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1 Introduction

Around 50% of all jobs are found through social contacts in the US, a trend which has remained constant over the past decades (Montgomery (1991), Topa (2011), Forbes (2011) and LinkedIn (2016)). This persistence is surprising, given that the internet has substantially facilitated job search, which implies that (i) workers have a much larger range of firms they can apply to and that (ii) referred candidates face many more competitors on average. Therefore, the likelihood of finding a job through personal contacts should decrease. Moreover, if we look at another prominent matching market, such persistence does not hold: in marriage markets, the internet seems to have crowded out search through personal contacts. Between 1995 and 2017, the share of people finding their partner through friends, relatives, coworkers, or neighbors dropped from 75 % to 41 % in the US (Rosenfeld et al., 2019). This raises the question of why the importance of referrals has remained unchanged in labor markets, despite the emergence of online search. To answer this question, I explore a new channel that makes referral applications favorable for both firms and workers. Moreover, I analyze how workers' chances of receiving a job offer through a referral are affected by aggregate search intensities to capture the impact of online search.

However, before describing this new mechanism, we briefly cover the existing theories on why referrals benefit job searchers to understand why they fall short of explaining the puzzle of why referrals have remained so persistent.[1] The first channel is that workers learn about more job opportunities. Accordingly, the rise of the internet and the corresponding reduction of search costs should have reduced the importance of referrals.

The second channel is related to network homophily. Since the referring employee has already been hired and must therefore be of high ability, the referred candidate is also likely to be capable. However, the assumption that referred candidates are of higher ability is rejected in the recent empirical literature (Burks et al. (2015), Pallais and Sands (2016), Brown et al. (2016)). Moreover, even if this assumption was to hold, a larger number of competitors implies that the likelihood that one competitor signals a higher ability than the referred candidate increases, which should have also reduced the prevalence of referrals over the past decades.

The third prominent channel is that referrals provide more information about the applicant, reducing uncertainty on the match quality. However, through the rise of social media and online newspapers, the internet has enhanced the firms' opportunities to find out more about their candidates, leading to a more level playing field between referred candidates and those who have submitted a regular application.

The fourth channel is that referred workers can be observed by their referrers on the job, which can mitigate moral hazard. However, moral hazard must have become a more important concern for this channel to explain the persistence of referrals in the presence of technological improvements. To the best of my knowledge, there is no evidence to support this.

This motivates exploring a new channel through which referrals provide advantages: alleviating congestion. As we will see in the literature review, a vast literature exists on congestion in decentralized matching markets as well as possible policy interventions on how to alleviate it. However, to the best of my knowledge, there is little focus on job referrals in that area.

I utilize the fact that referred candidates are much more likely to accept a job offer than other applicants. According to Burks et al. (2015), referred candidates accept the offer with a 3-7 percentage point higher likelihood, depending on the industry. On the other hand, in Pallais and Sands (2016), referred candidates accept with a 17 percentage point

[1] The literature corresponding to each channel is discussed in the literature review below.

higher likelihood: referred workers accept an offer with a likelihood of 68%, whereas non-referred candidates accept with 51%.

When choosing whom to make an offer to, firms face uncertainty over whether the candidate will accept their job offer. Since having offers turned down leads to costly delays in filling the vacancy, they need to consider which candidates are more interested in working for the firm.

There are several explanations for why referred candidates are more likely to accept a job offer. One is that workers can learn more about the vacancy through their friends than through any official job posting. Therefore, they have a better idea about how much they are interested in working for that company before applying. Moreover, a referral application involves a personal connection which impacts the acceptance likelihood in two ways: working with friends can be more desirable, and conversely, turning down a job offer is more costly since it can adversely impact the personal relationship: thanks to the referral, the firm has wasted time and effort in screening the candidate, which in turn can damage the referee's reputation inside the firm. Conversely, due to potential reputation damages, an employee will only want to make a referral if she believes that her friend or acquaintance is sufficiently interested in the job. Considering all these points, it is clear that, conditional on applying, referred applicants are, on average, more interested in taking the job than candidates applying online.[2]

Therefore, this paper explores the congestion channel by analysing how much the fact that some applicants are much more likely to accept matters, particularly with respect to the workers' search intensities. As we will see, in labor markets, firms suffer from (i) uncertainty over whether candidates accept their offers and hence whether they have apportioned their screening resources to the right candidates, and (ii) an *acceptance curse*. The latter is akin to the *winner's curse* in auction theory: with uncertainty over whether offers are accepted, the fact that a worker accepts a firm's offer transmits negative information about his ability. He may have accepted because other firms have found him unqualified, while the offering firm received a signal that had overstated his true ability.

In contrast, both problems are alleviated with referred candidates. Since referred candidates are more likely to accept, the risk of leaving the vacancy unfilled is lower, and the fact that the candidate has accepted transmits less negative information. Therefore, referred candidates have an edge over candidates who apply through the regular market.

As we will see, as workers search more intensely, a referred candidate has more competitors on average. However, on the other hand, as workers search more and hence apply to more firms, they are less interested in each firm on average. This, in turn, makes referred candidates stand out more among all applicants, which increases their chances. Therefore, overall, we have the surprising result that under some circumstances, workers are more likely to get a job through a referral if workers search more intensely, even though this increases the expected number of competitors. This helps explain why referrals have remained very important despite the emergence of vast search opportunities through the internet.

Overall, my paper provides three contributions. First, it uncovers another channel that explains the immense success of referrals. Second, it establishes the existence of an *acceptance curse* in labor markets with non-sequential search. Third, it shows how referrals can still be robust to an increase in search intensity and hence increased competition.

[2]Gee et al. (2017) find that strong ties are significantly more helpful for finding a job than weak ties while leaving the question of causality for further research. All the aforementioned explanations for why referred candidates are more likely to accept apply more to closer ties than weaker ties; hence, the mechanism presented in this paper is congruent with the evidence in Gee et al. (2017).

Related Literature. My paper broadly relates to two strands of literature. On the one hand, it is related to the literature on job referrals as it uncovers a new channel on how job searchers benefit from referrals. On the other hand, it relates to the literature on congestion in matching markets.

There exists a vast literature on the four channels mentioned above that make referrals beneficial for job searchers: The first one that focuses on how referrals provide workers with *additional application opportunities* is covered by Calvó-Armengol and Jackson (2004), Calvó-Armengol (2004), Wahba and Zenou (2005), and Calvó-Armengol and Jackson (2007). The second which assumes network homophily and thus a *higher ability* of referred candidates compared to candidates from the market is focused on in Montgomery (1991), Bolte et al. (2020), and Okafor (2020). The channel of *valuable information* about the match quality is covered by Simon and Warner (1992) and Dustmann et al. (2016)), and the *mitigation of moral hazard* by Kugler (2003), Heath (2018), and Dhillon et al. (2021).

Moreover, since congestion makes searching for a worker more tedious, both articles of Galenianos (2014, 2021) are related to my paper. In their setups, firms compare the search cost between hiring a referred candidate and hiring a candidate in a market where searching can be time-consuming. However, since their research focus differs from mine, their frameworks are also different, particularly because they assume that firms follow a different search approach: first, they choose whether to search for a referred or a market candidate. Afterwards, if they choose the latter, they try to match with the first worker they meet in the market. Thus, referred and market candidates do not directly compete for vacant jobs. In contrast, in my framework, firms receive all applications from both types of candidates simultaneously to capture the competition between referred and market candidates. This allows us to analyze the impact of lower search barriers: firms have more candidates to choose from, but markets are more congested.

The second relevant area this paper contributes to is congestion in decentralized matching markets. Albrecht et al. (2006), Galenianos and Kircher (2009), Kircher (2009), Engbom (2021) address the consequences of congestion, whereas Coles et al. (2013), Ashlagi et al. (2020), Kanoria and Saban (2020), Arnosti et al. (2021), and He and Magnac (2022) focus on policy interventions such as signaling, application costs, and application limits to alleviate congestion. Among those contributions, my setup is most similar to Albrecht et al. (2006), with the important extensions that workers differ in their ability and that some are endowed with a referral. Furthermore, Chade (2006) shows that an *acceptance curse* exists, albeit in a sequential instead of a simultaneous search setup.

Structure. Section 2 describes the framework of the labor market, and Section 3 its equilibrium properties. To understand the shortcomings of the labor market, both sections abstract from referrals. Afterwards, Section 4 analyzes the role of referrals. Section 5 briefly discusses the framework's assumptions and characterizes differences between labor markets and marriage markets, and Section 6 concludes.

