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The effect of stock splits on liquidity in a dynamic model

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Abstract

We develop a dynamic framework to detect the occurrence of permanent and transitory breaks in the illiquidity process. We propose various tests that can be applied separately to individual events and can be aggregated across different events over time for a given firm or across different firms. In an empirical study, we use this methodology to study the impact of stock splits on the illiquidity dynamics of the Dow Jones index constituents and the effects of reverse splits using stocks from the S&P 500, S&P 400 and S&P 600 indices. Our empirical results show that stock splits have a positive and significant effect on the permanent component of the illiquidity process while a majority of the stocks engaging in reverse splits experience an improvement in liquidity conditions.

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1 Introduction

One commonly held market explanation for why companies split their stock is the theory that this creates "wider" markets.[1] Reducing the price level should make it easier for a bigger pool of retail investors to buy into the stock and allows existing investors to sell part of their holdings more easily to other investors thereby increasing the investor base and the volume of transactions. This in turn should lead to improved liquidity conditions and reduce the cost of capital to companies. However, there are other theoretical arguments presented in Copeland (1979) that may point to a decrease in liquidity following a stock split, such as increases in real transaction costs.[2] In his empirical study, Copeland found the following: nonstationarities in trading behavior, volume increases less than proportionately, brokerage revenues increase, and increases in proportional bid-ask spreads following stock splits, i.e., liquidity worsened following stock splits on average. We will evaluate this latter finding with updated data and a different statistical method.

Many of the empirical studies about stock splits distinguish between short-term and long-term effects but do so in an informal way statistically. We use a recently developed time series model, Hafner et al. (2023) henceforth HLW, to capture this distinction formally. We base our analysis on the popular liquidity measure proposed by Amihud (2002), but rather than aggregating over daily measures as in the usual approach, we use the disaggregated daily measures directly. Our model is a member of the class of multiplicative error models (MEM) that have been applied to many different positive-valued financial time series including volatility, duration between trades, and transaction volume, see e.g. Engle (2002). We allow for a nonparametric time trend to capture the secular improvement in liquidity that has happened for most stocks from the 1960s to the present day. The MEM model and its applications and developments over the last 20 years are reviewed in Cipollini and Gallo (2022). Within our econometric model, we test the following hypotheses

$\mathbf{H}_0^1$: *Stock splits have no permanent effect on the level of liquidity against the*

[1] Announcing their 4 for 1 split in June of 2020, the Apple Board of Directors gave the reason " to make the stock more accessible to a broader base of investors."

[2] As Weld et al. (2009) say "commissions paid by investors on trading ten \$35 shares are about ten times those paid on a single \$350 share". Nowadays, trading costs for retail traders would not scale in this way but nevertheless the total costs would be higher.

alternative that they do have a permanent effect

$\mathbf{H}_0^2$: *Stock splits have no additional temporary effect on the level of liquidity against the alternative that they do have a temporary effect*

We consider individual event specific tests and tests that pool across different splits for the same firm and tests that pool also across firms. Our test of $\mathbf{H}_0^1$ is similar to that used in the regression discontinuity literature and the structural break literature - we look for discrepancy between forward looking trend estimates and backward looking trend estimates. Our test of $\mathbf{H}_0^2$ are designed in the spirit of event studies as e.g. in Fama et al. (1969), i.e., we look at abnormal liquidity and cumulative abnormal liquidity and determine whether these quantities are consistent with the null of no change. Our tests are robust to the presence of a permanent break and are looking additionally for short-term adjustments to the new level of liquidity. Since we focus on liquidity rather than firm valuation, we concentrate on the post split effects rather than the post announcement effects, although our test statistic are computed in some cases over periods including the announcement as well as the split itself.

We next discuss our empirical results. Our results broadly support findings in Copeland (1979) using a more recent sample of Dow Jones index component stocks. Specifically, we document that stock splits cause significant shifts in the long-term illiquidity trend while no additional significant effects on short-run liquidity dynamics are detected. This seems to suggest that the market quickly adjusts to the new pricing regime. The change in the long-term trend of illiquidity is predominantly positive, implying that liquidity conditions deteriorate in the long run after stock splits. We also investigate whether the effects of reverse stock splits on the illiquidity process are symmetric to the ones documented for stock splits. Our empirical study uses low-price stocks from the constituents of the S&P 500, S&P 400 and S&P 600 indices with a reverse split during the past 30 years. The results suggest that a majority of the low-price stocks engaging in reverse splits experience an improvement in liquidity conditions. However, we find limited effects on the short-run illiquidity dynamics. The fact that reverse splits result in a significant decrease in the illiquidity trend for the majority of those stocks is in line with the results in Han (1995). The pronounced improvement in liquidity conditions for our sample of stocks with low pre-event price levels is consistent with the evidence in Blau et al. (2023). This could be linked to the fact that short-selling activity increases after reverse splits in part because reverse splits ease the constraints on short selling for low-priced stocks (Kwan et al. (2015)).

Literature Review. There is a substantial literature on the effect of stock splits on firm valuation starting with Dolley (1933) who studied stock splits between 1921-1931 and found (split-adjusted) price increases around the time of the split. Fama et al. (1969) introduced a new methodology based on the market model for stock returns that we now call event study and applied it to 940 splits between 1927-1959. They argued that Dolley and other studies used windows that were too short and consequently did not control for the price appreciation trend established prior to the split decision and they find that most of the effect occurs before the split itself consistent with this sample selection interpretation: firms that take the decision to split their stock tend to have had a period of high price appreciation prior to their decision. Other studies following Fama et al. (1969) did find significant short-term/long-term effects on firm valuation. For example, Ikenberry et al. (1996) found a significant post-split excess return of 7.93 percent in the first year and 12.15 percent in the first three years for a sample of 1,275 two-for-one stock splits. These excess returns followed an announcement return of 3.38 percent, indicating that the market underreacts to split announcements and takes time to fully impound the event into prices. Weld et al. (2009) discuss reasons for why companies split their stock. These include: signaling managements confidence about the future, optimal trading ranges for retail investors, and optimal real tick size for market making. However, they find that none of the existing theories are able to explain the observed constant nominal stock prices over a large part of the 20th century (in contrast with the CPI, which went up manyfold over the same period). They conclude with the suggestion that the evidence is consistent with the idea that only customs and norms can explain the nominal price puzzle.[3]

Although the early focus of the empirical work was on firm valuation, there has been

[3] Since the time period considered by that article, it appears that some of the facts have changed regarding stock splits. Specifically, the frequency of stock splits has fallen considerably up to the present day, with some recent notable exceptions in the tech industry. Reflecting this reduction in the frequency of splits, the average price of large caps (S&P500) has increased from around $50 in 2000 to more than $200 in 2022, Mackintosh (2022).

a lot of subsequent work looking at other outcome variables. Ikenberry and Ramnath (2015) find that the stock market underreacts to the announcement of stock splits and report an abnormal return of 9% in the year following the announcement. Using specific oversampling techniques, Li et al. (2022) show that constructing portfolios from machine learning predictions of stock splits leads to abnormal monthly returns of up to 0.95%. Ohlson and Penman (1985) examine stock return volatilities before and after the ex-dates of stock splits. They find an increase of approximately 30% in the return standard deviations following the ex-date. This holds for daily and weekly returns and persists for a long while. Lamoureux and Poon (1987) find that of 215 splits, eighty-seven showed a statistically significant drop in split-adjusted, market-adjusted average daily volume, while twenty-seven exhibited a significant increase. Of forty-nine reverse splits, fifteen exhibit a statistically significant increase and two exhibit a significant decrease in split-adjusted, market-adjusted volume. Lakonishok and Lev (1987) report that trading volume temporarily increased on announcement day and decreased after the split announcement. Consistent with this, Huang et al. (2015) find that there is a liquidity improvement on announcement day, as well as in the period between announcement day and execution date. However, the liquidity declined after the Ex-date to the level before the announcement. The authors concluded that liquidity improvement is a short-lived effect of stock splits. On the other hand, other researchers (Chern et al. (2008); Guo et al. (2008); Yu and Webb (2009)) found that stock splits reduce bid-ask spreads, and increase the number of small traders who are attracted to the lower price on Ex-date, indicating liquidity improvement. Mohanty and Moon (2007) also found a significant improvement in the average trading volume, comparing 12 months post splits announcement with that for prior to announcements. Han (1995) studied the effect of reverse splits on liquidity using bid-ask spread, trading volume, and the number of non-trading days as liquidity proxies. He finds that liquidity significantly improves after reverse splits relative to a control group matched on industry, size, and share price. Lin et al. (2009) find contrarily that following forward stock splits a nontrading type of liquidity proxy improves. The upshot of this literature is that the evidence of the impact of stock splits on company-specific liquidity is, to this date, inconclusive.

The remainder of the paper is organized as follows. The following section recalls the Dynamic Autoregressive Liquidity (DArLiq) model introduced in HLW and defines a measure for the permanent effect. Section 3 presents the econometric methodology including estimation of the dynamic model and various tests of permanent and temporary effects. Sections 4 and 5 present an empirical application where we use our framework to analyze the effect of stock splits and reverse splits on the long-run trend and short-run illiquidity dynamics. Section 6 concludes. Some additional figures for the empirical analysis are collected in Appendix A.

