UNIVERSITY OF CAMBRIDGE
Faculty of Economics

CAMBRIDGE WORKING PAPERS IN ECONOMICS

# Understanding Program Complementarities: Estimating the Dynamic Effects of Head Start with Multiple Alternatives

Marc K. Chan    Antonio Dalla-Zuanna    Kai Liu

## Abstract

We use experimental data from the Head Start Impact Study to examine the effect of sequential participation in childcare programs on cognitive outcomes. Using a sequential threshold model, which accounts for selection beyond initial randomization, we estimate causal returns to program sequences, including joint and cross returns across skill investment programs. We then estimate a dynamic structural model as a juxtaposition and evaluate a counterfactual policy reform which limits individuals to one year of Head Start. Our results support Head Start implementation earlier in life, and support engaging low-ability children with center-based care and high-ability children with some home care.

## Reference Details

| | |
|---|---|
| CWPE | 2330 |
| Published | 20 March 2023 |
| Key Words | Head Start, program evaluation, treatment effects, complementarity, child care |
| Website | www.econ.cam.ac.uk/cwpe |

# Understanding Program Complementarities: Estimating the Dynamic Effects of Head Start with Multiple Alternatives

Marc K. Chan* Antonio Dalla-Zuanna † Kai Liu‡

June 20, 2022

**Abstract**

We use experimental data from the Head Start Impact Study to examine the effect of sequential participation in childcare programs on cognitive outcomes. Using a sequential threshold model, which accounts for selection beyond initial randomization, we estimate causal returns to program sequences, including joint and cross returns across skill investment programs. We then estimate a dynamic structural model as a juxtaposition and evaluate a counterfactual policy reform which limits individuals to one year of Head Start. Our results support Head Start implementation earlier in life, and support engaging low-ability children with center-based care and high-ability children with some home care.

*Faculty of Business and Economics, University of Melbourne. Email: marc.chan@unimelb.edu.au.
†Bank of Italy. Email: antonio.dalla.zuanna@gmail.com
‡Faculty of Economics, University of Cambridge and IZA. kai.liu@econ.cam.ac.uk

# 1 Introduction

Many government programs are evaluated by randomized experiments which offer program access to individuals via lottery. Perfect compliance is rarely achieved in these experiments. It is well known that the existence of close alternative programs complicates the experimental evaluation, especially if most control group individuals end up receiving similar services (Heckman, Hohmann, Smith, and Khoo, 2000). Another complication, which is less discussed in the literature but important for understanding the return to programs that span multiple periods, is that people also make different program choices in subsequent periods beyond the initial period of randomization. Those in the treatment group are not obliged to stay in the same government program and may switch to an alternative program later. Similarly, those in the control group who initially enrolled in an alternative program may switch to the government program later.

In this paper, we assess the returns to the Head Start Program for children who face multiple child-care options outside Head Start in multiple periods. Head Start (HS) is the largest and longest-running preschool education program for disadvantaged children (aged 3–5) in the United States. The Head Start Impact Study (HSIS), conducted in the fall of 2002 following a congressional mandate, analyzed the experimental impact of HS: applicants for HS were randomly assigned into either a treatment group where individuals were offered the opportunity to enroll in HS, or a control group where individuals had no access to HS for one year. Comparing the outcomes between the experimental treatment and control groups, Puma et al. (2010) find positive but relatively small impacts on children's test scores during the pre-school period.[1]

To motivate our methodological approach, it is useful to highlight some key data patterns in the HSIS age-3 cohort, i.e., children who entered the HSIS at age 3. First, almost 90% of control group children receive center-based care (HS or others) in at least one of the pre-school years. Second, treatment group children exhibit significant variations in the time spent in HS. Among individuals enrolled in HS at age 3, less than three-quarters stay in HS at age 4, and about 20% switch to other

[1]Other studies emphasize the heterogeneity in these results on the basis of the children's observable characteristics (e.g., Bitler, Hoynes, and Domina, 2014) and of the characteristics of the center offering HS (e.g., Walters, 2015). These positive effects are generally found to be fading out over time, but several studies that use non-experimental methods (not relying on HSIS data) find positive effects of HS on later outcomes, such as education, health and income (e.g., Ludwig and Miller, 2007; Deming, 2009; Carneiro and Ginja, 2014; De Haan and Leuven, 2020). Griffen and Todd (2017) compare experimental estimates derived from the HSIS data to nonexperimental estimates obtained from comparable nonexperimental data.

center-based care at age 4. These transitions make the experimental impact of Head Start, a multi-year program, difficult to interpret.

These observations suggest the need for a richer and more well-defined set of causal returns to HS beyond the experimental effects. To this end, we develop a framework for understanding the causal effects of *sequential program participation* in the context of multiple program options in multiple periods. When we define the returns to a program sequence, we distinguish between two types of returns: (1) the effect of a childcare program relative to home care at one age while fixing the rest of the program sequences to home care ("partial return"), and (2) the effect of a program at age 3 followed by another program (same or different) at age 4 relative to home care at both ages ("joint return"). The differential between a joint return and the sum of the relevant partial returns identifies the *cross return* between two programs, a concept closely related to dynamic complementarity in the child development literature (Cunha and Heckman, 2007). A positive cross return signals that the two programs are complementary, indicating that the return to one program is higher when combined with the other.

We combine the experimental variation of the HSIS with non-experimental methods to obtain estimates of the partial, joint and cross causal returns. Following the empirical approach in Heckman and Navarro (2007) and Heckman, Humphries, and Veramendi (2016), we begin by using a generalized Roy model to estimate a sequential choice model of program participation.[2] At each decision stage, an individual chooses one of the programs to participate in. Contrary to the experimental evaluation in which the decision rules are left unspecified, we characterize the decision rules using a flexible threshold model at each stage. By combining the sequential threshold model with outcomes, we can estimate ex post returns to a sequence of program participation and how these returns depend on both observed and unobserved variables.

We find that Head Start has higher average partial returns than other center-based care at both age 3 and age 4. In particular, our results suggest that one year of HS at age 3 or age 4 raises the average test score at the end of the pre-school period by 0.15 and 0.10 standard deviations (sd) respectively, whereas the average partial return to other center care is almost zero at age 3 and is only around 0.07

[2]Recent applications of this estimation approach include, for example, Fruehwirth, Navarro, and Takahashi (2016) (the effect of grade retention), Heckman, Humphries, and Veramendi (2018) (the effect of school choices on earnings and health) and Rodríguez, Saltiel, and Urzúa (2018) (the effect of on-the-job training).

sd at age 4. However, the high partial returns to HS do not translate to a high joint return to two consecutive years of HS. Indeed, the average joint return to HS at age 3 followed by other center care at age 4 is higher than the average joint return to two years of HS. This is due to a larger degree of complementarity between HS at age 3 and other center care at age 4, possibly due to a second year in HS giving competences which are more similar to those in the initial year as compared to other center care. Reversing the sequence – other center care at age 3 followed by HS at age 4 – instead generates a very low average joint return, highlighting the importance of the sequencing of program participation. Exploring heterogeneity by children's latent baseline ability (measured prior to the sequence), we find that low-ability children's outcome improves by enrolling in any formal childcare at both age 3 and 4 relative to any alternative program sequence involving the use of home care. On the contrary, for high-ability children, their joint returns are the highest if they enrol in HS at age 3 followed by home care at age 4.

We also use the model to analyze returns for HSIS compliers. We decompose compliers to the HS offer into 18 subgroups, all of whom respond to the offer by enrolling in HS at age 3 (by definition) but differ in terms of their choices at age 4 and their potential choice sequences without the offer. We find that about 60% of the HSIS compliers are path switchers, whose *entire* trajectory of program participation is changed due to the HS offer at age 3. The local average treatment effect (LATE) of the experiment (0.085 sd) is a weighted average of the returns for the 18 subgroups of compliers, whose returns and potential choice sequences differ widely; while the major subgroups of compliers display positive selection (on gains), the pattern of selection is mixed for other subgroups of compliers.

Although the sequential threshold model captures a variety of selection patterns, it does not distinguish contemporaneous payoffs from perceived continuation values. This means that, in general, we cannot use the model to conduct counterfactual analyses that involve a more fundamental change to program features, especially those that are time dependent. For instance, one important yet unanswered policy question from the HSIS is whether there is a benefit to having two years of HS rather than one year (Puma et al., 2010). Such a change in program feature cannot be analyzed using the sequential threshold model, and would require a more structural model which takes a stance on what agents are maximizing and their information sets. To complement the analysis of the sequential threshold model, we build and estimate a dynamic structural model of program participation that involves a forward-looking agent facing uncertainty in future payoffs. We examine the connections between a flexible sequential threshold model and a structural model with forward-looking agents, by juxtaposing both approaches and using a variety of (nested) specifications. The sequential threshold model serves as an empirical benchmark, as in Abbring and Heckman (2007) and Heckman, Humphries, and Veramendi (2018). The structural model has the advantage of examining ex ante choice decisions by distinguishing contemporaneous payoffs from perceived continuation values, identifying forward-looking behavior, and comparing counterfactuals that involve forward-looking behavior.

The estimated parameters of the structural model lend support for forward-looking behavior. We find that individuals incorporate the expected future outcomes as part of the information set and act upon it. Specifically, they are more likely to choose an option that yields better expected outcomes, keeping preferences fixed. We find that the results from the structural model align nicely with those from the sequential threshold model, and the estimated choice equations in the sequential threshold model reflect a counterbalance between current and future payoffs.[3]

We use the estimated structural model to evaluate a hypothetical policy experiment where the maximum duration of participation in HS is limited to one year, but individuals may choose whether to enrol at age 3 or 4. While the policy has limited impact on the average outcome of children, it exacerbates inequality. Individuals with low baseline ability perform substantially worse (relative to the status-quo of no limit in HS duration) while those with high baseline ability perform better. By removing the option of two consecutive years of HS participation, many individuals move away from HS at age 3 preemptively to "bank up" HS eligibility for possible enrolment at age 4. This pre-emptive response is more common among low-ability individuals and the majority of them switch to home care at age 3, which leads to worse outcomes. In comparison, pre-emptive responses are less common among high-ability individuals; those who respond pre-emptively tend to switch to home care, which leads to better outcomes. These results resonate with our earlier insights based on program returns that policy makers should engage low-ability individuals with two years of center-based care and high-ability individuals with at least one year of home care.

This paper is closely related to two papers analyzing the role of program substitution in program

[3]For instance, the flow utility from HS at age 3 increases with the child's baseline ability but the perceived continuation value declines with baseline ability, which provides a structural interpretation to the relatively small impact of baseline ability in the choice equation for HS in the sequential threshold model.

evaluation. Heckman, Hohmann, Smith, and Khoo (2000) show the importance of considering substitution and dropout biases in estimating the effects of job training. They point out that the experimental evaluation can only identify the effect of the program (the effect relative to the alternative training programs taken by individuals in the control group), but not the effect of training relative to no training. In Heckman, Hohmann, Smith, and Khoo (2000), the dropout behavior refers to dropping out of the program prior to receiving training, whereas we consider the issue arising from individuals who drop out after having enrolled in a program. In a recent paper that is closely related to ours, Kline and Walters (2016) use HSIS to estimate the returns and cost effectiveness of the HS program, taking into account that individuals may self-select into an alternative educational program that might be a close substitute to HS. Kline and Walters (2016) use a single-period selection model where both the choices and outcomes are measured within the first year of entry into HSIS (in Spring 2003). Their model implies that an expansion of HS may crowd out enrollments in other competing center care. In our paper, not only do we distinguish multiple outside options like Kline and Walters (2016) but we also analyze childcare choices and outcomes beyond the first year. This has non-trivial implications; for example, two programs are mutually exclusive within a period (hence they must be either substitutes or independent from each other), but across periods they can be complements. Our framework allows us to obtain estimates of cross returns between two different programs which would otherwise not be possible from a single-period model. In addition, we consider the dynamic structural model described above, which can identify important policy-relevant parameters which will not be possible to obtain using the sequential threshold model.

By considering a multiple-period framework, this paper also relates to a strand of the program evaluation literature which incorporates dynamic selection into programs, including Ham and LaLonde (1996), Eberwein, Ham, and LaLonde (1997), Abbring and Van den Berg (2003), Heckman and Navarro (2007) and Heckman, Humphries, and Veramendi (2016).[4] Studies in this literature only consider a binary program choice (one program versus no program). We contribute to this literature by combining a model of dynamic selection with multiple outside options in every decision period, which enables us

[4]Ham and LaLonde (1996) and Eberwein, Ham, and LaLonde (1997) consider the effect of participating in training programs on the duration of subsequent employment and unemployment spells. They assume that the treatment effect is constant with respect to the time spent in the program, whereas we are also interested in understanding the effect of program duration. Abbring and Van den Berg (2003) allow for the effect to change with the duration of the spell in the program, but this is at the expense of ruling out the heterogeneous effect of the treatment with respect to individuals' unobservable characteristics, which determine individuals' choices in terms of duration in the program. In our framework, we explicitly allow for this type of heterogeneity.

to explain richer substitution patterns and understand how the returns to one program may be affected by participation in another program.

Our interest in the cross returns between two childcare programs over time coincides with a small but growing literature which tests for dynamic complementarities using experimental or quasi-experimental variations. To secure identification, papers in this literature use two independent sources of exogenous variation, one for each type of human capital investment (see, among others, Bhalotra and Venkataramani, 2015; Johnson and Jackson, 2019; Rossin-Slater and Wüst, 2020; Goff, Malamud, Pop-Eleches, and Urquiola, 2022).[5] We complement this literature by presenting the partial, joint and cross returns to program participation that are easily comparable and interpretable for any subpopulation of interest (defined by observables or unobservables). Most papers in this literature report the "reduced-form" impact without accounting for program take-up, and hence do not identify the cross returns to program participation.[6] Among the few papers that attempt to use the two exogenous variations as instruments, the identified cross return is relevant only for a very specific subgroup of population which may not be policy relevant.[7]

The paper is organized as follows. Section 2 describes the HSIS and the sample, summarizes patterns of childcare choices and conducts an experimental evaluation of HS. In Section 3, we present the sequential threshold model and discuss identification and estimation. Section 4 reports the estimates of various parameters of program returns and their implications. Section 5 discusses the identification and estimation of a dynamic structural model of program participation, before presenting results of the structural parameters and counterfactual policy experiments. Section 6 concludes.

[5] This identification strategy is also known as the "lightning strikes twice": two identification strategies affecting the same cohort but at adjacent developmental stages (Almond and Mazumder, 2013).

[6] For instance, Rossin-Slater and Wüst (2020) find that the effect of *access* to preschool was smaller among those who had *access* to home visits during infancy. Goff, Malamud, Pop-Eleches, and Urquiola (2022) find weak evidence of negative effects of the interaction between exogenously driven improvements in family environment and being quasi-randomly *offered* a seat at a better school.

[7] Johnson and Jackson (2019) examine the complementarity between Head Start spending and K-12 spending. Although not directly related to program participation, they use exposure to the rollout of Head Start across counties as a shock to Head Start spending and the implementation of court-ordered school finance reforms as a shock to K-12 spending, finding a positive effect of the interaction of increased spending in education at different ages.

# 2 Experimental Evaluation of the Head Start Program

## 2.1 The Head Start Program and the Head Start Impact Study

Launched in 1965, Head Start (HS) is a U.S. federal program that offers year-long care to children between three and five years of age, with the aim of fostering their early reading and maths skills to be ready for school. It is funded by the Department of Health and Human Services, and in 2003 it enrolled almost one million children throughout the U.S. territory (Puma et al., 2010). Children who attend HS are not charged any fee. HS is administered by local agencies (both public and private), which compete for funding and are required to adhere to national standards. It is targeted to poor families (with income below the federal poverty line), but agencies are allowed to admit up to ten per cent of children from wealthier families.

The vast majority of HS providers offer center-based care. HS centers offer a large number of activities to support children's literacy and maths skills, as well as health-related services.[8] They also aim at supporting parents, with some centers providing transportation services for children and training/housing assistance for parents themselves; in addition, families receive at least two visits per year from HS staff.

In the 1998 reauthorization, the Congress mandated an evaluation of Head Start's impact on children's school readiness and parental practices that support children's development. The Head Start Impact Study (HSIS) followed this mandate: in fall 2002, around 5,000 newly entering children aged 3 and 4 were randomly assigned to a treatment group, which was offered HS, or a control group, which had no access to HS (but were free to select any other type of care available to them).[9] Children who were aged 3 at the time of randomization could spend up to two years in HS before starting kindergarten, while those who were aged 4 could spend one year in HS. The embargo period lasted one year only. Age-3 children in both the randomly assigned treatment group and the control group were allowed to re-apply to HS the following year when they turned 4 (in fall 2003). Hence, children who were assigned to the control group could eventually start HS in the following year when they turned 4 and those assigned to the treatment group could potentially change the type of care at age 4.

[8]For example, in the HSIS study about 90% of the children who attended HS at age 3 received hearing and vision screening and nutrition services.

[9]The sample of children in the HSIS consists of HS applicants to a nationally representative sample of 84 grantee/delegate agencies.

## 2.2 Sample Selection, Choice Set and Outcomes

We use data from the HSIS which follows children from the time of randomization until the third grade. Given our interest in the returns to sequential participation in childcare programs, we focus on children who joined HSIS at age 3 (i.e., the age-3 cohort). We combine information from three surveys. From the baseline survey conducted in Fall 2002 (at the time of randomization), we collect information including the random assignment, baseline characteristics of the children and characteristics of the HS centers which they applied for. From the first follow-up survey (Spring 2003), we collect information on childcare use, which we refer to as their childcare type at age 3 ($t = 1$). From the second follow-up survey (Spring 2004), we collect information on childcare use (referred to as childcare use at age 4 ($t = 2$)) and cognitive test scores.[10] As outcome we focus on the cognitive ability of children measured by the Peabody Picture Vocabulary Test (PPVT) taken in Spring 2004 at the end of the age-4 year (mean 3, s.d. 0.38).[11] PPVT measures the comprehension of spoken words in standard English, and its levels and scales are comparable across years. As measurements of children's baseline cognitive ability, we use the baseline survey to obtain the PPVT score (mean 2.30, s.d. 0.37) and the Woodcock-Johnson III (WJIII) Pre-Academic Skills score (mean 3.38, s.d. 0.21).[12]

We drop 573 children whose information on childcare type and test scores is not available. Our final sample consists of 1,876 children who were aged 3 in fall 2002. Descriptive statistics from our final sample indicate that the treatment and control groups are very similar across many dimensions – in demographic characteristics, baseline PPVT scores (taken in the baseline survey), and characteristics of the HS centers they applied to (see Appendix Table A1).[13] There is also a high fraction of teen mothers and mothers who were never married or were separated.

Children who did not choose HS had other program options aside from home care. These programs could be public or private and they differ from HS in several respects.[14] The following key characteristics

[10] To investigate the extent of within-year changes in program participation, we cross tabulate program participation using the Fall 2002 survey (at the start of the first HS year) and the Spring 2003 survey (at the end of the first HS year). We find a high degree of overlap in childcare use – over 90% of children report identical childcare use in the two surveys.

[11] We adjust the PPVT test scores by dividing the raw score by 100.

[12] The WJIII Pre-Academic Skills comprises three tests, i.e., Letter-Word Identification, Spelling, and Applied Problems (Puma et al., 2010). Both PPVT and WJIII scores obtained from HSIS data have been transformed using item response theory to make them comparable across ages.

[13] The HSIS data provided to us have 2,449 children in the age-3 cohort, 60% of whom are in the treatment group and 40% are in the control group. In our final sample, 62.8% are in the treatment group and 37.2% are in the control group.

[14] These can be private care or other federal or state-funded programs. For example, in the year 2002-2003, there were 738,000 children enrolled in state-funded preschool programs (Barnett, Hustedt, Robin, and Schulman, 2004).

differentiate HS from other center-based care obtained by children in our sample (Puma et al., 2010): (a) it offers more education-oriented activities and support for parents (see also Section 2.1), (b) health-related services are also included, (c) teachers are more likely to hold a bachelor's degree, have taken college ECE courses and receive training on a monthly basis, (d) centers are filled to capacity, and (e) centers are generally of higher quality as certified by specific indicators (ECERS-R, Arnett, and quality composite scores).

Table 1 shows the proportion of children in different types of care in our sample. The HS offer induced most children in the treatment group ($Z = 1$) to receive HS at $t = 1$ (85.1%), while a non-negligible proportion chose other center-based care (5.4%) or home care (9.5%). In the control group ($Z = 0$), 14.4% of children received HS at $t = 1$ even though they had no statutory access to HS (see Puma et al. (2010) for details), while 24.8% received other center-based care and 60.7% received home care. Overall, this implies an experimental compliance rate (the population share of compliers due to the random assignment) of 70.6%.

Table 1 also shows rich transition patterns in the type of care between $t = 1$ and $t = 2$. In both the control and treatment groups, among those who received HS at $t = 1$, slightly less than three quarters stay in HS at $t = 2$, about 20% transition to other center-based care, and about 6% transition to home care. By contrast, among those who were in other center-based care or home care at $t = 1$, the transition patterns are markedly different between the control and treatment groups. In the control group, among those who received other center-based care at $t = 1$, 37.6% transitioned to HS at t=2; this proportion is lower at 21.9% in the treatment group. The same pattern is observed in home care to HS transitions (49.8% in the control group versus 31.2% in the treatment group). These results support the notion of a rebound effect on HS receipt following the end of the embargo period. Yet even though the embargo period is over, control group children transition into HS at a considerably lower rate than the initial HS take-up rate of the treatment group (85.1%). Overall, the proportion receiving HS at $t = 2$ is 50.2% ($10.7 + 9.3 + 30.2$) in the control group, versus 66.2% ($62.0 + 1.2 + 3.0$) in the treatment group.

While the table shows that the majority of control group children chose home care at $t = 1$ (60.7%), this does not imply that they do not receive any formal care before starting kindergarten. The vast majority ($49.8 + 30.7 = 80.5\%$) transition to HS or other center-based care at $t = 2$. Considering that most children in the control group eventually enrolled in formal care prior to the kindergarten age, it makes the experimental impact of HS difficult to interpret even with random assignment to HS access.

### 2.3 Experimental Impact of the Head Start

Table 2 reports the experimental impact on PPVT score at age 4. Column (1) reports results from a regression of test score on a dummy for being in the experimental treatment group ($Z$). Consistent with Puma et al. (2010), we find that children offered HS gained 0.025 (2.5 points, or 0.06 standard deviations (sd)) in test score relative to those in the control group, although the gain is imprecisely estimated.[15] To increase precision, in column (3) we control for baseline characteristics of the child and the household.[16] The estimated experimental impact is of similar magnitude to that in column (1) and becomes statistically significant.

Given random assignment of HS offers, we can scale the experimental impact in column (1) (intent-to-treat estimates) by the proportion of HS compliers (those who respond to the HS offer by enrolling in HS) and obtain the local average treatment effect for the HS compliers (Imbens and Angrist, 1994). This estimate is equivalent to the instrumental variable estimate where the experimental assignment is used as an instrument for HS participation at age 3. Given the proportion of compliers (70.6%; see Section 2.2), the corresponding IV estimate implies that HS participation increases PPVT score at age 4 by 0.036.[17]

In column (2) we explore heterogeneity in the experimental impact by regressing test score on the treatment group indicator $Z$, and interactions of $Z$ with dummy variables for choosing other center care ($C$) or HS ($HS$) at $t = 2$. Because childcare choices at $t = 2$ are endogenous, these estimates do not have any causal interpretation. Nevertheless, results from this regression indicate heterogeneity in the impact of the HS offer depending on the type of care selected at $t = 2$. For instance, the estimated coefficient on the interaction of $Z$ and $C$ is large and positive, indicating that the effect of the HS offer is larger among individuals who choose other center care (relative to home care) at $t = 2$. Controlling

[15] Puma et al. (2010) reports that the HS offer raises PPVT raw score by 2.72 points (Exhibit 4.5), and this result is not significant. Their estimate is equivalent to 0.0272 in our analysis because we scale the raw PPVT score by 1/100.

[16] Similar to Kline and Walters (2016), we include controls for household size, number of siblings, dummies for whether the child is female, black, hispanic, use English as home language, living in urban area, living with both parents, in need of special education, child of a teen mother, child of a mother who never married or is separated, child of a mother with high school or more than high school.

[17] When we use 2SLS to recover the causal effect of attending HS at $t = 1$ for the compliers, controlling for the same characteristics, we obtain an estimate of 0.047, or 0.12sd, which is significant at the 5% level.

for covariates in this regression further increases the precision and magnitude of the coefficient on the interaction of $Z$ and $C$ (column (4)). Indeed, it appears that this particular subgroup drives the positive experimental effect because the outcomes of the treatment group individuals who choose either home care or HS at $t = 2$ are not so different from the outcomes of the control group (the estimated coefficients on $Z$ and $Z$ interacted with $HS$ are small). These results are suggestive that sequential participation in HS and other programs may be relevant in explaining the experimental impact.

## 3 A Sequential Model of Program Participation

We use a generalized Roy framework to estimate a threshold model of program participation and examine program returns. In the model, an individual chooses one program to participate at each of the two decision stages ($t = 1$ and $t = 2$) sequentially. Contrary to the experimental evaluation in which the decision rules are left unspecified, we characterize the decision rules using a flexible threshold model at each stage. By combining the sequential threshold model with outcomes, we can estimate ex post returns to a sequence of program participation and how these returns depend on both observed and unobserved variables.

The decision rules approximate a dynamic discrete choice model without specifying individuals' preferences or information sets. While this approach is flexible, it cannot identify certain parameters that are relevant to policy, such as ex ante program returns (i.e., the returns which individuals act on) and effects from certain counterfactual policy reforms. In Section 5, we extend this model into a dynamic structural discrete choice model by imposing additional structure on individuals' preferences and expectations.

**Choices.** We assume time is discrete and individual $i$ starts in period $t = 1$ in an initial state, $Z_i \in \{0, 1\}$, that has been randomly assigned by an experiment. If $Z_i = 1$, the individual has access to HS at $t = 1$; if $Z_i = 0$, she has no access to HS at $t = 1$ by default. In each of $t = 1$ and $t = 2$, the individual faces three program alternatives: HS ($h$), other center care ($c$), and home care ($n$).

Figure 1 shows the multistage decision framework. There are four decision nodes in total. Denote the set of decision nodes by $\mathcal{J} = \{\mathsf{o}, \mathsf{h}, \mathsf{c}, \mathsf{n}\}$, where $\mathsf{o}$ is the decision node at $t = 1$, and $\mathsf{h}$, $\mathsf{c}$ and $\mathsf{n}$ represent the decision node that the individual gets to at $t = 2$ after she chose $h$, $c$ and $n$ at t=1, respectively.

We specify a reduced-form threshold model for the choice process at each decision node. Upon arrival at node $j \in \mathcal{J}$, the perceived value from each alternative is given by:

$$U_{ij}^h = \psi_j^h + f_j(Z_i, \boldsymbol{X}_i, \theta_i) + \lambda_j^h \theta_i + \nu_{ij}^h \tag{1}$$

$$U_{ij}^c = \psi_j^c + \lambda_j^c \theta_i + \nu_{ij}^c \tag{2}$$

$$U_{ij}^n = 0 \tag{3}$$

where $U_{ij}^h$ is the value of HS, $U_{ij}^c$ is the value of other center care, and the value of home care ($U_{ij}^n$) is normalized to zero. The individual chooses an alternative that yields the largest value, and we define an indicator function $I_{ij}^k$, where $k \in \{h, c, n\}$, as follows:

$$I_{ij}^k = \begin{cases} 1 & \text{if and only if } U_{ij}^k \geq U_{ij}^m, \forall m \neq k, \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

so the decision is $k$ if and only if $I_{ij}^k = 1$. In practice, the individual starts from node $\mathsf{o}$ at $t = 1$ and reaches either node $\mathsf{h}$, $\mathsf{c}$ or $\mathsf{n}$ at $t = 2$ depending on her choice at $t = 1$. We can denote the individual's program choice at $t$ by $K_{it} \in \{h, c, n\}$ and her sequence of choices from period 1 up to period $t$ by $D_{it} \equiv (K_{i1}, K_{i2}, \ldots, K_{it})$. Given that nearly all children start kindergarten at age 5 ($t = 3$), we model program choices for up to 2 periods: $D_{it} = D_{i2}$ for all $t \geq 2$.[18]

The unobserved factor, $\theta_i$, is the key source of dynamic selection in this model and, together with other covariates, determines the dependence between selection into different programs and outcomes. The factor loadings, $\lambda_j^h$ and $\lambda_j^c$, vary flexibly across decision nodes, allowing for different types of individuals to select into a given program at different decision nodes.

For $t = 1$ ($j = \mathsf{o}$), we assume that

$$f_j(Z_i, \boldsymbol{X}_i, \theta_i) = \beta_j^{hsq} hsq_i + \beta_j^{trp} trp_i + \left(\beta_j^Z + \beta_j^{Zhsq} hsq_i + \beta_j^{Ztrp} trp_i + \beta_j^{Z\theta} \theta_i\right) Z_i \tag{5}$$

where $\boldsymbol{X}_i$ denotes a vector of HS center characteristics, including center quality ($hsq_i$) and center

[18] At $t = 3$, about 2 percent of children in our sample were in other center care and 1 percent were in Head Start or home care.

transportation availability ($trp_i$).[19] Being in the HSIS treatment group improves ease of access to HS, which is captured by coefficient $\beta_j^Z$. Empirically we find that the take-up rates of HS in the treatment group relative to the control group varies by center and individual characteristics.[20] To capture such heterogeneity, we interact $Z_i$ with the vector of HS center characteristics ($\boldsymbol{X}_i$) and the unobserved factor ($\theta_i$). Coefficients $\beta_j^{hsq}$ and $\beta_j^{trp}$ capture heterogeneity in HS enrollment probability by center characteristics for the control group. Similarly, coefficient $\lambda_j^h$ (in equation (1)) captures variations in HS enrollment in the control group by the unobserved factor. The experimental variation in HS and its interaction with HS center characteristics, which affect the value of HS, serve as exclusion restrictions in the model (discussed below).[21]

For $t = 2$, upon arrival at node $j \in \{\mathsf{h}, \mathsf{c}, \mathsf{n}\}$, the value of HS no longer depends on $Z_i$:

$$f_j(Z_i, \boldsymbol{X}_i, \theta_i) = \beta_j^{hsq} hsq_i + \beta_j^{trp} trp_i \tag{6}$$

We emphasize that $Z_i$ can affect the values at $t = 2$, which are node-specific, via changing the decision at $t = 1$. In fact, this is a key mechanism that our model examines.