2 Framework

Players. There are $v \in \mathbb{N}^+$ firms that post one vacancy each, and $u \in \mathbb{N}^+$ unemployed workers. Each worker can take up to one job, and each vacancy can be filled by at most one worker. Workers differ in terms of their ability $y \in \{0, 1\}$, and for simplicity, assume that half are of low ($y = 0$) and half of high ability ($y = 1$). Throughout this paper, we focus on markets with a large number of workers and firms. Therefore, $u, v \to \infty$, with

$\theta \equiv \frac{v}{u}$ denoting the labor market tightness.

Timing. The players move as follows:

1. Workers simultaneously choose their number of applications. They can apply to up to $a \in \{1, 2, ..., v\}$ firms
2. All applications are uniformly randomly distributed among all firms. They receive the applications and a noisy signal about each applicant's ability. Based on this, they make at most one offer
3. Workers receive offers and accept up to one. If they receive more than one offer, they uniformly randomize over which one to accept

Information. Each signal $\hat{y}$ on the ability is i.i.d. distributed within $[0, 1]$ according to the distribution functions G_1 and G_0 for high and low ability workers, respectively, with $G_y(1) = 1$ for both. G denotes the vector $\{G_1, G_0\}$. We will assume that $g_1'(\hat{y}) > 0 > g_0'(\hat{y})$ and $\frac{g_1(\hat{y})+g_0(\hat{y})}{2} = 1$ for any $\hat{y}$. The former condition implies that signals are informative, so the Monotonic Likelihood Ratio Property is satisfied. The latter is of a technical nature to make the framework more tractable.

Payoffs. While workers have no preference over firms and strictly prefer to be matched rather than remaining unmatched, the firms' ex-post payoff corresponds to the worker's ability $\pi^F = y \in \{0, 1\}$ if he has accepted. Remaining unmatched yields zero payoff.

3 Equilibrium

At stage 3, workers always accept a job offer and randomize with equal probability if they have received more than one offer. At stage 2, each firm receives its applications from stage 1.

Proposition 1. *In the unique equilibrium, the likelihood of receiving an offer from a firm is*

$$q_y = \int_0^1 e^{-\frac{a}{\theta}(1-\hat{y})} g_y(\hat{y}) d\hat{y} \qquad \textit{for } y \in \{0, 1\} \tag{1}$$

The proof is in Appendix A.2. Intuitively, this expression is derived based on two facts: first, firms always make an offer to the worker with the highest signal since they want to maximize their chances of hiring a worker with $y = 1$. Second, since $v, u \to \infty$ and all applications are distributed among firms uniformly randomly, firms receive applications according to the Poisson Distribution with parameter $\frac{a}{\theta}$, which denotes the average number of applications a firm receives.

Note that the likelihood that a firm receives no applications is $e^{-\frac{a}{\theta}}$. Therefore, if $\hat{y} = 0$, a worker would only receive an offer if he turns out to be the only applicant. Therefore, if $\frac{a}{\theta}$ is large, and thus the competition among workers is intense, the expected likelihood of receiving an offer exponentially increases in the signal since with higher search intensity, the risk of facing a competitor with a high signal increases.

With $a > 1$, some workers end up with multiple offers and thus uniformly randomize to decide which one to accept. This randomization is equivalent to randomly generating a preference order over all firms a worker applies to in stage 1, according to which he decides at stage 3 whose offer to accept. For instance, a firm's likelihood of being ranked 3^{rd} is $\frac{1}{a}$ which means that its offer is accepted if the worker has been rejected by his first two choices with likelihood $(1-q_y)^2$. Thus, the likelihood that a worker accepts a firm's offer is

$$r_y = \frac{1}{a}\sum_{x=0}^{a-1}(1-q_y)^x = \frac{1-(1-q_y)^a}{aq_y} \qquad \text{for } y \in \{0,1\} \tag{2}$$

which decreases with q_y.

Proposition 2. *For $a > 1$, firms face an acceptance curse:*

$$E(y|\hat{y}, accept) = \frac{\alpha r_1}{\alpha r_1 + (1-\alpha)r_0} < \alpha = E(y|\hat{y}) \tag{3}$$

which follows from $r_1 < r_0$ due to $q_1 > q_0$, with $\alpha \equiv \frac{g_1(\hat{y})}{2}$. The fact that an applicant accepts an offer lowers his expected ability because, in some likelihood, he only accepts because he has not sent a sufficiently large signal to other firms and hence has been rejected. Therefore, the firm takes into account that it might have overestimated the candidate's ability if he accepts its offer.

Based on this, we can decompose the expected payoff from making an offer as follows:

$$\alpha r_1 = \underbrace{\frac{\alpha r_1}{\alpha r_1 + (1-\alpha)r_0}}_{\text{expected ability if accepted}} - \underbrace{\frac{\alpha r_1(1-\alpha r_1-(1-\alpha)r_0)}{\alpha r_1 + (1-\alpha)r_0}}_{\text{rejection likelihood if high ability}} \tag{4}$$

Overall, firms suffer from congestion in two ways: a worker might not accept an offer, and even if he does, he is of lower ability in expectation.

4 Referrals

4.1 Primitives

Now assume that a fraction $p \in [0,1]$ of firms has employees who are connected with one unemployed worker and offer that worker to be referred for their job. Every worker can accept up to one referral, which implies that $\phi \equiv \frac{1-e^{-p\theta}}{\theta}$ denotes the share of firms that receive a referral application.

The assumption that workers can only accept up to one referral is based on the facts that (i) screening candidates is costly and that (ii) the referrer and the referred are connected through a social relationship. Firms would waste resources if they screened and interviewed candidates but ended up being turned down. Thus, to avoid a reputational damage of his social contact, the referred candidate is assumed to accept the offer with certainty. In line with this, we assume that he can only use at most one referral.

The extended setup's timeline is as follows:

1. Workers simultaneously choose their number of applications

2. Firms are ranked uniformly randomly. According to that ranking, firms sequentially send out referrals. Workers accept a referral if and only if they have not already accepted one

3. Firms receive all applications from the market and the referral simultaneously, combined with the candidates' ability signals, and choose whom to make an offer to

4. Workers accept at most one job offer

To understand how the upcoming results depend on the signal's information content, we will impose some structure on G whenever sensible. The class of signal distributions with **constant slope** that satisfy the assumptions $g_1' > 0 > g_0'$ and $\frac{g_1+g_0}{2} = 1$ are of the form:

$$g_1(\hat{y}) = \frac{2-b}{2} + b\hat{y} \qquad\qquad g_0(\hat{y}) = 2 - g_1(\hat{y}) = \frac{2+b}{2} - b\hat{y} \tag{5}$$

with $b \in (0, 2]$ measuring the signal's information content.

4.2 Benchmark: $p \to 0$

To pinpoint the mechanism through which search intensity affects the chance of receiving a job offer through a referral, we will first focus on $p \to 0$ which means that the share of referral applicants is negligible. In the next subsection, we will look at the case in which p is significantly above zero, such that firms take into account that an applicant from the regular market may also have been referred to another firm.

Define $Q^R \equiv \frac{q_0^R+q_1^R}{2}$ as the probability that a referred worker receives an offer on average, with q_0^R and q_1^R denoting the likelihood of a low-and high-ability worker to get a job offer through a referral, respectively. The following Theorem provides the main result of this paper.

Theorem 1. *Fix θ, G and assume that $p \to 0$. In equilibrium:*

1. *Q^R can increase in a*

2. *If g_1 has a constant slope: if q_0^R increases in a, then q_1^R increases in a*

3. *If g_1 has a constant slope: Q^R decreases in b*

The proof is provided in the Appendix. The *first part* states that the average likelihood of receiving a job offer through a referral application can increase if workers search more intensely. This might seem counterintuitive at first, given that higher search intensity leads to an increase in the number of expected competitors.

To understand this result, note that firms treat referred candidates' signals more favorably since those are assumed to accept a firm's offer with certainty: for a given $\hat{y}$, a market candidate yields an expected payoff of $\frac{g_1(\hat{y})}{2}r_1$, whereas a referred candidate yields $\frac{g_1(\hat{y})}{2}$. Therefore, if a referred candidate has a signal $\hat{y}_1$, he is treated equally to a market candidate with $\hat{y}_2 \geq \hat{y}_1$, with $\hat{y}_2 = g_1^{-1}(\frac{g_1(\hat{y}_1)}{r_1})$. This reflects the tradeoff that one firm faces: since firms care both about the expected ability and the chance that a candidate accepts the offer, they need to trade off the expected ability $\frac{g_1(\hat{y})}{2}$ with the acceptance likelihood r_1 of a high-skilled worker, to avoid remaining unmatched. Since referral candidates are assumed to always accept, firms view them favorably for two reasons: on the one hand, they provide the firm with certainty that they will accept the offer, and on the other hand, there is no *acceptance curse*.

