Political markets as equity price factors

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1 Introduction

This paper is concerned with the interaction and behaviour of prediction and financial markets around political events. That political risk affects asset prices is an established concept. Similarly, the information content and superior forecasting ability of political prediction markets is a settled matter in the literature. Could the ‘wisdom of the crowds’ from both these markets concerning political risk be used to derive pricing relationships between these two very different markets? This paper aims to answer this and other questions.

There are many studies in the literature that study single elections or other political events. However, studies that consider both political and financial markets, or multiple events, are few and far between. The papers that do exist typically only consider empirical and not theoretically derived relationships. There is a major gap in the literature for economically derived asset pricing models of prediction and financial markets that apply in a general setting. One working paper by this author, which shares some methodological aspects with this one, Auld (2022), attempts to bridge the gap between the two markets in the very particular situation of the hours directly after an election. But, the question remains, how do markets behave on a longer term, during the weeks and months preceding an election? This is the topic of this study.

The successful election night model of Auld (2022) used a key assumption. This was that asset prices are uniquely determined by the likelihood of a binary political outcome. Combined with market efficiency and risk neutrality, this led to the presence of a single stochastic trend and the presence of cointegration. However, it is difficult to argue that, outside of the unique hours directly following election, financial prices do not respond to the ebb and flow of other non-political information. One implication of the key assumption in the overnight model is that the conditional expectations of asset prices given binary political outcomes is fixed in time. This does not apply on a longer term basis. Conditional expectations of prices will change with the arrival of non-political information. Instead, the key assumption in this paper is that the difference, (or ratio), of the two conditional expectations is fixed. This is equivalent to assuming that the outcome of the elections has a fixed effect on the prospects of a company or financial asset. The resulting model yields a relationship between asset price returns and first differences in betting markets. This is no great surprise. A small number of studies have studied this relationship, albeit only for single events and only empirically. However, our model is based on economic principles and applies in a general setting. The residue of returns beyond that driven by the political factor is allowed to vary. This is due to the existence of other economic and commercial information beyond that related to the political event driving price changes. The distribution of returns is effectively partitioned
into a political component and a non-political component. Of course the overnight model in [Auld (2022)] is a special case of this model, where the variance of the non-political part is zero. The theory is quite naturally extended to equities, by modelling the non-political part of the returns with the ubiquitous 5 Fama–French factors, [Fama and French (2015)]. The result is a 6-factor augmented Fama–French characteristic factor model, with an additional factor being described by the betting markets and related to political risk. We test the model on six elections. Strong support is found for four, mixed results for one, and no evidence for a single election (the 2017 UK general election). The conclusion for the latter event is that this event was not meaningful for stock prices. We also find evidence that betting markets become more informative as the event approaches. This idea already has some support in the literature. An exponential weighting scheme, where observations closer to the election are weighted more heavily, improves and sharpens our results. Finally an inspection of the political factor loadings reveals some pleasing relationships between firm characteristics and political sensitivity. Internationalisation of revenue was a key explanatory factor. This is consistent with the hypothesis that firms that have a greater share of off-shore sales are more able to diversify domestic political risks. We also find geographical location and nationalisation risk under an opposition win also explain differences in political sensitivity.

The main contribution of this paper is the presentation of a general theoretical pricing model linking prediction markets with financial asset prices. It applied both for political events as well as any prediction market whereby the underlying event has an effect (through the conditional expectation) on asset prices. Secondly we discover firm characteristics that drive political sensitivity which both confirms and extends the existing literature on this topic. Finally, the discovery of a significant, albeit temporary, political equity factor contributes to the literature on equity factor modelling and portfolio choice.

The remainder of the paper proceeds as follows: Section 2 builds the theoretical model using the assumption of a fixed difference in conditional expectations of financial prices given the outcome of a binary political event. The empirical and testing framework is outlined in section 3. Section 4 tests the model on several recent elections and section 5 concludes our study.

2 Literature review

We are primarily interested in the relationships between, and behaviour of, prediction and financial markets, around political events. Relevant areas of literature include both the financial market effects of political events as well as election prediction markets in general.
The financial market implications of elections and political risk are well documented. Changes in the composition of government naturally brings about changes in policy. There are many studies that demonstrate either the effects on financial markets of election campaign periods or results. Three large multi-country studies demonstrate changes of asset price volatility around elections. Biakowski et al. (2008) studies 134 elections in 27 OECD countries from 1980 to 2004. Using a GARCH methodology they find that the relevant national stock exchange index volatility can easily double during the week after the election. Apparently ‘investors are surprised by the election outcome’. Kelly et al. (2016) find that this uncertainty is priced in the options market. They analyse data from 1990-2012 for options prices on either the national index, or an ETF tracking that index, for a sample of 20 countries. They find that prices and implied volatility are higher for options that span elections. They also document spillover effects from the election country to other international markets. Pantzalis et al. (2000) is another large multi country study. This paper finds significantly positive returns two weeks prior to election dates for elections in 33 countries between 1974 and 1995. The conclusion is that as election uncertainty is resolved, prices respond positively. Two later papers come to the opposite conclusion for US presidential elections. Goodell and Bodey (2012) consider how the Graham price to earnings (P/E) of the S&P500 index stocks, a valuation metric as well as a measure of consumer sentiment, changes during the campaign periods of US presidential elections. They find the measure worsens as the winner becomes clearer, according to the likelihood seen on the Iowa market (that is, as uncertainty reduces). They conclude that for the US, ‘during presidential election seasons, the market discovers its distaste for the economic policies of the likely winner’. The analysis is extended in Goodell and Vähämaa (2013) for the five presidential elections from 1992 to 1998. They consider the effects on the VIX, a measure of implied volatility of options on the S&P500 expiring in under one month. They find the VIX is positively associated with positive changes in the likelihood of the winner. This ‘indicates that the presidential election process engenders market anxiety as investors form and revise their expectations regarding future macroeconomic policy.’

There are also studies in the literature that demonstrate association between elections polls and asset prices, in the run up to an election. Gemmill (1992) considers the campaign period of the 1987 UK general election. In the paper the author derives the probability of a Conservative party win from polling data. They find that the ‘FTSE100 index was very closely related to the probability of a Conservative win’. Further, in the final two weeks before the voteshare options prices showed large increases in implied volatility. This was particularly the case for two nationalisation targets (of the opposition Labour party). Brander (1991) and Bernhard and Leblang (2006) study the 1988 Cana-
dian election. This was shortly after the implementation of the Canada-US FTA. The FTA was widely expected to increase trade between the two countries, being positive for the stock market. However, the opposition Liberal party was opposed to the agreement. Prices on the Toronto Stock Exchange (TSE) were found to be significantly positively related to polling numbers for the Conservative party during the campaign period.

We now briefly review the literature on political prediction markets. Prediction markets are exchange traded financial markets for the purpose of trading on the outcome of events. Election, or political, markets are prediction markets that are based on the outcome of elections. Modern examples of electronic election markets include University of Iowa’s Iowa Electronic Markets, introduced for the 1988 US presidential election, the University of British Columbia’s UBC Election Stock Market (now superseded by the Sauder School of Business Prediction Markets) and the Betfair Exchange, prices for which are used in this paper. There is a plethora of research on the accuracy of prediction, and election markets. For a recent review of prediction markets see Horn et al. (2014) and for political markets see Graefe (2016).

Studies, many of which are based on the Iowa and UBC markets, have demonstrated the remarkable accuracy of forecasts from election markets. There is a consensus in the literature that political markets outperform other methods including polling and expert predictions. Two early papers by Forsythe et al. demonstrate the outperformance of prediction markets when comparing final prices with final polling numbers for vote shares (despite the presence of judgement bias amongst traders). Forsythe et al. (1992) studies the Iowa market for the 1988 US presidential election. They find that ‘the market worked extremely well, dominating opinion polls’. By looking at the positions of different constituent groups they find that traders tended to place bets on their preferred candidates (indicating judgement bias). However, they conclude that prices are indeed set by the marginal trader. The implication being that even if a majority of participants are irrational and have misspecified beliefs, the existence of a small group of unbiased traders, perhaps arbitrageurs, keep prices in line. Oliven (2004) considers whether the Iowa market is skewed by biased participants. They conclude that market-making traders are more rational than price takers, the implication being that arbitrageurs are indeed making prices efficient. The analysis and conclusions are repeated for the 1993 Canadian federal election in Forsythe et al. (1995). Both these studies focus on shorter-term market predictions. A latter study, Joyce E. Berg and Rietz (2008), extends the analysis to longer-term forecasts. This paper compares vote share prices from the Iowa market with 964 polls over the five US presidential elections from 1988 to 2004. They find that the prediction market is closer than the polls 74% of the time. The average error in vote share

\footnote{Following the inclusion of Mexico in 1994 the CUSFTA became the North American Free Trade Association in 1994.}
for presidential candidates in the final five days of polling is 1.2% versus 1.64% for polls. Further ‘the market significantly outperforms the polls in every election when forecasting more than 100 days in advance’. This provides evidence that election markets are not only accurate at times close to a vote but have superior explanatory power months from an election.

The literature is clear that both election markets are powerful tools to forecast elections and that the outcome of elections have effects on asset prices. It naturally follows that prediction and financial markets should be related in some way. There are many event studies in the literature that consider elections, but they typically only consider either financial markets, or political markets and rarely both, and when they do they typically only uncover an empirical relationship and are not based on economic theory. The few studies in the literature that consider both types of market are discussed below.

There are two studies in the literature that directly relate political prediction market prices to the residuals of CAPM models. Both consider only the 2014 Scottish independence referendum. Acker and Duck (2015) find that the residuals of an estimated CAPM model are significantly positively related to several proxies for the likelihood of a vote to remain as part of the UK, one of which is a weighted sum of the Betfair exchange odds for ‘No’. Similarly Darby et al. (2019) study equities listed on the LSE that were headquartered in Scotland. They find that uncertainty betas help predict cross-sectional returns. Hanna et al. (2021) also use Betfair data to analysis how changes in the betting odds for ‘Leave’ influence financial markets for the Brexit referendum. They consider the period from January 2016 to the date the referendum was resolved (the early hours of June 24). Using high frequency data for trades on the Betfair exchange they regress short-term returns of GBPUSD and major UK and European stock indices on changes in betting prices during stock market opening hours. They find that changes in the odds for leave cause prices of UK equities and the pound to fall in the following 5 minutes. They also find some spillover effects to EU equity prices. However, the slopes of the regression appear somewhat small when compared to the sensitivities estimated in other studies, including in this paper. Finally we find only a single example of an economically derived relationship between prediction markets and asset prices in the literature which is Manasse et al. (2020). The authors build a simple portfolio model for currencies. This implies that currencies are cointegrated with betting prices for the period running up the to 2016 UK Brexit referendum. Under risk neutrality they find a linear cointegrating relationship but risk aversion leads to the presence of a risk factor related to uncertainty. This leads to a non-linear term of the betting market appearing in the cointegrating relationship. Auld (2022) builds a similar model in this paper. However that model only applies to the overnight session following an election. Manasse et al. (2020) find that currency and Betfair data are consistent with their model. However, we do not believe
the assumptions behind their model are valid. For it to be so, one would have to believe that the only determinant of the GBPUSD price in the weeks and months preceding the Brexit vote is the result of that vote. We believe this is not plausible. News and information beyond that relating to the referendum, including US economic releases, are likely to affect the British Pound and United States Dollar exchange rate in the period under study.

Contrary to the existing literature, this paper presents an asset pricing model of political and financial markets that is based on rigorous economic principles. It is for a particular setting, the session following an election, but applies to a general election. We believe the fact that there are no similar studies or theories presented in the literature speaks to the contribution of this work.

3 Theoretical model

We present a model linking the prices of political and financial markets that applies on longer term periods of weeks or months in the run up to an election.

We begin by outlining the scenario of the model. There exists a betting market which is liquid and trades multiple times a day in the run up to a scheduled event. Contracts are listed on the market relating to the outcome of a binary political event. Say $E = 1$ if this event occurs and $E = 0$ otherwise. The outcome of the election or event occurs at time $t = T$. For $t < T$ let $PB_t$ be the price of the contract that pays out when $E = 1$. There are $N$ financial assets indexed $i = 1, \ldots, N$ whose prices are $p_{it}$ at time $t$. For much of this section we consider only a single financial asset and then we label the price $p_t$. A unit time period ($\Delta t = 1$) is equal to one day.

3.1 Assumptions

There are again three main assumptions to our model:

CE The expected difference of the effect on asset prices between $E = 0$ and $E = 1$ is constant

EMHW The Efficient Market Hypothesis holds in the weak form

RN Risk Neutrality

The CE assumption is discussed below.

CE – Constant Effect of the political event

In the overnight model of [Auld 2022] it was assumed that the only determinant of the price of financial assets was the outcome of the political event. This led to conditional
expectations given the outcome of the election as being fixed in time. We wish to relax this assumption for longer periods but still impose the fact that the election has some fixed political effect on prices. For instance, ex-ante, the market may believe Trump winning an election would have a 10% effect on the stock price of the Mexican bank Inbursa, versus a Clinton win. This would be equivalent to

\[
\frac{\mathbb{E}_t (\text{INBURSA}_t | E = 1)}{\mathbb{E}_t (\text{INBURSA}_t | E = 0)} = 0.9 \quad \forall \ t < T. \quad (3.1)
\]

Similarly, for the duration of the election campaign, the market may have believed that the price would be 2.5 Pesos lower if Trump versus Clinton won. This could be expressed as

\[
\mathbb{E}_t (\text{INBURSA}_t | E = 1) - \mathbb{E}_t (\text{INBURSA}_t | E = 0) = 3 \quad \forall \ t < T. \quad (3.2)
\]

In either case some function of the two conditional expectations is fixed for the time period before the event, and not allowed to vary. We will be applying our model typically to a relatively small period of time. As such, as long as the stock price does not vary too much the above two formulations of the CE assumptions will be approximately equivalent. Taking logarithms of the first formulation would, for a general price \( p_t \), recover

\[
\log (\mathbb{E}_t (p_t | E = 1)) - \log (\mathbb{E}_t (p_t | E = 0)) = \gamma \quad \forall \ t < T.
\]

To make the assumption operational we make a further approximation. The conditional distribution of \( p_T | E \) at time \( t = T \) is likely to have variance of order much less than \( p_T \) during the period under study. This is because stocks typically vary by much less than order 100% over a period of a few weeks. As such, \( \log(p_t) \) will be approximately linear over the support of \( p_t | E \). Then

\[
\log (\mathbb{E}_t (p_t | E = i)) \approx \mathbb{E}_t (\log(p_t) | E = i) \quad \forall \ t < T.
\]

Thus if either of the above forms of the CE assumption for Imbursa hold (equations 3.1 and 3.2), the quantity

\[
\gamma_t = \mathbb{E}_t (\log(p_t) | E = 1) - \mathbb{E}_t (\log(p_t) | E = 0) = \gamma
\]

---

2Inbursa closed around 5.6% lower the day after Trump win, although it did initially trade significantly lower.