The alternative-specific idiosyncratic shocks, $\nu_{ij}^h$ and $\nu_{ij}^c$, are uncorrelated with the factor $\theta_i$ and other covariates in the model. They reflect unobserved shocks to demand for HS and other center care, respectively, relative to home care. They are assumed to be independent over time but may be correlated with each other within a given period, in which they follow a bivariate normal distribution with zero means, variance of 1 (normalization) for $\nu_{ij}^h$, variance of $\sigma_{\nu^c}^2$ for $\nu_{ij}^c$, and correlation coefficient $\rho_{hc}$.[22]

[19] The randomization of HSIS is implemented at HS center level where, at each HS center, some applicants are randomly assigned to the treatment group and some fall into the control group. $\boldsymbol{X}_i$ refers to the characteristics of the center that the individual has applied for. One concern is that the characteristics of the assigned HS center may be correlated with family characteristics. For instance, low-education mothers may tend to live closer to a low-quality HS center. In $\boldsymbol{X}_i$, we include "residualized" center characteristics, where a residual is obtained from a regression of one center characteristic on a set of family characteristics variables and then standardized. Therefore, the center characteristics in $\boldsymbol{X}_i$ have means zero and standard deviation 1, and are orthogonal to family characteristics.

[20] For instance, we find that lower-quality HS centers tend to admit control group individuals.

[21] We assume that $Z_i$ and $\boldsymbol{X}_i$ have no effect on the value of other center care (relative to home care) in period 1. Note that in a structural model with forward-looking agents, $\boldsymbol{X}_i$ may indirectly affect the values of care in period 1 via the expected value function (e.g., high-quality HS center raises the expected value of all modes of care now due to a possibility of choosing HS in the future). However, the symmetry of our decision tree and additional robustness checks suggest that any shifts in the expected value function have roughly the same magnitude in the "c" and "n" branches at $t = 1$. This motivates the exclusion restriction in the sequential choice model.

[22] Although the sequential setup is not most general, the sequential structure of the model naturally corresponds to the features in the HSIS data, making the perceived values of different choices easily comparable with the values estimated in the dynamic structural model in Section 5 (which is truly dynamic and also sequential as shocks are revealed over time).

**Potential outcomes.** Let $Y_{it}^d$ be the potential outcome evaluated at the end of period $t$ if individual $i$ is externally assigned to a sequence of programs $D_{it} = d$. As illustrated in Figure 1, if the outcome is measured beyond the terminal choice period ($t \geq 2$), then $d \in \{(h,h),(h,c),(h,n),(c,h),(c,c),(c,n),(n,h),(n,c),(n,n)\}$, where the couple $(j,j')$ indicates that individual is assigned to program $j$ at $t=1$ and $j'$ at $t=2$. The observed outcome can be linked to potential outcomes using a switching regression. For instance, when $t \geq 2$,

$$\begin{aligned}Y_{it} = Y_{it}^{n,n} &+ \Sigma_{j\in h,c,n}(Y_{it}^{h,j} - Y_{it}^{n,n})1(D_{it} = (h,j)) + \Sigma_{j\in h,c,n}(Y_{it}^{c,j} - Y_{it}^{n,n})1(D_{it} = (c,j)) \\ &+ \Sigma_{j\in h,c,n}(Y_{it}^{n,j} - Y_{it}^{n,n})1(D_{it} = (n,j))\end{aligned}$$

where $1(\cdot)$ is an indicator function.

We parameterize the potential outcomes as a linear combination of observables and unobservables:

$$Y_{it}^d = \alpha_t^d + \beta_t^d hsq_i + \gamma_t^d \theta_i + \varepsilon_{it}^d \tag{7}$$

Parameters determining the potential outcomes are time and node-specific. The potential-program-specific idiosyncratic error terms are $\varepsilon_{it}^d$. They are assumed identically distributed and independent from each other, the covariates $\mathbf{X}_i$, the unobserved factor, and the demand shocks in all decision nodes. The factor loadings capture how potential outcomes vary with the unobservable factor and reveal important information about a variety of selection schemes. They capture treatment effect heterogeneity that is related to unobservables governing selection into treatment (essential heterogeneity, Heckman, Urzua, and Vytlacil, 2006).[23] For instance, suppose that $\theta$ is positively associated with selection into HS and consider the potential outcome in period $t = 1$ ($d \in \{h,c,n\}$). If $\gamma_1^h = \gamma_1^c = \gamma_1^n = 0$, then potential outcomes are independent of the unobserved factor, implying no selection. If $\gamma_1^d = g > 0, \forall d \in \{h,c,n\}$, where $g$ is a constant, then there is no selection on gains into HS: those who are more likely to self-select into HS have high potential outcome regardless of their choices. If $\gamma_1^h > \gamma_1^n$, then there is selection into HS on gains.[24]

The model specification is also simpler, e.g., if we "flatten" it to a multinomial model instead we would allow $3 \times 3 - 1 = 8$ alternative-specific choice shocks to be mutually correlated without a priori restrictions.

[23] Also see Fruehwirth, Navarro, and Takahashi (2016) who point out that the factor loadings reveal whether there is essential heterogeneity.

[24] If $\theta$ is negatively associated with selection into HS, then there is selection into HS on gains when $\gamma_1^h < \gamma_1^n$.

**Measurement equations.** Following the literature on dynamic treatment effects (e.g., Heckman, Humphries, and Veramendi (2016)), we supplement the outcomes with a measurement system to proxy the unobserved factors and correct for the effects of measurement error in the proxy:

$$M_i^M = \alpha^M + \gamma^M \theta_i + e_i^M \tag{8}$$

where $M_i^M$ is a baseline test score that is observed around the time of randomization (in Fall 2002). We utilize two types of baseline test scores, the PPVT test ($M = pp$) and the WJIII test ($M = wj$, see Section 2.2).[25] Given that these measurements are observed at the time of randomization, the parameters affecting $M_i^M$ are invariant to potential program sequence $D$. The measurement error terms $e_i^M$ are independent of each other, the unobserved factor and all other shocks in the model. We thus proxy $\theta_i$ using baseline cognitive test scores to identify the interpretable sources of omitted variable bias and to determine how the unobservables determine the causal effects of program participation.[26]

## 3.1 Identification

Location and scale of the factor are not identified, so normalization on the factor is required (e.g., Heckman and Navarro (2007)). Specifically, the mean of the factor is normalized to zero (to fix the location), and, to fix the scale, the factor loading in the measurement equation for the baseline PPVT test score is fixed to one ($\gamma^{pp}$=1).[27] By linking the latent factor to the baseline PPVT test scores, the unobservable factor $\theta_i$ can be interpreted as the latent ability of the individual. Our identification argument is divided into four blocks: (1) measurement equations and the choice equation for $c$ at $t = 1$, (2) the choice equation for $h$ at $t = 1$, (3) all choice equations at $t = 2$, and (4) all outcome equations.

Let $\tilde{Z}_i$ be an exclusion variable in the choice equation for alternative $h$, i.e., $\tilde{Z}_i$ is excluded from the choice equations for $c$ and $n$ (in empirical analysis, $\tilde{Z}_i$ includes random assignment status $Z_i$, HS center characteristics $\boldsymbol{X}_i$, and interactions between $Z_i$ and $\boldsymbol{X}_i$). Define the residuals in the (centered)

[25] We also re-estimate the model by adding the WJIII Letter-Word Identification test score from the baseline survey as the third measurement and find our results are robust to the inclusion of the additional measurement. See Section 3.3 for details.

[26] In their preferred model, Kline and Walters (2016) instead control for baseline test scores as covariates in both the selection equations and potential outcomes equations. Therefore, differently from our model, the unobservables driving selection in Kline and Walters (2016) are interpreted as unobserved tastes (conditional on baseline test scores of children).

[27] We assume $\theta_i$ is unidimensional. See Fruehwirth, Navarro, and Takahashi (2016) for formal identification arguments when $\theta_i$ is multidimensional.

measurement equations by $\epsilon_i^M \equiv \gamma^M \theta_i + e_i^M, M \in \{pp, wj\}$ and the residual in the selection equation of $c$ at decision node $\mathsf{o}$ by $\mu_{i\mathsf{o}}^c \equiv \lambda_{\mathsf{o}}^c \theta_i + \nu_{i\mathsf{o}}^c$. By fixing $\tilde{Z}_i$ sufficiently small such that the choice model in $t = 1$ becomes a binary decision between $c$ and $n$, we can recover the joint distribution of $(\epsilon_i^{wj}, \epsilon_i^{pp}, \mu_{i\mathsf{o}}^c)$ because the measurements are not affected by subsequent program choices (Heckman and Smith, 1998; Fruehwirth, Navarro, and Takahashi, 2016).[28] Hence the following covariances are also identified:

$$Cov(\epsilon_i^{pp}, \epsilon_i^{wj}) = \gamma^{wj} \sigma_\theta^2 \tag{9}$$

$$Cov(\epsilon_i^{pp}, \mu_{i\mathsf{o}}^c) = \lambda_{\mathsf{o}}^c \sigma_\theta^2 \tag{10}$$

$$Cov(\epsilon_i^{wj}, \mu_{i\mathsf{o}}^c) = \lambda_{\mathsf{o}}^c \gamma^{wj} \sigma_\theta^2 \tag{11}$$

where $\sigma_\theta^2$ is the variance of the unobserved factor $\theta$. These covariance restrictions build on the conditional independence assumption where measurements and choices are independent from each other conditional on the unobserved factor. The ratio between $Cov(\epsilon_i^{wj}, \mu_{i\mathsf{o}}^c)$ and $Cov(\epsilon_i^{pp}, \mu_{i\mathsf{o}}^c)$ identifies $\gamma^{wj}$ and the ratio between $Cov(\epsilon_i^{wj}, \mu_{i\mathsf{o}}^c)$ and $Cov(\epsilon_i^{pp}, \epsilon_i^{wj})$ identifies $\lambda_{\mathsf{o}}^c$. Also, $\sigma_\theta^2$ can be identified from equation (9). Note that once the loadings $\gamma^{wj}$ and $\lambda_{\mathsf{o}}^c$ are identified, we can nonparametrically identify the marginal distributions of the unobservables $\theta_i$, $e_i^{pp}$, $e_i^{wj}$ and $\nu_{i\mathsf{o}}^c$ via Kotlarski (1967) (see Appendix Section A).

It remains to identify the parameters $\psi_{\mathsf{o}}^h$ and $\lambda_{\mathsf{o}}^h$ in the choice equation for $h$, and the conditional distribution of $\nu_{i\mathsf{o}}^h$ given $\nu_{i\mathsf{o}}^c$ (denoted by $F_{\nu_{i\mathsf{o}}^h | \nu_{i\mathsf{o}}^c}(\cdot)$). We impose a dependence structure between $\nu_{i\mathsf{o}}^h$ and $\nu_{i\mathsf{o}}^c$:[29]

$$\nu_{i\mathsf{o}}^h = \rho_{hc} \nu_{i\mathsf{o}}^c + \omega_{i\mathsf{o}} \tag{12}$$

where $\omega_{i\mathsf{o}}$, which is normalized by location to have zero mean, is independent of $\nu_{i\mathsf{o}}^c$ and all other unobservables and covariates in the model. Then, consider the probability of choosing $c$ at node $\mathsf{o}$

[28] The identification result in Heckman and Smith (1998) and Fruehwirth, Navarro, and Takahashi (2016) require an exclusion variable in the equation for $c$. In Appendix Section A, we provide an identification argument without an exclusion variable in the choice equation for $c$, and clarify that the cost is to impose a parametric distribution on $\nu_{i\mathsf{o}}^c$ (in which case the nonparametric identification of the distribution of $\nu_{i\mathsf{o}}^c$ becomes irrelevant, but all other results remain to hold).

[29] This can be relaxed to parametric nonlinear dependence structures. Note that if there is also an exclusion variable for $c$, then the joint distribution of $(\nu_{io}^h, \nu_{io}^c)$ is nonparametrically identified by fixing measurement $\epsilon_i^{pp} = \pm\infty$ and then vary each exclusion variable in $h$ and $c$ freely to trace out the distribution. See e.g., Matzkin (1993); Heckman and Smith (1998); Lewbel (2000) for details. Then, fix the exclusion variable for $c$ sufficiently small to reduce the model to binary choice ($h$ vs $n$) and vary measurement $\epsilon_i^{pp}$ to identify $\psi_{\mathsf{o}}^h$ and $\lambda_{\mathsf{o}}^h$.

given measurement $\epsilon_i^{pp}$ and exclusion variable $\tilde{Z}_i \neq \pm\infty$:[30]

$$
\begin{aligned}
&Pr(K_{i1} = c|\epsilon_i^{pp} = m, \tilde{Z}_i = z) \\
&= Pr(\psi_{\mathsf{o}}^c + \lambda_{\mathsf{o}}^c\theta_i + \nu_{i\mathsf{o}}^c \geq \psi_{\mathsf{o}}^h + \lambda_{\mathsf{o}}^h\theta_i + \tilde{Z}_i + \nu_{i\mathsf{o}}^h; \quad \psi_{\mathsf{o}}^c + \lambda_{\mathsf{o}}^c\theta_i + \nu_{i\mathsf{o}}^c \geq 0 \quad |\epsilon_i^{pp} = m, \tilde{Z}_i = z) \\
&= Pr(\omega_{i\mathsf{o}} \leq (1-\rho_{hc})\nu_{i\mathsf{o}}^c + (\psi_{\mathsf{o}}^c - \psi_{\mathsf{o}}^h) + (\lambda_{\mathsf{o}}^c - \lambda_{\mathsf{o}}^h)\theta_i - \tilde{Z}_i; \quad \nu_{i\mathsf{o}}^c \geq -\psi_{\mathsf{o}}^c - \lambda_{\mathsf{o}}^c\theta_i \quad |\epsilon_i^{pp} = m, \tilde{Z}_i = z) \\
&= \int_{-\infty}^{\infty}\int_{-\psi_{\mathsf{o}}^c - \lambda_{\mathsf{o}}^c\theta}^{\infty}\int_{-\infty}^{(1-\rho_{hc})\nu^c + (\psi_{\mathsf{o}}^c - \psi_{\mathsf{o}}^h) + (\lambda_{\mathsf{o}}^c - \lambda_{\mathsf{o}}^h)\theta - z} dF_{\omega_{i\mathsf{o}}}(\omega)dF_{\nu_{i\mathsf{o}}^c}(\nu^c)f_{\theta_i|\epsilon_i^{pp}}(\theta|m)d\theta \qquad (13)
\end{aligned}
$$

where the last equality is due to independence among $\omega_{i\mathsf{o}}$, $\nu_{i\mathsf{o}}^c$, $\theta_i$ and $\tilde{Z}_i$. The conditional density $f_{\theta_i|\epsilon_i^{pp}}(\theta|m) = \frac{f_{\epsilon_i^{pp}|\theta_i}(m|\theta)f_{\theta_i}(\theta)}{f_{\epsilon_i^{pp}}(m)} = \frac{f_{e_i^{pp}}(m-\theta)f_{\theta_i}(\theta)}{f_{\epsilon_i^{pp}}(m)}$ is identified from the densities of measurement $\epsilon_i^{pp}$, unobservables $\theta_i$ and $e_i^{pp}$. For the middle and inner integrals, the integration region is at the lower-right of the space of $(\nu_{i\mathsf{o}}^c, \omega_{i\mathsf{o}})$, bounded from the left by $-\psi_{\mathsf{o}}^c - \lambda_{\mathsf{o}}^c\theta_i$ and from above by a threshold line with gradient $1-\rho_{hc}$. Similarly, the conditional probability of choosing $n$ at node $\mathsf{o}$ is:

$$
Pr(K_{i1} = n|\epsilon_i^{pp} = m, \tilde{Z}_i = z) = \int_{-\infty}^{\infty}\int_{-\infty}^{-\psi_{\mathsf{o}}^c - \lambda_{\mathsf{o}}^c\theta}\int_{-\infty}^{-\rho_{hc}\nu^c - \psi_{\mathsf{o}}^h - \lambda_{\mathsf{o}}^h\theta - z} dF_{\omega_{i\mathsf{o}}}(\omega)dF_{\nu_{i\mathsf{o}}^c}(\nu^c)f_{\theta_i|\epsilon_i^{pp}}(\theta|m)d\theta \qquad (14)
$$

where the integration region is at the lower-left of the space of $(\nu_{i\mathsf{o}}^c, \omega_{i\mathsf{o}})$, bounded from the right by $-\psi_{\mathsf{o}}^c - \lambda_{\mathsf{o}}^c\theta_i$ and from above by a threshold line with gradient $-\rho_{hc}$. The space of $(\nu_{i\mathsf{o}}^c, \omega_{i\mathsf{o}})$ is partitioned into three regions $(h, c, n)$ with the three threshold lines intersecting at a single point. By varying $\tilde{Z}_i$ freely but keeping $\theta_i$ fixed, the two upper threshold lines will shift, which identifies the distribution $F_{\omega_{i\mathsf{o}}}(\cdot)$ nonparametrically as well as parameters $\rho_{hc}$ and $\psi_{\mathsf{o}}^h$. Then, by fixing $\tilde{Z}_i$ but varying measurement $\epsilon_i^{pp}$, the region of integration will change its boundaries (up to a mixture dictated by $f_{\theta_i|\epsilon_i^{pp}}(\cdot)$) which, given knowledge of $F_{\omega_{i\mathsf{o}}}(\cdot)$, identifies the remaining parameter $\lambda_{\mathsf{o}}^h$.

The intuition is further explained as follows. Given measurement $\epsilon_i^{pp}$, we know the approximate range of $\theta_i$ and alternative $c$'s value. In addition, the choice shocks follow the dependence structure in (12). Therefore, as the exclusion variable $\tilde{Z}_i$ varies, we can compute the change in probability of choosing each alternative even though three choices are involved. The resulting change in log-odds ratio between alternatives $c$ and $n$ informs $\rho_{hc}$. When we fix $\tilde{Z}_i$ but vary $\epsilon_i^{pp}$, the resulting change in

[30] As in Lewbel (2000), the coefficient on $\tilde{Z}_i$ is normalized to 1 and the distribution of $\omega_{i\mathsf{o}}$ is nonparametric. In empirical analysis, for ease of interpretation we normalize the variance of $\nu_{i\mathsf{o}}^h$ (assumed normal) to 1, and estimate the variance of $\nu_{i\mathsf{o}}^c$ and a linear function of $\tilde{Z}_i$.

choice probabilities informs $\lambda^h_{\mathsf{o}}$.

We now discuss identification of parameters in the second-period choice equations, at nodes $\mathsf{h}$, $\mathsf{c}$, $\mathsf{n}$. For illustration, we focus on node $\mathsf{h}$; the same argument applies to nodes $\mathsf{c}$ and $\mathsf{n}$. We assume the distributions of choice shocks are i.i.d. across nodes: $F_{\nu^c_{i\mathsf{h}}}(\cdot) = F_{\nu^c_{i\mathsf{o}}}(\cdot)$ and $F_{\omega_{i\mathsf{h}}}(\cdot) = F_{\omega_{i\mathsf{o}}}(\cdot)$. The conditional probability of choosing $c$ at node $\mathsf{h}$ given measurement $\epsilon^{pp}_i$ and exclusion variable $\tilde{Z}_i$ is then:

$$
\begin{aligned}
&Pr(K_{i2} = c | K_{i1} = h, \epsilon^{pp}_i = m, \tilde{Z}_i = z) \\
&= \int_{-\infty}^{\infty} \int_{-\psi^c_{\mathsf{h}} - \lambda^c_{\mathsf{h}}\theta}^{\infty} \int_{-\infty}^{(1-\rho_{hc})\nu^c + (\psi^c_{\mathsf{h}} - \psi^h_{\mathsf{h}}) + (\lambda^c_{\mathsf{h}} - \lambda^h_{\mathsf{h}})\theta - z\beta_{\mathsf{h}}} dF_{\omega_{i\mathsf{o}}}(\omega) dF_{\nu^c_{i\mathsf{o}}}(\nu^c) f_{\theta_i | K_{i1}, \epsilon^{pp}_i, \tilde{Z}_i}(\theta | h, m, z) d\theta
\end{aligned} \tag{15}
$$

Similarly, the conditional probability of choosing $n$ at node $\mathsf{h}$ is:

$$
\begin{aligned}
&Pr(K_{i2} = n | K_{i1} = h, \epsilon^{pp}_i = m, \tilde{Z}_i = z) \\
&= \int_{-\infty}^{\infty} \int_{-\infty}^{-\psi^c_{\mathsf{h}} - \lambda^c_{\mathsf{h}}\theta} \int_{-\infty}^{-\rho_{hc}\nu^c - \psi^h_{\mathsf{h}} - \lambda^h_{\mathsf{h}}\theta - z\beta_{\mathsf{h}}} dF_{\omega_{i\mathsf{o}}}(\omega) dF_{\nu^c_{i\mathsf{o}}}(\nu^c) f_{\theta_i | K_{i1}, \epsilon^{pp}_i, \tilde{Z}_i}(\theta | h, m, z) d\theta
\end{aligned} \tag{16}
$$

The parameters to be identified are the alternative-specific intercepts $(\psi^c_{\mathsf{h}}, \psi^h_{\mathsf{h}})$, loadings $(\lambda^c_{\mathsf{h}}, \lambda^h_{\mathsf{h}})$, and coefficient on the exclusion variable $(\beta_{\mathsf{h}})$. We can write the condition density $f_{\theta_i | K_{i1}, \epsilon^{pp}_i, \tilde{Z}_i}(\cdot)$ as

$$
\begin{aligned}
f_{\theta_i | K_{i1}, \epsilon^{pp}_i, \tilde{Z}_i}(\theta | h, m, z) &= \frac{Pr(K_{i1} = h | \theta_i = \theta, \epsilon^{pp}_i = m, \tilde{Z}_i = z) f_{\epsilon^{pp}_i | \theta_i}(m | \theta) f_{\theta_i}(\theta)}{Pr(K_{i1} = h | \epsilon^{pp}_i = m, \tilde{Z}_i = z) f_{\epsilon^{pp}_i}(m)} \\
&= \frac{Pr(K_{i1} = h | \theta_i = \theta, \tilde{Z}_i = z) f_{e^{pp}_i}(m - \theta) f_{\theta_i}(\theta)}{Pr(K_{i1} = h | \epsilon^{pp}_i = m, \tilde{Z}_i = z) f_{\epsilon^{pp}_i}(m)}
\end{aligned} \tag{17}
$$

where all the components are identified from the analysis on node $\mathsf{o}$ above. This conditional density captures all the selection into node $\mathsf{h}$. Because all the distributions in equations (15) and (16) have already been identified, we can simply identify the parameters $\psi^c_{\mathsf{h}}, \psi^h_{\mathsf{h}}, \lambda^c_{\mathsf{h}}, \lambda^h_{\mathsf{h}}, \beta_{\mathsf{h}}$ by varying measurement $\epsilon^{pp}_i$ and exclusion variable $\tilde{Z}_i$.

Finally, the factor loadings in the potential outcome equations can be identified by exploiting the experimental variation $Z$ (see also Kline and Walters (2016)). Utilizing the conditional independence assumption where the residuals in the choice and the outcome equation are independent conditional on the unobserved factor, the mean observed outcome conditional on program choice sequence $D_{it} = d$

can be written as:

$$E(Y_{it} \mid X_i = x, Z_i = z, D_{it} = d) = \alpha_t^d + \beta_t^d x + \gamma_t^d \lambda(x, z, d) \tag{18}$$

where $\lambda(x, z, d) \equiv E(\theta_i \mid X_i = x, Z_i = z, D_{it} = d)$ is a function of the selection equations and hence is an implicit function of the choice probabilities. Taking the difference of equation (18) between the treatment and the control group, we obtain:[31]

$$\gamma_t^d = \frac{E(Y_{it}|X_i = x, Z_i = 1, D_{it} = d) - E(Y_{it}|X_i = x, Z_i = 0, D_{it} = d)}{\lambda(x, 1, d) - \lambda(x, 0, d)} \tag{19}$$

Therefore, the factor loadings in the outcome equations ($\gamma_t^d$) can be identified as long as the denominator in equation (19) is non-zero. This condition will be satisfied if $Pr(D_{it} = d|X_i = x, Z_i = 1) \neq Pr(D_{it} = d|X_i = x, Z_i = 0), \forall d$, meaning that the HS experimental variation shifts choice probabilities at every sequence of program choice conditional on covariate value $X_i = x$.[32]

### 3.2 Estimated Parameters

The identification of the sequential threshold model relies on the conditional independence assumption of the factor structure and the exclusion restriction from the HS experimental variation. In particular, the identification does *not* require any distributional assumption on the unobserved factor. Nevertheless, to facilitate estimation, we assume that the unobserved factor follows a normal distribution, and consider sensitivity of our estimates to alternative distributional assumptions of the unobserved factor (see Section 3.3). We also assume that the potential-program-specific idiosyncratic error terms ($\varepsilon_{it}^d$) and the measurement error terms ($e_i^M$) are normally distributed.

The model is estimated using the method of maximum likelihood. A total of 60 parameters are estimated. Details of the estimation procedure are presented in Appendix Section D.2. The estimates and standard errors of the parameters of the selection equations are reported in Table 3. The normalization we impose implies that the unobservable factor $\theta_i$ is positively correlated with the baseline

[31] We have additional exclusion restrictions such as HS transportation availability ($trp$), which is excluded from all nine potential outcome equations, and HS quality ($hsq$) excluded from four potential outcome equations ($(c, c), (c, n), (n, c), (n, n)$).

[32] For example, the treatment shifts individuals into program choice sequences related to $h$, which modifies the initial selection mix of individuals in these sequences driven by the choice equations (See Section 2.2).

PPVT test score. In the first period (at the decision node $\mathsf{o}$), the factor loading in the choice equation for $h$ is small and insignificantly different from zero, whereas the factor loading in the choice equation for $c$ is positive and significant. This indicates that children choosing $c$ in the first period are positively selected based on ability (relative to those choosing $h$ and $n$). There is no selection based on ability for those choosing $h$ relative to $n$; however, because children are positively selected into $c$, there is negative selection into $h$ relative to those choosing $c$. As expected, being in the HSIS treatment group ($Z = 1$) increases the probability of enrolling in HS. Receiving the experimental offer increases the probability of HS enrollment more among those HS centers providing transportation services and rated as high quality. Interestingly, children in the control group ($Z = 0$) are less likely to enroll in high-quality HS centers.

We find that one year of experience in HS (at age 3) tends to increase the likelihood of participating in HS in the next period (age 4). Having enrolled in HS in $t = 1$ (at decision node $\mathsf{h}$), an individual with average ability is more likely to enroll in $h$ than $c$ (this can be verified from $\psi^h_{\mathsf{h}} > \psi^c_{\mathsf{h}}$ in combination with $\sigma_{\nu^c}$ and $\rho_{hc}$). The estimated coefficient on HS center quality is positive and significant, suggesting that individuals are more likely to continue to enroll in HS if the HS center is of high quality. Transportation services, on the other hand, becomes less important in determining their HS enrollment decision. Although imprecisely estimated, the factor loadings in the choice equations at decision node $\mathsf{h}$ imply that the positive correlation between HS enrollments in age 3 and age 4 is more pronounced for individuals whose ability is below the average ($\theta < 0$). For an average child who has enrolled in other center care at age 3 (at decision node $\mathsf{c}$), she is more likely to choose $c$ in age 4 instead of $h$. The value of option $h$ declines more rapidly with children's ability than the value of $c$ does, implying persistence in their choice of $c$ for the most able children. Finally, for an average child who did not enroll in any formal care in age 3 (at decision node $\mathsf{n}$), she is more likely to choose $h$ than $c$ in age 4.

Table 4 reports the estimates for the outcome and measurement equations. We define as $t = T$ the period when the outcome (PPVT test score) is observed (i.e., $T$ is the end of age-4 year). Given that the covariates in the outcome equation have unconditional means zero, the intercepts of the potential outcome equations ($\alpha^d_T$; first column) can be interpreted as average potential outcomes. Comparing the average potential outcomes by program sequence, we find that the average potential outcome if all children are externally assigned to HS at age 3 is the highest. Having enrolled in the HS program at age 3, their average potential outcome would be higher if they are assigned to other center care at age 4 instead of having another year of HS ($\alpha_T^{hc} > \alpha_T^{hh}$). However, this does not necessarily suggest that HS at age 4 has little return relative to other center care. For instance, if children were assigned to home care at age 3, then their average potential outcome would be maximized if they enroll in HS at age 4 ($\alpha_T^{nh} > \alpha_T^{nc}$). Section 4 analyzes in detail how to define returns to alternative combinations of programs. The estimated parameter on the HS quality in the outcome equation $hh$ is positive, indicating that center quality has a positive effect on children if they enroll in HS for two years. Center quality does not appear to affect potential outcomes if individuals only enroll in HS for one period.

The factor loadings in the potential outcome equation (second column in Table 4) combined with the estimates in the choice equation reveals patterns of selection. We find that $0 < \gamma_T^{cj} < \gamma_T^{nj}$, $\forall j \in \{h, c, n\}$, suggesting that high ability children will receive smaller potential test score gains from choosing $c$ (relative to choosing $n$). Recall that high ability children are more likely to choose $c$ in the first period relative to $n$. This indicates a reverse-Roy pattern of negative selection on test score gains. Similar to other center care, we also find $0 < \gamma_T^{hj} < \gamma_T^{nj}$, $\forall j \in \{h, c, n\}$, indicating that high ability children will receive smaller potential test score gains if they choose $h$ in the first period (relative to choosing $n$).[33]

From the estimated measurement equation, we can compute the signal to noise ratio, which captures the information content of each measure given the specification of the measurement system.[34] For the PPVT test score, the signal to noise ratio is 48%. For the WJIII test score, the signal to noise ratio is only 0.96%. Therefore, the PPVT scores include a substantial amount of information about the unobserved factor. At the same time, they also demonstrate the importance of allowing for measurement error.

To assess the fit of the model, in Table 5, we compare the sample moments for choices and outcome (PPVT score at age 4) to the moments predicted by the model, separating between the moments of the treated and control groups. The left panel of Table 5 compares the choice probabilities, and shows that the model matches the actual proportion well for both the treatment and control groups. Similarly, the model matches the outcome for the different subgroups. Furthermore, in Figure A1, we show that

[33] In the second period, our choice equation suggests negative selection into $h$ relative to $c$ for those at the decision node $\mathsf{c}$. As further evidence for negative selection on gains, we find that the factor loadings in the potential outcome at node $\mathsf{c}$ reveal that low-ability children also gain less from $h$ relative to $c$ (as $\gamma_T^{cc} < \gamma_T^{ch}$).

