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Target Setting in Contests with Sabotage

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We study a novel target prize contest between two heterogeneous contestants featuring sabotage. The contestants first choose a target prize should they win the contest, then exert two types of effort: (i) productive effort which directly enhances their performance; and (ii) destructive effort which reduces the opponent's performance. While both types of effort incur constant marginal costs (in the respective levels of effort), the productive effort's marginal cost is an increasing function of the target prize. We show that when contestants are allowed to choose their own target prize, they do not sabotage each other in any subgame perfect equilibrium.

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Target Setting in Contests with Sabotage

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Abstract

We study a novel *target prize contest* between two heterogeneous contestants featuring sabotage. The contestants first choose a target prize should they win the contest, then exert two types of effort: (i) *productive effort* which directly enhances their *performance*; and (ii) *destructive effort* which reduces the opponent's performance. While both types of effort incur constant marginal costs (in the respective levels of effort), the productive effort's marginal cost is an increasing function of the target prize. We show that when contestants are allowed to choose their own target prize, they do not sabotage each other in any subgame perfect equilibrium.

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1 Introduction

A contest is a game in which participants expend costly resources to win a valuable prize. For example, scholars in the same firm working hard to compete with each other for a promotion or pay rise, countries burning trillions during wars, and firms investing in R&D in patent races.

Following the work of Tullock (1980), the early papers in the contest literature all assume one-dimensional actions that directly improve each contestant's "performance" in a contest.[1]

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[1]For examples, Lazear and Rosen (1981); Hillman and Samet (1987); Hillman and Riley (1989); Hirshleifer (1989); Baye et al. (1996), and Skaperdas (1996) among others.

In a nutshell, each contestant exerts costly effort that increases their prospect of "winning" the contest. There are often two important simplifying assumptions imposed in these models.

First, the prize that each contestant receives as a result of their effort and performance is often assumed to be fixed and exogenously given. While the fixed prize assumption is proven to be useful analytically to allow the models to be tractable, it often fails to reflect many economic situations accurately. For example, in R&D contests, the prize of winning a patent race often depends on the actual quality of the product (Baik, 1994; Kräkel, 2004; Cohen et al., 2008). On the other hand, in war conflicts, the prize of the contest decreases in the contestants' efforts (Shaffer, 2006; Smith et al., 2014).

Chung (1996) and Chowdhury and Sheremeta (2011) provide the characterisation of the existence and uniqueness of Nash equilibria in a rent-seeking contest with an endogenous prize, which increases in the total effort of all contestants, between symmetric contestants facing linear effort costs. The case with asymmetric contestants is analysed in Hirai (2012), Hirai and Szidarovszky (2013), and Damianov et al. (2018), with the former two still sticking with linear effort costs and the latter assuming a convex cost.

Second, in each of these models, the contestant's only action is to decide their *productive effort*, that is, effort that directly contributes to their own performance and increases their odd of winning the contest. This ignores the issue of *destructive effort*, or *sabotage*, which improves a contestant's odd of winning by *reducing* the performance of other contestants. There is an abundance of evidence of sabotage both in laboratory experiments (Harbring and Irlenbusch, 2005; Harbring et al., 2007; Harbring and Irlenbusch, 2008, 2011) and field studies (Drago and Garvey, 1998; Balafoutas et al., 2012; Deutscher et al., 2013; Brown and Chowdhury, 2017). A general finding from these studies is that when an agent's own payoff depends on relative performance, there is an incentive to sabotage or to withhold from helping each other.

There are mainly two ways that sabotage is modelled in theoretical studies. The earliest theoretical work is by Lazear (1989), who models sabotage as effort that directly reduces the opponents' performance, and show a positive relation of the prize difference between the winner and loser(s) with incentive to sabotage. This approach has been adopted by studies including Konrad (2000), Chen (2003), Münster (2007), Gürtler (2008), and Doğan et al. (2019) among others. The second approach is to model sabotage as effort that increases the opponents' marginal cost of exerting productive effort. Amegashie (2012) shows that with this form of sabotage, contestants would only sabotage each other in equilibrium when the contest is sequential – in which they choose their destructive effort in the first stage then their productive effort in the second stage. Minchuk (2020) considers a combined approach of the two in a sequential contest, in which the contestants choose *both* forms of sabotage in the first stage then their productive effort in the second stage. They show that in such a contest, the contestants would only sabotage by increasing the contestants' productive effort cost.

We contribute by combining these two strands of the contest literature. We consider a two-player *target prize contest* in which the contestants choose their winning prize, as well as two types of effort: productive and destructive. In a two-stage game, the contestants first simultaneously choose a target prize that each of them gets should she wins the contest at Stage 1. Then, given their first-stage choices of target prize, they simultaneously choose their productive and destructive efforts at Stage 2. A contestant's own productive effort and the opponent's destructive effort then aggregate to her *performance*. Her *contest success function* (CSF), i.e., her winning probability, is given by her performance divided by the sum of both contestants' performance á la Tullock (1980).

In our model, the contestants face a novel strategic tension between setting a high or a low prize target, as we assume that the marginal cost of exerting productive effort in Stage 2 is an increasing function in the contestant's own chosen target prize. To our knowledge, the only existing research that allows contest prize to be a direct choice[2] of the contestants is de Roos et al. (2022), who consider a N-player symmetric contest in which each contestant chooses both her effort and prize. Similar to our model, their contestants' cost of participating the contest is increasing in both their effort and chosen prize. Our model differs from theirs in two main ways. First, we consider a sequential setting in our model, while target setting takes place before the contestants choose their efforts. Second, our model admits multidimensional efforts which each contestant can choose to exert both productive effort and destructive effort. Indeed, while their focus lies in establishing the sufficient conditions of equilibrium existence in a general model and the role of the number of contestants on equilibrium behaviour, we focus on how target setting and *prize-dependent* effort cost influence the contestants' incentive to sabotage.

While there are many possible applications to our model, throughout this paper we stick to a scenario in which two scholars with different abilities of the same lab work in different projects. Both scholars compete in a contest to be the "star performer" in the lab. In Section 3, we first solve for the scholar's optimal effort choices and their respective contest success functions (CSF), given any pair of target prizes at Stage 2. As mentioned previously, each scholar's marginal productive cost, c_r, is an increasing function of only her chosen target prize π_i, for each scholar i. More specifically, we assume that $c_r(\pi_i) = \frac{\gamma}{n}\pi_i^n$ for any $n \geqslant 1$. Thus, we allow c_r to be of any convexity in π_i, so long it is convex in π_i. Finally, we note that since c_r is independent of the actual level of productive effort exerted, our model does not deviate from the conventional approach to assume a constant marginal cost of productive effort. Therefore, our Stage 2 analysis is directly comparable with the existing research on sabotage in contests.

In sum, at Stage 2, the scholars would choose to sabotage each other if the marginal cost of doing so is sufficiently low. The threshold of sabotage cost depends on the scholars' abil-

[2]Additionally, contests in which the contestants have indirect influence to their prizes through their effort choices are also considered by studies including Cohen and Sela (2005), Bevia and Corchón (2006), and Matros and Armanios (2009), among others.

ities and the target prize. Our results lead to some surprising observations. First, while it is well-documented that lower-ability contestants tend to have a higher incentive to sabotage their higher-ability counterparts, we show that this effect can be mitigated when their effort costs depend on their prize. Indeed, we show that a scholar i's sabotage effort is greater than that of the other scholar j if and only if i has previously chosen a higher target prize. In other words, it is independent of the difference of their *abilities* but the difference of their *target prizes*. Second, given a pair of target prizes, we show that for the scholar who has chosen a higher target prize at Stage 1, there is an *encouragement effect* on the scholars' productive effort (i.e., they exert more productive effort when sabotaging is allowed versus when it is not, *ceteris paribus*). This is a stark contrast to the well-documented *discouraging effect* (Chen, 2003; Münster, 2007; Gürtler and Münster, 2010; Amegashie, 2012) in the literature. To our best knowledge, Doğan et al. (2019) is the only existing research with a model that features the encouragement effect in a *team contest*, which they attribute to the cause of the encouragement effect in their model. Our model thus provides a novel explanation to this phenomenon in contests between individuals.