3Inbursa was trading in the range 27–30 Pesos in the weeks prior to the election.
will be approximately fixed in time for \( t < T \). This is the mathematical form that our CE assumption takes.

The CE assumption will not hold for all assets all of the time. As an example, consider the case of a UK based bank unexpectedly announcing the opening of European operations during the Brexit referendum campaign. This may reduce the firm’s reliance on cross-border regulations between the UK and the EU, reducing any negative effect of a vote for Brexit. As such it would be reasonable to expect \( \gamma \) to reduce for this bank over the announcement. Nonetheless this would be a fairly rare and specific event. We consider that the CE assumption is a weak assumption for most assets for the majority of the time.

3.2 Implications

We derive a model of asset and political betting prices using the assumptions. In what follows below we assume a single asset price with price \( p_t \) at time \( t \).

\[ r_t = \log(p_t/p_{t-1}) = \log(p_t) - \log(p_{t-1}) = \Delta \log(p_t) \] the return of the asset on day \( t \).

3.2.1 Constant Effect (CE)

First we write down an equation linking the price of a financial asset today, and the expected value in the future. This is

\[ \log(p_t) = \mu_{Tt} + \mathbb{E}_t(\log(p_T)) \] (3.3)

\[ \mu_{Tt} = \mathbb{E}_t(\log(p_T) - \log(p_t)). \]

Simply put, the log price today is the expected log price of the asset price at some point in the future plus the expected appreciation of the asset. The purpose of this equation is to link future expected prices with the price today. Financial economics has many instances of such equations. These include the uncovered interest parity (UIP) relation applied to currencies discussed in [Auld (2022)]. Another example is the cash and carry relationship linking the arbitrage relationship between forward or futures and spot prices. The common feature of these relationships is that any changes in future expectations of an asset’s price are immediately reflected in today’s price.

The first difference of equation 3.3 recovers

\[ r_t = \Delta \log(p_t) = \Delta \mu_{Tt} + \Delta \mathbb{E}_t(\log(p_T)). \] (3.4)
For notational convenience write

\[ E^i_t = \mathbb{E}_t (p_T | E = i) \quad i = 1, 2 \]
\[ P^i_t = \mathbb{P}_t^i (E = i) \quad i = 1, 2 \]
\[ \gamma_t = E^1_t - E^0_t. \]

where \( \mathbb{P}_t^i (\cdot) \) represents the probability of an event as evaluated by participants in the financial markets.

Next we expand the expectation of the log price using the total law of expectation.

\[
\mathbb{E}_t (\log(p_T)) = P^1_t \cdot E^1_t + P^0_t \cdot E^0_t \\
= P^1_t \cdot E^1_t + (1 - P^1_t) \cdot E^0_t \\
= E^0_t + (E^1_t - E^0_t) \cdot P^1_t \\
= E^0_t + \gamma_t \cdot P^1_t
\]

Taking first differences gives an expression for the second term of equation 3.4

\[
\Delta \mathbb{E}_t (\log(p_T)) = \Delta E^0_t + \gamma_t \cdot P^1_t - \gamma_{t-1} \cdot P^1_{t-1} \\
= \Delta E^0_t + (\gamma_{t-1} + \Delta \gamma_t) \cdot P^1_t - \gamma_{t-1} \cdot P^1_{t-1} \\
= \Delta E^0_t + \gamma_{t-1} \cdot (P^1_t - P^1_{t-1}) + \Delta \gamma_t \cdot P^1_t \\
= \Delta E^0_t + \gamma_{t-1} \cdot \Delta P^1_t + \Delta \gamma_t \cdot P^1_t
\]

Note that \( \Delta E^0_t = \Delta E^1_t \) and that this quantity is the change in the expected future log price of the asset at time \( T, \log(p_T) \), that is not related to the political event. Further, our CE assumption is precisely that \( \Delta \gamma_t = 0 \) and \( \gamma_t = \gamma \forall t \). So CE ⇒

\[
\Delta \mathbb{E}_t (\log(p_T)) = \Delta E^0_t + \gamma \cdot \Delta P^1_t.
\]  

(3.5)

Thus the CE assumption implies that the expectation of the log price from time \( t - 1 \) to \( t \) can be split up into a change not related to the upcoming election (\( \Delta E^i_t \) \( i = 1 \) or 2)
and the change due to the political event \((\gamma, \Delta P_t^1)\). The latter term is simply a constant multiplied by the financial market’s evaluation of the likelihood of \(E = 1, P_t^1\). Assets that are expected to appreciate when \(E = 1\) have \(\gamma > 0\) whereas expected depreciation results in \(\gamma < 0\). The greater the sensitivity to the political event the larger the magnitude of \(\gamma\).

### 3.2.2 Cointegration model as a special case

Equation [3.5] decomposes asset returns into that due to changes in the likelihood of the outcome of the election \((\gamma, \Delta P_t^1)\), and that unrelated to the election \((\Delta E_t^0)\). A key assumption in the overnight model of [Auld 2022], relaxed here, is that only changes in the likelihood of the event have an effect on asset prices. This is equivalent to setting \(\Delta E_t^0 = 0\) in [3.5]. This now becomes

\[
\Delta E_t(\log(p_T)) = \gamma \cdot \Delta P_t^1. \tag{3.6}
\]

To first order, this is equivalent to

\[
\Delta E_t(p_T) = \gamma \cdot \Delta P_t^1.
\]

Replacing \(\gamma\) with \(\Delta p = E(p_T|E = 1) - E(p_T|E = 0)\), which is now fixed, recovers equation 3.3 of [Auld 2022], in differenced form. This equation, with the addition of market efficiency and risk neutrality, led to the derivation of the cointegrating relationship. Thus we see that the model of [Auld 2022] is in fact a special case of that presented in this paper, where the variance of factors affecting prices unrelated to the vote is zero.

### 3.2.3 Weak Market Efficiency (EWMH)

As in [Auld 2022], we use the market efficiency condition to equate the likelihoods of \(E = 1\) in the betting markets with those of the financial markets.

\[
P_f^l(E = 1) = P^B_t(E = 1)
\]

where \(P_f^l(\cdot)\) is the probability of an event as evaluated by participants in the betting market. Recall that for the contract that pays out £1 when \(E=1\)

\[
u(PB_t) = P^B_t(E = 1) \times u(£1)
\]

where \(u(\cdot)\) is the Bernoulli utility function. Normalising the utility function so that
\[ u(£1) = 1 \Rightarrow \]
\[ \mathbb{P}_t^B (E = 1) = u(PB_t). \]

Thus weak market efficiency \( \Rightarrow \)
\[ \mathbb{P}_t^f (E = 1) = \mathbb{P}_t^B (E = 1) = u(PB_t). \]

Taking first differences and substituting into equation \[3.5\] \( \Rightarrow \)
\[ \triangle E_t (log(p_T)) = \triangle E_t^0 + \gamma. \triangle u(PB_t). \] (3.7)

To recover an expression for the daily return of an asset we still need to consider the change in the expected return of the asset to time \( T \), \( \triangle \mu_T \). This can be achieved using the fact that market efficiency implies that returns have a martingale property. First write

\[ \mu_T = E_t (log(p_T) - log(p_t)) \]
\[ = E_t (log(p_T/p_t)) \]
\[ = E_t (log\left(\frac{p_T}{p_{T-1}} \cdot \frac{p_{T-1}}{p_{T-2}} \ldots \frac{p_{t+1}}{p_t}\right)) \]
\[ = E_t (log\left(\frac{p_T}{p_{T-1}}\right) + \ldots + log\left(\frac{p_{t+1}}{p_t}\right)) \]
\[ = E_t (log\left(\frac{p_T}{p_{T-1}}\right)) + \ldots + E_t (log\left(\frac{p_{t+1}}{p_t}\right)). \]

The final line uses the fact that the expectations are separable due to the martingale property of returns. Thus

\[ \triangle \mu_T = \mu_T - \mu_{t-1T} \]
\[ = \left( E_t (log\left(\frac{p_T}{p_{T-1}}\right)) + \ldots + E_t (log\left(\frac{p_{t+1}}{p_t}\right)) \right) \]
\[ - \left( E_{t-1} (log\left(\frac{p_T}{p_{T-1}}\right)) + \ldots + E_{t-1} (log\left(\frac{p_{t+1}}{p_{t-1}}\right)) \right). \]
If we assume an expected constant rate of daily return for the asset of \( \mu \) then
\[
E_t(\log(\frac{p_{t+1}}{p_t})) = E_{t-1}(\log(\frac{p_{t+1}}{p_{t+1-1}})) = \mu
\]
and
\[
\Delta \mu_T = \mu. \tag{3.8}
\]
Substituting equations 3.7 and 3.8 into equation 3.4 \( \Rightarrow \)
\[
r_t = \mu + \Delta E_0^t + \gamma \Delta u(PB_t). \tag{3.9}
\]
This is a non-linear equation linking the prices in betting markets and financial markets. Note that as \( u(\cdot) \) is monotonically increasing, \( \Delta u(PB_t) \) increases when the odds of \( E = 1 \) increases, and decreases if the odds decrease. Changes in the likelihood of \( E = 1 \) affect returns directly through the factor loading \( \gamma \). This is similar to the \( \beta \)s in the Capital Asset Pricing Model (CAPM). Whereas \( \beta \) in the CAPM relates to the sensitivity of the asset to the single factor market return, \( \gamma \) in our model represents the sensitivity to the political event.

### 3.2.4 Risk Neutrality (RN)

Risk neutrality results in a linear utility function \( u(\cdot) \). The chosen normalisation of the utility function leads to \( \Delta u(PB_t) = \Delta PB_t \), the daily change in the betting odds of \( E = 1 \). Substituting this into equation 3.9 results in the expression for returns
\[
r_t = \mu + \gamma \Delta PB_t + \zeta_t \tag{3.10}
\]
\[
\zeta_t = \Delta E_0^t
\]
\[
E(\zeta_t) = 0. \tag{3.11}
\]
This is effectively a factor model with a single observed factor \( \Delta PB_t \). The error \( \zeta_t \) is the residual in the daily return, less the expected rate of return \( \mu \) and less changes due to political effects related to the upcoming election. \( E(\zeta_t) = 0 \) follows from the fact that \( E_t^0 \) is the expectation at time \( t \) of the future asset price at time \( T \), and will not be expected to drift in either direction from one day to the next. Mathematically this is demonstrated as
\[
\zeta_{t+1} = \Delta E_0^t = E_{t+1}(p_T|E = i)) - E_t(p_T|E = i)).
\]
Using the law of iterated expectations \( \Rightarrow \)

\[
\mathbb{E}_t(\zeta_{t+1}) = \mathbb{E}_t(\mathbb{E}_{t+1}(p_T|E = i)) - \mathbb{E}_t(\mathbb{E}_t(p_T|E = i)) \\
= \mathbb{E}_t(p_T|E = i)) - \mathbb{E}_t(p_T|E = i)) \\
= \mathbb{E}_t(p_T|E = i) - \mathbb{E}_t(p_T|E = i)) \\
T > t \\
= 0.
\]

So \( \mathbb{E}(\zeta_{t+1}) = \mathbb{E}(\mathbb{E}_t(\zeta_{t+1})) = \mathbb{E}(0) = 0. \)

The model shows that under risk neutrality a regression of asset returns on changes in the related betting market prices will be significant. Simply put, betting markets linearly explain asset price returns.

### 3.2.5 Relaxation of assumptions

We first discuss relaxing risk neutrality. Equation 3.9 is the non-linear version of the factor model. In Auld (2022) a non-linear cointegration relationships for the overnight period following an event was derived in two ways. The first used a quadratic approximation to the utility function \( u(\cdot) \). The fact that any risk premium would vanish when the political event was certain \( (E = 0 \text{ or } 1) \) results in a quadratic approximation of the form,

\[
u(PB_t) \approx PB_t - \lambda R_t
\]

\[
R_t = PB_t(1 - PB_t).
\]

\( R_t \) is the risk factor, being largest at the point of greatest uncertainty \( PB_t = 0.5 \). Taking first differences recovers the linearised model

\[
r_t = \mu + \triangle E_t^0 + \gamma \triangle PB_t - \lambda \triangle R_t.
\]

A non-linear risk premium \( (\triangle R_t) \) is now present in the returns equation. Risky assets will have \( \lambda > 0 \) as their price will be lower when uncertainty is greatest. Monotonicity of \( u(\cdot) \) implies \( |\lambda| < |\gamma| \).