[34] The expression for the signal to noise ratio of measurement M is $\frac{(\gamma^M)^2 \sigma_\theta^2}{(\gamma^M)^2 \sigma_\theta^2 + \sigma_{eM}^2}$. It is the ratio of the variance of the latent factor (as estimated within the specification of our model) to the variance of each measurement.

the predicted and actual distribution of PPVT score overlap well.

### 3.3 Sensitivity Analysis

In Appendix Section B, we report sensitivity analysis of our estimates against functional form and distributional assumptions. We show that our parameter estimates are robust to the inclusion of additional baseline measurement (Appendix Section B.1) and relaxing the factor distribution used in estimation by modelling it as a mixture normal distribution (Appendix Section B.2). The choice equations in the sequential threshold model are approximations to the perceived value of each alternative. For forward-looking individuals, these perceived values are a combination of utility flow and perceived continuation value, meaning that the perceived continuation value may be nonlinear in parameters due to incorporation of uncertainty of future payoffs.[35] To allow for possible nonlinear selection by $\theta$, we also estimate a model by specifying a quadratic function of factor $\theta$ in the period-1 choice equations for $h$ and $c$ (see Appendix Section B.3). Our estimates are robust to relaxing the linearity assumption in the choice equations.

Finally, in Appendix Section B.4 we report estimates from the model where we use the simple average of the PPVT score and WJIII pre-academic skills score as the outcome variable (as in Kline and Walters (2016)). Although taking the average may not be ideal, it has the advantage that the outcome reflects information from multiple sources covering a broader set of skills.[36] Using the combined scores, we find that the estimated choice equations are very similar to those in the baseline model. A limited number of estimated parameters in the outcome equations become different, affecting the magnitude of the estimated program returns (see below) but the key patterns remain qualitatively similar. Notably, the outcome equations become more precisely estimated – standard errors are about one-third smaller on average.

[35] For instance, in the dynamic structural model constructed in Section 5, we show that the expected value function in the first decision period is a nonlinear function of parameters (also see Appendix Section D).

[36] While both tests measure cognitive skills, the PPVT test emphasizes listening comprehension and language skills, whereas the WJIII pre-academic skills measures a combination of reading and spelling skills and mathematical abilities. See Appendix Section B.4 for details.

## 4 Returns from Sequential Program Participation

Our estimated model yields a variety of ex-post counterfactuals and associated returns, i.e., differences in potential outcomes by different childcare sequences. Among those, we define and report two types of parameters in this section. The first is a set of parameters that are informative of the population-average joint, partial and cross returns from sequential program participation. Of particular interest are the cross returns, which indicate the extent of complementarity between HS and alternative childcare use. For the second set of parameters, we focus on the returns from enrolling in HS for those induced to change their choices when receiving the HS experimental offer (the compliers in the HSIS, see Section 2). The returns differ across compliers due to differences in the sequence of choices made when they are externally assigned to the treatment and control groups.

### 4.1 Joint, Partial and Cross Returns of Childcare Programs

Consider outcome $Y$ (in our application, PPVT score at the end of age-4 year) for an individual $i$ who is externally assigned to program $j$ at $t = 1$ and program $j'$ at $t = 2$, where $j, j' \in \{h, c\}$ ($j$ and $j'$ can be the same program). We define the *joint return* from a sequence of programs as the difference in the potential outcome upon completing sequence $(j, j')$ and the potential outcome when no program is assigned in both periods (home care at both $t = 1, 2$, i.e., $(n, n)$). The individual's joint return from the potential program sequence $(j, j')$ can be decomposed into

$$Y_i^{j,j'} - Y_i^{n,n} = \underbrace{(Y_i^{j,n} - Y_i^{n,n})}_{\text{Partial Return of } j \text{ at t=1}} + \underbrace{(Y_i^{n,j'} - Y_i^{n,n})}_{\text{Partial Return of } j' \text{ at t=2}} + \underbrace{[(Y_i^{j,j'} - Y_i^{n,j'}) - (Y_i^{j,n} - Y_i^{n,n})]}_{\text{Cross Return between } j \text{ and } j'}. \quad (20)$$

The first two terms are *partial returns*, which capture the effect of completing a program at a specific age as compared to home care at that age, while fixing the rest of the program sequences to home care. Note that the two partial returns may be different even when $j = j'$, because the effect of the program may depend on when the program is obtained, e.g., attending HS at age 3 may have a different effect than attending HS at age 4.

The last term measures the *cross return* from the two programs. The cross return arises because the joint return from program $j$ at $t = 1$ and program $j'$ at $t = 2$ may be different from the sum of their partial returns. A positive cross return would in fact signal that the two programs are *complements*, implying that the interaction of the two programs gives an even higher return than the combined partial returns from each single program. A negative cross return, on the other hand, implies that the two programs are *substitutes*, with the impact of one being offset by the presence of the other. This may signal that some of the competences learnt in one program are replicated by the other, thus resulting in no additional knowledge.[37]

Our model yields four individual-specific cross-return parameters, which are summarized in Table 6.[38] The two cells along the diagonal of the table measure the within-program cross returns. For instance, a positive cross return within the HS program ($h$) reveals that the return to $h$ at $t = 1$ is boosted by obtaining $h$ at $t = 2$. The cells off the diagonal are the between-program cross returns. The top-right cell measures the cross return between HS at $t = 1$ and other center care ($c$) at $t = 2$; a positive measure reveals that the return to $h$ at $t = 1$ is boosted by obtaining $c$ at $t = 2$. The bottom-left cell measures the cross return between other center care at $t = 1$ and HS at $t = 2$; a positive measure reveals that the return to $c$ at $t = 1$ is boosted by obtaining $h$ at $t = 2$.[39]

**Average partial, joint and cross returns for the population.** Figure 2 shows the average partial, joint, and cross returns from sequential program participation. These parameters are population averages of the individual-specific causal returns defined in Table 6, abstracting from any endogenous selection. They are useful to understand the overall mechanisms through which these different programs work and interact.[40]

The first column ($HH$) shows that enrolling in HS at either age 3 or age 4 has a positive effect on test scores as compared to home care. The average partial return is larger if HS is obtained at $t = 1$

[37]Our definition of program cross returns is closely related to the concept of dynamic complementarity in the context of the child development literature. Formally, one can define dynamic complementarity between investments at different ages for individual $i$ as

$$\frac{\partial^2 Y_{i,t}}{\partial x_{i,t} \partial x_{i,t-1}} = \frac{\partial^2 Y_{i,t}}{\partial x_{i,t} \partial Y_{i,t-1}} \frac{\partial Y_{i,t-1}}{\partial x_{i,t-1}},$$

where $x_{i,t}$ are the investment made at time $t$ (Cunha and Heckman, 2007; Aizer and Cunha, 2012). In our setting we explore complementarity (or substitutability) not only between investments made at different points in time but also between different *types* of investments, i.e. between different programs *relative to* a benchmark program (home care). Importantly, the cross returns in our context may vary across persons because they are defined from potential outcomes allowing for flexible individual-specific program returns.

[38]Clearly, all these outcomes are in potential terms, whereas for every individual $i$ only one outcome is observed, depending on the realized choice at each age.

[39]Symmetrically, one can also interpret the cross-return parameters (both within- and between-program) as the effect of receiving program $j$ at $t = 1$ on the return to program $j'$ at $t = 2$. This can be easily seen by switching the terms on the right hand side of equation 20: $(Y_i^{j,j'} - Y_i^{j,n}) - (Y_i^{n,j'} - Y_i^{n,n})$.

[40]Given the covariates in the potential outcome equations are normalized to have mean zero (see Section 3.2), these are computed using the intercepts in the outcome equations.

(0.06, or 0.15sd), relative to HS obtained at $t = 2$ (0.04, or 0.10sd). This column also shows that the average joint return from enrolling in HS at both $t = 1$ and $t = 2$ is positive yet smaller than the sum of the two average partial returns of HS. This indicates a negative within-program average cross return for HS, which signals that one period's HS enrollment is a substitute for another. In fact, the average joint return from two periods of HS (0.05, or 0.12sd) is even lower than the average partial return from HS obtained at $t = 1$.[41]

Turning to the second column ($HC$), we find that the average joint return from enrolling in HS at $t = 1$ and other center care at $t = 2$ is positive (0.07, or 0.19sd) and *higher* than the average joint return from two periods of HS. Combining with the evidence that the partial return of HS at $t = 1$ and the joint return of two periods of HS are both positive, we can conclude that enrollment in HS at age 3, no matter what is done afterwards, leads to higher average (joint) returns than receiving home care at both ages. However, we do not find evidence of cross-program complementarity between HS and other center care. The average joint return is again smaller than the sum of the two average partial returns, indicating that the education received in HS at age 3 reduces the effectiveness of other center care received at age 4. This between-program cross return is less negative than the within-program cross return for HS, so that the joint return of $(h, c)$ is still larger than the partial return from receiving HS at $t = 1$ only. This suggests that the competences offered by HS and by other center care are different, and HS in two periods substitute each other more than the substitution between HS at $t = 1$ and other center care at $t = 2$.

The last two columns ($CH$, $CC$) report average program returns where all individuals are assigned to other center care in the first period. In contrast with the large positive average partial return of HS at $t = 1$, the average partial return of other center care at $t = 1$ is close to zero. We find a large negative average cross return between other center care at $t = 1$ and HS at $t = 2$ (third column), relative to the small negative average cross return between HS at $t = 1$ and other center care at $t = 2$ (second column). This implies that the *sequence* of program participation is important. Although both program sequences of $(h, c)$ and $(c, h)$ assign individuals to one year of HS and one year of other center care, the average joint returns differ widely between the two program sequences: the former is much

[41]Note that all program returns are defined relative to the benchmark sequence $(n, n)$. The benchmark sequence $(n, n)$ is also a type of investment which may change the child's cognitive outcomes. The average joint return from $(h, h)$ being smaller than the average partial return of $(h, n)$ does not imply human capital destruction because $(h, n)$ is an alternative sequence of investments, rather than merely HS and no other investments.

higher than the latter. Differently from HS, enrollment in other center care at $t = 2$ increases the return of other center care at $t = 1$ (last column). This result must follow from the different characteristics of the different types of care. However, despite this positive within-program cross return, the average joint return from two periods of other center care (0.04, or 0.10sd) is still lower than any average joint returns involving HS in the first period.

Overall, the results from Figure 2 give two main messages: (1) HS has a higher average partial return than other center care, and (2) the average joint return from HS followed by other center care is higher than the average joint return from two periods of HS, due to a smaller degree of substitutability between HS at age 3 and other center care at age 4. As a consequence, if the objective of a policy maker is to choose a program at $t = 2$ in order to maximize the return of investment in HS at $t = 1$, our evidence suggests that offering a program which has features other than HS would be more desirable.

**Heterogeneity in program returns.** The average returns hide relevant heterogeneity. Figure 3 shows the heterogeneity in partial, joint, and cross returns from sequential program participation with respect to the unobserved factor. Each panel shows returns from a program sequence – (h,h), (h,c), (c,h), or (c,c) – for individuals at different percentiles of the distribution of the unobserved factor $\theta_i$. Under the baseline scenario of home care at both age 3 and 4 (i.e., fixing the program sequence to $(n, n)$), moving from the $10^{th}$ to the $90^{th}$ percentile of $\theta$ implies an increase in age-4 PPVT score by 0.99 (99 points).[42]

In all four program sequences, we find that the lowest-ability children gain the most from prolonged formal care participation – the joint return is highest among children with the lowest $\theta$, and it declines with $\theta$. In addition, the joint return remains positive for children with below-median $\theta$ and turns negative for children with above-median $\theta$. For example, in the (h,h) sequence, children at the 10th percentile of $\theta$ gain 0.172 (or 0.44sd), while children at the 90th percentile of $\theta$ lose 0.076 (or 0.20sd). This results in a narrowing of 0.248 in the outcome, which constitutes one-fourth of the 90/10 gap when individuals are assigned to home care in both periods instead.

The partial returns are also highest among lowest-ability children and decline with $\theta$, implying that assigning all children to at least one period of formal care would reduce the gap in educational outcomes

[42] This is computed from the (n,n) potential outcome equation: $2.955 + 1.511\theta$. At the 10th and 90th percentiles of $\theta$, the scores are 2.46 and 3.45, respectively.

between individuals of different ability. Notably, the partial return to HS at $t = 1$ is positive across all percentiles of $\theta$ and declines slowly with $\theta$ – children at the 10th and 90th percentiles of $\theta$ gain 0.080 and 0.044, respectively. This suggests that HS participation at age 3 alone (while fixing the rest of the program sequences to home care) can benefit all children, while also slightly decreasing the gap between differently capable children. The partial return to HS at $t = 2$ has a steeper negative gradient – children at the 10th percentile of $\theta$ gain 0.103, while children at the 90th percentile of $\theta$ lose 0.021. By contrast, the partial returns to other center care are positive for children with below-median $\theta$ and negative for children with above-median $\theta$, with the gradient being much steeper at $t = 2$ than $t = 1$.

In contrast to the negative average cross return shown in Figure 2, the cross return between HS at $t = 1$ and other center care at $t = 2$ are positive for children with a low value of $\theta$. Hence, for low-ability children, receiving other center care at $t = 2$ boosts returns to HS at $t = 1$, an evidence that is consistent with dynamic complementarity. This complementarity between HS at $t = 1$ and other center care at $t = 2$ further contributes to the high joint return from program sequence $(h, c)$ for low-$\theta$ children. Overall, we find that all within- and between-program cross returns are positive or close to zero among lowest-ability children and they decline with $\theta$.[43]

One interpretation to our findings is that the quality of home care differs by the baseline ability of the child. Relative to center care, existing evidence suggests that home care by low-education mothers (whose children are more likely to have low baseline ability) are more likely to be low quality.[44] As a result, home care is less effective than center care among low-education mothers, whereas home care provided by high-education mothers may be comparable to or even better than center care. This could explain the higher returns from participation in center-based care for low-ability children than high-ability children.

The joint returns together with the partial returns can inform the "optimal" pathways (defined in terms of ex-post returns) for a policy maker who can assign different groups of children to different paths to maximize their potential outcomes. For children who have below-median $\theta$, their joint returns

[43]We have also explored how program returns differ by the quality of the HS center. In Appendix Figure A2, we find that high-quality HS centers boost cross returns conditional on $\theta$. For instance, for low-$\theta$ children the estimated cross-return from two periods in high-quality HS is even positive, suggesting that a second period in high-quality HS boosts the returns in the first period. Similarly, the cross return from receiving HS at $t = 1$ and other center care at $t = 2$ also increases with the quality of the HS center.

[44]Studies have found that children from disadvantaged environments are exposed to a substantially less rich vocabulary than children from more advantaged families (Hart and Risley (1995), Fernald, Marchman, and Weisleder (2013)). Chan and Liu (2018) provide additional evidence from estimation of children's cognitive ability production function.

will be maximized if they are assigned to HS at $t = 1$ and other center care at $t = 2$. They also have large and positive returns from program sequences $(h, h)$, $(c, h)$ and $(c, c)$. Therefore, a policy maker should not engage these children with home care; instead, these children will gain from participation in any formal care program at both ages 3 and 4. On the contrary, for high-$\theta$ children, the social planner should engage them with at least one period of home care. Indeed, their optimal pathway is to enroll in HS at $t = 1$ followed by home care at $t = 2$, because the partial return from enrolling in HS at $t = 1$ is positive and all the joint returns depicted in Figure 3 are negative for these children.

## 4.2 Program Returns for HSIS Compliers

The program returns that we have presented so far are informative on how the programs differ and how they dynamically interact. However, they may not necessarily be the parameters that are relevant for policy. For instance, although we have shown that enrolling in HS or other center care improves the outcomes for children with low baseline ability, it does not necessarily mean that a policy intervention will successfully induce those children to receive formal childcare. If we consider a policy which impacts children in terms of the cost of care they attend, the policy relevant parameter is the effect for those who are induced to change their choice (Heckman and Vytlacil, 2001; Heckman, Humphries, and Veramendi, 2016, 2018). In this section we analyze the program returns for HSIS compliers, i.e., children who respond to the HS offer by enrolling in HS. Our estimated sequential choice model allows us to go beyond the Local Average Treatment Effect (LATE) by estimating average returns for specific types of compliers, distinguishing various margins of choice under multiple alternative choices over time.[45]

### 4.2.1 Compliance Types and Substitution Patterns in the HSIS

Building on the notations introduced in Section 3, let $D_i^z = (K_{i1}^z, K_{i2}^z)$ be individual $i$'s potential sequence of choices when the experimental offer is $Z_i = z$ (with $K_{it}^z$ being the potential choice at time $t$ with $Z_i = z$). For each individual, we observe either $D_i^0$ if the experimental variation puts her in the control group, or $D_i^1$ if she belongs to the treatment group. In the sequential threshold model, the experimental offer only affects the valuation for $h$ at $t = 1$; importantly, the experimental offer does

[45] The recent public finance literature has shown that the LATE is the relevant parameter (a sufficient statistic) for a thought experiment of a marginal random expansion in the HS program (see, e.g., Kline and Walters (2016); Hendren (2016)). Kline and Walters (2016) show that the average returns for specific type of compliers are necessary to evaluate such a marginal expansion when there is capacity constraint in the availability of care in other centers.

not affect the perceived value of $c$ and $n$ at $t = 1$, nor does it directly affect the perceived value of any childcare choices at $t = 2$ within a given decision node. These restrictions can be expressed by the following assumption:

**Assumption 1.** $K^0_{i1} \neq K^1_{i1} \Longrightarrow K^1_{i1} = h$.

Assumption 1 states that if individuals change their behavior at $t = 1$ as a consequence of being assigned to the treatment group, they do so by enrolling in HS. For example, this assumption rules out the possibility that individuals who select $c$ ($n$) in period 1 when assigned to the control group would switch to $n$ ($c$) when assigned to the treatment group.[46]

Under assumption 1, we can classify individuals into 27 subgroups, depending on their potential sequence of choices when in the treatment group and when in the control group. Each subgroup falls into one of the following compliance types:

- n-compliers type 1 (n-CP 1): $D^0_i = (n, j), D^1_i = (h, j),\ \forall j \in \{h, c, n\}$
- n-compliers type 2 (n-CP 2): $D^0_i = (n, j), D^1_i = (h, j'),\ \forall j, j' \in \{h, c, n\}$ *and* $j \neq j'$
- c-compliers type 1 (c-CP 1): $D^0_i = (c, j), D^1_i = (h, j),\ \forall j \in \{h, c, n\}$
- c-compliers type 2 (c-CP 2): $D^0_i = (c, j), D^1_i = (h, j'),\ \forall j, j' \in \{h, c, n\}$ *and* $j \neq j'$
- always-taker (AT): $D^0_i = (h, j), D^1_i = (h, j),\ \forall j \in \{h, c, n\}$
- never-taker (NT): $D^0_i = (j, j'), D^1_i = (j, j'),\ \forall j \in \{c, n\},\ j' \in \{h, c, n\}$

Table 7 summarizes all 27 subgroups in our context. Nine subgroups are always-takers (AT) or never-takers (NT), whose choices do not change by the HS offer. Eighteen subgroups are HSIS compliers; when assigned to the control group, nine of them would select home care at $t = 1$ (n-CP), while the remaining nine would select other center care at $t = 1$ instead (c-CP). We can further divide the n-CP and c-CP compliers into two types: (1) temporary switchers (type 1), where *only* the choice at $t = 1$ is changed due to the HS offer at $t = 1$, and (2) path switchers (type 2), where the entire trajectory of choice is changed due to the HS offer at $t = 1$.

[46] Similar assumptions are made in Kirkeboen, Leuven, and Mogstad (2016) and Kline and Walters (2016). Note that we impose this assumption only in period 1—beyond period 1 the experimental offer may have an indirect effect on the relative rank of $c$ and $n$ (via changing the first period choice).

The bottom panel of Table 7 reports the population shares of compliance types.[47] In the first period of the experiment, the HS offer shifts 51.0% of the population from $n$ to $h$ (n-CP compliers), and 19.6% from $c$ to $h$ (c-CP compliers).[48] Therefore, when considering only the first period, the HS program crowds out other center care and home care.

Considering potential choices in a multi-period framework enriches substitution patterns. We find that *path switchers* constitute about 60% ($\frac{12.11+29.87}{70.60}$) of all compliers – that is, more than half of the compliers also change the program choice at age 4 due to the HS offer at age 3. Among c-CP compliers, we find that 43% (or 8.32% of the population) change their period-2 choice from $c$ to $h$ ($D^0 = (c,c), D^1 = (h,h)$). This implies that receiving the HS offer not only reduces the demand for $c$ at $t = 1$ but also moves a substantial fraction of children away from $c$ towards $h$ at $t = 2$. Among n-CP compliers, we find that 22% (or 11.42% of the population) change their period-2 choice from $c$ to $h$ ($D^0 = (n,c), D^1 = (h,h)$), and 17% (or 8.76% of the population) change their period-2 choice from $n$ to $h$ ($D^0 = (n,n), D^1 = (h,h)$). Note that path switchers do not necessarily change their period-2 choice to HS. For example, some of them actually switch from $h$ to $c$ at $t = 2$, both among the c-CP compliers ($D^0 = (c,h), D^1 = (h,c)$, 7% of c-CP compliers) and the n-CP compliers ($D^0 = (n,h), D^1 = (h,c)$, 10% of the n-CP compliers).

#### 4.2.2 Returns for Compliers with Specific Counterfactual Alternatives

We can define the average return for each type of compliers:

$$\Delta_{d^0,d^1} = E[Y_i^{d^1} - Y_i^{d^0} | D_i^0 = d^0, D_i^1 = d^1] \tag{21}$$

[47] Given the estimated model, we simulate 30 paths of potential choices and associated outcomes for each individual in the sample, with ($Z = 1$) and without ($Z = 0$) the experimental offer. Each simulation consists of random draws of the unobserved factor and utility shocks. For each simulation path, we first simulate all 2*4=8 shocks across four nodes, and then switch Z from 0 to 1 keeping all shocks fixed.

[48] Given assumption 1, these numbers can be non-parametrically identified by computing $P(K_{i1} = n|Z_i = 0) - P(K_{i1} = n|Z_i = 1)$, for the n-CP compliers and $P(K_{i1} = c|Z_i = 0) - P(K_{i1} = c|Z_i = 1)$, for the c-CP compliers, where $K_{i1}$ is individual $i$'s choice at $t = 1$. According to this calculation, n-CP compliers are 51.2% of the population and c-CP compliers are 19.3%. See also Table 1, which shows that the HS offer reduces the share of children in other center care from 25% to 5%, and reduces the share of children in home care from 61% to 10%. Similar numbers are reported in Kline and Walters (2016) who, exploiting a sample which also includes children who are randomized at age 4, estimate that the proportion of n-CP compliers (in their words, *n*-compliers) is 45% and the proportion of c-CP compliers (*c*-compliers) is 23%.

where $d^0 \in \{(c,h),(c,c),(c,n),(n,h),(n,c),(n,n)\}$ and $d^1 \in \{(h,h),(h,c),(h,n)\}$ and $Y_i^{d^z}$ denotes the potential outcome for individual $i$ choosing $d^z$, with $z \in \{0,1\}$. Therefore, $\Delta_{d^0,d^1}$ gives the average treatment effect on a particular type of complier: those who choose one specific treatment $d^0$ when the instrument is turned off and choose $d^1$ when the instrument is turned on.

Table 7 reports the estimated average return (column 3) and the average value of $\theta$ (column 2) for each type of compliers. In column (4) we also report, for comparison, the average return for each specific pair of program sequences in the whole population, not only focussing on individuals choosing that specific pair of sequences. For illustration, consider individuals who choose $D_i^0 = (n,n)$ without an offer and $D_i^1 = (h,h)$ with an offer. In Figure 3, we have shown that low-$\theta$ children gain more from enrolling in HS for two periods relative to using home care throughout. Consistent with positive selection, we find that this type of compliers tend to have below-average values of $\theta$ (their average value of $\theta$ is -0.04). Consequently, the average effect of two periods of HS for this type of compliers is higher than the average population-wide return to two periods in HS relative to two periods in home care (0.067 compared to 0.048). Another type of compliers displaying strong positive selection are those with $D_i^0 = (c,c)$ and $D_i^1 = (h,h)$. This group is dominated by high-$\theta$ children, who gain more by moving the program sequence from $(c,c)$ to $(h,h)$ relative to low-$\theta$ children (see Figure 3). Such positive selection leads to the average return among this type of compliers being higher than the population-wide return of enrolling in program sequence $(h,h)$ relative to $(c,c)$ (0.021 compared to 0.009). Overall, we find that most compliers moving to two periods of HS (involving $D_i^1 = (h,h)$) are positively selected.

The selection pattern is mixed for other types of compliers. We do not find evidence of positive selection for those compliers moving to $D_i^1 = (h,c)$ from their respective counterfactual choices. For instance, the average return for compliers with $D_i^0 = (n,h)$ and $D_i^1 = (h,c)$ is 0.017, which is smaller than the average return of enrolling in program sequence $(h,c)$ relative to $(n,h)$ for the population (at 0.032). This suggests negative selection. Similar patterns of negative selection can be found for compliers moving to $(h,c)$ from the counterfactual choices of $(n,c)$ and $(n,n)$, whereas for those moving from the counterfactual choice of $(c,h)$ and $(c,c)$, there is no clear pattern of selection.

We show that the overall LATE estimated by exploiting the experimental variation as an IV is a weighted average of the returns from the different potential choices for each type of compliers, where the weights are given by their proportions within the overall complier group (see Appendix Section C).

In the last row of Table 7, we report that the overall LATE is 0.033, with the average composition in the overall complier group not different from the population (the average $\theta$ is close to 0). However, this overall LATE masks significant heterogeneity in the returns across different types of compliers. As we have shown, this heterogeneity is driven by heterogeneous returns to different combinations of program sequences in the population and by individuals with different returns choosing different childcare sequences in the HSIS experiment.

## 5 A Dynamic Structural Model with Forward-looking Behavior

Our sequential threshold model abstracts from many details of agents' optimization behavior. In general, we cannot use the model to conduct counterfactual analyses that involve a more fundamental change to program features, especially those that are time dependent. For instance, one policy counterfactual would be to restrict the maximum participation duration in HS from two years to one year. Such a change in program feature cannot be analyzed using the sequential threshold model, and requires a more structural dynamic discrete choice model which takes a stance on what agents are maximizing and their information sets.

We describe below a dynamic structural model that involves a forward-looking agent. This model is set up to be closely comparable to the sequential threshold model in Section 3. We then provide a methodological comparison of both approaches and illustrate the empirical importance of accounting for forward-looking behavior.

The structural model shares the same decision tree as the sequential threshold model: individual $i$ starts in $t = 1$ with an initial state of random assignment $Z_i \in \{0, 1\}$; in each of $t = 1$ and $t = 2$ faces three program alternatives (HS ($h$), other center care ($c$), home care ($n$)), which results in four decision nodes ($\mathcal{J} = \{\mathsf{o}, \mathsf{h}, \mathsf{c}, \mathsf{n}\}$) as defined earlier. The structural model's key distinction is that it focuses on ex ante choice decisions via an explicit information set and a model of expectations, which allows the perceived continuation value of each alternative to be examined. We first describe the utility function, and then information revelation and the intertemporal optimization problem. Upon arrival at node

$j \in \mathcal{J}$, the *utility flow* from each alternative is:

$$u_{ij}^h = \tilde{\psi}_j^h + \tilde{f}_j(Z_i, \boldsymbol{X}_i, \theta_i) + \tilde{\lambda}_j^h \theta_i + \tilde{\nu}_{ij}^h \tag{22}$$

$$u_{ij}^c = \tilde{\psi}_j^c + \tilde{\lambda}_j^c \theta_i + \tilde{\nu}_{ij}^c \tag{23}$$

$$u_{ij}^n = 0 \tag{24}$$

where we use lowercase $u$ and accent $\tilde{}$ to emphasize that these constructs are different to those in the sequential threshold model. Beyond $t = 2$, the individual receives a terminal value as a function of the period-$T$ outcome $Y_{iT}^d = \alpha_T^d + \beta_T^d hsq_i + \gamma_T^d \theta_i + \varepsilon_{iT}^d$ based on the realized program sequence $d = (k_1, k_2)$ (where $k_1, k_2 \in \{h, c, n\}$). We use $\bar{Y}_{iT}^d$ to denote the outcome exclusive of the idiosyncratic error $\varepsilon_{iT}^d$. The baseline measurements are $M_i^M = \alpha^M + \gamma^M \theta_i + e_i^M$ where $M \in \{pp, wj\}$ and $\gamma^{pp} = 1$. The outcome and measurement equations, which reflect technology, are without accent $\tilde{}$ as they are interpreted in the same manner as in the sequential threshold model. The preference, outcome and measurement shocks have the same statistical properties as in the sequential threshold model.[49]

All observed covariates and the factor $\theta_i$ are known to the individual throughout, but the shocks are revealed to her sequentially.[50] Upon arrival of node $j$, the individual chooses an alternative based on the realization of preference shocks at node $j$, comparing the current utility flow and expected future value embedded in each alternative. To express this intertemporal optimization problem in recursive form, consider an individual having chosen $K_{i1} = k_1 \in \{h, c, n\}$ at $t = 1$ and is at node $\mathsf{k}_1 \in \{\mathsf{h}, \mathsf{c}, \mathsf{n}\}$ (e.g., $\mathsf{k}_1 = \mathsf{h}$ if and only if $k_1 = h$) at $t = 2$:

$$V_{i2}(K_{i1} = k_1, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{k}_1}^h, \tilde{\nu}_{i\mathsf{k}_1}^c) := \max_{k_2 \in \{h,c,n\}} \left[ u_{i\mathsf{k}_1}^{k_2}(\boldsymbol{X}_i, \theta_i, \tilde{\nu}_{i\mathsf{k}_1}^h, \tilde{\nu}_{i\mathsf{k}_1}^c) + \kappa E_2 Y_{iT}^{k_1,k_2}(hsq_i, \theta_i) \right] \tag{25}$$

$$= \max_{k_2 \in \{h,c,n\}} \left[ u_{i\mathsf{k}_1}^{k_2}(\boldsymbol{X}_i, \theta_i, \tilde{\nu}_{i\mathsf{k}_1}^h, \tilde{\nu}_{i\mathsf{k}_1}^c) + \kappa \bar{Y}_{iT}^{k_1,k_2}(hsq_i, \theta_i) \right] \tag{26}$$

The value function $V_{i2}(\cdot)$ has state variables $K_{i1}$ $(= k_1)$, $\boldsymbol{X}_i$, $\theta_i$, $\tilde{\nu}_{i\mathsf{k}_1}^h$, $\tilde{\nu}_{i\mathsf{k}_1}^c$.[51] The terminal value scaling factor, $\kappa$, quantifies the extent to which the individual incorporates the expected future outcome in

[49] The preference shocks, $(\tilde{\nu}_{ij}^h,\ \tilde{\nu}_{ij}^c)$, follow a bivariate normal distribution with zero mean and covariance matrix $[1 \quad \tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}; \quad \tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c} \quad \tilde{\sigma}_{\nu^c}^2]$, and are independent across time and nodes. The outcome shocks $\varepsilon_{iT}^d \sim N(0, \sigma_\varepsilon^2)$ are independent across $d$. The measurement shocks $e_i^M \sim N(0, \sigma_e^2)$ are independent across $M$. Except the correlation between $\tilde{\nu}_{ij}^h$ and $\tilde{\nu}_{ij}^c$, all shocks are independent of one another.

