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Abstract

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Reference Details

2370 Cambridge Working Papers in Economics
2319 Janeway Institute Working Paper Series

Published 8 November 2023

Key Words Sovereign Default, Foreign Reserves, Self-Fulfilling Crises, Expectations, Discretionary Fiscal Policy, Debt Sustainability
JEL Code E43, E62, F34, H50, H63

Websites www.econ.cam.ac.uk/cwpe
www.janeway.econ.cam.ac.uk/working-papers

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*We thank our discussant Dmitry Mukhin and the participants in the 2023 CEPR International Macroeconomics and Finance Programme Meeting in London, the Cambridge Macro Workshop and the EUI Macro Working Group for comments. We gratefully acknowledge support by the Research Council of the EUI (2023 COVID-19 Project "Debt Sustainability: Towards a new blueprint for a comprehensive study of debt crises").
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Contents

1 Introduction

The significant buildup of international reserves by emerging markets stands out as a prominent puzzle in international economics. Countries accumulate abundant reserves as a liquidity buffer against disruptions in the international financial markets. However, holding reserves is costly—reserves pay a low return and, to the extent that large financial buffers reduce the social and economic costs of default, may raise borrowing costs for the government. Reserves could then be best employed to retire external debt, thereby lowering spreads and mitigating the risk of sovereign default. Why do countries accumulate a large amount of reserves? Why are they reluctant to run down their stock in the face of adverse shocks? Is there an optimal level of reserve accumulation?

Leading contributions in the literature have modelled reserves as a means to smooth consumption across default and repayment states, and high and low cost of borrowing states. Adopting this approach, a key benchmark result is derived by Alfaro and Kanczuk (2009), who show that, with one-period debt, the benefits of retiring external debt using reserves generally outweigh that of keeping reserves for consumption smoothing—if debt is short term, reserves do not play any quantitatively important role. Allowing for long-term bonds and risk-averse lenders, Bianchi et al. (2018) study the role of reserves as a means to insure against hikes in borrowing costs when large negative shocks hit the economy—as in this case the government can contain the issuance of new debt (at high interest rates) by running down the stock of reserves. Reserve management thus amounts to an optimal transfer of resources from good states of the world (where borrowing is cheap) to bad states (where borrowing is costly).[1] However, reserve accumulation is typically motivated by the need to address self-fulfilling debt crises and belief-driven hikes in borrowing costs—closer to models of crises arising in situations where market pricing and debt/fiscal policy are not unique in equilibrium.

In this paper we revisit the theory of reserve accumulation specifying a model in which a joint optimal policy of reserve and debt management shields the economy from belief-driven hikes in borrowing costs. Their accumulation is sized to the need to keep the equilibrium path of consumption smooth relative to paths that, without the optimal reserve and debt policy in place, would significantly deteriorate the equilibrium allocation reflecting non-fundamental uncertainty. Our work builds on Aguiar et al. (2022), who stress that unexpected shocks may lead governments to reconsider their willingness to repay upon concluding a debt auction—generating "intra-period uncertainty" about repayment. These authors show that, even if

[1] A common assumption in the literature is that default decisions are taken before auctioning new bonds, à la Eaton and Gersovitz (1981). Once the government has already committed to honor its outstanding liabilities before auctioning new debt, reserve holdings would not impinge on the *willingness* to repay in the same period.

lenders attach an infinitesimal probability to these post-auction shocks, their presence raises the possibility of belief-driven sovereign risk crises that can be highly detrimental to welfare and fiscal stability.[2] The need to manage 'intra-period' uncertainty prompts a novel and compelling perspective on reserve accumulation. Intuitively, reserve management moderates up to eliminating altogether such uncertainty by providing a liquidity buffer the government can count on after implementing its debt issuance plans. We show that the government is able to issue debt to finance the optimal level of reserve holdings without suffering a significant increase in default premia—i.e., facing little or no trade-off between intra-period and inter-temporal uncertainty.[3]

In the model, different types of equilibrium bond prices are possible. The first, dubbed the *Eaton-Gersovitz price*—prevails when lenders disregard concerns about intra-period uncertainty, and anticipate that the government will repay maturing debt in the current period in all circumstances. This price only reflects future default (inter-temporal) risk, which may rise with a higher stock of debt. Another equilibrium bond price is the *failed auction price*, equal to zero, characterizing the rollover crises studied by Cole and Kehoe (2000).[4] Most relevant for our analysis is the third type of prices, first characterized by Aguiar et al. (2022), reflecting default risk associated to intra-period shocks. These prices are termed *interior prices*, because they lie between the failed-auction and the Eaton-Gersovitz prices.[5] They are positive but lower than the Eaton-Gersovitz price, since they reflect both intra-period and inter-temporal default risk.

We show that the government can optimally manage reserves to keep the economy in a 'good' equilibrium where lenders consistently offer the Eaton-Gersovitz price (schedule). Over a reasonably large range of intermediate debt levels, through reserve accumulation the government brings the economy to operate in a region where abundant liquidity safeguards against intra-period risk and government bonds are traded at the near risk-free (Eaton-

[2]The theoretical core of our model lies in relaxing the assumption of Eaton and Gersovitz (1981) that the government commits to repay outstanding debt prior to the new debt auction—replacing it with the timing specified in Cole and Kehoe (2000) and Aguiar et al. (2022), which explicitly allows for a temporal gap between a debt auction and the subsequent decision on repayment of maturing bonds. Because of this gap, lenders participating in the auction may become wary that unforeseen shocks potentially influence repayment decisions after the auction is closed. Lenders' concern about repayment gives rise to the range of disruptive self-fulfilling hikes in sovereign risk extensively explored in Aguiar et al. (2022).

[3]This trade-off is central to the work by Aguiar et al. (2022). Abstracting from reserve accumulation, these authors show that, unless the outstanding stock of debt is very low, the government finds it optimal to eliminate intra-period uncertainty about repayment by 'over-issuing' debt, hence at the cost of higher interest rates associated with a larger stock of liabilities (creating "inter-temporal" uncertainty).

[4]Lenders coordinate on a zero price anticipating that, if the auction fails, the surplus adjustment required to honor maturing debt is too large and harsh, for the government not to default. The analyses in Eaton and Gersovitz (1981) and Cole and Kehoe (2000) formalize the equilibrium outcomes given these two distinct types of prices.

[5]Interior prices are self-fulfilling in a sense that the government, given those prices, may end up making default decisions depending on the realization of intra-period shocks.

Gerzovitz) price. The government will only refrain from holding reserves (and pursue austerity measures) at very low levels of debt, as in this case the economy is immune to belief-driven crises without relying on liquidity buffers.

We conduct our analysis in two steps. First, we rely on a stylized two-period model to offer a transparent analysis of the specific role reserve accumulation plays in ruling out belief-driven crises. Second, we generalize our results using an infinite-horizon model calibrated with short and long-term debt. Relying on the two-period model we show that, conditional on lenders being concerned about intra-period uncertainty, there exists a unique joint level of reserve accumulation and debt issuance that maximizes social welfare. A sub-optimal constraint on reserves worsens welfare in that either the economy remains vulnerable to instability due to intra-period uncertainty (if debt issuance is not re-optimized), or the government needs "over-issue" debt and eliminate intra-period default risk at the cost of increasing inter-temporal default risk, in line with the analysis of Aguiar et al. (2022).[6]

Relying on the infinite-horizon extension of the model, we show that the government has a strong welfare incentive to engage in precautionary accumulation of reserves to insure against future intra-period uncertainty (where it would refrain to do so in the two-period model). Our quantitative analysis suggests that the risk that lenders turn concerned about current and future intra-period uncertainty makes the government reluctant to run down reserves confronting negative fundamental shocks to output. In our simulation with short-term bonds, the government consistently holds reserves around 3% of GDP. It rarely reduces its holdings to zero—this only occurs in response to a long enough sequence of negative fundamental shocks to output.

While long-term bond maturities offer some insurance against intra-period risk, reserve accumulation remains pivotal to eliminate vulnerability to belief-driven crisis at low financial and welfare costs. In our calibration with long-term debt, the model explains reserve accumulation up to 2.4%, against the 3% benchmark with short-term debt. The high average holding is mostly due to intra-period uncertainty, but with long-term debt reserves are also used to smooth consumption in response to shocks to the fundamental. Relative to the optimal policy, constraining reserve accumulation to zero worsens welfare, raising average spreads (from .14 to 2.89 for the case of short-term debt), the frequency of default (from .14 to 2.52), and the volatility of consumption relative to that of output (from .93 to 1.29).

From a policy perspective, our results help explaining why governments with significant debt levels hold large amounts of reserves and cash balances, and appear reluctant to reduce their consistency even when their economies are hit by shocks. Figure 1 plots the median

[6] A numerical example based on the two-period model gives a clear sense of the importance of reserve policy. Namely, in our experiments, at an initial debt level of 27% of GDP, raising the holdings of reserves from zero to their optimal level reduces the yield spread on government bonds from 14% to about 1%. The gap is wider in our quantitative exercise below.

Figure 1: Trends in reserve to GDP ratio
Median and interquartile range for a sample of 59 countries

reserves holdings in percentage of GDP, along with the interquartile range for a sample comprising 59 of countries,[7] spanning the period from 1990 to 2022. Between 2010 and 2022, the countries in the sample keep reserve holdings above 10 percent of GDP, retaining them through downturns.[8] While, as theorized by Bianchi et al. (2018), liquidity buffers can (and should) be run down to smooth debt costs across good and bad states driven by fundamental uncertainty, our theory suggests that the liquidity buffer that mitigates short-run fundamental and non-fundamental uncertainty must be sized up to satisfy any belief-driven liquidity need that can potentially emerge off the equilibrium path.

The literature. This paper is related to a vast literature on self-fulfilling debt crises. Aguiar et al. (2022) enrich Cole and Kehoe (2000) by introducing intra-period uncertainty that gives rise to a rich set of self-fulfilling debt crises,[9] expanding beyond the canonical rollover crises in Cole and Kehoe (2000).[10] In our paper, we show that intra-period uncertainty provides a natural theoretical framework on which to build a theory of reserve accumulation.

As already mentioned, Alfaro and Kanczuk (2009) and Bianchi et al. (2018) study reserve accumulation building on the quantitative sovereign debt literature after Aguiar and

[7]In Appendix E, we provide the list of countries used in Figure 1.

[8]During the sample period episodes of large downturn include (but they are not limited to) the Great Recession, the European debt crises, the COVID-19 pandemic.

[9]A different strand of self-fulfilling debt crises literature, following Calvo (1988), is extensively explored in Corsetti and Dedola (2016), Ayres et al. (2018), Ayres et al. (2019), Lorenzoni and Werning (2019), and Ayres and Paluszynski (2022). See Corsetti and Maeng (2023) for a reappraisal.

[10]Recent papers that study rollover crises include Conesa and Kehoe (2014), Conesa and Kehoe (2017), Corsetti et al. (2017), Bianchi and Mondragon (2018), Bocola and Dovis (2019), Corsetti et al. (2020) and Galli (2021).

Gopinath (2006) and Arellano (2008) (in turn related to Eaton and Gersovitz (1981)). Alfaro and Kanczuk (2009) envision reserves as a means to transfer resources from repayment states to default states in future periods. With one-period bonds, they find that the optimal reserve holding is zero. Bianchi et al. (2018) investigate the function of reserves in hedging against rollover risk, an insurance role that arises only with debt maturity longer than one-period. Both of these studies emphasize the inter-temporal dimension of reserve management. Our work, instead, provides a novel perspective by focusing on short-run liquidity risks, bringing forwards the role of reserves in addressing disruptive belief-driven crises.

In this sense, our work is conceptually related to other strands of literature stressing the role of reserves as liquidity buffers against bank-run like sudden stops in market financing—as in Aizenman and Lee (2007) and Hur and Kondo (2016). Durdu et al. (2009) conduct a quantitative assessment in which reserve accumulation reduces the frequency and the severity of binding credit constraints.[11] Korinek and Servén (2016), Fanelli and Straub (2021) and Benigno et al. (2022) investigate reserves as a tool for stabilizing real exchange rates. Bianchi and Sosa-Padilla (2023) study how sovereign risk can amplify aggregate demand fluctuations, showing that reserve accumulation can serve as a stabilization tool.[12] Devereux and Wu (2022) focus on the role of reserves as a means to dampen the destabilizing effects of global shocks on the domestic economy, thereby reducing currency risk premia in domestic debt.[13] Bianchi and Sosa-Padilla (2022) explore the impact of constraints on reserve access and its relation to sovereign default. Bocola and Lorenzoni (2020) and Davis et al. (2023) study the role of reserves in eliminating self-fulfilling financial crises.[14]

This paper is organized as follows. Section 2 presents the model, analyzes the policy problem and characterizes equilibrium multiplicity. Section 3 studies how reserves policy impacts equilibrium pricing and allocation, and derive optimal reserve accumulation for a given debt pact. Section 4 discusses the optimal joint reserve and debt management policies of the government using intuitive graphical apparatus and a numerical example. Section 5 provides quantitative illustrations for both short- and long-term debt. Section 6 concludes.

[11] Caballero and Panageas (2008) and Jeanne and Rancière (2011) quantify the optimal reserve holdings and welfare benefits of reserve accumulation to hedge against sudden stop, where reserves are modelled as a state-contingent asset. Brunnermeier and Huang (2018) show that reserves reduce the severity of sudden stops but do not make them less likely.

[12] See also the Handbook chapter by Lorenzoni (2014) and Bianchi and Lorenzoni (2022).

[13] Amador et al. (2020) study the optimal exchange rate interventions at the zero-lower bound via reserve accumulation.

[14] Other related work on reserve accumulation includes Bussière et al. (2015), Qian (2017), Jung and Pyun (2016), Matsumoto (2022), Lutz and Zessner-Spitzenberg (2023), Jeanne and Sandri (2023), and Sosa-Padilla and Sturzenegger (2023). In the firm dynamic literature, our work is related to Xiao (2022), who studies how intra-period shocks motivate asset accumulation by firms.

2 A Two-period Model

In this and the next two sections, we rely on a two-period economy to facilitate a transparent and analytical discussion of the driving forces behind reserve holdings and crises. Tractability allows us to provide analytical insights on the role of reserves in shielding the economy from disruptive intra-period risk. In Section 5 we will offer a quantitative assessment.

We posit that the economy's saving and consumption decisions are all made by a sovereign. The government receives an endowment over two periods, indexed by $t \in \{0, 1\}$, issues non-state-contingent defaultable debt and accumulates reserves that pay the risk-free interest rate.

Endowment.—At $t = 0$, the economy starts with an endowment y_0 and initial debt b_0. In period 1, the economy receives a stochastic endowment $y_1 \in \mathbb{Y} \equiv [\underline{Y}, \bar{Y}]$, with $0 < \underline{Y} < \bar{Y} < \infty$. If default took place (either in period 0 or in period 1), output drops by a factor Z in period 1.[15]

Preferences.—The sovereign's preferences over aggregate consumption c_0 and c_1 are given by

$$u(c_0) + \beta \mathbb{E}_0[u(c_1)]$$

where $\beta \in (0, 1)$ and the differentiable utility function $u : \mathbb{R}^+ \to \mathbb{R}$ is strictly increasing and strictly concave. Each period, the government decides whether to default or to repay outstanding stock of debt. We describe the model environment backwards in what follows.