The second way the approximation was derived in Auld (2022) was to consider a utility-maximising representative investor. Said investor holds a proportion of her wealth \( \omega \) in the risky asset. In the case of risk aversion, an excess return over and above the
usual rate of return will be required for an investor to bear the additional political risk due to the upcoming vote. At time \( t \) this excess is \( \mathbb{E}_t(\log(p_T)) - \log(p_t) - \mu(T - t) \). We assume log mean variance preferences, such that

\[
U(w) = \omega \cdot [\mathbb{E}_t(\log(p_T)) - \log(p_t) - \mu(T - t)] - \frac{r}{2} \omega^2 \sigma^2.
\]

\( r \) is the coefficient of absolute risk-aversion and \( \sigma^2 \) is the portfolio variance \( (= R_t) \). The first order condition is

\[
\mathbb{E}_t(p_T) - p_t = \mu(T - t) + \omega r \sigma^2.
\]

Note that the LHS of the above equation is precisely the definition of \( \mu_{Tt} \) in equation 3.3. As \( \sigma^2 = PB_t(1 - PB_t) = R_t \) then

\[
\Delta \mu_{Tt} = \mu - \lambda \Delta R_t,
\]

where \( \lambda \) replaces \( -\omega r \). Using this expression for \( \Delta \mu_{Tt} \) in equation 3.4 recovers the non-linear equation 3.12 above. Using the utility maximisation argument introduces the political risk premium directly. This is through the effect of the additional utility required to hold the politically risky asset from time \( t \) to time \( T \). This is not to say that the asset becomes risk free post-election. Investors will be compensated for holding risky assets both pre- and post-election, through the usual rate of return \( \mu \), but prior to the election the premium required will be greater.

Both the above approaches demonstrate that relaxing risk neutrality introduces a second factor into our returns equation linked to the change in uncertainty, \( R_t \). This is analogous to the result that relaxing risk-neutrality for the election night model of Auld (2022) resulted in a non-linear cointegration relationship. The non-linear model is summarised as

\[
r_t = \mu + \gamma \cdot \Delta PB_t - \lambda \cdot \Delta R_t + \zeta_t \quad |\lambda| < |\gamma|
\]

\[
R_t = PB_t(1 - PB_t)
\]

\[
\mathbb{E}(\zeta_t) = 0.
\]

Next we turn to errors in market efficiency. We add the stationary error \( \epsilon_t \) to the market efficiency condition linking beliefs in the two markets:
\[ P_t^f(E = 1) = P_t^B(E = 1) + \epsilon_t = u(PB_t) + \epsilon_t. \]

Equation 3.7 becomes

\[ \Delta E_t(\log(p_T)) = \Delta E_t^0 + \gamma. \Delta u(PB_t) + \gamma. \Delta \epsilon_t \]

and equation 3.9 is varied to

\[ r_t = \mu + \Delta E_t^0 + \gamma. \Delta u(PB_t) + \gamma. \Delta \epsilon_t. \]

The linear version of the factor model is now

\[ r_t = \mu + \gamma. \Delta PB_t + \zeta_t \]
\[ \zeta_t = \Delta E_t^0 + \gamma. \Delta \epsilon_t \]

and the non-linear model is

\[ r_t = \mu + \gamma. \Delta PB_t - \lambda. \Delta R_t + \zeta_t \quad |\lambda| < |\gamma| \]
\[ \zeta_t = \Delta E_t^0 + \gamma. \Delta \epsilon_t. \]

If market efficiency generally holds, but with some stationary fleeting errors between beliefs about the likelihood of the political event, the residual error of the factor model will be serially correlated.

### 3.3 Extension to multi-factor Fama–French model

We now consider a large portfolio of stocks, with returns \( r_{it}, i = 1, \ldots, N \). The single political factor model is
\[ r_{it} = \mu_i + \gamma_i \Delta PB_t + \zeta_{it} \]
\[ \zeta_{it} = \Delta E_{it}^0 \]
\[ \mathbb{E}(\zeta_{it}) = 0 \]

The error \( \zeta_{it} \) is allowed, indeed expected, to be correlated across different stocks. Under strict market efficiency it will be serially uncorrelated but fleeting deviations from efficiency will result in auto-correlation. Nonetheless the model above allows a parsimonious description of the covariance matrix of returns when a political event is upcoming. \( \gamma_i \) is the political factor loading and is a measure of stock \( i \)'s sensitivity to the upcoming election. Ceteris paribus, stocks with higher or lower \( \gamma \)s will have higher or lower correlation. \( \mu_i + \xi_{it} \) is the part of the return not related to the political event. Indeed when there is no upcoming vote the returns model reduces to \( r_{it} = \mu_i + \xi_{it} \). This is a poor description of the covariance structure of the portfolio of stocks as there are no common factors. We now extend the model by applying the ubiquitous Fama–French factor model to describe this part of stock returns.

The Fama–French factor model for equities is:

\[ r_{it} - r_f = \alpha_i + \sum_{j=1}^{K} b_{ij} f_{jt} + \varepsilon_{it} \quad \mathbb{E}(\varepsilon_{it}) = 0 \quad i = 1, \ldots, n \quad j = N, \ldots, K. \]  

There are \( N \) financial assets and \( K \) factors. \( r_{it} \) is the return of the \( i \)th stock at time \( t \) and \( r_f \) is the risk free return. The factors \( f_{1t}, \ldots, f_{Kt} \) are \( K \) univariate random variables that vary with time. \( B \) is a \( N \times K \) constant matrix describing the loading of the factors within the space of the \( N \) stocks (the elements of which are \( \{b_{ij}\} \)). \( \varepsilon \) is a vector of shocks, assumed to be serially uncorrelated and weakly correlated across different stocks as \( N \to \infty \). The traditional market asset pricing model, the Capital Asset Pricing Model (CAPM), uses only one variable to describe the returns, being the market factor \( R_m \). Fama and French’s initial factor model\(^4\) had three factors. These added factors related to the return of small versus large cap stocks and the returns of cheap versus expensive companies (measured by book-to-value ratios). The model was extended to five factors in Fama and French (2015). A factor relating to operating profitability as measured by profits to assets was added as was one based on a measure

\(^4\) \( R_m \) is the return of the market as a whole.
of investment by the company. The key result of the model is used to describe the cross-section of returns across stocks. It follows from arbitrage pricing theory and is

$$\mu_i = \mathbb{E}(r_i) = r_f + \sum_{j=1}^{K} b_{ij} \mathbb{E}(f_{jt}).$$

Equivalently $\alpha_i = 0 \forall i$ in equation 3.15. This implication has been studied in numerous papers and contexts, too many to discuss here. It is also not the topic of this paper. We will not impose $\alpha_i = 0$. We simply wish to use the common factor approach to describe the non-political part of stock returns. Following the convention in the literature we write the excess return of stock $i$ as $Z_{it} = r_{it} - r_f$. The most simple way to proceed is to replace the non-political part of the return, $\mu_i + \zeta_{it}$, in our one factor political model (equation 3.14) with the general factor expression above (equation 3.15). The result is

$$Z_{it} = \alpha_i + f_{1t} b_{i1} + \cdots + f_{Kt} b_{iK} + \Delta PB_t \gamma_i + \eta_{it}. \quad (3.16)$$

The shock has been replaced by $\eta$ in the full model above to distinguish it from the Fama–French residual, $\varepsilon$. We now have a $K+1$ characteristic factor model which includes the additional political component.

$\Delta PB_t$ is unlikely to be uncorrelated with the factors $\{f_{jt}\}$. Indeed, the outcome of most political events will be expected to have an overall effect on the prices of stocks. Changes in the likelihood of $E = 1$ will be highly likely to affect the returns of the market $R_m$. When this is the case it is important to note that the factor loadings in the full political factor model (equation 3.16), will be different from those in the standard Fama–French model (equation 3.15). This is also the case for the political loadings $\{\gamma_i\}$. These loadings in the full model will differ from those in the single political factor model (equation 3.10). Only in the unlikely event of $\Delta PB_t$ being uncorrelated with $\{f_{jt}\}$ would loadings be equal between the full model and the sub-factor models.

In this paper we are not interested in the accurate estimation of factor loadings in any one of these particular models. The purpose of the investigation is to examine whether political markets are explanatory of stock returns. As long as both the political loadings $\{\gamma_i\}$ are non-zero in either model, and $\Delta PB_t$ is not collinear with $\{f_{jt}\}$, this will be the case. It will also follow that $\Delta PB_t$ will be explanatory of the Fama–French residuals $\{\varepsilon_{it}\}$. When collinearity occurs, loadings in the full model are not identifiable, but, more importantly, adding betting market information to the model does not help describe stock returns. This would be because changes in the betting markets would already be fully explained by the Fama–French factors.
Table 1: Theoretical model summary.

<table>
<thead>
<tr>
<th>Assumptions</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>CE+EMHW</td>
<td>$r_t \approx \mu + \gamma \Delta P B_t - \lambda \Delta R_t + \zeta_t$ $</td>
</tr>
<tr>
<td></td>
<td>$R_t = P B_t(1 - P B_t)$</td>
</tr>
<tr>
<td></td>
<td>$E(\zeta_t) = 0$</td>
</tr>
<tr>
<td>CE+EMHW+RN</td>
<td>$r_t = \mu + \gamma \Delta P B_t + \zeta_t$</td>
</tr>
<tr>
<td>Multiple Factor Extension:</td>
<td>$Z_{it} = \alpha_i + f_{1t}.b_{i1} + \cdots + f_{Kt}.b_{iK} + \Delta P B_t.\gamma_i + \eta_{it}$</td>
</tr>
<tr>
<td>Deviations from EMHW</td>
<td>$E(\zeta_\cdot \zeta_{t-1}) &lt; 0$, $E(\eta_{it} \eta_{it-1}) &lt; 0$</td>
</tr>
</tbody>
</table>

The results of our theoretical model are summarised in Table 1. We describe our testing strategy in the next section.

4 Empirical specification

Our theoretical model for stock returns is summarised as the full factor model

$$Z_{it} = \alpha_i + f_{1t}.b_{i1} + \cdots + f_{Kt}.b_{iK} + \Delta P B_t.\gamma_i + \eta_{it} \quad i = 1, \ldots, N, \ t \leq T.$$

The model shows that betting markets are explanatory of stock return $i$ if and only if $\gamma_i \neq 0$. Testing the validity of our model is equivalent to testing that the set of parameters $\{\gamma_i\}$ are significant. Rejecting $H_0 : \gamma_i = 0 \ \forall \ i$ would be strong evidence in favour for the model holding, stronger than assuming the model as the null hypothesis and failing to reject.

The model itself is a $K + 1$ characteristic factor model. The historical daily $K$ Fama–French factor returns are readily available from Professor K.R. French’s website⁴. Histories from around 1990 are provided for download. The final political factor $\Delta P B_t$ is available from betting markets for periods preceding a political event. In this paper we rely on data solely from the Betfair exchange platform. This acts like a limit order book. Contracts may be listed months and even years before an election. However, liquidity generally increases as the event approaches. Far out from an election, on most days there may be no trading at all. $\Delta P B_t$ appears in the model as a measurement of the changes

⁴https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html
in the likelihood of a political event $\mathbb{P}_t(E = 1)$. However, unless the betting market is trading and liquid then the measurement will not be valid. Care will be required to ensure there is sufficient liquidity when choosing which time period to apply the model to. In practice only a small number of months or a few weeks directly prior to the election will be considered.

One could proceed by estimating the full model above on this small period and attempting to generate significance for $\gamma$. However, this would be inefficient. The much longer history of the Fama–French factors would not be exploited for estimation.

First choose a period where we judge the betting market to be sufficiently liquid to be a valid measure of beliefs about the likelihood of the political event. Call this period the testing period. Say it starts at $T_1$ and ends at $T_2$. (In practice $T_2$ will be the day before the election result is announced, $T - 1$). Define a longer period starting at $T_0$ and ending at $T_1 - 1$. Call this the testing period. Fama–French factor data from both the testing and training periods will be used but only betting data from the testing set is considered. The approach is illustrated in Figure 1.

Fama–French factor loadings are estimated in the training window. Explanatory power of the betting market on the Fama–French residual is tested in the testing window.

Figure 1: Schematic of the empirical approach.

One way to proceed would be to estimate the following regression

$$Z_{it} = \tilde{\alpha}_i + f_{1t} \hat{b}_i + \cdots + f_{Kt} \hat{b}_i + [\delta_{it}, \{T_1, T_2\}] \cdot \tilde{\gamma}_i + \tilde{\eta}_{it} \quad t = T_0, \ldots, T_2 \quad (4.1)$$
where

\[ \delta_{t,T_1,T_2} = I \left( t \in \{T_1, T_2\} \right). \]

This replaces the political factor with \( \delta_{t,T_1,T_2} \times \Delta PB_t \). This forces the factor to zero outside of the liquid testing period. However, this is a flawed approach. Political risk may still be present prior to the testing period, and changing. An election, say, three or four months in the future may be well known and the prospects for each of the candidates varying. However, it just may be too far in the future to be in the minds of those that choose to place bets in a political market. Lack of participation in a betting market may make an evaluation of the likelihood of an election outcome unmeasurable. This is not the same as it being constant and unchanging though. Conducting a regression where it is assumed to be zero in the training period will lead to invalid results.

To overcome this issue, and use more of the history of the Fama–French factors, we proceed as follows:

1. Estimate the Fama–French loadings \( \hat{\alpha}, \hat{B} \) during the training period \( t = T_0, \ldots, T_1 - 1 \).

2. Evaluate the Fama–French estimated residuals \( \hat{\varepsilon}_{it} = Z_{it} - \hat{\alpha}_i - \sum_{j=1}^{K} \hat{b}_{ij} f_{jt} \) on the testing period \( t = T_1, \ldots, T_2 \).

3. Regress the estimated Fama–French residuals \( \hat{\varepsilon}_{it} \) on changes in the political factor \( \Delta PB_t \) on the testing period \( t = T_1, \ldots, T_2 \) without intercept.

The above approach involves the same regression as that of equation 4.1 under the null hypothesis. For most event studies it would also produce identical statistics under alternatives of interest. However, if the political factor is in fact changing in the training set then the two step approach has greater power. This is demonstrated in Appendix 6.

