=0)$	$p_{Z_\alpha}(a=0)$
GBPUSD[a]	1.5570	0.0937	–	0.126	0.099*
GBPUSD[b]	3.1833	-1.5309	-2.4388	0.008***	0.007***
GBPUSD[c]	0.7031	0.9464	0.9464	0.182	0.150

[a]The first stage regression is $p_t = \hat{p_0} + \hat{\triangle p} PB_t + \hat{\epsilon_t}$

[b]The first stage regression is $p_t = \hat{p_0} + \hat{\triangle p} PB_t + \hat{\lambda} PB_t.(1 - PB_t) + \hat{\epsilon_t}$

[c]The first stage regression is $p_t = \hat{p_0} + \hat{\triangle p}\,[PB_t + PB_t.(1 - PB_t)] + \hat{\epsilon_t}$

The residual regression is $\hat{\epsilon_t} = a\hat{\epsilon_{t-1}} + \eta_t$

Table 15: Results of linear cointegration tests for Scottish independence referendum where GBPUSD is shifted forward 30 minutes.

Symbol	$\hat{p_0}$	$\hat{\triangle p}$	$p_{Z_t}(a=0)$	$p_{Z_\alpha}(a=0)$
GBPUSD[a]	1.5580	0.0934	0.041**	0.030**

The first stage regression is $p_t = \hat{p_0} + \hat{\triangle p} PB_t + \hat{\epsilon_t}$

The residual regression is $\hat{\epsilon_t} = a\hat{\epsilon_{t-1}} + \eta_t$