Period 1.—If no default took place at $t = 0$, the government decides whether to default or not depending on the realized endowment y_1 and maturing debt b_1. The consumption when the repayment takes place at $t = 1$ is given by

$$c_1^R = y_1 - b_1 + a_1$$

where a_1 is the stock of reserves the government accumulated at $t = 0$.

If the government either has already defaulted in period 0, or decides to default at $t = 1$, the endowment is scaled by the penalty of defaulting ($Z < 1$). Hence, the aggregate consumption in a state of default c_1^D can be expressed as follows:

$$c_1^D = Zy_1 + a_1 \tag{1}$$

Upon default, the government retains control of its reserves.

[15] We assume that default at $t = 0$ does not lead to output loss at $t = 0$ for the sake of tractability.

A government with no history of defaulting repays debt if c_1^R is larger than c_1^D, i.e., when the benefit of repayment is larger than the cost of repayment, given by the following condition:

$$\underbrace{(1-Z)y_1}_{\text{benefit of repayment}} \geq \underbrace{b_1}_{\text{cost of repayment}} \tag{2}$$

It is worth stressing that, in our two-period model, for a given b_1, accumulated reserves a_1 have no effect on the cost and benefit of repayment at $t = 1$. That is to say, in equilibrium, the government's default decision at $t = 1$ only depends on the level of ounstanding debt b_1, not on the stock of accumulated reserves a_1. This feature of our two-period model simplifies the pricing of inter-temporal default risk, as this will depend only on the stock of b_1.

Period 0.—A government that decides to repay outstanding debt b_0 faces the following budget constraint:

$$c_0^R + q_a a_1 + b_0 = y_0 + q b_1$$

The government issues non-state-contingent defaultable debt b_1 and accumulates reserves a_1 that pays the risk-free interest rate in period 1.

The government makes its default decision after the bond auction. When the government defaults, under standard simplifying assumptions following Cole and Kehoe (2000) and Aguiar et al. (2022), the government fully defaults on its current (b_0) and future (b_1) obligations. Hence, the aggregate consumption in period 0 in a state of default consists of the endowment y_0 and the revenue from the period's bond auction qb_1, less the value of accumulated reserves $q_a a_1$:

$$c_0^D = y_0 + q b_1 - q_a a_1 \tag{3}$$

As shown below, in the model, the value of defaulting will depend on an intra-period shock ϵ.

Timing.—The sequence of fiscal and lenders' decision is summarized in Figure 2. The first period is subdivided in 3 sub-periods, $t = 0^1$; $t = 0^2$; $t = 0^3$. At $t = 0^1$, the economy starts out with an endowment y_0, maturing debt b_0; the exogenous shock determining the beliefs regime ρ_0 is also realized. The aggregate state s_0 is summarized by (y_0, b_0, ρ_0). The government decides how much new debt b_1 to issue and the amount of reserves a_1 to accumulate, given an equilibrium bond price schedule $q(s_0, b_1, a_1)$.

At $t = 0^2$, the bond auction takes place and lenders buy newly issued sovereign debt at price q. After the auction, the government decides whether to default ($z_0 = 0$) or to repay ($z_0 = 1$) at $t = 0^3$. Note that, the government makes the default decision after the auction á la Cole and Kehoe (2000). However, different from Cole and Kehoe (2000), we follow Aguiar et al. (2022) assuming that an intra-period shock ϵ affects the government's

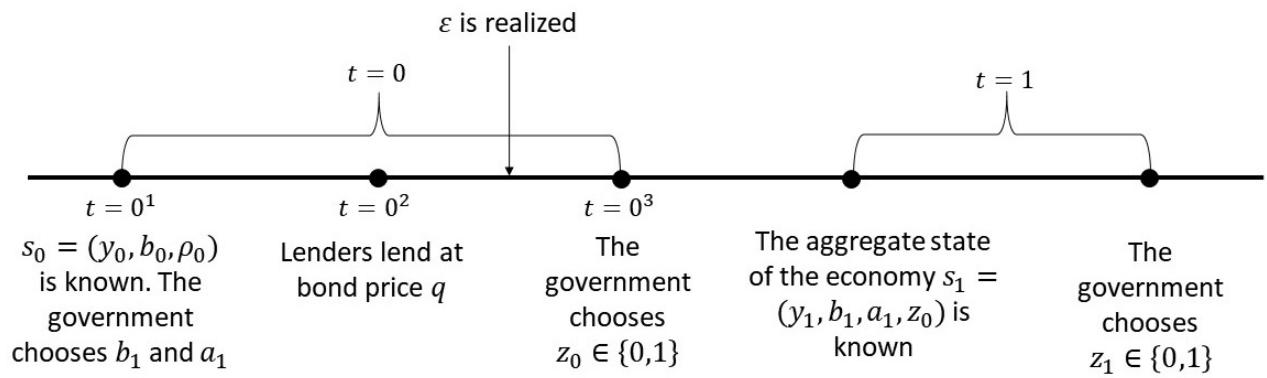

Figure 2: Timing

willingness to repay. This shock is realized after the auction but before the default decision—if the government defaults, it receives the utility $V^D(s_0, b_1, a_1) + \sigma\epsilon$, where $V^D(s_0, b_1, a_1)$ is given by

$$V^D(s_0, b_1, a_1) = u(\underbrace{y_0 + q(s_0, b_1, a_1)b_1 - q_a a_1}_{c_0^D \text{ from } (3)}) + \beta\mathbb{E}\big[u(\underbrace{Zy_1 + a_1}_{c_1^D \text{ from } (1)})\big]$$

The shock ϵ captures a variety of factors that generate uncertainty on the sovereign's willingness to repay within the period.[16] The parameter σ describes the size of intra-period risk. We posit that ϵ is distributed i.i.d over the interval $[0, 1]$, and the function $F(\epsilon)$ represents the cumulative distribution function (CDF) of intra-period shocks.

Our contribution consists of modelling reserve policies as a safeguard against the destabilizing effects of intra-period shocks. Recognizing the possibility of shocks occurring after the auction, the government may issue a substantial amount of defaultable debt and build up liquidity buffers to provide insurance.

In period $t = 1$, depending on the amount of maturing debt b_1 and the realization of output y_1, the government decides whether to repay or default. If a default has already occurred in $t = 0$, the economy consumes a penalized endowment Zy_1 and the accumulated reserves a_1.

Lenders—a continuum of measure one of lenders with "deep pockets" price defaultable sovereign debt in a risk-neutral manner, such that the expected return on sovereign debt is equal to the risk-free rate R^*. Additionally, lenders offer to sell reserves to a sovereign at the risk-free price, i.e., $q_a = (R^*)^{-1}$.

[16] For instance, the sovereign may exhibit reduced tolerance for cuts in social spending initially planned for the period.

2.1 The Government's Problem

In this subsection, we solve the government's problem in the two-period economy. The government's problem in the last period, characterized by (2), is straightforward: given b_1, default takes place at $t = 1$ when $y_1 < \frac{b_1}{1-Z}$. Working backward through period 0, if the government repays at $t = 0^3$, it obtains the value[17]

$$V^R(s_0, b_1, a_1) = u(y_0 - b_0 + q(s_0, b_1, a_1)b_1 - q_a a_1) + \beta\left(\int_{\underline{Y}}^{\frac{b_1}{1-Z}} u(Zy_1 + a_1)dy_1 + \int_{\frac{b_1}{1-Z}}^{\bar{Y}} u(y_1 - b_1 + a_1)dy_1\right)$$

If the government defaults, it receives $V^D + \sigma\epsilon$, where V^D is characterized by:

$$V^D(s_0, b_1, a_1) = u(y_0 + q(s_0, b_1, a_1)b_1 - q_a a_1) + \beta\left(\int_{\underline{Y}}^{\bar{Y}} u(Zy_1 + a_1)dy_1\right)$$

Therefore, the government repays at $t = 0^3$ if the following condition is satisfied:

$$\epsilon \leq \sigma^{-1} \underbrace{[V^R(s_0, b_1, a_1) - V^D(s_0, b_1, a_1)]}_{\equiv \Delta(s_0, b_1, a_1)}$$

where $\Delta(s_0, b_1, a_1)$ defines the net benefit of repayment when $\epsilon = 0$. Conditional on (s_0, b_1, a_1), the equilibrium probability of repayment at $t = 0^3$ is $F(\sigma^{-1}\Delta(s_0, b_1, a_1))$. The government's problem at $t = 0^1$ is then to choose b_1 and a_1 to maximize the expected utility at $t = 0^3$:

$$V(s_0) = \max_{b_1 \geq 0,\ a_1 \geq 0} \left\{ F(\sigma^{-1}\Delta(s_0, b_1, a_1))V^R(s_0, b_1, a_1) + \int_{\sigma^{-1}\Delta(s_0, b_1, a_1)}^{1} [V^D(s_0, b_1, a_1) + \sigma\epsilon]d\epsilon \right\} \tag{4}$$

2.2 The Lenders' Problem

Given that lenders are risk-neutral, the bond price must satisfy the break-even condition, equating the expected return on sovereign debt to the risk-free rate:

$$q(s_0, b_1, a_1) = (R^*)^{-1} \times F(\sigma^{-1}\Delta(s_0, b_1, a_1)) \times Prob\left(y_1 \geq \frac{b_1}{1-Z}\right) \tag{5}$$
$$\text{for } a_1, b_1 \in [0, \infty)$$

The bond pricing formula incorporates two distinct types of default risk inherent in the model. The term $F(\sigma^{-1}\Delta(s_0, b_1, a_1))$ captures the intra-period default risk at $t = 0^3$ after

[17] In the subsequent analysis, we assume, without loss of generality, that b_1 exceeds $(1 - Z)\underline{Y}$.

auction, while the term $Prob\Big(y_1 \geq \frac{b_1}{1-Z}\Big)$ reflects the probability of repayment in period 1, an inter-temporal default risk in line with (2). As already mentioned, a key advantage of our two-period model specification is that the probability of defaulting in period 1 is only affected by next period's maturing debt b_1—the amount of reserves outstanding in period 1 only matters indirectly through b_1.

2.3 Equilibrium

An equilibrium is a government value function $V(s_0)$ and policy functions $b_1(s_0)$ and $a_1(s_0)$, and an equilibrium price schedule $q(s_0, b_1, a_1)$ such that

1. $V(s_0)$, $b_1(s_0)$ and $a_1(s_0)$ solve the government's problem (4) given $q(s_0, b_1, a_1)$.
2. $q(s_0, b_1, a_1)$ satisfies the break-even condition in (5).

2.4 Equilibrium Bond Prices

In this subsection we show that, given the initial state (y_0, b_0) along with b_1 and a_1, there may exist multiple self-fulfilling equilibrium prices. Specifically, as in Aguiar et al. (2022), there are at most three types of bond prices that satisfy the bond pricing condition (5), labelled, respectively, *Eaton-Gersovitz price* ($F(\sigma^{-1}\Delta) = 1$), *Cole-Kehoe failed auction price* ($F(\sigma^{-1}\Delta) = 0$), and *interior prices* ($0 \leq F(\sigma^{-1}\Delta) \leq 1$).

Eaton-Gersovitz price. Assuming that the government will fully repay maturing debt at $t = 0^3$, the bond price depends only on the probability of defaulting next period. This price is the highest possible equilibrium price conditional on b_1 and a_1—denoted with the subscript EG. Provided that the repayment takes place at $t = 0^3$, we have:

$$q_{EG}(b_1, a_1) = (R^*)^{-1} \times Prob\Big(y_1 \geq \frac{b_1}{1-Z}\Big) \tag{6}$$

The utility of repaying, given the Eaton-Gersovitz price q_{EG}, is

$$V^R_{EG}(y_0, b_0, b_1, a_1) \equiv u(y_0 - b_0 + q_{EG}(b_1, a_1)b_1 - q_a a_1) + \\ \beta\Big(\int_{\underline{Y}}^{\frac{b_1}{1-Z}} u(Zy_1 + a_1)dy_1 + \int_{\frac{b_1}{1-Z}}^{\bar{Y}} u(y_1 - b_1 + a_1)dy_1\Big)$$

We define the associated utility of defaulting at the Eaton-Gersovitz price by $V^D_{EG}(Y, b_1, a_1) +$

$\sigma\epsilon$, where

$$V^D_{EG}(y_0, b_1, a_1) \equiv u(y_0 + q_{EG}(b_1, a_1)b_1 - q_a a_1) + \beta\left(\int_{\underline{Y}}^{\bar{Y}} u(Zy_1 + a_1)dy_1\right)$$

Note that, q_{EG} is valid if and only if

$$\tilde{\Delta}_{EG} \equiv V^R_{EG}(y_0, b_0, b_1, a_1) - V^D_{EG}(y_0, b_1, a_1) \geq \sigma \tag{7}$$

When the above condition is satisfied, by selling bonds at the highest rational price at auction, the government's decision on repaying maturing liabilities is not affected by intra-period risk.

Cole-Kehoe failed-auction price. Another possible equilibrium features the canonical rollover crises in Cole and Kehoe (2000), where b_1 is auctioned at zero price and the government defaults at the end of the period. We use subscript CK to denote Cole-Kehoe failed auction price:

$$q_{CK}(b_1, a_1) = 0 \tag{8}$$

Define the utility of repaying debt as

$$V^R_{CK}(y_0, b_0, b_1, a_1) \equiv u(y_0 - b_0 - q_a a_1) + \beta\left(\int_{\underline{Y}}^{\frac{b_1}{1-Z}} u(Zy_1 + a_1)dy_1 + \int_{\frac{b_1}{1-Z}}^{\bar{Y}} u(y_1 - b_1 + a_1)dy_1\right)$$

The corresponding utility of defaulting is $V^D_{CK}(y_0, b_1, a_1) + \sigma\epsilon$, where

$$V^D_{CK}(y_0, b_1, a_1) \equiv u(y_0 - q_a a_1) + \beta\left(\int_{\underline{Y}}^{\bar{Y}} u(Zy_1 + a_1)dy_1\right)$$

q_{CK} is a valid price if and only if the following condition is satisfied:

$$\tilde{\Delta}_{CK} \equiv V^R_{CK}(y_0, b_0, b_1, a_1) - V^D_{CK}(y_0, b_1, a_1) < 0 \tag{9}$$

Lenders' reluctance to roll over sovereign debt results in an excessively large and harsh cost of the surplus adjustment required to honour outstanding liabilities, relative to the cost of default. Opting for default after the auction becomes the preferred option, irrespective of the realization of intra-period shock ϵ, justifying the zero failed-auction price.

Interior prices. The third possible equilibrium bond price lies between q_{EG} and q_{CK}—referred

to an interior price in Aguiar et al. (2022). We label this price with subscript $ACCS$:

$$q_{ACCS}(b_1, a_1) = F(\tilde{\epsilon}) q_{EG}(b_1, a_1) \tag{10}$$

where $\tilde{\epsilon} \in [0, 1]$ is a value of intra-period shock, at which the government is indifferent between repayment and default.