As discussed in the previous section the Fama–French loadings estimated in step 1 above will not be the same as the loadings in the full factor model. This is due to the very likely correlation of \( \Delta PB_t \) with the other factors \( \{f_{1t}, \ldots, f_{Kt}\} \). However, significance of \( \gamma_i \) in the regression

\[ \hat{\varepsilon}_{it} = \alpha + \gamma_i \Delta PB_t + \xi_{it} \quad t = T_1, \ldots, T_2 \]

is equivalent to significance of \( \gamma_i \) in the full factor model. Note that we allow an intercept in the first stage regression as we do not impose the arbitrage pricing theory constraint. However, for the second stage we drop the intercept. This is because both \( \mathbb{E}(\hat{\varepsilon}_{it}) = 0 \) and
\( E(\Delta PB_t) = 0 \). The level of \( PB_t \) will change in the testing period but the unconditional expectation is zero. Of course there will be estimation error in \( \hat{\alpha}, \hat{B} \). This will lead to estimation error of \( \hat{\varepsilon}_{it} \). However, the error will be in the dependent variable in the second stage regression. Further, as the training and testing periods are disjoint, the estimation error will be independent of the residual of the final equation \( \xi_{it} \). No endogeneity will be present in the final step and estimates of \( \gamma_i \) will be unbiased, although there will be a reduction in precision.

### 4.1 Significance tests

In the next section we will test our model on a series of political events from recent years. For each event we will select an appropriate universe of stocks and a betting contract and follow the process described above. There are two tests we will perform which are outlined below.

#### 4.1.1 Portfolio test

The simplest test we apply is to consider the univariate regression for an equally weighted portfolio of the stocks of the chosen universe. If \( \varepsilon_{it} \) is the estimated Fama–French residual for stock \( i \) then define \( \bar{\varepsilon}_t = \frac{1}{N} \sum_{i=1}^{N} \varepsilon_{it} \), the average estimated residual. We then conduct the following regression

\[
\bar{\varepsilon}_t = \bar{\gamma} \cdot \Delta PB_t + \xi_t \quad t = T_1, \ldots, T_2.
\]

If \( \gamma_i = 0 \) \( \forall i \) then \( \bar{\gamma} = 0 \) in the above regression. Thus rejection of the null hypothesis \( \bar{\gamma} = 0 \) rejects the null hypothesis of insignificance of the betting market. This is a simple test to perform. It can also easily be made robust to serially correlated errors that may occur due to the presence in errors in market efficiency, as well as heteroskedasticity. However, the test is not robust to alternatives where the set of individual stock \( \gamma_i \)'s can take different signs.

#### 4.1.2 \( J_\alpha \) tests of Pesaran and Yamagata

The above portfolio test may suffer from a lack of power. There is loss of information by grouping all securities together in a single portfolio. We wish to use a test which can exploit information from a large number of stocks. As we are only conducting tests over periods where the betting market will be liquid, \( T \) is likely to be of the order of 10s, and

---

6There is an exception to \( E(\Delta PB_t) = 0 \). This is that the final Fama–French residual \( E(\varepsilon_{iT}) \) has positive expectation, as political risk, not present in the general factor model, disappears. We do not however, in practice, include the final period in the testing set. \( PB_t \) is at all other times a martingale.
certainly no more than 100. This is small compared to the number of stocks available. For example, there are around 5000 stocks listed in the US alone. There is the possibility of using large number of assets to generate significance but this will require a test to apply when $N > T$.

Recall that the key pricing implication of arbitrage pricing theory is that the excess returns, $\{\alpha_i\}$ in the regression equation 3.15 are zero. There is a large literature in empirical finance on testing this implication. Whereas the empirical question there is whether or not the intercepts in the panel model of excess stock returns on factors is zero, we are concerned with whether the slope coefficients $\{\gamma_i\}$ are significant. However, many of the existing methods translate simply when applied to the slope coefficient rather than the intercept. Reviews of the literature can be found in Black et al. (1972) and Fama and French (2004). Jensen (1968) was the first to use $t$-statistics to test the significance of $\alpha$ for a given security in the CAPM equation. However, when there are large numbers of stocks, individual tests lose their meaning. A joint test is sought. Further, the expected cross sectional dependence of the residuals leads to correlation of the individual $t$-statistics which means combining them into a single statistic is non-trivial. The standard test in the literature that addresses this problem is that of Gibbons et al. (1989). This is an exact test based on multivariate statistics. The test assumes normally distributed regression errors. It is also valid only when $N < T$. To overcome these limitations, typically monthly returns are used (to minimise deviations from non-normality), and securities are grouped into a small number of portfolios (to reduce $N$). Although superior to testing a single portfolio or stock, there is still loss of information, and potentially statistical power, due to grouping the securities into a smaller number of portfolios. There is also the possibility of introducing endogeneity when a large number of stocks are used as the portfolio return may become related to the market return factor $R_m$. Beaulieu et al. (2007) present a test of $\alpha_i = 0$. This is based on using simulation methods to calculate the test size for a wide class of non-normal distributions. Gungor and Luger (2009) and Gungor and Luger (2013) present distribution free non-parametric tests but are not robust to cross sectional dependence nor asymmetric distributions. Neither of these approaches is valid for $N > T$ though.

The problem of testing $\alpha_i = 0$ when the number of stocks is large relative to $T$ was solved in Pesaran and Yamagata (2012). This uses a normalised Wald statistic, $J_\alpha$, with a thresholding estimator on the cross-sectional error correlations which is robust to weak cross sectional dependence. Further, the test is demonstrated to be asymptotically valid in the case of non-normal errors. Monte Carlo evidence shows the test performs well in small samples. $T$ is tested at $T = 60$ and $T = 100$ and $N$ varied from 500 to as low as 50. Minimal size distortions are observed for the larger values of $N$ considered. These values of $N$ and $T$ are in the range of those used in our application. The test is based
on a Wald statistic of the individual t-statistics of univariate regressions, adjusted for (threshold applied) cross sectional correlations. It can be readily applied to testing the significance of the slope $\gamma_i$ and it is this test that will be primarily relied upon in the next section. Call this the $J_\gamma$ test. Improvements to the $J_\alpha$ (and other similar tests) have been proposed by Fan et al. (2015). They demonstrate that the $J_\alpha$ test lacks power against sparse alternatives where, for example, a finite number of stocks have significant $\alpha$s. They add a power enhancement term to the test statistic which vanishes asymptotically under the null but has power against such sparse alternatives. This may well be a relevant alternative when testing arbitrage pricing theory, where a small number of stocks may be responsible for failures in market efficiency. The empirical application in their paper, studying returns of the S&P500 index components, certainly suggests so. However, we do not believe this is relevant in our application. If betting markets are significant we expect explanatory effects beyond a small number of stocks, or particular industry sector. The number of stocks with significant $\alpha$s is likely to grow as $n \to \infty$. The $J_\alpha$ (or $J_\gamma$) tests should have power against the alternatives of interest and we will rely solely on them without power adjustment.

The test is summarised as follows. Consider the following factor model in the form of a panel data regression and stacked by cross-sectional regressions

$$y_t = \alpha + B f_t + u_t.$$ 

$y_i$ are the individual stock returns, and $f_t$ are $K$ known factors. $B$ is an $N \times K$ matrix of factor loadings. The Wald statistic for $\alpha = 0$ can be estimated as

$$W_\alpha = \sum_{i=1}^{N} t_{\alpha,i}^2$$

where $t_{\alpha,i}$ is the t-ratio of the intercept of the OLS regression of $(y_t)_i$ on intercept and $f_t$. In the case of cross sectionally independent errors it can be shown that under various regularity conditions

$$\mathbb{E}(W_\alpha) \to \frac{\nu N}{\nu - 2}$$

$$\mathbb{VAR}(W_\alpha) \to \frac{2N(\nu - 1)}{(\nu - 4)} \left( \frac{\nu}{\nu - 2} \right)^2$$

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as $N \to \infty$, $T \to \infty$ and $N/T \to \infty$ where $\nu = T - K - 1 > 4$. The following is an exactly standardised statistic

$$\hat{J}_{\alpha,1} = \frac{N^{-1/2} \sum_{i=1}^{N} \left( t_{\alpha,i}^2 - \frac{\nu}{\nu-2} \right)}{\left( \frac{\nu}{\nu-2} \right) \sqrt{\frac{2(\nu-1)}{(\nu-4)}}},$$

and thus is distributed as $N(0, 1)$ asymptotically under the null. The result holds in the case of non-normal errors. A second statistic $\hat{J}_{\alpha,2}$ is derived that is adjusted for correlation of the individual t-statistics when the errors are cross sectionally correlated. The adjustment is based on a consistent estimate of the correlation matrix of the disturbances $u_t$. Define

$$\hat{\rho}^2 = \frac{2}{N(N-1)} \sum_{i=2}^{N} \sum_{j=1}^{i-1} \hat{\rho}_{ij}^2 I(\hat{\rho}_{ij}^2 \geq \theta_N)$$

where $\hat{\rho}_{ij}$ is the sample correlation of the regression residuals $\hat{u}_t$ and $\theta_N$ is some threshold value. The latter is chosen so that the number of non-zero correlations decline steadily with $N$. We will follow the protocol set out in [Pesaran and Yamagata (2012)] with $\sqrt{\theta_N} = \Phi^{-1}(1 - \frac{p_N}{N})$ and $p_N = 0.1$. The adjusted statistic is now

$$\hat{J}_{\alpha,2} = \hat{J}_{\alpha,1} \times \frac{1}{\sqrt{\left[ 1 + (N-1) \hat{\rho}^2 \right]}} = \frac{N^{-1/2} \sum_{i=1}^{N} \left( t_{\alpha,i}^2 - \frac{\nu}{\nu-2} \right)}{\left( \frac{\nu}{\nu-2} \right) \sqrt{\frac{2(\nu-1)}{(\nu-4)}} \left[ 1 + (N-1) \hat{\rho}^2 \right]}.$$

[Pesaran and Yamagata (2012)] show that $\hat{J}_{\alpha,2}$ is asymptotically distributed as $N(0, 1)$ under various stricter regularity conditions and weak cross sectional correlation. They also show, via Monte Carlo simulation, small deviations from correct sizes for sample sizes similar to those used in this paper. Of course $|\hat{J}_{\alpha,2}| < |\hat{J}_{\alpha,1}|$ since it has lower variance due to the correlation adjustment of the t-statistics. $\hat{J}_{\alpha,2}$ will never reject the null when $\hat{J}_{\alpha,1}$ does not. We will not consider the first statistic at all as we do expect that errors will indeed have some correlation.

The test generalises simply to the slope parameter. We thus consider

$$\hat{J}_{\gamma,2} = \frac{N^{-1/2} \sum_{i=1}^{N} \left( t_{\gamma,i}^2 - \frac{\nu}{\nu-2} \right)}{\left( \frac{\nu}{\nu-2} \right) \sqrt{\frac{2(\nu-1)}{(\nu-4)}} \left[ 1 + (N-1) \hat{\rho}^2 \right]}.$$
where $t_{\gamma,i}$ is the t-statistic for slope in the regression of the Fama–French residual $\hat{\varepsilon}_{it}$, on intercept and changes in the betting market, $\triangle PB_t$.

This test has been demonstrated to have excellent properties for our setting where there may be non-normal errors and some correlation amongst stock residuals. The test however is not demonstrated to be robust in the case of serial correlation (possibly present due to inefficiencies) or heteroskedasticity. In practice we will test the residuals of the second stage regression for the presence of these effects. The lower power univariate portfolio test does have one advantage over this large $N$ test which is that it is simple to use robust errors.

5 Results

This section presents the empirical approach applied to real world data from six+ elections from recent years. To apply the method several choices will need to be made for each event. These include which particular betting contract to use, what portfolio of stocks to consider and what training and testing periods to use. A plot of the logarithm of trailing 7-day average daily volume on the Betfair exchanges for our chosen event is shown in Figure 2. Any key announcement concerning the event is also shown on the figure. As can be seen, liquidity explodes exponentially as the day of voting approaches, but can be very low a few months out. For example, the daily volumes trading on exchange are under £10k 5 months from several of the elections. We will need to treat data from lower liquidity days cautiously. Below, we discuss each event in more detail. Table 2 lists each election along with their chosen specifications for testing. Note that we test two portfolios for the Brexit referendum as well as the 2016 US presidential election. Also, 1, 3 and 5 factor models are available for each Fama–French choice, the 1 factor equating to CAPM.

5.1 Events

2014 Scottish independence referendum

The vote took place on 18th September 2014. The question on the ballot paper was ‘Should Scotland become an independent country’. To recap, polls showed a consistent lead for ‘No’ during July and August of that year. Polls tightened in September. There was even a poll that showed a small lead for ‘Yes’ published on 6th September. The pound and companies linked to Scotland depreciated on that day and the betting contract paying out £1 for ‘Yes’ rallied to 35p. The risk neutral Betfair implied probabilities can be seen from Figure 3. The figure shows the prices along with the chosen testing windows for
Figure 2: Liquidity on Betfair in the months prior to our chosen political events.
Figure 3: Betfair implied probabilities.
this and the other political events. Ultimately though, polls reverted and the ‘No’ side prevailed. As this was a binary Yes/No referendum the choice of betting contract is simple. We chose the ‘No’ bet in our model. For stocks we chose all companies listed on the LSE, from Q2 2014, domiciled in the UK. Many small less liquid companies will be removed from the portfolio according to a further screening step. This is discussed at the end of this subsection and is applied to all the portfolios studied in this paper. Fama–French daily factor returns are available for the European market and we utilise this as the most appropriate. The choice of testing period is not straightforward. The Scottish Government announced the date of the referendum on 21 March 2013, around 18 months prior to the vote itself. Liquidity on betting markets was muted though. Little more than around a few thousand pounds exchanged hands per day on Betfair through April, May and June 2014. The volume increased to around £10k per day in July and peaked at well over £1m a day in the week before the referendum. We chose to start the testing window on July 30th to give us sufficient time samples in the testing window. The volume did start to significantly increase from around £25k per day from this point.