[50] Note that the measurement shock is irrelevant to the decision problem as evident from the model above.

[51] For clarity, we write down the state variable $K_{i1}$, but it should be recognized that it plays the same role as in the sequential threshold model in that the choice at $t = 1$ changes the node at $t = 2$.

her information set and acts upon it (see discussion below).[52] Hence the decision problem at $t = 2$ is simply an intertemporal tradeoff between the current utility flow and the expected future outcome. At $t = 1$ (node $\mathsf{o}$), the value function is:

$$V_{i1}(Z_i, \boldsymbol{X}_i, \theta_i; \tilde{\nu}^h_{i\mathsf{o}}, \tilde{\nu}^c_{i\mathsf{o}}) := \max_{k_1 \in \{h,c,n\}} \left[ u^{k_1}_{i\mathsf{o}}(Z_i, \boldsymbol{X}_i, \theta_i, \tilde{\nu}^h_{i\mathsf{o}}, \tilde{\nu}^c_{i\mathsf{o}}) + \delta E_1 V_{i2}(k_1, \boldsymbol{X}_i, \theta_i; \tilde{\nu}^h_{i\mathsf{k}_1}, \tilde{\nu}^c_{i\mathsf{k}_1}) \right] \tag{27}$$

with state variables $Z_i$, $\boldsymbol{X}_i$, $\theta_i$, $\tilde{\nu}^h_{i\mathsf{o}}$, $\tilde{\nu}^c_{i\mathsf{o}}$; $\delta$ is the discount factor between $t = 1, 2$; $E_1(\cdot)$ is an expectation that integrates out the preference shocks at $t = 2$. Appendix Section D derives the semi-closed form solution to $E_1 V_{i2}(\cdot)$, which is a nonlinear function of parameters and incorporates uncertainty of preference shocks at t=2 and expected returns in outcomes.

**Identification of the Structural Model.** The structural model would be underidentified if there were no exclusion restrictions: the $EV$ relevant to each node and alternative will be subsumed into the intercepts ($\tilde{\psi}_j$), factor loadings ($\tilde{\lambda}_j$) and other parameters of the utility function, all of which are node-specific, i.e., only the ex post returns are identified.[53] We impose exclusion restrictions on $\tilde{f}_j(\cdot)$ in the utility function:

$$\tilde{f}_j(Z_i, \boldsymbol{X}_i, \theta_i) = \begin{cases} \tilde{\beta}^{trp}_j trp_i + \tilde{\beta}^{hsq}_j hsq_i (1 - Z_i) + \left( \tilde{\beta}^Z_j + \tilde{\beta}^{Ztrp}_j trp_i + \tilde{\beta}^{Z\theta}_j \theta_i \right) Z_i & \text{for} \quad j = \mathsf{o} \\ \tilde{\beta}^{trp}_j trp_i & \text{for} \quad j \in \{\mathsf{h}, \mathsf{c}, \mathsf{n}\} \end{cases} \tag{28}$$

where the difference to $f_j(\cdot)$ in the sequential threshold model is that, HS center quality ($hsq_i$) is excluded from the period-1 utility of the treatment group and period-2 utility for everyone. In short, we assume that HS quality drives current choices via expectations of better future outcomes. Nevertheless, we remain agnostic about whether HS quality affects the period-1 utility of the control group. As discussed in Section 3, period-1 HS enrolment in the control group may be largely determined by implicit barrier costs and excluding them from the period-1 utility may be a strenuous assumption.

[52] Note that, without loss of generality, the intercept of the terminal value function is normalized to zero. The intercept affects the terminal values of all states by an equal amount so it does not affect program choices. See also Keane and Wolpin (2001), Fang and Silverman (2009) and Chan and Liu (2018) who adopt a similar specification in discrete choice dynamic programming models.

[53] An alternative approach is to restrict the form of state dependence and unobserved heterogeneity by reducing the number of intercept and loading parameters in the utility function. This approach is popular in conventional structural models for theoretical or computational considerations, and not necessarily (or explicitly) for the purpose of identification. The intuition for identification is also less transparent when we impose these restrictions. Therefore, we keep the utility function as closely comparable to the sequential threshold model as possible, and rely solely on the continuous exclusion variable (HS quality) to identify $\delta$ and $\kappa$.

The exclusion of HS quality ($hsq_i$) from the utility function identifies the terminal value scaling factor $\kappa$ and discount factor $\delta$, which determine the perceived continuation values in ex ante choice decisions.[54] The exclusion of $hsq_i$ from the period-2 utility at nodes $\mathsf{h}, \mathsf{c}, \mathsf{n}$ identifies $\kappa$ (equation (26)). A test of $\kappa = 0$ versus $\kappa \neq 0$ informs whether the individual incorporates the expected future outcomes as part of her information set and acts upon it (Abbring and Heckman (2007)). In summary, $\kappa$ quantifies how sensitive the choices are to expected test scores.

Similarly, the exclusion of $hsq_i$ from the treatment group's period-1 utility identifies $\delta$ (equation (27)). Because the utility and $EV$ have the same scale in equation (27), $\delta$ has a conventional range between 0 and 1.[55] Empirically, we find that $\delta$ is harder to identify than $\kappa$. This may be due to the high HS takeup rate in the treatment group at $t = 1$ (85.1%). In the baseline structural model, we set $\delta = 1$. In a sensitivity analysis, we estimate $\delta$ jointly with other parameters in the structural model. The point estimate of $\delta$ is 0.94 (se=0.685), which provides reassurance that the exclusion restriction contains reasonable identifying information.

**Relationship with the Sequential Threshold Model.** Our main goal is to examine the connections between a flexible sequential threshold model and a structural model with forward-looking agents, by juxtaposing both approaches and using a variety of (nested) specifications. The sequential threshold model serves as an empirical benchmark, as in Abbring and Heckman (2007) and Heckman, Humphries, and Veramendi (2018). The structural model has the advantage of examining ex ante choice decisions by distinguishing contemporaneous payoffs from perceived continuation values, identifying forward-looking behavior, and comparing counterfactuals that involve perceived continuation values.

We acknowledge that this structural model is a simplified version of many richer models that incorporate other important behavioral mechanisms. We view this as a middle ground where certain key behavioral features such as perceived continuation values are explicitly modeled, but others are kept implicit and subsumed into the preference parameters. For example, consider the tradeoff between child outcome and the parent's labor supply.[56] A high-education mother (whose child may have higher $\theta_i$)

[54]More ideally, one would find experimental variation in a policy that affects expected future outcomes but not current utility; see, e.g., Chan (2017) for an example from welfare reform.

[55]Specifically, the utility scale at $t = 1, 2$, which reflects choices, are normalized with respect to the $h$ alternative ($var(\tilde{\nu}_{ij}^h) = 1$).

[56]See, for example, Griffen (2019) who estimates a full structural model of Head Start participation that captures

may be aware that putting her child in center care may worsen the child's outcome relative to taking care of the child on her own. However, she may still put her child in center care after weighing the benefits of working (e.g., better market opportunities or higher work preference) against the deterioration of child outcome. This tradeoff is subsumed into the factor loadings in the choice and outcome equations of our models.

### 5.1 Structural Model Estimation Results

Table 8 reports the estimates of the utility function and terminal value scaling factor $\kappa$ in the structural model.[57] As discussed in Section 3, the technological parameters in the baseline measurement equations, outcome equations and the factor distribution can be identified from the sequential threshold model. To facilitate comparison of the results in the choice equations, when we estimate the structural model we fix all technological coefficients to be the same as in the sequential threshold model (see Table 3).[58] Details of the estimation procedure are in Appendix Section D.

In the $t = 1$ (node $\mathsf{o}$) equation, the alternative-specific intercepts for $h$ and $c$ are considerably more negative (-1.601 and -2.232, respectively) than those in the sequential threshold model, which suggests that average control-group individuals: (1) prefer home care ($n$) if they consider period-1 utility only, and (2) have higher perceived continuation values for $h$ and $c$ than $n$. (Note that the muted estimates in the sequential threshold model is a result of counterbalance between current and future utility.) The factor loading ($\tilde{\lambda}$) for $c$ is considerably more positive (2.777) than that in the sequential threshold model, which suggests that high-$\theta$ control-group individuals: (1) prefer other center care ($c$) if they consider period-1 utility only, and (2) have lower perceived continuation values for $c$ than the other options. Indeed, as shown in Appendix Figure A3, we find that the flow utility of $c$ (relative to that of $n$) increases with $\theta$ whereas its continuation value decreases with $\theta$, with the total perceived value increasing in $\theta$. Similarly, for individuals in the treatment group, Appendix Figure A3 also shows that

this tradeoff explicitly. Like many other structural models, to maintain tractability the extra complexity comes at the expense of modeling certain parts of the model (outcome equation) in a highly stylized manner. Although our approach is less complex, it recognizes the sequential nature of participation in different programs and it models outcome equations flexibly.

[57]The structural model is less flexible than the sequential threshold model in the choice equations. The exclusion of $hsq_i$ results in four fewer parameters in the choice equations (one in each node $j \in \{\mathsf{o}, \mathsf{h}, \mathsf{c}, \mathsf{n}\}$).

[58]Nevertheless, we also estimate a "full" structural model including all 30 technological coefficients, which results in a total of 57 parameters to be estimated jointly. The estimated technological coefficients are very similar to those in the sequential threshold model. The terminal scaling factor $\kappa$ becomes less precisely estimated but remains positive and statistically significant. See Appendix Section B.5 for details.

the flow utility from $h$ increases with $\theta$ but the continuation value declines with $\theta$, leading to the total perceived value at $t = 1$ relatively flat with respect to $\theta$. This provides a structural interpretation to the relatively small factor loading in the choice equation for $h$ in the sequential choice model.[59]

In the $t = 2$ (nodes $\mathsf{h}, \mathsf{c}, \mathsf{n}$) equations, the estimates are similar to those in the sequential threshold model, except for the factor loadings. According to the alternative-specific intercepts of the flow utility in each period-2 node, average individuals in period 2 prefer $h$ if they chose $h$ in period 1, prefer $c$ if they chose $c$ in period 1, and prefer $h$ if they chose $n$ in period 1. By contrast, the factor loadings, which are more positive for $c$ than the other alternatives in all period-2 nodes, indicate that high-$\theta$ individuals in period 2 tend to prefer $c$ regardless of their choice in period 1.[60] Note also that the period-2 utilities form part of the perceived continuation values in period 1, which drives period-1 choice. For example, for high-$\theta$ individuals, choosing $h$ at $t = 1$ may be attractive because it opens up the option to choose their strongly preferred $c$ in period 2 (accounting for uncertainty in preference shocks).

The terminal value scaling factor $\kappa$ has a point estimate of 3.527 and is significant at the 1-percent level. This suggests that individuals incorporate the expected future outcomes as part of the information set and act upon it. Specifically, they are more likely to choose an option that yields better expected outcomes, keeping preferences fixed.[61]

### 5.2 Counterfactual policy experiment

One important policy debate about Head Start is the desirability of two-year enrolment versus one-year enrolment. While this depends on the specific criteria being evaluated, in Section 4 we presented some results based on the technological returns of various program sequences. In this section, we use the structural model to perform a hypothetical policy experiment, which restricts all treatment-group individuals to *at most* one period of HS only, while still allowing them to decide when to enrol in HS. We examine the effects of this policy on program choice in periods 1 and 2, the associated outcomes,

[59]Appendix Figure A3 can be easily modified to show the flow utility and continuation values of $h$ for individuals in the control group. The experimental offer $Z$ only affects the flow utility of HS at $t = 1$ (and does not affect the perceived continuation value).

[60]That is, individuals with different $\theta$ are subject to different degrees of "state dependence" in preferences. Hence perturbing initial childcare choice can have different consequences for subsequent choice among different types of individuals.

[61]To interpret its magnitude, consider an example of an individual in period 2 who chose home care in period 1. The expected outcome is $2.983 + 1.169\theta$ if she chooses other center care ("nc" in Table 4), versus $2.955 + 1.511\theta$ if she chooses home care ("nn"). This difference in expected outcomes is equivalent to a difference in period-2 utility of $3.527 \times [(2.983 - 2.955) + (1.169 - 1.511)\theta] = 0.099 - 1.206\theta$ between $c$ and $n$, implying that high-$\theta$ individuals are incentivized to choose $n$ over $c$ due to higher expected outcome of $(n, n)$ relative to $(n, c)$.

and utility.

Table 9 reports the results, which are based on 30 simulated paths per treatment-group individual under the baseline and counterfactual scenarios and keeping the realization of shocks identical across scenarios. Relative to the no-restriction baseline scenario (Column 1), the policy (Column 2) reduces period-1 HS enrolment by 32ppt (down from 85.2%), the majority of which switch to home care (+23.2ppt) and the rest to other center care (+8.8ppt). This suggests that many individuals move away from HS in period-1 preemptively, even though they are not impacted by the restriction directly. In period 2, the policy reduces HS enrolment by 48.7ppt (down from 68.5%), half of which switch to home care (+24.0ppt) and the other half to other center care (+24.7ppt). The effect is larger in period 2 because some individuals have become ineligible for HS.

Overall, in terms of program sequence, the elimination of the $(h, h)$ path (from 62.2% to 0%) results in the largest shift to $(h, n)$ (+17.8ppt) followed by $(h, c)$ (+12.5ppt). The shifts to $(n, h)$ (+10.5ppt) and $(c, h)$ (+3.0ppt) are considerable and reflect the tendency to "bank up" HS eligibility for possible enrolment in period 2 (note: the period-2 utility shocks are not yet realized in period 1). There are also shifts to $(n, c)$ (+7.0ppt), $(n, n)$ (+5.7ppt) and $(c, c)$ (+5.2ppt), which involve no HS enrolment at all. This is driven by forward-looking behavior under uncertainty – while the individual "banks up" HS eligibility in period 1, it ends up not being used due to particular realizations of period-2 utility shocks.[62]

The policy reduces the average outcome by 0.5 points (from 3.005 to 3.000, Columns 4 and 5), but increases the standard deviation of outcome by 1.9 points (from 0.379 to 0.398). The outcomes by program sequence reveal the complexity of selection and program effects. While the policy moves individuals from the $(h, h)$ path, which have lower average outcomes (see also Table 4), to other program sequences, some paths such as $(h, c)$ experience minimal changes in average outcomes, while others such as $(h, n)$ experience large reductions in average outcomes.

To gauge the importance of forward-looking behavior, we also consider the case where individuals

[62] In Appendix Table A2, we show that while the policy reduces the sum of present discounted ex-post utility (-0.56), it increases the period-1 utility slightly (+0.05) because the $(h, h)$ path, which yields more utility in period 2 than 1 due to state dependence in the preference for $h$ (see Table 8), is no longer feasible. In present-discounted terms, the policy reduces period-2 utility the most (-0.59) and reduces the terminal value marginally (-0.02)(includes scaling factor $\kappa$). We also show that, when individuals can only reoptimize at $t = 2$ (see discussions below), the sum of present discounted ex-post utility reduces by a larger amount (-0.67) due to the inability to smooth out the impact of the policy across $t = 1, 2$.

are assumed to *not* know about the enrolment limit policy at $t = 1$, and can only reoptimize at $t = 2$ when knowledge of the policy becomes part of the information set (Column 3). There is no pre-emptive response at $t = 1$: individuals switch program choice at $t = 2$ only, and the elimination of the $(h, h)$ path results in shifts to $(h, c)$ (+30.6ppt) and $(h, n)$ (+31.6ppt) exclusively. Relative to the full-response scenario, the policy yields less adverse outcomes in that the average outcome increases by 1.0 point (from 3.005 to 3.015, Column 6) and the standard deviation of outcome increases by 1.3 points (from 0.379 to 0.392). Note that this positive effect must stem from improvements in the outcomes of individuals who shifted from $(h, h)$ to $(h, c)$ and/or $(h, n)$; if there was only a compositional change, the effect on average outcomes would have been zero.

Figure 4 reports the distributional effect of the HS enrolment limit policy on outcomes. Individuals with low factor $\theta$, especially those at the bottom 20 percentiles, perform substantially worse (-5.5 points among the bottom 5 percentiles of $\theta$) while those with high $\theta$ perform better (+3.1 points among the top 5 percentiles of $\theta$). In Appendix Figure A4, we report the distributional effect separately for individuals whose HS center quality $hsq_i$ (measured at random assignment) is at the highest quartile and lowest quartile, respectively. The policy tends to worsen outcomes of individuals with high HS quality (up to -7.3 points) and improve outcomes of individuals with low HS quality (up to +4.2 points). This highlights the importance of HS quality. As a comparison, we also report the case where individuals can re-optimize at $t = 2$ only. The distributional effect is less extreme than the full-response scenario; in particular, the effect is much less adverse among low-$\theta$ individuals. This is because, in the full-response scenario, the pre-emptive switch from HS to other forms of care at $t = 1$ tends to yield disproportionately worse outcomes among low-$\theta$ individuals (see also heterogeneity in technological returns in Figure 3).

To further examine the source of increased inequality in outcomes, we report the effects on program choice separately for individuals whose $\theta$ is at the highest and lowest 10 percentiles, respectively (details are in Appendix Table A3). The pre-emptive response is much stronger among low-$\theta$ individuals; in period 1 the policy reduces HS enrolment by 41.9ppt (down from 86.4%), the majority of which switch to home care (+33.4ppt). Overall, low-$\theta$ individuals tend to switch from the $(h, h)$ path to $(n, *)$ paths, all of which make their outcomes worse. High-$\theta$ individuals exhibit weaker pre-emptive responses and they tend to switch from the $(h, h)$ path to $(h, c)$ or $(h, n)$ paths, the latter of which makes their outcomes better. In addition, some high-$\theta$ individuals switch from the $(h, h)$ path to $(n, *)$ paths, all of which make their outcomes better. These results resonate with our discussion in Section 4 that policy makers should engage low-$\theta$ individuals with multiple periods of center-based care and high-$\theta$ individuals with at least one period of home care.

# 6 Conclusion

This paper provides a comprehensive evaluation of the Head Start program, the largest preschool education program for disadvantaged children in the United States. Using the HSIS experimental data, we analyzed the causal effects of sequential childcare program participation when individuals face multiple childcare options in multiple periods. Our empirically-motivated threshold model accounts for selection into different modes of childcare use over time, for both the experimental treatment group and control group. Combining the sequential threshold model with outcomes, we estimated the model by exploiting the randomization of HS offer as a key source of identification and imposing a factor structure on the unobservables in characterizing the joint distribution of choices and outcomes.

We computed easily interpretable program returns from the estimated model. Relative to the estimated returns to HS that are available in the literature, our returns are defined on age-specific HS participation and use a specific alternative sequence of childcare use as benchmark comparison. For instance, our estimates of partial returns of HS indicate that, relative to using home care at both age 3 and age 4, one year of HS at either age 3 or age 4 (while fixing the rest of the program sequences to home care) raises the average test score at the end of the pre-school period by 0.15 sd and 0.10 sd, respectively. Two years of HS participation also improves children's average cognitive outcomes relative to home care at both ages, but the improvement is not as high as the combined partial returns to HS at age 3 and age 4. This is because a second year of HS participation lowers the return to HS in the first year, suggesting that returns to a single program spanning multiple periods are not necessarily linearly additive and separable across periods. In addition, we showed that there are interesting but nontrivial interactions between multiple programs in the context of sequential participation. For instance, compared with two years of HS, we found that children can achieve an even better average outcome if they are assigned to HS at age 3 followed by other center care at age 4. However, this does not necessarily imply that a combination of HS and other center care always yields better outcomes. Reversing the sequence –

other center care at age 3 followed by HS at age 4 – instead generates a very low average return.

We also demonstrated that the baseline ability of children matters significantly for program returns and policy. The partial return of HS at age 3, unlike other partial returns, is consistently positive for children with low and high baseline ability. The within- and between-program cross returns are positive or close to zero for children with low baseline ability, but are negative for children with high baseline ability. In order to improve overall cognitive outcomes, our estimates of program returns suggest that policy makers should engage low-ability individuals with two years of center-based care and high-ability individuals with at least one year of home care. One central question then is how to redesign HS to induce more individuals of heterogenous ability to choose their "socially desirable" program sequence. We analyzed a counterfactual policy reform which had not been considered before but deemed as an important unanswered question in the HSIS evaluation report: limiting the maximum duration of participation in HS to one year only (but keeping open the option to enrol at age 3 or 4). Using a dynamic structural model that involves a forward-looking agent who knows the imposed time limit from the beginning, we examined the effects of the policy on childcare choices and the associated outcomes. The policy reduces the average outcome by 0.5 points (-0.013 sd) but increases the standard deviation of outcome by 1.9 points (+0.05 sd), which is partly driven by pre-emptive responses at age 3 to "bank up" HS eligibility for possible enrolment at age 4. By way of comparison, had the policy not been known to individuals until age 4, which rules out pre-emptive responses, the policy yields less adverse outcomes in that the average outcome increases by 1.0 point (+0.026 sd). These results highlight the importance of the specification of information set and modeling forward-looking behavior, which is a strength of the dynamic structural model over the sequential threshold model considered in the first part of the paper.

Our modeling framework can be applied to other contexts, especially in the analysis of RCTs that rely on "encouragement design" (e.g., Duflo, 2001), where the randomization is based on providing access to or offer of a particular program. Similar to HS, in this type of experiments the duration of program participation may be endogenous, and participation in one program may affect decisions to participate in other programs in subsequent periods. Therefore, the experimental-oriented parameters may be hard to interpret and become less relevant for policy in the presence of multiple outside options and dynamic selection. In on-going work, Dalla-Zuanna and Liu (2019) apply the sequential choice framework developed in this paper to analyze the cross returns between different training programs.

# References

Abbring, J. H., and J. J. Heckman (2007): "Econometric evaluation of social programs, part III: Distributional treatment effects, dynamic treatment effects, dynamic discrete choice, and general equilibrium policy evaluation," *Handbook of econometrics*, 6, 5145–5303.

Abbring, J. H., and G. J. Van den Berg (2003): "The nonparametric identification of treatment effects in duration models," *Econometrica*, 71(5), 1491–1517.

Aizer, A., and F. Cunha (2012): "The production of human capital: Endowments, investments and fertility," Discussion paper, National Bureau of Economic Research.

Almond, D., and B. Mazumder (2013): "Fetal origins and parental responses," *Annual Review of Economics*, 5(1), 37–56.

Barnett, S., J. Hustedt, K. Robin, and K. Schulman (2004): "The State of Preschool 2004 Yearbook," *New Brunswick, NJ: National Institute for Early Education Research (NIEER).*

Berndt, E. K., B. H. Hall, R. E. Hall, and J. Hausman (1974): "Estimation and inference in nonlinear structural models," *Annals of Economic and Social Measurement*, 3(4), 653–665.

Bhalotra, S. R., and A. Venkataramani (2015): "Shadows of the captain of the men of death: Early life health interventions, human capital investments, and institutions," *Human Capital Investments, and Institutions (August 8, 2015).*

Bitler, M. P., H. W. Hoynes, and T. Domina (2014): "Experimental evidence on distributional effects of Head Start," Discussion paper, National Bureau of Economic Research.

Bonhomme, S., and J.-M. Robin (2009): "Consistent noisy independent component analysis," *Journal of Econometrics*, 149(1), 12–25.

Carneiro, P., and R. Ginja (2014): "Long-term impacts of compensatory preschool on health and behavior: Evidence from Head Start," *American Economic Journal: Economic Policy*, 6(4), 135–73.

Chan, M. K. (2017): "Welfare dependence and self-control: An empirical analysis," *The Review of Economic Studies*, 84(4), 1379–1423.

Chan, M. K., and K. Liu (2018): "Life-cycle and intergenerational effects of child care reforms," *Quantitative Economics*, 9(2), 659–706.

Cunha, F., and J. Heckman (2007): "The technology of skill formation," *American Economic Review*, 97(2), 31–47.

Dalla-Zuanna, A., and K. Liu (2019): "Understanding Program Complementarities: Estimating the Dynamic Effects of a Training Program with Multiple Alternatives," *Available at SSRN 3510450.*

De Haan, M., and E. Leuven (2020): "Head start and the distribution of long-term education and labor market outcomes," *Journal of Labor Economics*, 38(3), 727–765.

Deming, D. (2009): "Early childhood intervention and life-cycle skill development: Evidence from Head Start," *American Economic Journal: Applied Economics*, 1(3), 111–34.

Duflo, E. (2001): "Schooling and Labor Market Consequences of School Construction in Indonesia: Evidence from an Unusual Policy Experiment," *American Economic Review*, 91, 795–813.

Eberwein, C., J. C. Ham, and R. J. LaLonde (1997): "The impact of being offered and receiving classroom training on the employment histories of disadvantaged women: Evidence from experimental data," *The Review of Economic Studies*, 64(4), 655–682.

Fang, H., and D. Silverman (2009): "Time-Inconsistency and Welfare Program Participation: Evidence from the NLSY," *International Economic Review*, 50, 1043–1077.

Fernald, A., V. A. Marchman, and A. Weisleder (2013): "SES differences in language processing skill and vocabulary are evident at 18 months," *Developmental science*, 16(2), 234–248.

Fruehwirth, J. C., S. Navarro, and Y. Takahashi (2016): "How the timing of grade retention affects outcomes: Identification and estimation of time-varying treatment effects," *Journal of Labor Economics*, 34(4), 979–1021.

Goff, L., O. Malamud, C. Pop-Eleches, and M. Urquiola (2022): "Interactions between Family and School Environments: Access to Abortion and Selective Schools," *mimeo.*

Gough, P. B., and W. E. Tunmer (1986): "Decoding, reading, and reading disability," *Remedial and special education*, 7(1), 6–10.

Griffen, A. S. (2019): "Evaluating the effects of childcare policies on children's cognitive development and maternal labor supply," *Journal of Human Resources*, 54(3), 604–655.

Griffen, A. S., and P. E. Todd (2017): "Assessing the performance of nonexperimental estimators for evaluating Head Start," *Journal of Labor Economics*, 35(S1), S7–S63.

Ham, J. C., and R. J. LaLonde (1996): "The effect of sample selection and initial conditions in duration models: Evidence from experimental data on training," *Econometrica*, 64, 175–206.

Hart, B., and T. R. Risley (1995): *Meaningful differences in the everyday experience of young American children.* Paul H Brookes Publishing.

Heckman, J., N. Hohmann, J. Smith, and M. Khoo (2000): "Substitution and dropout bias in social experiments: A study of an influential social experiment," *The Quarterly Journal of Economics*, 115(2), 651–694.

Heckman, J. J., J. E. Humphries, and G. Veramendi (2016): "Dynamic treatment effects," *Journal of econometrics*, 191(2), 276–292.

——— (2018): "Returns to education: The causal effects of education on earnings, health, and smoking," *Journal of Political Economy*, 126(S1), S197–S246.

Heckman, J. J., and S. Navarro (2007): "Dynamic discrete choice and dynamic treatment effects," *Journal of Econometrics*, 136(2), 341–396.

Heckman, J. J., and J. A. Smith (1998): "Evaluating the Welfare State," *Econometrics and Economic Theory in the Twentieth Century: The Ragnar Frisch Centennial Symposium, edited by S. Strom*.

Heckman, J. J., and S. Urzua (2010): "Comparing IV with structural models: What simple IV can and cannot identify," *Journal of Econometrics*, 156(1), 27–37.

Heckman, J. J., S. Urzua, and E. Vytlacil (2006): "Understanding instrumental variables in models with essential heterogeneity," *The Review of Economics and Statistics*, 88(3), 389–432.

Heckman, J. J., and E. Vytlacil (2001): "Policy-relevant treatment effects," *American Economic Review*, 91(2), 107–111.

Hendren, N. (2016): "The policy elasticity," *Tax Policy and the Economy*, 30(1), 51–89.

Imbens, G. W., and J. D. Angrist (1994): "Identification and Estimation of Local Average Treatment Effects," *Econometrica*, 62(2), 467–75.

Johnson, R. C., and C. K. Jackson (2019): "Reducing inequality through dynamic complementarity: Evidence from Head Start and public school spending," *American Economic Journal: Economic Policy*, 11(4), 310–49.

Keane, M. (1994): "A computationally practical simulation estimator for panel data," *Econometrica: Journal of the Econometric Society*, pp. 95–116.

Keane, M. P., and K. I. Wolpin (2001): "The Effect of Parental Transfers and Borrowing Constraints on Educational Attainment," *International Economic Review*, 42(4), 1051–1103.

Kirkeboen, L. J., E. Leuven, and M. Mogstad (2016): "Field of study, earnings, and self-selection," *The Quarterly Journal of Economics*, 131(3), 1057–1111.

Kline, P., and C. R. Walters (2016): "Evaluating public programs with close substitutes: The case of Head Start," *The Quarterly Journal of Economics*, 131(4), 1795–1848.

Kotlarski, I. (1967): "On characterizing the gamma and the normal distribution," *Pacific Journal of Mathematics*, 20(1), 69–76.

Lewbel, A. (2000): "Semiparametric qualitative response model estimation with unknown heteroscedasticity or instrumental variables," *Journal of econometrics*, 97(1), 145–177.

LUDWIG, J., AND D. L. MILLER (2007): "Does Head Start improve children's life chances? Evidence from a regression discontinuity design," *The Quarterly journal of economics*, 122(1), 159–208.

MATZKIN, R. L. (1993): "Nonparametric identification and estimation of polychotomous choice models," *Journal of Econometrics*, 58(1-2), 137–168.

PUMA, M., ET AL. (2010): "Head Start Impact Study. Final Report.," *Administration for Children & Families, U.S. Department of Health and Human Services.*

REIERSØL, O. (1950): "Identifiability of a linear relation between variables which are subject to error," *Econometrica: Journal of the Econometric Society*, pp. 375–389.