2 The dynamic model for illiquidity

The daily illiquidity quantity is defined as $\ell_t = |R_t|/V_t$, where R is stock return and V is the dollar trading volume, which are both observable at the daily frequency. We suppose that this non-negative time series follows a multiplicative dynamic stochastic process

$$\ell_t = g(t/T)\lambda_t\zeta_t \tag{1}$$

$$B(L)\lambda_t = \omega + C(L)\ell^*_{t-1}, \tag{2}$$

where $g(.)$ is a positive but unknown function of rescaled time while $\ell^*_t = \ell_t/g(t/T) = \lambda_t\zeta_t$ is the rescaled liquidity, and ζ_t is a sequence of non-negative random variables with conditional mean one and finite unconditional variance denoted by σ^2_ζ. We here consider the special case where the lag polynomials satisfy $B(L) = 1 - \beta L$ and $C(L) = \gamma$, where $\beta, \gamma > 0$ and $\beta + \gamma < 1$ in which case the process λ_t is mean stationary. The component $g(.)$ captures the long-term slowly evolving trend in the process, which is necessary to include for many stocks and indexes due to the longer term improvements in liquidity that we have observed.[4] The dynamic process λ_t represents short-term stationary predictable variation around this trend, which is also needed for liquidity time series that have this moderately persistent deviations from the trend as we have documented in HLW. The last component ζ_t represents the new information, the shock driving the process. We adopt the multiplicative process rather than working with the logarithm of ℓ_t because there can be some zero observations

[4] Acharya and Pedersen (2005) recommend a modification of ℓ_t where return is multiplied by lagged price to take care of nonstationarity and winsorizing ℓ_t to reduce the effect of outliers. We explicitly model the trend to account for the nonstationarity. In HLW we discussed the issue of heavy tails and how to deal with them.

due to $R_t = 0$ even when trading has occurred with positive volume. Our methodology will allow for this feature rather than explicitly modelling it. When it comes to estimation, there is an identification issue because we can multiply and divide the two components g, λ by constants and obtain the same value of liquidity. To resolve this we suppose that $E(\lambda_t) = 1$, which is achieved by setting $\omega = 1 - \beta - \gamma$. The properties of this model and the overall estimation strategy are discussed in HLW.

The Amihud (2002) liquidity measure is formally defined as $A_t = \sum_{s \in I_t} \ell_s / n_t$. Usually, A is measured monthly, quarterly, or annually by averaging the daily values of ℓ_s within period I_t of length n_t days.[5] This low-frequency measure has been used in countless studies that compare different markets according to their trading costs, that try to identify the effect of regulatory and technical change on market functioning, and that try to measure improvements in market functioning over time. It has also been shown to be a priced factor in cross-sectional asset pricing tests, following Amihud (2002). Note that our trend function $g(t/T)$ is the model-specific counterpart of the low-frequency measure A_t. The key feature of our model is that we allow the detrended liquidity $\ell_t^* = \ell_t / g(t/T)$ to have nontrivial short-term predictability through the process λ_t so that shocks to liquidity have persistent effects on the level of liquidity relative to trend. In practice, both short-term and long-term predictability is present in this series, and it is important to take account of both features when carrying out inferential procedures and in predicting future liquidity.

Our model implies that $E(\ell_t \mid \mathcal{F}_{t-1}) = g(t/T)\lambda_t$ and $E(\ell_t) = g(t/T)$, which is the basis of our estimation strategy. It also implies that $\mathrm{var}(\ell_t \mid \mathcal{F}_{t-1}) = g(t/T)^2 \lambda_t^2 \sigma_{\zeta t}^2$, where $\sigma_{\zeta t}^2 = \mathrm{var}(\zeta_t \mid \mathcal{F}_{t-1})$. If we additionally assume that ζ_t is i.i.d., then $\mathrm{var}(\ell_t \mid \mathcal{F}_{t-1})$ is, apart from a constant, the square of $E(\ell_t \mid \mathcal{F}_{t-1})$, which may be rather restrictive, and so since we do not need this for estimation, we shall not require this preferring the weaker assumption that $\zeta_t - 1$ is a stationary martingale difference sequence.

We suppose that the liquidity trend function g is almost everywhere a smooth function

[5] Several studies have suggested modifications of the Amihud measure such as taking open-to-close returns instead of close-to-close (Barardehi et al. (2021)) or using functions of volatility instead of the absolute value of returns (Fong et al. (2018)). These alternative measures can be handled within our framework. In the empirical study, we focus on the Amihud measure based on the daily high-low price range as it is a more efficient low-frequency liquidity measure than the classic daily Amihud proxy, see Lacava et al. (2023). Our main empirical results are robust to the use of alternative liquidity measures.

of (rescaled) time, specifically, it has two continuous derivatives. The function g captures the long-run variation of liquidity, which is generally smoothly varying (and for most assets shows improvements over time). However, we allow the possibility of permanent shifts in liquidity (structural change) by allowing the function g to be discontinuous at a finite set of known points $u_1 = t_1/T, \ldots, u_m = t_m/T \in (0,1)$. For a given point $u \in (0,1)$ define the left and right limits of the function and its first two derivatives

$$\lim_{u \uparrow u} g^{(r)}(u) = g_-^{(r)}(u), \quad \lim_{u \downarrow u} g^{(r)}(u) = g_+^{(r)}(u), \quad r = 0, 1, 2,$$

which we assume are well defined and finite. We allow that $g_-^{(r)}(u_i) \neq g_+^{(r)}(u_i)$ for $i = 1, \ldots, m$ and $r = 0, 1, 2$, but for all $u \notin \{0, u_1, \ldots, u_m, 1\}$ we maintain that $g_-^{(r)}(u) = g_+^{(r)}(u) = g(u)$, for $r = 0, 1, 2$.[6] We adopt the convention that $g^{(r)}(.)$ are CADLAG (continuous from the right with limits from the left), that is, $g^{(r)}(u_i) = g_+^{(r)}(u_i)$, and so we may write for $r = 0, 1, 2$ and $u \in [0, 1]$

$$g^{(r)}(u) = g_0^{(r)}(u) + \mathcal{J}_i^{(r)} 1(u \in [u_i, u_{i+1}))$$

for some baseline function $g_0(.)$ that is twice continuously differentiable, where $1(.)$ is the indicator function, and $\left|\mathcal{J}_i^{(r)}\right| \leq C < \infty$. One question of interest is whether a break has occurred, and, if one has occurred, how big is the effect in terms of the level of g. The size of the jump at the point u_i is measured in level terms and in percentage terms, respectively, by:

$$\mathcal{J}_i = \mathcal{J}_i^{(0)} = g_+(u_i) - g_-(u_i), \quad \mathcal{J}_{\%i} = \frac{2\left(g_+(u_i) - g_-(u_i)\right)}{g_+(u_i) + g_-(u_i)}. \tag{3}$$

This is the magnitude of the permanent effect of the split on the level of liquidity of a given firm at time u_i (the effect that remains in the absence of further changes).

3 Econometric methodology

3.1 Estimation of the model

We observe a sample of daily illiquidities $\{\ell_t,\ t = 1, \ldots, T\}$ for a given firm. We first estimate the trend process by the local linear kernel smoother designed to be robust to

[6] Note that $g_+^{(r)}(0)$ and $g_-^{(r)}(1)$ are also assumed to be well defined finite quantities.

potential breaks at distinct points $0 < u_1 < u_2 < \cdots < u_m < 1$. Specifically, we define $\widehat{g}(u) = \widehat{\alpha}(u)$, where for $u \in [u_i, u_{i+1})$,

$$(\widehat{\alpha}(u), \widehat{\beta}(u)) = \arg\min_{\alpha,\beta} \sum_{t=1}^{T} K_h(t/T - u) \left\{ \ell_t - \alpha - \beta(t/T - u) \right\}^2 1\left(u_i \leq t/T < u_{i+1}\right). \quad (4)$$

Here, $K_h(.) = K(./h)/h$, where K is a kernel supported on $[-1, 1]$ and h is a bandwidth. This provides an automatic boundary adjustment that preserves the rate of convergence of the estimator at all points $u \in [0, 1]$. At interior points of the interval (u_i, u_{i+1}) the estimator is just the standard local linear with two-sided smoothing, but for points $u_i + ch$ with $c \in [0, 1]$ only time points to the right of u_i are included and at points $u_{i+1} - ch$ with $c \in [0, 1]$ only time points to the left of u_{i+1} are included. The estimator $\widehat{g}(u)$ is itself CADLAG and continuous everywhere except at the points $\{u_1, u_2, \ldots, u_m\}$, consistent with the posited behaviour of $g(u)$. Robinson (1989) established large sample properties of local constant kernel estimators in a similar time series setting without structural breaks, and Francisco-Fernández et al. (2003) establish uniform consistency. Kristensen (2012) and others have extended these results to settings with abrupt changes like ours. We carry out the estimation of the dynamic parameters of λ_t using this jump robust estimator. The estimator $\widehat{\alpha}(u)$ is not guaranteed to be positive with probability one, although in practice we have not yet encountered a violation. One could set a lower bound $\epsilon > 0$ and let $\widehat{g}(u) = \max\{\widehat{\alpha}(u), \epsilon\}$ or use some more sophisticated method to impose this restriction if necessary.

