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Online appendix:
https://www.econ.cam.ac.uk/research-files/docs/Internet_Appendix_Spinoffs_FinancialReview_Final.pdf

Reference Details

2347	Cambridge Working Papers in Economics
2312	Janeway Institute Working Paper Series
Published	30 June 2023
Keywords	Voluntary Firm Creation, Spin-Offs, Repeated Innovations
JEL-codes	M13, O31, O33
Websites	www.econ.cam.ac.uk/cwpe www.janeway.econ.cam.ac.uk/working-papers

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June 2023 ‡

forthcoming
The Financial Review

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Firms can voluntarily create independent firms to implement their technologically distant innovations and capture their value through capital markets. We argue that when firms repeatedly compete to make innovations, there is inefficient external implementation of innovations and "excessive" creation of such firms. This inefficiency is most exacerbated in the early stages of an industry, when the number of firms is still limited.

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‡We would like to thank the editor and reviewers as well as conference participants and discussants at the 11th (2018) annual conference on innovation economics at Northwestern Pritzker School of Law in Chicago, the 5th (2019) workshop on Corporate Governance at Toulouse Business School, and the 5th (2019) inter business school Finance seminar at Neoma, Reims, as well as seminar participants at Bristol University, Paris-Dauphine University, Edhec Business School, EM-Lyon, the Judge Business School in Cambridge, Luiss University in Rome, Skema Business School, for helpful comments.

1 INTRODUCTION

A firm which makes an innovation can implement it in its organization to generate cash. However, when the characteristics of the innovation are distant from those of the firm, the innovation would generate more cash if implemented in a new specialized organization, created with adapted characteristics.[1] The inventing firm, protected from expropriation by property rights, can obtain this outside value through capital markets: The firm voluntarily creates an independent firm that has all property rights to exploit the innovation and offers financial securities on it. We refer to the newly created firm as a spin-off firm.[2]

This rationale for firm creation is not ours. It is based on distance in the technological space and comes from the resource-based view of the firm. This paper argues that, when firms repeatedly compete to devise innovations, innovations lead too frequently to the voluntary creation of spin-off firms. The issue gradually fades away as the number of firms increases.

We model a dynamic industry in which firms repeatedly race with each other to make the next innovation. For a firm to have a chance of making the innovation, it employs research agents and compensates them for seeking to innovate. The inventing firm's decision to create a spin-off firm is characterized by an implementation threshold: the innovation is implemented inside the inventing firm when the distance between the characteristics of the innovation and the firm is less than a threshold; conversely, the innovation is implemented outside, in newly created firm, if the distance is larger. When the innovation game is played only once, the solution has a simple implementation threshold that is efficient.

In a repeated game, the following feature affects the implementation threshold: if an innovation is implemented inside the inventing firm, in the next race to make an innovation,

[1] In this paper, innovations refer to primary inventions and products whose competitive advantage is based on superior functional performance, offer high unit profit margin, and may require a reorientation of production facilities as well as corporate goals. These differ from deriving minor product and system improvements, whose value we take as embedded in that of a primary innovation. Because the successful exploitation of one innovation may require a costly reorientation of production facilities as well as corporate goals of the inventing firm, more aggregate profits can often be obtained if a new dedicated firm is created.

[2] In practice, when a firm voluntarily creates a firm and takes it to the market, it does so through a corporate spin-off or a carve-out: in a spin-off, the parent company distributes shares of the spun-off firm to its existing equityholders on a pro rata basis, in the form of a special dividend; in a carve-out, the parent company sells some or all of the shares in the new firm to the public through an initial public offering (IPO).

each existing firm employs again the same number of research agents; conversely, if the innovation is implemented in a spin-off firm, this new firm will employ additional research agents to compete for the next innovation.

The driving force behind this feature is a firm boundary friction: the research efforts of one agent (i) enhance the research benefits of all other agents in the firm and (ii) do not enhance the research benefit of agents in competing firms.[3] Then each firm suffers from the classic Holmstrom (1982) moral hazard in teams problem, as in equilibrium each agent provides a reduced level of effort. A firm does not seek to have a larger team of agents than others, as employing more agents would not increase its chance of winning the innovation race. Similarly, after the innovation is made, if it is implemented inside the inventing firm, existing firms do not benefit from employing more agents in the following race to innovate. Conversely, if the innovation is implemented in a spin-off firm, this wholly separate firm benefits from employing additional agents to build its own research team.

Capturing the outside market value of the innovation through a spin-off firm therefore entails additional benefits and costs, over building up a conglomerate firm:

– On the positive side, the market value of the innovation also includes the fraction of the aggregate expected value of further innovations, captured by the spin-off firm as it employs research agents. This market value can in turn be captured by the parent firm. The value captured by the parent firm is not an overall increment, but a transfer away from competitors of future innovations value.

– On the negative side, the emergence of an additional competitor imposes a negative cost on the parent firm, as the likelihood of making future innovations has to be shared amongst a larger number of firms.

These additional benefits and costs of firm creation would not be relevant (present) if the firm creation game were one-shot. They are however substantial in a dynamic settings.

[3]Knowledge flows between the agents of a firm and the output is common. Particularly when the output is a major innovation. Researchers have access to interim progress made by others. It is difficult to grant paternity on a final innovation to a subset of researchers and exclude others from rewards for success. The organization studies and organizational behaviour literatures highlights the significance of intra-firm knowledge transfers between insiders of a firm. In comparison, inter-firm knowledge transfers are limited. See Osterloh and Frey (2000), Argote and Ingram (2000), Argote, Ingram, Levine and Moreland (2000), Tsai (2001), Watson and Hewett (2006).

We show that under fairly general conditions the positive effect dominates the negative one and, as a result, some innovations are inefficiently implemented in spin-off firms. That is, there is "excessive" spin-off firm creation relative to the first-best case.

Our argument applies most to young and innovative industries: Spin-off firm creation is highest when the number of firms is still limited, innovations occur frequently, and setting up a firm is not costly. It is stronger when researchers are not given large equity stake incentives. The strength of the argument decreases as new firms are created. Spin-off firm creation gradually tends to the efficient level when the number of firms becomes large.

Our modelling makes that our analysis has clear limitations. In particular:

– Innovations are verifiable and we consider that intellectual property rights protect perfectly inventing firms from expropriation by their agents. Now, many innovations are non-verifiable and often lead to entrepreneurial spin-outs: In contrast to spin-offs which are firms voluntarily created by a parent firm, spin-outs are firms created by employees departing a parent firm to entrepreneurship without a formal transfer of ownership rights.[4] Some theories of spin-outs are based on asymmetries of information and private learning.[5] Others are based on imperfect evaluations of opportunities.[6]

– Innovations cannot be sold directly to other existing firms. However, some inventions are sold on the market for ideas.[7] A body of the strategy literature (see Casson (1982),

[4]Firms set-up by employees departing from a original firm are documented by Klepper and Sleeper (2005) and Sherer (2006) in the laser industry, by Agarwal, Echambadi, Franco and Sarkar (2004) and Franco and Filson (2006) for disk drives, by Klepper (2007) for cars, by Chatterji (2009) for medical devices and by Buenstorf and Klepper (2009) for tires. Babina and Howell (2018) document a causal effect of R&D investment on new firm creation by parent firm employees.

[5]In Anton and Yao (1995), agents generate ideas and do not reveal them to their principals in order to create their own firm. Firm creation cannot be prevented because there is adverse selection due to private discovery, agents have limited liability, and patents do not provide complete protection for the principals. In Agarwal, Echambadi, Franco and Sarkar (2004), Franco and Filson (2006) and Franco and Mitchell (2008) employees privately learn from their employers and then exploit this knowledge by creating a spin-out.

[6]In Klepper and Sleeper (2005) successful R&D investments generate opportunities that are accessible only to the firm and its employees and the occurrence of spin-outs is explained by the likelihoods that the firm and the employees recognise these opportunities. In Klepper (2007) and Klepper and Thompson (2010), a firm's strategy regarding implementation of opportunities is chosen by a team of decision makers who each imperfectly evaluate these opportunities. An employed manager chooses to start his firm when his disagreement with the firm strategy exceeds the cost of setting up a new firm.

[7]Shane (2001) finds that, primary inventions, more radical inventions, and inventions with a broader

Audretsch (1995), Christensen (1997)) studies the transfer of innovations between existing firms, through licensing agreements, strategic alliances, or acquisitions of start-ups.[8]
– No new firm is set up without an innovation and no firm disappears.

At the end of the paper, we discuss possible extensions of the model and argue that our qualitative conclusions on creation of spin-off firms and their dynamics should hold.

Related Literature.
Several theories have been proposed to rationalize the voluntary creation of value in spin-offs. In Habib, Johnsen and Naik (1997) and Nanda and Narayanan (1999), the increase in market value of the parent and the spun-off firms, comes from the fact that securities trading improves the quality of the information investors can infer from prices. In Scharfstein and Stein (2000), Rajan, Servaes and Zingales (2000) and Gertner, Powers and Scharfstein (2002) the gain in value comes from an improved internal allocation of capital. Chemmanur and Yan (2004) put forward the disciplining effects of spin-offs on firm management. Aron (1991) argues that spin-offs permit to provide better incentive contracts to management. Fulghieri and Sevilir (2011) argue that spin-offs enhance employee mobility across competing firms, thus improves employee incentives to exert innovation effort.

We uncover a motive for voluntary spin-offs related to the innovation process. The added value of a spin-off to an inventing firm (relative to building up a conglomerate firm) is that it allows to capture some of the value of future innovations from the firms it competes with to make innovations. The source of gain in value is the ability to provide incentives contracts to research agents in two separate firms, instead of just one.

Several theories have been proposed to rationalize the link between innovation and voluntary firm creation. A stream of theories comes from the resource-based view of the firm, developed in Wernerfelt (1984), Dierickx and Cool (1989), Chatterjee and Wernerfelt (1991), Peteraf (1993), and often attributed to Penrose (1959) and Chandler (1962). As mentioned, we

scope of patent protection are more likely to be commercialized through the creation of new firms.

[8] Gans, Hsu and Stern (2002) and Gans and Stern (2003) study the importance of imperfections in the market for ideas on the choice between competing on the product market with existing firms or selling the innovation to one of them. Arora and Fosfuri (2003) considers the impact of competition amongst technology holders on licensing. Arora, Fosfuri and Ronde (2013) study the management of licensing in corporations with internal conflicts of interests between business units. Akcigit, Celik and Greenwood (2016) develop an endogenous growth model where patents can be sold to another existing firm.

borrow the basic rationale for firm creation from this view. This view, ascribes high value to resources and capabilities that are scarce, specialised and imperfectly mobile. Then local dominance and high switching costs open-up benefits for firm creation when new products are away from the parent firm dominant area.[9]

Another stream of theories is about the strengths and weaknesses of internal versus external capital markets. Although not formulated in this context, the arguments directly extend to firm creation versus internal exploitation of innovations. In Gertner, Scharfstein and Stein (1994), with external financing, managers have higher ex ante effort incentives, because they are not vulnerable to ex post opportunistic behaviour by corporate headquarters. In Amador and Landier (2003), firm creation is attributed to the greater contractual flexibility of external versus internal financing. In Gromb and Scharfstein (2012), safety being bad for incentives, firm creation provides entrepreneurs with high-powered incentives ex ante. In Robinson (2008), headquarters can reallocate capital ex-post from low to high productivity projects, which can be ex-ante inefficient because of managerial incentives. Then strategic alliances facilitate the commitment to abandon winner-picking.