Based on this, the likelihood that a referred candidate receives an offer is:

$$Q^R = 1 - \bar{y} + \int_0^{\bar{y}} e^{-\frac{a}{\theta}\left[1-g_1^{-1}(\frac{g_1(\hat{y})}{r_1})\right]} d\hat{y} \tag{6}$$

With $\bar{y} = g_1^{-1}(g_1(1)r_1)$. Thus, if the candidate sends a signal above $\bar{y}$, he will always receive an offer. On the other hand, if the signal is below $\bar{y}$, he might be rejected if another competitor submits a sufficiently large signal.

Q^R depends on how many competitors a candidate faces and how much he stands out as a referred candidate. Thus, if a changes, Q^R is impacted in two ways:

$$\frac{dQ^R}{da} = \frac{\partial Q^R}{\partial a} + \frac{\partial Q^R}{\partial r_1}\frac{\partial r_1}{\partial a} \tag{7}$$

The first effect represents the increase in competition and is thus negative. On the other hand, $\frac{\partial Q^R}{\partial r_1}\frac{\partial r_1}{\partial a}$ denotes how much more the referred candidate stands out compared to his competitors if workers search more. As the following Lemma shows, this effect is positive:

Lemma 1. *If $p \to 0$, the high-ability workers' acceptance likelihood r_1 decreases in a*

To gain some intuition, recall that $r_1 = \frac{1}{a}\sum_{x=0}^{a-1}(1-q_1)^x$ and the interpretation that workers randomly generate a preference order among all firms they apply to at stage 1. As a increases, the likelihood of obtaining a high rank decreases, which lowers the chance that the candidate will accept the offer. On the other hand, q_1 decreases in a, which makes it more likely that workers are rejected by other firms and therefore accept an offer. The lemma states that the former effect dominates the latter effect.

Thus, as a increases, a referred worker faces more competitors. However, his competitors are less interested in the firm on average, which makes him stand out more.

The *second part* of the Theorem states that high-ability workers are more likely to benefit from an increase of search intensity a than low-ability workers. If low-ability workers' chances of receiving an offer increase, then high-ability workers' chances also increase. Intuitively, referred candidates always get an offer if they submit a strong signal beyond $\bar{y}$ and maintain good chances if their signal is close to $\bar{y}$. However, with a weak signal, the firm believes that the candidate is of low ability. Therefore, it prefers to make an offer to a market candidate with a much stronger signal, despite the risk of being rejected. And since high-ability workers send stronger signals in expectation, their chances tend to be more robust to an increase in competition.

The *third part* states that the average chance to receive an offer through a referral increases if ability signals are less informative. However, this is not trivial. On the one hand, a higher signal precision makes referrals stand out less since firms pay more attention to (informative) ability signals. However, on the other hand, with higher precision, high-ability candidates receive more offers on average, lowering their acceptance likelihood r_1 and thus enhancing the value of the referred candidates' higher acceptance likelihood. Nonetheless, as shown in the proof, the former effect dominates the latter.

For instance, if $b \to 0$ and hence signals are uninformative, every referred worker receives an offer with certainty if $r_1 < 1$. However, as b increases and therefore signals become more precise, firms take signals much more into account. Thus, the disadvantage stemming from a large number of competitors more likely outweighs the benefit of standing out in the applicant pool. Hence, a referred candidate with a low signal would receive no offer if signals are precise since firms believe him to be of low ability. In other words, chances of receiving a job through a referral are much more robust to higher search intensity if firms struggle to screen their candidates precisely.

Illustrations. Figure 1 illustrates the results. Two signal distribution functions with constant slopes are used. One is relatively precise ($b = 2$), and the other is noisy ($b = 1$).[3]

[3]If $b = 2$, a high-skilled worker submits a higher signal than a low-skilled worker to one firm with a likelihood of 83%, and if $b = 1$, with a likelihood of 67%.

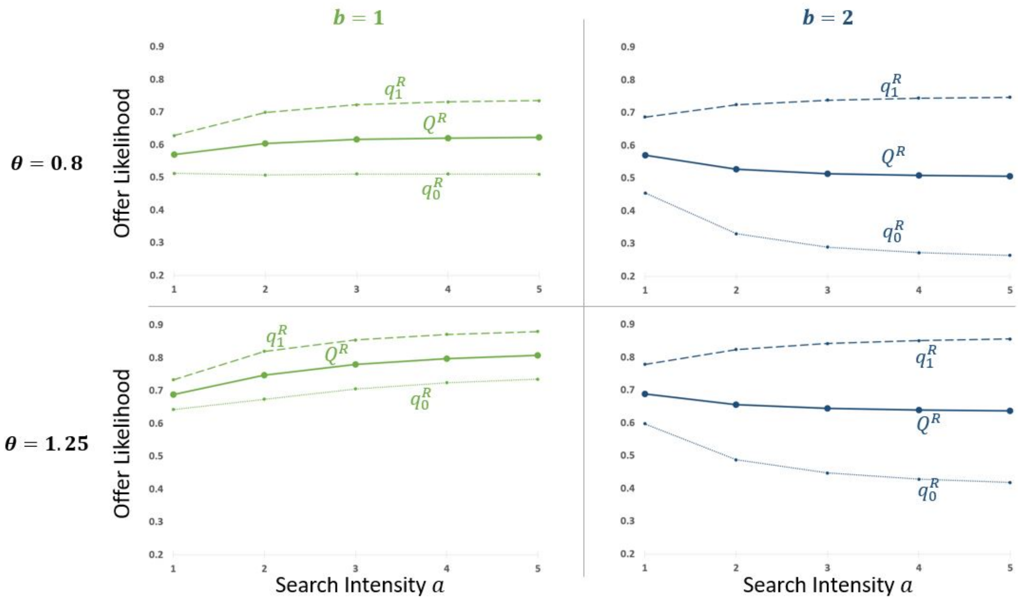

Figure 1: Example with different signal precisions and market imbalances

The top panel shows the likelihood that a referred candidate receives a job offer in an environment in which workers outnumber firms ($\theta = 0.8$), and the bottom panel shows an environment with more firms than workers. As we can see, Q^R can increase in a (Theorem 1). This is particularly the case for high-ability workers' changes q_1^R since they are likely to send a signal which leaves their market competitors without any chance.

Moreover, in line with Theorem 1, noisier signals lead to stronger increases of Q^R in a. Finally, higher values of θ disproportionally increase the likelihood of receiving an offer. The intuition is that a larger number of firms increases the chances of receiving an offer for workers through regular applications. Therefore, workers accept offers with a lower likelihood, which makes referred candidates much more appealing to firms.

4.3 General Case: $p > 0$

Now we focus on the more general case in which a considerable share of workers is referred. Since a referred worker accepts an offer from the market only if he is rejected by the firm that he has been referred to, the acceptance likelihood is

$$r_1 = \underbrace{(1 - \phi\theta q_1^R)}_{\text{A}} \underbrace{\frac{1-(1-q_1)^a}{aq_1}}_{\text{B}} \tag{8}$$

with $\phi \equiv \frac{1-e^{-\theta p}}{\theta}$ denoting the share of firms who have a referred candidate. While A denotes the likelihood that a candidate has no job offer through a referral, B represents the likelihood that a candidate accepts if he has applied to a firms.

With $p > 0$, the impact of a on Q^R is:

$$\frac{dQ^R}{da} = \underbrace{\frac{\partial Q^R}{\partial a}}_{\text{I}} + \Big(\underbrace{-\frac{\partial Q^R}{\partial r_1^M}}_{\text{II}}\Big) \underbrace{\frac{-(1-\phi\theta Q^R)\frac{\partial r_1^M}{\partial a} + \phi\theta r_1^M \frac{\partial q_1^R}{\partial a}}{1 + \phi\theta\frac{\partial q_1^R}{\partial r_1^M} + \frac{\partial q_1}{\partial r_1}(1-\phi\theta q_1^R)\frac{[1-(1-q_1)^a]-(1-q_1)^{a-1}aq_1}{aq_1^2}}}_{\text{III}} \tag{9}$$

$r_1^M \equiv \frac{1-(1-q_1)^a}{aq_1}$ expresses the likelihood that a candidate accepts if he has only applied to firms on the regular labor market. As with $p = 0$, (I) describes the direct impact of more competition which lowers Q^R.

However, with $p > 0$, the indirect effect becomes more sophisticated due to the interdependencies between the referral and regular markets. As the numerator of (III) shows, the acceptance likelihood r_1 is affected in two ways: it is lower since r_1^M decreases in a, and it increases since q_1^R is lowered by a through the direct effect of more competition. Moreover, as the denominator of (III) shows, forces exist that amplify and dampen the impact of a on Q^R, respectively: if the acceptance likelihood r_1 drops, q_1^R increases. This lowers r_1 by $\Delta q_1^R \phi\theta$, further increasing q_1^R and thus amplifying the impact.