Define the detrended liquidity $\widehat{\ell}_t^* = \ell_t/\widehat{g}(t/T)$, $t = 1, \ldots, T$. We use GMM to estimate the dynamic parameters $\theta = (\beta, \gamma)^\intercal$ from the conditional moment restriction $E(\ell_t^*|\mathcal{F}_{t-1}) = \lambda_t$, where $\ell_t^* = \ell_t/g(t/T)$, $t = 1, \ldots, T$. We work with the residual $\ell_t^*/\lambda_t(\theta) - 1$, which is a martingale difference sequence at the true parameter values $\beta = \beta_0, \gamma = \gamma_0$. In practice, we use $\widehat{\ell}_t^*/\widehat{\lambda}_t(\theta) - 1$, where $\widehat{\lambda}_t(\theta) = 1 - \beta - \gamma + \beta\lambda_{t-1} + \gamma\widehat{\ell}_{t-1}^*$. Then define $\rho_t(\theta, \widehat{g}) = z_{t-1}(\widehat{\ell}_t^*/\widehat{\lambda}_t(\theta) - 1)$ and

$$\widehat{\theta}_{GMM} = \arg\min_{\theta\in\Theta} \left\| M_T(\theta, \widehat{g}) \right\|_W, \quad M_T(\theta, \widehat{g}) = \frac{1}{T} \sum_{t=1}^{T} \rho_t(\theta, \widehat{g}), \quad (5)$$

where W is a weighting matrix, while $z_t \in \mathcal{F}_t$ are instruments, for example lagged values, while for a vector a and matrix W, $\|a\|_W^2 = a^\intercal W a$. This provides initial root-T consistent estimators of θ under the conditions of HLW, which are based on Chen et al. (2003). These conditions include undersmoothing (relative to what would be optimal for estimation of $g(u)$), that is, the bandwidth sequence h satisfies $Th^4 \to 0$.

Given consistent estimates of $\theta, g(.)$ one can improve both estimates for efficiency gains or simplicity of standard errors. Note that $E\left(\ell_t/\lambda_t\right) = g(t/T)$, which provides an alternative local moment condition for estimation of g, and one that is purged of the short-run variation induced by the dynamic process λ_t. We let $\widetilde{g}(u) = \widetilde{\alpha}(u)$, where $(\widetilde{\alpha}(u), \widetilde{\beta}(u))$ are defined as the minimizers of (4) with ℓ_t replaced by $\ell_t/\widehat{\lambda}_t$, where $\widehat{\lambda}_t = \widehat{\lambda}_t(\widehat{\theta}_{GMM}, \widehat{g})$, and $\widehat{\theta}_{GMM}, \widehat{g}(.)$ were estimated in the previous procedure. HLW show that the preliminary estimation of λ_t by $\widehat{\lambda}_t$ has no effect on the large sample distributions of the "pre-whitened" estimator $\widetilde{g}(u)$. As discussed in HLW, the benefit of working with $\widetilde{g}(u)$ is that its large sample variance is much simpler to estimate than the large sample variance of $\widehat{g}(u)$ (which requires long-run variance estimation), which tends to benefit inference in finite and large samples.

To estimate the jump size at points u_i we actually compute two different estimates of $g(u_i)$, a left sider and a right sider. Specifically, we let $\widetilde{g}_+(u_i) = \widetilde{\alpha}_+(u_i)$ and $\widetilde{g}_-(u_i) = \widetilde{\alpha}_-(u_i)$, $i = 1, \ldots, m$, where: $(\widetilde{\alpha}_+(u_i), \widetilde{\beta}_+(u_i))$ minimize (4) with ℓ_t replaced by $\ell_t/\widehat{\lambda}_t$, while $(\widetilde{\alpha}_-(u_i), \widetilde{\beta}_-(u_i))$ minimize (4) with ℓ_t replaced by $\ell_t/\widehat{\lambda}_t$ and $1\left(u_i \leq t/T < u_{i+1}\right)$ replaced by $1\left(u_{i-1} \leq t/T < u_i\right)$. The raw and percentage sizes of the jump are estimated by respectively

$$\widetilde{\mathcal{J}}_i = \widetilde{g}_+(u_i) - \widetilde{g}_-(u_i), \quad \widetilde{\mathcal{J}}_{\%i} = \frac{2\left(\widetilde{g}_+(u_i) - \widetilde{g}_-(u_i)\right)}{\widetilde{g}_+(u_i) + \widetilde{g}_-(u_i)}. \tag{6}$$

In principle one can and should use a different bandwidth sequence for these second round estimates reflecting the different bias/variance trade-off they face.

The large sample properties of the local linear estimators and the derived quantities can be written in terms of the equivalent kernel, see Fan and Gijbels (1996) (pp 70-72); in this case, the equivalent (right) boundary kernel for any point $u_i - ch$ with $c \in [0, 1]$ is

$$K_c^+(u) = (\alpha_0(c) + \alpha_1(c)u)K(u)1(u \in [-c, 1]), \tag{7}$$

$$\alpha_0(c) = \frac{\mu_{2,c}}{\mu_{0,c}\mu_{2,c} - \mu_{1,c}^2}, \ \alpha_1(c) = -\frac{\mu_{1,c}}{\mu_{0,c}\mu_{2,c} - \mu_{1,c}^2},$$

where $\mu_{j,c} = \mu_{j,c}(K) = \int_{-c}^1 K(u)u^j du$. Similarly, we can define the left boundary kernel $K_c^-(u)$. Our test is based on the case $c = 0$, and we denote $K^\pm = K_0^\pm$ and $\mu_j = \mu_{j,0}$. If the original kernel K is symmetric about zero, $||K^+||^2 = ||K^-||^2$, where $||K||^2 = \int K(u)^2 du$, which we shall assume from now on.

We can rewrite the model in terms of $\ell_t^\dagger = \ell_t/\lambda_t$ as follows:

$$\ell_t^\dagger = g(t/T) + g(t/T)\left(\zeta_t - 1\right), \tag{8}$$

where $\zeta_t - 1$ is a martingale difference sequence with finite unconditional variance σ_ζ^2, in which case $E(\ell_t^\dagger) = g(t/T)$ and $\mathrm{var}(\ell_t^\dagger) = g(t/T)^2\sigma_\zeta^2$. Under regularity conditions, the pointwise mean squared errors of $\widetilde{\mathcal{J}}_i, \widetilde{g}_\pm(u_i)$, are respectively:

$$MSE_\Delta(u_i) = \frac{h^4}{4}\mu_2^2(K^+)\left(g_+''(u_i) - g_-''(u_i)\right)^2 + \frac{1}{Th}\left\|K^+\right\|^2\left(g_+(u_i)^2 + g_-(u_i)^2\right)\sigma_\zeta^2, \tag{9}$$

$$MSE_\pm(u_i) = \frac{h^4}{4}\mu_2^2(K^+)\left(g_\pm''(u_i)\right)^2 + \frac{1}{Th}\left\|K^+\right\|^2 g_\pm(u_i)^2\sigma_\zeta^2, \tag{10}$$

and the optimal bandwidths are respectively:

$$h_{\Delta,opt}(u_i) = C_K\left(\frac{\left(g_+(u_i)^2 + g_-(u_i)^2\right)\sigma_\zeta^2}{\left(g_+''(u_i) - g_-''(u_i)\right)^2}\right)^{1/5} T^{-1/5}, \quad h_{\pm,opt}(u_i) = C_K\left(\frac{g_\pm(u_i)^2\sigma_\zeta^2}{g_\pm''(u_i)^2}\right)^{1/5} T^{-1/5},$$

where $C_K = (\left\|K^+\right\|^2/\mu_2^2(K^+))^{1/5}$ depends only on the kernel. Imbens and Kalyanaraman (2012) discuss and propose various methods for estimating the optimal bandwidth. Our approach is based on the so-called pilot method of Silverman (1986), Fan and Gijbels (1996). That is, we suppose that $g_+(u)$ is globally polynomial with parameters $\sum_{j=0}^p a_j^i u^j$, $p \geq 2$,on the interval $[u_i, u_{i+1})$, while $g_-(u)$ is globally polynomial with parameters $\sum_{j=0}^p a_j^{i-1} u^j$ on the interval $(u_{i-1}, u_i]$. We estimate the parameters a_j^i based on segmented least squares regression using data from the interval $[u_i, u_{i+1})$ and then plug in the estimated quantities. One issue arises when $g_+''(u_i) \simeq g_-''(u_i)$; in that case it may be better to use $h_{\pm,opt}(u_i)$.

3.2 Test of permanent effect

3.2.1 Single Split

We first consider the null hypothesis that $g_-(u_i) = g_+(u_i)$ versus the alternative that $g_-(u_i) \neq g_+(u_i)$ for a given u_i. One may also consider the "kinked" case where g is continuous but its first or higher order derivatives are not continuous, which is a more subtle change in the liquidity trend, but our focus is on jumps in the level of liquidity. This framework is essentially that of the regression discontinuity literature, see Cattaneo and Titiunik

(2022) for a survey; the main difference being that we have a fully specified dynamic model in the background that provides a framework for the construction of standard errors appropriate for the type of dependence found in this type of data and thereby suggests alternative estimates of the jumps. There is a large literature on testing for structural change in parametric models, Perron (1989), and in nonparametric regression, Muller (1992), Delgado and Hidalgo (2000). Indeed the regression discontinuity literature, Imbens and Lemieux (2008), draws on some of these ideas. We test for the presence of a discontinuity at u_i by computing an adjusted t-statistic based on the final stage one-sided local linear estimators.