In this scenario, if $\epsilon \leq \tilde{\epsilon}$, the government chooses repayment, while it opts for default if $\epsilon > \tilde{\epsilon}$. The corresponding bond price is therefore $F(\tilde{\epsilon}) q_{EG}(b_1, a_1)$, reflecting a non-zero default risk driven by intra-period shocks.

The utility of repaying when the bond price is q_{ACCS} is

$$V^R_{ACCS}(y_0, b_0, b_1, a_1) \equiv u(y_0 - b_0 + q_{ACCS}(b_1, a_1) b_1 - q_a a_1) + \beta \left(\int_{\underline{Y}}^{\frac{b_1}{1-Z}} u(Z y_1 + a_1) dy_1 + \int_{\frac{b_1}{1-Z}}^{\bar{Y}} u(y_1 - b_1 + a_1) dy_1 \right)$$

The utility of defaulting is $V^D_{ACCS} + \sigma\epsilon$, where

$$V^D_{ACCS}(y_0, b_1, a_1) \equiv u(y_0 + q_{ACCS}(b_1, a_1) b_1 - q_a a_1) + \beta \left(\int_{\underline{Y}}^{\bar{Y}} u(Z y_1 + a_1) dy_1 \right)$$

$\tilde{\epsilon}$ is the solution to the following equation:

$$\tilde{\Delta}_{ACCS} \equiv V^R_{ACCS}(y_0, b_0, b_1, a_1) - V^D_{ACCS}(y_0, b_1, a_1) = \sigma\tilde{\epsilon} \tag{11}$$

The following proposition describes the sufficient and necessary condition of the existence of interior prices q_{ACCS}, assuming $\sigma \to 0$.

Proposition 1. *For a given (y_0, b_0, b_1, a_1) and $\sigma \to 0$, (11) is satisfied iff (7) and (9) are simultaneously satisfied, i.e., there exists an interior equilibrium price iff q_{CK} and q_{EG} are both supportable.*

Proof. See Appendix A.1. □

Note that, Proposition 1 does not rule out the possibility of having more than one interior price. The subsequent proposition outlines that an arbitrarily small intra-period risk (i.e. $\sigma \to 0$) excludes the existence of multiple interior prices.

Proposition 2. *For a given (y_0, b_0, b_1, a_1), if $\sigma \to 0$, there exists at most one interior price. Namely, the solution $\tilde{\epsilon} \in [0, 1]$ to (11) is unique, if it exists.*

Proof. See Appendix A.2 □

The preceding propositions characterize the static multiplicity inherent in the model, followed by the corollary below.

Corollary 1. *The equilibrium price configuration when $\sigma \to 0$ is:*

1. *q_{EG} is the only equilibrium price*
2. *q_{CK} is the only equilibrium price*
3. *$\{q_{EG}, q_{CK}, q_{ACCS}\}$ can all be equilibrium prices—(7), (9) and (11) are all supported, and the corresponding $\tilde{\epsilon} \in [0, 1]$ is the unique solution to (11)*

3 Reserves and Equilibrium Allocation

In this section, we provide insight on how reserve accumulation a_1 impacts bond pricing and the equilibrium allocation. First, we carry out our analysis conditional on a given debt path, for each possible equilibrium bond price. Second, we elaborate on lenders' beliefs, which in equilibrium select the price schedule, and derive the optimal reserve policy given new issuance b_1 and the beliefs state ρ_0. Throughout the section, we focus on the case of arbitrarily small intra-period shocks, i.e., $\sigma \to 0$.

3.1 Bond Pricing and Welfare

In this subsection, we analyze bond pricing and welfare as a function of reserve accumulation. Our main results, stated in the two propositions below, are illustrated by the three panels in Figure 3. This figure is drawn for a given triplet (y_0, b_0, b_1), varying a_1. In particular, we select a value for (y_0, b_0, b_1) to ensure that both $q_{EG}(b_1, a_1)$ and $q_{ACCS}(b_1, a_1)$ can be equilibrium prices when $a_1 = 0$. We consider equilibria in which bonds are traded at each of these prices in turn.

Focus first on the solid blue line in the first panel of the figure, referred the Eaton-Gersovits prices q_{EG}. In the region $[0, \bar{a}]$, the bond price q_{EG} remains constant. This is because in our two-period model, for a given stock of debt b_1, inter-temporal default risk is unaffected by the choice of a_1 (see (2) and (6))—reserve accumulation influences within-period default risk only. Though reserve accumulation makes default a more attractive option in the current period, the condition (7) is still satisfied for all $a_1 \in [0, \bar{a}]$, thereby implying a constant bond price. However, note that when $a_1 > \bar{a}$, the government would default at $t = 0^3$ even if lenders lent at q_{EG}—q_{EG} is no longer supportable. We expand on the reason below.

Welfare conditional on the Eaton-Gersovitz price q_{EG} is illustrated in the second panel of Figure 3. In this panel, the two thin black lines, labelled V^D_{EG} and $V^D_{EG} + \sigma$, mark the

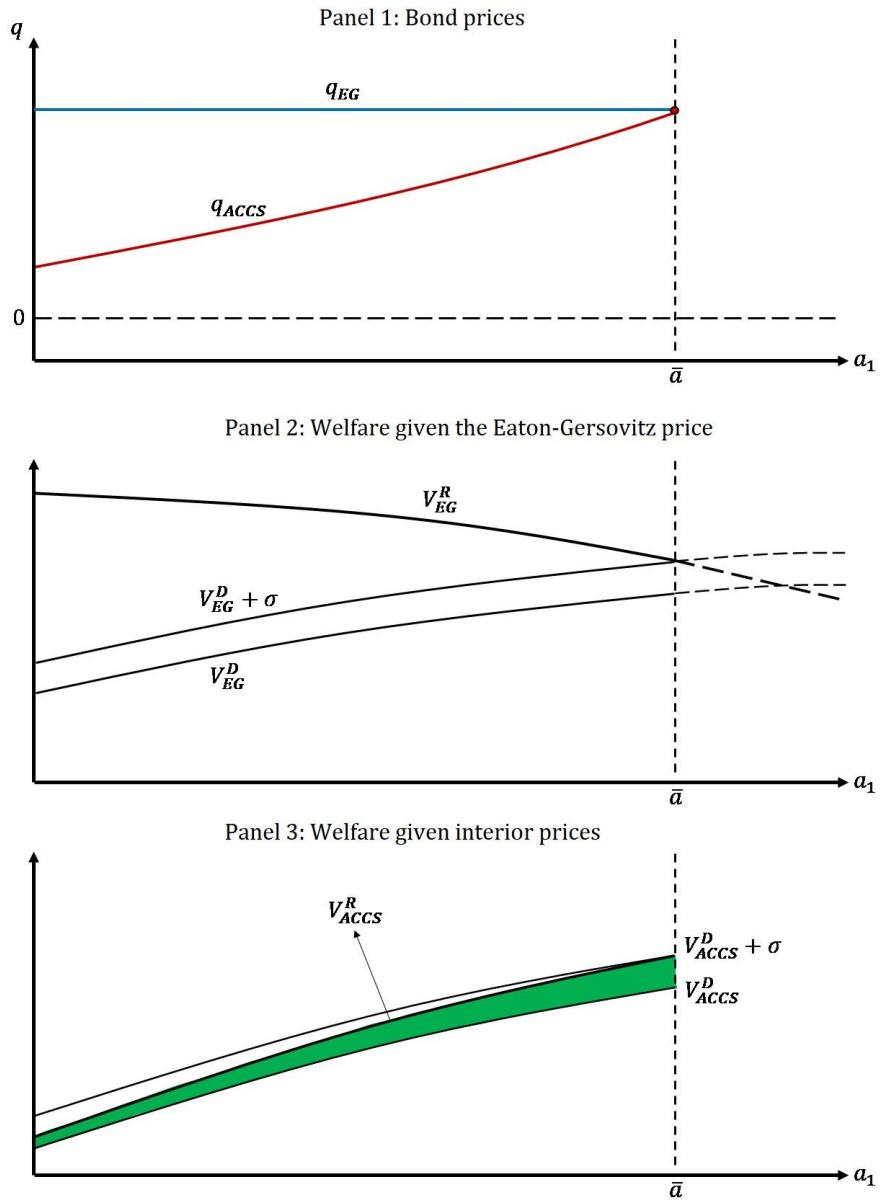

Figure 3: How reserve accumulation a_1 affect bond prices and welfare, given (y_0, b_0, b_1)

lower and the upper bound of default values at each level of reserves. Both V^D_{EG} and $V^D_{EG} + \sigma$ are increasing in a_1, since higher stocks of a_1 imply that the government has more resources to use in a state of default, which makes default more attractive. Conversely, the utility of repaying, labelled V^R_{EG}, is decreasing in a_1—since, given b_1, raising the stock of reserves requires a drop in current consumption.[18] As a_1 grows above $\bar{a}$, $V^D_{EG} + \sigma$ exceeds V^R_{EG} (i.e. (7) is not satisfied)—implying that q_{EG} is only supportable in the region $[0, \bar{a}]$. In accordance with Corollary 1, the only self-fulfilling price for $a_1 > \bar{a}$ is therefore a failed-auction (zero)

[18] For ease of exposition, we pick a value of b_1 that ensures this is the case. Indeed, for a sufficiently large b_1—larger than the optimal level—, V^R_{EG} would be hump-shaped

price. Intuitively, excessive reserve accumulation makes the temptation to default too strong. Lenders develop adverse expectations resulting in a rollover crisis à la Cole and Kehoe (2000).

Turning to equilibrium interior prices, in the next two propositions we establish that q_{ACCS} increases monotonically and approaches q_{EG} as $a_1 \uparrow \bar{a}$, as shown in the first panel of Figure 3.

Proposition 3. *For a given (y_0, b_0, b_1), if $\{q_{EG}, q_{CK}, q_{ACCS}\}$ can all be equilibrium prices for $a_1 \in \{a^1, a^2\}$ (with $a^1 < a^2$), q_{ACCS} is larger with larger reserve holdings a^2 and the corresponding probability of defaulting at $t = 0^3$ is lower.*

Proof. See Appendix A.3. □

Proposition 3 is silent on whether reserve accumulation can entirely eliminate intra-period default risk from bond pricing, as well as on whether an excessive accumulation of reserves may be detrimental. The next proposition establishes that, given b_0 and b_1, there is one level of reserves that completely eliminates within-period default risk. Reserves holdings above such level deteriorate lenders' expectations of post-auction repayment so much that the equilibrium price becomes the failed-auction price.[19]

Proposition 4. *For each (y_0, b_0, b_1), if q_{ACCS} is supportable at $a_1 = \tilde{a}$, there exists a unique level of reserves $\bar{a} \geq \tilde{a}$, that fully eliminates intra-period default risk in q_{ACCS}. Holding reserves above $\bar{a}$ gives rise to expectations of opportunistic default at $t = 0^3$, and therefore the only supportable price for $a_1 > \bar{a}$ is a failed auction price q_{CK}.*

Proof. See Appendix A.4. □

These results are illustrated in the bottom panel in Figure 3, which plots the utility associated with repayment at interior prices $V^R_{ACCS}(y_0, b_0, b_1, a_1)$—the thick solid line—, together with $V^D_{ACCS}(y_0, b_0, b_1, a_1)$ and $V^D_{ACCS}(y_0, b_0, b_1, a_1) + \sigma$—the two thin solid lines, that define the range of utility of defaulting.[20] Note a striking difference in comparison to the middle panel: $V^R_{ACCS}(y_0, b_0, b_1, a_1)$ is now increasing in a_1, and lies within the range of utility of defaulting, reflecting positive intra-period default risk after the auction.

The reason why V^R_{ACCS} is upward sloping in reserve holdings—despite the costs of reserve accumulation in terms of current consumption—lies in a general equilibrium effect. Conditional on the q_{ACCS} price schedule, for an equilibrium with interior prices to survive as

[19] Analogously, in Aguiar et al. (2022) there exists a level of debt issuance that fully eliminates intra-period uncertainty (at the cost of higher borrowing costs due to inter-temporal default risk). Newly issued debt above this level results in post-auction opportunistic default—the corresponding equilibrium bond price is therefore q_{CK}. See Section 4.3 for details.

[20] In the bottom panel of Figure 3, we plot the welfare given interior prices only in the range of $[0, \bar{a}]$, since the solution ($\tilde{\epsilon} \in [0, 1]$) to (11) exists only in such region—no interior prices exist for $a_1 > \bar{a}$.

reserve holdings rise, the government needs to become progressive more resilient to intra-period shocks—prompting lenders to offer higher interior prices.[21] Proposition 3 establishes that, as $a_1 \uparrow$, this general equilibrium effect on bond prices dominates—V^R_{ACCS} rises faster relative to V^D_{ACCS}, implying a rise in the likelihood of post-auction repayment (i.e. a larger $\tilde{\epsilon}$ in (11)). In the graph, we emphasize this point by highlighting in green color the set of intra-period shock values at which the government opts for repayment after the auction: this green region widens with reserve accumulation in the region $[0, \bar{a}]$.

The figure also shows that intra-period default risk is fully eliminated when $a_1 = \bar{a}$, at which point $V^R_{ACCS}(y_0, b_0, b_1, \bar{a})$ is equal to the upper bound of the default value $V^D_{ACCS}(y_0, b_1, \bar{a}) + \sigma$. This maps into the equilibrium bond price in the upper panel of Figure 3—as a_1 rises to $\bar{a}$, q_{ACCS} increases, eventually reaching its maximum value q_{EG} precisely at $a_1 = \bar{a}$.

3.2 Beliefs and Reserves Policy

With more than one equilibrium bond price, the role of beliefs state ρ_0 is to pick one of them. The belief state ρ_0 thus determines the equilibrium bond price schedule that the government takes as given at auction. In what follows, we characterize the government's optimal reserve policy functions, conditional on the amount of newly issued debt b_1 and an equilibrium price schedule picked by the belief state ρ_0.

In our analysis, we let lenders hold either "optimistic" or "concerned" beliefs—writing $\rho_0 = O$ and $\rho_0 = C$—, defined as follows:

- In a regime of optimistic beliefs, lenders approach the bond markets disregarding intra-period risk. For a given state (y_0, b_0, b_1, a_1), if (6) and (7) are simultaneously satisfied, lenders coordinate on $q_{EG}(b_1, a_1)$. If $q_{EG}(b_1, a_1)$ is not supportable, lenders coordinate on the failed-auction price $q_{CK}(b_1, a_1) = 0$.[22]

- In a regime of concerned beliefs, lenders approach the bond market "concerned" with default risk originating from intra-period uncertainty about repayment after the debt auction is closed. Hence, they coordinate on an interior price, if it exists—(10) and (11) must be simultaneously satisfied. Otherwise, they coordinate on a price that coincides with that under optimistic beliefs regimes.

We start with a Lemma stating that, for a given (y_0, b_0, b_1), if an interior price is supportable and $\rho_0 = C$, increasing reserve holdings leads to higher aggregate consumption and

[21] If lenders did not offer higher interior prices for larger reserve holdings, the government would default with probability one (regardless of the realization of intra-period shocks) after the auction. With a_1 rising in the region $[0, \bar{a}]$, the interior price has to increase for the equilibrium with the post-auction default driven by intra-period shocks to exist. This is the general equilibrium effect we refer to in the text.

