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CORPORATE CULTURE AND ORGANIZATIONAL FRAGILITY

MATTHEW ELLIOTT, BENJAMIN GOLUB, AND MATHIEU V. LEDUC

Abstract. Complex organizations accomplish tasks through many steps of collaboration among workers. Corporate culture supports collaborations by establishing norms and reducing misunderstandings. Because a strong corporate culture relies on costly, voluntary investments by many workers, we model it as an organizational public good, subject to standard free-riding problems, which become severe in large organizations. Our main finding is that voluntary contributions to culture can nevertheless be sustained, because an organization's equilibrium productivity is endogenously highly sensitive to individual contributions. However, the completion of complex tasks is then necessarily fragile to small shocks that damage the organization's culture.

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1. Introduction

For a large organization, such as a corporation, to successfully complete a complex project, such as designing a new product and bringing it to market, many constituent tasks must all be successfully completed. A typical such task can be completed only if several other, tailored input tasks are completed. Thus, a complex project requires many collaborations among workers to succeed, both within and across business units.

Collaborations may fail for many reasons: misunderstandings, insufficient or misallocated effort, a lack of trust, agency problems, and so on (Kreps, 1990). Corporate culture can mitigate these problems, e.g., by establishing and enforcing norms and supporting relational contracts.[1] It is thus an important determinant of successful collaborations within organizations.[2] We posit that collaborations are more likely to succeed in a better corporate culture—a broad notion that entails many aspects of the working environment.

Organizational culture is endogenous: its quality depends on individuals' costly efforts to maintain it, e.g., by articulating and adhering to organizational values, communicating relevant expectations and practices, and enforcing norms.[3] Such efforts can be viewed as costly contributions to a public good. They are thus subject to a free-riding problem; this problem becomes more severe as each worker's contribution becomes relatively small. Indeed, a standard analysis would suggest that workers' incentives to make voluntary contributions to any genuinely corporate (as opposed to more local) culture vanish as an organization becomes large, because their marginal impact becomes negligible while their marginal cost does not. This raises an important question: Why do workers exert voluntary effort to enhance the corporate cultures of large organizations?

We propose a perspective on these questions based on a network model of complex production within a large organization, adapting the model of Elliott, Golub, and Leduc (2022). The productive activity of the organization occurs via the completion of tasks. A worker completing a task typically relies on the completion of several types of essential subtasks; these are incorporated into the task via *collaborations* with other workers, whose success depends on the quality of the ambient corporate culture (among other determinants). Because any collaboration can fail, there are several substitutable subtasks of a given type. Each subtask can itself require further subtasks (reliant on other collaborations), and so on.

As an illustration, consider a marketing analyst performing the task of running surveys to market-test a potential feature. For this he needs input from engineers to supply technical specifications and input from data analysts on prices to test—two types of subtask. For each of these, the marketing analyst has access to several workers who can provide a subtask of that type. In turn, these workers rely on other organizational units. For instance, a data analyst needs the help of accounting staff to provide relevant records and a developer to write code. Thus, the marketing analyst's task relies, directly and indirectly, on many "upstream" successful

[1]See, e.g., Nguyen (2018) and Graham et al. (2022) for qualitative discussions and extensive empirical analysis.
[2]See, e.g., Kotter (2008). Similarly, Groysberg et al. (2018) find a positive association between the strength of a corporate culture, measured as employees' agreement about the culture's characteristics, and the efforts of workers.
[3]See Rahmandad and Repenning (2016) for a perspective on corporate culture, motivated by extensive evidence, as a stock variable that erodes and needs to be replenished in a changing environment. Bénabou, Falk, and Tirole (2018) model individual contributions supporting norms and narratives.

collaborations. We call a task complex if it is dependent on many levels of collaboration, with multiple types of subtasks being combined at each level.

We study how the level of corporate culture affects the probability of completing complex projects in such a network of overlapping collaborations.[4] We show that the probability of achieving a given complex task discontinuously increases from zero to a large positive number when the corporate culture increases past a threshold level of strength. Organizational performance is very sensitive to culture around this threshold. This implies a new mechanism generating incentives for voluntary, decentralized investment in culture even as an organization becomes large and the free-rider problem becomes severe: the marginal impact of contributions becomes large enough to compensate.

The analysis also implies a distinctive kind of fragility: In an equilibrium with positive contributions supported by this mechanism, the organization's performance must be very sensitive to an exogenous negative shock to corporate culture. This shock could be, for instance, a merger or a change in the company's top management (such as the arrival of a new CEO).[5] The theory thus suggests a novel account of how the complexity of an organization both sustains incentives for investment and makes it vulnerable, in equilibrium, to cultural disruptions. Section 4 discusses other implications and their relation to evidence.

Our approach relates to a literature on network theory and the provision of public goods (Bramoullé and Kranton, 2007), but builds on the distinctive fragility properties of large networks (Brummitt, Huremović, Pin, Bonds, and Vega-Redondo, 2017; Elliott, Golub, and Leduc, 2022; König, Levchenko, Rogers, and Zilibotti, 2022; Blume, Easley, Kleinberg, Kleinberg, and Tardos, 2013). At a conceptual level, a main message of our work is that there is an interesting interaction between these properties of complex production processes and the incentives of the agents involved, a theme also explored in recent work by, e.g., Levine (2012), Erol and Vohra (2022), and Dasaratha (2023). In Section 4.1, we discuss how our work relates to the existing literature on corporate culture.