On the other hand, the impact is dampened through the change of q_1. To see this, note that q_1 is impacted by r_1 since

$$q_1 = \int_0^1 e^{-\frac{a}{\theta}(1-\hat{y})}[1 - \phi + \phi max\{0, g_1^{-1}(g_1(\hat{y})r_1)\}]g_1(\hat{y})d\hat{y} \tag{10}$$

With likelihood ϕ, a market candidate faces a referred candidate as a competitor. Thus, he also faces the challenge of sending a signal exceeding the referred candidate's evaluation.

Therefore, if r_1 drops, regularly submitted applications are more likely rejected as referrals are more appreciated. This decreases q_1 and thus positively affects r_1. Hence, while the impact through q_1^R reinforces any changes of r_1, the impact through q_1 dampens them. If the former dominates the latter, the denominator of (III) is below one, and the impact amplifies overall.

As we can see, with $p > 0$, two markets exist in parallel: one regular labor market and one referral market. Both are highly interconnected, which means that the likelihood of receiving a job offer from one firm through the regular market q_y is strongly impacted by the likelihood that a high-ability worker receives an offer through a referral q_1^R.

Due to those interconnections, an equilibrium is only guaranteed to be unique for some choice of parameters: equilibria with low referral success rates and a high r_1 could coexist with equilibria with high referral success rates and low acceptance rates r_1.

To characterize the conditions for uniqueness, we need a measure for the signal's noise and thus consider the class of distributions with constant slope again. To guarantee uniqueness, the amplification described above must not be too strong, which is the case if signals are not too noisy and referrals are not too prevalent:

Proposition 3. *An equilibrium exists. Moreover, for distributions with a constant slope, the equilibrium is unique if* $\phi\theta \leq \frac{4b}{(2+b)^2}$.

The corresponding derivation is in Appendix A.3. To provide an example, if the market is balanced ($\theta = 1$) and signals are somewhat noisy ($b = 1$), the condition in Proposition 3 requires $p \leq -ln\left(\frac{5}{9}\right) \approx 0.59$, meaning that uniqueness is guaranteed as long as less than 59% of all firms have an employee who offers a referral.

While Condition 9 is much more challenging to analyze than the baseline with $p \to 0$, we can still take away one more important point about markets in which firms anticipate that applicants in the regular market may also have referrals. As the following proposition shows, as long as markets are not too tilted in favor of firms, referrals are even more robust to a if a substantial share of firms utilizes referrals.

Proposition 4. *For all distributions with a constant slope, there exists a cutoff* $\bar{\theta} < 1$. *For all* $\theta > \bar{\theta}$, Q^R *increases in p.*

Recall that $r_1 = (1 - \phi\theta q_1^R)r_1^M$. Intuitively, if more referrals are made, $(1 - \phi\theta q_1^R)$ decreases since firms anticipate that an applicant through the regular market is more likely

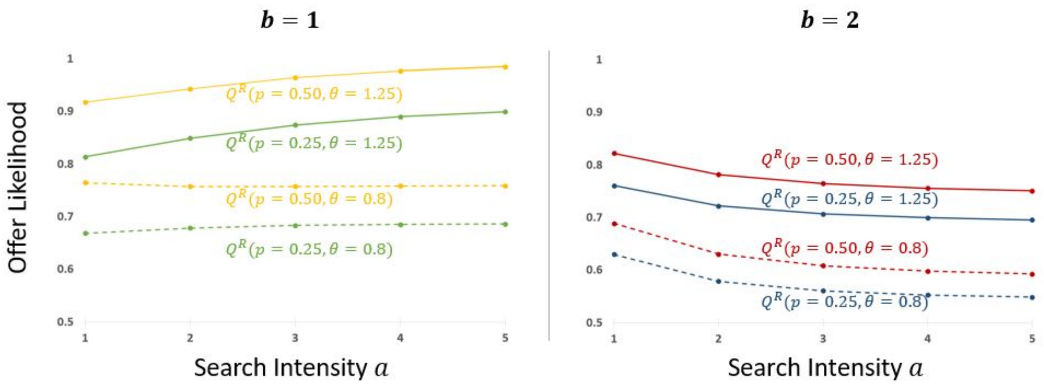

Figure 2: Example with different signal precisions b, market imbalances θ and network densities p

to have been referred and thus will turn down the offer. On the other hand, a higher prevalence of referrals increases the likelihood that a firm receives a referral application, which implies that any market candidate faces much fiercer competition. Therefore, q_1 decreases and r_1^M increases as workers applying through the market are more likely to accept. As this proposition states, the first impact dominates the latter as long as θ is not too small[4]. Overall, firms anticipate a lower acceptance likelihood, increasing the chances of securing a job through a referral.

Figure 2 illustrates this. As we can see, it is in line with our findings in Theorem 1: Q^R is lower if ability signals are more precise (b is higher) and can increase in a. Moreover, in line with Proposition 4, the chance of getting a job offer through a referral increases in p, which means that with more referrals circulating, firms anticipate that regular applicants might have been referred elsewhere, making their own referred candidate stand out even more. In other words, as referrals become more common, firms are more inclined to offer the job to referred candidates.

5 Discussion

This section briefly discusses a few critical assumptions made in this framework before discussing how key differences between labor and marriage markets affect the importance of personal ties.

5.1 Framework Assumptions

First, keeping the space of the ability y binary is highly simplified. However, if the space of y is much more generalized, firms might apply mixed strategies and may send an offer to workers with worse signals since they accept with a higher likelihood. While this is clearly also an interesting problem, it is not the focus of this paper. Nonetheless, even with a more general environment, the preferential treatment of referred candidates remains since they accept with a higher likelihood.

Second, assuming that firms can only send out one offer is another simplification to keep the results straightforward. However, even if we would allow firms to interview and make offers to numerous candidates, the main insights hold as long as interviewing

[4]Note that this condition does not state what happens for $\theta < \bar{\theta}$. Hence, it is possible that it always holds, even for $\theta \to 0$

is costly, which means that firms can only make an offer to a limited number of candidates. For instance, suppose we allow firms to make two offers and assume that the offering phase consists of two stages instead of one. Firms choose their two most promising candidates and send an offer to their first candidate at the first offer stage. Workers simultaneously receive their offers and accept at least one, following the rule in the main framework. In the second stage, rejected firms can make an offer to their second-chosen candidates. If those are still available on the market, they choose which offer to accept according to the rule in the main framework. Therefore, firms still face the issue that their vacancy may remain unfilled and that a candidate's acceptance reveals adverse information about his ability. Therefore, since referred candidates are more likely to accept the offer, the insights of the baseline framework also hold in that extended version.

Third, while the result that referred candidates stand out more if workers search more intensely (Theorem 1) holds for any signal distribution, we can only conduct further analysis by focusing on distributions with a constant slope. Using the parameter b as a measure of the signal's information content and/or the screening technology has a shortcoming: this approach does not cover distributions that make signals (almost) perfectly informative. The likelihood that a high-type player sends a signal that exceeds a low-type's signal is:

$$Pr(\hat{y}_1 > \hat{y}_0) = \int_0^1 (1 - \frac{2-b}{2}\hat{y} - \frac{b}{2}\hat{y}^2)(\frac{2+b}{2} - b\hat{y})d\hat{y} = \frac{b+3}{6} \in \left(\frac{1}{2}, \frac{5}{6}\right]$$

If $b = 2$, a high-ability worker sends out a higher signal than a low-ability worker with a likelihood of roughly 83%. Therefore, such distributions do not cover cases with more precise ability signals. However, on the other hand, a vast literature exists on ability signaling (for instance, based on Spence (1978)), and workers usually need to pass a probation period post-hiring. This strongly indicates that scenarios with highly precise screening are practically irrelevant.

Last, there is no bargaining. According to Hall and Krueger (2012) and Brenzel et al. (2014), only one-third of all employees report that they have explicitly bargained their wages. Therefore, abstracting from possible bargaining still captures most labor markets while keeping the analysis straightforward. Nonetheless, it is worthwhile to analyze this mechanism with flexible wages for future research.

5.2 Differences between labor Markets and Marriage Markets

As mentioned in the introduction, the importance of social contacts has decreased in marriage markets, which raises the question of why this paper's findings could not also be applied in such markets.

As modeled in the framework, firms care about hiring skilled workers and quickly filling their vacancies. Since referred candidates accept job offers much more likely, firms have incentives to offer a job to a referred candidate even if others have provided a better ability signal.

Therefore, firms are willing to cut corners to fill vacancies without delay. On the one hand, they want to hire employees quickly to keep their business operating. On the other hand, firms are not harmed significantly if they end up with an employee with mediocre skills. To understand this, note that in the long run, matches between workers and firms are (i) many-to-one and (ii) with transferable utility. The first attribute means that firms might be able to move unfit workers into other departments that correspond to their skills. The second attribute applies even if wages are not bargained upon before hiring because wages can change throughout employment. This means that firms are barely harmed if they hire a worker with mediocre skills because they can compensate him with a lower salary over time. In contrast, high-skilled employees would ask for a steeper career progression and higher salaries due to their outside option of finding a lucrative job.