**2016 Brexit referendum**

This is a binary vote and so the choice of betting contract is straight forward. We use the contract for ‘Leave’. On 20th February 2016 David Cameron announced the date of the referendum to parliament as 23rd June 2016. Liquidity improved on Betfair from this point with around £100k per day trading the week after the announcement, with volume peaking at around £50m on the overnight session following the vote. This makes our choice of testing period easier. We chose the day after the announcement to begin the testing window. The betting percentage odds during this period put the likelihood of leaving the EU from between the high teens to the high 30s. However, by the day of the vote there was a widespread (misplaced) belief that the country would vote to remain. The likelihood of a Leave vote bottomed out around 10%, just after the vote closed. We make the similar choices for the equity universe and Fama–French factors as the 2014 Scottish independence referendum, using stocks listed on the UK based LSE from Q1 2016. We also separately test a portfolio of stocks that are made up of EU27 based companies that earned at least 25% of their revenues in the UK according to their 2015 full year accounts.

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7Daily developed market factors are also available but the European centric-data set is preferred.
Table 2: Specifications for each political event.

<table>
<thead>
<tr>
<th>Event</th>
<th>Bet</th>
<th>Portfolios</th>
<th>Factors</th>
<th>Start Test</th>
<th>End Test</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>2014 Scottish independence referendum</td>
<td>No</td>
<td>Stocks listed on LSE in Q2 2014</td>
<td>Fama/French European</td>
<td>30-Jul-2014</td>
<td>18-Sep-2014</td>
<td></td>
</tr>
<tr>
<td>Brexit</td>
<td>Yes</td>
<td>Stocks listed on LSE in Q1 2016 European Stocks with &gt; 25% revenues from UK in 2015</td>
<td>Fama/French European</td>
<td>21-Feb-2016</td>
<td>23-Jun-2016</td>
<td></td>
</tr>
<tr>
<td>2017 UK General Election</td>
<td>Conservative Majority</td>
<td>Stocks listed on LSE in Q1 2017</td>
<td>Fama/French European</td>
<td>20-Apr-2017</td>
<td>8-Jun-2017</td>
<td></td>
</tr>
</tbody>
</table>

1 The test period ends on the day of the vote. The result is generally known one day later.

2016 US presidential election

The election of Donald Trump as the 45th president of the United States was the second political surprise of 2016. As election day is set under statute as ‘the Tuesday next after the first Monday in the month of November’, the date is known well in advance. However, the two candidates are not nominated until the summer before the vote. Recall that our model assumes a fixed difference between an expectation of a particular outcome (say $X$) and the complement of that outcome ($\overline{X}$). Even if one candidate is known, the complement, that is the opponent, may not be. Thus the expectation of the complement can change without the betting odds for the known candidate changing. For example, Donald Trump’s nomination came before the Democratic candidate. The odds of the particular Democratic candidate can change, effecting asset prices, without a corresponding change to the likelihood of Donald Trump winning. So our model will not hold. Until both candidates are known the model will not apply. The vote is not binary.

8 The vote for US president is in fact never completely binary. There is always the chance that a candidate may become incapacitated during the campaign. Running mates often trade with very small but positive likelihoods. The morbidly interested reader can look at the betting odds for Kamala Harris in the 2020 election to impute the market’s belief about the likelihood of the elderly president Biden passing during his campaign!

8 Trump was officially
nominated at the Republican national convention on 19th July, Clinton, on July 26th at the Democrat event. Liquidity for the election was much higher than the preceding events, with hundreds of thousands of pounds exchanging hands on Betfair around the time of the nominations. We are comfortable with using the betting markets from the day after both nominations were known. We chose the contract for Donald Trump for the model. Trump was (incorrectly) never seen as the stronger candidate. The percentage odds of him winning never rose above the mid 30s during the testing window. For this event we will use two different stock universes and test both. The first will be the index stocks of the S&P500 in Q2 of 2016; the second, publicly traded stocks domiciled in Mexico. We do not limit ourselves to companies listed on the major Mexican exchanges as many top Mexican stocks trade offshore. It was expected that a Trump win would be very bad for trade with Mexico and hence its markets. The relevant Fama–French factors for both these portfolios are the North American ones.

2017 UK General Election

The result of the 2016 Brexit referendum was a surprise. The ruling conservative party had a working majority of 17 in the house of commons. There was no election due under the Fixed-term Parliaments Act until May 2020. However, following the UK’s triggering of Article 50 of the Treaty on European Union, the Prime Minister Theresa May called a surprise election. This was to ‘strengthen her hand’ in negotiations with the EU. Ultimately though she failed in that aim. She lost her majority and following the result governed as the leader of a minority government with the support of the Democratic Unionist party.

Legislation to enact a general election was ratified by parliament on 19th April, 2017. The date was set for 8th June 2017. Until 18th April, the day before the announcement, liquidity for this event on Betfair was minimal, struggling to reach even £100 per day. However, trading jumped to around £100k a day once it became clear an election was imminent. It increased to over £1m per pay in the days before the vote. The choice of testing set is clear with this event as there is simply no liquidity prior to the date of the election’s announcement.

Until the snap election became public, the odds for a conservative majority were barely trading. For the few trades that did occur, the implied likelihood was around 50%. The lack of liquidity was likely due to the fact that the election was not scheduled for several years. Gamblers were neither interested in, nor informed about, the details of the election. This observation is remarkably consistent with the findings of Page and Clemen (2013). This is that prediction markets prices are significantly biased towards

\footnote{There are 650 seats in the house of commons and 17 is considered a small majority which makes governing difficult.}
50% for events far into the future. Until the announcement of the election date, the vote was indeed expected to be ‘far into the future’. Going into the campaign, opinion polls consistently showed a lead for the Conservatives. They tightened as the vote approached, Wikipedia (2017). Implied odds for a majority rallied to over 80% when the election was announced. They stayed in the range 78–95 until the vote. The choice of which contract to use is not straightforward. If the conservatives did not achieve a majority, then there were were potentially multiple alternatives. Bets for a ‘Labour majority’, a ‘hung parliament’ or ‘anything else’ were listed. However, the conservative’s lead over the Labour party was such that the likelihood of an actual Labour majority was very small. For the most part of the campaign, odds for this outcome hovered around 1–2.5%. It did increase a little as the polls tightened going into the election but was never implied to be higher than 4%, except for a few trades on election day. This means the event was close to binary with the majority of the likelihood distributed between a conservative majority and a hung parliament. However, we should note our model does not hold exactly. In terms of the universe of stocks, we will stick with all stocks headquartered in the UK listed on the LSE (in Q1 2017), with the European Fama–French factors.

2019 UK General Election

Following the UK government losing its majority in the house of commons in 2017 there was prolonged political deadlock. This led to Theresa May resigning as Prime Minister. Boris Johnson was elected as her replacement by the Conservative party in the summer of 2019. Johnson could not, though, convince the house of commons to pass a revised withdrawal agreement10. This caused him to call a snap election, the third general election in 4 years. Legislation was passed on 28th October 2019 and the date was set for 12th December. As was the case with the 2017 general election, trading on Betfair rose significantly over the announcement. For example, the day before only £2,600 traded, whereas the day after volume jumped to over £100k. Volumes generally increased from this point with over £2m per day trading in the final week. Given the uptick in volume we will again start the testing period the day after the announcement.

Throughout the campaign period, the conservatives held a strong lead over the Labour party (Wikipedia 2019). However, the implied odds of a Conservative majority was relatively low compared to opinion polls. It traded in the range 40–55% in the first fortnight of the campaign. This was likely due to the underperformance of the conservatives versus initial polling and expectations in the 2017 election. The odds did steadily increase to around 70–80% as Labour failed to significantly tighten the polls. The conservatives ultimately gained a landslide win with a huge majority of 80 seats in the house

10The withdrawal agreement was made between the UK and the EU and established the terms of the UK’s withdrawal from the EU.
of commons. We again use the bet for ‘Conservative majority’. The prices for a labour majority did initially imply odds as high as 6% but this soon dropped to the 2–3% range as they failed to make ground. The alternative to a conservative majority was again dominated by a hung parliament. We again use the same choices for the equity universe as the preceding UK events, with UK stocks listed on the LSE in Q3 2019.

2020 US presidential election

In 2020 the presidential election was scheduled for 3rd November. There was never any doubt that Trump, as sitting president, would run for a second term. However, there was a political battle for the heart of the Democratic party between establishment Joe Biden and the left winger Bernie Sanders. On June 5th Joe Biden gained the required number of delegates from the Democratic national convention to secure the nomination. Volume on Betfair for this event was in the hundreds of thousands of pounds a day at the time of the nomination. This increased to around £1m a day in the days immediately prior to the vote. As we now know, president Biden did not secure the Presidency the day after the election. In the days and weeks after election day there were numerous recounts. There were also several (seemingly baseless) legal challenges by the Trump campaign looking to overturn what was apparently a clear win for the Democratic ticket. Contracts continued to trade on Betfair until 14th December 2020. The electoral college confirmed Joe Biden’s victory in the election on that date. There is no doubt that there are numerous opportunities to study financial and political markets in the period 9th November to 14th December 2020. However, we do not believe the model presented in this paper will apply. This is because the prospects of a Joe Biden win and its alternative will have fundamentally changed. Instead of political risk relating to this election disappearing overnight on 8th November, it substantially increased. The prospect of Donald Trump retaining the presidency via a typical presidential election and one where the apparent rightful winner is deposed by the courts (or worse) is very different. The risk becomes less about what the electorate is voting for and more about the strength of US institutions. Political implications aside, the conditional expectations of asset prices given a Biden or Trump win certainly changed on 4th November 2020. Given this, we have no confidence the model will hold. We end the testing set on 3rd November. We use two equity universes to test this event: the index stocks of the S&P500 in Q2 of 2020 and the North American Fama–French factors.
5.2 Data sources and handling

**Equity data**

For each event we have selected a universe of stocks and Fama–French factors. Our aim is to ascertain if there is evidence to reject the null hypothesis $\gamma = 0$. The input price series we use to generate stock returns is the CRSP adjusted close data. This adjusts the price to account for dividends, stock splits, other distributions and rights issues\textsuperscript{11}. We use a variety of sources to conduct stock screens and source price data, including S&P Capital IQ, Thomson Reuters Datastream and Yahoo Finance. We do not exclude data points for days with specific company results or other announcements. This will no doubt introduce variance unrelated to either the political event or the Fama–French factors. Removing the corresponding return for such announcements would ‘clean’ the data set. Doing so would likely reduce the variance of the unexplained part of our regressions and potentially improve significance. It would also involve a considerable computational effort. We do not take this step, choosing to focus on testing a greater number of events, rather than a smaller number with cleaner data. This means there is an implicit assumption in our method. This is that the frequency of such stocks specific events is similar in training and testing sets. There is no reason to believe that this assumption is not valid.

Many of the stock universes we have chosen include all equities listed on a particular exchange, or domiciled in a particular country. This will include many illiquid and infrequently traded stocks. They may not respond to changes in the likelihood of a political event as they simply may not be traded. There may also be stocks included that do not exist for all of the training and testing sets. The estimates of the factor loadings may not be accurate for such stocks. To remove such securities we apply a filter to the portfolio. Firstly we remove any stocks that were not trading at the start of the training set. Secondly we only consider stocks that have trades, and non-zero returns, for 90% of days in both the training and testing sets. In practice this reduces the size of our stocks universes significantly.

**Betting data**

The betting company Betfair runs a platform called Betfair Exchange. This functions like a limit order book. For sporting, political and other events they list markets where orders for bets can be placed and executed. For each market, various selections are listed. Only one of the selections on each market will ultimately win. Participants can either ‘back’ or ‘lay’ each selection. The prices quoted are in terms of odds. Backers of the winners receive the odds multiplied by their stake, less a small percentage commission levied.

\textsuperscript{11}Adjustment methodology can be found at [https://www.crsp.org/products/documentation/crsp-calculations](https://www.crsp.org/products/documentation/crsp-calculations)
by Betfair. The payout, gross of commission, is paid by the participant that laid the relevant matched bet. The risk neutral probability, ignoring commission, for a selection with odds $o$ is $1/o$. In this paper we only quote probabilities and not odds and think of each bet as a binary contract. Figure 4 shows a screen shot of the Betfair market for the 2020 US presidential election on 21st November 2020. This was in the height of the political mayhem that followed the election when Donald Trump was contesting results with various legal challenges. It can be seen that the risk neutral probability of him winning is seen at almost 4%, although we believe the discount to the payout from a Biden win is largely due to risk and cost of capital matters. Note that all contenders apart from Trump and Biden have orders to lay at 1000 to 1 (1000 is the maximum odds allowed on the exchange). The market has reflected the removal of the other candidates from the presidential race.

The raw data we use from Betfair are matched trades on the exchange for the relevant market. For non-binary markets, which have multiple possible outcomes, such as the US presidential election shown in Figure 4, we only consider trades in a single selection (cor-
responding to $E = 1$). We handle binary events (Brexit and the Scottish independence referendum) differently. Here we use both selections, converting the alternative likelihood to the chosen selection via $P(E = 1) = 1 - P(E = 0)$. The market is sufficiently efficient that arbitrage opportunities do not exist by holding every selection on a market (doing so guarantees a payout). Trades also happen at multiple times throughout the day and at various prices. We need to convert the trades into a daily difference that aligns with the stock returns. We do this by choosing the trade that happens closest to the time of the relevant equity market close on each day.
Table 3: Regression of Fama–French factor returns on $\Delta PB_t$ for political events in the testing window.