Define the standard error and bias and the infeasible test statistic:

$$SE(u_i) = \sqrt{||K^+||^2 \left(\sigma_+^2(u_i) + \sigma_-^2(u_i)\right)/Th}, \quad b(u_i) = \frac{1}{2}h^2\mu_2\left(g_+''(u_i) - g_-''(u_i)\right),$$

$$\tau(u_i) = \frac{\widetilde{\mathcal{J}}_i - \mathcal{J}_i}{SE(u_i)}. \tag{11}$$

In practice, we replace $\sigma_\pm^2(u_i)$ by estimates $\widetilde{\sigma}_\pm^2(u_i)$, where

$$\widetilde{\sigma}_\pm^2(u_i) = \widetilde{g}_\pm(u_i)^2 \times \widehat{\sigma}_\zeta^2, \quad \widehat{\sigma}_\zeta^2 = \frac{1}{T}\sum_{t=1}^{T}\left(\widehat{\zeta}_t - \overline{\widehat{\zeta}}\right)^2.$$

Then let $\widetilde{\tau}(u_i)$ denote the feasible statistic with $SE(u_i)$ replaced by the estimated version $\widetilde{SE}(u_i)$. Under some conditions including the condition that $Th^5 \to \gamma$, where $\gamma \in [0, \infty)$ we have

$$\tau(u_i), \widetilde{\tau}(u_i) \Longrightarrow N(\rho_i, 1), \quad \rho_i = \lim_{T\to\infty} \frac{b(u_i)}{SE(u_i)}. \tag{12}$$

Note that the estimators subscripted $+$ are asymptotically independent of the estimators subscripted $-$, because $K^+ \times K^- = 0$, no matter what the correlation structure of the errors. Note that one can also base the test statistic on $\widetilde{\mathcal{J}}_{\%i}$, but since the asymptotic standard deviation of $\widetilde{g}_\pm(u_i)$ is proportional to $g_\pm(u_i)$, the test statistic would be identical.

In some cases, we may be testing for the effect of an event that takes place at the same time as other structural changes are affecting all stocks, such as during the Global Financial Crisis. In this case, we propose to include a control group to eliminate common trends at the change time. This amounts to a diff in diff test, Angrist and Pischke (2009). Specifically, suppose that we have a "treatment" stock labelled with an S subscript and a "control" stock labelled with an C subscript. We suppose that the dynamic model holds for both stocks and that ζ_{St} and ζ_{Ct} may be correlated. We define the diff-in-diff statistic as

$$\tau_{did}(u_i) = \frac{\left(\widetilde{g}_{S,+}(u_i) - \widetilde{g}_{S,-}(u_i)\right) - \left(\widetilde{g}_{C,+}(u_i) - \widetilde{g}_{C,-}(u_i)\right)}{SE_{did}(u_i)} \tag{13}$$

$$SE_{did}(u_i) = ||K^+||\sqrt{\frac{\left(\sigma^2_{S,+}(u_i) + \sigma^2_{C,+}(u_i) - 2\sigma_{S,C,+}(u_i)\right) + \left(\sigma^2_{S,-}(u_i) + \sigma^2_{C,-}(u_i) - 2\sigma_{S,C,-}(u_i)\right)}{Th}}$$

$$b_{did}(u_i) = \frac{h^2}{2} \times \int_0^1 u^2 K^+(u)du \times \left(\left(g''_{S,+}(u_i) - g''_{S,-}(u_i)\right) - \left(g''_{C,+}(u_i) - g''_{C,-}(u_i)\right)\right)$$

$$\widetilde{\sigma}_{S,C,\pm}(u) = \widetilde{g}_{S,\pm}(u_i)\widetilde{g}_{C,\pm}(u_i) \times \widehat{\sigma}_{\zeta_S,\zeta_C}, \quad \widehat{\sigma}_{\zeta_S,\zeta_C} = \frac{1}{T}\sum_{t=1}^{T}(\widehat{\zeta}_{St} - \overline{\widehat{\zeta}}_S)(\widehat{\zeta}_{Ct} - \overline{\widehat{\zeta}}_C)$$

and $\widetilde{\tau}_{did}(u_i)$ like $\tau_{did}(u_i)$ but with $\widetilde{SE}_{did}(u_i)$ in place of $SE_{did}(u_i)$. This corresponds to a test of the hypothesis that $g_{S,+}(u_i) - g_{S,-}(u_i) = g_{C,+}(u_i) - g_{C,-}(u_i)$, which imposes weaker assumptions than $g_{S,+}(u_i) - g_{S,-}(u_i) = 0$. Under this null hypothesis, $\tau_{did}(u_i), \widetilde{\tau}_{did}(u_i) \Longrightarrow N(\rho_{did}, 1)$, where $\rho_{did} = \lim_{T\to\infty}(\mid b_{did}(u_i) \mid /SE_{did}(u_i))$. We comment that the control group approach heavily relies on being able to find stock(s) that are not themselves influenced by the effect on the treatment group, i.e., where spillover effects from S to C are not anticipated. One may use a single stock as control or construct a synthetic control, see Abadie (2021).

We consider several approaches for inference. The central approach we adopt is to assume that the bias term $b(u_i)$ is of smaller order such that one does not need to account for it in the CLT and one does not need to provide estimates of $g''_{\pm}(u_i)$. This holds under the case that $g''_{+}(u_i) = g''_{-}(u_i)$, i.e., the curve has a level shift but the curvature from the left and the right are equal. It also holds when this condition is violated provided that $\gamma = \lim_{T\to\infty} Th^5 = 0$, which is referred to as the undersmoothing case. In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\widetilde{\tau}(u_i)$ with the normal critical values $\pm z_{\alpha/2}$ for a size α test.

An alternative approach is to consistently estimate the bias and subtract it from the estimator, which can be done in a number of ways either explicitly or implicitly but for consistent estimation of the bias another bandwidth has to be used. A popular method called jackknife is due to Schucany and Sommers (1977) in which we replace the estimator $\widetilde{g}_{\pm}(u_i; h)$ computed with whatever bandwidth h by $2\widetilde{g}_{\pm}(u_i; h/2) - \widetilde{g}_{\pm}(u_i; h)$, which removes the bias effect from the limiting distribution but raises the variance by making the implicit kernel of higher order and hence raising its L_2 norm, see Härdle and Linton (1994). Calonico et al. (2014) advocate an explicit bias correction with the same bandwidth used in the estimation of g. This bias correction eliminates the bias asymptotically but leads to an additional contribution to the variance, which needs to be accounted for. In the application we consider a parametrically guided bias correction based on a pilot parametric model, that is, we suppose that $g_+(u), g_-(u)$ are globally polynomial with parameters $\sum_{j=0}^{p} a_j^i u^j$ on the interval $[u_i, u_{i+1})$, in which case the bias is $b(u_i) = h^2\mu_2(K^+)\beta(u_i)/2$, where $\beta(u_i) = \sum_{j=2}^{p} j(j-1)(a_j^{i+1} - a_j^i)u_i^{j-2}$, and we estimate the parameters a_j^i by segmented global polynomial regression on the separate regimes. This approach is in the spirit of the rule of thumb approach to bandwidth selection; one does not need to adjust the standard error in our case because the estimates of a_j^i are root-T consistent.[7] In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\widetilde{\tau}(u_i) - \widetilde{b}(u_i)/\widetilde{SE}(u_i)$ with $\pm z_{\alpha/2}$ for a size α test.

Armstrong and Kolesár (2020) suggest an alternative approach that provides "honest" confidence intervals. In this case one estimates an upper bound on the bias terms. Specifically, let $\rho_{\max} = \max_{1\le i\le J} \sup_{\mathcal{G}} \lim_{T\to\infty}(\mid b(u_i) \mid /SE(u_i))$ be the upper bound over the class of functions $\mathcal{G}$. We let $\widetilde{\rho}_{\max} = \max_{1\le i\le J}(\mid \widetilde{b}(u_i) \mid /\widetilde{SE}(u_i))$ be the estimated upper bound described below. In this case we carry out the test of $\mathcal{J}_i = 0$ by comparing the statistics $\widetilde{\tau}(u_i)$ with $cv_{1-\alpha}(\widetilde{\rho}_{\max})$, where $cv_{1-\alpha}(\widetilde{\rho}_{\max})$ is the $1-\alpha$ critical value of the folded normal distribution $\mid N(\widetilde{\rho}_{\max}, 1) \mid$ for a size α test. In one implementation Armstrong and Kolesár (2020) suppose that $g_+(u), g_-(u)$ are globally polynomial with parameters $\sum_{j=0}^{p} a_j^i u^j$ on the interval $[u_i, u_{i+1})$, in which case the bias at the point u_i is $b(u_i) = h^2\mu_2(K^+)\beta(u_i)/2$, where $\beta(u_i)$ is defined above, and one estimates the parameters a_j^i by segmented global polynomial regression on the separate regimes.

Our tests are valid against all fixed departures for which $\mathcal{J}_i \neq 0$, since $\widetilde{\tau}(u_i) \xrightarrow{P} \infty$ (with $\mathcal{J}_i = 0$) in this case. They also have power against some small departures, specifically, power lies between zero and one for alternatives such that $\mathcal{J}_i = \delta_i \Delta_T$ for some sequence $\Delta_T \to 0$ such that $\sqrt{Th}\Delta_T \to \Delta \neq 0$. Our test also has power against local alternatives regarding the timing of the break (when there may be some small anticipation or delay in the effects), such as the break point occurring at $u_i \pm ch$ for some $c \in [0,1]$, see Hidalgo and

[7] Note that in our case there is also bias terms from $\widehat{\theta}_{GMM}$, which are of smaller order since we required that $Th^4 \to 0$ for that theory.

Seo (2013) for some discussion. One can also construct confidence intervals for $\mathcal{J}_i$ and $\widetilde{\mathcal{J}}_{\%i}$ using any of the above approaches without imposing $\mathcal{J}_i = 0$, since the distribution theory is stated in this general case.

To save space, we do not report the simulation results here, but keep them available upon request. We note that both the undersmoothing method and the honest confidence interval approach work well overall, especially when the sample size is large. In smaller samples (e.g. n=1000), the honest confidence interval approach performs slightly better than the undersmoothing approach. For the bias correction method, we observe that there is an over-rejection issue when the true trend does not have a break and this does not improve as the sample size increases. Therefore, a more sophisticated bias correction method should be considered but we do not pursue this direction in this paper.