The paper is organized as follows: Section 2 describes the set-up of the model. Section 3 examines the equilibrium outcome. Section 4 studies the dynamics of spin-off firm creation. Section 5 examines the implications on the dynamics of observable firm characteristics and discusses related empirical evidence. Section 6 assesses the extent to which the equilibrium behaviour is inefficient. Section 7 discusses possible extensions and concludes.

2 SET-UP

2.1 Firms, Innovations and Spin-Off Firm Creation

Consider an industry which evolves in discrete time starting from date 0. The set of discrete dates is either finite or countably infinite. We shall denote the horizon by T, where T is either finite or ∞. At any date, the industry consists of a finite set of firms (the set of firms may increase over time), with typical element denoted by f. Denote the set of firms existing

[9] In Cassiman and Ueda (2006), the firm rejects the commercialization of innovations that do not fit well with its internal resources. In Habib, Hege and Mella-Barral (2013), a new firm is created if the fit between the new product and its parent firm organization is not adequate.

at date t by $\mathcal{F}_t$. Assume $|\mathcal{F}_0| > 1$. All firms are financed by equity and capital markets are complete. For ease of exposition, consider that the equityholders of firm f delegate authority to one of them, we refer to as the principal p, and that p takes decisions in the best-interest of the equityholders of firm f.[10]

A firm is a specialized organization capable of (a) producing innovations (referred to as exploration) and (b) generating cash from these innovations (referred to as exploitation).[11] Each firm f has an expertise $x^f \in \mathbb{R}$, selected upon creation. A firm has and can only have one expertise. This expertise cannot be changed later.

Investors are unable to innovate. Crafting innovations requires skilled individuals, henceforth referred to as agents. There is a countable infinite number of agents, equally capable of producing innovation. The principal of a firm can offer an employment contract to as many agents as she wishes. Each agent who accepts the contract is provided access to the firm's organization to produce research, on its behalf. We assume:

Assumption 1. *Innovations are verifiable but cannot be sold off directly to non-inventing firms. Intellectual property rights (patents) protect perfectly the equityholders of the inventing firm from expropriation by employed agents or equityholders of other firms.*

Firms repeatedly race with each other to make the next innovation. One innovation i_t is to be made at every date $t \in \{1, \ldots, T\}$. All participants discount future cash flows at the unit-period rate of $\rho \in \mathbb{R}_{>0}$. Then, for given market conditions, a small (high) ρ can also be regarded as characterising an industry where innovations are frequent (rare).

In each race, all identify the upcoming innovation by a set of superior functional performances to be brought to the market. The challenge is one of feasibility. Firms race to be the first to patent an exploitation process which brings these target functional performances to the market. This consists of a feasible technical production process to be followed to generate cash from the innovation. Any such exploitation process has an associated ideal

[10] Any decision of p is taken maximizing the value of the equity of firm f, because at any time her payoff is a fraction of it. We do not model the incentives of managers and their private costs, ignoring conflicts of interests between shareholders and managers (governance problems) analyzed in Hart (1993). We do not model the market for managers. There exists a countable infinite number of equally competent investors willing to manage firms against their reservation wage of zero.

[11] The terminology exploration and exploitation is borrowed from March (1991).

firm expertise to have to generate most cash from the patented innovation.

We shall refer to the required firm expertise to exploit most profitably the date-t innovation i_t as the characteristic of the realized innovation and denote it by $x^{i_t} \in \mathbb{R}$. We measure the relative proximity between the ideal firm expertise demanded by innovation i_t and the actual expertise of firm f by

$$w(i_t, f) \;\; \equiv \;\; \exp\left[-\left|x^{i_t} \; - \; x^f\right|\right] \; . \tag{1}$$

The expertise of a firm, x^f, determines the characteristics of innovations it is (a) most capable of generating cash from and (b) most likely to make:

(a) Consider that, if firm f makes the date-t innovation i_t, the expected value at date t of the cash-flows f can generate exploiting i_t is simply equal to $w(i_t, f)$.

(b) Consider that date-t innovation characteristic x^{i_t} follows a distribution located around the expertise of the firm which invents it, x^f. For convenience, suppose that, if firm f makes the date-t innovation i_t, then the proximity $w(i_t, f)$ is uniformly distributed over $[0, 1]$, i.e. $g(w(i_t, f) = w) = 1$ for all $w \in [0, 1]$.[12]

If firm f makes the date-t innovation i_t, its principal p can surely decide to implement i_t in the firm, in which case firm f's equityholders obtain a payoff whose expected value at date t equals $w(i_t, f)$. However, the innovation does not have to be exploited in the inventing firm f. In some instances, an innovation is more profitably exploited in a new firm created specifically around it:

The advantage of creating a new firm, f^+, is that it gives the opportunity to choose the expertise of the new firm, x^{f^+}, *after* the date-t innovation i_t is made (knowing x^{i_t}). By setting a firm with expertise $x^{f^+} = x^{i_t}$, the creators of the new firm can insure that the proximity between the newly created firm f^+ and the innovation i, $w(i_t, f^+)$ equals 1. The disadvantage of creating a new firm is that it entails a cost $\kappa \in \mathbb{R}_{>0}$.

κ is the cost of setting up a specialized organization capable of producing and generating cash from a given innovation. The resource-based view of the firm posits that a firm's

[12] This amounts to assuming that x^{i_t} follows a Laplace distribution with probability density function

$$g_f(x^{i_t}) \;\; = \;\; \frac{1}{2} \exp\left[-\left|x^{i_t} \; - \; x^f\right|\right] \; . \tag{2}$$

organization is specialized to specific tasks and environments, hence has a limited ability or capacity to generate cash from diverse innovations. We modelled this considering that a firm f has only one expertise x^f and that expertise cannot be changed later. The latter consists of assuming that changing expertise is excessively costly. More specifically, it consists of assuming that for an inventing firm, the cost of setting up additional production facilities and orienting corporate goals to a technologically distant innovation, while preserving at the same time the exploitation value of existing products, exceeds κ.

Consider that these decisions can be taken and executed without delay at date t. Overall, the exploitation of the date-t innovation i_t, by a newly created firm f^+, can generate an alternative payoff whose expected value at date t equals $1-\kappa$.

Essentially, exploitation of a date-t innovation i_t within the inventing firm f can only be imperfect, because its' expertise was chosen before the innovation is made (at a date prior to t). Firm creation, is a costly way of overcoming this imperfection, because of the associated set-up cost κ. It is therefore optimal for the innovation i_t to be implemented at date t (a) in firm f when $w(i_t, f)$ is higher than a certain threshold level and (b) in a newly created firm f^+ when $w(i_t, f)$ is below that threshold. Notice that if $\kappa > 1$, the most profitable exploitation of the innovation never involves creating a new firm.

The equityholders of the inventing firm f capture the market value of the new firm using capital markets, through a spin-off: the principal p (on behalf of firm f's equityholders) issues separate shares on an independent firm f^+, created around the innovation with all property rights to exploit it, against a set-up cost κ. The expertise of the spin-off firm is set up to the ideal $x^{f^+} = x^{i_t}$ upon creation. All property rights on innovation i_t are transferred to the new firm. The equityholders of the new firm f^+ appoint a new principal p^+ to take management decisions in their best-interests.[13] Shares of firm f^+ can be traded at competitive prices (such as the seller extracts all the surplus from the purchaser).

[13]Nothing precludes the principal p^+ from being the principal p firm herself, in which case p creates firm f^+ and manages several independent separate firms, f and f^+. In the absence of monopoly rents and in a frictionless environment, the shareholders of firm f are indifferent between the two routes.

2.2 Innovation Process and Implementation

Only one firm amongst those existing at date $t-1$ can make the date-t innovation i_t. Suppose that after the implementation of the previous innovation i_{t-1} there are F firms existing at date $t-1$, i.e $F = |\mathcal{F}_{t-1}|$. The race to devise the date-t innovation i_t starts just after the race for previous innovation i_{t-1} ends at date $t-1$. This race involves two phases, each with decisions taken at two different dates: Phase A is related to the innovation process and involves decisions taken at date $t-1+\epsilon$; Phase B is related to the implementation of the innovation and involves a decision taken at date t. Specifically:

• Phase A - The innovation process. At date $t-1+\epsilon$:

– The principal of each existing firm f selects a set of agents and the unit-period incentive contract offered to each of these agents. Contracts are discussed just below.
Denote the set of agents who accept the offer by $\mathcal{A}^*_{f,t-1}$. This set of agents constitutes the research team of firm f. A research team must have a minimum number $\alpha \in \mathbb{Z}_{>0}$ of agents to have a chance of making the next innovation.[14]

– All agents exert a base effort incurring no associated private cost. Each agent $a \in \mathcal{A}^*_{f,t-1}$ can choose to exert an extra effort $e^a_{f,t-1} \in \mathbb{R}_{\geq 0}$ bearing a private cost $e^a_{f,t-1}$ at date $t-1+\epsilon$.

Extra efforts translate into research output and the probability that any firm makes the innovation depends on its research output relative to the total. Denoting the research output of firm f by $n_{f,t}$, we assume that it takes the following form:

$$n_{f,t} = \left(\sum_{a \in \mathcal{A}^*_{f,t-1}} e^a_{f,t-1} \right)^{\theta} , \tag{3}$$

if $|\mathcal{A}^*_{f,t-1}| \geq \alpha$; otherwise, $n_{f,t} = 0$. $\theta \in (0,1)$ is the rate of transformation of extra effort to research output.[15] Denoting $q_{f,t}$ the probability firm f makes the date-t innovation i_t:

$$q_{f,t} = \frac{n_{f,t}}{\sum_{f' \in \mathcal{F}_{t-1}} n_{f',t}} , \tag{4}$$

if there exists $f' \in \mathcal{F}_{t-1}$ such that $n_{f',t} \neq 0$; otherwise, $q_{f,t} = 1/F$.

• Phase B - Implementation of the innovation. At date t:

[14] α is introduced because generating a primary innovation typically requires more than one researcher. However, no result in this paper rests on the minimum number of agents α being strictly greater than one.

[15] θ is less than one, otherwise agents would not exert finite levels of extra effort.

The date-t innovation i_t happens. Its characteristic, x^{i_t}, is chosen according to the distribution $g_f(x^{i_t})$ in (2). Payments to employed agents according to contracts are made. Once payments are made, all control rights on the innovation are held by the principal of the firm.

– The principal p of the successful firm (the firm that innovates) f observes the characteristic, x^{i_t}, of the innovation i_t and decides to implement the innovation in firm f or in a new firm f^+. Clearly, the implementation decision at date t depends on $w(i_t, f)$, the relative proximity of i_t from the successful firm f.

If the date-t innovation is implemented in the successful firm, $\mathcal{F}_t$, the set of firms created up to date t, is equal to $\mathcal{F}_{t-1}$, hence the number of firms is unchanged, $|\mathcal{F}_t| = F$. Conversely, if the innovation is implemented in a spin-off firm f^+, $\mathcal{F}_t$ is equal to $\mathcal{F}_{t-1} \cup f^+$ and the number of firms increases by one, $|\mathcal{F}_t| = F + 1$. No new firm other than f^+ is set up at date t. Already created firms continue existing unchanged without incurring again set-up costs.

Contracts: The principal p of firm f selects the unit-period contract she offers each agent at date $t - 1 + \epsilon$, in order to incentivise agents to exert efforts. Contracts are incomplete in that an agent's effort level, $e^a_{f,t-1}$, and a firm's research output, $n_{f,t}$, are not contractible. Research output is common, hence contracts cannot single out ex-post a subset of more worthy agents and provide them higher benefits in case of innovation.

Contracts cannot impose penalties on the agents, hence contracted ex-post payoffs are non-negative. This directly implies that in any optimal contract, the payment promised to an agent, in case the firm's team of agents is unsuccessful in innovating, will be set to zero: The principal has no reason to give any reward for failing.