2. A task-based model of a large organization

2.1. **Model.**

2.1.1. *A network of tasks.* There is a directed, acyclic network $G = (V, E)$ in which each node $v \in V$ is a task and edges represent input relationships between tasks according to a technology that we now describe. There is a set $S_v \subseteq V$ of inputs for each task v. If S_v is nonempty, it is partitioned into m *types* of inputs,

$$S_v = S_{v,1} \cup S_{v,2} \cup \cdots \cup S_{v,m}.$$

Each $S_{v,t}$ has cardinality n, and these sets are disjoint. The interpretation is that each type t corresponds to a different kind of input needed (e.g., engineering specifications, pricing data analysis) and, for each such input, there are n distinct but substitutable tasks that can provide that input. For the modeling here we do not need to formally associate workers with tasks, but a natural interpretation is that different substitutable subtasks are associated with different

[4]Recall that corporate culture supports the performance of each collaboration.

[5]Indeed, the incompatibility of different corporate cultures is often blamed for failed mergers (Cartwright and Cooper (1993) and Kotter (2008)).

workers. However, the same workers may perform multiple tasks throughout a network. We discuss worker payoffs in Section 3.[6]

Directed links $(v, v') \in E$ between v and some $v' \in S_v$ are called *collaborations* (note that links are directed from tasks to their inputs). The interpretation is that a worker performing task v can receive the result of task v', performed by another worker, as an input. Each such collaboration may be *operational* (denoted by $\phi_{vv'} = 1$) or not ($\phi_{vv'} = 0$).

Each task may be successful or not; let s_v be the indicator variable of whether task v is successful. The main technological assumption is that a task is successful if and only if, for each $t \in \{1, \ldots, m\}$, there is at least one subtask $v' \in S_{v,t}$ which is successful and such that $\phi_{vv'} = 1$, so that there is an operational collaboration for v to source the result of the task. Given a realization ϕ of which links are operational, we let $s^*(\phi)$ be the maximal vector s consistent with the technology of production just described; such a vector exists by Tarski's fixed point theorem. This gives the set of all tasks that can be completed given exogenous constraints.

We assume there is a "root" node M that is not an input into any other task. This can be interpreted as the main task. A node not requiring any inputs is called a leaf. We say a network has $L < \infty$ layers if the distance from M to each leaf[7] is $L - 1$. In that model, all leaf tasks are always successful. We say the network has $L = \infty$ if it has no leaves—an idealized model of highly complex operation where there is no definite point at which interdependencies are "cut off."

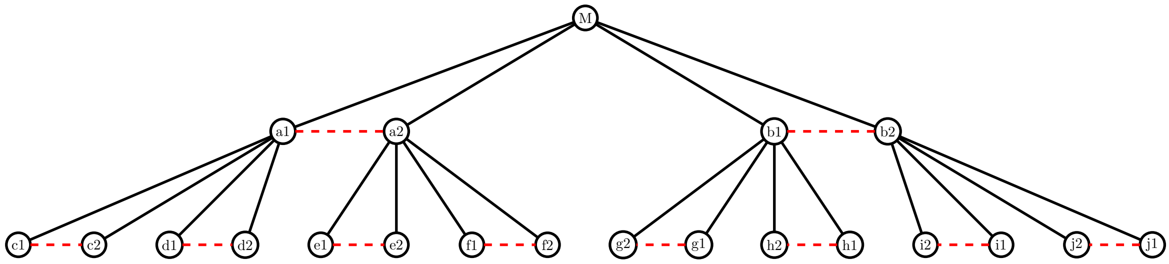
2.1.2. *An illustration.* Figure 1(A) illustrates part of a task network. Each task requires the completion of two types of subtasks. Each $S_{v,t}$ contains two substitutable tasks. The task M needs a type a and a type b task as inputs. Either task $a1$ or $a2$ can serve the purpose, and similarly for task type b, and so on.

Figure 1(B) illustrates a possible realization of which collaborations would be operational if needed: these are represented by the retained links relative to Figure 1(A). Note that although the collaboration $(M, a2)$ is operational, the task M cannot be completed. This is because task $a2$ cannot itself be completed.

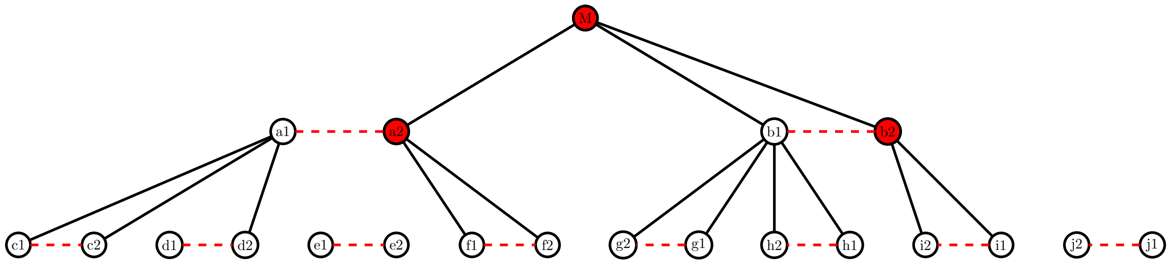
2.1.3. *Distribution of operational collaborations.* We now introduce randomness into the realizations of which collaborations are operational. Let $\pi \in (0, 1)$ denote the probability that a given collaboration is operational, and let all these realizations be independent. We call π the strength or level of corporate culture. In the context of Figure 1, this is the probability that a link present in Panel (A) is kept in Panel (B). We take this to be a one-dimensional summary of the strength of the corporate culture. In the context of a collaboration between a worker performing a task and another worker providing a required subtask, the idea is that a failure of operation represents a failure of an informal contract, preventing the subtask from being incorporated successfully into the task seeking to use it. The simplest interpretation is that this outcome is realized when the worker handling task v seeks help from one handling v',

[6]In the supplementary online appendix, available at http://dx.doi.org/10.2139/ssrn.4325317, we embed this simple model in a more general one—where each task can require a different number of subtasks and a different number of potential task providers—and show that the results presented throughout this paper are robust to such elaborations.

[7]Note this definition imposes that each path has the same length—which is done for simplicity, but could be relaxed.