<table>
<thead>
<tr>
<th>European Factors</th>
<th>American Factors</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope</td>
<td>Error</td>
</tr>
<tr>
<td><strong>2014 Scottish independence ref.</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MktRF</td>
<td>1.15</td>
<td>3.24</td>
</tr>
<tr>
<td>SMB</td>
<td>-0.21</td>
<td>1.06</td>
</tr>
<tr>
<td>HML</td>
<td>-0.67</td>
<td>1.31</td>
</tr>
<tr>
<td>RMW</td>
<td>0.89</td>
<td>1.17</td>
</tr>
<tr>
<td>CMA</td>
<td>-0.70</td>
<td>0.89</td>
</tr>
<tr>
<td><strong>2016 Brexit referendum</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MktRF</td>
<td><strong>-16.28</strong></td>
<td>5.00</td>
</tr>
<tr>
<td>SMB</td>
<td><strong>3.16</strong></td>
<td>1.90</td>
</tr>
<tr>
<td>HML</td>
<td>-0.83</td>
<td>1.81</td>
</tr>
<tr>
<td>RMW</td>
<td>0.57</td>
<td>1.47</td>
</tr>
<tr>
<td>CMA</td>
<td>0.34</td>
<td>0.62</td>
</tr>
<tr>
<td><strong>2017 UK GE</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MktRF</td>
<td>0.23</td>
<td>4.92</td>
</tr>
<tr>
<td>SMB</td>
<td>0.35</td>
<td>1.88</td>
</tr>
<tr>
<td>HML</td>
<td><strong>5.34</strong></td>
<td>2.15</td>
</tr>
<tr>
<td>RMW</td>
<td><strong>-3.41</strong></td>
<td>1.77</td>
</tr>
<tr>
<td>CMA</td>
<td><strong>2.70</strong></td>
<td>1.39</td>
</tr>
<tr>
<td><strong>2019 UK GE</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>MktRF</td>
<td>-0.30</td>
<td>1.66</td>
</tr>
<tr>
<td>SMB</td>
<td>0.91</td>
<td>0.98</td>
</tr>
<tr>
<td>HML</td>
<td>-1.30</td>
<td>1.25</td>
</tr>
<tr>
<td>RMW</td>
<td>0.22</td>
<td>0.60</td>
</tr>
<tr>
<td>CMA</td>
<td>0.64</td>
<td>0.74</td>
</tr>
</tbody>
</table>

Heteroskedastic robust errors are used.
Fama–French factor data

Fama–French daily factor returns are available for download from Professor French’s website. We source data for the five factor model for both the training and testing periods. The factors are labelled MktRF, the market return, SMB, the small minus big size companies, HML, high minus low value stocks, RMW, robust minus weak operating profitability and CMA, conservative minus aggressive investment.

In terms of the size of the training set, we settle on using two years worth of data. We use the data immediately prior to the testing set to estimate the Fama–French loadings. We judge this period sufficient to estimate the factor loadings. We avoid choosing a longer training set to avoid the possibility of structural breaks in stock loadings. Over long periods of time the factor loadings may change for a stock. For example, small companies can become large ones.

5.3 Results

Firstly we examine relationships between the factor returns and the betting markets. We regress each factor on $\Delta PB_t$ and an intercept in the event window for each political event. Note we exclude the day after the election. This is because both the changes in betting odds, and market moves, can be very large on that day. Including this day will dominate the OLS regression. Further, returns, and Fama-Franch factors, are calculated based on market close. If we were to include the day after the election, it would be better to look at the market open as this will be the first snapshot of prices post-result. There can be large intra-day volatility the day after the outcome becomes known. For example, after the 2016 US presidential election the market rout that followed the surprise win was totally reversed within the trading day. This followed a reassuring morning address from president Trump. If there is a relationship between the betting and stock markets we want to test that it is persistent over a reasonable length of time and not just when the election result becomes known. Results for the slope for these regressions are shown in Table 3. Errors robust to heteroskedasticity are used. The betting markets do not explain the variation in the European factors for the Scottish independence referendum. This is not surprising as this was a UK based event and the factors are based on the whole of the European market. Correspondingly though for the Brexit referendum, there is a significant relationship at the 99% level for the market return, and at the 90% level for the SMB factor. Brexit was an event that had potential effects beyond the UK and could affect many large European economies and exporters. (The Fama–French European factor covers both the UK and mainland Europe.) The estimated slopes in the regressions suggests a difference in price of the average European company of 17%
(±5%) between the UK remaining in the EU and leaving. Smaller companies would be expected to outperform by 3% (±1.9%) under Brexit, perhaps as smaller EU companies are less likely to export to the UK than large ones. The 2017 UK general election has surprising results. MktRF and SMB do not appear related to the betting market, but the other three factors do. This election was called by the Prime Minister Theresa May to ‘strengthen her hand’ in any Brexit negotiations. The outcome of this election could have been expected to have an effect on the type of deal the UK and the EU would ultimately strike. It then follows that the likelihood of Theresa May gaining a majority could have an effect on stock prices throughout Europe. However, it is puzzling that effects were seen on stock prices according to their relative value, operating profitability and investment strategy, but not on the overall level of the market. Further, no significant effects were seen on the Fama–French factors in the general election two years later, where the new Prime Minister Boris Johnson was vowing to ‘Get Brexit Done’. Turning now to the 2016 US presidential elections, we see significant effects in the SMB and CMA factors. The relationship between MktRF and $\Delta PB_t$ is not seen as significant though. This is a surprise given the huge sell off in asset prices observed on the night of the election itself when Donald Trump unexpectedly won. We do note that the estimated slope coefficient does suggest the market would be 5.2% (±4.8%) lower if Trump were to win. For the 2020 election, no significant effects are seen on the Fama–French factors from the betting market. To experiment to see if the betting markets become more informative as the election approaches we re-run the regressions, but begin the event window only a calendar month before the dates of the election. The new results are shown in Table 4 and are striking. All factors are now significant for the 2016 election bar RMW and MktRF and CMA are significant for the 2020 election. Biden is now seen as having a rather unbelievable positive effect of around 25% (±8%) on the stock market if he were to replace Trump as president. This suggests that the betting markets become more informative as the election nears. Note that liquidity also exponentially increases during this time too.

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13The S&P500 future initially sold off by 4.8% overnight. The odds for Trump increased from around 20% to 99% during this time.
Table 4: Regression of Fama–French factor returns on $\Delta PB_t$ for US presidential elections from the month prior to election day.

<table>
<thead>
<tr>
<th></th>
<th>2016 US election</th>
<th></th>
<th>2020 US election</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Slope</td>
<td>Error</td>
<td>tStat</td>
<td>pValue</td>
</tr>
<tr>
<td>MktRF</td>
<td>-12.03**</td>
<td>(5.92)</td>
<td>-2.03</td>
<td>0.042</td>
</tr>
<tr>
<td>SMB</td>
<td>-4.89**</td>
<td>(2.22)</td>
<td>-2.20</td>
<td>0.028</td>
</tr>
<tr>
<td>HML</td>
<td>4.45</td>
<td>(2.96)</td>
<td>1.50</td>
<td>0.133</td>
</tr>
<tr>
<td>RMW</td>
<td>1.82</td>
<td>(1.60)</td>
<td>1.14</td>
<td>0.254</td>
</tr>
<tr>
<td>CMA</td>
<td>5.09***</td>
<td>(1.61)</td>
<td>3.16</td>
<td>0.002</td>
</tr>
</tbody>
</table>

$T = 22$  
Heteroskedastic robust errors are used.

5.3.1 Significance tests

We now turn to results of significance tests for the model. As with the Fama–French factor regressions, we exclude the day after the election from the testing period. Including the day after in the regressions would almost certainly generate significance for any events with surprise results. Returns will likely have very large values on that day. However this does not test that the model holds over a general period and would relate to significance ‘after the fact’. We apply the steps in the empirical specification for the residuals of the K=0, 1 and 5 factor Fama–French models. For K=0 we simply regress stock returns, less their mean from the training set, on $\Delta PB_t$. K=1 corresponds to residuals from the CAPM model. Both the mean weighted portfolio and individual stock Pesaran and Yamagata tests are run. Results are shown in Table 5. We perform various diagnostics to check for serial correlation and heteroskedasticity.

From the theoretical pricing model recall that the presence of serial correlation in the regression errors is consistent with errors in market efficiency. It would also invalidate the results of the $J^2$ test. Ljung Box statistics are calculated for all regressions, including the multivariate form of the statistic. There is little evidence of serial correlation from the mean weighted portfolio regressions. Of the 24 regressions conducted a single one has a significant Ljung-Box statistic at the 95% level. This is for the one factor model for the 2016 US presidential election. This is not repeated for the other factor models, nor for the individual stock regressions for that election. We ascribe little relevance to this observation. There is also no evidence for serial correlations from the portfolio regressions with no significant Ljung-Box statistics. The conclusion is that the data demonstrate no evidence of market failure. Chapters 1 and 2 found deviations of weak market efficiency.
Table 5: Results of the significance tests for \( \gamma \).

<table>
<thead>
<tr>
<th></th>
<th>Univariate regressions(^a)</th>
<th>Individual stock regressions</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( K )</td>
<td>( \tilde{L} )</td>
</tr>
<tr>
<td>2014 Scottish independence referendum</td>
<td>0</td>
<td>0.025</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.017</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.028</td>
</tr>
<tr>
<td>2016 Brexit referendum - LSE stocks</td>
<td>0</td>
<td>-0.147**</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-0.028*</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-0.036</td>
</tr>
<tr>
<td>2016 Brexit referendum - European exporters to UK</td>
<td>0</td>
<td>-0.133***</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.010</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.024</td>
</tr>
<tr>
<td>2016 US presidential election - S&amp;P500 constituents</td>
<td>0</td>
<td>-0.041</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.009</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.005</td>
</tr>
<tr>
<td>2016 US presidential election - Mexican stocks</td>
<td>0</td>
<td>-0.156***</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-0.158***</td>
</tr>
<tr>
<td>2017 UK general election - LSE stocks</td>
<td>0</td>
<td>-0.088</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>-0.012</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>-0.011</td>
</tr>
<tr>
<td>2019 UK general election - LSE stocks</td>
<td>0</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.014</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.014</td>
</tr>
<tr>
<td>2020 US presidential election - S&amp;P500 constituents</td>
<td>0</td>
<td>0.083</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>0.004</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>0.006</td>
</tr>
</tbody>
</table>

\(^a\) Heteroskedastic robust errors are used for the univariate portfolio regression
\(^b\) Ling-Li serial correlation test p-value
\(^c\) Ling-Li multivariate heteroskedasticity test p-value
\(^d\) Engle ARCH test p-value

41
of the order of minutes to tens of minutes. It is not surprising that no inefficiency was found here. We consider only less frequent daily data. Discovering an inefficiency on that time-scale would be a much more surprising result.\footnote{Given the number of regressions performed, we would expect a small number of seemingly significant Ljung Box statistics at the 95% level under the null hypothesis of no serial correlation.}

Turning now to heteroskedasticity we conduct both the Breusch-Pagan and Engle ARCH tests for the univariate portfolio regressions. The Engle test provides some evidence of deviations from homoskedasticity in these univariate regressions. We note though that only one event, the 2019 UK general election, is significant, at the 95% level, once the first market wide factor has been controlled for. However, results of these regressions will be valid as heteroskedasticity robust errors are employed. For the $J_{\gamma,2}$ tests we conduct the \cite{Ling Li 1997} test. We observe a single significant value at the 95% level for the zero factor model in the Brexit test. Again we do not ascribe much relevance to this finding as it is not repeated for the higher factor models. In general we are confident that conclusions drawn from the modified Pesaran and Yamagata $J_{\gamma,2}$ tests are valid.

In short, we observe highly significant values of $\gamma$ for four of the six events studied. We also note that, as expected, the $J_{\gamma,2}$ tests have higher power than the univariate mean-weighted portfolio test. Significance of the portfolio test is only ever found when the $J_{\gamma,2}$ is significant, but the opposite is not true. We discuss the results below in more detail.

For the 2014 Scottish independence referendum, $J_{\gamma,2}$ is significant at the 95% level for $K=0$ and at the 99.9% level for $K=1$ and 5. This is consistent with the conclusions of \cite{Darby et al. 2019} and \cite{Acker and Duck 2015} that showed that the CAPM residuals are related to betting odds. However, we go further and demonstrate that there is still explanatory power in the betting markets when the additional Fama–French factors are controlled for. We note that the average value of $\gamma$ implies an average decrease in the price of UK stocks of 2.5% between Scotland voting for independence and voting to remain in the UK. The regressions on the residuals of the higher factor models estimates the decline relative to the European wide market. This is because the factors used are Europe wide. The underperformance of UK stocks relative to their European peers is estimated at 1.7–1.8% given a vote for independence versus Scotland remaining in the UK.

The 2016 Brexit referendum also produced highly significant findings for UK based equities listed on the LSE. All the models produced $J_{\gamma,2}$ significant at the 99.9% level. Two of the three univariate regressions are also significant at the 99.9% level for $K=0$, and at the 90% level for $K=1$. Estimated values of $\gamma$ imply very steep falls in stocks given a vote for Brexit. The $K=0$ model suggests an overall decline of 15% (individual regressions) relative to a vote to Remain. The $K=1$ and 5 models estimate declines for
UK stocks relative to the overall European market of around 2.6–2.8%. Results for the Brexit referendum for the European exporters portfolio are interesting. Recall that the betting markets were highly explanatory for this European wide market factor. $\gamma$ is significant for $K=0$ but once we control for the first market factor, MktRF, they become insignificant. The significance seen for European exporters in the raw returns is due to $\Delta PB_t$ driving the wider European market. However, $\Delta PB_t$ does not explain the CAPM residual, even for European companies that generated over 25% of their revenues in the UK in 2015. These companies were not expected to perform significantly differently than the wider European stock market given a vote for Brexit. We note that the estimated fall for these European exporters of 13% is consistent with both the estimated fall of UK stocks (around 15%) and the estimated underperformance of UK stocks (3%) to within standard errors. Recall also that a regression of the European wide MktRF factor on the betting market for this event was significant. The estimated effect of a 1% change in the odds of Brexit led to a fall of 0.16% ± 0.05%. Again this is broadly consistent with the fall of European exporters estimated in the portfolio regressions.

The 2016 US presidential election was a major event for Mexico. Changes in the likelihood of a Trump victory were highly significant for Mexican stocks, both outright and relative to the North American market. All univariate and multivariate tests were significant at the 99.9% level. The difference between a Clinton and a Trump win was estimated to have an average effect on Mexican stock prices of 18%. The decline relative to the North American market was estimated at around 16%. We note that for the $J_{\gamma,2}$ test $N < T$. Although the test is not asymptotically valid in such cases, Monte Carlo simulation of test statistics presented in Pesaran and Yamagata (2012) suggest good performance for similar values of $N$ and $T$ when $N < T$. They also demonstrate for these values of $N$ and $T$ superior performance to the standard multivariate test of Gibbons et al. (1989). Furthermore we do not need to rely on the modified test of Pesaran and Yamagata here as the weaker mean-weighted portfolio test generated significance anyway. For the historical S&P500 constituents our tests do not generate significance, even for $K=0$. This is particularly surprising given the sell off on the night of the election itself when Trump’s surprise win became apparent. This is a surprising result, but not dissimilar to the finding that $\Delta PB_t$ did not explain MktRF when regressing over the full event window. This will be explored further later.