It is, however, our Stage 1 result that proves to be the most surprising. Knowing what efforts will be chosen at Stage 2 in response to the chosen target prizes, the scholars know exactly the set of target prizes that would induce sabotaging at Stage 2 and at what level. We show that in any subgame perfect equilibrium, neither scholar will be sabotaging at all. With our choice of the c_r being a monomial of any degree, we can claim that this result is robust to any convexity of c_r in π_i. In other words, when the scholars are able to set their own target prize, the problem of sabotaging can be fully mitigated. Our Stage 1 result thus provides a novel policy recommendation on reducing sabotage, in oppose to various different ways of either reducing the benefit of sabotaging directly via manipulating winning and losing prizes[3] or increasing the cost of sabotage.[4]

Throughout this paper, we adopt the conventional definition of performance, aggregating the scholar's ability, productive effort, and her opponent's sabotage effort. In Section 5, we consider an alternative definition which also takes into account the target prize that the scholar has chosen. This modification captures the idea that tackling more difficult tasks (and thus higher prize) is often rewarded and acknowledged. We show that when a scholar's performance takes into account her chosen target prize, it weakens the trade off between setting high versus low target. However, our main result of no sabotage in SPE is robust to this alternative setup.

The rest of the paper is structured as follows. Section 2 introduces the model of endogenous prize contest with sabotage. Section 3 analyses the Stage 2 equilibrium of the model given a pair of target prizes. Section 4 solves for the subgame perfect target prizes and thus

[3]For examples, see Drago and Turnbull (1991) and Charness et al. (2014).

[4]See Krakel (2000) and Curry and Mongrain (2009), among others.

the subgame perfect equilibria of the contest. Section 5 considers an alternative definition of performance that rewards high target prizes. Section 6 concludes and discusses some possible ways to extend this model.

2 The Model

Consider a target prize contest with sabotage. Two contestants, scholar 1 and scholar 2, work on separate projects in a lab and compete with each other to be the standout performer of the lab. They participate in a contest by first choosing their respective *target* prizes $\pi_i \in \mathbb{R}_+$, before choosing their *productive effort* $r_i \in \mathbb{R}_+$ and *destructive effort* (i.e., sabotage) $s_i \in \mathbb{R}_+$, where $i \in N \equiv \{1, 2\}$. Let $r = (r_i, r_j)$ and $s = (s_i, s_j)$.

We define i's *performance* by:

$$q_i(r_i, s_j) \equiv \frac{\alpha_i r_i}{1 + s_j}, \tag{1}$$

where $i \neq j \in N$, and $\alpha_i, \alpha_j \in \mathbb{R}_{++}$ represents the *ability* of each scholar. In words, a scholar's performance is increasing in her ability and her own productive effort, and decreasing in her opponent's destructive effort. Throughout this paper, we assume that scholar 1 is the high-ability scholar, thus $\alpha_1 > \alpha_2$. Given the performance, scholar i's *contest success function*, i.e., her *winning probability*, is given by

$$p_i(q_i, q_j) \equiv \frac{q_i}{q_i + q_j}. \tag{2}$$

p_i is increasing in r_i and s_i but decreasing in r_j and s_j. If scholar i wins the contest, she receives the prize $\pi_i \in \mathbb{R}_+$ that she has previously chosen. Targeting a higher prize is costly, however, as we assume that the marginal cost of exerting productive effort c_r to be constant in r_i, but to be a convex function of the target prize:

$$c_r(\pi_i) = \frac{\gamma}{n}\pi_i^n \tag{3}$$

for some $\gamma > 0$ and $n \geqslant 1$. The value of n governs the convexity of the productive marginal cost function. We impose the lower bound of n to ensure that c_r is always convex in π_i, which reflects the conventional wisdom that it is harder to increase the difficulty of an already difficult task (and hence resulting in higher prize). γ is a positive scaling parameter which can be used to assess the impact of the target prize on the c_r. Finally, we assume a constant marginal cost of destructive efforts c_s.

Collecting all building blocks, given (r, s), scholar i's ex-ante expected utility function is given by

$$u_i(r_i, r_j, s_i, s_j, \pi_i) \equiv \frac{q_i(r_i, s_j)}{q_i(r_i, s_j) + q_j(r_j, s_i)}\pi_i - \frac{\gamma}{n}\pi_i^n r_i - c_s s_i. \tag{4}$$

The timing of this games is as follows: At Stage 1, the scholars simultaneously chooses a target prize π_i. Observing the prizes chosen, each scholar then chooses at Stage 2 her productive and destructive efforts (r_i, s_i). The solution concept of this game is the subgame perfect Nash equilibrium (SPE).

We take a moment here to discuss some of the modelling choices we adopted. First, we assume c_r to be a n-degree monomial. This functional form gives us a good level of flexibility while preserving the tractability of analysis. By allowing n to take the value of any strictly positive real number, we allow a wide range of convexity of c_r in π_i. Indeed, as we show in the next section, the scholar's Stage 2 equilibrium behaviour does depend on n.

Second, here we assume that the target prizes in the contest to be solely the choices of the contestants. This reminiscences the situation where each scholar comes up with a goal for the coming year after fixed-period reviews with the lab manager. Another notable feature of our model is that the scholar's CSFs are not directly dependent on the target prizes – since their performance q_i and q_j are not directly dependent on the target prizes.[5] In Section 5, we consider an alternative definition of performance which takes into account the difficulty of the task (thus the target prize).

3 Effort Choices Given Target Prizes

By backward induction, we first solve Stage 2, where the scholars choose their efforts given some target prizes. Alternatively, this section can be viewed as a standalone analysis of an exogenous prize contest with contestants facing an increasing productive effort cost in their prizes.

First, we solve for the equilibrium efforts $r^* \equiv (r_i^*, s_i^*)$ given (π_i, π_j) for some given target prizes π_i and π_j. At this stage, each scholar i solves the following maximisation problem:

$$\max_{r_i, s_i} \; u_i(r_i, r_j, s_i, s_j, \pi_i) = \frac{q_i(r_i, s_j)}{q_i(r_i, s_j) + q_j(r_j, s_i)}\pi_i - \frac{\gamma \pi_i^n}{n} r_i - c_s s_i. \tag{5}$$

The next lemma and proposition show the existence and the outline of an interior equilibrium of efforts at Stage 2, given π_i and π_j.

Lemma 1. *An interior Stage 2 equilibrium in which both scholars sabotage each other can exist only if*

$$\frac{\pi_i^{n-2}\alpha_i}{\pi_j^{n-2}\alpha_j} \in [1/3, 3]. \tag{6}$$

[5] An example situation where this setup makes sense is that each scholar is only judged on whether their project is successful. Each can choose whether to acquire the help of an additional third scholar also in the lab, which would make the project less costly to work on. However, introducing another scholar means that any subsequent publications will not be solo-authored, hence discounting the prize.

Lemma 1 provides a necessary condition for an interior equilibrium, which requires for some given target prizes (π_i, π_j), the scholars' abilities α_i and α_j are not too far apart. This is intuitive as a much higher-ability scholar would not find herself threatened by the much lower-ability scholar, and would not find it profitable to invest additional effort in sabotaging. Likewise, the lower-ability scholar would face a low CSF with or without sabotaging, thus would also find it not profitable to invest in sabotaging.