[22] Note that, according to Corollary 1, there is no interior price if q_{EG} is not supportable given that $\sigma \to 0$, and therefore lenders consistently offer zero price to the government if (7) is not satisfied.

lower intra-period default risk.

Lemma 1. *For a given (y_0, b_0, b_1), consider two possible reserve holdings a^1 and a^2, with $a^1 < a^2$. If interior prices are supportable when $a_1 \in \{a^1, a^2\}$, the following conditions hold*

$$q_{ACCS}(b_1, a^1) < q_{ACCS}(b_1, a^2) \tag{12}$$

$$q_{ACCS}(b_1, a^1)b_1 - q_a a^1 < q_{ACCS}(b_1, a^2)b_1 - q_a a^2 \tag{13}$$

Proof. See Appendix A.5. □

The first inequality (12) states that the intra-period risk of a^2 is less than that of a^1—corresponding $\tilde{\epsilon}^1$ and $\tilde{\epsilon}^2$ from equation (11) satisfy $F(\tilde{\epsilon}^2) > F(\tilde{\epsilon}^1)$, i.e., the probability of defaulting at $t = 0^3$ is lower when the government holds more reserves. The second inequality (13) states that accumulating reserves increases interior bond prices q_{ACCS} so much (by reducing default risk at $t = 0^3$) that, in equilibrium, aggregate consumption in period 0 is higher, the larger the reserve holdings.

Proposition 5 below establishes that, all else equal, the government strictly prefers less intra-period default risk, as this implies higher consumption at $t = 0$.

Proposition 5. *For a given (y_0, b_0, b_1) with $\rho_0 = C$, consider two possible reserve holdings a^1 and a^2 that support interior prices, with $a^1 < a^2$, implying (12) and (13) in Lemma 1 are satisfied. Then, a^2 strictly dominates a^1 as a policy choice.*

Proof. See Appendix A.6. □

The optimal reserve holding conditional on a given triplet (y_0, b_0, b_1) with $\rho_0 = C$ follows as a corollary of the above. The corollary 2 states that the government always prefers to accumulate a sufficient amount of reserves to fully eliminate intra-period default risk.

Corollary 2. *For a given (y_0, b_0, b_1) with $\rho_0 = C$, suppose that there exists a level of reserve holdings $\bar{a}$ that fully eliminates intra-period default risk, such that $q_{EG}(b_1, \bar{a}) = q_{ACCS}(b_1, \bar{a})$. Any reserve holdings $\tilde{a}$ smaller than $\bar{a}$ satisfy (12) and (13): (i) $q_{ACCS}(b_1, \tilde{a}) < q_{ACCS}(b_1, \bar{a})$ and (ii) $q_{ACCS}(b_1, \tilde{a})b_1 - \tilde{a} < q_{ACCS}(b_1, \bar{a})b_1 - \bar{a}$, and the government finds $\bar{a}$ strictly preferable to $\tilde{a}$.*

When $a_1 = \bar{a}$, the auction takes place at the Eaton-Gersovitz price, and the government raises more revenues than at any level of reserve holdings lower than $\bar{a}$.

We conclude with an analysis of an economy under optimistic beliefs. The following proposition identifies sufficient conditions for any positive stock of reserves to be suboptimal conditional on $\rho_0 = O$.

Proposition 6. *Accumulating any positive amount of reserves is sub-optimal for the government when $\rho_0 = O$, if either condition below is satisfied*

- *The government is sufficiently impatient.*
- *The current endowment y_0 is lower than the lowest possible endowment after default in period 1, i.e., $y_0 < Z\underline{Y}$.*

In either case, the government should refrain from accumulating reserves and, instead, utilize any outstanding stock of reserves to pay down debt and reduce borrowing costs. As a special case, hence, our model nests the findings by Alfaro and Kanczuk (2009), where (with one-period debt) reserves play no (quantitatively relevant) role in equilibrium if $\rho_0 = O$. We return to this result in our two-period numerical illustration in Section 4.3.

Discussion. The key result of this section is that, given (y_0, b_0, b_1), when $\rho_0 = C$ (and interior prices are sustainable), the optimal policy of the government is to accumulate a sufficient amount of reserves $\bar{a}$ to fully eliminate intra-period uncertainty. Increasing reserve holdings entails a direct cost in terms of current-period consumption, but at the same time improves the bond prices monotonically via general equilibrium effect (see Figure 3 and Proposition 3). Hence the government can borrow at lower cost to contain the effect of accumulating reserves on current consumption. Lemma 1 shows that the benefits in terms of higher bond prices dominate over the relevant range, implying that the government optimally raises reserves up to $\bar{a}$, fully eliminating intra-period uncertainty (see Corollary 2). At this point, the revenue from bond issuance is at its peak.

Conditional on holding the optimal levels of reserves $\bar{a}$, the government always faces the Eaton-Gersovitz price under concerned beliefs regime. By contrast, when $\rho_0 = O$, the government also faces the Eaton-Gersovitz price but without any needs to accumulate reserves. That is to say, in equilibrium, irrespective of whether lenders are optimistic or concerned, the government invariably confronts the Eaton-Gersovitz price (schedule). The difference between these two beliefs regimes lies in the equilibrium reserve holdings.

4 Joint Reserve and Debt Policy

We now bring the two-period model to bear on how, at different levels of initial debt, a welfare-maximizing government would jointly set the optimal level of reserve holdings a_1 and debt issuance b_1 conditional on each beliefs regime. We illustrate our main results for economies with a high and a low level of initial debt—first using the graphical apparatus in Figures 4 and 5, then using a numerical example. We show that, by holding reserves, the

government can optimally mitigate intra-period uncertainty without suffering a significant rise in borrowing costs—in spite of the additional debt issued to finance reserve accumulation.

As in the previous section, we restrict our analysis to the case where $\sigma \to 0$. We denote with $a^*(s_0, b_1)$ the optimal level of reserves given the aggregate state $s_0 = (y_0, b_0, \rho_0)$ and the amount of newly issued bonds b_1. The two figures 4 and 5—drawn for different levels of initial debt—plot bond prices against bond issuance b_1, conditional on different regimes of beliefs and levels of reserve accumulation. Blue solid lines correspond to the optimistic regime $\rho_0 = O$, under which there is no need for reserves. Red solid lines correspond to the concerned beliefs regime, $\rho_0 = C$, under which a welfare-maximizing government would hold the optimal level of reserves $a_1 = a^*$. Green solid lines reproduce the case of concerned beliefs banning reserve holdings, i.e., setting $a_1 = 0$. This last case allows us to compare our results with Aguiar et al. (2022).

4.1 Economies with a High Initial Debt

Starting with Figure 4—drawn for a high level of initial debt,—note that the equilibrium bond price is always the "failed-auction" one (zero price) if *new* debt issuance is either lower than $\underline{b}_{EG}$ or higher than $\bar{b}_{EG}$. For the relatively high level of initial debt, the government will never choose to issue new debt outside the range $[\underline{b}_{EG}, \bar{b}_{EG}]$. Outside this range, either new borrowing would fall short to produce enough resources to prevent sub-optimal contraction in consumption ($b_1 < \underline{b}_{EG}$) or the government would borrow up so much up to making the temptation to default and use the revenue in the current period too strong to resist ($b_1 > \bar{b}_{EG}$). As depicted in Figure 4, in either case default at $t = 0$[3] would be unavoidable.[23]

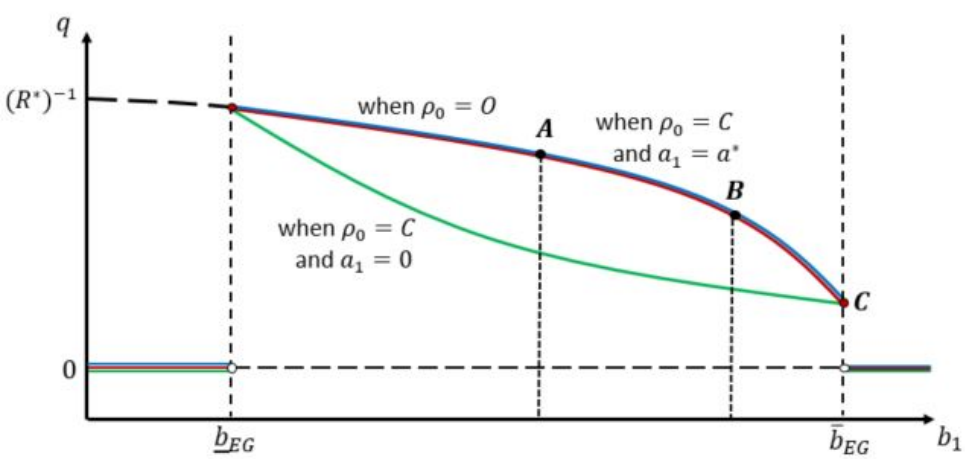

Figure 4: Equilibrium and bond price schedules at high initial debt

[23]Figure 4 represents initial debt levels that lie in the crisis zone studied in Cole and Kehoe (2000)—if lenders coordinate on the Cole-Kehoe failed auction price, the government unambiguously defaults.

The government will instead choose its optimal new debt issuance b_1 within the range $[\underline{b}_{EG}, \bar{b}_{EG}]$. In this range, the Eaton-Gersovitz price q_{EG} is always supportable conditional on the beliefs regime $\rho_0 = O$. Under optimism, the optimal level of reserves is 0—there are no benefits from reserve accumulation in this case. The equilibrium is at point $\boldsymbol{A}$, corresponding to the optimal level of new issuance b_1^A.

Strikingly, over the same debt range, bonds also trade at the Eaton-Gersovitz price when lenders turn concerned, provided the government pursues the optimal reserve policy. Figure 4 illustrates the results outlined in Corollary 2. For each level of debt issuance b_1, given that reserve holdings are optimized to a^*, the intra-period default risk is completely eliminated in equilibrium: the bond price schedule conditional on $\rho_0 = C$ and $a_1 = a^*$ coincides with the one conditional on $\rho_0 = O$. In equilibrium the government issues more debt (relative to $\boldsymbol{A}$) as it needs to finance reserves to fully eliminate intra-period uncertainty. The equilibrium is at point $\boldsymbol{B}$, corresponding to the optimal level of new issuance b_1^B.

This is in sharp contrast to the case in which lenders are concerned ($\rho_0 = C$) but holding reserves is constrained to zero by assumption ($a_1 = 0$)—under this constraint, the predictions of our two-period model align with the findings in Aguiar et al. (2022). With no access to reserves, the government optimally eliminates intra-period uncertainty by "over-borrowing".[24] In equilibrium, the government optimally issues $\bar{b}_{EG}$—the maximum in the range $[\underline{b}_{EG}, \bar{b}_{EG}]$, significantly larger than b_1^A.[25] With debt issuance large enough to rule out intra-period risk, the bond auction also closes at the Eaton-Gersovitz price. But since ruling out intra-period default risk comes at the cost of high debt accumulation, inter-temporal default risk correspondingly rises—driving down the equilibrium Eaton-Gersovitz price to its lowest level—corresponding to the point $\boldsymbol{C}$ in the figure. Comparing the point $\boldsymbol{B}$ and $\boldsymbol{C}$ in the figure highlights the key benefit of reserve accumulation: the government can eliminate intra-period risk without engaging in a significant "over-borrowing" (the point $\boldsymbol{C}$), and hence the equilibrium inter-temporal default risk and the corresponding borrowing costs are much lower.

4.2 Economies with a Low Initial Debt

Comparing economies with a relatively low (Figure 5) and a relatively high (Figure 4) initial stock of debt, two key results stand out. First, the bond price is positive also at very low levels of *new* debt issuance. Actually, as long as new debt issuance does not exceed $\underline{b}_{CK}$, lenders consistently lend at the Eaton-Gersovitz price regardless of beliefs regimes. This is

[24] Banning reserve accumulation, in equilibrium, the government never issues debt at interior prices that reflect intra-period default risk. See Aguiar et al. (2022) for details.

[25] Given that the government cannot accumulate reserves and intra-period shocks are arbitrarily small (i.e. $\sigma \to 0$), the point $\boldsymbol{C}$ strictly dominates other points in the region $[\underline{b}_{EG}, \bar{b}_{EG}]$ in Figure 4. The proof is in Appendix B.

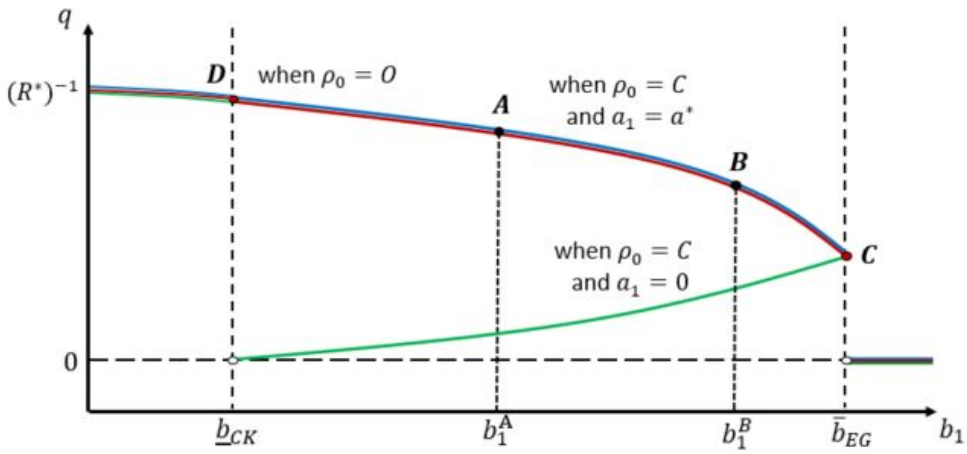

Figure 5: Equilibrium and bond price schedules at low initial debt

due to the fact that, with a low outstanding stock of debt, the government would strictly prefer to repay maturing debt even when facing a rollover crisis (a zero failed auction price).[26] In accordance with Corollary 1, the only self-fulfilling debt price when the amount of debt issuance lies at $[0, \underline{b}_{CK}]$ is therefore the Eaton-Gersovitz price.

Second, with concerned beliefs, it may be possible to have equilibria in which under-issuance (or "deleveraging")—relative to an equilibrium under optimistic beliefs, corresponding to point $\boldsymbol{A}$ in the figure—is an effective strategy to rule out intra-period risk. The reason is as follows. We have seen above that, if the government issues new debt in the region $[0, \underline{b}_{CK}]$, lenders consistently offer the Eaton-Gersovitz price regardless of their beliefs regimes, hence there is no need to accumulate reserves. If, instead, the government issues in the region $(\underline{b}_{CK}, \bar{b}_{EG}]$, the government will need to optimally build up reserves to rule out intra-period default risk and secure the Eaton-Gersovitz price in equilibrium. Because of the need for reserve, then, all points in the region $(\underline{b}_{CK}, \bar{b}_{EG}]$ may be dominated by a candidate equilibrium with moderate debt issuance and no reserve accumulation—see point $\boldsymbol{D}$ corresponding to $\underline{b}_{CK}$ in Figure 5. At this point, the contraction in current consumption due to "austerity" (under-issuance relative to point $\boldsymbol{A}$) may be more than compensated by the large "saving" on reserve accumulation—no longer needed to eliminate intra-period uncertainty. In general, it is possible to construct examples in which the equilibrium may lie at either point $\boldsymbol{B}$ or $\boldsymbol{D}$.