Our final significant event was the 2019 UK general election. Boris Johnson was the firm favourite to win this election promising to ‘Get Brexit Done’. His opponent Jeremy Corybn had doubled down on his left wing policies after failing to win the 2017 election. He launched what was seen as the most left wing set of policy ideas seen for a generation, Pickard (2019). Policies included raising taxes sharply for companies and higher earners as well as nationalising key UK companies. A failure for Boris Johnson to
gain a majority may have opened up the possibility of a reversal of Brexit as opposition parties were suggesting a second referendum. However, for asset markets, Jeremy Corbyn was now apparently seen as the bigger threat. Also, failure to gain a majority could likely continue the intense political uncertainty that had dogged the UK for the preceding few years. A conservative majority was seen as significantly positive for UK stocks, both outright and relative to the European market. $J_{\gamma, 2}$ is significant for all three models (at the 99.9% level). Estimated values of $\lambda$ imply an average premium for stocks of 1.4% under a conservative majority.

The two events that did not generate significant $\gamma$ are the 2017 UK general election and the 2020 US presidential election. It may be the case for these events that either, one, $\gamma = 0$, and the political event is not informative for asset prices, or two, that the elections are informative but that noise in the data fail to generate statistically significant regression coefficients due to $\gamma$ being small relative to the variance of the full model residual $\eta$. We cannot be certain of the answer. However, it is plausible that neither event is in fact, for the average stock, informative for prices. $\gamma = 0$ appears consistent with the very muted reaction seen in the markets for the 2017 General election on what was in effect a shock result. For example the FTSE100 blue chip index opened the morning after the election at 7,450 which was exactly where it closed the night before on election day! Prior to this election there was political gridlock due to disagreements about Brexit. Although the opposition Labour party was seen as less friendly to business, power for them would have made the likelihood of remaining in the EU more likely. Results for the market were not clear. However, by the time of the 2019 election, opposition policies had become either more extreme or more progressive, depending upon one’s political viewpoint. Either way, in 2019 the implications for the UK stock prices from a Corbyn win were generally seen as being more negative than from the certainty of Brexit under the more generally perceived business friendly incumbents. The possibility that the outcome of the 2017 election was not informative for stock prices at all, and that $\lambda = 0$, is also suggested by the very small values of the t-statistics in the stock by stock regressions. This is $-0.07$ (or $-0.13$ for K=1) and $\sqrt{t^2} = 0.99 - 1.00$. Similarly for the 2020 presidential election, as the US stock market had been on a roar during Trump’s presidency (contrary to 2016 fears), implications for stocks were not at all clear either in that election. Either result is less surprising than the failure of the 2016 presidential election to generate significance for the S&P500 constituents.

5.3.2 Weighted regression tests

The failure to demonstrate that betting markets significantly explained the returns of the S&P500 stock returns (and their Fama–French residuals) was a puzzling result for the 2016 US presidential election. This is a similar result though to the failure of the
Table 6: Results for weighted regressions, with the weight increasing by 5% each week.

### Univariate regressions

<table>
<thead>
<tr>
<th>K</th>
<th>$\hat{\beta}$</th>
<th>$\sigma_\hat{\beta}$</th>
<th>t</th>
<th>p</th>
<th>$R^2$</th>
<th>pLjung</th>
<th>pEngle</th>
<th>pLLn</th>
</tr>
</thead>
</table>

- **2014 Scottish independence referendum**
  - 0: 0.023 (0.030) 0.78 0.438 0.167 0.234 0.575 0.142
  - 1: 0.014 (0.017) 0.82 0.412 0.192 0.693 0.543 0.143
  - 5: 0.014 (0.016) 0.90 0.366 0.223 0.824 0.276 0.347
  - N = 200, T = 87

- **2016 Brexit referendum - LSE stocks**
  - 0: -0.170** (0.029) -5.93 <0.001 0.289 0.666 0.946 0.228
  - 5: -0.028* (0.015) -1.84 0.065 0.037 0.823 0.588 0.388
  - N = 215, T = 87

- **2016 US presidential election - S&P500 constituents**
  - 0: 0.345 0.897 0.601 0.893 -0.175 -2.34 2.73
  - 5: 0.003 (0.019) 0.15 0.882 0.003 0.205 0.211 0.689
  - N = 50, T = 15

### Individual stock regressions

<table>
<thead>
<tr>
<th>$\hat{\gamma}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\beta}$</th>
<th>pLjung</th>
<th>pEngle</th>
<th>pLLn</th>
</tr>
</thead>
</table>

- **2016 US presidential election - S&P500 constituents**
  - 0: -0.080** (0.036) -2.33 0.026 0.068 0.032 0.587 0.061
  - 1: 0.008 (0.007) 1.10 0.273 0.013 0.333 0.298 0.764
  - 5: 0.004 (0.006) 0.62 0.537 0.003 0.569 0.720 0.822
  - N = 488, T = 74

### 2017 UK general election - LSE stocks

<table>
<thead>
<tr>
<th>$\hat{\gamma}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\beta}$</th>
<th>pLjung</th>
<th>pEngle</th>
<th>pLLn</th>
</tr>
</thead>
</table>

- **2019 UK general election - LSE stocks**
  - 0: 0.016 (0.025) 0.65 0.518 0.011 0.679 0.527 0.874
  - 1: 0.014 (0.017) 0.81 0.418 0.019 0.784 0.338 0.042
  - 5: 0.015 (0.019) 0.78 0.433 0.019 0.970 0.414 0.045
  - N = 242, T = 37

### 2020 US presidential election - S&P500 constituents

<table>
<thead>
<tr>
<th>$\hat{\gamma}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\beta}$</th>
<th>$\hat{\beta}$</th>
<th>pLjung</th>
<th>pEngle</th>
<th>pLLn</th>
</tr>
</thead>
</table>

- **Univariate Ljung-Box serial correlation test p-value**
- **Breusch-Pagan heteroskedasticity test p-value**
- **Engle ARCH test p-value**
- **Ling-Li multivariate heteroskedasticity test p-value**

---

* Equally weight portfolio regressions
* Ljung-Box serial correlation test p-value
* Breusch-Pagan heteroskedasticity test p-value
* Engle ARCH test p-value
* Ling-Li multivariate heteroskedasticity test p-value
betting markets to explain the returns of the first market factor, MktRF, over the same period. However, significance was found when regressing MktRF over the shorter period of the month preceding the election, both for this election and the 2020 presidential election. Could it be that the betting market’s explanatory power increases closer to an election? This would be consistent with the findings of [Page and Clemen (2013)]. They demonstrated the performance of prediction markets is negatively correlated with time to expiry of the market. As the election nears it tends to dominate the news cycle and will more likely be on the minds of potential bettors. Betting volumes also increase hugely.

To explore this idea we apply a weighted regression scheme to our significance tests, with the weights increasing as the election nears. The $J_\gamma$ tests of Pesaran and Yamagata allow this as long as the same weighting is used for each individual stock regression. The weighted estimates of correlation coefficients are also required when adjusting for cross sectional dependence of returns. This is trivial to apply. A weighting scheme will need to be chosen. It would be natural to weight by the volume traded in the betting markets. However, as Table 2 shows, the liquidity can increase by several orders of magnitude during the testing period, and peaks just before the election. Using raw volume will simply put all the weight on the final few days. Using the logarithm of volume could be a choice but probably does not increase sharply enough given the large increases in volume. Rather than come up with a scheme whereby we make some choice of function of volume to use, we will simply weight in the time dimension. We chose to increase the weighting exponentially by a relatively modest 5% per week. Regressions and significance tests are run for both individual stocks as well as the equally weighted portfolio. Results are shown in Table 6.

For the previously significant stock universe and event pairs, results are repeated but with increased significance. Absolute values of t-statistics and estimates of $J_\gamma,2$ are higher. Values of $\lambda$ are also similar. We will not in general discuss these results individually. One meaningful change is with the European exporters portfolio during Brexit. Not only has the significance of the betting markets improved for the K=0 model (and become significant in the univariate test) but the K=1 and 5 models now produce significant results (at the 99.9% level). A small outperformance of these stocks versus the European average given Brexit over Remain of 1.4–1.9% is estimated. This is surprising as this author expected European exporters to the UK to underperform given a Brexit scenario. There may be other factors at play here, such as these companies exporting more outside Europe, potentially mitigating a downturn in EU revenues, than the average European company.

With regards to the S&P500 universe in the 2016 presidential election, results are now highly significant using the weighted regression. All $J_\gamma,2$ tests are significant at the 99.9% level and the K=0 univariate equally weighted portfolio test is significant at the 95% level.
It appears that the information content of the betting markets may indeed improve as the election is approached. The K=0 models imply a fall in price of the average S&P500 stock of 8% given a win for Trump versus Biden. The K=1 and 5 models actually predict a small outperformance of 0.4–0.8% of the S&P500 versus the North American market as a whole. This may be highly significant but it is an economically small effect.

The 2017 UK general election remains insignificant using the weighted regressions. Again very small values of t-statistics are found in the stock by stock regressions. $t$ is in the range $-0.04$ to $-0.11$ and $\sqrt{t^2} = 1.06$. We conclude that our model is not relevant for this event and indeed $\lambda = 0$.

Finally we see that using a weighted regression generates some significance for S&P500 stocks in the 2020 US presidential election. For K=0 the results are significant at the 99.9% level for the stock by stock test and at the 95% level for the equally weighted portfolio regression. The typical index stock is expected to outperform by 20% given a Biden win over a Trump win. The K=5 model also generates a significant $J_{\gamma,2}$ at the 99% level. However, the predicted outperformance of the index stocks of 0.3% versus the wider North American market is economically insignificant. We note that using the weighted scheme has generated significance for the S&P500 portfolios for both US elections studied in this paper. Betting markets do explain the moves in outright stock returns. However, despite the betting markets having significant explanatory effects on the residuals of the Fama–French model for these index stocks the predicted effects are very small and economically insignificant.

In general, using significance tests based on the weighted regressions has sharpened our results. Events that were deemed to be significant using the standard regression over the whole testing period remain significant but more so. We have also demonstrated that the betting markets explain the returns of S&P500 stocks using the weighted scheme. This suggest that the information content increases as the election nears. This is a not unexpected result given the explosion in trading volumes on betting exchanges as the election is approached. However, we do not generate results of economically meaningful magnitude for the Fama–French residuals of the S&P500 index stocks. Finally our model does not appear to hold in that $\gamma = 0$ for the 2017 UK general election. The results of that election did not seem clear for Brexit and hence stock prices given the particular political situation in the UK at that time.

5.3.3 Political factor loading characteristics

Next we turn to an investigation of the political factor loadings $\gamma$ and how they vary with any common observable characteristics of the stocks. We will examine how $\gamma$ varies for the 5 factor model. This will identify the political sensitivity of individual stocks, controlled for common characteristics related to the Fama–French factors. Before we begin we need
Table 7: Regressions of factor loadings $\gamma$ against stock characteristics for different events.

<table>
<thead>
<tr>
<th>Dependent Variable</th>
<th>Estimate</th>
<th>Error</th>
<th>tStat</th>
<th>pValue</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>2014 Scottish independence referendum</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.030**</td>
<td>0.012</td>
<td>-2.56</td>
<td>0.011</td>
</tr>
<tr>
<td>% 2013 UK revenue</td>
<td>0.080***</td>
<td>0.018</td>
<td>4.54</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>$I$(HQ in Scotland)</td>
<td>0.073**</td>
<td>0.036</td>
<td>2.03</td>
<td>0.044</td>
</tr>
<tr>
<td></td>
<td>$N = 158, R^2 = 0.136$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2016 Brexit referendum - UK Stocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>0.036**</td>
<td>0.014</td>
<td>2.50</td>
<td>0.013</td>
</tr>
<tr>
<td>% 2015 developed Europe revenue</td>
<td>-0.098***</td>
<td>0.021</td>
<td>-4.78</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td></td>
<td>$N = 183, R^2 = 0.112$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2016 Brexit referendum - EU Exporters</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.000</td>
<td>0.023</td>
<td>0.00</td>
<td>0.999</td>
</tr>
<tr>
<td>% 2015 UK revenue</td>
<td>0.044</td>
<td>0.050</td>
<td>0.87</td>
<td>0.385</td>
</tr>
<tr>
<td></td>
<td>$N = 101, R^2 = 0.008$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2016 US presidential election - S&amp;P500</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>0.058***</td>
<td>0.013</td>
<td>4.50</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>% 2015 US revenue</td>
<td>-0.075***</td>
<td>0.017</td>
<td>-4.41</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td></td>
<td>$N = 461, R^2 = 0.041$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2016 US presidential election - Mexican stocks</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.094</td>
<td>0.063</td>
<td>-1.50</td>
<td>0.156</td>
</tr>
<tr>
<td>% 2015 US revenue</td>
<td>-0.199</td>
<td>0.174</td>
<td>-1.15</td>
<td>0.271</td>
</tr>
<tr>
<td></td>
<td>$N = 16, R^2 = 0.086$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>2019 UK general election</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.041*</td>
<td>0.021</td>
<td>-1.97</td>
<td>0.051</td>
</tr>
<tr>
<td>% 2016 UK revenue</td>
<td>0.111***</td>
<td>0.033</td>
<td>3.41</td>
<td>&lt;0.001</td>
</tr>
<tr>
<td>$I$(Corbyn to nationalise)</td>
<td>0.004</td>
<td>0.050</td>
<td>0.07</td>
<td>0.944</td>
</tr>
<tr>
<td></td>
<td>$N = 202, R^2 = 0.071$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>20206 US presidential election - S&amp;P500</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Intercept</td>
<td>-0.030</td>
<td>0.019</td>
<td>-1.56</td>
<td>0.120</td>
</tr>
<tr>
<td>% 2015 US revenue</td>
<td>0.033</td>
<td>0.025</td>
<td>1.32</td>
<td>0.187</td>
</tr>
<tr>
<td></td>
<td>$N = 404, R^2 = 0.002$</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>
to caveat our results. We regress the loadings against some observable characteristics. However, not all stocks had the necessary data we sought. As such we cannot discount the possibility of some sample selection bias in what we report. However, most events had reasonably good data coverage and the relationships uncovered were generally strong. We are confident of the direction of the relationships revealed if not their exact magnitude.