On the other hand, marriage markets usually do not fulfill either attribute: they are one-to-one and with non-transferable utility. Therefore, people's gain from having an excellent matching partner is higher, which makes waiting more reasonable. Given this, it is unsurprising that internet search has become so important: it provides many more opportunities to find an outstanding match.

6 Conclusion

This paper addresses the question of why personal ties still play a dominant role in job search, despite the additional search opportunities created by the internet, which should have led to a more level playing field. To do this, my mechanism focuses on one channel which has been barely considered in the referral literature so far: alleviating congestion. To understand the shortcomings of conventional labor markets, I first set up a labor market framework with simultaneous search in which workers vary in ability. Like in the real world, workers can apply to firms without committing to accept their offers, whereas firms need to provide offers with the commitment of hiring the worker. As we see, this leads to two issues: if firms' offers are turned down, they have wasted costly time searching for workers. Moreover, since an acceptance implies that other firms may have rejected that worker, firms face an *acceptance curse*, akin to the *winner's curse* in auctions.

Therefore, since referred workers are more likely to accept job offers, they stand out among all candidates and thus have better chances of receiving an offer. As the framework shows, if search barriers vanish and workers apply to more firms, the likelihood that workers will accept one particular job offer is also lower. Thus, while higher search intensity implies that (i) a referred applicant expects to face more competitors and thus more applicants with a strong signal of their ability, it also means that (ii) all applicants are less interested in the firm on average. This makes the referred applicant stand out more. Therefore, regardless of the workers' aggregate search intensity, the likelihood that a worker ends up with a job through his referring contact remains high.

In my framework, referred candidates have a significant edge over regular applicants because they can credibly signal their interest in a job. Therefore, job platform designers might consider allowing candidates to be endowed with a limited number of signals they can attach to their applications (for instance, see Coles et al. (2010) or Lee and Niederle (2015)). This would alleviate congestion and lead to a more level playing field among workers with and without referrals. Moreover, firms that receive many applications could tackle the described issues at the root by increasing application barriers and thus discourage uninterested workers from applying in the first place.

References

Albrecht, James, Pieter A Gautier, and Susan Vroman, "Equilibrium directed search with multiple applications," *The Review of Economic Studies*, 2006, *73* (4), 869–891.

Arnosti, Nick, Ramesh Johari, and Yash Kanoria, "Managing congestion in matching markets," *Manufacturing & Service Operations Management*, 2021, *23* (3), 620–636.

Ashlagi, Itai, Mark Braverman, Yash Kanoria, and Peng Shi, "Clearing matching markets efficiently: informative signals and match recommendations," *Management Science*, 2020, *66* (5), 2163–2193.

Bolte, Lukas, Nicole Immorlica, and Matthew O Jackson, "The Role of Referrals in Inequality, Immobility, and Inefficiency in Labor Markets," *Immobility, and Inefficiency in Labor Markets (January 1, 2020)*, 2020.

Brenzel, Hanna, Hermann Gartner, and Claus Schnabel, "Wage bargaining or wage posting? Evidence from the employers' side," *Labour Economics*, 2014, *29*, 41–48.

Brown, Meta, Elizabeth Setren, and Giorgio Topa, "Do informal referrals lead to better matches? Evidence from a firm's employee referral system," *Journal of Labor Economics*, 2016, *34* (1), 161–209.

Burks, Stephen V, Bo Cowgill, Mitchell Hoffman, and Michael Housman, "The value of hiring through employee referrals," *The Quarterly Journal of Economics*, 2015, *130* (2), 805–839.

Calvó-Armengol, Antoni, "Job contact networks," *Journal of economic Theory*, 2004, *115* (1), 191–206.

_ and Matthew O Jackson, "The effects of social networks on employment and inequality," *American economic review*, 2004, *94* (3), 426–454.

_ and _ , "Networks in labor markets: Wage and employment dynamics and inequality," *Journal of economic theory*, 2007, *132* (1), 27–46.

Chade, Hector, "Matching with noise and the acceptance curse," *Journal of Economic Theory*, 2006, *129* (1), 81–113.

Coles, Peter, Alexey Kushnir, and Muriel Niederle, "Preference signaling in matching markets," *American Economic Journal: Microeconomics*, 2013, *5* (2), 99–134.

_ , John Cawley, Phillip B Levine, Muriel Niederle, Alvin E Roth, and John J Siegfried, "The job market for new economists: A market design perspective," *Journal of Economic Perspectives*, 2010, *24* (4), 187–206.

Dhillon, Amrita, Vegard Iversen, and Gaute Torsvik, "Employee referral, social proximity, and worker discipline: Theory and suggestive evidence from India," *Economic Development and Cultural Change*, 2021, *69* (3), 1003–1030.

Dustmann, Christian, Albrecht Glitz, Uta Schönberg, and Herbert Brücker, "Referral-based job search networks," *The Review of Economic Studies*, 2016, *83* (2), 514–546.

Engbom, Niklas, "Contagious unemployment," Technical Report, National Bureau of Economic Research 2021.

Forbes, "Networking Is Still The Best Way To Find A Job, Survey Says," `https://www.forbes.com/sites/susanadams/2011/06/07/networking-is-still-the-best-way-to-find-a-job-survey-says/` 2011. Accessed: 2021-11-03.

Galenianos, Manolis, "Hiring through referrals," *Journal of Economic Theory*, 2014, *152*, 304–323.

_ , "Referral networks and inequality," *The Economic Journal*, 2021, *131* (633), 271–301.

_ **and Philipp Kircher**, "Directed search with multiple job applications," *Journal of economic theory*, 2009, *144* (2), 445–471.

Gee, Laura K, Jason Jones, and Moira Burke, "Social networks and labor markets: How strong ties relate to job finding on Facebook's social network," *Journal of Labor Economics*, 2017, *35* (2), 485–518.

Hall, Robert E and Alan B Krueger, "Evidence on the incidence of wage posting, wage bargaining, and on-the-job search," *American Economic Journal: Macroeconomics*, 2012, *4* (4), 56–67.

He, Yinghua and Thierry Magnac, "Application costs and congestion in matching markets," *The Economic Journal*, 2022, *132* (648), 2918–2950.

Heath, Rachel, "Why do firms hire using referrals? Evidence from Bangladeshi garment factories," *Journal of Political Economy*, 2018, *126* (4), 1691–1746.

Kanoria, Yash and Daniela Saban, "Facilitating the search for partners on matching platforms," *Management Science (to appear)*, 2020.

Kircher, Philipp, "Efficiency of simultaneous search," *Journal of Political Economy*, 2009, *117* (5), 861–913.

Kugler, Adriana D, "Employee referrals and efficiency wages," *Labour economics*, 2003, *10* (5), 531–556.

Lee, Soohyung and Muriel Niederle, "Propose with a rose? Signaling in internet dating markets," *Experimental Economics*, 2015, *18*, 731–755.

LinkedIn, "2016 US & Canada Talent Trends," `https://business.linkedin.com/talent-solutions/job-trends/2016-talent-trends-home/2016-us-and-canada-talent-trends` 2016. Accessed: 2021-11-03.

Montgomery, James D, "Social networks and labor-market outcomes: Toward an economic analysis," *The American economic review*, 1991, *81* (5), 1408–1418.

Okafor, Chika O, "All things equal? social networks as a mechanism for discrimination," *arXiv preprint arXiv:2006.15988*, 2020.

Pallais, Amanda and Emily Glassberg Sands, "Why the referential treatment? Evidence from field experiments on referrals," *Journal of Political Economy*, 2016, *124* (6), 1793–1828.

Rosenfeld, Michael J, Reuben J Thomas, and Sonia Hausen, "Disintermediating your friends: How online dating in the United States displaces other ways of meeting," *Proceedings of the National Academy of Sciences*, 2019, *116* (36), 17753–17758.

Simon, Curtis J and John T Warner, “Matchmaker, matchmaker: The effect of old boy networks on job match quality, earnings, and tenure,” *Journal of labor economics*, 1992, *10* (3), 306–330.

Spence, Michael, “Job market signaling,” in “Uncertainty in economics,” Elsevier, 1978, pp. 281–306.

Topa, Giorgio, “Labor markets and referrals,” in “Handbook of social economics,” Vol. 1, Elsevier, 2011, pp. 1193–1221.

Wahba, Jackline and Yves Zenou, “Density, social networks and job search methods: Theory and application to Egypt,” *Journal of Development Economics*, 2005, *78* (2), 443–473.