(A)



(B)

FIGURE 1. (A) Structure of potential collaborations within an organisation and required subtasks. (B) Successful task completion after collaboration success has been determined. Collaborations that fail have been removed. Red-shaded vertices are those tasks that cannot be completed because they do not have access to a key subtask.

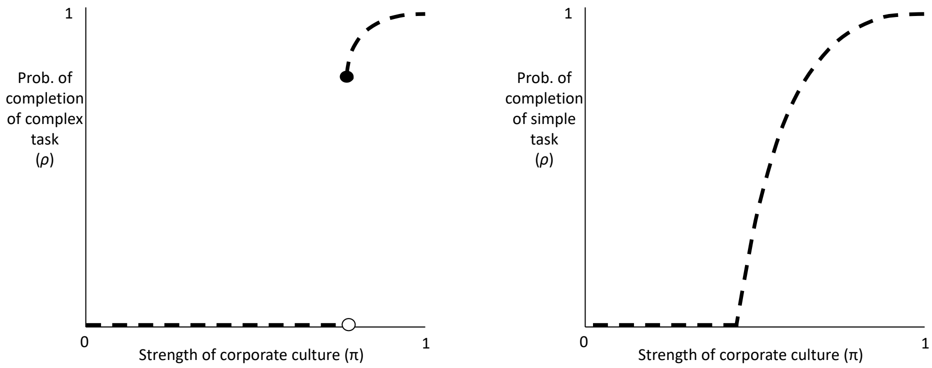
but it may also be that v' is a task done at some point in the past (e.g., uploading records to a database) and the uncertainty is over whether this was done in a way useful for task v. A stronger corporate culture can help avoid failures by reducing misunderstandings, strengthening norms of cooperation, motivating workers to follow best practices such as documenting their work, etc.

2.2. **Sensitivity of an organization to corporate culture.** Suppose first that the strength of the corporate culture is exogenous. We will study how the probability of successful completion of task M depends on it. We call this probability r: formally, the probability that $s^*_M(\phi) = 1$. Assume for the moment that the number of layers in the task tree network is infinite ($L = \infty$).

We now claim that r satisfies the following equation:

$$r = (1 - \underbrace{(1 - \pi r)^n}_{\text{Probability a given subtask cannot be completed}})^m. \tag{1}$$

This equation is relatively straightforward to derive. Consider the first subtask—say of type t. The probability of a subtask of this type, say t_1, being provided successfully is πr. This is the probability that the subtask is completed successfully, which by symmetry is also r, multiplied by the probability that the collaboration to use it is operational, which is π. Consequently, the probability that this does not happen is $(1 - \pi r)$ and the probability that no subtask of type t can be provided is $(1 - \pi r)^n$. Hence, the probability of being able to source some subtask of this type is $1 - (1 - \pi r)^n$. The probability of being able to source *all* m types of subtasks is



(A) (B)

FIGURE 2. (A) Probability of successful completion of a *complex* task when all task providers require two subtasks ($m = 2$) and have two potential providers of each subtask ($n = 2$). (B) Probability of successful completion of a *simple* task when all task providers require a single subtask ($m = 1$) and have two potential providers of this subtask ($n = 2$).

$(1 - (1 - \pi r)^n)^m$. This is therefore the probability of the event that the complex task M can be successfully completed.

Eq. (1) can have multiple fixed points and $r = 0$ is always among them. However, it is the maximal fixed point that corresponds to $s^*(\phi)$, the set of tasks that can technologically be completed.[8] We call this maximal fixed point $\rho(\pi)$. In Figure 2(A), we plot $\rho(\pi)$. The key fact about this plot is that the probability of successful completion of a complex task is *discontinuous* in the strength of the corporate culture π. The probability of successful completion is 0 below a threshold corporate culture strength π_{crit}, but then increases discontinuously to more than 70% in the example examined here.

We formalize this property in the following lemma.

Lemma 1. Let $m \geq 2$ and $n \geq 1$. Then, there is a value $\pi_{\text{crit}} \in (0, 1]$ such that: (i) $\rho(\pi) = 0$ for all $\pi < \pi_{\text{crit}}$, (ii) $\rho(\pi)$ has a unique point of discontinuity at π_{crit}, (iii) $\rho(\pi)$ is strictly increasing for all $\pi \geq \pi_{\text{crit}}$ and (iv) $\lim_{\pi \downarrow \pi_{\text{crit}}} \frac{\partial \rho(\pi)}{\partial \pi} = \infty$, $\lim_{\pi \uparrow 1} \frac{\partial \rho(\pi)}{\partial \pi} = 0$, and $\frac{\partial \rho(\pi)}{\partial \pi}$ is otherwise finite.

This relationship already has some stark implications. As corporate culture improves, the completion of any task first becomes possible at some threshold level of corporate culture strength. Moreover, the *reliability* of the organization—defined as the probability that a task is completed—can instantaneously increase from 0 to a large positive number as corporate culture crosses the threshold π_{crit}. This implies that small reductions in corporate culture can have a large negative impact on production and profits for complex organizations.

We call a task *complex* if $m \geq 2$, so that at least two types of input are required at each stage of production (and L is large). Complexity is key to the discontinuity. If instead the task to be completed is *simple*, requiring only one type of subtask at each stage ($m = 1$), then the probability of successful task completion ρ is continuous in π. Figure 2(B) illustrates this.

[8]This can also be obtained by taking the large-L limit of finite-layer models; see Elliott et al. (2022, II.C).

An analogous analysis can be carried out for finite depths[9], $L < \infty$, and we denote by $\rho_L(\pi)$ the probability of completion for the root task M. The functions ρ_L converge uniformly to the function ρ studied here (Elliott, Golub, and Leduc, 2022, Lemma SA2).