Equilibria with under-issuance are also possible if reserve accumulation is ruled out by assumption—the green lines in the figure. Interior prices are supportable in the range $(\underline{b}_{CK}, \bar{b}_{EG}]$.[27] In equilibrium, the government strictly prefers to fully eliminate intra-period

[26] Figure 5 corresponds to the outstanding stock of debt in the safe zone studied in Cole and Kehoe (2000)—(9) is not satisfied as long as b_1 is smaller than $\underline{b}_{CK}$.

[27] Proposition 1 states that, given (y_0, b_0, b_1, a_1), if both zero and the Eaton-Gersovitz price are supportable,

uncertainty: when the initial stock of debt is low enough, it can do so not only by "over-issuing" $b_1 = \bar{b}_{EG}$, corresponding to the equilibrium point $\boldsymbol{C}$, but also by "under-issuing" $b_1 = \underline{b}_{CK}$, corresponding to the equilibrium point $\boldsymbol{D}$.[28] Globally, the repayment value may be larger at either point—$\boldsymbol{D}$ or $\boldsymbol{C}$. The case in the figure illustrates that, for a low enough initial debt, deleveraging may be the preferred option by a government without access to reserve accumulation.

4.3 A Numerical Illustration

We conclude with an assessment of the model predictions based on a numerical example. In our exercise, we assume an arbitrarily small intra-period shocks (i.e. $\sigma \to 0$) and an impatient government (i.e. a low value of β)—all the other parameter values are detailed in Appendix C. Figure 6 depicts the choice of debt b_1 and reserves a_1 in period 0. Analogous to Figure 4 and 5, the color-coded lines—blue, red, and green—represent, respectively, the policy functions under (i) $\rho_0 = O$, (ii) $\rho_0 = C$ & $a_1 = a^*$, and (iii) $\rho_0 = C$ & $a_1 = 0$.

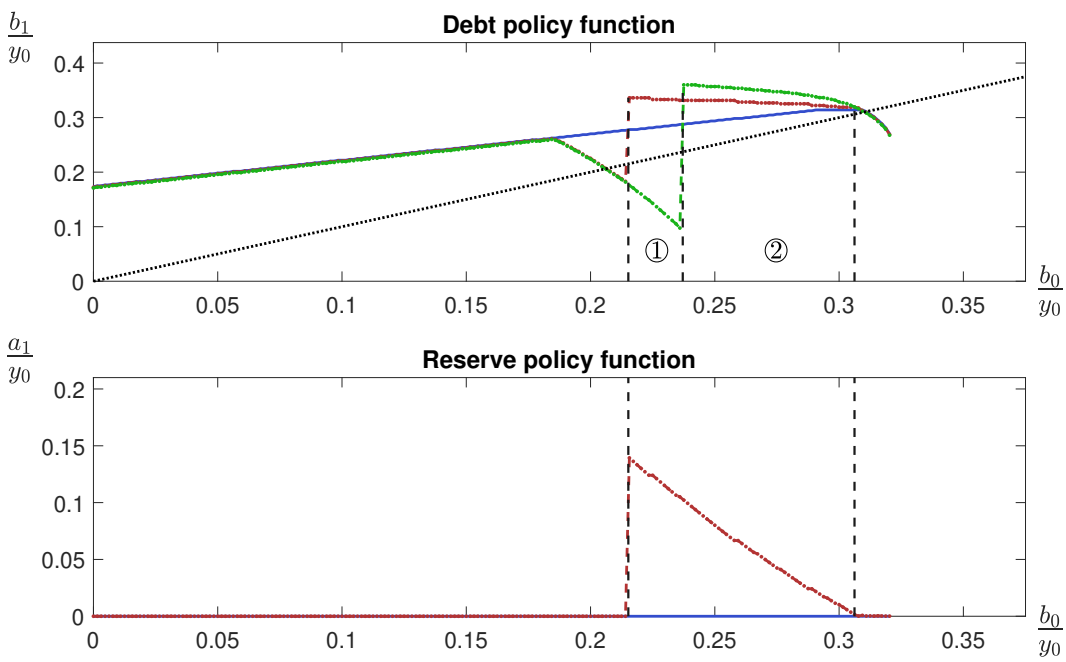

Figure 6: Policy functions

Looking at Figure 6, note that at very low levels of initial debt, the debt policy overlaps

interior prices exist. In the region $(\underline{b}_{CK}, \bar{b}_{EG}]$, both q_{CK} and q_{EG} survive, and therefore interior prices exist. Note that, as $b_1 \downarrow \underline{b}_{CK}$, interior prices approach zero. This is because, at $\underline{b}_{CK}$, the government faces a point of indifference between default and repayment at the Cole-Kehoe failed auction price.

[28] Opting for $\bar{b}_{EG}$ strictly dominates debt issuance in the range $(\underline{b}_{CK}, \bar{b}_{EG})$ and it results in lower intra-period default risk. We provide proofs in Appendix B.

across beliefs. Optimal debt issuance in this case is contained enough that the bond price schedule is the same regardless of the beliefs regime (see Figure 5 for b_1 lower than $\underline{b}_{CK}$). In our numerical example, the corresponding debt region is between 0% and 18% of GDP. The equilibrium is again unique for debt level above 31%. High government "impatience" and a large outstanding stock of debt lead the government to invariably raise new issuance to its largest possible level $\bar{b}_{EG}$—supporting current-period consumption irrespective of beliefs regimes. For debt levels in these two 'tails', reserves are not 'useful'.

Reserves make a difference at intermediate levels of debt. Allowing for the option to accumulate reserves—red lines—, we find that the government also prefers deleveraging for initial debt levels between 18% and 22%. In terms of Figure 5, the candidate equilibrium point $\boldsymbol{D}$ is preferred to the other candidate equilibrium point $\boldsymbol{B}$. Yet reserve accumulation allows the government to discontinue deleveraging already when debt reaches 22% of GDP. In other words, over the region ①, the government switches to the optimal reserve accumulation policy where it would keep pursuing austerity without the option to hold reserves. Note that, in the region ①, the amount of reserve accumulation and debt issuance needed to support the Eaton-Gersovitz price is at its peak. Correspondingly, the probability of defaulting in the subsequent period is at its highest level, and equilibrium bond prices at their lowest level (see bond prices in the region ① of Figure 7).

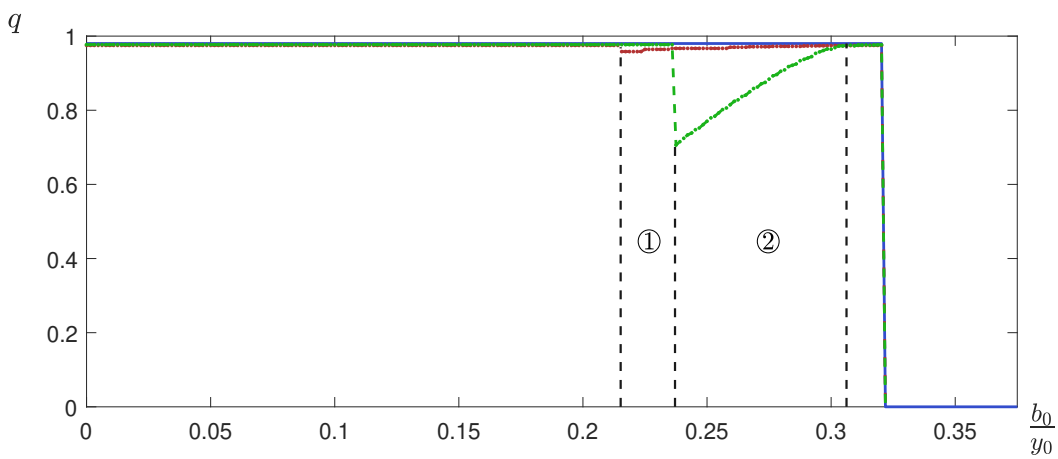

Figure 7: Equilibrium bond prices

Differences in bond issuance policies narrow when the initial debt is in the region marked ②. In this region, if lenders are concerned, the government issues more debt relative to $\rho_0 = O$, regardless of the feasibility of reserve accumulation. Nonetheless, here is where the pivotal role of reserves in mitigating inter-temporal default risk becomes most apparent. Reserves rule out a significant "over-issuance", since they provide an effective insurance against post-auction default risk. As shown in Figure 7, under the optimal reserve policy, the equilibrium interest spread (bond price) is significantly lower (higher). In our numerical example, when

the initial debt-to-output ratio is .27, the equilibrium bond spread without reserve holdings stands at 13.8% (equivalent to 0.863 of the bond price depicted in the figure). By contrast, optimal reserve accumulation reduces the spread down to 0.9% (0.971 of the bond price in the figure).

The bottom panel of Figure 6 shows that, across the regions ① and ②, the equilibrium reserve holdings fall as the stock of initial debt becomes larger. The reason is that, conditional on lenders being "concerned", the lower the stock of initial debt, the higher the spikes in spreads required to generate a credible threat of post-auction default. In other words, under concerned beliefs, the government confronts higher (belief-driven) intra-period default risk at lower initial debt levels. The stock of reserves required to eliminate this risk is correspondingly higher.[29] This feature of the equilibrium, inherent in the (self-fulfilling) nature of intra-period risk, maps into the steady decline in equilibrium bond prices shown in Figure 7, particularly within the region ① and ②. The equilibrium bond prices exhibit a downward trend for lower initial debt levels, as a consequence of the larger debt issuance depicted in the upper panel of Figure 6.

5 Infinite Horizon Model

A key result from the two-period model is that, confronted with concerned beliefs, the government may optimally build up reserves, thereby completely eliminating intra-period uncertainty. In this section we show that this result holds in a quantitative infinite-horizon version of the model, whether debt is short- or long term—the full specification is in Appendix D. In contrast to the two-period case, in an infinite-horizon model intra-period uncertainty motivates a precautionary buildup of reserves under optimistic beliefs. While long maturities help containing vulnerability to belief-driven crises, reserve and debt management remains as a key policy strategy. In our quantitative exercises, equilibrium reserve holdings exhibit only a small degree of fluctuation in response to fundamental shocks to output. The government maintains a positive amount of reserves either as a precaution (under optimistic beliefs), or as a shield (under concerned beliefs) against current-period intra-period uncertainty. As in the two-period numerical example, we set $\sigma \to 0$.

[29] Similar considerations explain why over-issuance is larger at low levels of debt when reserve accumulation is not possible. This is depicted by the green lines in the upper panel of Figure 6, specifically within the region ②.

5.1 Calibration

In line (and for comparability) with other quantitative work on sovereign debt crises, we calibrate our model to Mexico.[30] Mexico serves as a suitable reference due to the characteristics of its business cycle, similar to those of other emerging economies. Following the literature we introduce a non-discretionary minimum public spending $\bar{g}$ in the utility function. A lower bound on government consumption limits the government's ability to adjust fiscal policy to negative shocks.[31] The utility function takes the following form:

$$u(c) = \frac{(c - \bar{g})^{1-\gamma}}{1 - \gamma}$$

The model period is a year. As is standard in quantitative sovereign default studies, we set the relative risk aversion of the sovereign γ to 3, discount factor equal to 0.8, the risk-free interest rate to 1%. Some parameters are chosen drawing on evidence. The level of non-discretionary public spending $\bar{g}$ is set to 0.15, to match the average level of public expenditure to GDP in Mexico. Following Aguiar and Gopinath (2006) and Aguiar and Gopinath (2007), endowment process is governed by two exogenous processes—a trend g_t and transitory component m_t: $\ln Y_t - \ln Y_{t-1} = g_t + m_t$, where $g_t = (1-\rho_g)\mu_g + \rho_g g_{t-1} + \sigma_g \epsilon_{g,t}$, $m_t = \sigma_m(\epsilon_{m,t} - \epsilon_{m,t-1})$, with $\epsilon_g, \epsilon_m \sim N(0,1)$. We use annual Mexican real GDP data from 1990 to 2020 and estimate $\mu_g = 0.013$, $\sigma_g = 0.011$, $\rho_g = 0.41$, and $\sigma_m = 0.019$.

Default leads to a temporary exclusion from financial markets and a proportional loss in output, d. After a default, the sovereign may re-enter international financial market with a constant probability of 0.2. On average, this leads to an exclusion period of 5 years. Following Aguiar et al. (2022), we adjust the cost of defaulting d to match the debt due to GDP ratio. In 1994, the maturity of government debt due prior to the peso crisis was mostly less than a year. Hence, with short-term bonds, we target debt due in 1994, accounting for 35% of annual GDP. In our one-period debt example, we set $d = 0.123$.

The belief state ρ_t follows an i.i.d. process each period. Specifically, the probability of $\rho_t = C$ is set at 3.33%, i.e., on average lenders become concerned every 30 years.

In the long-term debt model, in line with the approach of Chatterjee and Eyigungor (2012), we model bonds that mature probabilistically. In each period, a bond pays a coupon κ and has a probability λ of maturing. The flow of debt payments is therefore $(\kappa + \lambda)B$, where λ is the inverse of maturity. We set the coupon rate κ at 9%. The average maturity of government debt in our sample periods is 4 years (in line with the evidence for Mexico), which implies $\lambda = 0.25$. Analogous to short-term debt calibration, we target the debt payment to

[30] See, for instance, Bianchi et al. (2018) for details.

[31] Bocola and Dovis (2019), Conesa and Kehoe (2017), and Corsetti and Maeng (2023) employ a similar approach to constrain the government's capacity for discretionary deleveraging facing negative shocks.

GDP ratio, which requires $d = 0.176$.

Table 1: Simulated Moments

	One-period bonds			Long-term bonds		
	Optimistic $\rho \in \{O\}$	Baseline $\rho \in \{O, C\}$	No reserves $\rho \in \{O, C\}$	Optimistic $\rho \in \{O\}$	Baseline $\rho \in \{O, C\}$	No reserves $\rho \in \{O, C\}$
Debt Payment/GDP (%)	34	35	33	34	35	33
Default Frequency (%)	0.10	0.14	2.52	0.18	0.25	1.63
Mean of $r - r^*$ (%)	0.10	0.14	2.89	0.19	0.20	1.33
SD$(r - r^*)$ (%)	0.11	0.18	2.15	0.05	0.08	0.59
σ_c/σ_y	1.11	0.93	1.29	1.51	1.49	2.02
Mean of Reserve-to-GDP (%)	0	3.07	N/A	0.01	2.41	N/A

5.2 Short-term Debt

To enhance comparability with the results of the two-period model, we start with an analysis of the infinite-horizon model retaining the assumption that debt is short term. The first three columns of Table 1 reports simulated moments for our calibration with one-period bonds. Mirroring Figure 6 and 7, column 2 refers to our baseline specification with reserves; in column 1 we posit that beliefs are always optimistic; in column 3 we rule out reserve accumulation as a policy option.