3.2.2 Multiple Splits

We provide a joint test of the null hypothesis of no breaks at any of the u_i versus the alternative of one or more breaks. We may also aggregate across a sample of firms $j = 1, \ldots, n$ with breaks at times u_{ji}, $i = 1, \ldots, m_j$, so we suppose that N is the total number of events being considered, where $N \leq \sum_{j=1}^{n} m_j$. We consider either of the statistics

$$W = \sum_{i=1}^{N} \widetilde{\tau}(u_i)^2, \quad M = \max_{1 \leq i \leq N} \left| \widetilde{\tau}(u_i) \right|, \tag{14}$$

Provided the splits occur at different times the above arguments regarding the asymptotic variances follow. Under the null hypothesis of no breaks anywhere, W is asymptotically distributed as $\sum_{i=1}^{N} (Z_i + \rho_i)^2$ and M is asymptotically distributed as $\max_{1 \leq i \leq N} |Z_i + \rho_i|$, where Z_i are i.i.d. standard normal random variables (the individual t-statistics are mutually independent in large samples given the physical separation between u_i and u_j).

An alternative approach is to work with a directional test. Suppose that we pool the jumps across the splits (either for a given firm or across firms) as follows

$$\widetilde{\tau}_w = \frac{\sum_{i=1}^{N} w_i \widetilde{\tau}(u_i)}{\sqrt{\sum_{i=1}^{N} w_i^2}}, \tag{15}$$

where w_i is a (possibly stochastic) weighting scheme such as market cap or equal weighting. Then we may show that under the null hypothesis, we have (as $T \to \infty$ for N

fixed) $\widetilde{\tau}_w \Longrightarrow N(\rho_w, 1)$, where $\rho_w = \sum_{i=1}^N w_i \rho_i / \sqrt{\sum_{i=1}^N w_i^2}$. The individual statistics are uncorrelated across distinct points u_i. We may test the null hypothesis by comparing $\widetilde{\tau}_w$ with the normal critical values in the undersmoothing case or by comparing $\widetilde{\tau}_w$ with critical values $cv_{1-\alpha}(\widetilde{\rho}_{w,\max})$, where $cv_{1-\alpha}(\widetilde{\rho}_{w,\max})$ is the $1-\alpha$ critical value of the folded normal distribution $\mid N(\widetilde{\rho}_{w,\max}, 1) \mid$ for a size α test. Here, the worst case ratio is $(\sum_{i=1}^N w_i / \sqrt{\sum_{i=1}^N w_i^2}) \times \max_{1 \le i \le N} \sup_{\mathcal{G}} \lim_{T\to\infty} (\mid b(u_i) \mid / SE(u_i))$. This test is more directional in its intent, and will not reject all null hypotheses, only those for which the discontinuities tend to go in the same direction, i.e., for which $\sum_{i=1}^N w_i \mathcal{J}_i \neq 0$. This is similar to the principle underlying the variance ratio tests and the usual way in which event studies are conducted through cumulative abnormal returns etc.

3.3 Test of temporary effects

We next consider how to allow for temporary effects or rather short-term adjustments that eventually die out. We do this by including dummy variables in the dynamic equation, that is, we let

$$\lambda_t = \omega + \beta \lambda_{t-1} + \sum_{j=1}^J \alpha_j D_{jt} + \gamma \ell_{t-1}^*,$$

where D_{jt} is a dummy variable that is one in period t_j and zero otherwise. To allow for anticipation effects and slow transmission we focus on times around the known intervention point, that is, if t_j is a stock split day, we include dummy variables for $t_j - E, \ldots, t_j + E$ for some event window $\mathcal{E}$ of length $J = 2E + 1$. With multiple splits we include dummy variables around all the key dates. Under this modelling assumption the level of the process λ_t is affected for all $t \geq t_1$, with a flexible effect between t_1 and t_J, but after t_J the effect decreases rapidly as $t - t_J \to \infty$ and the long run effect of the intervention is zero. In this case, it is not possible to consistently estimate the parameters α_j, although the estimated parameters give an indication of the temporary effects direction and magnitude.

We instead will propose a test of the null hypothesis that $\alpha_1 = \ldots = \alpha_J = 0$ against the alternative under the assumption of i.i.d. shocks ζ_t. In fact our test is also valid when $\zeta_t - 1$ is only a stationary mixing martingale difference sequence. Our test is based on the residuals from the null estimation. Here, we just consider the single event setting and take a simple approach. We suppose that the event window is given by $\{t_1 - E, \ldots, t_1 + E\}$.

Define the residuals $\widehat{\zeta}_t = \ell_t/\widetilde{g}(t/T)\widehat{\lambda}_t$, $t = 1, \ldots, T$, where the estimation of $\widehat{\theta}$ and $\widetilde{g}(\cdot)$ are described above. Under our conditions, these residuals are asymptotically equivalent (as $T \to \infty$) to the true unobserved ζ_t. We define abnormal illiquidity and cumulative abnormal illiquidity at times $\tau = 1, \ldots, J$ as follows:

$$\widehat{AIL}_\tau = \widehat{\zeta}_{t_1-E+\tau} - 1, \quad \widehat{CAIL}(\tau) = \sum_{s=0}^{\tau-1} \widehat{AIL}_s. \tag{16}$$

We do not use the usual normal critical values here because this is not likely to be a good assumption in view of the fact that $\zeta_t \geq 0$ and that the event window is typically short, i.e., E is finite. We replace them by nonparametrically estimated critical values. Suppose that $\zeta_t - 1$ is a stationary mixing process with marginal distribution F that is unknown and let F_{w_τ} denote the marginal distribution of the stationary mixing series $\{w_{r,\tau}\}$, where $w_{r,\tau} = \sum_{s=0}^{\tau-1}(\zeta_{r+s} - 1)$. We estimate the distributions F and F_{w_τ} based on the data not including the event window, $S = \{1, \ldots, T\} \setminus \{t_1 - E, \ldots, t_1 + E\}$. Specifically, letting $\widehat{w}_{r,\tau} = \sum_{s=0}^{\tau-1}(\widehat{\zeta}_{r+s} - 1)$, we define

$$\widehat{F}_{\widehat{w}_\tau}(x) = \frac{1}{T_S} \sum_{t \in S} 1\left(\widehat{w}_{t,\tau} \leq x\right), \quad x \in \mathbb{R},$$

where T_S is the cardinality of the set S, and $\widehat{F}(x) = \widehat{F}_{\widehat{w}_0}(x)$. Then define the critical values $\widehat{F}^{-1}_{\widehat{w}_\tau}(\alpha/2), \widehat{F}^{-1}_{\widehat{w}_\tau}(1 - \alpha/2)$. We reject the null hypothesis if $\widehat{CAIL}(\tau)$ is outside the interval $[\widehat{F}^{-1}_{\widehat{w}_\tau}(\alpha/2), \widehat{F}^{-1}_{\widehat{w}_\tau}(1 - \alpha/2)]$ for $\tau = 1, \ldots, J$. There is a large literature about estimation of distribution functions of residuals in time series settings, see for example Koul and Ling (2006) and Escanciano (2010). Their results can be used to show that as $T \to \infty$, $\widehat{F}_{\widehat{w}_\tau}(x) \xrightarrow{P} F_{w_\tau}(x)$ and $\widehat{F}^{-1}_{\widehat{w}_\tau}(x) \xrightarrow{P} F^{-1}_{w_\tau}(x)$, and therefore the rejection frequency converges to α under the null hypothesis and to one under the alternative hypothesis.

We may aggregate across events (same firm different events or across firms) and obtain a CLT when the number of events being aggregated across is large. Provided the timing of the stock splits across firms does not coincide very much, the standard errors can be based on the "as if independence" assumption. Specifically, let

$$\widehat{AAIL}(\tau) = \sum_{i=1}^{N} w_i \widehat{AIL}_{i\tau}, \quad \widehat{ACAIL}(\tau) = \sum_{i=1}^{N} w_i \widehat{CAIL}_i(\tau) \tag{17}$$

denote the aggregated abnormal and cumulated abnormal liquidity across a total number of events N. Then provided N and T are large, these statistics are asymptotically normally distributed and can be compared with the critical values $z_{\alpha/2}\sqrt{\sum_{i=1}^{N} w_i^2 \widehat{\sigma}_{\zeta i}^2}$ and $z_{\alpha/2}\sqrt{\tau \sum_{i=1}^{N} w_i^2 \widehat{\sigma}_{\zeta i}^2}$, respectively, where $\widehat{\sigma}_{\zeta i}^2$ is the estimated variance of the corresponding ζ for the particular firm event.

Our tests are applied using the jump consistent estimate of the trend and so the null hypothesis here includes the possibility of a permanent change to liquidity at the specified points. We should expect to see that adjustments, if any, should be relatively quick if markets are efficient and so there should not be much of a role for slow dynamic responses.

4 Empirical study: stock splits

4.1 Data description

In our application, we use the proposed framework to analyze whether stock splits have permanent and temporary effects on the illiquidity process. We consider historical daily price and volume data for the Dow Jones index component stocks.[8] The sample period starts from each asset's first available data point (after June 15, 1992) until December 31, 2023 and there are in total 76 splits. We plot in Figure 1 the number of splits by year. We observe that stock splits happen more often during periods of strong market performance. For example, numerous splits took place during the build-up phase of the dot-com bubble between 1992 and 2000, and very few splits occurred in the aftermath of its collapse. Likewise, we can see that there were hardly any splits during the Global Financial Crisis. This empirical evidence suggests that periods of high price appreciation could be one of the factors motivating firms' decision to split their stock, in line with Fama et al. (1969).