Since a firm agent's effort level influences the likelihood of devising the innovation, $q_{f,t}$, but not the characteristics of the innovation itself, x^{i_t}, the optimal incentive contract is simple: If the agents are successful in making the innovation i_t, each agent a employed by firm f receives a fixed compensation $b_{f,t} \in \mathbb{R}_{\geq 0}$.

The repeated game is detailed in the Appendix.

3 EQUILIBRIUM BEHAVIOUR

We derive a symmetric Markov perfect Bayesian equilibrium strategy that is anonymous with respect to the identity of the players. The full details of the equilibrium strategy can be found in Theorem 1, in the Appendix. All proofs can be found in an internet Appendix.[16] Here, we begin describing the three properties of the equilibrium outcome. We then provide the equilibrium spin-off firm creation and value of future innovations. These are all the features and results we need to analyse the dynamics discussed later in the paper.

3.1 Properties of the Equilibrium Outcome

For any date-t innovation i_t with F existing firms, where $t \in \{1, \ldots, T\}$, the outcome induced by the equilibrium strategy in Theorem 1, has the following three properties:

Property 1. *The principal p of each existing firm f employs α agents to seek to make the date-t innovation i_t.*

Property 2. *If they make the date-t innovation i_t, the α successful agents share a fraction ϕ_F of the value of the innovation at date t, where*

$$\phi_F \;=\; \theta \left(1 - \frac{1+\theta}{F+\theta}\right) \; . \tag{5}$$

Property 3. *The probability each firm f makes the date-t innovation i_t is equal to $1/F$.*

The intuition behind Property 1 is as follows:

From (3) and (4), the research efforts of one agent enhance the research benefits of all other agents in the firm. Contracts cannot isolate-out low effort exerting agents from innovation benefits. Then in equilibrium, each agent provides a reduced level of effort. A firm must employ α agents or more to have a chance of making the next innovation, but having more than α agents would not increase the aggregate level of effort provided by the agents of the firm. Essentially, no firm benefits from employing more than α agents because the production of research suffers from a Holmstrom (1982) moral hazard in teams problem. No firm seeks to have a larger research team than that other firms, as doing so would not increase its chance of making the next innovation.

[16]The internet appendix is available in the supporting materials section online.

The above holds irrespective of the history of innovations implemented in a firm. It follows that when a firm f makes an innovation and the innovation is implemented inside the inventing firm, each existing firm employs again the same number of agents in the following race to innovate than they did in the previous one.

From (3) and (4), the research efforts of one agent do not enhance the research benefit of agents in competing firms either. Then, when a firm f makes an innovation and its principal finds it optimal to incur the set-up cost κ to exploit the innovation in an adapted organization, it is more profitable to create a separate firm f^+ instead of an extra division. The equity of a spin-off firm f^+ has higher market value than an extra division of firm f, because the former comprises the value of employing another team of α agents, distinctly outside of firm f. Doing so the research efforts of one agent employed by firm f^+ do not enhance the research benefits the α agents employed by firm f. Contracting agents in a new firm is valuable, because it segments the moral hazard in teams problem across two firms.

The intuition behind Properties 2 and 3 is as follows:

The equilibrium compensation $\hat{b}_{F,t}$ offered to each agent at date t when there are F firms conditional on success in (27) equals to a fraction $\frac{\phi_F}{\alpha} = \frac{\theta}{\alpha}\left(1 - \frac{1+\theta}{F+\theta}\right)$ of the expected value of the upcoming innovation. In later periods, the number of firms increases each time an innovation is implemented in a new firm. As the number of firms increases, ϕ_F tends to θ.

In equilibrium, all firms employ α agents, all agents are provided the same compensation and exert equal extra effort. All firms have equal probability of making the next innovation.

3.2 Spin-Off Firm Creation and Value of Future Innovations

At date t, (1) the date-t innovation i_t is made, (2) the principal of successful firm takes the implementation decision, and (3) future innovations consist of innovations $i_{t+1}, \ldots, i_T$. We now establish the implementation threshold of the date-t innovation i_t and the expected value at date t of future innovations to the equityholders of a firm.

Spin-Off Firm Creation: *If at date $t \in \{1, \ldots T\}$ firm f makes innovation i_t then i_t is implemented in the firm if $w(i_t, f) \geq \hat{\omega}_{F,t}$ and in a spin-off firm if $w(i_t, f) < \hat{\omega}_{F,t}$, where the* *threshold*

$$\hat{\omega}_{F,t} \;\equiv\; \max\{0\,,\, \min\{\omega^*_{F,t}\,,\,1\}\,\}\;, \qquad (6)$$
$$\omega^*_{F,t} \;\equiv\; 1-\kappa\,+\,2\,V^E_{F+1,t}\,-\,V^E_{F,t}\;,$$

$V^E_{F,t}$ and $V^E_{F+1,t}$ are the expected value at date t of future innovations (innovations i_{t+1}, ..., i_T) to the equityholders of any firm, if the number of firms is F and $F+1$, respectively.

The implementation threshold, $\hat{\omega}_{F,t}$ in (6), characterizes the implementation decision of the innovation devised at date t. This decision is taken at date t by the principal p of the firm which just made the innovation i_t. When deciding, the principal p knows that the proximity of this innovation to the inventing firm f is, $w(i_t, f)$. Having paid their compensation to each of her agents and holding all property rights on the innovation, she examines the two options available to her at that point.

– If the innovation is implemented in her firm f, the payoff to the equityholders of firm f at date t is the sum of the exploitation value of innovation i_t in the inventing firm f at date t (equal to the proximity $w(i_t, f)$), plus the expected value at date t of future innovations to the equityholders of firm f with an unchanged number of firms, $V^E_{F,t}$.
– If the innovation is implemented in a spin-off firm f^+, the payoff to the equityholders of firm f at date t is the sum of the market value at date t of the equity of the new firm f^+ at date t (equal to the sum of the exploitation value of innovation i_t in a new firm at date t with ideal proximity $w(i, f^+) = 1$, minus the set-up cost κ, plus the expected value at date t of future innovations to the equityholders of the new firm f^+ with one more firm $V^E_{F+1,t}$), plus the expected value at date t of future innovations to the equityholders of the parent firm f with one more firm $V^E_{F+1,t}$.

Since from (6),

$$\omega^*_{F,t}\;+\;V^E_{F,t}\;\;=\;\;1-\kappa\,+\,V^E_{F+1,t}\;+\;V^E_{F+1,t}\;, \qquad (7)$$

it follows that the principal p (who maximizes the payoff to the equityholders of firm f at date t) is indifferent between implementing inside (LHS) and outside (RHS) if and only if the proximity of the innovation to the firm, $w(i_t, f)$, is equal to the threshold value $\omega^*_{F,t}$. The implementation threshold, $\hat{\omega}_{F,t} = \max\{0, \min\{\omega^*_{F,t}\,, 1\}\,\}$ in (6) is simply a transformation of the threshold $\omega^*_{F,t}$ which ensures that $\hat{\omega}_{F,t} \in [0, 1]$. The inventing firm implements the innovation inside if the proximity w is greater than $\hat{\omega}_{F,t}$ in (6) and in a spin-off firm otherwise.

Equity Value of Future Innovations: *The expected value at date t of future innovations (innovations $i_{t+1}, \ldots, i_T$) to the equityholders of any firm with F firms, satisfies*

$$V^E_{F,t} \;=\; (1-\phi_F)\left[\frac{\bar\pi(\hat\omega_{F,t+1}) + \hat\omega_{F,t+1}\; V^E_{F+1,t+1}}{(1+\rho)\,F}\right] + \frac{(1-\hat\omega_{F,t+1})\,V^E_{F,t+1} + \hat\omega_{F,t+1}\,V^E_{F+1,t+1}}{1+\rho}\;, \tag{8}$$

$$\text{where}\;\; \bar\pi(\omega) \;\equiv\; \int_\omega^1 w\,dw \;+\; \omega\,(1-\kappa) \;\;=\;\; (1-\kappa)\,\omega \;+\; \frac{1-\omega^2}{2}\;, \tag{9}$$

and ϕ_F is given by (5). If the horizon T is finite, then $V^E_{F,T} = 0$. If T is infinite, then $V^E_{F,t} = V^E_F$, for any date t, for some $V^E_F \in \mathbb{R}_{\geq 0}$.

The intuition is as follows: At date t, the first future innovation, i_{t+1}, will be made at date $t+1$. Its implementation threshold is $\hat\omega_{F,t+1}$. So with probability $1-\hat\omega_{F,t+1}$, the number of firms at date $t+1$ will not be altered. With probability $\hat\omega_{F,t+1}$, the number of firms at date $t+1$ will be increased by one. Then, the second term on the RHS of the expression of $V^E_{F,t}$ in (8) is the expected value at date t to the equityholders of a firm of what at date $t+1$ future innovations will consist of (innovations $i_t+2, \ldots, i_T$).

For any $\omega \in [0,1]$, the term $\bar\pi(\omega) = \int_\omega^1 w\,dw \,+\, \omega\,(1-\kappa)$ is the expected payoff of an innovation, if the principal of the innovating firm uses an implementation threshold ω. Then, given that for innovation i_{t+1}, all firms will use an implementation threshold $\hat\omega_{F,t+1}$, we have that $\bar\pi(\hat\omega_{F,t+1}) + \hat\omega_{F,t}\; V^E_{F+1,t+1} = \int_{\hat\omega_{F,t+1}}^1 w\,dw + \hat\omega_{F,t+1}\,(1-\kappa+V^E_{F+1,t+1})$ is the expected proceeds from the first future innovation to the firm f which makes this innovation at date $t+1$. To see this, note that for any proximity $w(i_{t+1},f) \geq \hat\omega_{F,t+1}$ firm f obtains w and for any proximity $w(i_{t+1},f) < \hat\omega_{F,t+1}$ firm f obtains the market value of the equity of a spin-off firm at date $t+1$, which equals $1-\kappa+V^E_{F+1,t+1}$.

As the probability a given firm devises the next innovation is $1/F$ and the discount rate is ρ, the squared bracket term $\frac{\bar\pi(\hat\omega_{F,t+1})+\hat\omega_{F,t+1}\;V^E_{F+1,t+1}}{(1+\rho)\,F}$ in (8) is the expected value at date t of the innovation i_{t+1} to be devised at date $t+1$, to one firm.

Each firm gives incentive compensations to its agents worth a fraction ϕ_F of the value of the firm. The equity holders of a given firm therefore obtain the remaining fraction $1-\phi_F$ of the expected value at date t of future innovations, to one firm. The expected value at date t of future innovations to any agent a at date t, is then $V^a_{F,t} = \frac{\phi_F}{\alpha}\;\frac{V^E_{F,t}}{1-\phi_F}\;\left(1-\frac{\theta}{\alpha}\left(1-\frac{1}{F}\right)\right)$. The reduction factor $1-\frac{\theta}{\alpha}\left(1-\frac{1}{F}\right)$ reflects the costs of efforts privately incurred by agent a.

4 DYNAMICS OF SPIN-OFF FIRM CREATION

To examine how the industry develops as spin-off firms are created, we first analyse the extent to which the innovation implementation threshold, $\hat{\omega}_{F,t}$ in (6) depends on the number of firms, F. We then analyse the dynamics of innovation implementation threshold, $\hat{\omega}_{F,t}$, with respect to other parameters of the industry, θ, ρ and κ.