For the 188 stocks listed on the LSE that survived our selection process for the 2014 Scottish independence referendum, 158 of them published easy to access percentages of revenues for the UK in their 2013 full year accounts. A greater sensitivity to independence is likely for stocks that generate more of their revenues on shore as the UK economy would likely suffer more than international peers. thirteen of the companies were also headquartered in Scotland. These latter stocks would be much more likely to affected by independence than companies based in the rest of the UK.

Table 7 shows the results of a regression of $\gamma$ against these two stock characteristics. Results for similar regressions from the other events are also shown in the table. Both characteristics coefficients are significant. For every additional 1% revenue earned onshore in 2013, $\gamma$ would be 0.0008 higher. This means the sensitivity of the stock price to the election increases 8 basis points (0.08%) from that 1% additional UK based revenue. By sensitivity here we mean the expected difference of prices between Scotland leaving or remaining as part of the UK. The coefficient of the ‘HQ in Scotland’ indicator implies an increased sensitivity due to being based in Scotland of 7.3%. Darby et al. (2019) demonstrated that sensitivity to betting odds helped predict cross sectional returns of Scottish based companies. The work in this paper goes further and shows that Scottish headquartered companies had significantly more risk to the 2014 Scottish independence referendum than other UK firms.

For stocks listed on the LSE in the Brexit referendum, we use percentage of revenue generated in developed European markets published in the 2015 full year accounts. This includes the revenue earned in the UK. This developed EU markets figure is found to be very similar to the revenue figure for the UK only. Most of the revenue earned by UK companies in developed European markets is earned in the UK. Joint regression of $\gamma$ on both these figures is problematic due to co-linearity. We chose the European number to include any effects on exporters to the EU (who are likely to be also affected by Brexit). These figures are available for 183 of the 219 stocks used in the regressions. European revenue is significantly explanatory for $\gamma$ at the 99.9% level. The political sensitivity increases by about 10 basis points for every additional percent of developed European revenue earned. Put another way, for every additional percent of revenue earned outside of developed European markets, stocks are expected to have prices 8 basis points higher in price under Brexit (versus Remain). We also regress $\gamma$ on revenue earned in the UK in 2015 for the EU exporters portfolio (this was available for all the exporters). No
relationship was found. The UK sales figure was not informative with a very low $R^2$ of 0.008 reported for the regression.

For the 2016 election where Trump won the presidency, we study both the S&P500 index stocks as well as Mexican companies. For the index stocks we source percentage of sales earned locally in the US in 2015 (we hypothesise that exporters would have more sensitivity to a Trump win given the potential for increased trade disruption). The characteristic was available for 461 of the 488 index stocks that were used in the significance tests. The local percentage of revenue figure was highly significant with γ falling 7.5 basis points for every additional percent of revenue earned locally. This is puzzling, as this means γ was increasing with increasing exports and that exporters would be likely to perform better under a Trump versus a Clinton presidency. We have no explanation for this result but do note that the estimated values of γ for this model had on average an expectantly positive sign, although the effect was very small (β = 0.004).

For the Mexican portfolio we source the percentage of revenue earned in the US in 2015. Unfortunately this was only reported for 16 of the 51 stocks used. The regression does not generate significant results, not unexpectedly since N is so low. The coefficient is estimated at −0.199 ± 0.174 ($p = 0.271$). This suggests a Mexican stock that earns all revenue from the US is estimated to have prices 20% lower under Trump versus Clinton than a Mexican company that earns no revenue in the US.

For the 2017 UK General election we do not proceed as we have concluded our model does not hold ($γ = 0$). For the 2019 UK General election portfolio we use the 2015 reported percentage revenue earned in the UK. This is available for 202 of the 243 stocks considered. Jeremy Corbyn, the opposition Labour party leader, also promised to nationalise 17 UK based companies, if he won, Rees (2019). We regress γ on the UK revenue percentage and an indicator based on whether or not the company would be nationalised under Labour. UK Revenue is significant at the 99.9% level. Political sensitivity increases with UK revenue, 11 basis points for every additional percentage point revenue earned onshore in 2018. This means UK focused businesses were expected to perform relatively poorly had the Conservatives failed to get a majority, relative to exporters. The coefficient on the indicator of nationalisation risk under Corbyn is not significant, but is estimated at 3.9%. We note that the average γ for stocks identified as being nationalised under Corbyn is 0.055 ± 0.12 versus 0.012 ± 0.17 for the other stocks. We thus test equivalence of the distributions of γ amongst stocks conditional on nationalisation risk versus conditional on no nationalisation risk. We apply a one-sided two-sample Kolmogorov-Smirnov test. The null is that the two samples come from the same distribution versus an alternative that the CDF for the sample given nationalisation risk is higher. The Kolmogorov–Smirnov test statistic is 0.325 with a p-value of 0.028. The 17 stocks Corbyn named as nationalisation targets are significantly more sensitive
to the result of the election than others at the 95% level.

Finally we seek a relationship for the S&P500 index stocks for the 2020 US presidential election. 2019 local percentage revenue is available for 404 of the 487 stocks used in the model estimation stage. This is not explanatory for $\gamma$ with a very low $R^2$ reported of 0.002 for the regression.

Elections often involve domestic or geographical risks. It is likely that companies that earn more revenue offshore or internationally are more able to diversify away from election risks. Some evidence for this hypothesis was present in the literature for Brexit, Hill et al. (2019). Internalisation was found to decrease the sensitivity of UK stock prices to the betting odds for Brexit as well as the day-after-result return. In this paper we have extended these findings, testing this idea for all six events. We included a proxy for national revenue (or developed EU revenue for Brexit), finding that internalisation is indeed significantly explanatory for four of the six events. These results both complement Hill et al. (2019) and improve them. Not only do we consider more events, we check the explanatory power after the five Fama–French factors have been controlled for.

Similarly we have repeated an observation made in one of the oldest studies of elections risk and financial markets (Gemmill, 1992). For the 1987 UK general election this paper showed there was a much larger increase as the election was approached in option volatility of stocks that were at risk of nationalisation under the Labour opposition. History appears to have repeated itself in 2019. The then Labour leader Jeremy Corbyn threatened to nationalise various utility and other companies if he won. Here we measure political risk as the sensitivity to betting odds of the Conservatives failing to get a majority, finding the risk significantly larger for companies at risk of nationalisation.

6 Conclusion

The information content of political prediction markets is well documented. As is the effect on asset prices of elections and other political risks. Given this, it is natural to ask is there a way to formally link prices between political markets and financial markets? Auld (2022) sought to answer this question in the very particular circumstances of the overnight session following a vote. In this paper we seek to describe the relationship between these two types of markets in the weeks and months leading up to a political event. There are a small number of examples in the literature that study this question (see Manasse et al. (2020), Hanna et al. (2021), Acker and Duck (2015) and Darby et al. (2019)). However, they consider only a single event and typically study only empirical relationships. We build a theoretical model for the asset pricing relationship from the ‘ground-up’ using economic assumptions. That we recover a relationship between asset price returns and the first difference of betting markets is perhaps no surprise. Indeed
other papers have studied this relationship empirically. However, this work differs in that we have a theoretical basis for the model, and also test it on multiple events. Indeed the main contribution of this paper is to present a model that is grounded in economic assumptions and can be applied to any political event\textsuperscript{15}.

To build the model we make a key modification to the assumptions of the cointegrating model of the preceding section. This is that the difference of the conditional expectations of asset prices (given the result of the election) is fixed, as opposed to the conditional expectations themselves being fixed. This leads to the relationship in first differences. The variance of the returns of asset prices are separated into a political part, explained by political markets, and a residue, related to commercial, economic and other non-political factors. In fact the cointegration model of the Auld (2022) is a special case of this model, where this latter residue has zero variance. The model is naturally extended using the Fama–French factors to describe the variance of the non-political residue. The resulting model is an extended characteristic factor model, where all factors, both Fama–French and political, via betting markets, are observed.

We test the model on six recent elections. We find strong evidence in favour of our model for four of the six events. One election has mixed results (the 2020 presidential election). A weighting scheme was required there to generate significance. Data closer to the election was weighted more highly than data far from the vote. Although a modification of the original model, this can be explained by political markets having greater information content the closer we are to an election. Some justification for the approach is from Page and Clemen (2013) which showed that the forecasting ability of prediction markets is negatively correlated with time to expiry. Further, betting volumes also increase exponentially as the election approaches. We find the weighting scheme sharpens the results of the four other events found to be significant with the unweighted scheme. Significance of the political factor loading, \( \gamma \), increased under the weighted approach. This is consistent with the huge increases in volumes observed on betting platforms as the election drew near. We find no support for our model for a single event, the 2017 UK general election. We conclude that this election is not informative for asset prices. In the model we believe the weighting on the political factor, \( \gamma \), is in fact zero.

An exploration of the factor loadings reveals some pleasing relationships. Consistent with Hill et al. (2019) we find that domestic (or EU based) revenue is a strong explanatory factor of political risk. Indeed, this characteristic was significant for four of the six elections studied. This provides evidence for the hypothesis that companies with a greater reliance on international sales are more able to diversify domestic political risk. We

\textsuperscript{15}The model applies in more general settings. It can be used for events with a prediction market and whose outcome has an effect on asset prices.
find that the location of company headquarters, and nationalisation risk under a given outcome, are also found to increase $\gamma$ markedly.

References


Appendix

Power of two-step empirical specification versus a single regression approach

In this section we demonstrate that using our two step approach to testing political betting significance (outlined in section 4) has superior power than using a single larger factor model regression (equation 4.1). The latter (falsely) assumes an unchanging political likelihood on the training set. The key assumption is that the political factor, although not necessarily observable, has non-zero variance in some part of the training period. To show this consider the simple univariate regression

\[ y_t = \alpha + \beta x_t + \epsilon_t \quad t = 1, \ldots, T_1, T_1 + 1, \ldots, T_2 \]  

with \( \mathbb{E}(\epsilon_t) = \sigma_\epsilon \). Assume that this is the correct model and that \( \beta \neq 0 \). Data is available for \( y_t \) for entire period \( t = 1, \ldots, T_2 \). No data for \( x_t \) exists for the first period \( t = T_1 + 1, \ldots, T_2 \). Consider the regression on the period where we have data for both \( y_t \) and \( x_t \), \( (t = T_1 + 1, \ldots, T_2) \). This is analogous to regressing the estimated Fama-French residual on changes in the political markets on the testing set only. The t-statistic for \( \beta \) in this regression is

\[
t_\beta = (T_2 - T_1) \frac{\sum_{i=T_1+1}^{T_2} x_i (y_i - \frac{1}{(T_2-T_1)} \sum_{i=T_1+1}^{T_2} y_i)}{\sum_{i=T_1+1}^{T_2} \epsilon_{it}^2} \rightarrow (T_2 - T_1) \frac{\text{COV}(y_t, x_t)}{\sigma_\epsilon}.
\]

Next define

\[
\tilde{x}_t = I(t > T_1) x_t
\]

which is \( x_t \) forced erroneously to zero on the first set \( T = 1, \ldots, T_1 \) where data is lacking. Now consider a regression of \( y_t \) on \( \tilde{x}_t \) on the whole period for \( t = 1, \ldots, T_2 \). This is analogous to the single regression empirical approach where we jointly estimate the Fama-French loadings with a false assumption that the political likelihood is unchanging. The relevant t-statistic for \( \beta \) is now
\[ t_\beta = T_2 \frac{\sum_{i=1}^{T} \hat{x}_i (y_i - \frac{1}{T_2} \sum_{i=T_1+1}^{T} y_t)}{\sum_{i=1}^{T} \hat{\epsilon}_t^2} = \frac{\sum_{i=1}^{T} \hat{x}_i (y_i - \frac{1}{T_2} \sum_{i=T_1+1}^{T} y_t)}{\frac{1}{T_2} \sum_{i=T_1+1}^{T} \hat{\epsilon}_t^2} = (T_2 - T_1) \frac{\frac{1}{T_2} \sum_{i=1}^{T} \hat{x}_i (y_i - \bar{y})}{\frac{1}{T_2} \sum_{i=T_1+1}^{T} \hat{\epsilon}_t^2} \rightarrow (T_2 - T_1) \frac{\text{COV}(y_t, x_t)}{\sigma_\epsilon}. \]

Note that \( t_\beta \) is very similar to \( t_\beta \) and differs only in the denominators \( \sigma_\epsilon \) and \( \sigma_\tilde{\epsilon} \). However, we have assumed that

1. \( \beta \neq 0 \)
2. \( \{ x_1, \ldots, x_{T_1} \} \) is not everywhere zero
3. Equation (6.1) represents the true model.

However, if the model is indeed true it cannot provide a worse fit than the model

\[ y_t = \alpha + \beta \hat{x}_t + \tilde{\epsilon}_t \quad t = 1, \ldots, T_2. \]

When assumptions 1 and 2 above hold, the fit must be strictly worse. Thus \( \sigma_\tilde{\epsilon} > \sigma_\epsilon \). The limit of the second t-statistic has smaller absolute size than that of the first t-statistic. Thus under an alternative hypothesis \( \beta \neq 0 \) the second t-statistic will never reject the null hypothesis \( \beta = 0 \) unless the first t-statistic does. This is exactly analogous to showing that jointly estimating the Fama-French residuals along with the political component, whilst forcing the political factor to be zero, has lower power than the two step approach we employ.