We summarize in Table 1 the frequency of different split sizes. The vast majority of the splits in the sample are two-for-one, followed by a handful of 1.5-to-one splits. The occurrence of large stock splits is less frequent and they could be motivated by index inclusion reasons as the Dow Jones weights its constituents based on their stock price.[9]

[8]The composition of the Dow Jones index is based on the constituents as of January 12, 2024. The daily price and volume data as well as the stock split information are retrieved from the CRSP database.

[9]During the considered sample period, the only large stock split was by Apple, which joined the Dow Jones index in 2015 after undergoing a seven-to-one stock split in June 2014. The split brought Apple's

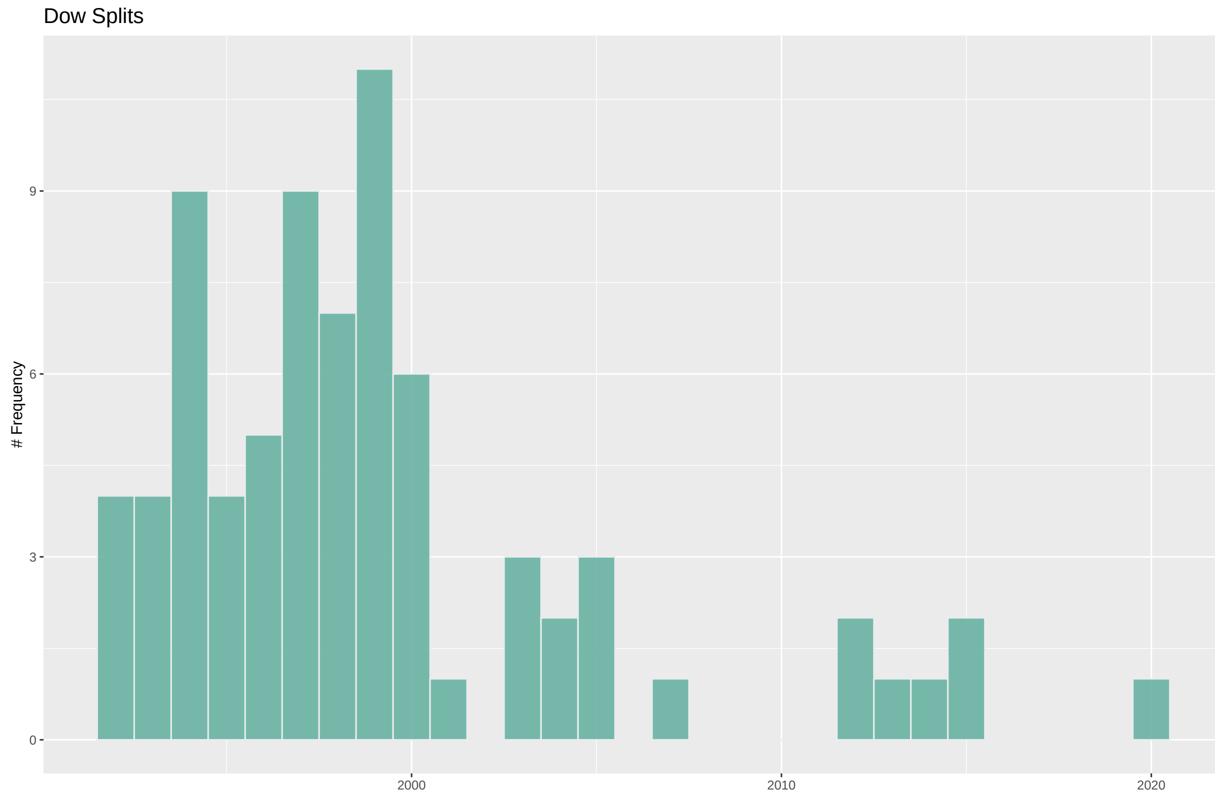

Figure 1: Number of splits per year.

We focus on the most common split factor in our empirical study, i.e. the two-to-one stock splits, leaving us with 24 of the 30 index constituents for the analysis below.[10]

Table 1: Distribution of different split sizes.

Split Size	<1.5	1.5	2	3	4	>4
Number	1	7	62	2	3	1
Total	76					

4.2 Test for permanent effects

We assess whether there is a permanent shift in illiquidity at a given stock split time u_0. This is achieved by testing for potential discontinuities in the long-run trend function g at

stock price into a more comparable range to the other constituents.

[10]Goldman Sachs did not have any stock split over our sample period and the remaining five companies had splits with other sizes than the two-to-one factor.

u_0. More specifically, we estimate the $g^{\pm}(u_0)$ functions using the local linear approach and construct the test statistics $\tau(u_0)$ as developed in Section 3.2. To facilitate the computation of the variance of $\hat{g}^{\pm}(u_0)$, we work with the improved estimator obtained by smoothing out ℓ_t, i.e. $\ell_t/\hat{\lambda}_t$. We first look at the average jump size in percentage terms for each stock as defined in (3), i.e. $\widetilde{\mathcal{J}}_{\%w}$. The values of the statistics are reported in the second and third rows of Table 2 where the row with subscript "BC" stands for the case with bias correction. The first row provides the number of splits for each stock. We observe that overall the percentage changes in the illiquidity trend level between pre and post stock splits are above 20% for a majority of the stocks. This suggests that there are pronounced negative effects of stock splits on long-run liquidity conditions.

Table 2: Average statistics for testing permanent breaks in the liquidity series.

	UNH	MSFT	HD	AMGN	MCD	CAT	BA	HON
# of splits	5	5	1	3	2	3	1	2
$\widetilde{\mathcal{J}}_{\%w}$	43%	37%	35%	49%	11%	31%	-4%	43%
$\widetilde{\mathcal{J}}_{\%w}^{BC}$	31%	30%	34%	45%	7%	27%	10%	46%
p_W^{US}	0.00	0.00	0.05	0.00	0.26	0.00	0.78	0.00
p_W^{BC}	0.00	0.00	0.04	0.00	0.08	0.00	0.55	0.00
	TRV	AAPL	JPM	JNJ	WMT	IBM	PG	CVX
# of splits	2	2	1	2	2	2	2	2
$\widetilde{\mathcal{J}}_{\%w}$	58%	107%	239%	42%	32%	31%	42%	31%
$\widetilde{\mathcal{J}}_{\%w}^{BC}$	55%	145%	239%	51%	37%	39%	42%	25%
p_W^{US}	0.00	0.00	0.00	0.00	0.06	0.00	0.00	0.00
p_W^{BC}	0.00	0.00	0.00	0.00	0.03	0.00	0.00	0.00
	MRK	MMM	NKE	KO	CSCO	INTC	VZ	WBA
# of splits	1	2	5	2	5	5	1	3
$\widetilde{\mathcal{J}}_{\%w}$	33%	57%	9%	20%	63%	64%	64%	23%
$\widetilde{\mathcal{J}}_{\%w}^{BC}$	36%	55%	10%	25%	62%	66%	77%	27%
p_W^{US}	0.00	0.00	0.04	0.03	0.00	0.00	0.00	0.15
p_W^{BC}	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.05

Note: $\widetilde{\mathcal{J}}_{\%w}$ is the average jump in percentage as defined in (3). p_W is the p-value of the aggregated statistic $W = \sum_{i=1}^{m} \tau(u_i)^2$. US stands for undersmoothing and BC stands for bias correction.

Table 3: Average statistics for directional tests of permanent breaks in the liquidity series.

	UNH	MSFT	HD	AMGN	MCD	CAT	BA	HON
τ_w^{US}	5.73	8.78	1.99	4.00	1.64	3.61	-0.28	6.96
$\tau_{w,MC}^{US}$	2.93	4.40	1.99	4.12	1.56	3.62	-0.28	8.44
τ_w^{BC}	5.16	7.86	2.39	4.30	1.84	3.90	0.78	9.43
$\tau_{w,MC}^{BC}$	2.78	2.97	2.39	4.73	1.77	3.18	0.78	11.06
τ_w^{HCI}	5.46	7.69	2.13	4.27	2.37	3.84	0.60	9.83
$cv_{1-\alpha}^{HCI}$	5.34	4.84	2.02	3.74	2.96	3.66	1.99	2.89
$\tau_{w,MC}^{HCI}$	3.18	3.24	2.13	4.58	2.26	3.33	0.60	11.41
$cv_{1-\alpha,MC}^{HCI}$	3.88	3.29	2.02	3.22	2.67	3.33	1.99	2.70
	TRV	AAPL	JPM	JNJ	WMT	IBM	PG	CVX
τ_w^{US}	7.43	8.40	29.08	5.81	2.37	3.94	7.48	4.08
$\tau_{w,MC}^{US}$	6.58	5.29	29.08	6.09	1.98	4.24	7.00	3.52
τ_w^{BC}	10.49	12.49	36.44	7.65	3.13	5.77	9.38	4.58
$\tau_{w,MC}^{BC}$	9.53	8.25	36.44	7.87	2.67	6.44	8.69	4.22
τ_w^{HCI}	10.50	12.05	36.35	7.25	2.71	5.54	9.43	5.09
$cv_{1-\alpha}^{HCI}$	2.90	3.45	1.97	2.91	2.90	3.07	2.90	3.00
$\tau_{w,MC}^{HCI}$	9.36	8.04	36.35	7.50	2.28	6.40	8.66	4.63
$cv_{1-\alpha,MC}^{HCI}$	2.58	3.27	1.97	2.73	2.61	2.85	2.85	2.60
	MRK	MMM	NKE	KO	CSCO	INTC	VZ	WBA
τ_w^{US}	7.53	6.91	2.22	2.64	5.51	7.91	3.58	2.01
$\tau_{w,MC}^{US}$	7.53	6.75	2.84	2.62	4.95	6.31	3.58	1.91
τ_w^{BC}	10.50	8.64	2.60	4.19	6.60	9.47	5.37	2.92
$\tau_{w,MC}^{BC}$	10.50	8.48	2.86	3.95	6.17	6.83	5.37	2.57
τ_w^{HCI}	10.76	9.19	2.09	4.22	6.75	9.64	5.26	2.74
$cv_{1-\alpha}^{HCI}$	2.02	3.14	5.14	2.93	5.17	4.97	1.97	3.58
$\tau_{w,MC}^{HCI}$	10.76	8.90	2.57	3.87	6.47	7.05	5.26	2.69
$cv_{1-\alpha,MC}^{HCI}$	2.02	2.89	3.86	2.85	3.57	3.61	1.97	3.05

Note: τ_w is the statistic computed as in (15) and is asymptotically $N(0,1)$ under the null hypothesis. US stands for undersmoothing and BC stands for bias correction. HCI stands for honest confidence interval where we compare the τ_w statistics with $cv_{1-\alpha}^{HCI}$. MC indicates that the statistics are aggregated based on the inverse of market capitalization on different stock split dates.