As shown in the first column, default is rare when lenders always have optimistic beliefs ($\rho \in \{O\}$).[32] Consistent with the findings of Alfaro and Kanczuk (2009), the optimal policy of the government is to accumulate zero reserves (see also our two-period numerical exercise in Section 4.3).

Conversely, as shown in the third column, the economy experiences a significant increase in the frequency of default, mean spread, and spread volatility (relative to both the first and the second column) when the government is constrained in its ability to hold reserves. Given that reserve holdings are restricted to zero, unless debt is very low, concerned beliefs prompt the government to engage in a significant over-issuance of debt (see Aguiar et al. (2022)). A significant over-issuance drives up both the mean and the standard deviation of spreads, hence the frequency of default. Without reserves as an additional policy instrument to smooth consumption across states and periods, consumption experiences much higher

[32]Note that this result follows from the assumption of linear (non-convex) default costs, that implicitly rule out debt forgiveness in low output state. This is a well-known feature of sovereign default model with linear default costs. See Aguiar et al. (2022) for details.

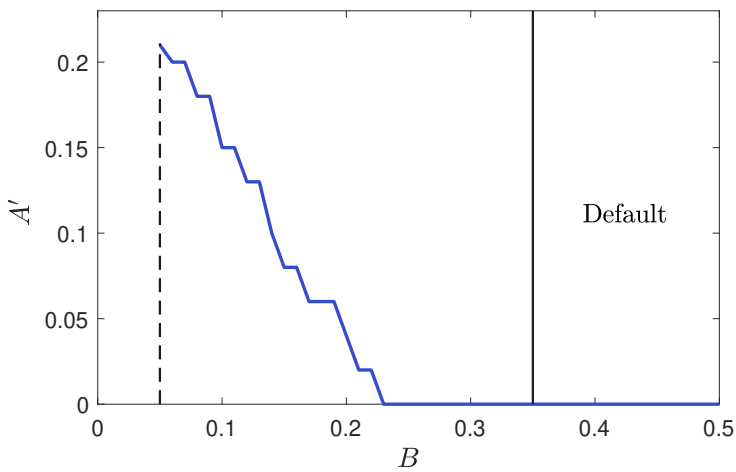

Figure 8: Reserve policy function, when $\rho = O$ in the baseline

volatility—the ratio of the standard deviations of the Hodrick-Prescott-detrended logarithm of consumption to the logarithm of output on the fifth row is the highest among the three specifications (we expand on the economics of consumption volatility below).

The second column—our baseline—illustrates the pivotal role played by optimal reserve holdings in addressing the risk and volatility arising from concerned beliefs. The combined debt and reserve management policy results in a decreased frequency of default, a lower average spread, and reduced spread volatility in comparison to the third column of the table.

Most notably, in spite of a relatively low probability of confronting concerned beliefs (a beliefs crisis once every thirty years), our baseline specification predicts a substantial accumulation of reserves, equivalent to 3.07% of GDP. In a multiperiod setting, an economy starting out with zero reserves would accumulate them not only when lenders turn concerned at intermediate levels of debt (as discussed in the case in the two-period model), but also as a precautionary measure, even under optimistic beliefs.

To illustrate this point, Figure 8 depicts the reserve policy function under optimistic beliefs (blue line) , evaluated at the mean growth rate of endowment and *zero initial holdings of reserves*. When the outstanding debt is low enough, in a world with short-term bonds there is no need to accumulate reserves. At very low levels of debt, it is optimal for the government to keep new issuance contained, so that the equilibrium remains unique both in the current and in the next period, whether or not lenders are concerned today or turn concerned in the future. But as soon as the initial debt rises above .05, dynamically, the economy is no longer immune to beliefs-driven crisis. Over the range [.05, .22], the government accumulates reserves in a precautionary manner, to insure against future state with concerned beliefs. The precautionary buildup of reserves reaches its highest at the lower bound of the range

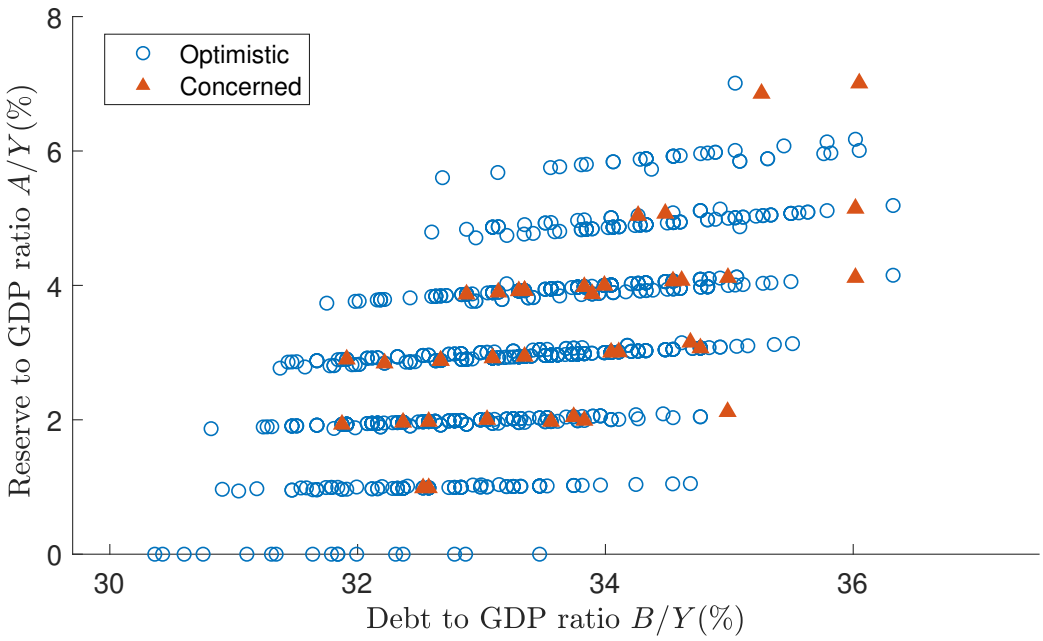

Figure 9: Simulation results

[.05, .22], and monotonically decreases as debt approaches .22.

Model simulations show that the government optimally brings the economy to operate in a region where debt remains comprised between .30 and .35 and reserves oscillate around .03. This is shown in Figure 9, which reports sample simulation results for 1000 periods using our baseline. Blue dots refer to a simulation paths for debt and reserves when lenders hold optimistic beliefs, whereas red triangle dots indicate paths under concerned beliefs. Regardless of beliefs regimes, the government holds reserves, either as a precaution against future concerns of lenders (optimistic), or to shield against intra-period risk today (concerned). We find that reserve holdings rarely drop to zero—this occurs only in 2% of periods in our simulation, in response to a sequence of negative fundamental shocks to output.[33]

A striking result from our baseline specification is that the volatility of consumption relative to output is close to but below unity (lower than in the economy with permanent optimism in the first column). In the model, the key driver of consumption volatility is a low discount factor (i.e., relative social impatience), which leads to front-loading adjustment in consumption.[34] While in equilibrium the accumulation of reserves is mostly financed by

[33]We find that reserves drop from a positive number to zero when the economy is hit by a sequence of negative output shocks, through which the government has progressively less capacity to hold reserves to insure against intra-period uncertainty.

[34]Front-loading due to impatience, together with limited commitment results in consumption converging towards an ergodic distribution. A lower value of discount factor (i.e. a government with less patience) generally leads to a larger consumption volatility. See the Handbook chapter by Aguiar and Amador (2014) for details.

debt issuance, it also requires some contraction in consumption—the government behaves "as if" it were more patient.

5.3 Long-term Debt

The simulated moments with long-term debt are presented in the last three columns of Table 1. In an economy where lenders are always optimistic (the fourth column of the table, with $\rho \in \{O\}$), the default frequency remains low due to the assumption of linear cost of default, yet higher than in the one-period debt specification (the first column). This reflect the risk of debt dilution—prompting lenders holding long-term bonds to require a higher return. Higher borrowing costs in turn prompt the government to borrow further into the default region.

Notably, as shown in the fourth column of the table, in spite of a permanent state of optimistic beliefs, the government does accumulates some (small) reserves, up to 0.01% of GDP. The reason resonates with the argument in Bianchi et al. (2018): with long-term debt, the government accumulates reserves to hedge against a potential rise in borrowing costs in the future driven by output fluctuations. As in our work we focus on a different motive to hold reserves, quantitatively, the amount of reserve holdings is negligible in our specification without intra-period risk.[35]

The benefits from issuing long-term debt are apparent in the last column of the table, where we drop the assumption of permanent optimism. With a constraint on reserve accumulation, default frequency, bond spread, and standard deviation of the spread are all lower with long-term bonds, compared to the one-period bond economy (in the third column). Since only a small portion of debt matures each period, the government has lower borrowing needs. This reduces the government's incentive to default post-auction, ultimately enlarging the region that supports the unique Eaton-Gersovitz price in equilibrium.

Results for our baseline with long-term bonds are in the fifth column of the table. Reserve holdings amount to 2.41 percent of GDP, lower than in the one-period bond economy (3.07% of GDP). This is because a longer debt maturity already provides some protection against self-fulfilling intra-period uncertainty. Consequently, the need to accumulate reserves decreases. The 2.41% reserve holdings is mostly driven by the need to protect the economy against intra-period uncertainty. Only a small part is accounted for hedging against variations in borrowing costs with output fluctuations (see the case $\rho \in \{O\}$).

[35] In Bianchi et al. (2018), risk premium shocks play a crucial role in generating high levels of reserve accumulation. During periods of low risk premiums, the government accumulates reserves as a safeguard against sudden spikes in risk premiums. Also, the discount factor is much higher than the one assumed in our exercises.

6 Conclusion

With discretionary policymakers, lenders may become concerned with the possibility of default just after the government issues new debt. Even if post-auction default is seen as a very low probability event, the uncertainty it generates enhances bond markets vulnerability to disruptive self-fulfilling sovereign risk crises (Aguiar et al. (2022)). We argue that post-auction uncertainty about repayment lays the foundations for a theory of reserve accumulation as a key building block of a cost-effective debt management strategy. We show that, unless the outstanding stock of debt is very low, welfare-maximizing governments would optimally accumulate enough reserves to rule out post-auction default in all circumstances. Reserve management is the crucial policy tool that enables governments to eliminate (or at least mitigate, if reserve accumulation is constrained) intra-period risk without suffering a significant increase in borrowing costs—in spite of the fact that their buildup is financed in large part by issuing new debt.

Our model shows that reserves are useful regardless of whether debt is short or long term, and helps explaining why countries hold a large stock of reserves and appear reluctant to drive them down during adverse shocks. While reserves can indeed be used to smooth out fundamental shocks that materialize in equilibrium, their stock needs to be sized up to satisfy liquidity needs that may arise off-equilibrium driven by self-fulfilling expectations of post-auction default.

Our theory calls attention to the type of risk that arguably motivates the high demand for reserves at global level in a world of relatively free capital mobility. From a policy perspective, however, by no means our analysis weakens the argument in favor of international sharing arrangements on reserve management. "Self-insurance" via reserve accumulation requires governments to expand debt and hold a significant amount of resources in low interest bearing assets. International arrangements that reduce the need for self-insurance would obviously be welfare improving.

These considerations identify promising areas of further investigation, aiming at exploring policy trade-offs that can improve the design of effective policy institutions and strategies at domestic and international levels. For analytical clarity, in this paper we presented the novel theory in its essence, using a stripped-down frameworks. We leave to future research a generalization of our analysis in a richer setting.

References

Aguiar, M. and M. Amador (2014). Sovereign Debt. In *Handbook of International Economics*, Volume 4, Chapter 11, pp. 647–687. Elsevier B.V.

Aguiar, M., S. Chatterjee, H. L. Cole, and Z. Stangebye (2022). Self-Fulfilling Debt Crises, Revisited. *Journal of Political Economy 130*(5), 1147–1183.

Aguiar, M. and G. Gopinath (2006). Defaultable debt, interest rates and the current account. *Journal of International Economics 69*, 64–83.

Aguiar, M. and G. Gopinath (2007). Emerging market business cycles: The cycle is the trend. *Journal of Political Economy 115*(1), 69–102.

Aizenman, J. and J. Lee (2007). International Reserves: Precautionary versus Mercantilist Views, Theory and Evidence. *Open Economies Review 18*(2), 191–214.

Alesina, A., M. D. Broeck, A. Prati, G. Tabellini, M. Obstfeld, and S. Rebelo (1992). Default Risk on Government Debt in OECD Countries. *Economic Policy*, 428–463.

Alfaro, L. and F. Kanczuk (2009). Optimal reserve management and sovereign debt. *Journal of International Economics 77*(1), 23–36.

Amador, M., J. Bianchi, L. Bocola, and F. Perri (2020). Exchange Rate Policies at the Zero Lower Bound. *Review of Economic Studies 87*(4), 1605–1645.

Arellano, C. (2008). Default Risk and Income Fluctuations in Emerging Economies. *American Economic Review 98*(3), 690–712.

Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2018). Sovereign Default: The Role of Expectations. *Journal of Economic Theory 175*, 803–812.

Ayres, J., G. Navarro, J. P. Nicolini, and P. Teles (2019). Self-Fulfilling Debt Crises with Long Stagnations. *Federal Reserve Bank of Minneapolis, No. 757*.

Ayres, J. and R. Paluszynski (2022). Multiplicity in Sovereign Default Models: Calvo meets Cole-Kehoe. *Manuscript*.

Benigno, G., L. Fornaro, and M. Wolf (2022). Reserve Accumulation, Growth and Financial Crises. *Journal of International Economics 139*, 103660.

Bianchi, J., J. C. Hatchondo, and L. Martinez (2018). International Reserves and Rollover Risk. *American Economic Review 108*(9), 2629–2670.

Bianchi, J. and G. Lorenzoni (2022). *The Prudential Use of Capital Controls and Foreign Currency Reserves*, Volume 6. Elsevier B.V.

Bianchi, J. and J. Mondragon (2018). Monetary Independence and Rollover Crises. *NBER Working Paper No. 25340*.

Bianchi, J. and C. Sosa-Padilla (2022). On Wars, Sanctions and Sovereign Default. *NBER Working Paper No. 29989*.

Bianchi, J. and C. Sosa-Padilla (2023). Reserve Accumulation, Macroeconomic Stabilization, and Sovereign Risk. *Review of Economic Studies rdad075*.

Bocola, L. and A. Dovis (2019). Self-Fulfilling Debt Crises: A Quantitative Analysis. *American Economic Reivew 109*(12), 4343–4377.

Bocola, L. and G. Lorenzoni (2020). Financial Crises, Dollarization, and Lending of Last Resort in Open Economies. *American Economic Review 110*(8), 2524–2557.

Brunnermeier, M. K. and L. Huang (2018). A Global Safe Asset for and from Emerging Market Economies. *NBER Working Paper No. 25373*.

Bussière, M., G. Cheng, M. D. Chinn, and N. Lisack (2015). For a few dollars more: Reserves and growth in times of crises. *Journal of International Money and Finance 52*, 127–145.