4.1 Dynamics as the Industry Develops

To start, consider the case where there the model is one-shot (hence only one innovation is made at date $T = 1$). As the game is one-shot, when the implementation decision of the single innovation i_1 is taken at date 1, the value of future innovations is equal to zero ($V^E_{F,T=1} = 0$). Hence, the principal p of the firm which makes the date-1 innovation i_1 implements the innovation in firm f if and only if $w(i_1, f) \geq \check{\omega}$, where

$$\check{\omega} \equiv \max\{1 - \kappa; 0\} , \tag{10}$$

and $1-\kappa$ is the exploitation value of the innovation in a newly created firm net of firm set-up cost. Henceforth, we shall refer to $\check{\omega}$ as the implementation threshold in the one-shot game.

In our dynamic framework with horizon T, the implementation threshold at any date t with F firms is $\hat{\omega}_{F,t} = \max\{0, \min\{\omega^*_{F,t}, 1\}\}$ where $\omega^*_{F,t} = 1 - \kappa + 2V^E_{F+1,t} - V^E_{F,t}$. Clearly, if the firm set-up cost κ is very large, there is just no creation of spin-off firms. In these circumstances, the argument developed in the paper becomes simply irrelevant. If on the contrary κ is more moderate and the equilibrium implementation threshold $\hat{\omega}_{F,t} \in (0, 1)$, then $\hat{\omega}_{F,t}$ differs from $1 - \kappa$ by $V^E_{F+1,t} - (V^E_{F,t} - V^E_{F+1,t})$. This difference is zero if t is the final date of a finite horizon model, as once the last innovation i_T is been made at date T, the value of future innovations is zero. However, when t is not a final date ($t < T$) then the difference, given by $V^E_{F+1,t} - (V^E_{F,t} - V^E_{F+1,t})$, is not necessarily zero. Effectively, $\hat{\omega}_{F,t}$ may differ from $1 - \kappa$ for two different reasons:

1. The value at date t of the equity of a spin-off firm f^+ is worth more than $1 - \kappa$. Investors on capital markets are willing to pay an extra amount $V^E_{F+1,t}$ for holding this equity at date t. Issuing shares on a spin-off firm f^+ allows the equityholders of the

inventing firm f to internalize not only the value of exploiting innovation i_t, but also the value of future innovations to the equityholders of the new firm f^+. This pushes an inventing firm to create a new firm more frequently.

2. The creation of a spin-off firm f^+ however reduces the value of future innovations to the equityholders of the inventing firm f. This is because an additional firm will be competing to devise further innovations. The value of the equity of the inventing firm f at date t is reduced by $V^E_{F,t} - V^E_{F+1,t}$. This pushes an inventing firm to create a new firm less frequently.

Consider the simplest case where these dynamic forces are present, which is when the game is repeated once. When $T = 2$, one implementation decision is made at date 1 (that of innovation i_1), a second one is made at date 2 (that of innovation i_2), and the game ends. We compute the implementation thresholds at different dates by working backwards. Since date 2 is final date, the implementation threshold at date 2 is the static one, i.e. $\hat{\omega}_{F,2} = \breve{\omega}$ in (10), for all F. Then, by (8), the value of at date 1 of future innovations (only innovation i_2) to the equityholders of any firm is equal to $V^E_{F,1} = \frac{1-\phi_F}{F}\,\frac{\breve{\pi}}{1+\rho}$, for any F, where

$$\breve{\pi} \;\equiv\; \bar{\pi}(\breve{\omega}) \;=\; \frac{1 + (\max\{1-\kappa; 0\})^2}{2}\,. \tag{11}$$

Hence, it follows from (6), that the implementation threshold at date 1 is

$$\hat{\omega}_{F,1} \;=\; \max\{0, \min\{\omega^*_{F,1}\,, 1\}\}\,, \quad \text{with} \;\; \omega^*_{F,1} \;=\; 1-\kappa \,+\, \frac{\Gamma_F\,\breve{\pi}}{1+\rho}\,, \tag{12}$$

$$\text{where} \qquad \Gamma_F \;\equiv\; 2\left(\frac{1-\phi_{F+1}}{F+1}\right) \;-\; \frac{1-\phi_F}{F}\,. \tag{13}$$

Let $\overline{\kappa}$ be level of firm set-up cost κ such that $\omega^*_{F,1} = 0$. We have

$$\overline{\kappa} \;=\; 1 \,+\, \frac{\Gamma_F}{2\,(1+\rho)}\,. \tag{14}$$

We show that $\Gamma_F > 0$ for all $F > 1$ (see (**??**) in the internet appendix). Next, we provide the following characterisation for the implementation threshold $\hat{\omega}_{F,1}$.

Proposition 1 (Two-Innovation Model)**.** *Suppose the innovation game is played twice. If $\kappa \geq \overline{\kappa}$, then $\hat{\omega}_{F,1} = \breve{\omega} = 0$ for any $F > 1$. If $\kappa < \overline{\kappa}$ the implementation threshold of the first innovation, $\hat{\omega}_{F,1}$, has the following properties:*

(i) $\lim_{F \to +\infty} \hat{\omega}_{F,1} = \breve{\omega}$*;*

(ii) $\hat{\omega}_{F,1} > \breve{\omega}$ *for any* $F > 1$*;*

(iii) $\hat{\omega}_{F,1} \geq \hat{\omega}_{F+1,1}$ *for any* $F > 2$*. The inequality is strict if* $\hat{\omega}_{F,1} < 1$*.*

The proof of Proposition 1 is in the internet appendix. Clearly, if setting up a firm is very costly, no sequel firm is ever created; $\overline{\kappa}$ in (14) is the lower bound to the value of κ above which no firm is created. To see the intuition for parts (i)-(iii) in the above proposition, note that by (12), when $\hat{\omega}_{F,1} \in (0,1)$, the difference between $\hat{\omega}_{F,1}$ and $1-\kappa$ is given by $\frac{\Gamma_F \breve{\pi}}{1+\rho}$ and this term depends on F only through the term Γ_F. Part (i) of Proposition 1 holds because when the number of firms is large, Γ_F is small (effectively the value of future innovations to one firm go to zero when the number of firms $F \to \infty$). To understand parts (ii) and (iii) of Proposition 1, consider the case when θ is arbitrary close to 0 (i.e. the case in which the impact of agents' extra efforts in exploration is small). Then Γ_F is approximately equal to $\frac{2}{F+1} - \frac{1}{F}$. But this implies that $\hat{\omega}_{F,1} > 1-\kappa$, as $\frac{2}{F+1} - \frac{1}{F} > 0$ for any $F > 1$. Furthermore, $\hat{\omega}_{F,1} > \hat{\omega}_{F+1,1}$, because $\Gamma_F - \Gamma_{F+1}$ is approximately $\{\frac{2}{F+1} - \frac{1}{F}\} - \{\frac{2}{F+2} - \frac{1}{F+1}\}$ which is always positive for any $F > 2$.[17]

The characterisation of the implementation threshold $\hat{\omega}_{F,1}$ has the following features. Part (i) of Proposition 1 implies that this difference in the level of firm creation is small if the number of firms is large. Part (ii) of Proposition 1 implies that the level of firm creation at date 1 in the two-innovation model exceeds that in the one-shot one. Part (iii) of Proposition 1 implies that the likelihood of firm creation is smaller the larger the number of firms. Do analogous properties hold when the number of innovations is greater than two?

It is easy to show the property in part (i) of Proposition 1 holds independently of the horizon T. This is because, for any date t, $\hat{\omega}_{F,t} = 1 - \kappa + 2V^E_{F+1,t} - V^E_{F,t}$ when $\hat{\omega}_{F,1} \in (0,1)$ and the values of future innovations to one firm $V^E_{F+1,t}$ and $V^E_{F,t}$ vanish as $F \to \infty$.

It is much less obvious that the properties in part (ii) and (iii) of Proposition 1 extend when the number of innovations is greater than two. The implementation threshold $\hat{\omega}_{F,1}$ at date 1 depends on $2V^E_{F+1,1} - V^E_{F,1}$ which in turn depends on all relevant future implementation thresholds. For arbitrary finite T, the relevant future implementation thresholds are $\hat{\omega}_{F',t'}$, where $t' \in \{2, \ldots T\}$ and $F' \in \{F, \ldots F + t' - 1\}$. With two innovations (i.e. $T = 2$)

[17]When $F = 2$ the term $\{\frac{2}{F+1} - \frac{1}{F}\} - \{\frac{2}{F+2} - \frac{1}{F+1}\} = 0$. In fact the claim that $\hat{\omega}_{F,1} > \hat{\omega}_{F+1,1}$ does not hold when $F = 2$ as $\Gamma_2 - \Gamma_3 = \frac{-\theta\,(1+\theta)}{(2+\theta)\,(3+\theta)\,(4+\theta)} < 0$.

the only relevant future implementation thresholds are $\hat{\omega}_{F,2}$ and $\hat{\omega}_{F+1,2}$ and both of these terms equal $\breve{\omega}$. With more than two innovations, the difficulty is that the relevant future implementation thresholds before the last date (for $t' < T$) depend on the number of firms and take complicated values.

Consider then the limit case where the number of innovations T is infinite. The infinite horizon model is arguably the most natural one and can give a full sense of the magnitude of dynamic effects. In the infinite horizon model, the equilibrium described in Theorem 1 is time independent; hence, for the ease of notation, we drop subscripts t in the infinite horizon model and refer to $V^E_{F,t}$, $V^a_{F,t}$, $\hat{\omega}_{F,t}$ and $\omega^*_{F,t}$ by V^E_F, V^a_F, $\hat{\omega}_F$ and ω^*_F, respectively.

Next we provide a set of sufficient conditions that ensure that the properties of the threshold mentioned in the previous subsection hold in the infinite horizon model.

Proposition 2 (Infinite Horizon Model)**.** *Suppose the innovation game is infinitely repeated. If* $\kappa \geq 1 + \frac{\Gamma_F}{2\rho}$ *then* $\hat{\omega}_F = \breve{\omega} = 0$ *for any* $F > 1$*. If* $\kappa < 1 + \frac{\Gamma_F}{2\rho}$ *the implementation threshold,* $\hat{\omega}_F$*, has the following properties:*

(i) $\lim_{F\to+\infty} \hat{\omega}_F = \breve{\omega}$;

(ii) $\hat{\omega}_F > \breve{\omega}$ *for any* $\kappa > 1 - \sqrt{\frac{F\,\Gamma_F}{1-\phi_F}}$ *and* $F > 1$;

(iii) $\hat{\omega}_F \geq \hat{\omega}_{F+1}$ *for any* $\kappa > 1 - \sqrt{\frac{\Gamma_F - \Gamma_{F+1}}{\frac{1-\phi_F}{F} + 2\,\frac{1-\phi_{F+2}}{F+2}}}$ *and* $F > 2$*. The inequality is strict if* $\hat{\omega}_F < 1$.

The proof of Proposition 2 is in the internet appendix. We also show that $\frac{F\,\Gamma_F}{1-\phi_F} > 0$ for all $F > 1$ and $\frac{\Gamma_F - \Gamma_{F+1}}{\frac{1-\phi_F}{F} + 2\,\frac{1-\phi_{F+2}}{F+2}} > 0$ for all $F > 2$.

The sufficient conditions in parts (ii) and (iii) of Proposition 2 consist of κ not being too small. When on the contrary the cost of setting-up a firm κ is very small, each innovation results in the creation of an additional firm as $\hat{\omega}_F$ is always close to 1. We could not develop a clear sufficient condition for the dynamics of $\hat{\omega}_F$ in this case. When κ is larger than a certain level (as stipulated in parts (ii) and (iii) of Proposition 2), the properties of the threshold mentioned in the previous subsection hold in the infinite horizon model.[18]

[18] To get a sense of the sufficient conditions in Proposition 2, consider the case when θ is arbitrary close to 0. Then parts (ii) and (iii) of Proposition 2 can be respectively written as:

- $\hat{\omega}_F > \breve{\omega}$ for any $\kappa > 1 - \sqrt{\frac{F-1}{F+1}}$ and $F > 1$;
- $\hat{\omega}_F > \hat{\omega}_{F+1}$ for any $\kappa > 1 - \sqrt{\frac{F-2}{(F+1)\,(3\,F+2)}}$ and $F > 2$.