We then compute the test statistic $\tau(u_i^j)$ for the i^{th} split of stock j using the three approaches developed in Section 3.2. Namely, we consider the undersmoothing case ($\tau^{US}(u_i^j)$), the bias correction approach ($\tau^{BC}(u_i^j)$ and the honest confidence interval case where we compare the test statistics $\tau^{HCI}(u_i^j)$ without bias correction with the critical value obtained based on the worst-case-scenario bias $cv_{1-\alpha}^{HCI}$. We aggregate the test statistics for each company j to provide a joint test of the null hypothesis of no breaks at any point of the illiquidity series for this stock.[11] The fourth and fifth rows of Table 2 report the p-values for the statistic $W^j = \sum_{i=1}^{m} \tau\left(u_i^j\right)^2$ for the case of undersmoothing and the bias correction approach respectively. The p-values corresponding to the W^j statistics on stock split dates in the undersmoothing case are below 5% for 20 of the 24 Dow Jones index constituents considered in our analysis. When considering the approach with bias correction, the p-values suggest that the effects for 22 out of 24 stocks are significant.[12] These results indicate strong evidence against the null hypothesis of no long-term effect on liquidity from the stock split events.

The first two rows of Table 3 provide the directional average statistics defined as in (15) using an equal weighting scheme (τ_w) and one based on the inverse of market capitalization ($\tau_{w,MC}$) respectively. The next two rows report their counterparts with bias correction. In these two cases, the τ_w statistics should be compared to the critical values from a standard normal distribution, i.e. ± 1.96 for $\alpha = 5\%$. The last three rows report the τ_w statistics together with the computed critical values for $\alpha = 5\%$ using the honest confidence interval approach. The values of the τ_w statistic are above its corresponding critical value for most stocks, confirming that the pre- and post-split long-run illiquidity trends are significantly different from each other on average. Lastly, we note that the τ_w statistics are all positive with one exception being Boeing. This indicates an increase in the illiquidity trend and thus a corresponding worsening in stock liquidity conditions following a split. Our results are in line with the empirical evidence in Copeland (1979) who also documents a permanent decrease in stock liquidity after a split.

[11] The test statistics $\tau_i^j(u)$ for the individual splits are presented in Appendix A.1.

[12] The four exceptions in the undersmoothing case are Home Depot (HD), McDonald (MCD), Boeing (BA) and Walgreens (WBA). For Home Depot and Walgreens, they are significant in the bias correction case.

4.3 Test for temporary effects

We plot in Figure 2 the aggregated test statistics for the temporary effects across all splits for all considered firms as defined in (17). The blue line represents the aggregated Cumulative Abnormal Illiquidity (ACAIL) test statistics at horizons ranging from 45 days before to 45 days after the event, i.e. $\widehat{ACAIL}(\tau), \quad \tau = 1, \ldots, 91$. The red lines are the 2.5% and 97.5% quantiles, i.e. $z_{\alpha/2}\sqrt{\tau \sum_{i=1}^{N} w_i^2 \widehat{\sigma}_{\zeta i}^2}$. We find very limited evidence supporting a statistically significant effect of stock splits on short-term illiquidity. In addition, there is no clear direction for the temporary effects.

We also plot in Figures 12 and 13 (Appendix A.2) the aggregated test statistics across events of a given firm as defined in (17). The results on the direction of the temporary effects for different firms are mixed, with most stocks exhibiting insignificant increases and decreases in liquidity over the event window. These unclear patterns are consistent with the mixed evidence in previous literature on the effect of stock splits on market liquidity, with some authors reporting liquidity improvements following a split (Lamoureux and Poon (1987)) and others documenting a reduction in liquidity or muted effect in the post-split period (Lakonishok and Lev (1987)). A notable exception is the Apple stock, which experiences a significant increase in cumulative abnormal illiquidity before the split execution date persisting in the post-split period. This is in line with the prediction from the signaling theory of Brennan and Copeland (1988) who model a firm's decision to split its stock as a costly signal about its prospects, which is associated with a (at least) temporary decrease in the stock liquidity.

To summarize, our empirical evidence suggests that stock splits have an overall significant permanent effect on the long-run trend level of illiquidity but very limited effects on the short-run illiquidity dynamics. The documented increase in stocks' long-run illiquidity following a split challenges somewhat the predictions from the optimal price range and optimal tick size (Angel (1997)) theories that splitting firms should experience an increase in the liquidity of their stock in the long run. See also Goyenko et al. (2006) for an in-depth discussion of the short- and long-run liquidity effects of stock splits.

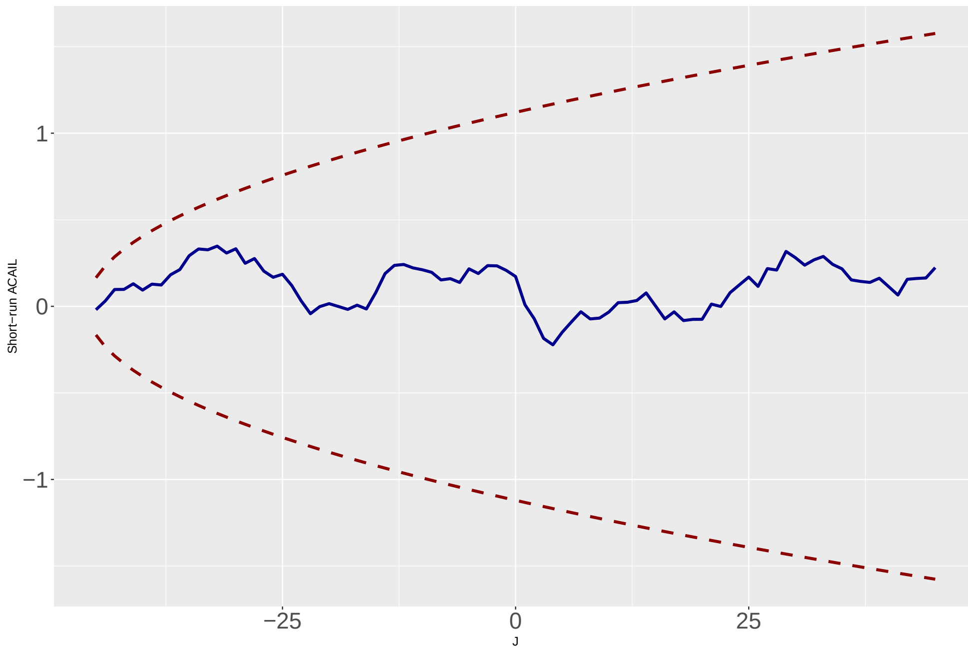

Figure 2: Test for temporary effects. The blue line is the aggregated test statistic and the red lines are the corresponding 2.5% and 97.5% quantiles.

5 Reverse stock splits

In this section, we investigate whether the effects of reverse stock splits on the illiquidity process are symmetric to the ones documented for stock splits in Sections 4.2 and 4.3. The analysis in the literature shows that the reverse splits have a more pronounced improvement in liquidity conditions for low-price stocks.[13] This could be because reverse splits ease the constraints on short selling for low-priced stocks, see e.g. Kwan et al. (2015). Therefore, in this part of the analysis, we focus our attention on the constituents of the S&P 500, S&P 400 and S&P 600 indices with a pre-event price level below \$5. In total, we have 53 stocks in our sample and there is only one reverse split for each stock in the sample. The sample period starts from each asset's first available data point (after June 15, 1992) until December 31, 2023.

We estimate the $g^{\pm}(u_0)$ functions using the local linear approach and construct the

[13]Our results are in line with these findings. We observe that reverse splits improve liquidity for most stocks with a low pre-event price, but they tend to have the opposite effect on stocks with higher pre-event price levels.

test statistics $\tau(u_0)$ for the permanent effect introduced in Section 3.2.[14] The results for the test statistic $\tau(u^j)$ for each stock j are reported in the second row of Table 4 for the test statistic computed without bias correction. We observe for 40 out of 53 stocks that there is a negative effect of the reverse splits on the illiquidity trend – corresponding to an improvement in liquidity conditions. Among those, there are 32 stocks for which the decrease in the illiquidity trend after a reverse split is statistically significant.[15] The fact that reverse splits result in a significant decrease in the illiquidity trend for the majority of the stocks considered is in line with the results in Han (1995). The pronounced improvement in liquidity conditions for our sample of stocks with low pre-event price levels is consistent with the evidence in Blau et al. (2023) that short-selling activity increases after reverse splits – in part because reverse splits ease the constraints on short selling for low-priced stocks (Kwan et al. (2015)).