Caballero, R. J. and S. Panageas (2008). Hedging Sudden Stops and Precautionary Contractions. *Journal of Development Economics 85*(1-2), 28–57.

Calvo, G. A. (1988). Servicing the Public Debt: The Role of Expectations. *American Economic Review 78*(4), 647–661.

Chatterjee, S. and B. Eyigungor (2012). Maturity, Indebtedness and Default Risk. *American Economic Review 102*(6), 2674–2699.

Cole, H. L. and T. J. Kehoe (2000). Self-Fullfiling Debt Crises. *The Review of Economic Studies 67*(1), 91–116.

Conesa, J. C. and T. J. Kehoe (2014). Is it too late to Bail Out the Troubled Countries in the Eurozone? *American Economic Review: Papers and Proceedings 104*(5), 88–93.

Conesa, J. C. and T. J. Kehoe (2017). Gambling for Redemption and Self-Fulfilling Debt Crises. *Economic Theory 64*(4), 707–740.

Corsetti, G. and L. Dedola (2016). The Mystery of the Printing Press: Monetary Policy and Self-Fulfilling Debt Crises. *Journal of the European Economic Association 14*(6), 1329–1371.

Corsetti, G., A. Erce, and T. Uy (2017). Debt Sustainability and the Terms of Official Support. *ADEMU Working Paper*.

Corsetti, G., A. Erce, and T. Uy (2020). Official Sector Lending Strategies During the Euro Area Crisis. *The Review of International Organisations 15*, 667–705.

Corsetti, G. and S. H. Maeng (2023). Debt Crises, Fast and Slow. *Robert Schuman Centre for Advanced Studies Research Paper 2023/15*.

Davis, J. S., M. B. Devereux, and C. Yu (2023). Sudden Stops and Optimal Foreign Exchange Intervention. *Journal of International Economics 141*, 103728.

Devereux, M. B. and S. P. Y. Wu (2022). Foreign Reserves Management and Original Sin. *NBER Working Paper No. 30418*.

Durdu, C. B., E. G. Mendoza, and M. E. Terrones (2009). Precautionary Demand for Foreign Assets in Sudden Stop Economies: An Assessment of the New Mercantilism. *Journal of Development Economics 89*(2), 194–209.

Eaton, J. and M. Gersovitz (1981). Debt with Potential Repudiation: Theoretical and Empirical Analysis. *The Review of Economic Studies 48*(2), 289–309.

Fanelli, S. and L. Straub (2021). A Theory of Foreign Exchange Interventions. *Review of Economic Studies 88*(6), 2857–2885.

Galli, C. (2021). Self-fulfilling Debt Crises, Fiscal Policy and Investment. *Journal of International Economics 131*, 103475.

Hur, S. and I. O. Kondo (2016). A Theory of Rollover Risk, Sudden Stops, and Foreign Reserves. *Journal of International Economics 103*, 44–63.

Jeanne, O. and R. Rancière (2011). The Optimal Level of International Reserves For Emerging Market Countries: A New Formula and Some Applications. *Economic Journal 121*(555), 905–930.

Jeanne, O. and D. Sandri (2023). Global Financial Cycle and Liquidity Management. *Journal of International Economics*.

Jung, K. M. and J. H. Pyun (2016). International Reserves for Emerging Economies: A Liquidity Approach. *Journal of International Money and Finance 68*, 230–257.

Korinek, A. and L. Servén (2016). Undervaluation through Foreign Reserve Accumulation: Static Losses, Dynamic Gains. *Journal of International Money and Finance 64*, 104–136.

Lorenzoni, G. (2014). International Financial Crises. In *Handbook of International Economics*, Volume 4, pp. 689–740. Elsevier B.V.

Lorenzoni, G. and I. Werning (2019). Slow Moving Debt Crises. *American Economic Review 109*(9), 3229–3263.

Lutz, F. and L. Zessner-Spitzenberg (2023). Sudden Stops and Reserve Accumulation in the Presence of International Liquidity Risk. *Journal of International Economics 141*(I), 103729.

Matsumoto, H. (2022). Foreign Reserve Accumulation, Foreign Direct Investment, and Economic Growth. *Review of Economic Dynamics 43*, 241–262.

Qian, X. W. (2017). International Reserves and the Maturity of External Debt. *Journal of International Money and Finance 73*, 399–418.

Sosa-Padilla, C. and F. Sturzenegger (2023). Does it matter how central banks accumulate reserves? Evidence from sovereign spreads. *Journal of International Economics 140*.

Xiao, J. (2022). Borrowing to Save and Investment Dynamics. *Review of Economic Studies, Conditionally accepted*.

A Proofs

A.1 Proof of Proposition 1

First, we show that, when (7) and (9) are satisfied, (11) is also satisfied, given that $\sigma \to 0$. Define $\tilde{\Delta}(Q)$ by

$$\tilde{\Delta}(Q) \equiv u(y_0 - b_0 + Qb_1 - q_a a_1) - u(y_0 + Qb_1 - q_a a_1) + \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \left[u(y_1 - b_1 + a_1) - u(Zy_1 + a_1)\right] dy_1 \tag{14}$$

Clearly, $\tilde{\Delta}(q_{EG}) = \tilde{\Delta}_{EG}$ and $\tilde{\Delta}(q_{CK}) = \tilde{\Delta}_{CK}$. That is to say, as (7) and (9) are both satisfied (with $\sigma \to 0$), the following condition holds

$$\tilde{\Delta}(q_{EG}) \geq 0 \text{ and } \tilde{\Delta}(q_{CK}) < 0$$

If $\tilde{\Delta}(q_{EG}) = 0$, q_{EG} solves (11)—an interior price exists, equal to the Eaton-Gersovitz price. If $\tilde{\Delta}(q_{EG}) > 0$, by the intermediate value theorem, the above inequalities imply existence of $\tilde{q}$ that satisfies

$$\tilde{\Delta}(\tilde{q}) = 0$$

which solves (11).

We now show that, if interior prices are supportable, both q_{CK} and q_{EG} are also supportable. We proceed the proof by contradiction. We posit that either q_{CK}, or q_{EG}, or both are not supportable, when interior prices are supportable. That is to say:

i. $\tilde{\Delta}(q_{EG}) \geq 0$ and $\tilde{\Delta}(q_{CK}) \geq 0$ or

ii. $\tilde{\Delta}(q_{EG}) < 0$ and $\tilde{\Delta}(q_{CK}) < 0$ or

iii. $\tilde{\Delta}(q_{EG}) < 0$ and $\tilde{\Delta}(q_{CK}) \geq 0$

Note that

$$\frac{\partial \tilde{\Delta}(Q)}{\partial Q} > 0$$

Hence, if i. $\tilde{\Delta}(q_{EG}) \geq 0$ and $\tilde{\Delta}(q_{CK}) \geq 0$, as $\tilde{\Delta}(q_{CK}) \geq 0$ and $\frac{\partial \tilde{\Delta}(Q)}{\partial Q} > 0$, $\tilde{\Delta}(q_{EG})$ is unambiguously larger than 0. If $\tilde{\Delta}(q_{CK}) > 0$, there does not exist $\tilde{q} \in [q_{CK}, q_{EG}]$ that satisfies $\tilde{\Delta}(\tilde{q}) = 0$. When $\tilde{\Delta}(q_{CK}) = 0$, the government is indifferent between defaulting and repaying given the failed-auction bond price q_{CK}. We posit, in (11), that the government repays confronting indifference. Hence, though q_{CK} is a solution to (11), given that the government repays given $q_{CK} = 0$, q_{CK} cannot be an interior price.

When ii. $\tilde{\Delta}(q_{EG}) < 0$ and $\tilde{\Delta}(q_{CK}) < 0$, there does not exist $\tilde{q} \in [q_{CK}, q_{EG}]$ that satisfies $\tilde{\Delta}(\tilde{q}) = 0$. Also, iii. $\tilde{\Delta}(q_{EG}) < 0$ and $\tilde{\Delta}(q_{CK}) \geq 0$ contradicts with $\frac{\partial \tilde{\Delta}(Q)}{\partial Q} > 0$—$\tilde{\Delta}(q_{EG})$ is supposed to be larger than $\tilde{\Delta}(q_{CK})$.

A.2 Proof of Proposition 2

Proposition 1 states that, given $\sigma \to 0$, there exist interior prices, iff both (7) and (9) are satisfied. Hence, given that interior prices exist, from (14), we know that

$$\tilde{\Delta}(q_{EG}) \geq 0 \text{ and } \tilde{\Delta}(q_{CK}) < 0 \tag{15}$$

The following inequality is satisfied when $\sigma \to 0$

$$\frac{\partial \tilde{\Delta}(Q)}{\partial Q} > 0$$

That is to say, the function $\tilde{\Delta}(Q)$ in (14) is monotonic with respect to Q. Hence, if interior prices exist, i.e., (15) is satisfied, the monotonicity of $\tilde{\Delta}(Q)$ guarantees the uniqueness of the interior price in the region $(q_{CK}, q_{EG}]$, according to the intermediate value theorem.

A.3 Proof of Proposition 3

For a given (y_0, b_0, b_1, a_1), suppose an interior price exists, i.e., (11) is satisfied. Define $G(\tilde{\epsilon}, a_1)$:

$$\begin{aligned} G(\tilde{\epsilon}, a_1) \equiv \sigma\tilde{\epsilon} - \tilde{\Delta}_{ACCS} =& \sigma\tilde{\epsilon} - u(y_0 - b_0 + q_{EG}(b_1, a_1)F(\tilde{\epsilon})b_1 - q_a a_1) + u(y_0 + q_{EG}(b_1, a_1)F(\tilde{\epsilon})b_1 - q_a a_1) \\ &+ \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u(Zy_1 + a_1) - u(y_1 - b_1 + a_1)\big] dy_1 = 0 \end{aligned}$$

With $\sigma \to 0$, it gives

$$\begin{aligned} \frac{\partial G}{\partial \tilde{\epsilon}} &= q_{EG}(b_1, a_1) b_1 f(\tilde{\epsilon}) \Big(u'(y_0 + q_{EG}(b_1, a_1)F(\tilde{\epsilon})b_1 - q_a a_1) - u'(y_0 - b_0 + q_{EG}(b_1, a_1)F(\tilde{\epsilon})b_1 - q_a a_1) \Big) < 0 \\ \frac{\partial G}{\partial a_1} &= \Big(q_a - \frac{\partial q_{EG}(b_1, a_1)}{\partial a_1} F(\tilde{\epsilon}) b_1 \Big) \Big(u'(y_0 - b_0 + q_{EG}(b_1, a_1)F(\tilde{\epsilon})b_1 - q_a a_1) \\ &\quad - u'(y_0 + q_{EG}(b_1, a_1)F(\tilde{\epsilon})b_1 - q_a a_1) \Big) + \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u'(Zy_1 + a_1) - u'(y_1 - b_1 + a_1)\big] dy_1 \end{aligned}$$

As $\frac{\partial q_{EG}(b_1, a_1)}{\partial a_1} = 0$, due to the fact that $u'(y_0 - b_0 + q_{EG}(b_1, a_1)F(\tilde{\epsilon})b_1 - q_a a_1) > u'(y_0 + q_{EG}(b_1, a_1)F(\tilde{\epsilon})b_1 - q_a a_1)$ and $\int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u'(Zy_1 + a_1) - u'(y_1 - b_1 + a_1)\big] dy_1 > 0$, we have $\frac{\partial G}{\partial a_1} > 0$.

According to implicit function theorem

$$\frac{d\tilde{\epsilon}}{da_1} = -\frac{\partial G/\partial a_1}{\partial G/\partial \tilde{\epsilon}} > 0$$

That is to say, when an interior price is supportable, increasing reserves would lead to lower within-period default risk.

A.4 Proof of Proposition 4

For a given (y_0, b_0, b_1), define $H(a_1)$

$$\begin{aligned} H(a_1) \equiv \sigma - \tilde{\Delta}_{EG} =& \sigma - u(y_0 - b_0 + q_{EG}(b_1, a_1)b_1 - q_a a_1) + u(y_0 + q_{EG}(b_1, a_1)b_1 - q_a a_1) \\ &+ \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u(Zy_1 + a_1) - u(y_1 - b_1 + a_1)\big] dy_1 \end{aligned}$$

We take $\sigma \to 0$. As q_{ACCS} is supportable when a reserve holding is $\tilde{a}$, q_{EG} is also supportable at $\tilde{a}$. In other words, (7) is satisfied, which gives

$$H(\tilde{a}) \leq 0$$

Given the Eaton-Gersovitz bond price q_{EG}, the aggregate consumption at $t = 0$ when the government repays is equal to $y_0 - b_0 + q_{EG}(b_1, a_1)b_1 - q_a a_1$, and non-negative consumption constraints imply that

$$a_1 < \underbrace{\frac{y_0 - b_0 + q_{EG}(b_1, a_1)b_1}{q_a}}_{\equiv \overline{\overline{a}}}$$

In other words, when reserve holdings are sufficiently close to $\overline{\overline{a}}$, the aggregate consumption in period 0 given full repayment almost reaches zero, strictly dominated by default at $t = 0$[3], implying

$$\lim_{a_1 \to \overline{\overline{a}}} H(a_1) >> 0$$

As a function u is continuous, according to intermediate value theorem, there exists $\bar{a} \in [\tilde{a}, \overline{\overline{a}})$ that satisfies

$$H(\bar{a}) = 0$$

which completes the proof of the existence of $\bar{a}$ that fully eliminates intra-period default risk, as the solution to (11) is $\tilde{\epsilon} = 1$ when $a_1 = \bar{a}$. We now turn to the proof of uniqueness of $\bar{a}$.

Note that the following condition is satisfied:

$$\frac{\partial H}{\partial a_1} = q_a\Big(u'(y_0 - b_0 + q_{EG}(b_1, a_1)b_1 - q_a a_1) - u'(y_0 + q_{EG}(b_1, a_1)b_1 - q_a a_1)\Big) + \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u'(Zy_1 + a_1) - u'(y_1 - b_1 + a_1)\big] dy_1 > 0$$

The above condition shows the monotonicity of $H(a_1)$, which guarantees the uniqueness of solution $H(\bar{a}) = 0$.

We now show that accumulating reserves above $\bar{a}$ leads to a failed auction, and the only self-fulfilling price in this case is a failed-auction price q_{CK}. As $H(\bar{a}) = 0$ and $\frac{\partial H}{\partial a_1} > 0$, the following condition is satisfied:

$$H(a_1) > 0 \text{ for } a_1 \in (\bar{a}, \bar{\bar{a}}) \Rightarrow \sigma > \tilde{\Delta}_{EG} \text{ for } a_1 \in (\bar{a}, \bar{\bar{a}})$$

That is to say, q_{EG} is not supportable for $a_1 \in (\bar{a}, \bar{\bar{a}})$. According to Corollary 1, the only self-fulfilling price in this case is a failed auction price q_{CK}.