A Appendix

A.1 Proof of Theorem 1

Part (i) To show that Q^R can increase, note that according to the following Lemma, $\frac{\partial Q^R}{\partial r_1}\frac{\partial r_1}{\partial a} \geq 0$:

Lemma 2. *If $p \to 0$, r_1 decreases in a*

To show that the decrease of r_1 as well as the impact of r_1 on Q^R can be strong enough, consider a simple example with $g_1(\hat{y}) = \frac{2-b}{2} + b\hat{y}$ and $b \to 0$. With $a = 1$, $Q^R = \theta(1 - e^{-\frac{1}{\theta}}) < 1$. With $a = 2$, $r_1 < 1$ and thus $Q^R = 1$ since signals are not informative such that firms always choose the referred candidate.

Part (ii) Since $q_y^R = 1 - G_y(\bar{y}) + \int_0^{\bar{y}} e^{-\frac{a}{\theta}(1-g_1^{-1}(\frac{g_1(\hat{y})}{r_1}))} g_y(\hat{y})d\hat{y}$, the derivative is

$$\frac{dq_y^R}{da} = \int_0^{\bar{y}} \underbrace{e^{-\frac{a}{\theta}(1-g_1^{-1}(\frac{g_1(\hat{y})}{r_1}))}}_{c(\hat{y})} \left[\underbrace{\frac{a}{\theta}\frac{\partial g_1^{-1}(\frac{g_1(\hat{y})}{r_1})}{\partial r_1}\frac{dr_1}{da} - \frac{1}{\theta}(1 - g_1^{-1}(\frac{g_1(\hat{y})}{r_1}))}_{d(\hat{y})} \right] g_y(\hat{y})d\hat{y} \quad (11)$$

Since $g_1(\hat{y})$ and $g_0(\hat{y})$ both have a constant slope, $g_1^{-1}(\cdot)$ is also linear. Thus, there is a threshold $\tilde{y} \in [0, \bar{y}]$ at which $d(\hat{y}) = 0$, $d(\hat{y}) < 0$ for $\hat{y} \in [0, \tilde{y})$ and $d(\hat{y}) > 0$ for $\hat{y} \in (\tilde{y}, \bar{y}]$.

Assume that $\frac{dq_0^R}{da} \geq 0$. Note that $g_0(\hat{y}) > g_1(\hat{y})$ for $\hat{y} \in [0, 0.5)$ and $g_0(\hat{y}) < g_1(\hat{y})$ for $\hat{y} \in (0.5, 1]$. Therefore, we distinguish between two cases:

1. $\bar{y} \leq 0.5$. Since $\int_0^{\bar{y}} adg_0(\hat{y})d\hat{y} \geq 0$, this also holds for G_1 because $\int_0^{\bar{y}} c(\hat{y})d(\hat{y})\Delta_0(\hat{y})d\hat{y} > 0$ since $\Delta_0(\hat{y}) \equiv g_1(\hat{y}) - g_0(\hat{y})$ is increasing in $\hat{y}$.

2. $\bar{y} > 0.5$. Define $\tilde{y} : d(\tilde{y}) = 0$

   (a) $\tilde{y} \leq 0.5$. This implies that Condition 11 changes by

$$\underbrace{\int_{0.5}^{\bar{y}} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}}_{+} + \underbrace{\int_{\tilde{y}}^{0.5} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}}_{-} + \underbrace{\int_0^{\tilde{y}} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}}_{+}$$

Note that $\int_{\tilde{y}}^{0.5} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}$ would be lowest if $\tilde{y} = 0$. Nonetheless, even in that case $\int_{0.5}^{\bar{y}} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y} > -\int_{\tilde{y}}^{0.5} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}$ it would hold as Condition 11 holds and $\Delta_0\hat{y}$ increases in $\hat{y}$.

(b) $\tilde{y} > 0.5$. This implies that Condition 11 changes by

$$\underbrace{\int_{\tilde{y}}^{\bar{y}} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}}_{+} + \underbrace{\int_{0.5}^{\tilde{y}} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}}_{-} + \underbrace{\int_{0}^{0.5} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}}_{+}$$

Since Condition 11 holds and $\Delta_0\hat{y}$ increases in $\hat{y}$,
$\int_{\tilde{y}}^{\bar{y}} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y} > -\int_{0.5}^{\tilde{y}} c(\hat{y})d(\hat{y})\Delta_0\hat{y}d\hat{y}$.

Part (iii) Recall that $Q^R = 1 - \bar{y} + \int_0^{\bar{y}} e^{-\frac{a}{\theta}(1-g_1^{-1}(\frac{g_1(\hat{y})}{r_1}))}d\hat{y}$. Taking the derivative with respect to b, it decreases if

$$\int_0^{\bar{y}} e^{-\frac{a}{\theta}(1-min\{1,g_1^{-1}(\frac{g_1(\hat{y})}{r_1})\})}\left[-\frac{dr_1}{db}\left(\frac{\hat{y}}{r_1^2} + \frac{2-b}{2br_1^2}\right) - \frac{1-r}{rb^2}\right]d\hat{y} < 0$$

Clearly, this condition is more difficult to satisfy with a larger $\hat{y}$, so let us examine it at $\hat{y} = \bar{y} \equiv g_1^{-1}(g_1(1)r_1) = \frac{2+b}{2b}r_1 - \frac{2-b}{2b}$. Thus, a sufficient condition is

$$\begin{aligned}
-\frac{dr_1}{db}\frac{\frac{2+b}{2b}}{r_1} - \frac{1-r_1}{r_1b^2} &\leq 0\\
\frac{r_1-(1-q_1)^{a-1}}{q_1}\frac{1}{2}\Delta q_1\frac{\frac{2+b}{2b}}{r_1} - \frac{1-r_1}{r_1b^2} &\leq 0\\
\frac{r_1-(1-q_1)^{a-1}}{1-r_1}\frac{\Delta q_1}{q_1} &\leq \frac{4}{b(2+b)}\\
\frac{r_1-(1-q_1)^{a-1}}{1-r_1} &\leq \frac{4}{\frac{\Delta q_1}{q_1}b(2+b)}
\end{aligned}$$

With $\Delta q_1 = q_1(b=2) - q_1(b \to 0)$. Denote $A \equiv \frac{4}{b(2+b)\frac{q}{\Delta q}}$. Moreover, use the following Lemma:

Lemma 3. $r_1 - (1-q_1)^{a-1} \leq 1 - r_1$

Proof. Recall that $r_1 = \frac{1}{a}\sum_{x=0}^{a-1}(1-q_1)^x$. Note that $(1-q_1)^x$ is convex in x. Therefore, $\frac{1+(1-q_1)^{a-1}}{2} \geq \frac{1}{a}\sum_{x=0}^{a-1}(1-q_1)^x = r_1$. □

Thus, $\frac{r_1-(1-q)^{a-1}}{1-r_1} \leq 1$ and therefore $A \geq 1$ is sufficient. Note that A decreases in b since

$$\frac{dA}{db} = \frac{4}{\Delta q b(2+b)}\left(-\frac{2+2b}{b(2+b)}q_1 + 0.5\Delta q_1\right) \leq 0$$

since $\frac{4+4b}{b(2+b)} > 1 > \frac{\Delta q}{q}$ with the latter following from $\frac{\Delta q}{q} \leq \frac{q(b=2)-q(b\to 0)}{q(b\to 0)} = \frac{q(b=2)}{q(b\to 0)} - 1 < 1$ since $\frac{q(b=2)}{q(b\to 0)} < 2$.
With $b = 2$, $A = \frac{0.5}{1-\frac{q(b\to 0)}{q(b=2)}}$. As we know from above, $\frac{q(b\to 0)}{q(b=2)} > 0.5$, such that $A > 1$ always holds. □

A.2 Proof of Proposition 1

We need to show that

$$q_y = \int_0^1 e^{-\frac{a}{\theta}(1-\hat{y})} g_y(\hat{y}) d\hat{y} \qquad \text{for } y \in \{0,1\}$$

and do this in two steps: the first one is established by the following Lemma:

Lemma 4. *Firms always make an offer to the candidate with the highest signal*

Proof. A firm's ex-ante payoff from a candidate with signal $\hat{y}$ is $\frac{g_1(\hat{y})}{2} r_1$. Since r_1 is independent from $\hat{y}$ and $g_1'(\hat{y}) > 0$, the ex-ante payoff strictly increases in $\hat{y}$ □

Second, we need to establish that with $v, u \to \infty$, firms receive applications according to the Poisson Distribution. This has been done by Albrecht et al. (2006), who show that the likelihood of receiving an offer from a firm is

$$\lim_{u,v\to\infty} q_y = 1 - \sum_{x=0}^{\infty} Pr_y(rejected|x)h(x)$$

with x denoting the actual number of competitors for one vacancy, and $h(x) = \frac{(\frac{a}{\theta})^x e^{-\frac{a}{\theta}}}{x!}$.