3. Endogenizing corporate culture

So far, corporate culture π has been modeled as exogenous. In this section, we model the endogenous determination of π based on workers' investments in corporate culture.

3.1. A model of endogenous investment.

3.1.1. *Worker effort and the determination of culture.* There are k workers. Worker i can exert effort $x_i \in [0, 1]$ toward maintaining corporate culture. The cost of investment x_i is $c(x_i)$ and is borne by worker i. Let $x = (x_1, x_2, ..., x_k)$ denote the profile of investment choices of the k workers. The corporate culture strength is $\pi_k(x, \underline{\pi})$, a function increasing in investments x_i and in the baseline level of corporate culture $\underline{\pi}$, such that $\pi_k(\underline{x}, \underline{\pi}) = \underline{\pi}$ where $\underline{x}_i = 0$ for all i. If different workers have different roles in the organization, then their impact on corporate culture may also be different (e.g., managers may have a greater weight). Thus, let corporate culture strength be determined by the average investment and the baseline level $\underline{\pi}$ via the following formula:

$$\pi_k(x, \underline{\pi}) = \underline{\pi} + \sum_i w_{k,i} h(x_i) \tag{2}$$

where $w_{k,i} > 0$ is the weight[10] of agent i in this average and $\sum_i w_{k,i} = 1$. Here $h : [0, 1] \to [0, \overline{h}]$, with $\overline{h} = 1 - \underline{\pi}$, is an increasing and weakly concave function with $h(0) = 0$ and $h(1) = \overline{h}$, modeling potentially decreasing returns to investments.[11]

Our main results focus on the large-organization limit, where the number of workers k grows large. We will assume that, in this limit, the marginal impact each worker has on corporate culture becomes negligible:

Assumption 1. $\max_i w_{k,i} \to 0$ as $k \to \infty$.

We assume $\underline{\pi} < \pi_{\text{crit}}$, so that when individuals make no effort in maintaining the corporate culture, the probability that each given collaboration is functional is too low to allow a complex task to be successfully completed, per Lemma 1. We also make the following assumption:

Assumption 2. $c(x_i)$ is smooth and convex, satisfying $c(0) = 0$; $c'(x_i) > 0$ for any $x_i > 0$; $c'(0) = 0$; and $\lim_{x_i \uparrow 1} c'(x_i) = \infty$.

3.1.2. *The investment game.* The timing of the game is as follows. At time 0, workers simultaneously choose their investments x. At time 1, each link in the collaboration network is operational with probability $\pi_k(x, \underline{\pi})$. The network realization determines whether the root task M can be completed successfully.

Workers' payoffs depend on the reliability, ρ, of the network—the probability of completing the root task M. Let each worker i enjoy a (potentially different) benefit $a_i > \underline{a} > 0$ once the

[9]The interested reader can also consult the online appendix, which contains further discussion.

[10]For example, if e is the total number of nodes in a tree network and $e_{k,i}$ is the number of such nodes occupied by worker i, then we could let $w_{k,i} = e_{k,i}/e$.

[11]In the online appendix, we show that our results are robust to more general functional forms for $\pi_k(x, \underline{\pi})$.

complex task has been completed. Thus, worker i's expected payoff can be expressed as

$$U_i(x_i, x_{-i}) = \underbrace{a_i \rho(\pi_k(x, \underline{\pi}))}_{a_i \times \text{ Probability of successful task completion}} - \underbrace{c(x_i)}_{\text{Cost of investment}}. \quad (3)$$

While individuals have different positions within the organization, their interests are somewhat aligned: all workers derive some incremental benefits from the successful completion of the complex task (e.g., through bonuses, stock options, future career success, etc.). The dependence of all payoffs on a single root task is assumed just for simplicity. More realistically, different workers might be rewarded for different tasks. As long as each is rewarded for the completion of some high-L task or tasks, our analysis extends readily.

3.2. **Equilibrium analysis.** As k grows large, the marginal impact of each worker's effort on corporate culture becomes negligible. The individual marginal costs of contributing, on the other hand, do not vanish. This raises the question of whether workers would ever voluntarily contribute to raise the strength of corporate culture beyond the exogenous baseline $\underline{\pi}$, since the free-riding problem is so severe.

Figure 2(A) shows how in spite of this, equilibrium contributions to corporate culture can be considerable. We illustrate the key logic in the idealized case of $L = \infty$, where tasks branch indefinitely; the interested reader can consult the online appendix, where we discuss that one can state an analogue of our results for a large, finite L. With no investments, the probability of successful task completion would be 0. Now suppose each worker i were to make an investment x_i such that $h(x_i) = \pi_{\text{crit}} - \underline{\pi}$. Then, corporate culture would be at the point of discontinuity, π_{crit}, where $\rho(\pi)$ has infinite derivative by Lemma 1(iv). At such a point, for a large, finite number of workers, the marginal benefits of investment are higher than the marginal costs of investment. Therefore, there is a corporate culture level π slightly above π_{crit} at which marginal benefits equal marginal costs for all workers. This yields an equilibrium with positive, voluntary contributions.

At such an equilibrium, marginal benefits from investment must equal marginal costs:

$$a_i \frac{\partial \rho}{\partial \pi_k} \frac{\partial \pi_k}{\partial x_i} = a_i \frac{\partial \rho}{\partial \pi_k} w_{k,i} h'(x_i) = c'(x_i), \ \forall i.$$

For this to hold, we need $\partial \rho / \partial \pi_k$ evaluated at the equilibrium value of π to be large enough as k grows (recalling that $\max_i w_{k,i} \to 0$ as $k \to \infty$ and thus the marginal impact of each worker's investment on π also diminishes to 0). Thus, the equilibrium will approach the point of discontinuity (where $\partial \rho / \partial \pi_k$ grows arbitrarily large) from above, and hence be on the edge of the precipice.