Now, let us define the two relevant *threshold sabotaging costs* for each scholar i, $\bar{c}_s^i$ and $\hat{c}_s^i$, be:

$$\bar{c}_s^i(\alpha_i, \alpha_j, \pi_i, \pi_j) \equiv \frac{\alpha_i \alpha_j \pi_i^{n-1} \pi_j^{n-2}}{(\alpha_i \pi_j^{n-2} + \alpha_j \pi_i^{n-2})^2}; \tag{7}$$

$$\hat{c}_s^i(\alpha_i, \alpha_j, \pi_i, \pi_j) \equiv \frac{\alpha_i \alpha_j \pi_i^{n} \pi_j^{n-1}}{(\pi_j^{n-1}\alpha_i + \pi_i^{n-1}\alpha_j)^2}. \tag{8}$$

As it will be shown in Proposition 1, $\bar{c}_s^i$ is the relevant threshold for scholar i to be willing to sabotage when scholar j also sabotages; whereas $\hat{c}_s^i$ is the relevant threshold when scholar j does not. The following lemma shows a useful relationship between $\bar{c}_s^j$ and $\hat{c}_s^i$.

Lemma 2. *$\bar{c}_s^j \leqslant \hat{c}_s^i$ if and only if $\pi_i \geqslant \pi_j$. It holds for equality if $\pi_i = \pi_j$.*

The reason why we highlight this relationship and not others is because $\hat{c}_s^i$ is only relevant when one of the scholar (say scholar j) would not want to sabotage when the other scholar does (i.e., when $c_s < \bar{c}_s^j$). In this case $\bar{c}_s^i$ is no longer a relevant threshold, as it is given that scholar j is not sabotaging. We are now ready to present the Stage 2 equilibrium.

Proposition 1. *Suppose $\pi_i \geqslant \pi_j$, where $i, j = 1, 2$ and $i \neq j$. Then $\bar{c}_s^j < \bar{c}_s^i$ and the scholars choose efforts as follows:*

1. *If $c_s < \bar{c}_s^j$ for both $i = 1, 2$, the CSF, chosen efforts, and resultant performance of each scholar i are given by*

$$p_i^* = \frac{\pi_j^{n-2}\alpha_i}{\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j}, \quad r_i^* = \frac{n\alpha_i\alpha_j\pi_j^{n-2}}{\gamma\pi_i(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2},$$
$$s_i^* = \frac{\alpha_i\alpha_j\pi_i^{n-1}\pi_j^{n-2}}{c_s(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2} - 1, \quad q_i^* = \frac{n\alpha_i c_s}{\gamma\pi_i^{n-1}\pi_j}. \tag{9}$$

2. *If $c_s \in \left[\bar{c}_s^j, \hat{c}_s^i\right)$, only scholar i sabotages while scholar j does not. The CSF, chosen efforts*

and resultant performance are given by

$$\tilde{p}_i = 1 - \sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}, \quad \tilde{r}_i = \frac{n\tilde{p}_i\tilde{p}_j}{\gamma\pi_i^{n-1}}, \quad \tilde{s}_i = \frac{\pi_i\tilde{p}_i\tilde{p}_j}{c_s} - 1, \quad \tilde{q}_i = \frac{n\alpha_i\tilde{p}_i\tilde{p}_j}{\gamma\pi_i^{n-1}}.$$

$$\tilde{p}_j = \sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}, \quad \tilde{r}_j = \frac{n\tilde{p}_i\tilde{p}_j}{\gamma\pi_j^{n-1}}, \quad \tilde{s}_j = 0, \quad \tilde{q}_j = \frac{n\alpha_j c_s}{\gamma\pi_i\pi_j^{n-1}}. \tag{10}$$

This case is not present if $\pi_i = \pi_j$.

3. *If* $c_s \geqslant \hat{c}_s^i$, *the chosen efforts and resultant performances for each* $i = 1, 2$ *are given by*

$$p_i' = \frac{\pi_j^{n-1}\alpha_i}{\pi_j^{n-1}\alpha_i + \pi_i^{n-1}\alpha_j}, \quad r_i' = \frac{n\alpha_i\alpha_j\pi_j^{n-1}}{\gamma(\pi_j^{n-1}\alpha_i + \pi_i^{n-1}\alpha_j)^2}, \quad s_i' = 0, \quad q_i' = \alpha_i r_i'. \tag{11}$$

The cases for $\pi_i < \pi_j$ *are analogous.*

The chosen efforts presented in Proposition 1 would be the Nash equilibrium of the game if the prizes were exogenously given. In this case, our model captures an exogenous prize contest which each scholar's cost of investing in productive effort increases in their winning prize. In loose term, our Stage 2 solutions provide some insights on the desirability of "*high-risk, high reward*" strategies in contests. By choosing a higher target prize, the increased cost increases the risk of participating a contest as the stake becomes higher.

In sum, given π_i and π_j, there can be three cases: (i) both scholars sabotage each other; (ii) one scholar sabotages while the other does not; (iii) neither sabotage each other. The condition that determines whether a scholar would sabotage hinge on c_s being sufficiently low.

We now highlight some interesting observation from Proposition 1.

Corollary 1. *Across all three cases, scholar i exerts higher productive effort than scholar j if and only if* $\pi_i > \pi_j$.

Corollary 2. *When both scholars sabotage,* $s_i^* > s_j^*$ *if and only if* $\pi_i > \pi_j$.

In other words, given (π_i, π_j), the scholar's sabotaging efforts are independent of the difference of their abilities but their chosen target prizes, provided that they are not too far apart as stated in Lemma 1. This result contrasts some of the established results in the literature of sabotaging in exogenous prize contests. It is well-established that the lower-ability contestant often has more incentive to sabotage than the higher-ability scholar (Chen, 2003; Münster, 2007; Gürtler, 2008; Deutscher et al., 2013).

Among these papers, it is particularly worth-discussing the difference between us and Deutscher et al. (2013). In their paper, they assume the marginal return *increases* in the contestant's ability. They too assume that sabotaging and productive efforts are substitutes as in our model. With this assumption, they established that the lower-ability contestants would exert less productive effort and more sabotage. In contrast, we do not assume the return to differ between different-ability scholars, but we assume the *marginal cost* of productive effort to be increasing in a contestant's target prize. We argue that while their assumption suits more in settings in which different-ability contestants are asked to perform the same tasks or tasks with similar difficulties, while our model focuses on the contestants given different tasks. In a combined model that imposes both assumptions, it is likely that we see an intermediate solution between theirs and ours.

Another interesting observation we can draw is about the productive effort exerted by the scholars, especially when sabotage is available versus when it is not. Let p_i^{NS} be the CSF of scholar i if sabotage is not allowed. In this case, even if we have $c_s < \min\left\{\bar{c}_s^i, \bar{c}_s^j\right\}$, the scholars will still choose $p_i^{NS} = p_i'(\pi_i^{NS}, \pi_j^{NS})$ as described in Part 2 of Proposition 1. Similarly for r_i^{NS} and q_i^{NS}. Then by comparing r_i^* and r_i^{NS}, we find that it is possible for the scholars to exert more in productive effort when they sabotage each other than when they do not.

Proposition 2. *Fix some (π_i^{NS}, π_j^{NS}) such that $\pi_i^{NS} > \pi_j^{NS}$ and $c_s < \min\left\{\bar{c}_s^i, \bar{c}_s^j\right\}$. There is an encouragement effect of sabotage for scholar i if and only if $\pi_i > \pi_j$.*

The term *encouragement effect* is defined as the increased incentive to exert productive effort when sabotage is allowed compared to when it is not allowed, holding other factors constant.

The opposite effect, the *discouragement effect* is well-documented in the literature of sabotage in exogenous prize contests, as seen in Chen (2003); Münster (2007); Gürtler (2008); Gürtler and Münster (2010) among others. Proposition 2 then suggests that it is possible to obtain an opposite result when each contestant's productive effort cost is increasing in her target prize for the contestant with the higher winning prize.