Making use of Lemma 4 as well as the convergence to the Poisson distribution, we can derive the likelihood of receiving a job offer from one particular firm. Given that the unconditional distribution is uniform as $\frac{g_0(\hat{y})+g_1(\hat{y})}{2} = 1$ always holds, the likelihood of being rejected with x competitors is $1 - \hat{y}^x$ for a given $\hat{y}$.

Therefore,

$$1 - \sum_{x=0}^{\infty}\left[\int_0^1 1 - \hat{y}^x g_y(\hat{y})d\hat{y}\right]\frac{(\frac{a}{\theta})^x e^{-\frac{a}{\theta}}}{x!} = 1 - \int_0^1 \sum_{x=0}^{\infty}(1-\hat{y}^x)\frac{(\frac{a}{\theta})^x e^{-\frac{a}{\theta}}}{x!} g_y(\hat{y})d\hat{y} =$$
$$1 - \int_0^1 \sum_{x=0}^{\infty} \frac{(\frac{a}{\theta})^x e^{-\frac{a}{\theta}}}{x!} - \frac{(\frac{a}{\theta}\hat{y})^x e^{-\frac{a}{\theta}}}{x!} g_y(\hat{y})d\hat{y} = 1 - \int_0^1 1 - e^{\frac{a}{\theta}\hat{y}} e^{-\frac{a}{\theta}} g_y(\hat{y})d\hat{y} =$$
$$\int_0^1 e^{-\frac{a}{\theta}(1-\hat{y})} g_y(\hat{y})d\hat{y} \square$$

A.3 Proof of Proposition 3

Existence. The variable summarising the equilibrium is r_1. Based on this, for a given θ, a, ϕ and G, both q_1 and q_1^R are determined. Recall that

$$F \equiv r_1 - (1 - \phi\theta q_1^R) r_1^M = 0$$

has to hold in equilibrium. If we use $r_1 = 0$, then clearly $F < 0$. Conversely, for $r_1 = 1$, $F > 0$ since $r_1^M < 1$ for $a > 1$. F is continuous in r_1, so according to the Intermediate Value Theorem, there exists an r_1 where $F = 0$.

Uniqueness. Applying contraction mapping, an equilibrium is unique if

$$1 > \underbrace{-\phi\theta r^m \frac{\partial q_1^R}{\partial r_1}}_{+} \underbrace{-\frac{\partial q_1}{\partial r_1}(1-\phi\theta q_1^R)\frac{[1-(1-q_1)^a]-(1-q_1)^{a-1}aq_1}{aq_1^2}}_{-}$$

Therefore, the change of q_1^R must not be too large. If g_1 is of constant slope, q_1^R can be rewritten as

$$q_1^R = 1 - G_1(\bar{y}) + \int_{\underline{y}}^{1} e^{-\frac{a}{\theta}(1-\hat{y})} f_1(\hat{y}) d\hat{y}$$

with

$$\begin{aligned}
\bar{y} &= g_1^{-1}\left(g(1) r\right) = \frac{2+b}{2b} r_1 - \frac{1}{b} + 0.5 = \frac{2+b}{2b} r_1 - \frac{2-b}{2b} \\
\underline{y} &= g_1^{-1}\left(\frac{g(0)}{r_1}\right) = \frac{1-0.5b}{b r_1} - \frac{1}{b} + 0.5 = \frac{2-b}{2b r_1}(1-r_1) \\
f_1(\hat{y}) &= r_1 g_1(\hat{y}) \frac{\bar{y}}{1-\underline{y}} = r_1^2 g_1(\hat{y})
\end{aligned}$$

Intuitively, the referred worker's signal distribution is shifted upward for all signals below $\bar{y}$. The lower bound is $\underline{y}$, which is the converted signal if $\hat{y} = 0$. Thus, $f_1(\hat{y})$ consists of two parts: one, through the condition $g_1(\hat{y}_2) = r_1 g_1(\hat{y}_1)$ which denotes the inequality condition of a firm receiving a signal $\hat{y}_2$ from a referred and $\hat{y}_1$ from a regular applicant, respectively. Second, the scaling $\frac{\bar{y}}{1-\underline{y}}$ is included since the rewritten signal distribution is transformed from $[0, \bar{y}]$ into $[\underline{y}, 1]$.

Therefore,

$$\begin{aligned}
\frac{\partial q_1^R}{\partial r_1} =& - g(\bar{y}) \frac{\partial \bar{y}}{\partial r_1} + 2 r_1 \int_{\underline{y}}^{1} e^{-\frac{a}{\theta}(1-\hat{y})} g_1(\hat{y}) d\hat{y} - e^{-\frac{a}{\theta}(1-\underline{y})} g(\underline{y}) \frac{\partial \underline{y}}{\partial r_1} r_1^2 = \\
& - r_1 \frac{(2+b)^2}{4b} + 2 r_1 \int_{\underline{y}}^{1} e^{-\frac{a}{\theta}(1-\hat{y})} g_1(\hat{y}) d\hat{y} - e^{-\frac{a}{\theta}(1-\underline{y})} g(\underline{y}) \frac{\partial \underline{y}}{\partial r_1} r_1^2 = \\
& - 2 r_1 \left(\frac{(2+b)^2}{8b} - \int_{\underline{y}}^{1} e^{-\frac{a}{\theta}(1-\hat{y})} g_1(\hat{y}) d\hat{y} \right) + e^{-\frac{a}{\theta}(1-\underline{y})} \frac{2-b}{2 r_1} r_1^2 \frac{2-b}{2 b r_1^2} = \\
& - 2 r_1 \left(\frac{(2+b)^2}{8b} - \int_{\underline{y}}^{1} e^{-\frac{a}{\theta}(1-\hat{y})} g_1(\hat{y}) d\hat{y} \right) + e^{-\frac{a}{\theta}\left(\frac{r_1(2+b)-(2-b)}{2 b r_1}\right)} \frac{(2-b)^2}{4 b r_1} \leq 0
\end{aligned}$$

Therefore, a sufficient condition for uniqueness is $1 \geq \phi\theta \frac{(2+b)^2}{4b}$.□

A.4 Proof of Proposition 4

First define $\bar{q}_1 := \int_0^1 e^{-\frac{a}{\theta}(1-\hat{y})} g(\hat{y})_1 d\hat{y}$ and $q_1^B := \int_0^1 e^{-\frac{a}{\theta}(1-\hat{y})} max\{0, g_1^{-1}(g_1(\hat{y}) r_1) g_1(\hat{y})\} d\hat{y}$, such that $q_1 = (1-\phi)\bar{q} + \phi q_1^B$.

If ϕ changes, Q^R is impacted through any changes in r_1. Note that r_1 decreases in ϕ iff

$$\begin{aligned}
\theta q_1^R r_1^M &\geq (1 - \phi\theta q_1^R)(r_1^M - (1-q_1)^{a-1}) \frac{\partial q_1}{\partial \phi} \frac{1}{q_1} \\
1 - \frac{(1-q_1)^{a-1}}{r_1^M} &\leq \frac{\theta q_1^R}{(1-\phi\theta q_1^R)} \frac{(1-\phi)\bar{q}_1 + \phi q_1^B}{q_1^B} \\
1 - \frac{\theta q_1^R}{(1-\phi\theta q_1^R)} \frac{(1-\phi)\bar{q}_1 + \phi q_1^B}{q_1^B} &\leq \frac{(1-q_1)^{a-1}}{r_1^M} \\
1 - (1-q_1)^a &\leq \frac{(1-q_1)^{a-1} a q_1}{1 - \frac{\theta q_1^R}{(1-\phi\theta q_1^R)} \frac{(1-\phi)\bar{q}_1 + \phi q_1^B}{q_1^B}} \qquad (12)
\end{aligned}$$

with the last line using $r_1^M = \frac{1-(1-q_1)^a}{aq_1}$. For $a = 1$, $1 - (1 - q_1)^a = aq_1(1 - q_1)^{a-1}$ and the condition trivially holds. For $a > 1$, $1 - (1 - q_1)^a > aq_1(1 - q_1)$ since $\frac{\partial r_1^M}{\partial q_1} < 0$ for $a > 1$. Therefore, this condition holds only if the denominator on the right-hand side is small enough. Clearly, a sufficient condition for Condition 12 to hold is

$$\frac{\theta q_1^R}{1 - \phi\theta q_1^R} \frac{(1-\phi)\bar{q}_1 + \phi q_1^B}{q_1^B} \geq 1$$
$$\leftrightarrow \theta \frac{q_1^R\left[(1-\phi)\bar{q}_1 + 2\phi q_1^B\right]}{q_1^B} \geq 1$$

In the following two paragraphs, in some instances, computations are necessary to confirm the magnitude of some parameters. Whenever this is the case, (*) is added. The corresponding code is available upon request.