Now, consider an exogenous[12] shock that reduces, even slightly, the baseline level of corporate culture strength $\underline{\pi}$ after the agents' choice of investment (the interpretation is that we are considering a timescale on which agents do not have time to adjust their investments to compensate for the change). This shock could be, for instance, the announcement of a merger or change in management, or any other change in the work environment that reduces collaboration effectiveness. Because the equilibrium configuration was on the edge of the precipice, such a

[12]In Section 3 of the online appendix, we further show that our results are also robust to *anticipated* shocks to the baseline corporate culture strength $\underline{\pi}$, i.e. when workers take into account the possibility that such exogenous shocks may occur.

shock will result in a collapse, or at least a severe reduction, in the organization's probability of completing complex tasks.

To formalize this discussion, we now define a strong notion of fragility, which requires the total collapse of the organization's ability to complete the complex task following a small exogenous shock to corporate culture strength.

Definition 1 (Fragility)**.** An equilibrium profile x^* is *ϵ-fragile* if $\rho(\pi_k(x^*, \underline{\pi} - s)) = 0$, for any $s > \epsilon > 0$. That is, the probability of completing the complex project falls to 0 following a negative shock of size greater than ϵ to the baseline corporate culture strength $\underline{\pi}$.

We can now state our main result.

Proposition 1.

(a) There is a $\underline{k}$ such that for $k > \underline{k}$: (i) There exists a no-contribution equilibrium with $x_i^* = 0$ for all i. (ii) If $\underline{a}$ is sufficiently large[13] and $w_{k,i}/w_{k,j}$ is uniformly bounded, there is a positive-investment equilibrium—one with $x_i^* > 0$ for all i.

(b) For any $\epsilon > 0$, there exists $\underline{k}(\epsilon)$ such that for $k > \underline{k}(\epsilon)$, any positive-investment equilibrium x^* satisfies $\pi_k(x^*, \underline{\pi}) \leq \pi_{\text{crit}} + \epsilon$ and is ϵ-fragile.

The key force supporting the positive-investment equilibrium is that the sensitivity of ρ to the level of culture π makes each worker's marginal effect on task completion sufficient to incentivize positive investment. That sensitivity also makes the outcome fragile to global shocks. In practice, the organization may have access to "backup" processes that produce a lower-value substitute in case it is incapable of supporting complex production (see Section 4.2). We also note that fragility coexists with a robustness to the idiosyncratic shocks that make many collaborations fail to operate. Indeed, the redundancy built into the network (i.e. the fact that each subtask can be potentially completed by up to n workers) makes the system quite robust to local shocks.

4. Discussion and Extensions

4.1. **Connections and contrasts with other theories of corporate culture.** Several economic theories of corporate culture have been presented in the literature (for example, Camerer and Vepsalainen (1988), Kreps (1990), Akerlof and Kranton (2005), Akerlof et al. (2020), Kets (2021) and Gibbons et al. (2021)). We do not view these as alternatives to our approach, but rather as other facets of a complex concept. Broadly, these theories are consistent with how we view corporate culture: as supporting common understandings and norms that help address agency problems and other frictions. The novelty of our approach is in the interaction between the complex, nested nature of the task network and the model of culture as a public good.

An important feature of our model is that the culture that agents invest in is global, i.e., at the level of the organization. Of course, some investments are more local in nature, e.g., developing deeper understandings with specific collaborators. Elliott, Golub, and Leduc (2022) models investments only in one's own relationships in a closely related model of production; the sensitivity of ρ to global culture plays *no* direct role in agents' investment incentives there. This

[13]The required lower bound on $\underline{a}$ does not depend on k.

important theoretical contrast means that, in this other model, positive equilibrium investments can put the outcome well above the precipice—something that cannot occur in the present model.

The takeaway from this comparison is that if purely local investment incentives are strong enough, then collaborations can enable complex production without investment in broader corporate culture. The theory developed in the present paper is relevant in case this does not happen, so that voluntary investment in organization-wide culture matters. This is especially salient in large organizations where new collaborations arise frequently outside the context of pre-existing close relationships—due to, e.g., the complex and shifting nature of tasks as well as turnover.[14] Broad corporate culture then becomes critical for fostering productivity.

The endogenous sensitivity of outcomes to individual investments may have interesting interactions with other issues in the theory of organizations. For example, theories of multilateral enforcement rely on threats of a broad breakdown of cooperation to deter the violation of norms or promises (Levin, 2002). Such "grim trigger" coordinations by many workers to withdraw investment may not always be plausible. In contrast, if equilibrium performance is endogenously highly sensitive to individuals' or small groups' investments, as in our model, such a deterrent may become more credible.

4.2. **Selecting the complexity of the organization.** We now provide a minimal elaboration of our benchmark complex organization model to illustrate some phenomena that arise when we endogenize the complexity of projects.