4.2 Dynamics w.r.t. other Characteristics of the Industry

We now analyse the dynamics of innovation implementation threshold, $\hat{\omega}_{F,t}$, with respect to other exogenous parameters of the model which characterize the industry. These are (i) the rate of transformation of agents extra effort to research output, θ, (ii) the rate at which all participants discount future cashflows, ρ, and (iii) the cost of setting up a specialized organization capable of producing and generating cash from a given innovation, κ.

Consider again the simplest case where time considerations are present, which is when the game is repeated once ($T = 2$). In the two-innovations model, $\hat{\omega}_{F,1}$, the implementation threshold at date 1 remains simple and is given in (12). Here, we can establish the comparative statics of the innovation implementation threshold, with respect to θ, ρ, and κ:

Proposition 3 (Two-Innovation Model)**.** *Suppose the innovation game is played twice. The implementation threshold of the first innovation, $\hat{\omega}_{F,1}$, has the following properties:*

$$(i) \qquad \frac{\partial\, \hat{\omega}_{F,1}}{\partial\, \theta} \leq 0 \qquad \textit{and} \qquad \lim_{\theta \to 1} \hat{\omega}_{F,1} = \breve{\omega}\ ; \tag{15}$$

$$(ii) \qquad \frac{\partial\, \hat{\omega}_{F,1}}{\partial\, \rho} \leq 0 \qquad \textit{and} \qquad \lim_{\rho \to +\infty} \hat{\omega}_{F,1} = \breve{\omega}\ ; \tag{16}$$

$$(iii) \qquad \frac{\partial\, \hat{\omega}_{F,1}}{\partial\, \kappa} \leq 0 \qquad \textit{and} \qquad \hat{\omega}_{F,1} = \breve{\omega} = 0 \quad \textit{if} \quad \kappa \geq \overline{\kappa}\ . \tag{17}$$

The proof of Proposition 3 is in the internet Appendix. The intuition is as follows:

When the rate of transformation of extra effort in research output θ is small, firms benefit little from providing high-powered incentive contracts to the agents. The equityholders of a firm capture the largest fraction of the value of future innovations made by their firm, because the principal (on behalf of equity holders) only promises a small fraction $\phi_F = \theta\left(1 - \frac{1+\theta}{F+\theta}\right)$ of this value to their agents in equilibrium. Similarly (but more directly), when the rate at they discount future cashflows ρ is small and/or when the cost of setting up a spin-off firm κ is small, the expected value at date t of future innovations to the equityholders of any firm, $V^E_{F,t}$ in (8) also high.

This has a bearing on the implementation decision of an inventing firm. When θ, ρ, and κ are small, the gain in expected value at date t from future innovations (innovations i_{t+1}, $\ldots$, i_T) to the equityholders of the inventing firm if it chooses to spin-off its innovation i_t, $V^E_{F+1,t} - (V^E_{F,t} - V^E_{F+1,t})$, is high. Then, the extent to which the implementation threshold of the date-t innovation, $\hat{\omega}_{F,t}$, exceeds the one-shot threshold $\breve{\omega}$ because the gain in expected value from future innovations is high. As θ, ρ, or κ increase, this gain in expected value from future innovations when choosing to spin-off shrinks. This explains the comparative statics in Parts (i), (ii) and (iii) of Proposition 3. As $\theta \to 1$ and $\rho \to +\infty$, the gain is annihilated and the implementation threshold tends to the one-shot threshold $\breve{\omega}$. Similarly, when the cost of setting up a spin-off firm κ becomes larger than a lower bound value (equal to $\overline{\kappa}$ in (14) in the two-innovations model), the inventing firm never chooses to spin-off its innovation. Then, both $\hat{\omega}_{F,1}$ and $\breve{\omega}$ are equal to zero.

Proposition 3 establishes this logic in the two-innovations model, considering the implementation decision at date 1 of the firm which makes the first innovation i_1. The gain in expected value from future innovations when choosing to spin-off only comes from the single future innovation i_2. Although the logic seems to extend straightforwardly to cases when the number of innovations is greater than two, we were unable to analytically establish analytically similar results for the general case $T > 2$.

In the absence of analytical results, we carried out some numerical simulations in the infinite horizon model, to confirm these dynamics. This also gives a sense of the impact on the implementation threshold, when the set of future innovations is large.[19] Figure 1 shows the impact of θ and ρ on the dynamics of the implementation threshold, around a central case $\{\theta; \rho; \kappa\} = \{10\%; 10\%; 0.5\}$. In all cases we observe that $\hat{\omega}_F > \breve{\omega}$.
– In Panel (a), ρ and κ are fixed. The implementation threshold $\hat{\omega}_F$ decreases in θ, for any given number of firms F. For a given θ, $\hat{\omega}_F$ is largest and most decreasing in F, when the rate of transformation of extra effort in research output θ is smallest.
– In Panel (b), θ and κ are fixed. The implementation threshold $\hat{\omega}_F$ decreases in ρ, for any given number of firms F. For a given ρ, $\hat{\omega}_F$ is largest and most decreasing in F, when the discount rate ρ is smallest (recall that a small ρ can be also be regarded as representing an industry in which innovations happen frequently).

[19]Given that $\lim_{F \to +\infty} V^E_F = 0$ we approximate the dynamics by setting $V^E_{\overline{F}} = V^E_{\overline{F}+1} = 0$, for a large $\overline{F}$ (we took $\overline{F} = 200$). Then we recursively worked backwards by solving, for any F, for $(\hat{\omega}_{F-1}, V^E_{F-1})$ from $(\hat{\omega}_F, V^E_F)$, using the equations $V^E_{F-1} = \frac{1-\phi_{F-1}}{F-1} \left[\frac{\bar{\pi}(\hat{\omega}_{F-1}) + \hat{\omega}_{F-1}\, V^E_F}{1+\rho}\right] + \frac{(1-\hat{\omega}_{F-1})\, V^E_{F-1} + V^E_F}{1+\rho}$ and $\hat{\omega}_{F-1} \equiv \max\{0\,,\, \min\{1 - \kappa\, +\, 2\, V^E_F\, -\, V^E_{F-1}\,,\, 1\}\,\}$.

5 IMPLICATIONS AND EVIDENCE

The essential conclusion we can draw from the analysis in Section 4 is that under reasonable circumstances, the following result holds.

Result 1. *The equilibrium implementation threshold, $\hat{\omega}_{F,t}$, exceeds the one-shot threshold $\breve{\omega}$ defined in (10), when $F > 1$. The dynamics of $\hat{\omega}_{F,t}$ are as follows:*
1 – *$\hat{\omega}_{F,t}$ decreases with F. The difference between $\hat{\omega}_{F,t}$ and $\breve{\omega}$ vanishes as F becomes large.*
2 – *$\hat{\omega}_{F,t}$ is decreasing in the rate of transformation of agents extra effort to research output, $\theta \in (0, 1)$. $\hat{\omega}_{F,t} - \breve{\omega}$ tends to 0, as θ tends to 1.*
3 – *$\hat{\omega}_{F,t}$ is decreasing in the discount rate, $\rho \in \mathbb{R}_{>0}$. $\hat{\omega}_{F,t} - \breve{\omega}$ tends to 0, as ρ becomes large.*
4 – *$\hat{\omega}_{F,t}$ is decreasing in the cost of setting up a firm, $\kappa \in \mathbb{R}_{>0}$. $\hat{\omega}_{F,t} = \breve{\omega} = 0$ when κ is larger than a certain lower bound value.*

5.1 Implications

Differences across factors, F, θ, ρ, and κ, are observable as follows:

• In our set-up, the number of firms F can only increase over time as no existing firm disappears. Therefore, F captures the maturity of the industry, with young (mature) industries being characterised by a small (large) number of firms.
• The rate of transformation of agents extra effort to research output, θ, is not immediately observable. However, θ directly determines the extent of incentive contracts provided by the principals to their agents. When θ is small (high), the agents are given a small (high) fraction $\phi_F = \theta\left(1 - \frac{1+\theta}{F+\theta}\right)$ of the shares of the firm.
• ρ is the unit-period discount rate and the unit-period is the time that elapses between two innovations in the industry. Then, for given market conditions, a small (high) ρ corresponds to an industry where more (less) innovations are made per year.

Next we examine the implications of Result 1 on the dynamics of each of three observable resultants of the implementation threshold $\hat{\omega}_{F,t}$: (i) the frequency with which spin-off firms are created, (ii) the focus of firms and (iii) the profitability of firms, in the industry.

Frequency. In terms of our earlier notation, if at date t firm f makes innovation i_t, the proximity of that innovation is given by $w(i_t, f)$. In equilibrium a spin-off firm is then created if and only if $w(i_t, f) \in (0; \hat{\omega}_{F,t})$. From (2), this proximity $w(i_t, f)$ is uniformly distributed over all possible proximities, for all firm f. It follows that the probability a spin-off firm is created is simply equal to the prevailing implementation threshold, i.e. $\text{Prob}\left[w(i_t, f) < \omega_{F,t}\right] = \hat{\omega}_{F,t}$. Hence, we can conclude the following.

Implication 1 (Frequency). *When Result 1 holds, an innovation leads more frequently to the creation of a spin-off firm, when:*

1 – *the industry is young with a small number of participating firms;*

2 – *the fraction of shares of the firm held by research agents is small;*

3 – *the industry is one where innovations are frequent;*

4 – *the cost of setting up a new firm is small.*

Focus. A firm is focused when it exploits neighbouring innovations. In our set-up, the proximity of an innovation i_t to a firm f is measured by $w(i_t, f)$. Then, for any date t with F firms, let $\mathcal{I}^f_{F,t}$ denote the set of innovations firm f has made and implemented inside the firm throughout its history up to and including date t. A simple measure of firm f's focus at date t with F firms is the average of the proximities of innovations $i_\tau \in \mathcal{I}^f_{F,t}$:

$$\frac{1}{|\mathcal{I}^f_{F,t}|} \sum_{i_\tau \in \mathcal{I}^f_{F,t}} w(i_\tau, f) \; . \tag{18}$$

Given that, at any date $t' \leq t$, with $F' \leq F$ any successful firm f implements the date-t' innovation $i_{t'}$ inside the firm when $w(i_{t',f}) \in [\hat{\omega}_{F',t'}, 1]$, it follows that the above measure of focus is increasing in $\hat{\omega}_{F',t'}$ for each $F' \leq F$ and $t' \leq t$. We can state the following.[20]

Implication 2 (Focus). *When Result 1 holds, the focus of firms is higher, when:*

1 – *the industry is young with a small number of participating firms;*

2 – *the fraction of shares of the firm held by research agents is small;*

3 – *the industry is one where innovations are frequent;*

4 – *the cost of setting up a new firm is small.*

Profitability. For any date t and any number of firms F, we have that (i) the value at date t of exploiting innovation i_t in the inventing firm f, is equal to the proximity $w(i_t, f)$, (ii) firm

[20]The definition $\mathcal{I}^f_{F,t}$ purposefully excludes the initial innovation a firm exploits when it is created from the set (that innovation was made by the parent firm). Otherwise firms would all start with a measure of focus in (18) equal to 1. Result 2 would seem to be a direct consequence of this.

f implements the date-t innovation only if $w(i_t, f) \in (\hat{\omega}_{F,t}, 1)$ and (iii) $w(i_t, f)$ is uniformly distributed over all possible proximities. Then each firm's profit from implementing the date-t innovation equals on average to $(\hat{\omega}_{F,t} + 1)/2$. We can state the following.