RODRÍGUEZ, J., F. SALTIEL, AND S. S. URZÚA (2018): "Dynamic Treatment Effects of Job Training," Discussion paper, National Bureau of Economic Research.

ROSENBAUM, S. (1961): "Moments of a truncated bivariate normal distribution," *Journal of the Royal Statistical Society: Series B (Methodological)*, 23(2), 405–408.

ROSSIN-SLATER, M., AND M. WÜST (2020): "What is the added value of preschool for poor children? Long-term and intergenerational impacts and interactions with an infant health intervention," *American Economic Journal: Applied Economics*, 12(3), 255–86.

TALLIS, G. M. (1961): "The moment generating function of the truncated multi-normal distribution," *Journal of the Royal Statistical Society: Series B (Methodological)*, 23(1), 223–229.

WALTERS, C. R. (2015): "Inputs in the production of early childhood human capital: Evidence from Head Start," *American Economic Journal: Applied Economics*, 7(4), 76–102.

WENDLING, B. J., F. A. SCHRANK, AND A. J. SCHMITT (2007): "Educational interventions related to the Woodcock-Johnson III tests of achievement (Assessment Service Bulletin No. 8)," *Rolling Meadows, IL: Riverside Publishing.*

Table 1: Proportion of Children in Different Childcare Programs

| Program Sequence | | Proportion (%) | | | |
|---|---|---|---|---|---|
| $t=1$ | $t=2$ | $Z=0$ | | $Z=1$ | |
| Head Start ($h$) | Head Start ($h$) | 10.7 | *74.3* | 62.0 | *72.9* |
| | Other Center Care ($c$) | 2.7 | *18.8* | 18.1 | *21.3* |
| | Home Care ($n$) | 1.0 | *6.9* | 5.0 | *5.9* |
| Total ($h$) | | 14.4% | | 85.1% | |
| Other Center Care ($c$) | Head Start ($h$) | 9.3 | *37.6* | 1.2 | *21.9* |
| | Other Center Care ($c$) | 13.9 | *56.1* | 3.7 | *68.8* |
| | Home Care ($n$) | 1.6 | *6.4* | 0.5 | *9.4* |
| Total ($c$) | | 24.8% | | 5.4% | |
| Home Care ($n$) | Head Start ($h$) | 30.2 | *49.8* | 3.0 | *31.2* |
| | Other Center Care ($c$) | 18.6 | *30.7* | 2.8 | *29.5* |
| | Home Care ($n$) | 11.9 | *19.6* | 3.7 | *39.3* |
| Total ($n$) | | 60.7% | | 9.5% | |

*Notes* : This table reports the percentages of children selecting different combinations of childcare programs, separately for the control ($Z=0$) and for the treatment group ($Z=1$). Numbers in italic show the proportions selecting the program at $t=2$ (age 4) *conditional* on the choice made at $t=1$ (age 3). "Total (•)" rows show the overall proportion making each choice at $t=1$ (i.e., the sum of the three rows above).

Table 2: Experimental Impact of Head Start: Reduced Form Estimates

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| $Z$ | 0.025 | 0.015 | 0.033** | -0.009 |
| | (0.018) | (0.040) | (0.015) | (0.032) |
| $Z \times C$ | | 0.066 | | 0.100*** |
| | | (0.043) | | (0.035) |
| $Z \times HS$ | | -0.010 | | 0.027 |
| | | (0.039) | | (0.032) |
| Controls | NO | NO | YES | YES |
| Constant | 2.983*** | 2.983*** | 2.878*** | 2.883*** |
| | (0.015) | (0.015) | (0.047) | (0.047) |
| Observations | 1,876 | 1,876 | 1,876 | 1,876 |
| R-squared | 0.001 | 0.005 | 0.359 | 0.363 |

*Notes* : $Z=1$ if the child is assigned to the treatment group, $C=1$ if the child is in other center care at $t=2$, $HS=1$ if the child is in HS at $t=2$. Each column is from a separate regression of the outcome variable (PPVT score at the end of age-4 year) on a dummy variable for having been randomly assigned to HS (columns (1) and (3)) and its interaction with dummy variables for the different childcare use at $t=2$ (columns (2) and (4)). Columns (3) and (4) include additional controls to the regression. We include controls for household size, number of siblings, dummies for whether the child is female, black, hispanic, use English as home language, living in urban area, living with both parents, in need of special education, child of a teen mother, child of a mother who never married or is separated, child of a mother with high school or more than high school. Standard errors are in parentheses. Significance level (t-test for the parameter being 0): *** 1%, ** 5%, * 10%

Table 3: Estimates of the Parameters of the Choice Equations from the Sequential Threshold Model

| Decision Node ($j$) | Covariate | Perceived Value | | | | | |
|---|---|---|---|---|---|---|---|
| | | Head Start ($U_{ij}^h$) | | | Other center care ($U_{ij}^c$) | | |
| o ($t=1$) | Intercept | -0.604 | (0.169) | *** | -1.131 | (0.267) | *** |
| | Factor $\theta$ | -0.011 | (0.285) | | 1.008 | (0.444) | ** |
| | Z | 1.914 | (0.171) | *** | | | |
| | $\theta \times$ Z | 0.114 | (0.358) | | | | |
| | hsq | -0.197 | (0.055) | *** | | | |
| | trp | -0.149 | (0.052) | *** | | | |
| | hsq $\times$ Z | 0.320 | (0.069) | *** | | | |
| | trp $\times$Z | 0.154 | (0.071) | ** | | | |
| h ($t=2$) | Intercept | 1.538 | (0.079) | *** | 0.337 | (0.166) | ** |
| | Factor $\theta$ | -0.376 | (0.338) | | 0.359 | (0.526) | |
| | hsq | 0.099 | (0.045) | ** | | | |
| | trp | 0.019 | (0.043) | | | | |
| c ($t=2$) | Intercept | 1.518 | (0.290) | *** | 1.878 | (0.322) | *** |
| | Factor $\theta$ | -2.071 | (1.075) | * | -1.081 | (1.156) | |
| | hsq | 0.118 | (0.097) | | | | |
| | trp | -0.216 | (0.113) | * | | | |
| n ($t=2$) | Intercept | 0.609 | (0.123) | *** | -0.065 | (0.146) | |
| | Factor $\theta$ | 0.192 | (0.329) | | -0.034 | (0.565) | |
| | hsq | 0.050 | (0.048) | | | | |
| | trp | 0.081 | (0.056) | | | | |
| Other utility parameters: | $\sigma_{\nu^c}$ | 2.037 | (0.401) | *** | | | |
| | $\rho_{hc}$ | 0.718 | (0.166) | *** | | | |

*Notes* : N=1876. Log-likelihood = -3827.97. Trp: HS transport availability. Hsq: HS center quality. The model uses PPVT score at the end of age-4 year as the outcome. Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): *** 1%, ** 5%, *10%.

Table 4: Estimates of the Parameters of the Potential Outcome and Measurement Equations from the Sequential Threshold Model

| | Intercept | | | Covariate Factor $\theta$ | | | HS quality | | |
|---|---|---|---|---|---|---|---|---|---|
| Potential outcome by program sequence: | | | | | | | | | |
| $h, h$ | 3.003 | (0.014) | *** | 1.134 | (0.094) | *** | 0.025 | (0.012) | ** |
| $h, c$ | 3.028 | (0.024) | *** | 0.981 | (0.113) | *** | -0.002 | (0.026) | |
| $h, n$ | 3.017 | (0.050) | *** | 1.457 | (0.188) | *** | -0.011 | (0.043) | |
| $c, h$ | 2.947 | (0.045) | *** | 1.032 | (0.191) | *** | 0.009 | (0.046) | |
| $c, c$ | 2.994 | (0.030) | *** | 0.951 | (0.127) | *** | | | |
| $c, n$ | 2.955 | (0.146) | *** | 1.380 | (0.376) | *** | | | |
| $n, h$ | 2.996 | (0.024) | *** | 1.323 | (0.120) | *** | 0.015 | (0.018) | |
| $n, c$ | 2.983 | (0.030) | *** | 1.169 | (0.123) | *** | | | |
| $n, n$ | 2.955 | (0.036) | *** | 1.511 | (0.139) | *** | | | |
| Baseline measurement: | | | | | | | | | |
| PPVT | 2.296 | (0.009) | *** | 1.000 | - | | | | |
| WJIII | 3.375 | (0.005) | *** | 0.080 | (0.024) | *** | | | |
| Standard deviations: | | | | | | | | | |
| Factor $\theta$ | 0.256 | (0.012) | *** | | | | | | |
| Baseline PPVT error | 0.266 | (0.010) | *** | | | | | | |
| Baseline WJIII error | 0.208 | (0.004) | *** | | | | | | |
| Outcome error | 0.237 | (0.013) | *** | | | | | | |

*Notes* : N=1876. Log-likelihood = -3827.97. The model uses PPVT at the end of age-4 year as the outcome. The parameters of the outcome equation differ depending on the program sequence (see equation 7). The program sequence is reported in the first column. $h$ is for HS, $c$ is for other center care, $n$ is for home care. The first letter refers to the program at $t = 1$, the second to the program at $t = 2$ (for instance, $h, h$ denotes HS both at $t = 1$ and at $t = 2$). The parameters for the baseline measurement differ depending on which measurement is taken into account (either PPVT or WJIII, see equation 8). Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): ***1%, ** 5%, *10%.

Table 5: Goodness of Fit of the Sequential Threshold Model

| | | Proportions | | | Outcome | | |
|---|---|---|---|---|---|---|---|
| | | Program $t = 2$ | | | | | |
| | | $h$ | $c$ | $n$ | $h$ | $c$ | $n$ |
| | Program $t = 1$ | Control Group | | | Control Group | | |
| Simulation | $h$ | 10.4 | 2.9 | 0.9 | 2.94 | 3.03 | 3.07 |
| Data | $h$ | *10.7* | *2.7* | *1.0* | *2.90* | *3.03* | *3.25* |
| Simulation | $c$ | 8.9 | 14.8 | 1.6 | 2.92 | 3.07 | 3.24 |
| Data | $c$ | *9.3* | *13.9* | *1.6* | *2.91* | *3.05* | *3.25* |
| Simulation | $n$ | 27.8 | 18.4 | 14.2 | 2.99 | 2.95 | 2.91 |
| Data | $n$ | *30.2* | *18.6* | *11.9* | *3.01* | *2.95* | *2.94* |
| | | Treatment Group | | | Treatment Group | | |
| Simulation | $h$ | 62.2 | 17.9 | 5.0 | 2.98 | 3.07 | 3.09 |
| Data | $h$ | *62.0* | *18.1* | *5.0* | *2.99* | *3.07* | *3.04* |
| Simulation | $c$ | 1.7 | 3.1 | 0.4 | 2.95 | 3.08 | 3.33 |
| Data | $c$ | *1.2* | *3.7* | *0.5* | *3.04* | *3.10* | *3.34* |
| Simulation | $n$ | 4.4 | 2.9 | 2.4 | 2.99 | 2.95 | 2.94 |
| Data | $n$ | *3.0* | *2.8* | *3.7* | *2.98* | *2.99* | *2.89* |

*Notes* : Numbers in italics are the population equivalents (computed directly from the data). Each cell shows the proportion or the average outcome (PPVT score at the end of age-4 year) for the group of children who at $t = 1$ enrol in the program reported in the second column and at $t = 2$ enrol in the program reported in the second row, separately for the treatment and for the control group. $h$ is HS, $c$ is other center care, $n$ is home care.

Table 6: Within- and Between-Program Cross Returns

| | Program at $t = 2$ | |
|---|---|---|
| Program at $t = 1$ | Head Start ($h$) | Other Center Care ($c$) |
| Head Start ($h$) | $(Y_i^{hh} - Y_i^{nh}) - (Y_i^{hn} - Y_i^{nn})$ | $(Y_i^{hc} - Y_i^{nc}) - (Y_i^{hn} - Y_i^{nn})$ |
| Other Center Care ($c$) | $(Y_i^{ch} - Y_i^{nh}) - (Y_i^{cn} - Y_i^{nn})$ | $(Y_i^{cc} - Y_i^{nc}) - (Y_i^{cn} - Y_i^{nn})$ |

Table 7: Proportion, Mean of the Unobserved Factor and Returns for each Compliance Group

| $D_i^0$ | $D_i^1$ | Compliance Type | CP-Type | Proportion (1) | $\theta$ (2) | Return (3) | Population Return (4) |
|---|---|---|---|---|---|---|---|
| (h,h) | (h,h) | AT | | 10.45 | -0.05 | 0 | 0 |
| (h,c) | (h,c) | AT | | 3.07 | 0.01 | 0 | 0 |
| (h,n) | (h,n) | AT | | 0.90 | 0.01 | 0 | 0 |
| (c,h) | (h,h) | c-CP | 1 | 4.97 | -0.04 | 0.056 | 0.056 |
| (c,h) | (h,c) | c-CP | 2 | 1.39 | 0.01 | 0.080 | 0.081 |
| (c,h) | (h,n) | c-CP | 2 | 0.42 | 0.02 | 0.075 | 0.070 |
| (c,h) | (c,h) | NT | | 1.77 | 0.00 | 0 | 0 |
| (c,c) | (h,h) | c-CP | 2 | 8.32 | 0.05 | 0.021 | 0.009 |
| (c,c) | (h,c) | c-CP | 1 | 2.48 | 0.10 | 0.037 | 0.034 |
| (c,c) | (h,n) | c-CP | 2 | 0.74 | 0.12 | 0.084 | 0.045 |
| (c,c) | (c,c) | NT | | 3.12 | 0.10 | 0 | 0 |
| (c,n) | (h,h) | c-CP | 2 | 0.93 | 0.19 | 0.005 | 0.048 |
| (c,n) | (h,c) | c-CP | 2 | 0.31 | 0.24 | -0.021 | 0.073 |
| (c,n) | (h,n) | c-CP | 1 | 0.07 | 0.25 | 0.079 | 0.062 |
| (c,n) | (c,n) | NT | | 0.38 | 0.26 | 0 | 0 |
| (n,h) | (h,h) | n-CP | 1 | 17.32 | -0.02 | 0.012 | 0.007 |
| (n,h) | (h,c) | n-CP | 2 | 4.94 | 0.04 | 0.017 | 0.032 |
| (n,h) | (h,n) | n-CP | 2 | 1.26 | 0.06 | 0.029 | 0.021 |
| (n,h) | (n,h) | NT | | 4.43 | 0.00 | 0 | 0 |
| (n,c) | (h,h) | n-CP | 2 | 11.42 | -0.03 | 0.025 | 0.020 |
| (n,c) | (h,c) | n-CP | 1 | 3.13 | 0.02 | 0.042 | 0.045 |
| (n,c) | (h,n) | n-CP | 2 | 0.95 | 0.02 | 0.041 | 0.034 |
| (n,c) | (n,c) | NT | | 2.94 | -0.02 | 0 | 0 |
| (n,n) | (h,h) | n-CP | 2 | 8.76 | -0.04 | 0.067 | 0.048 |
| (n,n) | (h,c) | n-CP | 2 | 2.55 | 0.02 | 0.062 | 0.073 |
| (n,n) | (h,n) | n-CP | 1 | 0.65 | -0.02 | 0.064 | 0.062 |
| (n,n) | (n,n) | NT | | 2.33 | -0.02 | 0 | 0 |
| Total | | AT | | 14.42 | -0.03 | 0 | |
| | | NT | | 14.98 | 0.02 | 0 | |
| | | c-CP | 1 | 7.52 | 0.01 | 0.050 | |
| | | c-CP | 2 | 12.11 | 0.06 | 0.031 | |
| | | n-CP | 1 | 21.10 | -0.01 | 0.018 | |
| | | n-CP | 2 | 29.87 | -0.01 | 0.040 | |
| | | *CP* | *overall* | *70.60* | *0.00* | *0.033* | |

*Notes* : $D_i^0$ and $D_i^1$ refer to the potential program sequence without and with the HS offer, respectively. Columns (1) and (2) report proportions and the average baseline ability ($\theta$) of each compliance group. Column (3) reports the average return (in terms of PPVT score at the end of age-4 year) to program sequence $D_i^1$ relative to $D_i^0$ within each type of complier. Column (4) reports the difference in average potential outcome between program sequence $D_i^1$ and $D_i^0$ in the population. See Section 4.2 for definitions of the compliance groups. The final row "*CP Overall*" refers to all the compliers from the HSIS experiment (who select $h$ at $t = 1$ if there is an offer and either $c$ or $n$ at $t = 1$ if there is no offer). Column (3) in this row is the average return for all the compliers and hence corresponds to the LATE, which can also be computed dividing the reduced-form estimate in Table 2 by the proportion of compliers.

Table 8: Estimates of the Parameters of the Utility Function from the Structural Model

| Decision Node ($j$) | Covariate | Flow Utility: Head Start ($u_{ij}^h$) | | | Other center care ($u_{ij}^c$) | | |
|---|---|---|---|---|---|---|---|
| o ($t=1$) | Intercept | -1.601 | (0.229) | *** | -2.232 | (0.511) | *** |
| | Factor $\theta$ | 0.457 | (0.455) | | 2.777 | (0.730) | *** |
| | Z | 1.988 | (0.160) | *** | | | |
| | $\theta \times$ Z | 0.193 | (0.362) | | | | |
| | trp | -0.165 | (0.064) | ** | | | |
| | trp $\times$ Z | 0.173 | (0.070) | ** | | | |
| | hsq $\times$ (1-Z) | -0.249 | (0.057) | *** | | | |
| h ($t=2$) | Intercept | 1.564 | (0.104) | *** | 0.316 | (0.156) | ** |
| | Factor $\theta$ | 0.703 | (0.451) | | 2.002 | (0.689) | *** |
| | trp | 0.019 | (0.043) | | | | |
| c ($t=2$) | Intercept | 1.390 | (0.312) | *** | 1.629 | (0.340) | *** |
| | Factor $\theta$ | -0.744 | (0.735) | | 0.427 | (0.911) | |
| | trp | -0.165 | (0.087) | * | | | |
| n ($t=2$) | Intercept | 0.401 | (0.161) | ** | -0.151 | (0.141) | |
| | Factor $\theta$ | 0.890 | (0.367) | ** | 1.115 | (0.614) | * |
| | trp | 0.072 | (0.056) | | | | |
| Other utility parameters: | | | | | | | |
| | $\tilde{\sigma}_{\nu^c}$ | 1.843 | (0.456) | *** | | | |
| | $\tilde{\rho}_{hc}$ | 0.615 | (0.190) | *** | | | |
| | $\kappa$ | 3.527 | (1.174) | *** | | | |

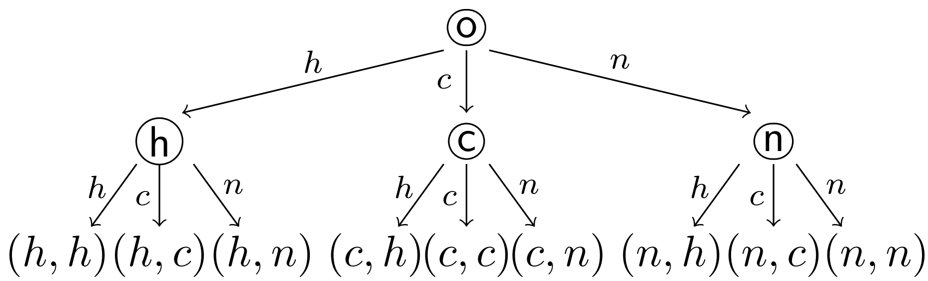
*Notes* : N=1876. Log-likelihood = -3834.25. trp: HS transport availability. hsq: HS center quality. The model uses PPVT score at the end of age-4 year as the outcome, as in the baseline sequential threshold model. It fixes the following estimates of parameters to that model: all parameters in the baseline measurement equations ($\alpha^M$ (2 parameters), $\gamma^M$ (1), $\sigma_e$ (2)); factor sd ($\sigma_\theta$ (1)); all parameters in the terminal value / outcome equation ($\alpha_T^M$ (9), $\beta_T^D$ (5), $\gamma_T^D$ (9), $\sigma_\varepsilon$ (1)). Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): *** 1%, ** 5%, *10%.

Table 9: Effects of Limiting Head Start Enrolment to One Period Only

| | Program choice (%) | | | Average outcome | | |
|---|---|---|---|---|---|---|
| | Baseline | Counterfactual | | Baseline | Counterfactual | |
| | | Full resp. | Partial resp. | | Full resp. | Partial resp. |
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Choices (t=1): | | | | | | |
| $h$ | 85.2 | -32.0 | 0.0 | | | |
| $c$ | 5.3 | +8.8 | 0.0 | | | |
| $n$ | 9.6 | +23.2 | 0.0 | | | |
| Choices (t=2): | | | | | | |
| $h$ | 68.5 | -48.7 | -62.2 | | | |
| $c$ | 24.0 | +24.7 | +30.6 | | | |
| $n$ | 7.5 | +24.0 | +31.6 | | | |
| Program sequence: | | | | | | |
| $h, h$ | 62.2 | -62.2 | -62.2 | 2.98 | - | - |
| $h, c$ | 18.0 | +12.5 | +30.6 | 3.07 | -0.01 | -0.03 |
| $h, n$ | 4.9 | +17.8 | +31.6 | 3.10 | -0.06 | -0.09 |
| | | | | | | |
| $c, h$ | 1.8 | +3.0 | 0.0 | 2.96 | -0.04 | 0.00 |
| $c, c$ | 3.1 | +5.2 | 0.0 | 3.07 | -0.03 | 0.00 |
| $c, n$ | 0.4 | +0.6 | 0.0 | 3.37 | -0.12 | 0.00 |
| | | | | | | |
| $n, h$ | 4.5 | +10.5 | 0.0 | 2.99 | -0.04 | 0.00 |
| $n, c$ | 2.9 | +7.0 | 0.0 | 2.94 | -0.02 | 0.00 |
| $n, n$ | 2.2 | +5.7 | 0.0 | 2.92 | -0.05 | 0.00 |
| | | | | | | |
| Overall mean | | | | 3.005 | 3.000 | 3.015 |
| Overall s.d. | | | | 0.379 | 0.398 | 0.392 |

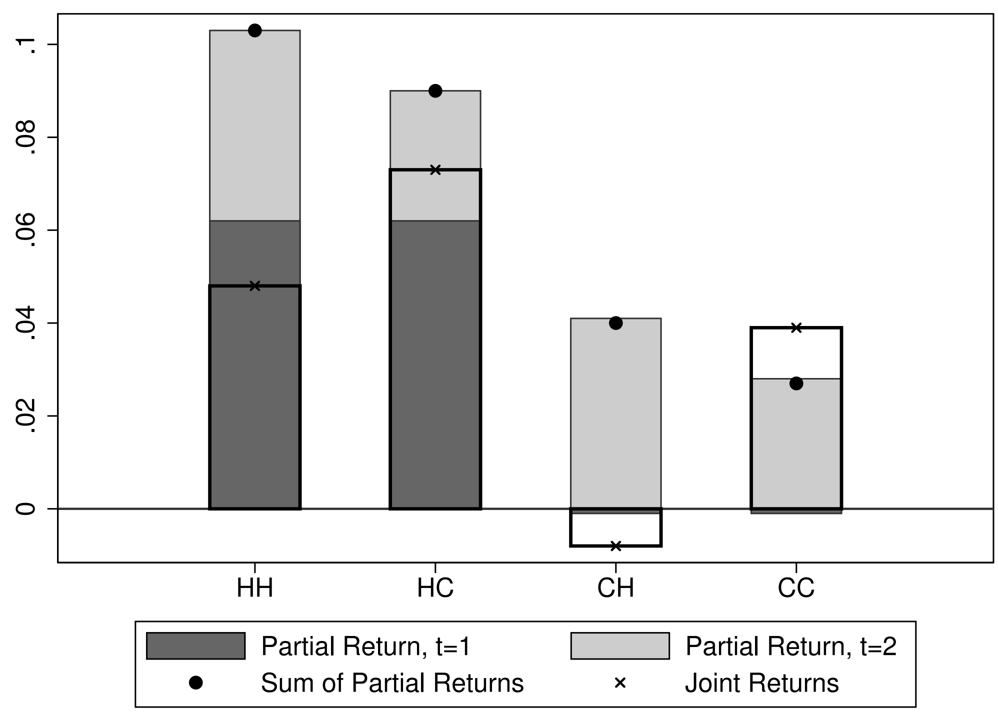
*Notes* : All scenarios are simulated from the structural model with 30 paths per treatment-group individual. The "Baseline" columns show the proportion of the population making each choice and their average outcome under the baseline scenario, i.e. when individuals are allowed to select two periods of HS. The "Counterfactual" columns report the difference in the proportion of population selecting each childcare option and in the outcomes between the baseline scenario and the counterfactuals: "Full resp." (full response) allows individuals to select at most one period in HS and assumes that they can reoptimize both at $t = 1$ and $t = 2$; "Partial resp." (partial response) assumes individuals at $t = 1$ do not know about the HS enrolment limit, so they can reoptimize at $t = 2$ only.

Figure 1: The Sequential Decision Framework with Multiple Alternatives



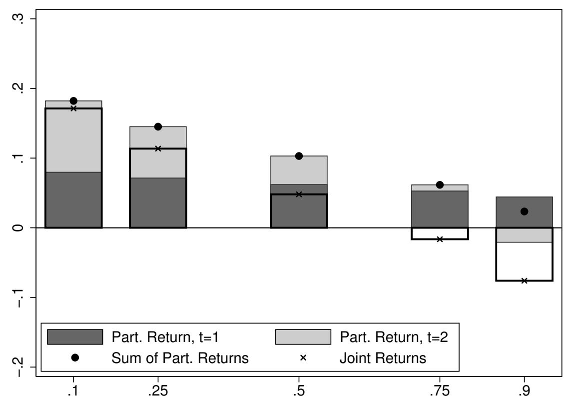

*Notes* : ⦿ refers to a decision node. $(j, j')$ refers to the program sequence where program $j \in (h, c, n)$ is obtained at period $t = 1$ and $j' \in (h, c, n)$ is obtained at period $t = 2$.

Figure 2: Average Partial, Joint and Cross Returns from Sequential Program Participation

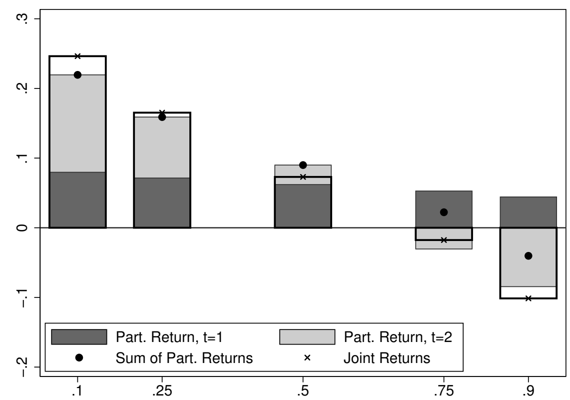



*Notes* : The outcome analyzed is the PPVT test score obtained at the end of age-4 year. Each bar represents the joint and partial return of different program sequences. The first bar (HH) shows the return of receiving HS both at $t = 1$ and at $t = 2$. The second bar (HC) shows the return of receiving HS at $t = 1$ and other center care at $t = 2$. The third bar (CH) shows the return of receiving other center care at $t = 1$ and HS at $t = 2$. The fourth bar (CC) shows the return of receiving other center care in both periods. The cross returns corresponds to the difference between the joint return and the sum of the partial returns. All these returns use two periods in home care as the baseline program sequence for comparison (see Section 4.1 for the exact definition of partial, joint and cross-returns).

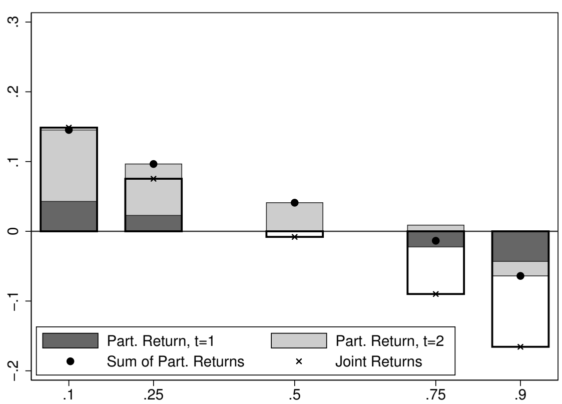
Figure 3: Partial, Joint and Cross Returns from Sequential Program Participation, Different Values of $\theta$



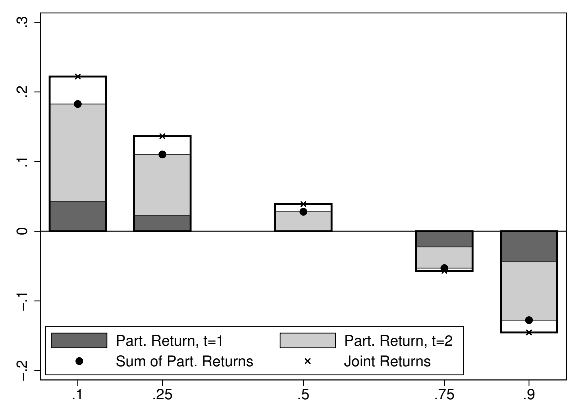

(a) HH




(b) HC



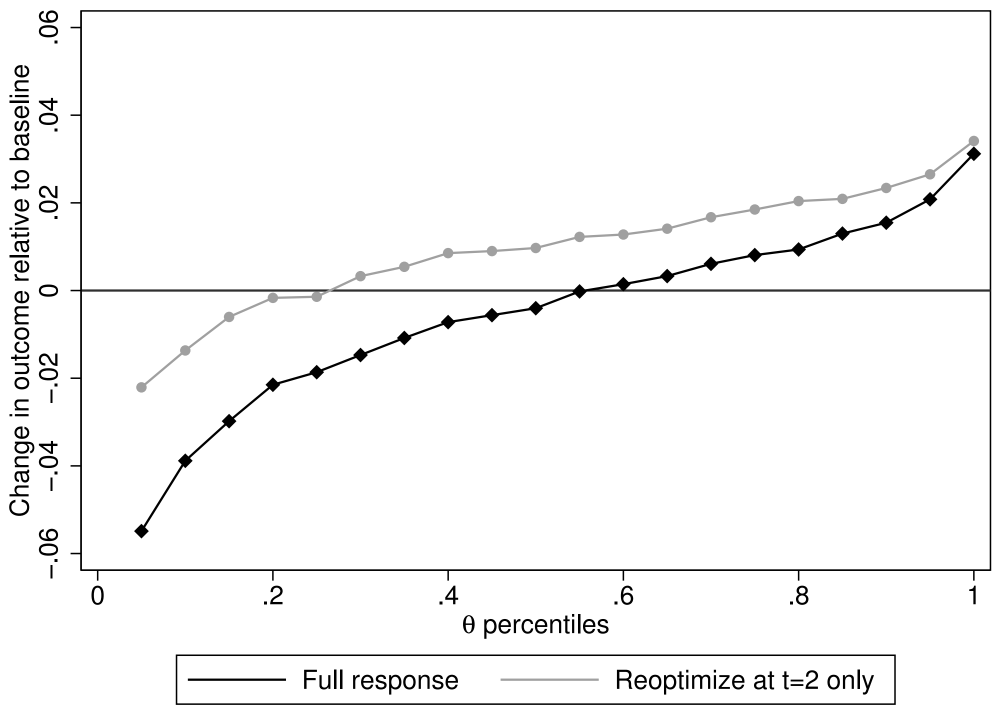

(c) CH




(d) CC

*Notes* : The outcome analyzed is the PPVT test score obtained at the end of age-4 year. Within each panel, the different bars represent the joint and partial returns of the same program sequence, but for different levels of baseline ability $\theta$ ("Part. Return" in label stands for "Partial Returns"). In particular, we report the returns for the 10th, the 25th, the 50th, the 75th and the 90th percentile of the $\theta$ distribution. Panel (a) is the return for two periods in HS (HH), panel (b) for one period in HS followed by other center care (HC), panel (c) for one period in other center care followed by HS (CH) and panel (d) for two periods in other center care (CC).