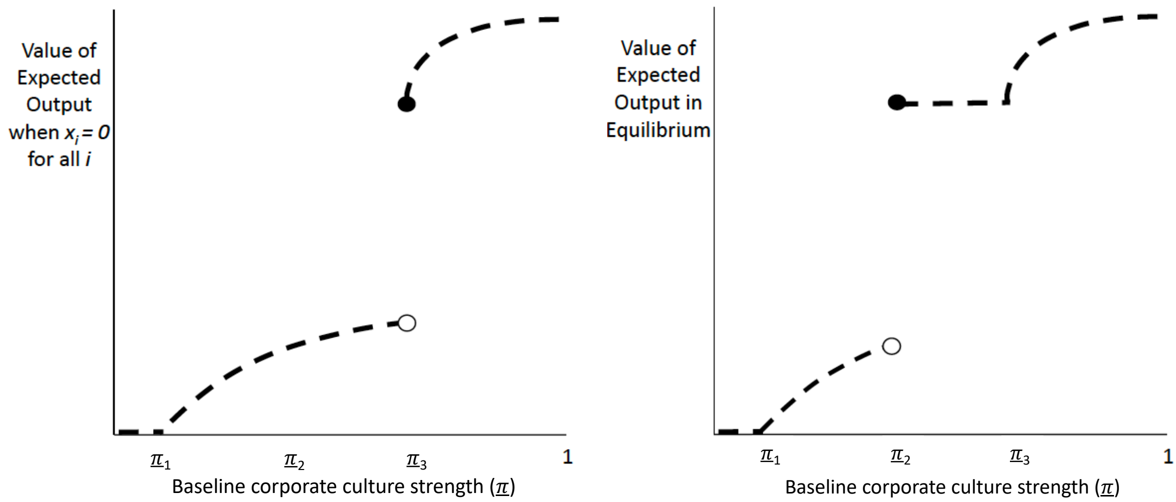
Suppose that the management of a firm can choose between a simple project and a complex project, before the game we have studied is played. For the simple project, each task, including the completion of the final project, requires a single type of subtask to be completed. For the complex project, each task requires two types of subtask. In both cases, there are two potential collaborations for each type of subtask, so that $n = 2$. We assume the complex project is considerably more valuable than the simple one, if completed, and again consider the $L = \infty$ case, with a large number k of workers. Finally, we assume that the organization works on the project with the highest expected value; when there are multiple equilibria for a given project, the most productive equilibrium is selected.

As shown in Figure 3(A), there is a key threshold for the strength of corporate culture below which the completion of any task, either simple or complex, fails for sure. We denote this threshold $\pi_1 > 0$. Further, as there is no discontinuity in the probability of successful completion when the simple task is performed, whenever the simple task is selected there will be no contributions to corporate culture strength, and its equilibrium level will be the baseline level $\underline{\pi}$.

Now we consider the existence of the positive contribution equilibrium when the task is complex. By Proposition 1, we know that a positive contribution equilibrium can exist for the complex task when completing this complex task is efficient. Thus, in such a case, there is some threshold[15] $\underline{\pi}_2 \geq 0$ such that shutdown of the complex task is efficient and there is no positive contribution equilibrium when $\underline{\pi} \leq \underline{\pi}_2$.

[14] Rahmandad and Repenning (2016) surveys evidence that such forces require constant replenishment of a stock of corporate culture.

[15] We assume that $\underline{\pi}_2 > \underline{\pi}_1$, although this does depend on the parametrization of the problem.



(A) (B)

FIGURE 3. Choosing between a simple and complex project: Panel (A) shows the value of expected output conditional on there being no contributions to corporate culture ($x = 0$) as baseline corporate culture strength varies. Panel (B) shows the value of expected equilibrium output as baseline corporate culture strength varies and contributions x^* are those made in equilibrium.

A third key threshold that has been discussed at length is the threshold on actual (rather than baseline) corporate culture strength, at which the probability of successful completion of the complex task becomes positive for the first time. We had so far denoted it by π_{crit}. For this section, denote this threshold by $\underline{\pi}_3$ and note that $\underline{\pi}_3 > \underline{\pi}_2$. This threshold will be important in equilibrium because for a baseline strength $\underline{\pi} > \underline{\pi}_3$, the discontinuity in the probability of successful task completion will be irrelevant for incentivizing individual contributions, and there will be a unique equilibrium in which no contributions are made.

We can now consider what happens in equilibrium as the baseline level of corporate culture strength $\underline{\pi}$ increases, under the assumptions that for a given task the most productive equilibrium is selected and that, given this, the most productive task is selected. This is shown in Figure 3(B). First, for sufficiently low $\underline{\pi}$, there is no output. This corresponds to the case in which the baseline corporate culture strength is sufficiently low that even the simple task cannot be successfully completed. Then, once the threshold $\underline{\pi}_1$ is passed, the simple task is used to generate positive output, but there will be no individual contributions towards corporate culture in equilibrium. Next, the threshold $\underline{\pi}_2$ is passed. At this point, the firm switches to the complex task and the equilibrium also switches to the positive contributions equilibrium. Endogeneously, corporate culture strength $\pi(x^*, \underline{\pi})$ is now equal to $\underline{\pi}_3$ (which, as we said, is equal to the critical level π_{crit}), and around this point there is a discontinuous increase in output. Output, however, does remain fragile and will collapse following a small shock to corporate culture. As the baseline corporate culture strength keeps increasing, lower private investments will be made in equilibrium but the overall equilibrium corporate culture strength will remain at $\underline{\pi}_3$. In this range, improvements in baseline corporate culture strength perfectly displace private effort. Finally, once the threshold $\underline{\pi}_3$ is passed, there will be no equilibrium contributions to corporate culture, but the later will nevertheless become more robust to shocks—larger shocks will be required to cause a collapse in the firm's ability to complete tasks.

This simple extension of our model is consistent with there being a positive association between the ability of firms to complete complex tasks and the strength of corporate culture. It also suggests that, in equilibrium, the returns to improving the baseline corporate culture vary in a nonmonotonic and interesting way. The returns from passing one of the thresholds $\underline{\pi}_1$ or $\underline{\pi}_2$ can be very large, while other improvements have little or no impact. The impact on workers' incentives to contribute to the public good are also interesting. They have no incentive to do so below $\underline{\pi}_2$ or above $\underline{\pi}_3$, but between these two thresholds make strictly positive investments.

4.3. **Leadership.** In this section, we model a choice of corporate culture at the organizational level and consider problems that may arise when corporations face changing environments.