Implication 3 (Profitability)**.** *When Result 1 holds, firms obtain (on average) higher profits from the innovations they implement, when:*

1 − *the industry is young with a small number of participating firms;*

2 − *the fraction of shares of the firm held by research agents is small;*

3 − *the industry is one where innovations are frequent;*

4 − *the cost of setting up a new firm is small.*

All these effects fade away in the long run: given that $\lim_{F \to +\infty} \hat{\omega}_{F,t} = \breve{\omega}$, the frequency with which spin-off firms are created, the focus and the profitability of firms become stable.

5.2 Evidence

There is substantial evidence that spin-offs create value.[21] John and Ofek (1995) and Berger and Ofek (1999) find that their is value creation primarily when the parent firm focuses on its core operations in the process. Desai and Jain (1999) and Daley, Mehrotra, V. and Sivakumar (1997), show that this value of focus in spin-offs is larger when the core operations of the spin-off firm are distant from those of the parent firm.

Our intention here is not to claim that any specific empirical evidence is driven only by the effect we have studied. As mentioned in the introduction, several theories have been proposed to rationalize the voluntary creation of spin-off firms and its link to innovation. There are also several theories of entrepreneurial spin-outs. Next, we just note that our results are in line with a series of empirical facts established in fairly general contexts.

Gompers, Lerner and Scharfstein (2005) carried out an extensive empirical study of spawning, which includes spin-offs and spin-outs. Actually, the interpretation they give

[21] The empirical literature, starting with Hite and Owers (1983), Miles and Rosenfeld (1983), Schipper and Smith (1983) and Alexander, Benson and Kampmeyer (1984), documents positive abnormal stock returns for parent firms on the announcement of a corporate spin-off. Cusatis, Miles and Woolridge (1993) document that both spin-offs and their parents experience significantly positive abnormal returns for up to three years beyond the spin-offs' announcement date.

of their results is more about spin-outs than spin-offs, as they argue that younger, venture backed firms give their employees better skills and networks, which allows them to create new firms. Still, our Implications 1, 2 and 3 are in line with their findings that (i) the frequency with which firms generate new firms decreases with age, (ii) more profitable firms generate more new firms, and (iii) diversified firms generate less new firms than focused ones.[22]

There is also substantial evidence that firms decline in profitability and lose focus with age.[23] A very natural explanation for the decline in profitability is increasing product market competition. Several agency explanations have also been proposed: One is that managerial private benefits lead to empire building; Another is risk-shifting, whereby shareholders of mature levered firms privately benefit from firm cashflows being invested in risky growth opportunities. We acknowledge that these rationales are possibly stronger determinants than ours and that our model has totally silenced these effects. We simply note that Implications 1, 2 and 3 are in line with these empirical findings.

6 ANALYSIS OF INEFFICIENCIES

Next we study the extent to which the equilibrium outcome is inefficient. The equilibrium behaviour is the result of a series of private optimizations by different participants with conflicting interests: The incentive contracts offered by each individual firm "pushes" its agents to exert extra effort in the race to innovate; Each agent chooses his level of extra effort considering his private costs; Each individual firm has incentive to do spinoffs because this captures a bigger piece of the innovation "pie" at the expense of rival firms. The equilibrium outcome is therefore unlikely to maximize the aggregate payoff of all players.

Let $W_{F,t}$ denote the value at date $t \in \{1, \dots, T\}$ of future innovations to all investors

[22]They observe that firms which report just one top 3-digit SIC segment have firm creation levels that are 19% higher than those operating in multi segments.

[23]Eisenberg, Sundgren and Wells (1998), Majumdar (1997), Loderer and Waechli (2009), Loderer and Waelchli (2015), Loderer, Stulz, and Waelchli (2016) find that firms' profitability decreases with age. Denis Denis and Sarin (1997) find that firms become less focused with age. Lang and Stulz (1994) find that firm diversification and Tobin's q are negatively related and Berger and Ofek (1995) find that operating margin and ROA profitability measures are lower for diversified companies. Lins and Servaes (1999) find similar results in Japan and the United Kingdom. Note that Campa and Kedia (2002) find that the diversification discount is reduced once the endogeneity of the diversification decision is controlled for.

and all available agents, when the number of existing firms is F. Thus, $W_{F,t}$ is the sum of expected proceeds from innovations i_{t+1} to i_T (devised at dates $t+1$ to T), minus the sum of all discounted costs of efforts expected to be exerted by all agents, discounted to date t, under the equilibrium behaviour. In the internet appendix, we show:

Proposition 4. *The expected value at date t of future innovations (innovations i_{t+1}, ..., i_T) to all players with F firms equals*

$$W_{F,t} \;=\; F\,\left[V^E_{F,t} \,+\, \alpha\, V^a_{F,t}\right] \,+\, U^a_{F,t}\ , \tag{19}$$

where $V^E_{F,t}$ satisfies (8) and $V^a_{F,t} = \frac{\phi_F}{\alpha}\,\frac{V^E_{F,t}}{1-\phi_F}\,\left(1 - \frac{\theta}{\alpha}\left(1 - \frac{1}{F}\right)\right)$, and $U^a_{F,t}$ satisfies

$$U^a_{F,t} \;=\; \frac{\hat{\omega}_{F,t+1}\,\alpha\, V^a_{F+1,t+1}}{1+\rho} \;+\; \frac{(1-\hat{\omega}_{F,t+1})\, U^a_{F,t+1} + \hat{\omega}_{F,t+1}\, U^a_{F+1,t+1}}{1+\rho}\ , \tag{20}$$

and $U^a_{F,T} = 0$.

$W_{F,t}$ in (19) is the sum of two terms. $F\,\left[V^E_{F,t} \,+\, \alpha\, V^a_{F,t}\right]$ is the value of future innovations to the existing industry participants (i.e. all equityholders and all employed agents of the existing firms). $U^a_{F,t}$ is the value of future innovations to all the agents who are unemployed at date t, but will be recruited at dates $t+1$ and later. $U^a_{F,t}$ is written recursively as the sum of the value of future innovations to the agents who will be employed at date $t+1$ if innovation i_{t+1} leads to the creation of an extra firm (with probability $\hat{\omega}_{F,t+1}$, a new firm will be created, this firm will employ α unemployed agents, and each of these will obtain a value $V^a_{F+1,t+1}$ from future innovations) plus the value of future innovations to all the agents who will still be unemployed at date $t+1$, but will be recruited at later dates.

Next, we characterise first-best strategies. That is, we establish features which must hold for a strategy to yield the highest aggregate payoff of all players. This will enable us to examine the extent to which the equilibrium outcome path is inefficient. We obtain:

Proposition 5. *Under a first best strategy, no private cost of extra effort is borne by any agent at any date $t \in \{1, \ldots, T\}$.*

If at any date $t \in \{1, \ldots, T\}$ firm f makes innovation i_t then i_t is implemented in the firm if $w(i_t, f) \geq \breve{\omega}$ and in a spin-off firm if $w(i_t, f) < \breve{\omega}$, where the first best firm implementation threshold equals $\breve{\omega}$ defined in (10). The expected value at date t of future innovations (innovations i_{t+1}, ..., i_T) to all players with F firms under a first best strategy equals

$$\breve{W}_t \;=\; \frac{\breve{\pi}}{\rho}\left[1 \;-\; \frac{1}{(1+\rho)^{T-t}}\right]\ , \tag{21}$$

where $\breve{\pi}$ is defined in (11).

To assess the significance of equilibrium inefficiencies we examine the extent to which the equilibrium value of future innovations, $W_{F,t}$, falls short of its first best value, $\breve{W}_t$. We can break down the difference $\breve{W}_t - W_{F,t}$ as follows:

Proposition 6. *The agency costs expected at date t to result from the equilibrium strategy*

$$\breve{W}_t \; - \; W_{F,t} \;\; = \;\; C^e_{F,t} \; + \; C^\omega_{F,t} \; , \tag{22}$$

where $C^e_{F,t}$ and $C^\omega_{F,t}$ satisfy

$$C^e_{F,t} \;=\; \frac{\theta\,\phi_F}{\alpha}\left(1-\frac{1}{F}\right)\left[\frac{\bar{\pi}(\hat{\omega}_{F,t+1}) + \hat{\omega}_{F,t+1}\;V^E_{F+1,t+1}}{1+\rho}\right] + \frac{(1-\hat{\omega}_{F,t+1})\,C^e_{F,t+1} + \hat{\omega}_{F,t+1}\,C^e_{F+1,t+1}}{1+\rho}, \tag{23}$$

$$C^\omega_{F,t} \;=\; \frac{\breve{\pi} \;-\; \bar{\pi}(\hat{\omega}_{F,t+1})}{1+\rho} \;+\; \frac{(1-\hat{\omega}_{F,t+1})\,C^\omega_{F,t+1} + \hat{\omega}_{F,t+1}\,C^\omega_{F+1,t+1}}{1+\rho} \;, \tag{24}$$

with $C^e_{F,T} = C^\omega_{F,T} = 0$, for any F, and $\bar{\pi}(.)$, $\breve{\pi}$ and $\hat{\omega}_F$ are given in (9), (11) and (6).

Remember that at date t, future innovations consists of innovations $i_{t+1}, \; \ldots, \; i_T$. The first one, i_{t+1}, will be made at date $t+1$ and its implementation threshold is $\hat{\omega}_{F,t+1}$.

The term $C^e_{F,t}$ is the well known agency cost of excessive efforts: Firms are engaged in a race to innovate and as a result the principal of each firm induces her agents to exert inefficient equilibrium extra efforts, in order not to fall behind others. Whereas the first best would be attained if all the agents of all firms exerted the base level of effort (incurring no associated private cost), in equilibrium agents are induced to exert an extra effort, $e^a_{f,t-1} > 0$.[24] The first term in the RHS of (23) is the sum of the costs incurred by all agents in extra efforts at date $t+\epsilon$ to devise the first future innovation i_{t+1} when there are F firms. The agency cost of excessive efforts, $C^e_{F,t}$, is the discounted sum of these private costs of agents, over the sequence of innovations i_{t+1} to i_T.

The term $C^\omega_{F,t}$ is much less discussed. It refers to an agency cost of inefficient firm creation: $\breve{\pi}$ is the expected value of exploiting an innovation under the first best implementation threshold $\breve{\omega}$ in (10). In comparison, $\bar{\pi}(\hat{\omega}_{F,t+1})$ is the expected value of exploiting the date

[24] In the Appendix, we establish and discuss that the extra effort $e^a_{f,t-1}$ exerted by agents in equilibrium equals $\hat{e}_{F,t-1}$ in (30), where $\hat{e}_{F,t-1} > 0$.

$t+1$ innovation under the equilibrium implementation threshold $\hat{\omega}_{F,t+1}$ in (6). As discussed in Section 4, the implementation threshold $\hat{\omega}_{F,t}$ differs from $\check{\omega}$. This difference in implementation thresholds results in a difference between expected exploitation values $\bar{\pi}(\hat{\omega}_{F,t})$ and $\check{\pi}$. Figure **??** in the internet Appendix illustrates this. The agency cost of suboptimal creation of firms, $C_{F,t}$, is the discounted sum of these differences in expected exploitation values, over the sequence of innovations i_{t+1} to i_T.