For $a > 1$, assume first that $\phi = 0$, in which case it can be shown that it always holds for any b and $a > 1$. First, computations show that for $a \in \{2, 3, ..., 100\}$ it always holds (*). To show that it also holds for any $a > 100$, note that $q_1^R > 1 - G(\bar{y}) > 1 - \bar{y}$ and $\frac{\bar{q}_1}{q_1^B} > \frac{1}{\bar{y}}$. Therefore, $q_1^R \frac{\bar{q}_1}{q_1^B} > \frac{1-\bar{y}}{\bar{y}}$ which is above 1 as long as $\bar{y} \leq \frac{1}{2}$. Using $\bar{y} = \frac{2+b}{2b} r_1 - \frac{2-b}{2b}$, we need $r_1 \leq \frac{2}{2+b}$ to have $\bar{y} \leq \frac{1}{2}$. As computations show (*), at $a = 100$, this condition holds for any b. Since by Lemma 1, r_1 decreases with a, this also holds with any $a > 100$.

To show that it also holds for any $\phi > 0$, assume that there exists some $\phi' > 0$ where it does not hold. Denote $F \equiv \frac{q_1^R\left[(1-\phi)\bar{q}_1 + 2\phi q_1^B\right]}{q_1^B} - 1$. For ϕ' to exist, it is necessary that there exists some $\phi'' = min\{\phi : F = 0, \frac{\partial F}{\partial \phi} < 0\}$. This can only be the case if $\frac{\left[(1-\phi)\bar{q}_1 + 2\phi q_1^B\right]}{q_1^B}$ has sufficiently decreased in ϕ since $q_1^R(\phi = \phi') > q_1^R(\phi = 0)$. This implies that $\bar{q}_1 > 2q_1^B$ and thus $\frac{\left[(1-\phi)\bar{q}_1 + 2\phi q_1^B\right]}{q_1^B} > 2$. Since, however, q_1^R is always beyond 0.5 at $\phi = 0$ (*), $F(\phi') > 0$ which leads to a contradiction.

Clearly, this condition is also satisfied for any $\theta > 1$ since

$$\frac{q_1^R\left[(1-\phi)\bar{q}_1 + 2\phi q_1^B\right]}{q_1^B} + \theta \frac{\frac{\partial q_1^R}{\partial \theta}\left[(1-\phi)\bar{q}_1 + 2\phi q_1^B\right] q_1^B}{{q_1^B}^2} +$$
$$\theta \frac{q_1^R q_1^B [(1-\phi)\frac{\partial \bar{q}_1}{\partial \theta} + \frac{\partial q_1^B}{\partial \theta}]}{{q_1^B}^2} - \theta \frac{\frac{\partial q_1^B}{\partial \theta} q_1^R \left[(1-\phi)\bar{q}_1 + 2\phi q_1^B\right]}{{q_1^B}^2} > 0$$

which follows from $\theta \frac{\partial q_1^B}{\partial \theta} < q_1^B$ if we apply the logic from the proof of Lemma 1, given that $g_1^{-1}(g_1(\hat{y})r_1)$ increases in $\hat{y}$.

Therefore, $\frac{dr_1}{d\phi} < 0$ for any $\theta \geq 1$. Based on this, we can define the set $S^\theta := \{\theta : \theta = \frac{q_1^B}{q_1^R\left[(1-\phi)\bar{q}_1 + 2\phi q_1^B\right]}\}$ and $S = S^\theta \cap \mathbb{R}^+$. If S^θ is empty, then $\bar{\theta} = 0$. Otherwise, $\bar{\theta} = maxS < 1.\Box$

A.5 Proof of Lemma 1

For notational convenience, define $m_1 \equiv 1-(1-q_1)^a$ as the likelihood that a worker receives at least one job offer. Note that

$$\frac{dr_1}{da} = \frac{\frac{dm_1}{da}aq_1 - m_1[q_1 + a\frac{\partial q_1}{\partial a}]}{a^2q_1^2} =$$
$$\frac{(1-q_1)^a[-ln(1-q_1)+\frac{\partial q_1}{\partial a}\frac{aq_1}{1-q_1}]aq_1 - m_1[q_1+a\frac{\partial q_1}{\partial a}]}{a^2q_1^2} =$$
$$\frac{-q_1[m_1 + ln(1-q_1)(1-q_1)^a a] - a\frac{\partial q_1}{\partial a}[m_1 - aq_1(1-q_1)^{a-1}]}{a^2q_1^2} < 0$$

since $m_1 + ln(1-q_1)(1-q_1)^a a > m_1 - aq_1(1-q_1)^{a-1} > 0$ and $q_1 > -a\frac{\partial q_1}{\partial a}$. This holds as we can show that:

1. $m_1 + ln(1-q_1)(1-q_1)^a a > m_1 - aq_1(1-q_1)^{a-1} > 0$
2. $q_1 > -a\frac{\partial q_1}{\partial a}$

For the first part, note that $-ln(1-q) > \frac{q}{1-q}$ as $q > 0$. Furthermore, $\frac{dr_1}{dq_1} = -\frac{m_1 - aq_1(1-q_1)^{a-1}}{aq_1^2} < 0$ as $r_1 = \frac{1}{a}\sum_{x=0}^{a-1}(1-q_1)^x$ clearly decreases in q_1.

For the second part, note that

$$\frac{d(aq_1)}{da} = a\frac{\partial q_1}{\partial a} + q_1 = \int_0^1 e^{-\frac{a}{\theta}(1-\hat{y})}[1-\frac{a}{\theta}(1-\hat{y})]g_1(\hat{y})d\hat{y}$$

Define $\check{y} \equiv 1 - \frac{\theta}{a}$. With a uniform distribution, $aq_1 = a\frac{\theta}{a}(1-e^{-\frac{a}{\theta}}) = \theta(1-e^{-\frac{a}{\theta}})$ which clearly increases in a. Therefore, $\frac{d(aq_1)}{da} = a\frac{\partial q_1}{\partial a} + q_1 > 0$. Put differently,

$$\int_{\check{y}}^1 e^{-\frac{a}{\theta}(1-\hat{y})}[1-\frac{a}{\theta}(1-\hat{y})]d\hat{y} - \int_0^{\check{y}} e^{-\frac{a}{\theta}(1-\hat{y})}[\frac{a}{\theta}(1-\hat{y})-1]d\hat{y} > 0 \tag{13}$$

Now, we will show that this implies that $\frac{daq_1}{da} > 0$ for any $g_1(\hat{y})$ which satisfied MLRP and $\frac{g_0(\hat{y})+g_1(\hat{y})}{2} = 1$. Define $\Delta\hat{y} \equiv g_1(\hat{y}) - 1$, $f(\hat{y}) \equiv e^{-\frac{a}{\theta}(1-\hat{y})}[1-\frac{a}{\theta}(1-\hat{y})]$ and $\tilde{y} = min\{\hat{y} : g_1(\hat{y}) = 1\}$.

1. $\check{y} > \tilde{y}$. This implies that Condition 13 changes by

$$\underbrace{\int_{\check{y}}^1 f(\hat{y})\Delta\hat{y}d\hat{y}}_{+} + \underbrace{\int_{\tilde{y}}^{\check{y}} f(\hat{y})\Delta\hat{y}d\hat{y}}_{-} + \underbrace{\int_0^{\tilde{y}} f(\hat{y})\Delta\hat{y}d\hat{y}}_{+}$$

Since Condition 13 holds and $\Delta\hat{y}$ increases in $\hat{y}$, we have $\int_{\check{y}}^1 f(\hat{y})\Delta\hat{y}d\hat{y} > -\int_{\tilde{y}}^{\check{y}} f(\hat{y})\Delta\hat{y}d\hat{y}$

2. $\check{y} \leq \tilde{y}$. This implies that Condition 13 changes by

$$\underbrace{\int_{\tilde{y}}^1 f(\hat{y})\Delta\hat{y}d\hat{y}}_{+} + \underbrace{\int_{\check{y}}^{\tilde{y}} f(\hat{y})\Delta\hat{y}d\hat{y}}_{-} + \underbrace{\int_0^{\check{y}} f(\hat{y})\Delta\hat{y}d\hat{y}}_{+}$$

Note that $\int_{\check{y}}^{\tilde{y}} f(\hat{y})\Delta\hat{y}d\hat{y}$ would be lowest if $\check{y} = 0$. Since $\int_0^1 \Delta\hat{y}d\hat{y} = 0$ and both $f(\hat{y})$ and $\Delta\hat{y}$ increase with $\hat{y}$, $\int_{\tilde{y}}^1 f(\hat{y})\Delta\hat{y}d\hat{y} + \int_{\check{y}}^{\tilde{y}} f(\hat{y})\Delta\hat{y}d\hat{y} > 0$ always holds.□