To formalize the idea that the culture into which workers are asked to invest is a strategic choice of the organization, suppose each organization chooses a corporate culture $\theta_j \in \Theta$, where Θ is a finite set.To capture the received wisdom that a corporate culture must be tailored to an organization (i.e., that one culture does not fit all), we let each firm j have a mapping

$$\underline{\pi}_j : \Theta \to [0,1]$$

that determines the baseline corporate culture strength as a function of its culture choice.[16]

Thus, in a new stage zero of our game, each firm chooses

$$\theta_j^* \in \operatorname*{argmax}_{\theta_j \in \Theta} \underline{\pi}_j(\theta_j),$$

anticipating profits from play of the most productive equilibrium of the game studied in our main analysis. (Note a firm's expected profits are always weakly increasing in $\underline{\pi}_j$.) We now use this framework to consider some challenges firms might face. First, suppose two complex organizations j and j' merge. For simplicity, we suppose that operations in the firm remain independent, but we assume that the merged firm must then choose a single corporate culture to fit both organizations. In this case, unless there exists a $\theta^* = \theta_j^* = \theta_{j'}^*$ that is optimal for both organizations, at least one of the organizations will have to operate with a corporate culture that is a worse fit for it: Either $\underline{\pi}_j$, $\underline{\pi}_{j'}$ or both will decrease. There are then two possibilities. If workers' effort choices can adjust, there may be an equilibrium in which their effort increases to exactly offset the decrease in baseline corporate culture levels. In this case, there will be no change to the organization's ability to execute complex tasks. However, there may not be such an equilibrium. If $\underline{\pi}_j$ is sufficiently low, the unique equilibrium is for the workers to exert no effort, resulting in the organization's inability to complete complex tasks. Production may also fail if the workers do not have time to adjust their effort levels. Thus the theory predicts that mergers pose a substantial risk to the ability of large organizations to execute complex tasks, and this risk is larger when the corporate cultures of the merging firms are initially less aligned. The incompatibility of corporate cultures is often blamed for failed mergers (Cartwright and Cooper, 1993; Kotter, 2008) and is of interest as a mechanism

[16]A line of work in the management literature presents a typology of corporate cultures consistent with this modeling approach. See, for example, Groysberg et al. (2018).

explaining which firms can successfully integrate (Gorodnichenko, Kukharskyy, and Roland, 2018).[17]

A second challenge we consider is a change of leadership in a large organization. An important role that leaders can play is in setting the corporate culture and coordinating the direction of organization-wide efforts (Gibbons and Henderson, 2012; Bolton et al., 2013). We can think of the choice set of available cultures as specific to the leadership: i.e. Θ_j depends on the leadership of firm j. Thus, after a change in leadership, the initial θ_j^* might no longer be feasible. This can create opportunities as well as risks. The new leadership might be able to choose a better corporate culture for the organization, improving $\underline{\pi}_j$. If the organization initially has a $\underline{\pi}_j$ such that the zero-contribution equilibrium is being played, then any change in equilibrium selection this brings about will be weakly beneficial. However, if a positive contribution equilibrium is being played, then a change in corporate culture could result in zero contributions. Indeed, a change in leadership might mean that this is the only equilibrium. Again, such a change would undermine the ability of the corporation to undertake complex tasks, to its detriment.

Some observations are consistent with this. First, organizations with strong corporate cultures seem to often appoint leaders from within, thus helping to maintain the corporate culture. Second, when strong leaders depart unexpectedly, an organization's stock price often falls substantially (Quigley, Crossland, and Campbell, 2017). This could reflect concerns that the new leadership cannot maintain a successful corporate culture.

5. Proofs

Proof of Lemma 1. From Eq. (1), if $r = \rho(\pi)$ and $r > 0$ then we can manipulate (1) to yield

$$\pi = \Pi(r) := \frac{1 - (1 - r^{1/m})^{1/n}}{r}.$$

Lemma SA1 of Elliott, Golub, and Leduc (2022) establishes all but the first statement in (iv). It does this by showing that Π has a unique minimum at a point $(r_{\text{crit}}, \pi_{\text{crit}})$ and that Π is the inverse of ρ on the domain $\pi \in [\pi_{\text{crit}}, 1]$. To check (iv), note that the previous sentence implies $\frac{d\rho(\pi)}{d\pi}|_{\pi \downarrow \pi_{\text{crit}}} = 1/\frac{d\Pi(r)}{dr}|_{r \downarrow r_{\text{crit}}}$. Since Π is continuously differentiable and has a minimum at $r = r_{\text{crit}}$, its derivative must be 0 there. □

Proof of Proposition 1. We first prove part (b). Consider worker i's expected utility in an infinite-length ($L = \infty$) tree:

$$U_i(x_i, x_{-i}) = a_i \rho(\pi_k(x, \underline{\pi})) - c(x_i).$$

By the chain rule, worker i's marginal utility from increasing his investment is

$$\frac{dU_i(x)}{dx_i} = a_i \left(\frac{\partial \rho(\pi_k(x, \underline{\pi}))}{\partial \pi_k(x, \underline{\pi})} \right) \left(\frac{\partial \pi_k(x, \underline{\pi})}{\partial x_i} \right) - c'(x_i). \tag{4}$$

The first term is the marginal benefits to i of increasing x_i and the second term is i's marginal cost.

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[17] As corporate culture is hard to measure, field evidence establishing its causal effect is difficult to obtain. Weber and Camerer (2003) conduct a laboratory experiment. Under an interpretation of corporate culture consistent with ours, they find that it is possible to establish norms and understandings within subject groups separately; once groups are merged together, performance declines markedly.

Denoting $\frac{\partial\rho(\pi_k(x,\underline{\pi}))}{\partial\pi_k(x,\underline{\pi})}\Big|_{\pi_k(x,\underline{\pi})=\pi}$ by $\rho'(\pi)$, and noting that $\frac{\partial\pi_k(x,\underline{\pi})}{\partial x_i} = w_{k,i}h'(x_i)$, the first-order optimality condition ($\frac{dU_i(x)}{dx_i} = 0$) can be stated as

$$a_i w_{k,i} h'(x_i)\rho'(\pi) = c'(x_i), \tag{5}$$

where

$$\pi = \underline{\pi} + \sum_i w_{k,i}h(x_i). \tag{6}$$

A positive investment equilibrium x^* must satisfy Eqs. (5) and (6) for all i such that $x_i > 0$, and have $\pi_k(x,\underline{\pi}) > \underline{\pi}$.