Given that $\lim_{F\to\infty}\hat{\omega}_{F,t} = \check{\omega}$, it follows that the agency costs of inefficient firm creation shrinks to zero as the number of firms become large: $\lim_{F\to\infty} C^{\omega}_{F,t} = 0$. However, since the total sum of costs of efforts do not vanish as $F\to\infty$, the agency costs of excessive efforts does not: $\lim_{F\to\infty} C^{e}_{F,t} = \frac{\theta^2}{\alpha}\check{W}_{F,t} > 0$.[25]

While agency costs of extra efforts are more significant than agency costs of inefficient firm creation when the number of firms is large, this is often not the case with small number of firms. Figures **??** and **??** in the internet Appendix exhibits the magnitude of these two agency costs as a fraction of first best value, $\frac{C^{\omega}_{F,t}}{\check{W}_t}$ and $\frac{C^{e}_{F,t}}{\check{W}_t}$, as the number of firms progresses, for the numerical applications of the infinite horizon model carried out in Section 4.

7 CONCLUDING REMARKS AND EXTENSIONS

We considered a principal-agent model of an industry where firms repeatedly compete to innovate and the number of firms increases endogenously, as parent firms create spin-off firms. Comparing the equilibrium behaviour with the first-best strategy, we conclude that the repetition of the race to innovate leads to the inefficient external implementation of some innovations. Excessive creation of spin-off firms is highest in early stages of the industry.

In order to establish our results we made several assumptions on firm creation. Below we would like to briefly discuss three of our assumptions.

Transferring innovations amongst existing firms. We assumed that an innovation cannot be sold on the market for ideas to a non-inventing existing firm. However, it may be that a non-inventing firm has an expertise closer to the characteristic of the innovation than the inventing firm. In this case the inventing firm might prefer to sell her innovation to this

[25] Given that $\lim_{F\to\infty}\phi_F = \theta$, $\lim_{F\to\infty}\bar{\pi}(\hat{\omega}_{F,t}) = \check{\pi}$ and $\lim_{F\to\infty} V^{E}_{F+1,t+1} = 0$, we have $\lim_{F\to\infty}\frac{C^{e}_{F,t}}{\check{W}_{F,t}} = \frac{\theta^2}{\alpha}$

existing firm. So far we have not allowed for this possibility for simplicity. We think our qualitative conclusions on excessive creation of firms and their dynamics extend to allowing the inventing firms selling their innovation to existing firms.

To see this suppose that the principal of any inventing firm can sell the invention to any principal of an existing firm and that any such trade is competitive i.e. the seller can extract all the surplus from the purchaser.[26] Extended selling opportunities do not change the attractiveness of creating a new independent firm: If the principal p of the successful firm f implements the innovation in a spin-off firm, the payoff of the equityholders of firm f is still equal to $1-\kappa+V^E_{F+1,t+1}$. In contrast, if p implements the date-t innovation i_t in the inventing firm or sells the innovation to an existing firm, the payoff to the equityholders of firm f at date t is $\bar{w}(i_t)+V^E_{F,t}$, where $\bar{w}(i_t)\equiv\max_{f'\in\mathcal{F}_t}\{w(i_t,f')\}$ denotes the proximity at date t between the expertise demanded by innovation i_t and the expertise of the firm most capable of exploiting it in the set of existing firms (compare the payoff $\bar{w}(i_t)+V^E_{F,t}$ in this case with the payoff $w(i_t,f)+V^E_{F,t}$ which the equityholders of firm f would have obtained if the innovation could not be sold to an existing firm). It follows that when $\bar{w}(i_t)$ is lower than the threshold

$$\hat{\omega}^{Extd}_{F,t} \;\equiv\; \max\{0\,,\,\min\{\omega^{*Extd}_{F,t}\,,\,1\}\}\;,\quad\text{with}\quad \omega^{*Extd}_{F,t}\;\equiv\;1-\kappa\,+\,2\,V^E_{F+1,t}\,-\,V^E_{F,t}\;, \tag{25}$$

the principal p spins off the innovation in a newly created firm. When $\bar{w}(i_t)$ is greater or equal to $\hat{\omega}^{Extd}_{F,t}$, the innovation is implemented in one of the existing firms.[27]

Clearly the analysis is exactly the same as before except that proximity of the innovation $\omega(i_t,f)$ has to be replaced by $\bar{w}(i_t)$. The threshold $\hat{\omega}^{Extd}_{F,t}$ in (25) has an identical relationship to κ and values of innovations to equityholders as $\hat{\omega}_{F,t}$ in (6).[28] As in the main model, the

[26] The assumption that the seller can extract all surplus from any existing firm may not be reasonable. A better framework for modelling such trades may be multi-person bargaining as principals of existing firms are likely to have some bargaining power in such a situation. Our set-up of allowing the inventing firm to sell only to outsiders avoids these complications.

[27] Akcigit, Celik and Greenwood (2016) construct a measure of the relative distance between two patents, comparing the number of instances other patents cite only one of the two patents to the number of instances other patents cite both. They find that with a patent sale, the distance between the patent and the historical portfolio of patents of the firm which owns this patent is on average decreased by 0.15. It takes on average 5.5 years to sell a patent and 16% of the patents are sold to another existing firm.

[28] Since $\bar{w}(i_t)\geq\omega(i_t,f)$, if the two thresholds $\hat{\omega}^{Extd}_{F,t}$ and $\hat{\omega}_{F,t}$ were the same, then p would less likely

threshold $\hat{\omega}_{F,t}^{Extd}$ exceeds the optimal level $\breve{\omega}$ in (10), if $2\,V_{F+1,t+1}^{E} - V_{F,t+1}^{E} \in (0,1)$. Hence, as long as the latter holds there would also be excessive firm creation in this new set up.

Adapting the model to spin-outs and entrepreneurship. In our set-up, firm creation is effectively decided by the equityholders of the inventing firm. Specifically, we have assumed that intellectual property rights protect them perfectly from expropriation by their agents. The principal of the firm decides whether to implement the innovation inside the inventing firm or to voluntarily transfer these property rights to a new independent firm. However, often in the context of start-ups, firm creation is done by employees becoming entrepreneurs without formal transfer of ownership rights. It would be interesting in future research to adapt the set-up, so that equityholders are not fully protected by property rights and agents can create firms to implement their innovations, without the consent of the firm which employs them. In such an environment, writing compensation contracts is not necessarily needed to provide agents with incentives to exert effort: each employed agent exerts effort because it increases his chances of starting a new firm exploiting a distant innovation which the inventing firm would implement less profitably.

Firm creation without an innovation. We assumed that no one sets up a new firm at date t without holding an innovation i_t. A simple justification for this assumption may come from differences in set-up cost of firms when one has an innovation in hand compared to when one does not: The former may be small relative to the latter because when an innovation i_t is made, exploitation processes specifically adapted to the characteristics of the innovation, x^{i_t} are also developed. These specific exploitation processes are part of the innovation i_t and reduce the cost of setting up a firm with expertise x^{i_t}.

Even if set-up costs are the same with or without an innovation in hand, it may still be the case that in equilibrium no one wishes to create a firm without an innovation in hand. We conjecture that our results extends to the case when all (with and without an innovation at hand) can set-up firms at a common set-up cost for a significant set of parameter values.

sell-out innovation i_t, if she can also sell it to existing principals. However, the two thresholds may not be the same, as the equity values $V_{F+1,t}^{E}$ and $V_{F,t}^{E}$ are larger if the principal can also sell to existing principals.

APPENDIX

F could be finite or infinite. However, due to fixed cost of setting up a firm κ, F will be finite in equilibrium. $\mathcal{F}_0$ is not a singleton, which ensures that the following race to innovate starts with innovation i_1. All agents are penny-less and have a reservation value equal to zero. Denote the set of available agents by $\mathcal{N}^A$.

Game:

The following game for date-t innovation i_t is repeated for each $t \in \{1, \dots, T\}$.

- At date $t - 1 + \epsilon$:

1 − The principal p of each existing firm f selects $\mathcal{A}_{f,t-1} \subset \mathcal{N}^A$, the set of agents she wishes to employ to make innovation i_t, and makes a take-it-or-leave-it contract offer $b_{f,t} \in \mathbb{R}_{\geq 0}$ to each agent in $\mathcal{A}_{f,t-1}$ simultaneously (hence all the bargaining power is with the principal). Denote $A_{f,t-1} \equiv |\mathcal{A}_{f,t-1}|$ the number of agents offered employment in firm f at date $t - 1 + \epsilon$ to make the date-t innovation.

2 − The solicited agents receive the offers made by the principals. We assume for simplicity that if more than one firm proposes to the same agent, the agent receives only one offer.

3 − Each agent individually responds to the offer he receives. At the time the agent receives an offer from f at date $t - 1 + \epsilon$, the agent knows, in addition to past history up to date $t - 1$, the proposal of firm f and the set of agents, $\mathcal{A}_{f,t-1}$ who receive offers from firm f at date $t - 1 + \epsilon$, but he does not know the contracts offered by other firms and the set of agents to which other firms make an offer to.[29]

We shall denote the response of the agent a to firm f proposal by $r^a_{f,t-1}$, where $r^a_{f,t-1} = 1$ refers to a accepting the offer and $r^a_{f,t-1} = 0$ refers to rejecting the offer. Denote $\mathcal{A}^*_{f,t-1} = \{a' \in \mathcal{A}_{f,t-1} \mid r^a_{f,t-1} = 1\}$ the set of agents who accept the offer from f at date $t - 1 + \epsilon$ (so $\mathcal{A}^*_{f,t-1} \subseteq \mathcal{A}_{f,t-1}$).

We assume that the principal p of a firm f which employed a set of agents $\mathcal{A}^*_{f,t-2}$ to make the previous innovation i_{t-1}, has a biais for offering to employ again these agents to make innovation i_t.[30] Hence in stage **1** above, $\mathcal{A}_{f,t-1} \subset \mathcal{A}^*_{f,t-2}$ if $A_{f,t-1} \leq |\mathcal{A}^*_{f,t-2}|$, and

[29] The details of the extensive form information are not important for the results. We have assumed these partly for realism and partly to simplify the exposition.

[30] A natural justification for this assumption is that switching to a non experimented agent typically involves adjustment costs. This is more natural than assuming that any agent can only be employed once,

$\mathcal{A}_{f,t-1} \supset \mathcal{A}^*_{f,t-2}$ if $A_{f,t-1} \geq |\mathcal{A}^*_{f,t-2}|$: if the principal p wants to expand the number of employees, all the firm's existing employees receive an offer; if she wishes to shrink the number of agents, it only makes offer to existing employees.

4 – All agents exert a base effort incurring no associated private cost. Each agent $a \in \mathcal{A}^*_{f,t-1}$ can choose to exert an extra effort $e^a_{f,t-1} \in \mathbb{R}_{\geq 0}$ to make innovation i_t, bearing a private cost $e^a_{f,t-1}$ at date $t-1+\epsilon$.

- At date t:

5 – The research output of firm f resulting from extra efforts of its agents is $n_{f,t}$ in (3) if $|\mathcal{A}^*_{f,t-1}| \geq \alpha$ and 0 otherwise. If the research output from extra effort of some firm is positive, the probability that any firm makes the innovation depends on its research output relative to the total is $q_{f,t}$ given in (4), if there exists $f' \in \mathcal{F}_{t-1}$ such that $n_{f',t} \neq 0$. If F firms employ at least α agents and all agents only exert the base effort, the probability firm f makes the innovation i_t is $q_{f,t} = 1/F$.

6 – The date-t innovation i_t happens. Its characteristic, x^{i_t}, is chosen according to the distribution $g_f(x^{i_t})$ in (2). Only one innovation happens. Payments to employed agents according to contracts, $b_{f,t}$, are made. The principal of the inventing firm has all property rights on the innovation.