Figure 4: Effect of Limiting Head Start Enrolment to One Period Only for Different Values of $\theta$




*Notes* : The outcome analyzed is the PPVT test score obtained at the end of age-4 year. Each curve represents deviations in outcomes from the baseline scenario in the treatment group, e.g., zero means no effect of the HS enrolment limit on the outcome. On each curve, each marker represents one ventile of the distribution of $\theta$ (i.e., baseline ability. E.g. the leftmost marker represents individuals with the lowest 5 percentiles of $\theta$, the second leftmost marker represents individuals between the 5-10 percentiles, and so on). All scenarios are simulated from the structural model with 30 paths per treatment-group individual. "Full response": allow individuals to reoptimize at t=1 and t=2. "Reoptimize at t=2 only": assume individuals at t=1 do not know about the HS enrolment limit, so they can reoptimize at t=2 only.

# ONLINE APPENDIX

## A Further Note on Identification

The section provides details of an identification argument without an exclusion variable in the choice equation for $c$. We clarify that the cost is to impose a parametric distribution on $\nu^c_{i\mathsf{o}}$, and all other results remain to hold. First, consider the (centered) measurement equations:

$$\epsilon_i^{pp} \equiv \theta_i + e_i^{pp} \tag{A.1}$$

$$\epsilon_i^{wj} \equiv \gamma^{wj}\theta_i + e_i^{wj} \tag{A.2}$$

Following Fruehwirth, Navarro, and Takahashi (2016), higher-order cross moments can be used to identify $\gamma^{wj}$ (see also Bonhomme and Robin (2009)):

- If $\theta_i$ is asymmetric: use $E((\epsilon_i^{pp})^2\epsilon_i^{wj}) = \gamma^{wj}E(\theta_i^3)$ and $E(\epsilon_i^{pp}(\epsilon_i^{wj})^2) = (\gamma^{wj})^2E(\theta_i^3)$ to identify $\gamma^{wj} = \frac{E(\epsilon_i^{pp}(\epsilon_i^{wj})^2)}{E((\epsilon_i^{pp})^2\epsilon_i^{wj})}$;
- If $\theta_i$ is symmetric and kurtotic (i.e., $E(\theta_i^4) \neq 3[E(\theta_i^2)]^2$): obtain $(\gamma^{wj})^2 = \frac{E(\epsilon_i^{pp}(\epsilon_i^{wj})^3)-3E(\epsilon_i^{pp}\epsilon_i^{wj})E((\epsilon_i^{wj})^2)}{E((\epsilon_i^{pp})^3\epsilon_i^{wj})-3E(\epsilon_i^{pp}\epsilon_i^{wj})E((\epsilon_i^{pp})^2)}$ where the sign of $\gamma^{wj}$ is the same as that of $E(\epsilon_i^{pp}\epsilon_i^{wj})$ (see Fruehwirth, Navarro, and Takahashi (2016), Appendix B3);

and then the distributions of $\theta_i$, $e_i^{pp}$ and $e_i^{wj}$ are nonparametrically identified via Kotlarski (1967).[63]

As in the main text, let $\tilde{Z}_i$ be an exclusion variable in the choice equation for alternative $h$, and fix $\tilde{Z}_i$ sufficiently small such that the choice model becomes a binary decision between $c$ and $n$. The probability of choosing $c$ at node $\mathsf{o}$ given measurement $\epsilon_i^{pp}$ is:

$$\begin{aligned} Pr(K_{i1} = c|\epsilon_i^{pp} = m, \tilde{Z}_i = -\infty) &= Pr(\psi^c_{\mathsf{o}} + \lambda^c_{\mathsf{o}}\theta_i + \nu^c_{i\mathsf{o}} \leq 0|\theta_i + e_i^{pp} = m, \tilde{Z}_i = -\infty) \\ &= \frac{1}{f_{\epsilon_i^{pp}}(m)}\int_{-\infty}^{\infty} F_{\nu^c_{i\mathsf{o}}}(-\psi^c_{\mathsf{o}} - \lambda^c_{\mathsf{o}}(m-e))dF_{e_i^{pp}}(e) \end{aligned} \tag{A.3}$$

due to independence among $\nu^c_{i\mathsf{o}}$, $e_i^{pp}$ and $\tilde{Z}_i$; where $f_{\epsilon_i^{pp}}(\cdot)$ is the density of measurement $\epsilon_i^{pp}$, and $F_{\nu^c_{i\mathsf{o}}}(\cdot)$ and $F_{e_i^{pp}}(\cdot)$ are the CDFs of unobservables $\nu^c_{i\mathsf{o}}$ and $e_i^{pp}$, respectively. Assuming a parametric distribution for $F_{\nu^c_{i\mathsf{o}}}(\cdot)$ (e.g., standard normal), $\psi^c_{\mathsf{o}}$ and $\lambda^c_{\mathsf{o}}$ are identified by varying $m$.[64]

It is worth noting a special case where $\theta_i$, $e_i^{pp}$ and $e_i^{wj}$ are all normally distributed. In this case, $\gamma^{wj}$ is not identified from equations (A.1) and (A.2) alone (see e.g., Reiersøl (1950)).[65] However, by supplementing the measurement equations with the choice equation for $c$ (again fixing $\tilde{Z}_i = -\infty$), and

[63] By Kotlarski (1967), if $X_1, X_2$ and $X_3$ are independent real-valued random variables, and $Y_1 \equiv X_1 - X_2$ and $Y_2 \equiv X_1 - X_3$, then, if the characteristic function of $(Y_1, Y_2)$ does not vanish, the joint distribution of $(Y_1, Y_2)$ determines the distributions of $X_1$, $X_2$ and $X_3$ up to location. See Fruehwirth, Navarro, and Takahashi (2016) for details.

[64] If there is an exclusion variable for alternative $c$, then $\nu^c_{i\mathsf{o}}$ is nonparametrically identified.

[65] If $\theta_i$ is normal but $e_i^{pp}$ and $e_i^{wj}$ are nonnormal, $\gamma^{wj}$ is still identified from equations (A.1) and (A.2) alone (Reiersøl (1950)).

assuming $F_{\nu_{io}^c}(\cdot)$ is standard normal, all parameters in this three-equation system ($\gamma^{wj}$, $\sigma_\theta^2$, $\sigma_{e^{pp}}^2$, $\sigma_{e^{wj}}^2$, $\psi_o^c$, $\lambda_o^c$) are identified.[66]

# B Robustness checks

We conduct a range of robustness checks to gauge the sensitivity of our conclusions to various modeling and empirical specifications classified into five broad categories: (1) measurement, (2) factor distribution, (3) choice equations, (4) outcome and (5) structural model.

## B.1 Robustness to measurement system

We expand the measurement system by adding the baseline WJIII Letter-Word Identification test score as the third measurement. The estimation results are reported in Tables A4 and A5. The estimated choice equations are similar to those in the baseline model. In the outcome equations, the intercept estimates are similar to those in the baseline model and the standard errors are about 10 percent smaller ($\hat{\alpha}^{c,n}$ becomes larger but it remains imprecisely estimated). While the factor loadings become about 15 percent smaller, their relative magnitudes remain similar to those in the baseline model. The standard deviation of factor $\theta$ becomes 5 percent larger. The stability of these results is noteworthy given the non-trivial change in the measurement system: the loading on the baseline WJIII pre-academic score doubles in size (0.168; a signal-to-noise ratio of 4.3%), while the loading on the baseline WJIII Letter-Word Identification score is 0.40 (a signal-to-noise ratio of 23.3%).

## B.2 Robustness to factor distribution

We relax the factor distribution used in estimation by modelling it as a mixture normal distribution instead. The estimation results are reported in Tables A6 and A7. The second mixture has a probability weight of 0.075 (se=0.052) with a mean of 0.293 (se=0.238) and standard deviation of 0.264 (se=0.14). The first mixture has a probability weight of 0.925 with a mean of -0.024 (implied by the zero-mean normalization of the factor distribution) and standard deviation of 0.248 (se=0.018). The implied standard deviation of the mixture normal distribution is 0.263, which is 2.7% larger than that in the baseline model. Although the likelihood ratio test rejects the normal factor model at the 10% significance level (p-value=0.063),[67] simulations suggest that the overall shape of the mixture distribution is visually very similar to a normal distribution except for a slightly thicker right tail. The other estimated parameters in the model remain similar to those in the baseline model, except for two differences. First, the sd of the idiosyncratic shock for alternative $c$ ($\sigma_{\nu^c}$) and the correlation coefficient

[66] The same intuition as in Section 3.1 applies. For example, first concentrate out the following parameters using second moments: $\gamma^{wj} = \frac{Cov(\epsilon_i^{pp}, \epsilon_i^{wj})}{\sigma_\theta^2}$; $\sigma_{e^{pp}}^2 = Var(\epsilon_i^{pp}) - \sigma_\theta^2$; $\sigma_{e^{wj}}^2 = Var(\epsilon_i^{wj}) - \frac{(Cov(\epsilon_i^{pp}, \epsilon_i^{wj}))^2}{\sigma_\theta^2}$. Then, use equation (A.3) evaluated at multiple values of $\epsilon_i^{pp}$, which captures the comovement of $\epsilon_i^{pp}$ and the tendency to choose $c$, to concentrate out $\psi_o^c$, $\lambda_o^c$ as functions of $\sigma_\theta^2$. Finally, use $Pr(K_{i1} = c | \epsilon_i^{wj} = m, \tilde{Z}_i = -\infty)$, which is a function of $\gamma^{wj}, \psi_o^c$, $\lambda_o^c$, $\sigma_{e^{wj}}^2$, to solve for $\sigma_\theta^2$.

[67] $LR = 2 \times (-3824.32 - (-3827.97)) = 7.3 \sim \chi^2(3)$ under the null hypothesis that all the parameters in the second mixture are zero.

of alternative-specific idiosyncratic shocks ($\rho_{hc}$) become larger (0.967). Second, the outcome and choice parameters related to program sequence (c,n) have changed in the direction of stronger selection into (c,n).[68]

### B.3 Robustness to choice equation specification

The choice equations in the sequential threshold model are approximations to the perceived value of each alternative. For forward-looking individuals, these perceived values are a combination of utility flow and perceived continuation value; even if the flow is linear in parameters, the perceived continuation value may be nonlinear in parameters due to incorporation of uncertainty of future payoffs (see Section 5 and Appendix Section D.1). To assess this threat of functional form misspecification, we specify a quadratic function of factor $\theta$ in the period-1 choice equations for $h$ and $c$ to allow for nonlinear selection by $\theta$.[69] The estimation results are reported in Tables A8 and A9. The quadratic coefficients are 0.265 (se=0.77) and -0.035 (se=1.455) in the choice equations for $h$ and $c$, respectively, and they are not jointly statistically significant according to the likelihood-ratio test (p-value=0.7). The other estimated parameters in the model remain similar to those in the baseline model, except that (as in the case of the mixture normal model above) the outcome and choice parameters related to program sequence (c,n) have changed in the direction of stronger selection into (c,n).

### B.4 Robustness to outcome definition

First, we follow Kline and Walters (2016) and use the simple average of the PPVT score and WJIII pre-academic skills score as the outcome variable.[70] Although taking the average may not be ideal, it has the advantage that the outcome reflects information from multiple sources covering a broader set of skills. The Peabody Picture Vocabulary Test (PPVT; Third Edition) aims at measuring the child's listening comprehension for the spoken words in standard English. Children are provided with several pictures and asked to identify the one which best represents the word orally presented (a Spanish version - Test de Vocabulario en Imagenes Peabody, TVIP - is used for Spanish speaking children). The Woodcock-Johnson III Tests of Achievement used in our estimates, instead, is a combination of the results on three different tests, two of which measure the pre-reading ability of a child, while the third is a test of the mathematical abilities of the child.[71]

[68] Specifically, $\hat{\alpha}^{cn}$ becomes smaller but $\hat{\gamma}^{cn}$ becomes larger; the intercepts of the period-2 choice equations for $h$ and $c$ at node c become more positive and the factor loadings in both equations become more negative. Note that a low proportion of individuals choose sequence (c,n) in the data.

[69] As discussed in Section 5, the period-2 value function $V_{i2}(.)$ can be expressed as a linear function of $\theta$, therefore we focus on the threat of misspecification at $t = 1$.

[70] We do not use standardized scores like they do. Our PPVT and WJIII scores at the end of age-4 year are defined in the same manner as the baseline PPVT and WJIII scores used in the measurement equations.

[71] Specifically, the two tests for pre-reading skills are the Letter-Word Identification test and the Spelling test. The Letter-Word Identification test measures the child's ability to recognize letters and words, while the Spelling test measures children's ability to copy letters and, as the items progress in difficulty, to write different letters. The math test is the Applied Problems test, where a problem is orally described to the child and she must recognize which procedure to follow and then count or perform some calculation.

The estimation results are reported in Tables A10 and A11. The estimated choice equations are similar to those in the baseline model. Notably, the outcome equations become more precisely estimated – standard errors are one-third smaller on average.[72] The standard deviation of the outcome error is also one-third smaller. In the measurement system, the loading on the baseline WJIII pre-academic score doubles in size (0.159; a signal-to-noise ratio of 3.4%) but, interestingly, it is still rather small given that the outcome gives equal weights to PPVT and WJIII scores. This suggests that the latent ability captured by the factor $\theta$ remains most relevant to the skills embedded in PPVT.

Note that while both tests measure cognitive skills, they have different roles in education theory. For example, reading comprehension as a high-level skill is widely viewed as a function of language comprehension and word recognition/decoding (e.g., Gough and Tunmer, 1986). The Letter-Word Identification and Spelling tests in WJIII pre-academic skills measure word recognition/decoding. The PPVT is involved with language development and lexical knowledge more closely related to language comprehension – it is conceivable that such knowledge, which may be gained at early stages via extensive interactions with the mother, lays the foundation for development in word recognition/decoding, which is a skill that is more likely to be acquired in more formal learning environments.[73]

With this in mind, we turn to Figures A5 and A6 which report the partial, joint and cross returns from various program sequences. At the population average, the returns are more positive than those in the baseline model, but the key patterns remain qualitatively similar (see Section 4.1) with the exception that CC no longer exhibits positive within-program cross return. Regarding heterogeneity of returns by $\theta$, we find two main differences: (1) the partial return of HS at age 3 is *increasing* in $\theta$, and (2) the partial return of other center care at age 4 remains positive for all $\theta$ (but still declines by $\theta$). These differences are consistent with the notion that WJIII measures a higher-level skill that can be picked up more easily by children with high baseline ability via formal care. Yet the other patterns and the key message remain the same: a policy maker should engage low-ability children with two years of formal care and high-ability children with one year of home care.

### B.5 Robustness to structural model specification

To facilitate comparison with the sequential threshold model, the baseline structural model fixes the values of all technological coefficients to the sequential threshold model and estimates the rest of the parameters. While the quadratic-factor sensitivity analysis above provides credibility to the estimated technological coefficients from the sequential threshold model, we estimate the "full" structural model including all 30 technological coefficients, which results in a total of 57 parameters to be estimated. The estimation results are reported in Tables A12 and A13. The estimated technological coefficients are very similar to those in the sequential threshold model and they are estimated with similar precision. The terminal scaling factor $\kappa$ becomes larger (7.851) and less precisely estimated (se=4.035), and it is

[72] The intercepts become 10 percent larger (WJIII scores are higher than PPVT scores, although they are not comparable) while the loadings become 20 percent smaller.

[73] This is consistent with the different modes of suggested educational interventions, e.g., "explicit, systematic phonics instruction" for enhancing decoding and "creating a language- and experience-rich environment" for enhancing language development and lexical knowledge (see Wendling, Schrank, and Schmitt, 2007).

statistically significant at the 10% level. The key message remains the same: individuals incorporate the expected future outcomes as part of the information set and act upon it by choosing an option that yields better expected outcomes, keeping preferences fixed. Yet the large point estimate of $\kappa$ has quantitative implications for coefficients in the choice equations. The factor loadings in the utility flow become imprecisely estimated; this is because the large $\kappa$ magnifies the perceived continuation values by $\theta$, which now dominate the choice decisions. In summary, by fixing the technological coefficients to those in the sequential threshold model, we can achieve higher efficiency in the estimation of the structural model, the main task of which is to decompose the overall perceived value into utility flow and perceived continuation value. The sequential threshold model is useful for structural estimation, and both approaches cross-validate and complement each other.

# C Relation to LATE

Let $Y_i^{d^z}$ denote individual $i$'s potential outcome under program sequence $d^z$, i.e., the potential sequence she chooses with experimental offer $Z_i = z \in \{0, 1\}$. Exploiting the initial random assignment as an instrument, the IV estimate can be written using a Wald estimator:

$$LATE \equiv \frac{E[Y_i|Z_i=1] - E[Y_i|Z_i=0]}{E[1(K_{i,1}=h)|Z_i=1] - E[1(K_{i,1}=h)|Z_i=0]} = E\left[Y_i^{d^1} - Y_i^{d^0}|K_{i,1}^0 \neq h, K_{i,1}^1 = h\right], \quad \text{(C.1)}$$

where $1(K_{i,1}=h)$ is an indicator function which takes the value of 1 if the individual $i$'s observed choice in the first period is HS and 0 otherwise. $K_{i,1}^1$ and $K_{i,1}^0$ denote her potential choices at $t=1$ with and without the HS offer, respectively. The IV estimate identifies the LATE, giving the average effect of HS participation for compliers who would choose HS at $t=1$ only if assigned to the experimental treatment group (Imbens and Angrist, 1994).

With multiple alternatives, it is well known that the LATE is the weighted sum of return to HS for compliers drawn from different margins: those drawn into HS from home care and those drawn into HS from other center care (in the terminology of Section 4.2.1 the first group consists of n-compliers and the second group consists of c-compliers).[74] Consider the case where individuals choose their childcare type at $t=1$ only. Following Kline and Walters (2016), we can decompose the LATE into a weighted average of treatment effects for different groups of compliers, each of which measures the effect of HS for compliers drawn from a specific counterfactual alternative:

$$LATE = \frac{\pi_n}{\pi_{CP}} \underbrace{E[Y_i^h - Y_i^n|K_{i,1}^0 = n, K_{i,1}^1 = h]}_{\text{n-compliers returns}} + \frac{\pi_c}{\pi_{CP}} \underbrace{E[Y_i^h - Y_i^c|K_{i,1}^0 = c, K_{i,1}^1 = h]}_{\text{c-compliers returns}}. \quad \text{(C.2)}$$

where $Y_i^k$ is the potential outcome for individual $i$ when assigned to childcare type $k$ at $t=1$ (irrespective of the assignment at $t=2$). The weights are given by the relative proportion of children induced into HS from each specific counterfactual alternative: $\frac{\pi_n}{\pi_{CP}}$ is the proportion of compliers drawn from

[74]See, for example, Heckman and Urzua (2010) and Kline and Walters (2016).

home care and $\frac{\pi_c}{\pi_{CP}}$ is the proportion of compliers drawn from other center care.

As we discussed in Section 4.2.1, the n-compliers and c-compliers can each be further divided into 9 subgroups depending on the potential choices made at $t = 2$ with and without the HS offer. Extending the decomposition in Equation C.2 to multiple periods, we show that the average return for n-compliers and the average return for c-compliers can each be further decomposed into a weighted average of the returns for 9 different subgroups, leading to following decomposition result:

$$
\begin{aligned}
LATE = \sum_{d^1}\sum_{d^0}\frac{\pi_{d^0,d^1}}{\pi_{CP}}E\left[Y_i^{d^1} - Y_i^{d^0}|D_i^0 = d^0, D_i^1 = d^1\right] = \sum_{d^1}\sum_{d^0}\frac{\pi_{d^0,d^1}}{\pi_{CP}}\Delta_{d^0,d^1}, \\
\forall\ d^0 \in \{(c,h),(c,c),(c,n),(n,h),(n,c),(n,n)\},\ d^1 \in \{(h,h),(h,c),(h,n)\}
\end{aligned}
\tag{C.3}
$$

where $\Delta_{d^0,d^1}$ is the average treatment effect for a particular type of complier (those who choose program sequence $d^0$ when the instrument is turned off and choose $d^1$ when the instrument is turned on), $\pi_{CP}$ is the overall fraction of compliers and $\pi_{d^0,d^1}$ is the population proportion of the subgroup selecting $d^0$ when in the control group and $d^1$ when treated.

# D Computational and estimation procedures

## D.1 Computation of the expected value function

For node $j \in \mathcal{J} = \{\mathsf{o},\mathsf{h},\mathsf{c},\mathsf{n}\}$, let $\bar{u}_{ij}^k$ denote the alternative-specific utility flow, *exclusive* of the preference shock, for alternative $k \in \{h,c,n\}$: $\bar{u}_{ij}^h = \tilde{\psi}_j^h + \tilde{f}_j(Z_i,\boldsymbol{X}_i,\theta_i) + \tilde{\lambda}_j^h\theta_i$; $\quad \bar{u}_{ij}^c = \tilde{\psi}_j^c + \tilde{\lambda}_j^c\theta_i$; $\bar{u}_{ij}^n = 0$. Now consider the decision problem at $t = 2$ for an individual having chosen $K_{i1} = k_1 \in \{h,c,n\}$ at $t = 1$ and is at node $\mathsf{k}_1 \in \{\mathsf{h},\mathsf{c},\mathsf{n}\}$ (e.g., $\mathsf{k}_1 = \mathsf{h}$ if and only if $k_1 = h$). Conditional on the state variables $K_{i1}(= k_1)$, $\boldsymbol{X}_i$, $\theta_i$, the probabilities for program choice at $t = 2$, $K_{i2}$, are (suppress $\boldsymbol{X}_i$ and $\theta_i$ for notational simplicity):

$$
\begin{aligned}
Pr(K_{i2} = h|K_{i1} = k_1) = Pr[&\tilde{\nu}_{i\mathsf{k}_1}^h - \tilde{\nu}_{i\mathsf{k}_1}^c > (\bar{u}_{i\mathsf{k}_1}^c - \bar{u}_{i\mathsf{k}_1}^h) + \kappa(\bar{Y}_{iT}^{k_1,c} - \bar{Y}_{iT}^{k_1,h}); \\
&\tilde{\nu}_{i\mathsf{k}_1}^h > -\bar{u}_{i\mathsf{k}_1}^h + \kappa(\bar{Y}_{iT}^{k_1,n} - \bar{Y}_{iT}^{k_1,h})|K_{i1} = k_1] \qquad (D.1) \\
Pr(K_{i2} = c|K_{i1} = k_1) = Pr[&\tilde{\nu}_{i\mathsf{k}_1}^c - \tilde{\nu}_{i\mathsf{k}_1}^h > (\bar{u}_{i\mathsf{k}_1}^h - \bar{u}_{i\mathsf{k}_1}^c) + \kappa(\bar{Y}_{iT}^{k_1,h} - \bar{Y}_{iT}^{k_1,c}); \\
&\tilde{\nu}_{i\mathsf{k}_1}^c > -\bar{u}_{i\mathsf{k}_1}^c + \kappa(\bar{Y}_{iT}^{k_1,n} - \bar{Y}_{iT}^{k_1,c})|K_{i1} = k_1] \qquad (D.2) \\
Pr(K_{i2} = n|K_{i1} = k_1) = 1 - &Pr(K_{i2} = h|K_{i1} = k_1) - Pr(K_{i2} = c|K_{i1} = k_1) \qquad (D.3)
\end{aligned}
$$

which are computed by the Geweke-Hajivassiliou-Keane (GHK) simulator (Keane, 1994). For each $k_1 \in \{h,c,n\}$, we then compute the expected value function $E_1V_{i2}(\cdot)$, which is a probability-weighted average of the value of each future path:

$$
\begin{aligned}
& E_1 V_{i2}(K_{i1}=k_1;\tilde{\nu}^h_{i\mathsf{k}_1},\tilde{\nu}^c_{i\mathsf{k}_1}) \\
=& \sum_{k_2\in\{h,c\}} Pr(K_{i2}=k_2|K_{i1}=k_1)E_1\left(\bar{u}^{k_2}_{i\mathsf{k}_1}+\kappa\bar{Y}^{k_1,k_2}_{iT}+\tilde{\nu}^{k_2}_{i\mathsf{k}_1}|K_{i2}=k_2,K_{i1}=k_1\right)+Pr(K_{i2}=n|K_{i1}=k_1)\kappa\bar{Y}^{k_1,n}_{iT} \\
=& \kappa\bar{Y}^{k_1,n}_{iT}+\sum_{k_2\in\{h,c\}} Pr(K_{i2}=k_2|K_{i1}=k_1)\left(\bar{u}^{k_2}_{i\mathsf{k}_1}+\kappa\left(\bar{Y}^{k_1,k_2}_{iT}-\bar{Y}^{k_1,n}_{iT}\right)\right) \\
& +\sum_{k_2\in\{h,c\}} Pr(K_{i2}=k_2|K_{i1}=k_1)E_1(\tilde{\nu}^{k_2}_{i\mathsf{k}_1}|K_{i2}=k_2,K_{i1}=k_1) \qquad \text{(D.4)}
\end{aligned}
$$

The first two terms reflect the weighted average of the *deterministic* value of each path. The last term contains expectations of truncated distributions of preference shocks, i.e., selection into alternatives that have larger realizations of shocks. To compute these expectations we invoke an analytical result in Rosenbaum (1961) and Tallis (1961):

$$
\begin{aligned}
Pr(X>a,Y>b)E(Y|X>a,Y>b) &= \left[r\phi(a)\Phi\left(-\frac{b-ra}{\sqrt{1-r^2}}\right)+\phi(b)\Phi\left(-\frac{a-rb}{\sqrt{1-r^2}}\right)\right] \\
&\equiv \bar{\Lambda}(a,b,r) \qquad \text{(D.5)}
\end{aligned}
$$

where $X$ and $Y$ follow a standard bivariate normal distribution with zero means, unit variances and correlation coefficient $r$; $\phi(\cdot)$ and $\Phi(\cdot)$ are the PDF and CDF of the standard normal distribution, respectively. We have:

$$
\begin{aligned}
& Pr(K_{i2}=h|K_{i1}=k_1)E_1(\tilde{\nu}^h_{i\mathsf{k}_1}|K_{i2}=h,K_{i1}=k_1) \\
=& Pr(K_{i2}=h|K_{i1}=k_1)E_1(\tilde{\nu}^h_{i\mathsf{k}_1}|\frac{\tilde{\nu}^h_{i\mathsf{k}_1}-\tilde{\nu}^c_{i\mathsf{k}_1}}{\sqrt{\tilde{\sigma}^2_{\nu^c}-2\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}+1}}>a_1;\quad \tilde{\nu}^h_{i\mathsf{k}_1}>b_1;\quad K_{i1}=k_1) \\
=& \bar{\Lambda}(a_1,b_1,r_1) \qquad \text{(D.6)}
\end{aligned}
$$

where $a_1=\frac{(\bar{u}^c_{i\mathsf{k}_1}-\bar{u}^h_{i\mathsf{k}_1})+\kappa(\bar{Y}^{k_1,c}_{iT}-\bar{Y}^{k_1,h}_{iT})}{\sqrt{\tilde{\sigma}^2_{\nu^c}-2\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}+1}}$, $b_1=-\bar{u}^h_{i\mathsf{k}_1}+\kappa(\bar{Y}^{k_1,n}_{iT}-\bar{Y}^{k_1,h}_{iT})$, and $r_1=\frac{1-\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}}{\sqrt{\tilde{\sigma}^2_{\nu^c}-2\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}+1}}$ is the correlation coefficient between $\frac{\tilde{\nu}^h_{i\mathsf{k}_1}-\tilde{\nu}^c_{i\mathsf{k}_1}}{\sqrt{\tilde{\sigma}^2_{\nu^c}-2\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}+1}}$ and $\tilde{\nu}^h_{i\mathsf{k}_1}$. Similarly,

$$
\begin{aligned}
& Pr(K_{i2}=c|K_{i1}=k_1)E_1(\tilde{\nu}^c_{i\mathsf{k}_1}|K_{i2}=c,K_{i1}=k_1) \\
=& \tilde{\sigma}_{\nu^c}Pr(K_{i2}=c|K_{i1}=k_1)E_1(\frac{\tilde{\nu}^c_{i\mathsf{k}_1}}{\tilde{\sigma}_{\nu^c}}|\frac{\tilde{\nu}^c_{i\mathsf{k}_1}-\tilde{\nu}^h_{i\mathsf{k}_1}}{\sqrt{\tilde{\sigma}^2_{\nu^c}-2\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}+1}}>a_2;\quad \frac{\tilde{\nu}^c_{i\mathsf{k}_1}}{\tilde{\sigma}_{\nu^c}}>b_2;\quad K_{i1}=k_1) \\
=& \tilde{\sigma}_{\nu^c}\bar{\Lambda}(a_2,b_2,r_2) \qquad \text{(D.7)}
\end{aligned}
$$

where $a_2=\frac{(\bar{u}^h_{i\mathsf{k}_1}-\bar{u}^c_{i\mathsf{k}_1})+\kappa(\bar{Y}^{k_1,h}_{iT}-\bar{Y}^{k_1,c}_{iT})}{\sqrt{\tilde{\sigma}^2_{\nu^c}-2\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}+1}}$, $b_2=\frac{-\bar{u}^c_{i\mathsf{k}_1}+\kappa(\bar{Y}^{k_1,n}_{iT}-\bar{Y}^{k_1,c}_{iT})}{\tilde{\sigma}_{\nu^c}}$, and $r_2=\frac{\tilde{\sigma}_{\nu^c}-\tilde{\rho}_{hc}}{\sqrt{\tilde{\sigma}^2_{\nu^c}-2\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}+1}}$ is the correlation coefficient between $\frac{\tilde{\nu}^c_{i\mathsf{k}_1}-\tilde{\nu}^h_{i\mathsf{k}_1}}{\sqrt{\tilde{\sigma}^2_{\nu^c}-2\tilde{\rho}_{hc}\tilde{\sigma}_{\nu^c}+1}}$ and $\frac{\tilde{\nu}^c_{i\mathsf{k}_1}}{\tilde{\sigma}_{\nu^c}}$.