Now consider a sequence (indexed by k) of positive contribution equilibria. There must be a $\delta > 0$ and sequence $i(k)$ so that, for each k, $x_{i(k)} \geq \delta$ and and Eq. (5) holds with $i = i(k)$; otherwise $\lim_k \pi_k(x(k),\underline{\pi}) = \underline{\pi}$ and nobody has an incentive to make positive contributions by Lemma 1. Along this sequence, the right-hand side of (5) is bounded away from 0, while $w_{k,i(k)} \to 0$ by Assumption 1 and h' is uniformly bounded for $x_i \geq \delta$. Thus for (5) to hold it must be that $\rho'(\pi_k(x(k),\underline{\pi})) \to \infty$, which by Lemma 1 is possible only if $\lim_k \pi_k(x(k),\underline{\pi}) = \pi_{\text{crit}}$.

As a result, there is a $\underline{k}(\epsilon)$ so that if $k \geq \underline{k}(\epsilon)$ then $\pi_k(x(k),\underline{\pi}) < \pi_{\text{crit}} + \epsilon$. A shock $s > \epsilon$ to the baseline corporate culture strength $\underline{\pi}$ results in a corporate culture strength $\pi_{\text{crit}} - s < \pi_{\text{crit}}$ and thus in $\rho(\pi_{\text{crit}} - s) = 0$ by Lemma 1, establishing the claim about fragility.

Turning now to part (a), we now show that there always exists a zero contribution equilibrium when k is large enough. We consider the candidate profile $x = \vec{0}$ and the deviations $(x_i, x_{-i}) = (x_i, \vec{0})$ where i takes action x_i and everyone else takes action 0. When k is large enough, then for any $x_i \in (0,1]$,

$$a_i\rho(\pi_k((x_i,\vec{0}),\underline{\pi})) = 0 < c(x_i),$$

and hence the benefit of investing is smaller than the cost. Indeed, $c(x_i) > 0$ for $x_i > 0$, while $a_i\rho(\pi_k((x_i,\vec{0}),\underline{\pi})) = 0$ for all $x_i \in [0,1]$ when $w_{k,i}$ is small enough, since then $\pi_k((1,\vec{0}),\underline{\pi}) = \underline{\pi} + w_{k,i}h(1) < \pi_{\text{crit}}$. It follows that $x = \vec{0}$ is an equilibrium.

Finally, we show the existence of positive-contribution equilibria. Define $g(x_i) = \frac{c'(x_i)}{h'(x_i)}$, which is increasing.[18] Note since $\lim_{x_i\to 0} g(x_i) = 0$ and $\lim_{x_i\to 1} g(x_i) = \infty$, then it follows that $\lim_{y\to 0} g^{-1}(y) = 0$ and $\lim_{y\to\infty} g^{-1}(y) = 1$.

For $\pi \geq \pi_{\text{crit}}$, define $X_i(\pi) = g^{-1}\Big(a_i w_{k,i}\rho'(\pi)\Big)$. Now define[19]

$$P(\pi) = \underline{\pi} + \sum_i w_{k,i}h\Big(X_i(\pi)\Big) = \underline{\pi} + \sum_i w_{k,i}h\Big(g^{-1}\Big(a_i w_{k,i}\rho'(\pi)\Big)\Big). \tag{7}$$

Fix any k. Note $P(1) = \underline{\pi}$. Moreover, by what we have said about g above, as $\pi \to \pi_{\text{crit}}$ from above, we have $P(\pi) \to 1$. Therefore, using continuity of the right-hand side of (7) and the intermediate value theorem, there is a fixed point of P with $\pi \geq \pi_{\text{crit}}$. From now on, let $\pi^*(k)$ denote the largest such fixed point, which satisfies $\pi^*(k) > \pi_{\text{crit}}$. To this $\pi^*(k)$ corresponds an investment profile given by $x^*(k)$ with $x_i^*(k) = X_i(\pi^*(k)) = g^{-1}\Big(a_i w_{k,i}\rho'(\pi^*)\Big)$ that satisfies the first-order conditions for investment.

[18]This follows from the fact that $g(x_i) = \frac{c'(x_i)}{h'(x_i)}$ is increasing, which in turn follows from $c'(x_i)$ being increasing while $h'(x_i)$ is weakly decreasing.

[19]The motivation is that using (6), an equilibrium corporate culture strength π such that all agents make positive investments is a fixed point of P.

Each individual's investment problem is concave in own investment x_i in the domain of x_i such that $\pi_k((x_i, x^*_{-i}(k)), \underline{\pi}) \geq \pi_{\text{crit}}$. Thus there is exactly one interior optimum among such x_i. To establish this is a global optimum, we check i's payoff is higher at $X_i(\pi^*(k))$ than the payoff of 0 obtained by any x_i with $\pi_k((x_i, x^*_{-i}(k)), \underline{\pi}) < \pi_{\text{crit}}$ (if this is possible given others' contributions). To check this, we first note that there is an $\overline{x}$ such that $X_i(\pi^*(k)) < \overline{x} < 1$ for all i; otherwise, by the assumption on weight ratios, all $X_i(\pi^*(k))$ would tend to 1, so $\pi^*(k)$ would approach 1, contradicting (b). Now since $\pi^*(k) \to \pi_{\text{crit}}$ by (b), it suffices to check that $a_i\rho(\pi_{\text{crit}}) - c(\overline{x})$ is positive, which we have by the assumption that $\underline{a}$ is large enough. □

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