To understand this dynamic, note that when $\pi_i > \pi_j$, it is more costly for scholar i to exert productive effort than scholar j. But when sabotages come in play, scholar i has higher incentive to sabotage than scholar j in order to make up for her higher $c_r(\pi_i)$. The more competitive behaviour by scholar i thus decreases scholar j's incentive to compete. This thus further encourages scholar i to be more competitive to maximise her winning chance, and she does so by exerting higher productive effort.[6]

To our best knowledge, Doğan et al. (2019) is the only existing research with a model that features the encouragement effect. In their model, two teams each consisting of two players engage in a contest similar to ours. The collective-effort nature of the model resulted in a solution in which one player in each team specialises in exerting productive effort and the other destructive effort. This specialisation resulted in a higher collective productive effort

[6]Since the marginal benefit from both types of effort is diminishing, it works best for i to increase r_i here rather than keep increasing s_i.

from each team. Our model thus provide an alternative explanation when the contestant's productive effort cost is increasing in her winning prize.

4 Endogenous Target Prizes

Moving to Stage 1, anticipating the Stage 2 equilibrium effort choices, each scholar then chooses her target prize π_i. Given the Stage 2 equilibrium choices, by simple algebra, each scholar's expected utility is given as follows, split in three different regimes:

1. If both scholars choose to sabotage each other, the expected utility function becomes:

$$\begin{aligned} u_i^*(\pi_i, \pi_j) &= \frac{\pi_j^{n-2}\alpha_i}{\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j}\pi_i - \frac{2\alpha_i\alpha_j\pi_i^{n-1}\pi_j^{n-2}}{(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2} + c_s \\ &= p_i^*\pi_i - 2\pi_i p_i^* p_j^* + c_s. \end{aligned} \tag{12}$$

2. If scholar i sabotages but scholar j does not, their expected utility functions are, respectively,

$$\begin{aligned} \tilde{u}_i(\pi_i, \pi_j) &= \pi_i\left(1 - \sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\right)^2 - 2\pi_i\sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\left(1 - \sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\right) + c_s; \quad (13) \\ &= \tilde{p}_i\pi_i - 2\pi_i\tilde{p}_i\tilde{p}_j + c_s \\ \tilde{u}_j(\pi_i, \pi_j) &= \frac{\alpha_j}{\alpha_i}\left(\frac{\pi_i}{\pi_j}\right)^{n-2} c_s. \quad (14) \end{aligned}$$

3. If both scholar choose not to sabotage, the expected utility function becomes:

$$\begin{aligned} u_i'(\pi_i, \pi_j) &= \left(\frac{\pi_j^{n-1}\alpha_i}{\pi_j^{n-1}\alpha_i + \pi_i^{n-1}\alpha_j}\right)^2 \pi_i. \\ &= p_i'^2\pi_i. \end{aligned} \tag{15}$$

To find the SPE of the game, we first fix the scholars to be acting in Stage 2 according to any of the 3 regime listed above, and solve for the Stage 1 equilibrium prizes within each regime. Then, we solve for the condition for which the scholar chooses to be each respective regime. For example, in Stage 1, scholar i solves:

$$\max_{\pi_i} \frac{\pi_j^{n-2}\alpha_i}{\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j}\pi_i - \frac{2\alpha_i\alpha_j\pi_i^{n-1}\pi_j^{n-2}}{(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2} + c_s$$

subject to $c_s \leqslant \min\{\bar{c}_s^i, \bar{c}_s^j\}$, the requirement from Stage 2. Note that when this Stage 2 constraint becomes binding, at least one scholar does not sabotage and the resultant Stage 2 equilibrium transits to the second regime. First, we present the set of (π_i', π_j') that is admissible in any SPE where the scholars do not sabotage and the resultant expected utilities.

Lemma 3. *(π_i', π_j') admits a non-sabotaging SPE such that neither scholar sabotages each other if and only if:*

$$\pi_i^{n-1}\alpha_j = \pi_j^{n-1}\alpha_i \quad \textit{and} \quad c_s < \frac{\min\{\pi_i', \pi_j'\}}{4}. \tag{16}$$

As a result, the CSF of each scholar is always equal in any non-sabotaging SPE: $p_i' = p_j' = 1/2$. The expected utility of each scholar i is given by

$$u_i'(\pi_i', \pi_j') = \frac{\pi_i}{4}.$$

This result implies that in any non-sabotaging SPE, any ability difference is completely mitigated by the choices of effort induced by the target prizes. With Lemma 3, we obtain our main result for Stage 1, and thus the entire game of contest.

Proposition 3. *When the scholars have full control over their target prizes, then neither scholar sabotages in any SPE.*

Proof: To understand the reasoning behind this result, we first inspect regime 1. For a *sabotaging equilibrium* to exist, i.e., if both scholars are to sabotage each other, their expected utility is given by Equation 12. Using the expressions presented in Proposition 1, this can be rewritten in terms of p_i^* and p_j^* as follows:

$$\begin{aligned} u_i^*(p_i^*, p_j^*, \pi_i, \pi_j) &= p_i^*\pi_i - \frac{\gamma\pi_i^n}{n}\frac{np_i^*p_j^*}{\gamma\pi_i^{n-1}} - c_s\left(\frac{\pi_i p_i^* p_j^*}{c_s} - 1\right) \\ &= \pi_i p_i^*(p_i^* - p_j^*) + c_s. \end{aligned} \tag{17}$$

This means that for any (π_i^*, π_j^*), unless it induces $p_i^* = p_j^*$, at least one of the scholar would have the incentive to deviate from these target prizes. When $p_i^* = p_j^* = 1/2$, the resultant expected utility is $u_i^* = c_s$ and the threshold sabotage cost for this regime becomes:

$$\begin{aligned} \bar{c}_s &= \min\left\{\bar{c}_s^i, \bar{c}_s^j\right\} \\ &= \frac{\min\left\{\pi_i^*, \pi_j^*\right\}}{4}. \end{aligned}$$

This means that the Stage 2 requirement for the scholars to sabotage each other directly contradicts the Stage 1 requirement for them to actually prefer sabotaging each other, per Lemma 3. The argument for the non-existence of any *semi-sabotaging equilibrium* is analgous.

First, note that any semi-sabotaging equilibrium requires:

$$\begin{aligned} c_s < \min\{\hat{c}_s^i, \hat{c}_s^j\} &= \frac{\alpha_i \alpha_j \pi_i^{n-1} \pi_j^{n-1} \min\{\pi_i, \pi_j\}}{(\pi_j^{n-1}\alpha_i + \pi_i^{n-1}\alpha_j)^2} \\ &= p_i' p_j' \min\{\pi_i, \pi_j\} \\ &= p_i'(1 - p_i') \min\{\pi_i, \pi_j\}. \end{aligned}$$

It is obvious that the maximised value of $p_i' p_j'$ is 1/4 when $p_i' = p_j'$. Then by the same reasoning, the Stage 1 and Stage 2 requirements for a semi-sabotaging equilibrium contradict each other. ■

Proposition 3 suggests that when a contestant has full control over her target prize of winning the contest, any incentive to sabotage is gone. Intuitively, Equation 17 shows that the only reason the scholars would sabotage lies in gaining an edge over each other in their CSF. But when they have full control over the prizes, by backward induction, when scholar i anticipates herself to be sabotaged by scholar j and now will have a lower CSF than scholar j, she can always adjust her target prize to migitate the CSF difference. This result has a significant welfare implication.

It is obvious that any sabotaging activity is a waste of resources and thus inefficient. Policies introduced by past literature have been focusing on either decreasing the benefits from sabotages or increasing the cost of sabotages. Lazear (1989) proposed that a contest moderator should reduce the gap between winning and losing prizes.