This analytical result reduces the computational burden of the expected value function, which now largely involves the computation of choice probabilities in equations (D.1) and (D.2) for each $k_1\in\{h,c,n\}$. Because $Z_i$ and $\boldsymbol{X}_i$ differ for each individual, we solve the dynamic programming problem for each individual in the sample separately.

### D.2 Model estimation procedure

The model is estimated by the method of maximum likelihood. We first describe the estimation of the structural model, and then describe the estimation of the sequential threshold model as a special case. In both models, the researcher observes the individual's experimental treatment group status $Z_i$, covariates $\boldsymbol{X}_i$, baseline measurements $M_i^M$ ($M \in \{pp, wj\}$), program choice sequence $(K_{i1}, K_{i2})$, and outcome $Y_{iT}$.

The parameters of the structural model are listed as follows: preferences in utility ($\tilde{\psi}_j^h$, $\tilde{\psi}_j^c$, $\tilde{\lambda}_j^h$, $\tilde{\lambda}_j^c$, $\tilde{\beta}_j^{(\cdot)}$ for $j \in \{\mathsf{o}, \mathsf{h}, \mathsf{c}, \mathsf{n}\}$; $\tilde{\sigma}_{\nu^c}$, $\tilde{\rho}_{hc}$), terminal value scaling factor and discount factor ($\kappa$, $\delta$), baseline measurement equations ($\alpha^M$, $\sigma_e^M$ for $M \in \{pp, wj\}$; $\gamma^{wj}$), outcome equations ($\alpha_T^d$, $\beta_T^d$, $\gamma_T^d$ for $d \in \{(h,h), (h,c), (h,n), (c,h), (c,c), (c,n), (n,h), (n,c), (n,n)\}$; $\sigma_\varepsilon$), and the factor distribution ($\sigma_\theta$).

For each iteration in the parameter space, computation of the likelihood for individual $i$ consists of three nested loops. The inner loop computes the likelihood for baseline measurements and for choices and outcomes at $t = 1, 2, T$ given the individual's state variables, factor $\theta_i$, and the expected value function obtained from the backward recursion procedure. The middle loop carries out the backward recursion procedure of the dynamic programming problem given the individual's state variables and factor $\theta_i$. The outer loop integrates out the likelihood with respect to the factor distribution.

For the inner loop, the choice probability at $t = 1$ (node $j = \mathsf{o}$) is:

$$
\begin{aligned}
&Pr(K_{i1} = k_1 | Z_i, \boldsymbol{X}_i, \theta_i) \qquad\qquad \text{(D.8)}\\
&= \begin{cases}
Pr[\tilde{\nu}_{i\mathsf{o}}^h - \tilde{\nu}_{i\mathsf{o}}^c > (\bar{u}_{i\mathsf{o}}^c - \bar{u}_{i\mathsf{o}}^h) + \delta(E_1 V_{i2}(c, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{c}}^h, \tilde{\nu}_{i\mathsf{c}}^c) - E_1 V_{i2}(h, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{h}}^h, \tilde{\nu}_{i\mathsf{h}}^c)); & \\
\quad \tilde{\nu}_{i\mathsf{o}}^h > -\bar{u}_{i\mathsf{o}}^h + \delta(E_1 V_{i2}(n, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{n}}^h, \tilde{\nu}_{i\mathsf{n}}^c) - E_1 V_{i2}(h, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{h}}^h, \tilde{\nu}_{i\mathsf{h}}^c)) | Z_i, \boldsymbol{X}_i, \theta_i] & \text{if } k_1 = h \\
Pr[\tilde{\nu}_{i\mathsf{o}}^c - \tilde{\nu}_{i\mathsf{o}}^h > (\bar{u}_{i\mathsf{o}}^h - \bar{u}_{i\mathsf{o}}^c) + \delta(E_1 V_{i2}(h, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{h}}^h, \tilde{\nu}_{i\mathsf{h}}^c) - E_1 V_{i2}(c, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{c}}^h, \tilde{\nu}_{i\mathsf{c}}^c)); & \\
\quad \tilde{\nu}_{i\mathsf{o}}^c > -\bar{u}_{i\mathsf{o}}^c + \delta(E_1 V_{i2}(n, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{n}}^h, \tilde{\nu}_{i\mathsf{n}}^c) - E_1 V_{i2}(c, \boldsymbol{X}_i, \theta_i; \tilde{\nu}_{i\mathsf{c}}^h, \tilde{\nu}_{i\mathsf{c}}^c)) | Z_i, \boldsymbol{X}_i, \theta_i] & \text{if } k_1 = c \\
1 - Pr(K_{i1} = h | Z_i, \boldsymbol{X}_i, \theta_i) - Pr(K_{i1} = c | Z_i, \boldsymbol{X}_i, \theta_i) & \text{if } k_1 = n
\end{cases}
\end{aligned}
$$

and the choice probability at $t = 2$ (node $j = \mathsf{k}_1 \in \{\mathsf{h}, \mathsf{c}, \mathsf{n}\}$) is:

$$
\begin{aligned}
&Pr(K_{i2} = k_2 | K_{i1} = k_1, Z_i, \boldsymbol{X}_i, \theta_i) \qquad\qquad \text{(D.9)}\\
&= Pr(K_{i2} = k_2 | K_{i1} = k_1, \boldsymbol{X}_i, \theta_i) \\
&= \begin{cases}
Pr[\tilde{\nu}_{i\mathsf{k}_1}^h - \tilde{\nu}_{i\mathsf{k}_1}^c > (\bar{u}_{i\mathsf{k}_1}^c - \bar{u}_{i\mathsf{k}_1}^h) + \kappa(\bar{Y}_{iT}^{k_1,c}(\boldsymbol{X}_i, \theta_i) - \bar{Y}_{iT}^{k_1,h}(\boldsymbol{X}_i, \theta_i)); & \\
\quad \tilde{\nu}_{i\mathsf{k}_1}^h > -\bar{u}_{i\mathsf{k}_1}^h + \kappa(\bar{Y}_{iT}^{k_1,n}(\boldsymbol{X}_i, \theta_i) - \bar{Y}_{iT}^{k_1,h}(\boldsymbol{X}_i, \theta_i)) | K_{i1} = k_1, \boldsymbol{X}_i, \theta_i] & \text{if } k_2 = h \\
Pr[\tilde{\nu}_{i\mathsf{k}_1}^c - \tilde{\nu}_{i\mathsf{k}_1}^h > (\bar{u}_{i\mathsf{k}_1}^h - \bar{u}_{i\mathsf{k}_1}^c) + \kappa(\bar{Y}_{iT}^{k_1,h}(\boldsymbol{X}_i, \theta_i) - \bar{Y}_{iT}^{k_1,c}(\boldsymbol{X}_i, \theta_i)); & \\
\quad \tilde{\nu}_{i\mathsf{k}_1}^c > -\bar{u}_{i\mathsf{k}_1}^c + \kappa(\bar{Y}_{iT}^{k_1,n}(\boldsymbol{X}_i, \theta_i) - \bar{Y}_{iT}^{k_1,c}(\boldsymbol{X}_i, \theta_i)) | K_{i1} = k_1, \boldsymbol{X}_i, \theta_i] & \text{if } k_2 = c \\
1 - Pr(K_{i2} = h | K_{i1} = k_1, \boldsymbol{X}_i, \theta_i) - Pr(K_{i2} = c | K_{i1} = k_1, \boldsymbol{X}_i, \theta_i) & \text{if } k_2 = n
\end{cases}
\end{aligned}
$$

where we suppress the arguments of $\bar{u}_{ij}^{(\cdot)}$ for notational simplicity and the choice probability at $t = 2$ does not depend on $Z_i$ when conditioned on $K_{i1} = k_1$ (see Section 5 for specifications). We compute each choice probability using the GHK simulator. Define the likelihood contribution of individual $i$ given the observables and unobserved factor $\theta_i$ as

$$\begin{aligned} L_i(\theta) \equiv\, & Pr(K_{i1}=k_1|Z_i=z, \boldsymbol{X}_i=x, \theta_i=\theta)Pr(K_{i2}=k_2|K_{i1}=k_1, \boldsymbol{X}_i=x, \theta_i=\theta)\times \\ & f_{Y_{iT}|K_{i1},K_{i2},hsq_i,\theta_i}(y|k_1,k_2,hsq,\theta) f_{M_i^{pp}|\theta_i}(m^{pp}|\theta) f_{M_i^{wj}|\theta_i}(m^{wj}|\theta) \end{aligned} \tag{D.10}$$

where $f_{Y_{iT}|K_{i1},K_{i2},hsq_i,\theta_i}(\cdot)$, $f_{M_i^{pp}|\theta_i}(\cdot)$, and $f_{M_i^{wj}|\theta_i}(\cdot)$ are the conditional density functions of the outcome and baseline measurement equations, respectively. We then compute $\ell_i \equiv \ln \int_{-\infty}^{\infty} L_i(\theta) f_{\theta_i}(\theta) d\theta$ where $f_{\theta_i}(\cdot)$ is the density function of the unobserved factor and the integration is carried out using the Gauss-Hermite quadrature. The log-likelihood function is $\ell \equiv \sum_{i=1}^{N} \ell_i$. Denote the parameter vector by $\Theta$ and the maximum likelihood estimates by $\hat{\Theta}$. We compute the standard errors using the outer-product-of-gradient (OPG) estimator (sometimes called the BHHH estimator (Berndt, Hall, Hall, and Hausman (1974))): $\widehat{var}(\hat{\Theta}) = [\frac{1}{N}\sum_{i=1}^{N}\big(\frac{\partial}{\partial\Theta}\ell_i(\hat{\Theta})\big)\big(\frac{\partial}{\partial\Theta}\ell_i(\hat{\Theta})\big)']^{-1}$, which is consistent for the inverse Fisher information.

The sequential threshold model is estimated separately, but its estimation procedure is the same as in the structural model except that there is no dynamic programming involved (effectively, both $\kappa$ and $\delta$ are zero). The sequential threshold model contains the following parameters in the perceived value equations (in lieu of flow utility parameters, terminal value scaling factor and discount factor): $\psi_j^h$, $\psi_j^c$, $\lambda_j^h$, $\lambda_j^c$, $\beta_j^{(\cdot)}$ for $j \in \{\mathsf{o},\mathsf{h},\mathsf{c},\mathsf{n}\}$; $\sigma_{\nu^c}$, $\rho_{hc}$. The parameters in the baseline measurement equations, outcome equations, and the factor distribution are defined in the same manner as in the structural model. For each iteration in the parameter space, computation of the likelihood for individual $i$ consists of two nested loops only. The inner loop computes the likelihood for baseline measurements and for choices and outcomes at $t = 1, 2, T$ given the individual's state variables and factor $\theta_i$ (equations (D.8), (D.9) and (D.10)). The outer loop integrates out the likelihood with respect to the factor distribution.

# Appendix Tables and Figures

Table A1: Descriptive Statistics in the Baseline Survey

| | Control Group | Treatment Group | Difference |
|---|---|---|---|
| Household Size | 4.53 | 4.51 | 0.03 |
| Female =1 | 0.50 | 0.51 | -0.01 |
| Black =1 | 0.35 | 0.38 | -0.03 |
| Hispanic =1 | 0.33 | 0.33 | 0.00 |
| English as Home Language =1 | 0.76 | 0.75 | 0.00 |
| In Need of Special Education =1 | 0.10 | 0.13 | -0.03** |
| PPVT Score at Baseline | 2.31 | 2.29 | 0.02 |
| WJIII Pre-Academic Score at Baseline | 3.37 | 3.38 | 0.00 |
| Number of Siblings | 1.33 | 1.39 | -0.06 |
| Both Parents in Household =1 | 0.47 | 0.48 | -0.01 |
| Teen Mother =1 | 0.18 | 0.14 | 0.04** |
| Mother Not Married =1 | 0.42 | 0.44 | -0.02 |
| Mother Separated =1 | 0.14 | 0.14 | 0.00 |
| Mother Education: | | | |
| High School Diploma =1 | 0.33 | 0.35 | -0.02 |
| More than High School =1 | 0.30 | 0.31 | -0.02 |
| Urban Area =1 | 0.81 | 0.82 | -0.02 |
| HS Center Quality Index | 0.70 | 0.70 | 0.00 |
| HS Transportation Available =1 | 0.69 | 0.68 | 0.01 |
| N | 698 | 1,178 | |

*Notes* : This table reports the mean value of different individual characteristics comparing the control to the treatment group. The center quality index (obtained directly from the HSIS data) and the availability of transportation refer to the center of random assignment. Note that the center quality and the transportation variables used in actual estimation are residualized and standardized so to have standard deviation 1 (see Section 3 for details). Significance level (t-test for the difference between the average for the treatment group and the one for the control group=0): *** 1%, ** 5%, * 10%

Table A2: Effects of Limiting Head Start Enrolment to One Period Only on the Present Value of ex-post Discounted Utility

| | Baseline | Counterfactual | |
|---|---|---|---|
| | | Full resp. | Partial resp. |
| First period | 0.50 | +0.05 | 0.00 |
| Second period | 1.69 | -0.59 | -0.71 |
| Terminal period | 10.80 | -0.02 | +0.03 |
| Total | 12.99 | -0.56 | -0.67 |

*Notes* : All scenarios are simulated from the structural model with 30 paths per treatment-group individual. The "Baseline" column shows the average present discounted utility of the population in each period under the baseline scenario, i.e. when individuals are allowed to select two periods of HS. The "Counterfactual" columns report the difference in the present discounted utility between the baseline scenario and the counterfactuals: "Full resp." (full response) allows individuals to select at most one period in HS and assumes that they can reoptimize both at $t = 1$ and $t = 2$; "Partial resp." (partial response) assumes individuals at $t = 1$ do not know about the HS enrolment limit, so they can reoptimize at $t = 2$ only.

Table A3: Effects of Limiting Head Start Enrolment to One Period Only, by Children's Ability

| | Low-ability children | | | High-ability children | | |
|---|---|---|---|---|---|---|
| | Baseline | Counterfactual | | Baseline | Counterfactual | |
| | | Full resp. | Partial resp. | | Full resp. | Partial resp. |
| Program choice (%) | (1) | (2) | (3) | (4) | (5) | (6) |
| Choices (t=1): | | | | | | |
| $h$ | 86.4 | -41.9 | 0.0 | 83.2 | -23.3 | 0.0 |
| $c$ | 3.1 | +8.5 | 0.0 | 8.9 | +9.3 | 0.0 |
| $n$ | 10.5 | +33.4 | 0.0 | 7.9 | +14.0 | 0.0 |
| Choices (t=2): | | | | | | |
| $h$ | 76.4 | -52.3 | -70.6 | 60.2 | -45.6 | -54.5 |
| $c$ | 17.7 | +25.9 | +33.9 | 30.5 | +22.3 | +27.2 |
| $n$ | 5.9 | +26.3 | +36.6 | 9.3 | +23.3 | +27.3 |
| Program sequence: | | | | | | |
| $h,h$ | 70.6 | -70.6 | -70.6 | 54.5 | -54.5 | -54.5 |
| $h,c$ | 12.5 | +11.4 | +33.9 | 22.6 | +12.9 | +27.2 |
| $h,n$ | 3.3 | +17.2 | +36.6 | 6.1 | +18.3 | +27.3 |
| $c,h$ | 1.4 | +4.3 | 0.0 | 1.8 | +2.1 | 0.0 |
| $c,c$ | 1.6 | +4.1 | 0.0 | 5.6 | +5.6 | 0.0 |
| $c,n$ | 0.1 | +0.1 | 0.0 | 1.6 | +1.6 | 0.0 |
| $n,h$ | 4.4 | +14.0 | 0.0 | 3.9 | +6.8 | 0.0 |
| $n,c$ | 3.6 | +10.4 | 0.0 | 2.4 | +3.8 | 0.0 |
| $n,n$ | 2.5 | +9.0 | 0.0 | 1.6 | +3.4 | 0.0 |

*Notes* : All scenarios are simulated from the structural model with 30 paths per treatment-group individual. The "Baseline" columns show the proportion of the population making each choice under the baseline scenario, i.e. when individuals are allowed to select two periods of HS. The "Counterfactual" columns report the difference in the proportion of population selecting each childcare option between the baseline scenario and the counterfactuals: "Full resp." (full response) allows individuals to select at most one period in HS and assumes that they can reoptimize both at $t = 1$ and $t = 2$; "Partial resp." (partial response) assumes individuals at $t = 1$ do not know about the HS enrolment limit, so they can reoptimize at $t = 2$ only. Columns (1) – (3) ("Low-ability children") report the results for children in the $10^{th}$ percentile of the $\theta$ distribution, while Columns (4)–(6) ("High-ability children") report the results for the children in the $90^{th}$ percentile of the $\theta$ distribution.

Table A4: Estimates of the Parameters of the Choice Equations from the Sequential Threshold Model, Expanding the Measurement System to Include Three Baseline Scores

| Decision Node ($j$) | Covariate | Perceived Value | | | | | |
|---|---|---|---|---|---|---|---|
| | | Head Start ($U_{ij}^h$) | | | Other center care ($U_{ij}^c$) | | |
| o ($t=1$) | Intercept | -0.681 | (0.182) | *** | -1.044 | (0.318) | *** |
| | Factor $\theta$ | -0.002 | (0.265) | | 1.062 | (0.405) | *** |
| | Z | 1.988 | (0.143) | *** | | | |
| | $\theta \times$ Z | 0.253 | (0.343) | | | | |
| | hsq | -0.209 | (0.056) | *** | | | |
| | trp | -0.160 | (0.054) | *** | | | |
| | hsq $\times$ Z | 0.331 | (0.071) | *** | | | |
| | trp $\times$ Z | 0.164 | (0.073) | ** | | | |
| h ($t=2$) | Intercept | 1.509 | (0.122) | *** | 0.345 | (0.227) | |
| | Factor $\theta$ | -0.420 | (0.284) | | 0.463 | (0.437) | |
| | hsq | 0.102 | (0.045) | ** | | | |
| | trp | 0.014 | (0.044) | | | | |
| c ($t=2$) | Intercept | 1.310 | (0.453) | *** | 1.664 | (0.495) | *** |
| | Factor $\theta$ | -1.429 | (0.851) | * | -0.359 | (0.955) | |
| | hsq | 0.096 | (0.095) | | | | |
| | trp | -0.207 | (0.111) | * | | | |
| n ($t=2$) | Intercept | 0.553 | (0.153) | *** | -0.059 | (0.220) | |
| | Factor $\theta$ | 0.086 | (0.303) | | -0.173 | (0.485) | |
| | hsq | 0.051 | (0.050) | | | | |
| | trp | 0.085 | (0.059) | | | | |
| Other utility parameters: | $\sigma_{\nu^c}$ | 1.860 | (0.515) | *** | | | |
| | $\rho_{hc}$ | 0.634 | (0.170) | *** | | | |

*Notes* : N=1876, Log-likelihood = -3525.61. Trp: HS transport availability. Hsq: HS center quality. The model uses PPVT scores at the end of age-4 year as the outcome. The measurement system includes baseline PPVT scores, baseline WJIII pre-academic score and baseline WJIII Letter-Word Identification test scores. Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): *** 1%, ** 5%, *10%.

Table A5: Estimates of the Parameters of the Potential Outcome and Measurement Equations from the Sequential Threshold Model, Expanding the Measurement System to Include Three Baseline Scores

| | Intercept | | | Covariate Factor $\theta$ | | | HS quality | | |
|---|---|---|---|---|---|---|---|---|---|
| Potential outcome by program sequence: | | | | | | | | | |
| $h, h$ | 3.005 | (0.010) | *** | 0.960 | (0.067) | *** | 0.024 | (0.012) | * |
| $h, c$ | 3.026 | (0.021) | *** | 0.782 | (0.102) | *** | 0.005 | (0.027) | |
| $h, n$ | 3.014 | (0.044) | *** | 1.190 | (0.177) | *** | -0.016 | (0.039) | |
| $c, h$ | 2.938 | (0.039) | *** | 0.794 | (0.195) | *** | 0.009 | (0.046) | |
| $c, c$ | 2.997 | (0.027) | *** | 0.786 | (0.113) | *** | | | |
| $c, n$ | 3.028 | (0.146) | *** | 1.392 | (0.470) | *** | | | |
| $n, h$ | 3.015 | (0.022) | *** | 1.183 | (0.109) | *** | 0.015 | (0.018) | |
| $n, c$ | 2.996 | (0.027) | *** | 1.031 | (0.122) | *** | | | |
| $n, n$ | 2.953 | (0.031) | *** | 1.305 | (0.132) | *** | | | |
| Baseline measurement: | | | | | | | | | |
| PPVT | 2.299 | (0.007) | *** | 1.000 | - | | | | |
| WJIII | 3.375 | (0.005) | *** | 0.168 | (0.026) | *** | | | |
| WJIII (word) | 2.939 | (0.006) | *** | 0.400 | (0.029) | *** | | | |
| Standard deviations: | | | | | | | | | |
| Factor $\theta$ | 0.269 | (0.007) | *** | | | | | | |
| Baseline PPVT error | 0.252 | (0.006) | *** | | | | | | |
| Baseline WJIII error | 0.204 | (0.002) | *** | | | | | | |
| Baseline WJIII (word) error | 0.195 | (0.002) | *** | | | | | | |
| Outcome error | 0.272 | (0.006) | *** | | | | | | |

*Notes* : N=1876. Log-likelihood = -3525.61. The model uses PPVT scores at the end of age-4 year as the outcome. The parameters of the outcome equation differ depending on the program sequence (see equation 7). The program sequence is reported in the first column. $h$ is for HS, $c$ is for other center care, $n$ is for home care. The first letter refers to the program at $t = 1$, the second to the program at $t = 2$ (for instance, $h, h$ denotes HS both at $t = 1$ and at $t = 2$). The measurement system includes baseline PPVT scores, baseline WJIII pre-academic score and baseline WJIII Letter-Word Identification test scores. The parameters for the baseline measurement differ depending on which measurement is taken into account (PPVT, WJIII pre-academic and WJIII Letter-Word Identification). Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): ***1%, ** 5%, *10%.

Table A6: Estimates of the Parameters of the Choice Equations from the Sequential Threshold Model, with Factor Distributed as Mixture Normal

| Decision Node ($j$) | Covariate | Perceived Value | | | | | |
|---|---|---|---|---|---|---|---|
| | | Head Start ($U_{ij}^h$) | | | Other center care ($U_{ij}^c$) | | |
| o ($t=1$) | Intercept | -0.307 | (0.078) | *** | -1.378 | (0.234) | *** |
| | Factor $\theta$ | 0.226 | (0.214) | | 1.244 | (0.470) | *** |
| | Z | 1.628 | (0.093) | *** | | | |
| | $\theta \times$ Z | -0.051 | (0.297) | | | | |
| | hsq | -0.127 | (0.040) | *** | | | |
| | trp | -0.083 | (0.036) | ** | | | |
| | hsq $\times$ Z | 0.250 | (0.058) | *** | | | |
| | trp $\times$ Z | 0.084 | (0.059) | | | | |
| h ($t=2$) | Intercept | 1.569 | (0.061) | *** | 0.236 | (0.181) | |
| | Factor $\theta$ | -0.284 | (0.322) | | 0.568 | (0.548) | |
| | hsq | 0.088 | (0.043) | ** | | | |
| | trp | 0.013 | (0.043) | | | | |
| c ($t=2$) | Intercept | 2.068 | (0.382) | *** | 2.442 | (0.431) | *** |
| | Factor $\theta$ | -3.355 | (1.529) | ** | -2.598 | (1.675) | |
| | hsq | 0.131 | (0.094) | | | | |
| | trp | -0.215 | (0.111) | * | | | |
| n ($t=2$) | Intercept | 0.715 | (0.061) | *** | -0.126 | (0.164) | |
| | Factor $\theta$ | 0.161 | (0.299) | | -0.086 | (0.626) | |
| | hsq | 0.040 | (0.039) | | | | |
| | trp | 0.067 | (0.046) | | | | |
| Other utility parameters: | $\sigma_{\nu^c}$ | 2.590 | (0.199) | *** | | | |
| | $\rho_{hc}$ | 0.967 | (0.039) | *** | | | |

*Notes* : N=1876, Log-likelihood = -3824.32. Trp: HS transport availability. Hsq: HS center quality. The model uses PPVT scores at the end of age-4 year as the outcome. Factor $\theta$ is assumed to follow a mixture normal distribution. Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): *** 1%, ** 5%, *10%.

Table A7: Estimates of the Parameters of the Potential Outcome and Measurement Equations from the Sequential Threshold Model, with Factor Distributed as Mixture Normal

| | Intercept | | | Covariate Factor $\theta$ | | | HS quality | | |
|---|---|---|---|---|---|---|---|---|---|
| Potential outcome by program sequence: | | | | | | | | | |
| $h, h$ | 3.005 | (0.014) | *** | 1.096 | (0.104) | *** | 0.025 | (0.012) | ** |
| $h, c$ | 3.029 | (0.023) | *** | 0.903 | (0.120) | *** | -0.002 | (0.025) | |
| $h, n$ | 3.029 | (0.051) | *** | 1.399 | (0.197) | *** | -0.011 | (0.043) | |
| $c, h$ | 2.959 | (0.046) | *** | 1.065 | (0.215) | *** | 0.008 | (0.047) | |
| $c, c$ | 3.000 | (0.030) | *** | 0.917 | (0.129) | *** | | | |
| $c, n$ | 2.632 | (0.236) | *** | 1.913 | (0.618) | *** | | | |
| $n, h$ | 2.998 | (0.025) | *** | 1.289 | (0.129) | *** | 0.015 | (0.018) | |
| $n, c$ | 2.986 | (0.030) | *** | 1.136 | (0.130) | *** | | | |
| $n, n$ | 2.960 | (0.037) | *** | 1.446 | (0.152) | *** | | | |
| Baseline measurement: | | | | | | | | | |
| PPVT | 2.298 | (0.009) | *** | 1.000 | - | | | | |
| WJIII | 3.375 | (0.005) | *** | 0.084 | (0.024) | *** | | | |
| Factor $\theta$ distribution (Mixture Normal): | | | | | | | | | |
| Mean 1 | -0.024 | - | | | | | | | |
| SD 1 | 0.248 | (0.018) | *** | | | | | | |
| Weight 1 | 0.925 | - | | | | | | | |
| Mean 2 | 0.293 | (0.238) | | | | | | | |
| SD 2 | 0.264 | (0.140) | * | | | | | | |
| Weight 2 | 0.075 | (0.052) | | | | | | | |
| Mean $\theta$ | 0.000 | - | | | | | | | |
| SD $\theta$ | 0.263 | - | | | | | | | |
| Standard deviations: | | | | | | | | | |
| Baseline PPVT error | 0.260 | (0.011) | *** | | | | | | |
| Baseline WJIII error | 0.208 | (0.004) | *** | | | | | | |
| Outcome error | 0.243 | (0.014) | *** | | | | | | |

*Notes* : N=1876. Log-likelihood = -3824.32. The model uses PPVT scores at the end of age-4 year as the outcome. The parameters of the outcome equation differ depending on the program sequence (see equation 7). The program sequence is reported in the first column. $h$ is for HS, $c$ is for other center care, $n$ is for home care. The first letter refers to the program at $t = 1$, the second to the program at $t = 2$ (for instance, $h, h$ denotes HS both at $t = 1$ and at $t = 2$). The parameters for the baseline measurement differ depending on which measurement is taken into account (either PPVT or WJIII, see equation 8). Factor $\theta$ is distributed as mixture normal and we report weights, mean and s.d. of each mixture, plus the overall s.d. (the overall mean is normalized to 0). Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): ***1%, ** 5%, *10%.

Table A8: Estimates of the Parameters of the Choice Equations from the Sequential Threshold Model, Non-Linear Function of $\theta$ in the Choice Equation

| Decision Node ($j$) | Covariate | Perceived Value | | | | | |
|---|---|---|---|---|---|---|---|
| | | Head Start ($U_{ij}^h$) | | | Other center care ($U_{ij}^c$) | | |
| o ($t=1$) | Intercept | -0.613 | (0.171) | *** | -1.146 | (0.282) | *** |
| | Factor $\theta$ | 0.037 | (0.299) | | 1.236 | (0.507) | ** |
| | $\theta \times \theta$ | 0.265 | (0.770) | | -0.035 | (1.455) | |
| | Z | 1.907 | (0.168) | *** | | | |
| | $\theta \times$ Z | 0.089 | (0.376) | | | | |
| | hsq | -0.195 | (0.054) | *** | | | |
| | trp | -0.146 | (0.051) | *** | | | |
| | hsq $\times$ Z | 0.318 | (0.068) | *** | | | |
| | trp $\times$ Z | 0.151 | (0.070) | ** | | | |
| h ($t=2$) | Intercept | 1.539 | (0.076) | *** | 0.334 | (0.168) | ** |
| | Factor $\theta$ | -0.363 | (0.344) | | 0.412 | (0.541) | |
| | hsq | 0.096 | (0.044) | ** | | | |
| | trp | 0.018 | (0.043) | | | | |
| c ($t=2$) | Intercept | 2.010 | (0.447) | *** | 2.388 | (0.480) | *** |
| | Factor $\theta$ | -4.103 | (1.618) | ** | -3.170 | (1.608) | ** |
| | hsq | 0.143 | (0.105) | | | | |
| | trp | -0.226 | (0.119) | * | | | |
| n ($t=2$) | Intercept | 0.615 | (0.118) | *** | -0.063 | (0.147) | |
| | Factor $\theta$ | 0.195 | (0.338) | | -0.049 | (0.586) | |
| | hsq | 0.049 | (0.048) | | | | |
| | trp | 0.080 | (0.056) | | | | |
| Other utility parameters: | $\sigma_{\nu^c}$ | 2.058 | (0.383) | *** | | | |
| | $\rho_{hc}$ | 0.729 | (0.158) | *** | | | |

*Notes* : N=1876, Log-likelihood = -3827.62. Trp: HS transport availability. Hsq: HS center quality. The model uses PPVT scores at the end of age-4 year as the outcome. Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): *** 1%, ** 5%, *10%.