A.5 Proof of Lemma 1

(12) can be shown directly by Proposition 3. In what follows, we complete the proof of (13). For a given (y_0, b_0, b_1, a_1), suppose an interior price exists, i.e., (11) is satisfied. Define $I(N, a_1)$:

$$I(N, a_1) \equiv \sigma\tilde{\epsilon} - \tilde{\Delta}_{ACCS} = \sigma\tilde{\epsilon} - u(y_0 - b_0 + N) + u(y_0 + N) + \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u(Zy_1 + a_1) - u(y_1 - b_1 + a_1)\big] dy_1 = 0$$

where $N \equiv q_{ACCS}(b_1, a_1)b_1 - q_a a_1$ denotes net revenue. Take $\sigma \to 0$, it gives

$$\frac{\partial I}{\partial N} = u'(y_0 + N) - u'(y_0 - b_0 + N) < 0$$
$$\frac{\partial I}{\partial a_1} = \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u'(Zy_1 + a_1) - u'(y_1 - b_1 + a_1)\big] dy_1 > 0$$

Therefore

$$\frac{dN}{da_1} = -\frac{\partial I/\partial a_1}{\partial I/\partial N} > 0$$

In other words, $q_{ACCS}(b_1, a_1)b_1 - q_a a_1$ increases with larger reserve holdings.

A.6 Proof of Proposition 5

For a given (y_0, b_0, b_1), if a^i $(i \in \{1, 2\})$ support interior prices, according to (11) we have

$$V^R_{ACCS}(y_0, b_0, b_1, a^i) = V^D_{ACCS}(y_0, b_0, b_1, a^i) + \sigma\tilde{\epsilon}^i$$

The expected payoff from a^i at $t = 0^1$ is

$$\begin{aligned}
&\mathbb{E}\{V^R_{ACCS}(y_0, b_0, b_1, a^i), V^D_{ACCS}(y_0, b_1, a^i) + \sigma\epsilon\} \\
=&\mathbb{E}\{V^D_{ACCS}(y_0, b_1, a^i) + \sigma\tilde{\epsilon}^i, V^D_{ACCS}(y_0, b_1, a^i) + \sigma\epsilon\} \\
=&V^D_{ACCS}(y_0, b_1, a^i) + \sigma\mathbb{E}\max\{\tilde{\epsilon}^i, \epsilon\}
\end{aligned}$$

As $q_{ACCS}(b_1, a^2)b_1 - q_a a^2 > q_{ACCS}(b_1, a^1)b_1 - q_a a^1$ from (13), we have $V^D_{ACCS}(y_0, b_0, b_1, a^2) > V^D_{ACCS}(y_0, b_0, b_1, a^1)$. This, with the fact that $\tilde{\epsilon}^2 > \tilde{\epsilon}^1$ from (12), a^2 strictly dominates a^1.

B Debt Management Banning Reserve Accumulation

Suppose that reserve accumulation is not feasible (i.e. $a_1 = 0$). The following proposition states that, when facing interior prices, the government strictly prefers larger debt issuance as it generates larger debt revenue.

Proposition B.1. *For a given (y_0, b_0) and $a_1 = 0$, consider that interior prices are supportable when $b_1 \in \{b^1, b^2\}$ with $b^1 < b^2$. Then, b^2 strictly dominates b^1 as a policy choice.*

Proof. Taking $a_1 = 0$ and $\sigma \to 0$, define $J(N, b_1)$, where $N \equiv q_{ACCS}(b_1, 0)b_1$

$$J(N, b_1) \equiv \sigma\tilde{\epsilon} - \tilde{\Delta}_{ACCS} = -u(y_0 - b_0 + N) + u(y_0 + N) + \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u(Zy_1) - u(y_1 - b_1)\big] dy_1$$

If b_1 supports interior prices, $J(N, b_1) = 0$ and therefore

$$\begin{aligned}
\frac{\partial J}{\partial N} &= u'(y_0 + N) - u'(y_0 - b_0 + N) < 0 \\
\frac{\partial J}{\partial b_1} &= \beta \int_{\frac{b_1}{1-Z}}^{\bar{Y}} \big[u'(y_1 - b_1)\big] dy_1 > 0
\end{aligned}$$

Hence, according to implicit function theorem

$$\frac{dN}{db_1} = -\frac{\partial J/\partial b_1}{\partial J/\partial N} > 0$$

That is to say, increasing debt issuance strictly increases net debt revenue $N = q_{ACCS}(b_1, 0)b_1$.

This implies that net debt revenue from b^i ($i \in \{1,2\}$) satisfies $N^2 > N^1$. The expected payoff from b^i at $t = 0^1$ is

$$\begin{aligned}
&\mathbb{E}\{V^R_{ACCS}(y_0, b_0, b^i, 0), V^D_{ACCS}(y_0, b^i, 0) + \sigma\epsilon\} \\
=&\mathbb{E}\{V^D_{ACCS}(y_0, b^i, 0) + \sigma\tilde{\epsilon}^i, V^D_{ACCS}(y_0, b^i, 0) + \sigma\epsilon\} \\
=&V^D_{ACCS}(y_0, b^i, 0) + \sigma\mathbb{E}\max\{\tilde{\epsilon}^i, \epsilon\} \\
=&u(y_0 + N^i) + \beta\left(\int_{\underline{Y}}^{\bar{Y}} u(Zy_1)dy_1\right)
\end{aligned}$$

The government strictly prefers larger N^i at $t = 0^1$. Hence, b^2 strictly dominates b^1 as a policy choice, and the equilibrium level of debt issuance is $\bar{b}_{EG}$ in the region that supports interior prices (e.g. $[\underline{b}_{EG}, \bar{b}_{EG}]$ in Figure 4).

□

C Parameter Values

In this section, we lay out parameter values in Table 2 that we set for the numerical illustration in Section 4.3. We postulate that the government's utility adheres to a constant relative risk aversion (CRRA) utility function.

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma}$$

We set the relative risk-aversion parameter equal to $\gamma = 3$. Mean output at $t = 1$ is normalized to 1, larger than the current period output. This normalization ensures that, in the majority of cases, the realized output at $t = 1$ exceeds y_0, thus motivating the government to prioritize borrowing over saving in period 0. Consistently, we assign a discount factor as low as $\beta = 0.75$. This further reinforces the rationale for consumption smoothing by borrowing.[36]

Default entails both output and utility losses. Output losses, which only occur in period 1, are set equal to 0.3, higher than the value estimated in Alesina et al. (1992). The difference is due to the fact that we abstract from output losses in period 0. To ensure that a reasonable stock of initial debt is sustainable, we introduce a one-time utility loss at $t = 0$, denoted as v, triggered by default. By setting v equal to 0.234, the intitial debt is sustainable up to

[36]Low period-0 output and high government impatience ensure that the benefits from retiring external debt using reserves outweigh that from holding reserves for inter-temporal consumption smoothing purposes. Namely, in our numerical example, the government never accumulates reserves to smooth consumption inter-temporally. See Proposition 6 for details.

32% in percentage of GDP.[37] The world risk-free interest rate R^* is set to 2%.

Table 2: Parameter values

y_0	Output in period 0	0.75
$1-Z$	Default penalty in period 1	0.3
β	Discount factor	0.75
γ	Relative risk aversion	3
$\sigma_{\log(y)}$	Standard deviation of $\log(y_1)$	0.05
$\mu_{\log(y)}$	Mean of $\log(y_1)$	-$\frac{1}{2}\sigma^2_{\log(y)}$
R^*	Risk-free interest rate	0.02
v	Default utility loss in period 0	0.234

D Infinite-horizon Model with Short-term Debt

In this section, we expand the two-period model of Section 2 to an infinite-horizon model. We describe the model environment in a recursive manner, abstracting from re-entry to the international borrowing after default. The aggregate state is summarized by $S = (\Gamma_{-1}, g, \epsilon_m, \rho, z_{-1}, B, A)$. Γ_{-1} denotes a previously accumulated trend component in the endowment process; g is a trend growth rate; ϵ_m is a transitory component in the endowment process; ρ describes the belief state in the current period, which is governed by an exogenous sunspot process; z_{-1} indicates whether the government has defaulted in the past ($z_{-1} = 0$) or not ($z_{-1} = 0$); B and A denote, respectively, debt and reserves maturing in the current period. Though in this section we assume that the country with a history of default is unable to borrow from lenders permanently—we allow for re-entry in the international bond markets after default in our quantitative exercise in Section 5.

Endowment.—The endowment process is governed by a permanent shock g and a temporary shock ϵ_m.

$$Y = \Gamma e^{\sigma_m \epsilon_m}$$

$$\Gamma = \Gamma_{-1} e^{g}$$

Timing.—In each period, the timing follows the approach adopted in our two-period model:

[37] Alternatively, we could ensure enough debt sustainability by setting the default penalty $1-Z$ at an exceptionally high level. However, such an elevated default cost would encourage the government to accumulate reserves for the purpose of consumption smoothing through default states at $t = 1$—an economically meaningful use of reserves which however would not be the one we focus our paper on.

1. The aggregate state $S = (\Gamma_{-1}, g, \epsilon_m, \rho, z_{-1}, B, A)$ is known. Given that the government has no history of defaulting, it decides B' and A' at the beginning of the period.

2. The bond auction takes place. Lenders lend at risk-neutral bond price Q.

3. Intra-period shocks ϵ are realized.

4. The government decides whether to repay or to default.

As in our two-period model specification, we solve the model backwards, starting from the default decision at the end of the period.

The government.—With no history of defaulting ($z_{-1} = 1$), the government decides whether to default ($z = 0$) or not ($z = 1$) after the auction.

$$V^{end}(S, B', A') = \max_{z \in \{0,1\}} zV^R(S, B', A') + (1 - z)(V^D(S, B', A') + \sigma\epsilon)$$

where $V^R(S, B', A')$ and $V^D(S, B', A')$ are

$$\begin{aligned} V^R(S, B', A') &= u(Y - B + A - q_A A' + QB') + \beta\mathbb{E}[V(S')|S] \\ V^D(S, B', A') &= u(Y + A - q_A A' + QB') + \beta\mathbb{E}[V^{aut}(S')|S] \end{aligned}$$

V^{aut} denotes the utility without access to international lending markets, characterized by

$$V^{aut}(S) = \max_{A'} u(Y + A - q_A A') + \beta\mathbb{E}[V_{aut}(S')|S]$$

Note that, upon defaulting, the government loses access to the international bond market but retains access to accumulated reserves. The government repays at the end of the period if the following condition is satisfied:

$$\epsilon \leq \sigma^{-1} \overbrace{[V^R(S, B', A') - V^D(S, B', A')]}^{\equiv \Delta(S, B', A')}$$

The within-period repayment probability after the auction is characterized by $F(\sigma^{-1}\Delta)$, where $F(\cdot)$ is the cumulative distribution function of ϵ.

At the beginning of the period, the government chooses B' and A' to maximize expected utility at the end of the period:

$$V(S) = \max_{B', A'} \left\{ F(\sigma^{-1}\Delta(S, B', A'))V^R(S, B', A') + \int_{\sigma^{-1}\Delta(S, B', A')}^{1} \left[V^D(S, B', A') + \sigma\epsilon\right] dF(\epsilon) \right\} \tag{16}$$

Lenders.—Lenders are risk-neutral. Let $\mathcal{B}(S)$ and $\mathcal{A}(S)$ denote the government's optimal debt and reserve policy functions for a given state S. Optimal pricing must satisfy the break-even condition below, reflecting both intra-period and inter-temporal default risk:

$$Q(S, B', A') = (R^*)^{-1} \times \overbrace{F(\sigma^{-1}\Delta(S, B', A'))}^{\text{intra-period default risk}} \times \overbrace{\mathbb{E}[F(\sigma^{-1}\Delta(S', \mathcal{B}(S'), \mathcal{A}(S')))|S]}^{\text{inter-temporal default risk}} \tag{17}$$

Equilibrium.—An equilibrium consists of a government value function $V(S)$ and policy functions $\mathcal{B}(S)$ and $\mathcal{A}(S)$, with equilibrium bond price schedule $Q(S, B', A')$ such that

(i) $V(S)$, $\mathcal{B}(S)$ and $\mathcal{A}(S)$ solve the government's problem (16) given $Q(S, B', A')$.

(ii) $Q(S, B', A')$ satisfies the break-even condition in (17).

Detrending.—Due to the trend in the endowment process, the state variables are non-stationary. We "detrend" all non-stationary variables by dividing them with the trend growth factor Γ_{-1}. For any variable X, we denote $x = X/\Gamma_{-1}$ as the detrended value. The state variable after detrending is $s = (g, \epsilon_m, \rho, z_{-1}, b, a)$. We characterize a detrended government's and lenders' problems in what follows.

The government at the end of the period decides whether to default or not:

$$V^{end}(s, b', a') = \max_{z \in \{0,1\}} zV^R(s, b', a') + (1 - z)(V^D(s, b', a') + \sigma\epsilon)$$

where $V^R(s, b', a')$ and $V^D(s, b', a')$ are

$$\begin{aligned} V^R(s, b', a') &= u(e^{g+\sigma_m\epsilon_m} - b + a - e^g q_A a' + e^g Q b') + \beta\mathbb{E}[V(s')|s] \\ V^D(s, b', a') &= u(e^{g+\sigma_m\epsilon_m} + a - e^g q_A a' + e^g Q b') + \beta\mathbb{E}[V^{aut}(s')|s] \end{aligned}$$

V^{aut} is characterized by

$$V^{aut}(s) = \max_{a'} u(e^{g+\sigma_m\epsilon_m} + a - e^g q_A a') + \beta\mathbb{E}[V_{aut}(s')|s]$$

Debt repayment condition is summarized by:

$$\epsilon \leq \sigma^{-1} \overbrace{[V^R(s, b', a') - V^D(s, b', a')]}^{\equiv\Delta(s,b',a')}$$

At the beginning of the period, the government chooses b' and a':

$$V(s) = \max_{b', a'} \left\{ F(\sigma^{-1}\Delta(s, b', a'))V^R(s, b', a') + \int_{\sigma^{-1}\Delta(s,b',a')}^{1} \left[V^D(s, b', a') + \sigma\epsilon\right] dF(\epsilon) \right\}$$

Lenders price the government debt in a risk-neutral manner, see below:

$$Q(s, b', a') = (R^*)^{-1} \times F(\sigma^{-1}\Delta(s, b', a')) \times \mathbb{E}[F(\sigma^{-1}\Delta(s', \mathbf{b}(s'), \mathbf{a}(s')))|s]$$

E List of Countries

Our list includes 59 countries: Argentina, Australia, Austria, Bangladesh, Brazil, Bulgaria, Canada, Chile, China, Colombia, Croatia, Denmark, Hungary, Ecuador, Egypt, France, Georgia, Germany, Greece, Iceland, India, Indonesia, Isreal, Italy, Japan, Jordan, Kazakhstan, Laos, Libya, Malaysia, Mexico, Mongolia, Morocco, Nepal, Netherland, New Zealand, Nigeria, Norway, Pakistan, Peru, Philippines, Poland, Portugal, Romania, Saudi Arabia, Senegal, Spain, South Africa, South Korea, Sri Lanka, Sweden, Switzerland, Tanzania, Thailand, Tonga, Turkey, United Kingdom, Uzbekistan, Vietnam. We acquire reserves data of these countries from the International Financial Statistics, labelled as "International Liquidity: Total Reserves Minus Gold".