On the former category, Drago and Turnbull (1991) proposed to include a threat of not hosting the contest when the incentive to sabotage becomes large. Experimental results obtained by Charness et al. (2014) suggest that sabotaging could be discouraged if the contestants have reduced information on their performance ranking. On the latter category, Curry and Mongrain (2009) shows how punishing any winner who sabotages other contestants by stripping the prize can mitigate the wrongdoing. This is supported by empirical evidence provided by the likes of Harbring and Irlenbusch (2011) and Vandegrift and Yavas (2011).

In the present paper, not only does the key assumption we imposed that $c_r(\pi_i)$ is increasing in π_i not lower the benefits from sabotaging, it *increases* them – as the target prize increases, the benefits of winning the contest increases along with the cost of exerting productive effort increases. This makes sabotaging becomes even more beneficial.

Our result thus offers a novel way to mitigate sabotages. By letting the contestants to set their own targets, they automatically steer away from any sabotaging activities. When a contestant chooses a higher target prize, the benefit she gets from this is even higher when she chooses *not* to sabotage.

5 Target Prize-adjusted Performance

Throughout this paper, we have been assuming that each scholar's performance is independent of the target prize. This choice of a more conventional performance function allows us to compare directly our results with the existing literature. In this section, we consider an alternative definition of performance, which takes into account the different target prizes. One can think that the different target prizes reflect the *difficulties* of the tasks, and the contest is more rewarding to those who takes on a higher risk with a higher target. Our following results show that it does not make a huge qualitative difference, adjusting performance by target prizes or not.

Formally, let the target-prize adjusted performance be:

$$q_i^A(r_i, s_j, \pi_i) \equiv \frac{\alpha_i \pi_i r_i}{1 + s_j} \tag{18}$$

and the redefined CSF be:

$$p_i^A(q_i^A, q_j^A) \equiv \frac{q_i^A}{q_i^A + q_j^A}. \tag{19}$$

and each scholar solves the following problem:

$$\max_{r_i, s_i} \; u_i^A(r_i, r_j, s_i, s_j, \pi_i) = \frac{q_i^A(r_i, s_j, \pi_i)}{q_i^A(r_i, s_j, \pi_i) + q_j^A(r_j, s_i, \pi_j)} \pi_i - \frac{\gamma \pi_i^n}{n} r_i - c_s s_i. \tag{20}$$

Furthermore, let the relevant threshold sabotage cost for both scholars for this case be:

$$\bar{c}_s^{A,i}(\alpha_i, \alpha_j, \pi_i, \pi_j) \equiv \frac{\alpha_i \alpha_j \pi_i^{n-2} \pi_j^{n-3}}{(\pi_j^{n-3}\alpha_i + \pi_i^{n-3}\alpha_j)^2}; \tag{21}$$

$$\hat{c}_s^{A,i}(\alpha_i, \alpha_j, \pi_i, \pi_j) \equiv \frac{\alpha_i \alpha_j \pi_i^{n-1} \pi_j^{n-2}}{\left(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j\right)}, \tag{22}$$

defined in the similar ways as in Section 3. It is straightforward to state an adjusted version of Lemma 2 as well.

Lemma 4. $\bar{c}_s^{A,j} \leqslant \hat{c}_s^{A,i}$ *if and only if* $\pi_i \geqslant \pi_j$. *It holds for equality if* $\pi_i = \pi_j$.

We then can solve for the Stage 2 equilibrium with the adjusted performance function.

Proposition 4. *Suppose* $\pi_i \geqslant \pi_j$. *If each scholar's performance is adjusted by their target prizes, then given* (π_i, π_j), *the Stage 2 equilibria are given as follows:*

1. *If* $c_s < \bar{c}_s^A$, *the CSF, chosen efforts, and resultant performance of each scholar i are given*

by:

$$p_i^{A*} = \frac{\pi_j^{n-3}\alpha_i}{\pi_j^{n-3}\alpha_i + \pi_i^{n-3}\alpha_j}, \quad r_i^{A*} = \frac{n\alpha_i\alpha_j\pi_j^{n-4}}{\gamma\pi_i(\pi_j^{n-3}\alpha_i + \pi_i^{n-3}\alpha_j)^2},$$
$$s_i^* = \frac{\alpha_i\alpha_j\pi_i^{n-2}\pi_j^{n-3}}{c_s(\pi_j^{n-3}\alpha_i + \pi_i^{n-3}\alpha_j)^2} - 1, \quad q_i^* = \frac{n\alpha_i c_s}{\gamma\pi_i^{n-2}\pi_j}. \tag{23}$$

2. *If* $c_s \in \left[\bar{c}_s^{A,j}, \hat{c}_s^{A,i}\right)$*, the CSF, chosen efforts, and resultant performance of each scholar i are given by:*

$$\tilde{p}_i^A = 1 - \sqrt{\frac{\pi_i^{n-3}\alpha_j c_s}{\pi_j^{n-2}\alpha_i}}, \quad \tilde{r}_i^A = \frac{n\tilde{p}_i^A\tilde{p}_j^A}{\gamma\pi_i^{n-2}}, \quad \tilde{s}_i^A = \frac{\pi_i\tilde{p}_i^A\tilde{p}_j^A}{c_s} - 1, \quad \tilde{q}_i^A = \frac{n\alpha_i\tilde{p}_i^A\tilde{p}_j^A}{\gamma\pi_i^{n-2}}.$$
$$\tilde{p}_j^A = \sqrt{\frac{\pi_i^{n-3}\alpha_j c_s}{\pi_j^{n-2}\alpha_i}}, \quad \tilde{r}_j^A = \frac{n\tilde{p}_i^A\tilde{p}_j^A}{\gamma\pi_j^{n-1}}, \quad \tilde{s}_j^A = 0, \quad \tilde{q}_j^A = \frac{n\alpha_j c_s}{\gamma\pi_i\pi_j^{n-2}}. \tag{24}$$

3. *If* $c_s \geqslant \hat{c}_s^{A.i}$*, the chosen efforts and resultant performances for each* $i = 1, 2$ *are given by*

$$p_i^{A\prime} = \frac{\pi_j^{n-2}\alpha_i}{\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j}, \quad r_i^{A\prime} = \frac{n\alpha_i\alpha_j\pi_j^{n-2}}{\gamma\pi_i(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2}, \quad s_i^{A\prime} = 0, \quad q_i^{A\prime} = \alpha_i\pi_i r_i^{A\prime}. \tag{25}$$

The proof of Proposition 4 follows the exact same stpes as Proposition 1, and is thus omitted.

The first thing we observe from Proposition 4 is that with the adjusted performance function, the nature of the solution does not change much. The expression of each equilibrium variable has roughly the same form, but the power attached to each π_i and π_j term in each equilibrium variable is decreased by 1 as an adjustment to the additional π_i in q_i^A. It is ambiguous whether or not each equilibrium variable is now greater or smaller than the original counterpart, as it depends on whether π_i and π_j are greater or smaller than 1. However, it is obvious that our main Stage 2 finding on the encouragement effect importantly remains true.

Moving on to Stage 1, we obtain a similar no-sabotage result as before.

Proposition 5. *If each scholar's performance is adjusted by their target prizes, no scholar sabotages in an SPE.*

The logic behind this proposition is the same as in Proposition 3. However, since scholar i's expected utility in a sabotaging equilibrium is now

$$u_i^A(p_i^{A*}, p_j^{A*}, \pi_i, \pi_j) = \pi_i p_i^{A*}\left(p_i^{A*} - p_j^{A*}\frac{\pi_i}{\pi_j}\right) + c_s,$$

the sabotaging equilibrium now requires $p_i^* = p_j^* \frac{\pi_i}{\pi_j}$. This means the resultant expected utility in a sabotaging equilibrium is always c_s. Similarly, we can also compute the expected utility of the scholars in a non-sabotaging equilibrium:

$$\begin{aligned} u_i^{A'}(p_i^{A'}, p_j^{A'}, \pi_i, \pi_j) - p_i^{A'}\pi_i - \frac{\gamma\pi_i^n}{n} r_i^{A'} \\ = p_i^{A'}\pi_i(1 - p_j^{A'}), \end{aligned}$$

which is maximised at $p_i^{A'} = p_j^{A'} = 1/2$, and $u_i^A(1/2, 1/2, \pi_i, \pi_j) = \frac{\pi_i}{4}$.