Table A9: Estimates of the Parameters of the Potential Outcome and Measurement Equations from the Sequential Threshold Model, Non-Linear Function of $\theta$ in the Choice Equation

| | Covariate | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | Intercept | | | Factor $\theta$ | | | HS quality | | |
| Potential outcome by program sequence: | | | | | | | | | |
| $h, h$ | 3.004 | (0.014) | *** | 1.155 | (0.091) | *** | 0.025 | (0.012) | ** |
| $h, c$ | 3.027 | (0.024) | *** | 1.002 | (0.113) | *** | -0.002 | (0.026) | |
| $h, n$ | 3.017 | (0.050) | *** | 1.470 | (0.186) | *** | -0.011 | (0.043) | |
| $c, h$ | 2.946 | (0.046) | *** | 1.105 | (0.200) | *** | 0.009 | (0.047) | |
| $c, c$ | 2.990 | (0.031) | *** | 0.996 | (0.141) | *** | | | |
| $c, n$ | 2.629 | (0.180) | *** | 1.840 | (0.458) | *** | | | |
| $n, h$ | 2.998 | (0.024) | *** | 1.360 | (0.119) | *** | 0.016 | (0.018) | |
| $n, c$ | 2.984 | (0.030) | *** | 1.207 | (0.124) | *** | | | |
| $n, n$ | 2.956 | (0.035) | *** | 1.561 | (0.137) | *** | | | |
| Baseline measurement: | | | | | | | | | |
| PPVT | 2.295 | (0.009) | *** | 1.000 | - | | | | |
| WJIII | 3.375 | (0.005) | *** | 0.078 | (0.024) | *** | | | |
| Standard deviations: | | | | | | | | | |
| Factor $\theta$ | 0.252 | (0.011) | *** | | | | | | |
| Baseline PPVT error | 0.269 | (0.009) | *** | | | | | | |
| Baseline WJIII error | 0.208 | (0.004) | *** | | | | | | |
| Outcome error | 0.233 | (0.012) | *** | | | | | | |

*Notes* : N=1876. Log-likelihood = -3827.62. The model uses PPVT scores at the end of age-4 year as the outcome. The parameters of the outcome equation differ depending on the program sequence (see equation 7). The program sequence is reported in the first column. $h$ is for HS, $c$ is for other center care, $n$ is for home care. The first letter refers to the program at $t = 1$, the second to the program at $t = 2$ (for instance, $h, h$ denotes HS both at $t = 1$ and at $t = 2$). The parameters for the baseline measurement differ depending on which measurement is taken into account (either PPVT or WJIII, see equation 8). Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): ***1%, ** 5%, *10%.

Table A10: Estimates of the Parameters of the Choice Equations from the Sequential Threshold Model, using the Average of PPVT and WJIII Pre-Academic Skills Scores at the End of Age-4 Year as the Outcome

| Decision Node ($j$) | Covariate | Perceived Value | | | | | |
|---|---|---|---|---|---|---|---|
| | | Head Start ($U_{ij}^{h}$) | | | Other center care ($U_{ij}^{c}$) | | |
| o ($t=1$) | Intercept | -0.676 | (0.176) | *** | -1.035 | (0.285) | *** |
| | Factor $\theta$ | 0.015 | (0.302) | | 1.102 | (0.460) | ** |
| | Z | 1.980 | (0.168) | *** | | | |
| | $\theta \times$ Z | 0.159 | (0.373) | | | | |
| | hsq | -0.209 | (0.056) | *** | | | |
| | trp | -0.152 | (0.053) | *** | | | |
| | hsq $\times$ Z | 0.332 | (0.070) | *** | | | |
| | trp $\times$ Z | 0.157 | (0.072) | ** | | | |
| h ($t=2$) | Intercept | 1.506 | (0.105) | *** | 0.353 | (0.154) | ** |
| | Factor $\theta$ | -0.259 | (0.327) | | 0.693 | (0.527) | |
| | hsq | 0.099 | (0.045) | ** | | | |
| | trp | 0.020 | (0.044) | | | | |
| c ($t=2$) | Intercept | 1.407 | (0.344) | *** | 1.780 | (0.364) | *** |
| | Factor $\theta$ | -2.057 | (1.049) | ** | -1.266 | (1.125) | |
| | hsq | 0.111 | (0.097) | | | | |
| | trp | -0.205 | (0.107) | * | | | |
| n ($t=2$) | Intercept | 0.555 | (0.148) | *** | -0.048 | (0.138) | |
| | Factor $\theta$ | 0.142 | (0.354) | | -0.116 | (0.551) | |
| | hsq | 0.050 | (0.050) | | | | |
| | trp | 0.087 | (0.059) | | | | |
| Other utility parameters: | $\sigma_{\nu^c}$ | 1.849 | (0.458) | *** | | | |
| | $\rho_{hc}$ | 0.637 | (0.194) | *** | | | |

*Notes* : N=1876, Log-likelihood = -3114.93. Trp: HS transport availability. Hsq: HS center quality. The model uses the simple average of PPVT and WJIII Pre-Academic Skills scores at the end of age-4 year as the outcome. Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): *** 1%, ** 5%, *10%.

Table A11: Estimates of the Parameters of the Potential Outcome and Measurement Equations from the Sequential Threshold Model, using the Average of PPVT and WJIII Pre-Academic Skills Scores at the End of Age-4 Year as the Outcome

| | Intercept | | | Covariate Factor $\theta$ | | | HS quality | | |
|---|---|---|---|---|---|---|---|---|---|
| Potential outcome by program sequence: | | | | | | | | | |
| $h, h$ | 3.350 | (0.009) | *** | 0.813 | (0.058) | *** | 0.021 | (0.008) | *** |
| $h, c$ | 3.382 | (0.018) | *** | 0.803 | (0.074) | *** | -0.007 | (0.019) | |
| $h, n$ | 3.365 | (0.034) | *** | 1.180 | (0.123) | *** | 0.021 | (0.028) | |
| $c, h$ | 3.313 | (0.034) | *** | 0.859 | (0.137) | *** | 0.023 | (0.027) | |
| $c, c$ | 3.362 | (0.022) | *** | 0.728 | (0.072) | *** | | | |
| $c, n$ | 3.314 | (0.124) | *** | 1.114 | (0.286) | *** | | | |
| $n, h$ | 3.332 | (0.017) | *** | 0.927 | (0.079) | *** | 0.000 | (0.012) | |
| $n, c$ | 3.355 | (0.021) | *** | 0.968 | (0.079) | *** | | | |
| $n, n$ | 3.274 | (0.024) | *** | 1.099 | (0.091) | *** | | | |
| Baseline measurement: | | | | | | | | | |
| PPVT | 2.296 | (0.009) | *** | 1.000 | - | | | | |
| WJIII | 3.375 | (0.005) | *** | 0.159 | (0.024) | *** | | | |
| Standard deviations: | | | | | | | | | |
| Factor $\theta$ | 0.244 | (0.010) | *** | | | | | | |
| Baseline PPVT error | 0.275 | (0.008) | *** | | | | | | |
| Baseline WJIII error | 0.206 | (0.004) | *** | | | | | | |
| Outcome error | 0.153 | (0.008) | *** | | | | | | |

*Notes* : N=1876. Log-likelihood = -3114.93. The model uses the simple average of PPVT and WJIII Pre-Academic Skills scores at the end of age-4 year as the outcome. The parameters of the outcome equation differ depending on the program sequence (see equation 7). The program sequence is reported in the first column. $h$ is for HS, $c$ is for other center care, $n$ is for home care. The first letter refers to the program at $t = 1$, the second to the program at $t = 2$ (for instance, $h, h$ denotes HS both at $t = 1$ and at $t = 2$). The parameters for the baseline measurement differ depending on which measurement is taken into account (either PPVT or WJIII, see equation 8). Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): ***1%, ** 5%, *10%.

Table A12: Estimates of the Parameters of the Utility Function from the Structural Model, Jointly Estimating the Choice, Outcome and Measurement Equations.

| Decision Node ($j$) | Covariate | Flow Utility Head Start ($u^h_{ij}$) | | | Other center care ($u^c_{ij}$) | | |
|---|---|---|---|---|---|---|---|
| o ($t=1$) | Intercept | -1.874 | (0.579) | *** | -2.291 | (1.246) | * |
| | Factor $\theta$ | 0.866 | (1.660) | | 3.215 | (2.955) | |
| | Z | 2.029 | (0.158) | *** | | | |
| | $\theta \times$ Z | 0.141 | (0.378) | | | | |
| | trp | -0.175 | (0.066) | *** | | | |
| | trp $\times$ Z | 0.170 | (0.072) | ** | | | |
| | hsq $\times$ (1-Z) | -0.334 | (0.070) | *** | | | |
| h ($t=2$) | Intercept | 1.554 | (0.433) | *** | 0.282 | (0.454) | |
| | Factor $\theta$ | 2.095 | (1.909) | | 4.115 | (2.456) | * |
| | trp | 0.019 | (0.044) | | | | |
| c ($t=2$) | Intercept | 1.640 | (1.257) | | 1.582 | (1.184) | |
| | Factor $\theta$ | 1.266 | (3.356) | | 2.538 | (3.335) | |
| | trp | -0.163 | (0.092) | * | | | |
| n ($t=2$) | Intercept | 0.191 | (0.417) | | -0.240 | (0.400) | |
| | Factor $\theta$ | 1.742 | (1.357) | | 2.766 | (1.887) | |
| | trp | 0.062 | (0.058) | | | | |
| Other utility parameters: | | | | | | | |
| | $\tilde{\sigma}_{\nu^c}$ | 1.680 | (0.503) | *** | | | |
| | $\tilde{\rho}_{hc}$ | 0.563 | (0.210) | *** | | | |
| | $\kappa$ | 7.851 | (4.035) | * | | | |

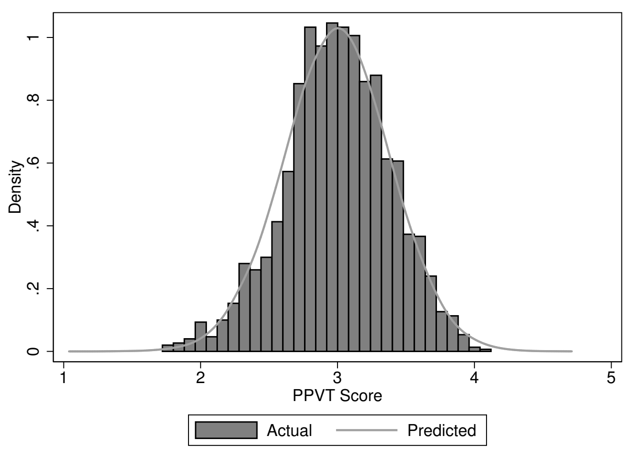
*Notes* : N=1876. Log-likelihood = -3830.42. The model jointly estimates the choice, outcome and measurement equations. Hence, differently from the estimates reported in Section 5, we do not fix the 30 technological coefficients of the outcome and measurement equations to the levels estimated by the sequential threshold model. trp: HS transport availability. hsq: HS center quality. The model uses PPVT score at the end of age-4 year as the outcome. Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): *** 1%, ** 5%, *10%.

Table A13: Estimates of the Parameters of the Potential Outcome and Measurement Equations from the Structural Model, Jointly Estimating the Choice, Outcome and Measurement Equations

| | Intercept | | | Covariate<br>Factor $\theta$ | | | HS quality | | |
|---|---|---|---|---|---|---|---|---|---|
| Potential outcome by program sequence: | | | | | | | | | |
| $h, h$ | 3.004 | (0.014) | *** | 1.136 | (0.094) | *** | 0.024 | (0.011) | ** |
| $h, c$ | 3.025 | (0.024) | *** | 0.982 | (0.112) | *** | 0.005 | (0.008) | |
| $h, n$ | 3.014 | (0.051) | *** | 1.455 | (0.187) | *** | 0.017 | (0.011) | |
| $c, h$ | 2.942 | (0.044) | *** | 1.004 | (0.183) | *** | 0.004 | (0.010) | |
| $c, c$ | 2.998 | (0.030) | *** | 0.951 | (0.125) | *** | | | |
| $c, n$ | 2.995 | (0.159) | *** | 1.376 | (0.387) | *** | | | |
| $n, h$ | 2.997 | (0.024) | *** | 1.338 | (0.121) | *** | 0.011 | (0.008) | |
| $n, c$ | 2.983 | (0.030) | *** | 1.178 | (0.123) | *** | | | |
| $n, n$ | 2.957 | (0.036) | *** | 1.529 | (0.139) | *** | | | |
| Baseline measurement: | | | | | | | | | |
| PPVT | 2.296 | (0.009) | *** | 1.000 | - | | | | |
| WJIII | 3.375 | (0.005) | *** | 0.080 | (0.024) | *** | | | |
| Standard deviations: | | | | | | | | | |
| Factor $\theta$ | 0.255 | (0.012) | *** | | | | | | |
| Baseline PPVT error | 0.266 | (0.009) | *** | | | | | | |
| Baseline WJIII error | 0.208 | (0.004) | *** | | | | | | |
| Outcome error | 0.236 | (0.013) | *** | | | | | | |

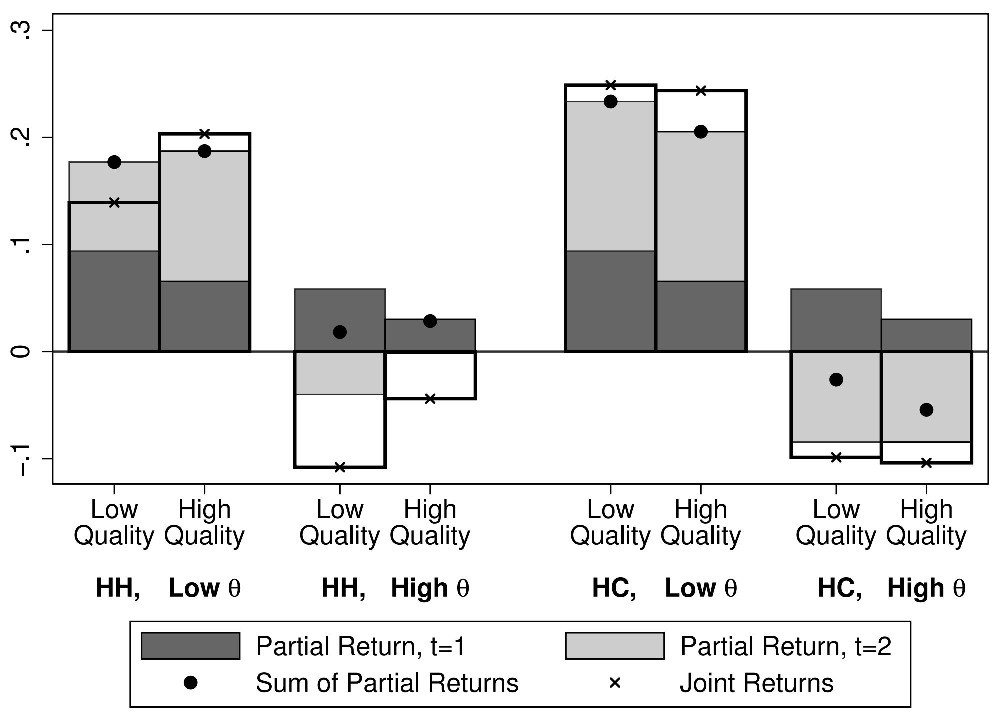
*Notes* : N=1876. Log-likelihood = -3830.42. The model uses PPVT scores at the end of age-4 year as the outcome. The model jointly estimates all parameters from the choice, outcome and measurement equations. The parameters of the outcome equation differ depending on the program sequence (see equation 7). The program sequence is reported in the first column. $h$ is for HS, $c$ is for other center care, $n$ is for home care. The first letter refers to the program at $t = 1$, the second to the program at $t = 2$ (for instance, $h, h$ denotes HS both at $t = 1$ and at $t = 2$). The parameters for the baseline measurement differ depending on which measurement is taken into account (either PPVT or WJIII, see equation 8). Standard errors are in parentheses. Significance level (t-test for testing if each parameter=0): ***1%, ** 5%, *10%.

Figure A1: Goodness of Fit, Distribution of PPVT Score at Age 4



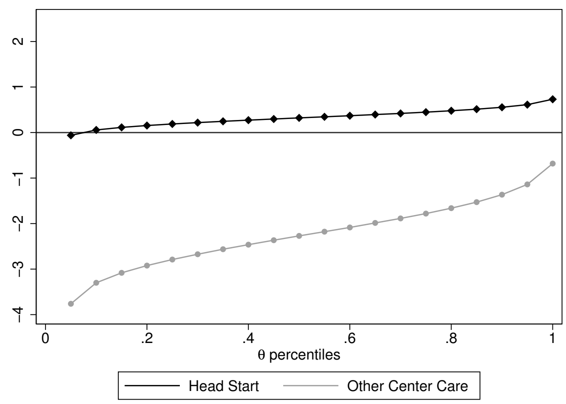

*Notes* : Comparison between the actual distribution of PPVT score obtained at the end of age-4 year as in the population and the distribution of the same PPVT score as predicted by our sequential threshold model.

Figure A2: Partial, Joint and Cross Returns from Sequential Program Participation, by Different HS Center Quality

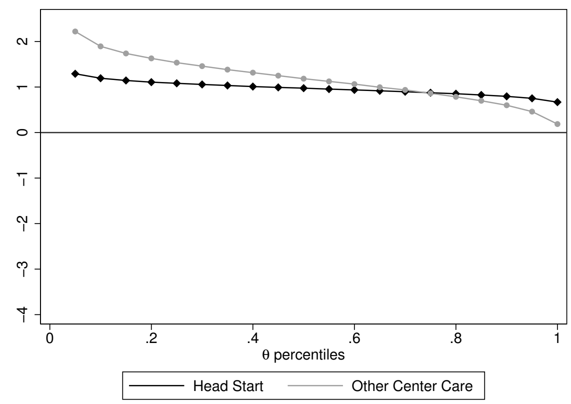



*Notes* : The outcome analyzed is the PPVT test score obtained at the end of age-4 year. The first four bars show the return from two periods in HS, by different values of baseline ability $\theta$ and different center quality. The last four bars show the returns from one period in HS and one in other center care by different values of baseline ability $\theta$ and different center quality. Low $\theta$ corresponds to the $10^{th}$ percentile of the baseline ability distribution, High $\theta$ corresponds to the $90^{th}$ percentile. As explained in Section 3, we define the quality of a HS center as the residualized quality due to the concern that the characteristics of the center may be correlated with family characteristics. We take the residuals of a regression of the HS center quality on household size, number of siblings, dummies for whether the child is female, black, hispanic, use English as home language, living in urban area, living with both parents, in need of special education, child of a teen mother, child of a mother who never married or is separated, child of a mother with high school or more than high school. In this graph, we define a low quality center as one with quality corresponding to the $10^{th}$ percentile of the residualized quality distribution, while high quality is a center with quality corresponding to the $90^{th}$ percentile.

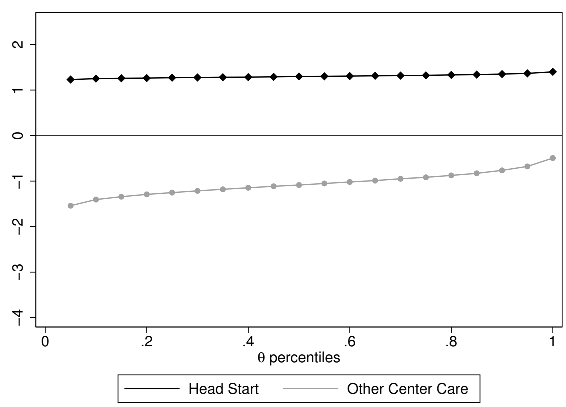
Figure A3: Flow Utility, Perceived Continuation Value and Overall Perceived Value of Attending Head Start and Other Center Care at $t = 1$ Relative to Home Care for Different Values of $\theta$




(a) Flow Utility Relative to Home Care ($n$)



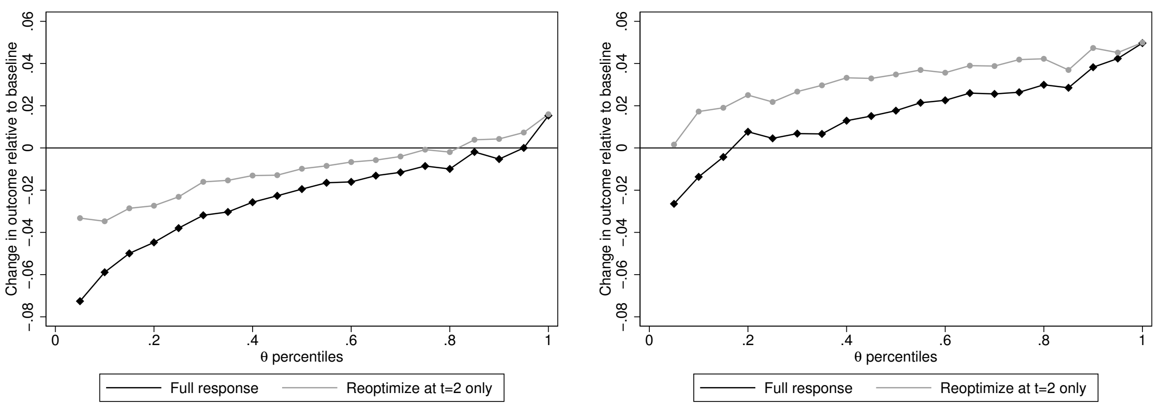

(b) Continuation Value Relative to Home Care ($n$)




(c) Overall Perceived Value Relative to Home Care

*Notes* : On each curve, each marker represents one ventile of the distribution of $\theta$ (i.e. baseline ability). Panel (a) shows the flow utility of attending HS or other center care at $t = 1$ as compared to the flow utility from home care. Panel (b) shows the perceived continuation value of attending HS or other center care at $t = 1$ as compared to the perceived continuation value of home care. Panel (c) is the sum of the values in Panels (a) and (b) and shows the difference between the total perceived value of attending HS or other center care and the total perceived value of home care. All results are derived for treatment group individuals from the structural model.

Figure A4: Effect of Limiting Head Start Enrolment to One Period Only for Different Values of $\theta$, High and Low Quality of the HS Center

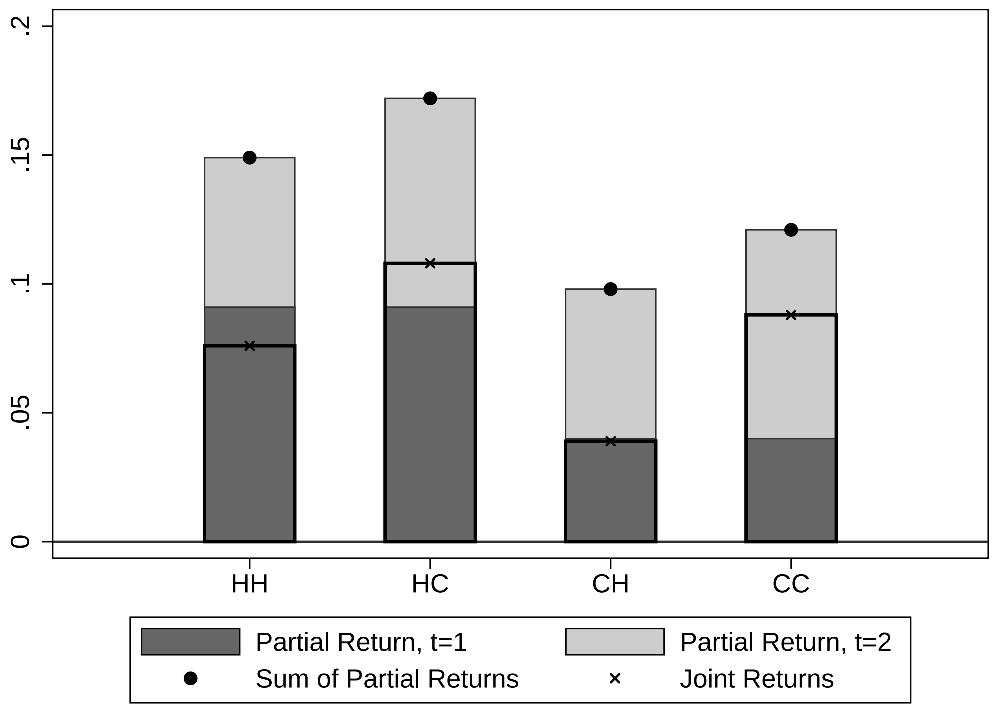



(a) High Quality HS Center

(b) Low Quality HS Center

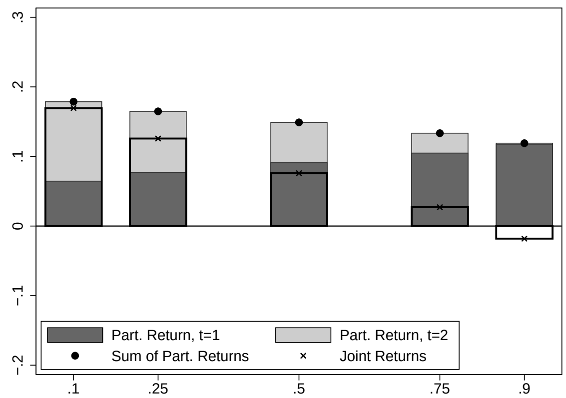
*Notes* : The outcome analyzed is the PPVT score obtained at the end of age-4 year. Each curve represents deviations in outcomes from the baseline scenario in the treatment group, e.g., zero means no effect of the HS enrolment limit on the outcome. On each curve, each marker represents one ventile of the distribution of $\theta$ (i.e. baseline ability). Panel (a) shows the result for high quality HS centers, while panel (b) is for low quality (where high quality is defined as the top quartile of the quality distribution and low quality as the bottom quartile). All scenarios are simulated from the structural model with 30 paths per treatment-group individual. "Full response": allow individuals to reoptimize at t=1 and t=2. "Reoptimize at t=2 only": assume individuals at t=1 do not know about the HS enrolment limit, so they can reoptimize at t=2 only.

Figure A5: Average Partial, Joint and Cross Returns from Sequential Program Participation, using the Average of PPVT and WJIII Pre-Academic Skills Scores at the End of Age-4 Year as the Outcome

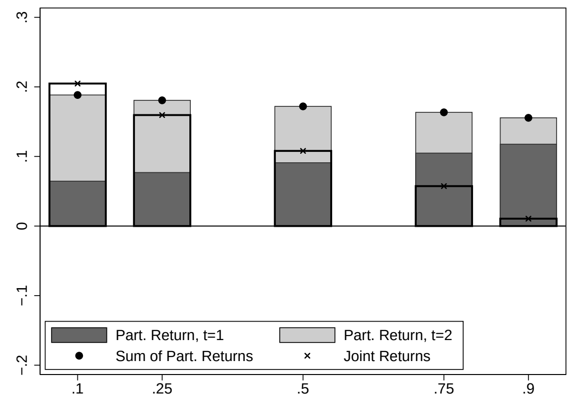



*Notes* : The outcome analyzed is the simple average of PPVT and WJIII Pre-Academic Skills scores at the end of age-4 year. Each bar represents the joint and partial return of different program sequences. The first bar (HH) shows the return of receiving HS both at $t = 1$ and at $t = 2$. The second bar (HC) shows the return of receiving HS at $t = 1$ and other center care at $t = 2$. The third bar (CH) shows the return of receiving other center care at $t = 1$ and HS at $t = 2$. The fourth bar (CC) shows the return of receiving other center care in both periods. The cross returns corresponds to the difference between the joint return and the sum of the partial returns. All these returns use two periods in home care as the baseline program sequence for comparison (see Section 4.1 for the exact definition of partial, joint and cross-returns).

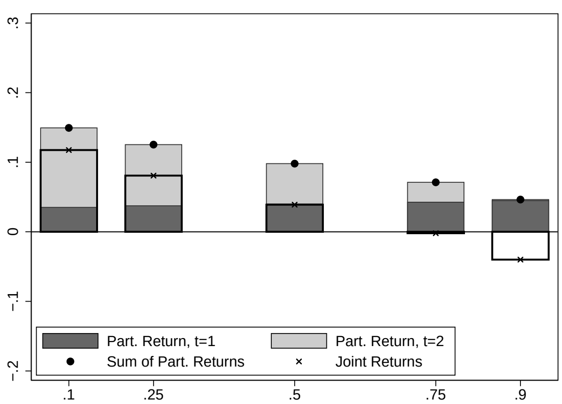
Figure A6: Partial, Joint and Cross Returns from Sequential Program Participation, using the Average of PPVT and WJIII Pre-Academic Skills Scores at the End of Age-4 Year as the Outcome, Different Values of $\theta$



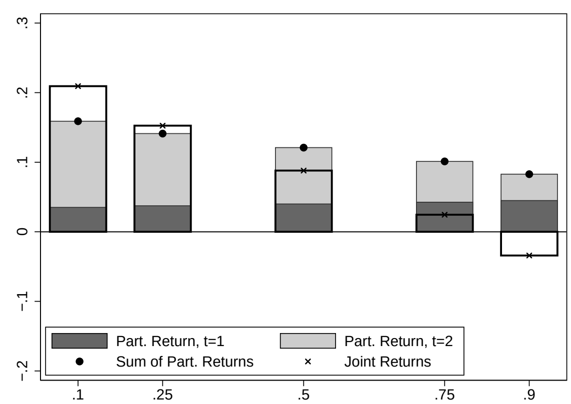

(a) HH




(b) HC




(c) CH




(d) CC

*Notes* : The outcome analyzed is the simple average of PPVT and WJIII Pre-Academic Skills scores at the end of age-4 year. Within each panel, the different bars represent the joint and partial returns of the same program sequence, but for different levels of baseline ability $\theta$ ("Part. Return" in label stands for "Partial Returns"). In particular, we report the returns for the 10th, the 25th, the 50th, the 75th and the 90th percentile of the $\theta$ distribution. Panel (a) is the return for two periods in HS (HH), panel (b) for one period in HS followed by other center care (HC), panel (c) for one period in other center care followed by HS (CH) and panel (d) for two periods in other center care (CC).