7 – The principal p of the successful firm (the firm that innovates) f decides at date t to implement the innovation in firm f or in a spin-off firm f^+. At the time the principal p makes this decision at date t, she knows, in addition to the previous history of play before date t and the identity of the innovation i_t, the set of agents who accepted her employment offer, but she does not know the contracts offered by other firms and the set of agents to which other firms made an offer to. We shall denote the decision of the principal of the successful firm by $d_{f,t}$ where, $d_{f,t} = 1$ refers to implementing the innovation in the successful firm f and $d_{f,t} = 0$ refers to implementing the innovation in a sequel firm f^+. Clearly, for any innovation i_t, $d_{f,t}$ depends on $w(i_t, f)$, the relative proximity of i_t from the successful firm f.

When a spin-off firm f^+ is created, all property rights on the innovation i_t are transferred to the new firm. The expertise of the firm is set to $x^{f^+} = x^{i_t}$ upon creation, against a set-up cost κ at date t. Equity on the new firm f^+ is issued at date t. A new principal p^+

or assuming some random matching for each race.

is appointed to manage firm f^+ in the best-interests of its equityholders. Already created firms continue existing unchanged without incurring again set-up costs. The principal p^+ is not the principal of an existing firm. No new firm other than f^+ is set up.

Finally, in the case when T is finite, the game ends after innovation i_T at date T.

The timeline is illustrated in the internet Appendix, Figure **??**.

If innovation i_t is made by firm f, the principal p therefore pays $A_{f,t-1}\, b_{f,t}$ at date t to her agents, and the equity of firm f gains the remaining of the innovation value: In case p decides to implement the innovation within firm f, the innovation value is the value of the proceeds from exploitation. In case the innovation is implemented in a newly created firm f^+, the innovation value is the market value of the spin-off firm's equity.

Equilibrium Outcome:

To state our result, we introduce the following notation: For any $F > 1$, any $(x, y) \in \mathbb{R}^2_{\geq 0}$,

$$G_F(x \,|\, y) \quad \equiv \quad \frac{(1+\rho)\; x^{1-\theta}\left[x^\theta + (F-1)\, y^\theta\right]^2}{\theta\,(F-1)\, y^\theta}\;. \tag{26}$$

Theorem 1. *There exists a symmetric Markov perfect Bayesian equilibrium strategy $\hat{E}$ such that for any $t \in \{1, \ldots, T\}$ and state $s = (\mathcal{F}, \mathcal{A})$ with $|\mathcal{F}| = F > 1$, the following holds in equilibrium:*

(i) At date $t-1+\epsilon$, any firm already in operation employs the same α agents and any firm just set-up at date $t-1$ chooses α agents randomly amongst those not already employed;

(ii) The offer of any firm to any agent consists of a payment at date t in case of success

$$\hat{b}_{F,t} \quad \equiv \quad \frac{\phi_F}{\alpha}\left(\bar{\pi}(\hat{\omega}_{F,t}) + \hat{\omega}_{F,t}\, V^E_{F+1,t}\right)\;, \tag{27}$$

where $\bar{\pi}(\omega)$ and ϕ_F are given by (9) and (5);

(iii) Every agent accepts any offer $b \in \mathbb{R}_{\geq 0}$;

(iv) For any $b \in \mathbb{R}_{\geq 0}$, the extra effort exerted at date $t-1+\epsilon$ by any agent in any firm when the firm employs $A \geq \alpha$ agents is $e_{F,t-1}(A,b)$ where

$$e_{F,t-1}(A,b) = \begin{cases} e^*_{F,t-1}(A,b) & \textit{if } b > 0\,; \\ 0 & \textit{otherwise}\,. \end{cases} \tag{28}$$

$e^*_{F,t-1}(A,b)$ is the unique solution to

$$G_F(A\, e^*_{F,t-1}(A,b) \,|\, \alpha\, \hat{e}_{F,t-1}) \quad = \quad b \tag{29}$$

$$\textit{and} \quad \hat{e}_{F,t-1} \quad \equiv \quad \frac{\theta}{\alpha}\left(1 - \frac{1}{F}\right)\;\frac{\hat{b}_{F,t}}{(1+\rho)\,F}\;. \tag{30}$$

The effort level of any agent in a firm is zero if the firm employs less than α agents;

(v) *At date t, for any f, if firm f makes the date-t innovation i_t then i_t is implemented in the firm if $w(i_t, f) \geq \hat{\omega}_{F,t}$ and in a spin-off firm if $w(i_f, f) < \hat{\omega}_{F,t}$, where*

$$\hat{\omega}_{F,t} \;\equiv\; \max\{0\,,\, \min\{\omega^*_{F,t}\,,\, 1\}\}\;, \quad \textit{with} \quad \omega^*_{F,t} \;\equiv\; 1-\kappa \,+\, 2\,V^E_{F+1,t} \,-\, V^E_{F,t}\;. \tag{31}$$

(vi) *The expected value at date t of future innovations (innovations $i_{t+1}, \ldots, i_T$) to the equityholders of any firm and any agent a, $V^E_{F,t}$ and $V^a_{F,t}$, respectively, satisfy*

$$V^E_{F,t} \;=\; (1-\phi_F)\left[\frac{\bar{\pi}(\hat{\omega}_{F,t+1}) + \hat{\omega}_{F,t+1}\; V^E_{F+1,t+1}}{(1+\rho)\,F}\right] + \frac{(1-\hat{\omega}_{F,t+1})\,V^E_{F,t+1} + \hat{\omega}_{F,t+1}\,V^E_{F+1,t+1}}{1+\rho}\;, \tag{32}$$

$$V^a_{F,t} = \frac{\phi_F}{\alpha}\left(1-\frac{\theta}{\alpha}\left(1-\frac{1}{F}\right)\right)\left[\frac{\bar{\pi}(\hat{\omega}_{F,t+1}) + \hat{\omega}_{F,t+1}\; V^E_{F+1,t+1}}{(1+\rho)\,F}\right] + \frac{(1-\hat{\omega}_{F,t+1})V^a_{F,t+1} + \hat{\omega}_{F,t}V^a_{F+1,t+1}}{1+\rho}, \tag{33}$$

$$\textit{and} \quad \begin{cases} \textit{if } T \textit{ is finite,} & \textit{then } V^E_{F,T} \;=\; V^a_{F,T} \;=\; 0\,; \\ \textit{if } T \textit{ is infinite,} & \textit{then } V^E_{F,t} \;=\; V^E_{F} \textit{ and } V^a_{F,t} \;=\; V^a_{F}\;,\; \textit{for any date } t\;, \\ & \qquad \textit{for some } V^E_F \in \mathbb{R}_{\geq 0} \textit{ and some } V^a_F \in \mathbb{R}_{\geq 0}\;. \end{cases} \tag{34}$$

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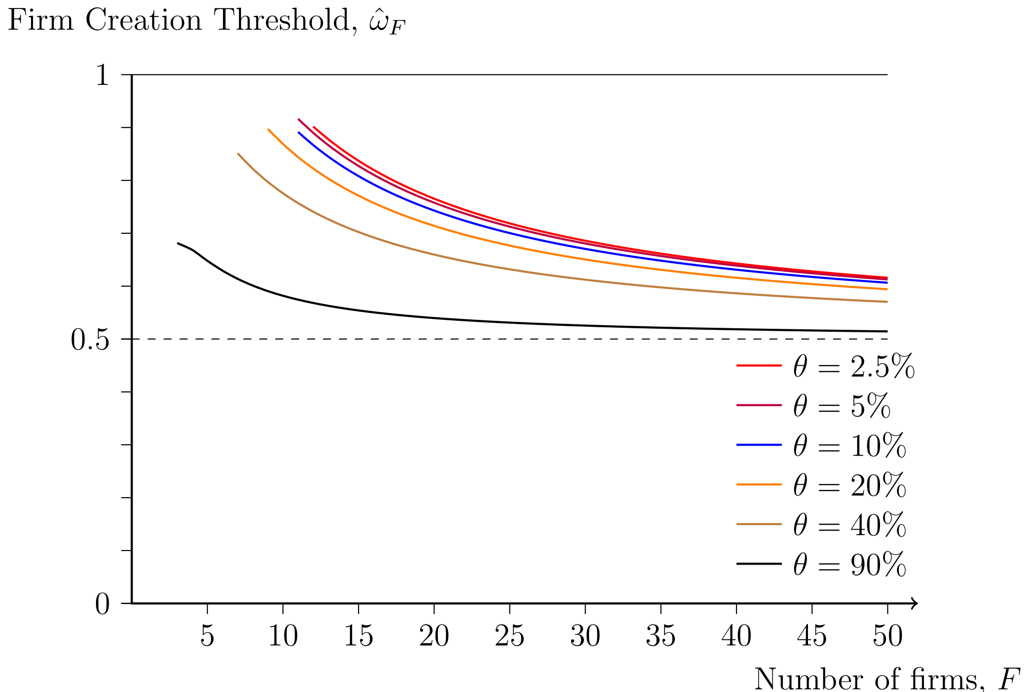
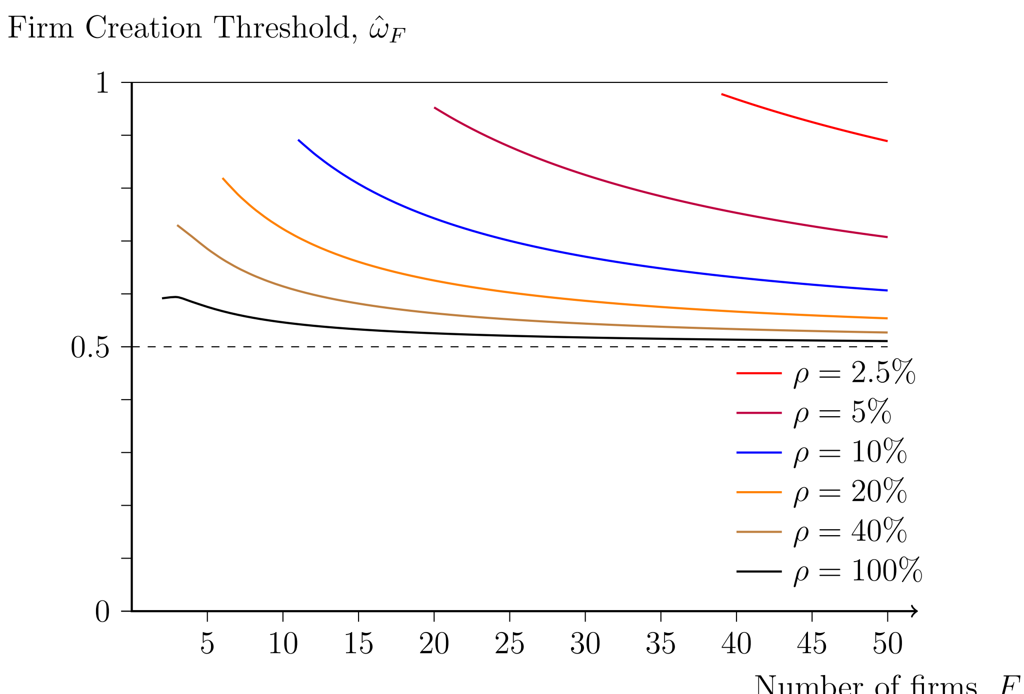


(a) across θ, for $\rho = 10\%$ and $\kappa = 1/2$.



(b) across ρ, for $\theta = 10\%$ and $\kappa = 1/2$.

Figure 1: Dynamics of Firm Creation.