Given the choices of (π_i, π_j), the Stage 2 requirement for a sabotaging equilibrium is therefore

$$\begin{aligned} \bar{c}_s^A &\leqslant \min\left\{\bar{c}_s^{A,i}, \bar{c}_s^{A,j}\right\} \\ &= p_i^{A*} p_j^{A*} \min\{\pi_i, \pi_j\} \end{aligned}$$

As before, the RHS is maximised when $p_i^{A*} = p_j^{A*} = 1/2$. But even then, we obtain the same contradiction between Stage 1 and Stage 2 requirements for a sabotaging equilibrium.

In conclusion, we show that our main results are robust in this variation which each scholar's performance is adjusted by their target prize.

6 Conclusion

This paper studies a two-player asymmetric target prize contest with sabotage. We show that when the contestants have full control over their winning prize (subject to its impact on the marginal cost of productive effort), no contestant sabotages in any SPE. This result is robust to any level of convexity within the specified functional form of $c_r(\pi_i)$, as well as an alternative definition of *performance* that takes into account the target prize.

Our findings rely on the assumption that each contestant has full control over her winning prize. If this assumption does not hold, it is possible that the scholars end up at some target prizes such that it is optimal for them to sabotage at Stage 2 instead. And when that is true, we show that there is an encouragement effect on productive effort for the scholar who has set a higher target. Indeed, in real-life situation, contestants are unlikely to be able to assume full control on their targets. In the story of the two scholars, the target prize could be a result of discussions with the lab manager during periodic reviews sessions. The present paper thus provides a benchmark of the extreme case when the lab manager has no control over the prizes. For future research, it would be interesting to study an extended model that fully explores the role of the lab manager, for example, an incentive scheme designed by the lab manager to maximise the overall performance of the scholars.

A Proofs

A.1 Proofs of Proposition 1 and Lemma 1

We begin by solving for the first-order conditions to prove Proposition 1, then examine the second-order conditions to prove Lemma 1.

Part 1: The first-order conditions of scholar i's problem stated in Equation 5 are given by:

$$0 = \frac{q_j \pi_i}{(q_i + q_j)^2} \frac{\alpha_i}{1 + s_j} - \frac{\gamma}{n} \pi_i^n$$
$$0 = \frac{q_i \pi_i}{(q_i + q_j)^2} \frac{\alpha_j r_j}{(1 + s_i)^2} - c_s.$$

Simple algebra allows to re-arrange the conditions into:

$$1 + s_j = \frac{n \alpha_i q_j}{\gamma \pi_i^{n-1} (q_i + q_j)^2}; \qquad (\mathrm{FOC}_i^r)$$
$$1 + s_i = \frac{q_i q_j \pi_i}{c_s (q_i + q_j)^2}. \qquad (\mathrm{FOC}_i^s)$$

By symmetry, we also have:

$$1 + s_i = \frac{n \alpha_j q_i}{\gamma \pi_j^{n-1} (q_i + q_j)^2}; \qquad (\mathrm{FOC}_j^r)$$
$$1 + s_j = \frac{q_i q_j \pi_j}{c_s (q_i + q_j)^2}. \qquad (\mathrm{FOC}_j^s)$$

Dividing FOC_i^s by FOC_j^s and dividing FOC_j^r by FOC_i^r

$$\frac{1 + s_i}{1 + s_j} = \frac{\pi_i}{\pi_j} = \frac{\alpha_j q_i \pi_i^{n-1}}{\alpha_i q_j \pi_j^{n-1}}$$
$$\Rightarrow \frac{q_i}{q_j} = \frac{\pi_j^{n-2} \alpha_i}{\pi_i^{n-2} \alpha_j}.$$

Similarly, dividing FOC_j^r by FOC_i^s gives:

$$\frac{1 + s_i}{1 + s_i} = \frac{n \alpha_j q_i}{\gamma \pi_j^{n-1} (q_i + q_j)^2} \frac{c_s (q_i + q_j)^2}{q_i q_j \pi_i}$$
$$\Rightarrow q_j^* = \frac{n \alpha_j c_s}{\gamma \pi_i \pi_j^{n-1}}.$$

Likewise, we have

$$q_i^* = \frac{n \alpha_i c_s}{\gamma \pi_i^{n-1} \pi_j}.$$

Thus we have:

$$p_i^* = \frac{q_i^*}{q_i^* + q_j^*} = \frac{\pi_j^{n-2}\alpha_i}{\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j}.$$

Plugging q_i^*, and q_j^* back into FOC_j^r, we get:

$$\begin{aligned} 1 + s_i^* &= \frac{n^2\alpha_i\alpha_j c_s}{\gamma^2\pi_i^{n-1}\pi_j^{n-2}} \frac{1}{\left(\frac{n\alpha_j c_s}{\gamma\pi_i\pi_j^{n-1}} + \frac{n\alpha_i c_s}{\gamma\pi_i^{n-1}\pi_j}\right)^2} \\ &= \frac{\alpha_i\alpha_j\pi_i^{n-1}\pi_j^{n-2}}{c_s(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2}. \end{aligned}$$

By symmetry, we also have

$$1 + s_j^* = \frac{\alpha_i\alpha_j\pi_i^{n-2}\pi_j^{n-1}}{c_s(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2}.$$

We can then also pin down the productive efforts:

$$\begin{aligned} r_i^* &= \frac{q_i^*(1 + s_j^*)}{\alpha_i} \\ &= \frac{n\alpha_i c_s}{\gamma\alpha_i\pi_i^{n-1}\pi_j} \frac{\alpha_i\alpha_j\pi_i^{n-2}\pi_j^{n-1}}{c_s(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2} \\ &= \frac{n\alpha_i\alpha_j\pi_j^{n-2}}{\gamma\pi_i(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2}. \end{aligned}$$

Finally, we identify the condition for this case to be the Stage 2 equilibrium, which requires $s_i > 0$:

$$\begin{aligned} s_i^* > 0 &\Leftrightarrow \frac{\alpha_i\alpha_j\pi_i^{n-1}\pi_j^{n-2}}{c_s(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2} > 1 \\ &\Leftrightarrow c_s < \frac{\alpha_i\alpha_j\pi_i^{n-1}\pi_j^{n-2}}{(\pi_j^{n-2}\alpha_i + \pi_i^{n-2}\alpha_j)^2} \equiv \bar{c}_s^i. \end{aligned} \tag{26}$$

This completes the proof of Part 1 of Proposition 1, providing that the second-order condition holds.

Part 2: when Equation 26 is not satisfied for one of the scholar, she would optimally choose not to exert both types of efforts. It is obvious that a scholar would never exert destructive effort without also exerting productive effort. Let this scholar be scholar j. Then the FOCs from both scholars' maximisation problems are:

$$1 = \frac{n\alpha_i q_j}{\gamma \pi_i^{n-1}(q_i + q_j)^2}; \qquad (\text{FOC}_i^r)$$

$$1 + s_i = \frac{q_i q_j \pi_i}{c_s(q_i + q_j)^2}.; \qquad (\text{FOC}_i^s)$$

$$1 + s_i = \frac{n\alpha_j q_i}{\gamma \pi_j^{n-1}(q_i + q_j)^2}. \qquad (\text{FOC}_j^r)$$

By FOC_i^s by FOC_j^r gives:

$$\tilde{q}_j = \frac{n\alpha_j c_s}{\gamma \pi_i \pi_j^{n-1}},$$

which is the same as in Part 1. Plug $\tilde{q}_j$ in FOC_i^r gives:

$$\begin{aligned}
(\tilde{q}_i + \tilde{q}_j)^2 &= \frac{n\alpha_i}{\gamma \pi_i^{n-1}} \frac{n\alpha_j c_s}{\gamma \pi_i \pi_j^{n-1}} \\
&= \frac{n^2 \alpha_i \alpha_j c_s}{\gamma^2 \pi_i^n \pi_j^{n-1}} \\
\Rightarrow \tilde{p}_j &= \frac{\tilde{q}_j}{\tilde{q}_i + \tilde{q}_j} = \frac{\frac{n\alpha_j c_s}{\gamma \pi_i \pi_j^{n-1}}}{\sqrt{\frac{n^2 \alpha_i \alpha_j c_s}{\gamma^2 \pi_i^n \pi_j^{n-1}}}} \\
&= \sqrt{\frac{\pi_i^{n-2} \alpha_j c_s}{\pi_j^{n-1} \alpha_i}}.
\end{aligned}$$

Then we also have:

$$\begin{aligned}
1 + \tilde{s}_i &= \frac{\tilde{q}_i \tilde{q}_j \pi_i}{c_s(\tilde{q}_i + \tilde{q}_j)^2} = \frac{\pi_i \tilde{p}_i \tilde{p}_j}{c_s} \\
&= \frac{\pi_i}{c_s} \sqrt{\frac{\pi_i^{n-2} \alpha_j c_s}{\pi_j^{n-1} \alpha_i}} \left(1 - \sqrt{\frac{\pi_i^{n-2} \alpha_j c_s}{\pi_j^{n-1} \alpha_i}}\right)
\end{aligned}$$

and

$$
\begin{aligned}
\tilde{q}_i &= \tilde{p}_i(\tilde{q}_i + \tilde{q}_j) \\
&= \left(1 - \sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\right)\sqrt{\frac{n^2\alpha_i\alpha_j c_s}{\gamma^2\pi_i^n\pi_j^{n-1}}} \\
&= \frac{n\alpha_i}{\gamma\pi_i^{n-1}}\left(1 - \sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\right)\sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}} \\
&= \frac{n\alpha_i}{\gamma\pi_i^{n-1}}\tilde{p}_i\tilde{p}_j.
\end{aligned}
$$

Then we can back out $\tilde{r}_i$ and $\tilde{r}_j$:

$$
\begin{aligned}
\tilde{r}_i &= \frac{\tilde{q}_i}{\alpha_i} \\
&= \frac{n}{\gamma\pi_i^{n-1}}\tilde{p}_i\tilde{p}_j \\
&= \frac{n}{\gamma\pi_i^{n-1}}\left(1 - \sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\right)\sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}} \\
\tilde{r}_j &= \frac{\tilde{q}_j(1+\tilde{s}_i)}{\alpha_j} \\
&= \frac{n\tilde{p}_i\tilde{p}_j}{\gamma\pi_j^{n-1}} \\
&= \frac{n}{\gamma\pi_j^{n-1}}\left(1 - \sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\right)\sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}.
\end{aligned}
$$

Finally, we again identify the condition on c_s for this case to be equilibrium, which requires

$\tilde{s}_i$ to be positive:

$$\begin{aligned}
\tilde{s}_i > 0 \Leftrightarrow & \frac{\pi_i}{c_s}\sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\left(1-\sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}}\right) > 1 \\
\Leftrightarrow & c_s < \pi_i\sqrt{\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i}} - \pi_i\frac{\pi_i^{n-2}\alpha_j c_s}{\pi_j^{n-1}\alpha_i} \\
\Leftrightarrow & c_s\left(1+\frac{\pi_i^{n-1}\alpha_j}{\pi_j^{n-1}\alpha_i}\right) < \pi_i\sqrt{\frac{\pi_i^{n-2}\alpha_j}{\pi_j^{n-1}\alpha_i}}\sqrt{c_s} \\
\Leftrightarrow & \sqrt{c_s} < \frac{\pi_i\sqrt{\frac{\pi_i^{n-2}\alpha_j}{\pi_j^{n-1}\alpha_i}}}{1+\frac{\pi_i^{n-1}\alpha_j}{\pi_j^{n-1}\alpha_i}} \\
\Leftrightarrow & c_s < \frac{\pi_i^2\frac{\pi_i^{n-2}\alpha_j}{\pi_j^{n-1}\alpha_i}}{\left(1+\frac{\pi_i^{n-1}\alpha_j}{\pi_j^{n-1}\alpha_i}\right)^2} \\
\Leftrightarrow & c_s < \frac{\alpha_i\alpha_j\pi_i^n\pi_j^{n-1}}{(\pi_j^{n-1}\alpha_i+\pi_i^{n-1}\alpha_j)^2} \equiv \hat{c}_s^i.
\end{aligned}$$

That is, we require

$$\frac{\alpha_i\alpha_j\pi_i\pi_j^2}{(\alpha_i+\alpha_j)^2} \leqslant c_s < \frac{\alpha_i\alpha_j\pi_i^2\pi_j}{(\pi_j\alpha_i+\alpha_j\pi_i)^2}.$$

This completes the proof of Part 2 of the proposition, providing that the SOC holds.

Part 3: If c_s does not satisfy either Part 1 or Part 2, it must mean that neither scholar has incentive to sabotage each other. Thus, they exert productive effort only. The FOCs are:

$$1 = \frac{n\alpha_i q_j}{\gamma\pi_i^{n-1}(q_i+q_j)^2}; \tag{FOC$_i^r$}$$

$$1 = \frac{n\alpha_j q_i}{\gamma\pi_j^{n-1}dc74(q_i+q_j)^2}. \tag{FOC$_j^r$}$$

Equating them gives:

$$q_j = \frac{\alpha_j\pi_i^{n-1}}{\alpha_i\pi_j^{n-1}}q_i.$$

Plug this back into the FOC gives:

$$\begin{aligned}
q_i^2\left(1+\frac{\alpha_j\pi_i^{n-1}}{\alpha_i\pi_j^{n-1}}\right)^2 &= \frac{n\alpha_j q_i}{\gamma\pi_j} \\
\Rightarrow q_i' &= \frac{n\alpha_i^2\alpha_j\pi_j^{n-1}}{\gamma(\pi_j^{n-1}\alpha_i+\pi_i^{n-1}\alpha_j)^2}.
\end{aligned}$$

By symmetry, we also get:

$$q_j' = \frac{n\alpha_i\alpha_j^2\pi_i^{n-1}}{\gamma(\pi_j^{n-1}\alpha_i + \pi_i^{n-1}\alpha_j)^2}$$

Thus we have

$$r_i' = \frac{q_i'}{\alpha_i} = \frac{n\alpha_i\alpha_j\pi_j^{n-1}}{\gamma(\pi_j^{n-1}\alpha_i + \pi_i^{n-1}\alpha_j)^2};$$
$$r_j' = \frac{q_j'}{\alpha_j} = \frac{n\alpha_i\alpha_j\pi_i^{n-1}}{\gamma(\pi_j^{n-1}\alpha_i + \pi_i^{n-1}\alpha_j)^2}.$$

This completes the proof of Part 3 and thus the whole proposition, conditioning on the second-order condition. For Part 3, we can easily see that the second-order condition is satisfied, as

$$\frac{\partial^2 u_i}{\partial r_i^2} = -\frac{2q_j\alpha_i^2\pi_i}{(q_i+q_j)^2} < 0.$$

The same can be said for the non-sabotaging scholar j in Part 2.

We now turn to Lemma 1 to show that the SOC indeed holds in Part 1 and in Part 2 for the sabotaging scholar i. The second derivatives of scholar i's maximisation problem are given by:

$$\begin{aligned}
\frac{\partial^2 u_i}{\partial r_i^2} &= -\frac{2q_j}{(q_i+q_j)^2}\frac{\alpha_i^2\pi_i}{(1+s_i)^2} < 0; \\
\frac{\partial^2 u_i}{\partial s_i^2} &= \frac{2q_iq_j^2}{(q_i+q_j)^3}\frac{\pi_i}{(1+s_i)^2} - \frac{2q_iq_j\pi_i}{(1+s_i)^3}\frac{q_i\pi_i}{(q_i+q_j)^2} \\
&= -\frac{2q_i^2q_j\pi_i}{(q_i+q_j)^3(2+s_i)^2} < 0; \\
\frac{\partial^2 u_i}{\partial r_i \partial s_i} &= \frac{\alpha_i\pi_i}{1+s_j}\left(\frac{q_i-q_j}{(q_i+q_j)^3}\right)\frac{\alpha_j r_j}{(1+s_i)^2} \\
&= \frac{\alpha_i\pi_i}{(1+s_i)(1+s_j)}\frac{q_j(q_i-q_j)}{(q_i+q_j)^3}.
\end{aligned}$$

From the first two we can see that the first principle minor of the Hessian matrix is negative. It remains to be shown that the determinant of the Hessian matrix, i.e., the second principle minor, is positive.

$$\begin{aligned}\Delta &= \frac{2q_j}{(q_i+q_j)^2}\frac{\alpha_i^2\pi_i}{(1+s_i)^2}\frac{2q_i^2q_j\pi_i}{(q_i+q_j)^3(2+s_i)^2} - \frac{\alpha_i^2\pi_i^2}{(1+s_i)^2(1+s_j)^2}\frac{q_j^2(q_i-q_j)^2}{(q_i+q_j)^6}\\ &= \frac{4q_i^2q_j^2\alpha_i^2\pi_i^2}{(q_i+q_j)^6(1+s_i)^2(1+s_j)^2} - \frac{q_j^2(q_i-q_j)^2\alpha_i\pi_i}{(q_i+q_j)^6(1+s_i)^2(1+s_j)^2}\\ &= \frac{q_j^2\alpha_i^2\pi_i^2}{(q_i+q_j)^6(1+s_i)^2(1+s_j)^2}\Big(4q_i^2-(q_i-q_j)^2\Big).\end{aligned}$$

This is weakly positive if and only if

$$\begin{aligned}&4q_i^2 \geqslant (q_i-q_j)^2\\ \Leftrightarrow &2q_i > |q_i-q_j|\\ \Leftrightarrow &q_i \in \left[\frac{q_j}{3}, 3q_j\right].\end{aligned}$$

Recall that in Part 1,

$$\frac{q_i^*}{q_j^*} = \frac{n\alpha_i c_s}{\gamma\pi_i^{n-1}\pi_j}\frac{\gamma\pi_i\pi_j^{n-1}}{n\alpha_j c_s} = \frac{\pi_i^{n-2}\alpha_i}{\pi_j^{n-2}\alpha_j},$$

the above condition thus becomes

$$\frac{\pi_i^{n-2}\alpha_i}{\pi_j^{n-2}\alpha_j} \in [1/3, 3].$$

This completes the proof of Lemma 1. ■

A.2 Proofs of Corollary 1, Corollary 2, and Proposition 2

Proofs of Corollary 1 and Corollary 2: To prove the corollaries, one simply needs to observe the following:

$$\begin{aligned}r_i^* &= \frac{np_i^*p_j^*}{\gamma\pi_i^{n-1}},\\ \tilde{r}_i &= \frac{n\tilde{p}_i^*\tilde{p}_j^*}{\gamma\pi_i^{n-1}},\\ r_i' &= \frac{np_i'p_j'}{\gamma\pi_i^{n-1}}\end{aligned}$$

for both $i = 1, 2$. Similarly, we also have:

$$\begin{aligned} s_i^* &= \frac{\pi_i p_i^* p_j^*}{c_s} - 1, \\ \tilde{s}_i^* &= \frac{\pi_i \tilde{p}_i \tilde{p}_j}{c_s} - 1, \\ \bar{c}_s^i &= \pi_i p_i^* p_j^*, \\ \hat{c}_s^i &= \pi_i p_i' p_j'. \quad \blacksquare \end{aligned}$$

Proof of Proposition 2: First, note that r_i^{NS} takes the same functional form as r_i'. Then, comparing the two productive effort, we have

$$\begin{aligned} r_i^* > r_i^{NS} \Leftrightarrow & \frac{n\alpha_i\alpha_j \left(\pi_j^{NS}\right)^{n-2}}{\gamma \pi_i^{NS} \left(\left(\pi_j^{NS}\right)^{n-2} + \left(\pi_i^{NS}\right)^{n-2} \alpha_j\right)^2} > \frac{n\alpha_i\alpha_j \left(\pi_j^{NS}\right)^{n-1}}{\gamma \left(\left(\pi_j^{NS}\right)^{n-1} \alpha_i + \left(\pi_i^{NS}\right)^{n-1} \alpha_j\right)^2} \\ \Leftrightarrow & \frac{\left(\left(\pi_j^{NS}\right)^{n-1} \alpha_i + \left(\pi_i^{NS}\right)^{n-1} \alpha_j\right)^2}{\pi_i^{NS}\pi_j^{NS}} > \left(\left(\pi_j^{NS}\right)^{n-2} + \left(\pi_i^{NS}\right)^{n-2} \alpha_j\right)^2 \\ \Leftrightarrow & \left(\pi_j^{NS}\right)^{n-1} \alpha_i + \left(\pi_i^{NS}\right)^{n-1} \alpha_j > \sqrt{\pi_i^{NS}\pi_j^{NS}} \left(\left(\pi_j^{NS}\right)^{n-2} \alpha_i + \left(\pi_i^{NS}\right)^{n-2} \alpha_j\right) \\ \Leftrightarrow & \left(\pi_i^{NS}\right)^{n-2} \alpha_j \left(\pi_i^{NS} - \sqrt{\pi_i^{NS}\pi_j^{NS}}\right) > \left(\pi_j^{NS}\right)^{n-2} \alpha_i \left(\sqrt{\pi_i^{NS}\pi_j^{NS}} - \pi_j^{NS}\right) \\ \Leftrightarrow & \pi_i^{NS} > \pi_j^{NS}. \quad \blacksquare \end{aligned}$$

A.3 Proof of Lemma 3

At Stage 1 regime 3, each worker solves:

$$\max_{\pi_i} \; u_i'(\pi_i, \pi_j, p_i', p_j') = p_i'^2 \pi_i$$

The FOC is given by:

$$\begin{aligned} 0 = \frac{\partial u_i'}{\partial \pi_i} &= 2 p_i \pi_i \frac{\partial p_i'}{\partial \pi_i} + p_i^2 \\ &= -2\pi_i \frac{\pi_j^{2(n-1)} \alpha_i^2}{\pi_j^{n-1}\alpha_i + \pi_j^{n-1}\alpha_j} \frac{\pi_i^{n-2}\alpha_j(n-2)}{(\pi_j^{n-1}\alpha_i + \pi_j^{n-1}\alpha_j)^2} + \left(\frac{\pi_j^{n-1}\alpha_i}{\pi_j^{n-1}\alpha_i + \pi_j^{n-1}\alpha_j}\right) \\ &= p_i^2 \left(1 - 2(n-1) p_j\right) \\ \Leftrightarrow p_j' &= \frac{1}{2(n-1)}. \end{aligned}$$

Then by symmetry, scholar j's FOC requires:

$$p_i = \frac{1}{2(n-1)} = p_j.$$

Thus, in equilibrium we must have $p_i' = p_j' = 1/2$ and

$$\pi_i^{n-1}\alpha_j = \pi_j^{n-1}\alpha_i.$$

The resultant expected utility is thus:

$$u_i' = \left(\frac{1}{2}\right)^2 \pi_i = \frac{\pi_i}{4}. \quad \blacksquare$$

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