We also consider the robustness of our results to the inclusion of a control group to account for other events that can potentially impact all stocks at the event time (see Equation (13) in Section 3.2). The test statistics for the synthetic control approach (Abadie (2021)) are reported in the third row of Table 4. Our conclusion that reverse stock splits significantly decrease the illiquidity trend level is robust to controlling for common changes in stock trends around the event time via the synthetic control approach.

[14]See also Section 4.2 for additional detail on the empirical implementation.

[15]We also consider the robustness of our results to the use of the bias-corrected and the honest confidence interval approaches. In the bias correction case, 37 out of the 53 stocks experienced improved liquidity conditions after a reverse split, and the improvement is statistically significant for 34 of them. When adopting the honest confidence interval, 36 out of the 53 stocks experienced improved liquidity conditions with 32 being statistically significant.

Table 4: Statistics for testing permanent breaks in the liquidity series.

	AAON	ACLS	AIG	ARWR	ASRT	BANR	BCEI	BCOR	BKNG	C	CAR
Split size	1-4	1-4	1-20	1-10	1-4	1-7	1-111.6	1-10	1-6	1-10	1-10
τ_w	-0.09	-2.71	-21.37	-10.53	4.42	15.40	-8.41	-7.87	-11.77	8.64	-17.14
$\tau_{w,SC}$	-2.63	-1.02	-6.75	-9.52	3.98	5.02	-14.07	-3.48	-8.38	0.15	-23.24

	CBB	CCOI	CIEN	CIVI	COO	CPE	CPF	CSII	CYTK	EPAC	EXPR
Split size	1-5	1-20	1-7	1-111.6	1-3	1-10	1-20	1-10	1-6	1-5	1-20
τ_w	-0.01	-2.20	3.83	-8.41	0.49	-2.71	-9.85	-6.97	-24.56	-14.86	-0.46
$\tau_{w,SC}$	3.77	-3.49	-6.72	-14.53	0.44	-1.51	-16.14	-9.19	-17.34	-6.90	-0.63

	FBP	FTR	HAFC	HPR	HSKA	IART	KEM	KLXE	LCI	LPI	MSTR
Split size	1-15	1-15	1-8	1-50	1-10	1-2	1-3	1-5	1-4	1-20	1-10
τ_w	-4.26	-1.90	-7.65	-4.10	-7.48	4.55	-3.93	1.02	-2.19	-7.99	-19.93
$\tau_{w,SC}$	-4.11	-0.86	-10.26	-1.59	-8.29	5.17	-1.94	2.64	-2.28	-7.75	-9.44

	MTH	NEU	ODP	OPCH	PFBC	PPBI	RRC	SANM	SBCF	SNV	SPPI
Split size	1-3	1-5	1-10	1-4	1-5	1-5	1-15	1-6	1-5	1-7	1-25
τ_w	-0.63	-4.48	-4.76	1.27	-4.15	0.47	-1.33	-13.75	-8.22	-1.69	6.36
$\tau_{w,SC}$	0.19	-7.12	1.09	1.06	-4.49	0.61	-2.30	-9.02	-7.76	-9.33	8.23

	SSP	THRM	TISI	UCBI	UFI	UIS	VIAV	XPO	ZD
Split size	1-3	1-5	1-10	1-5	1-3	1-10	1-8	1-4	1-4
τ_w	12.78	1.05	-2.22	0.66	-6.77	-18.41	-2.24	-0.09	-3.33
$\tau_{w,SC}$	19.07	0.58	-1.53	-3.78	-12.32	-10.75	-0.20	0.25	-3.66

Note: SC stands for synthetic control.

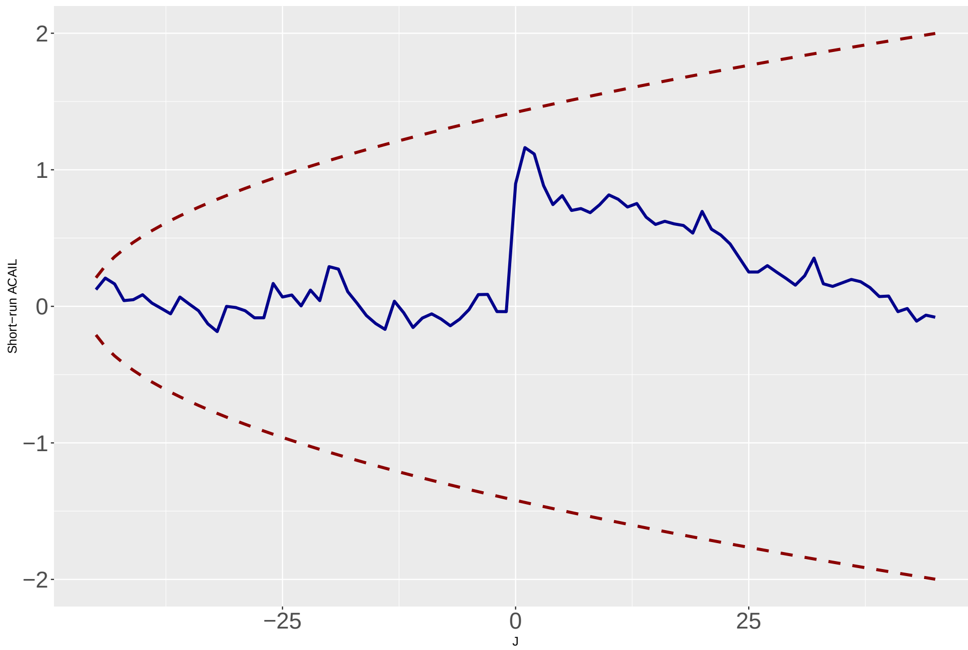

Figure 3: Test for temporary effects. The blue line is the aggregated test statistic and the red lines are the corresponding 2.5% and 97.5% quantiles.

We plot in Figure 3 the average test statistics for the temporary effects of the reverse stock splits aggregated across all firms.[16] The results echo our analysis in Section 4.3 for stock splits, with limited evidence supporting a statistically significant effect of reverse splits on short-term illiquidity. However, we do observe a short-lived worsening in liquidity conditions around the execution date although it is not significant. The test statistics for the temporary effects of individual reverse splits are plotted in Figures 14 to 19 (Appendix A.3). We can see that only a handful of stocks exhibit significant changes in cumulative abnormal illiquidity during the event window and, as in the stock split case, no clear pattern emerges for the impact of reverse splits on short-term illiquidity. We note however that the worsening in liquidity conditions observed for some stocks before the execution date is consistent with reverse splits signaling a lack of confidence from executives in the prospects of their firm (Han (1995)).[17]

To summarize, our empirical evidence suggests that reverse stock splits have an overall significant permanent effect on the long-run trend level of illiquidity but limited effects on the short-run illiquidity dynamics. The documented decrease in stocks' long-run illiquidity

[16]See Sections 3.3 and 4.3 for additional detail.

[17]See e.g. NewMarket Corporation and Avis Budget Group Inc.

following a reverse split is the symmetric image of our results in Section 4.2 for stock splits and confirms earlier evidence in the literature such as Han (1995).

6 Conclusions

We propose a framework to detect the occurrence of permanent and transitory breaks in the illiquidity process. Our approach builds on the class of dynamic semiparametric models introduced in Hafner et al. (2023), which flexibly capture long-term trends with a nonparametric component and short-run variations with an autoregressive component. We develop various tests that can either be applied separately to individual events or can be aggregated across different events – over time for a given firm and/or across different firms. The test for permanent breaks in the long-run component of the illiquidity process is built on differences between forward- and backward-looking trend estimates, which is similar to the approach used in the regression discontinuity and structural break literature. The test for transitory breaks in the short-term illiquidity dynamics is inspired by the event-study approach pioneered by Fama et al. (1969) and is robust to the presence of permanent breaks in the long-run component of illiquidity.

Equipped with this testing framework, we revisit the long-standing debate surrounding the impact of stock splits on firm liquidity. Using a sample of 24 stocks from the Dow Jones index over the period 1992-2023, we find that stock splits have an overall significant permanent effect on the long-run trend level of illiquidity but very limited effects on the short-run illiquidity dynamics. Our results are in line with previous studies documenting a permanent decrease in stock liquidity after a split (e.g. Copeland (1979)) and challenge the common view that stock splits should increase the potential pool of investors and lead to improved liquidity conditions. Finally, we investigate whether the effects of reverse stock splits on the illiquidity process are symmetric to the ones documented for stock splits. Using a sample of 53 low-price stocks from the constituents of the S&P 500, S&P 400 and S&P 600 indices over the same period, we find that reverse splits result in an overall significant decrease in long-run illiquidity but with limited effects on the short-run illiquidity dynamics. The improvement in liquidity conditions is quite pronounced for our sample of stocks with low pre-event price levels. This is consistent with evidence that short-selling activity increases after reverse splits (Blau et al. (2023)) for low-priced stocks as constraints are eased.

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Appendices

A Additional tables and figures

A.1 Permanent and temporary effects for each stock split

For each split of each stock, we plot in Figures 4 to 11 the results of testing for permanent breaks and temporary effects of stock splits. The upper panel of each figure presents the results for permanent effect. The test statistic is marked by a red dot while the two bars indicate the 2.5% and 97.5% quantiles. We can observe that most of the test statistics on stock split dates are outside the critical value bands. This suggests that, overall, stock splits have positive and significant effects on the long-run trend level of the illiquidity process.

The lower panel of each figure presents the results for temporary effects where the blue line represents the test statistic together with the 2.5% and 97.5% quantiles marked by the red lines.[18] The figures show that the effect of stock splits on the short-term dynamics of liquidity is rarely significant with only very few exceptions.

[18]Note that for the split events preceded by another one we only consider the period after the first stock split event for the computation of the quantiles.

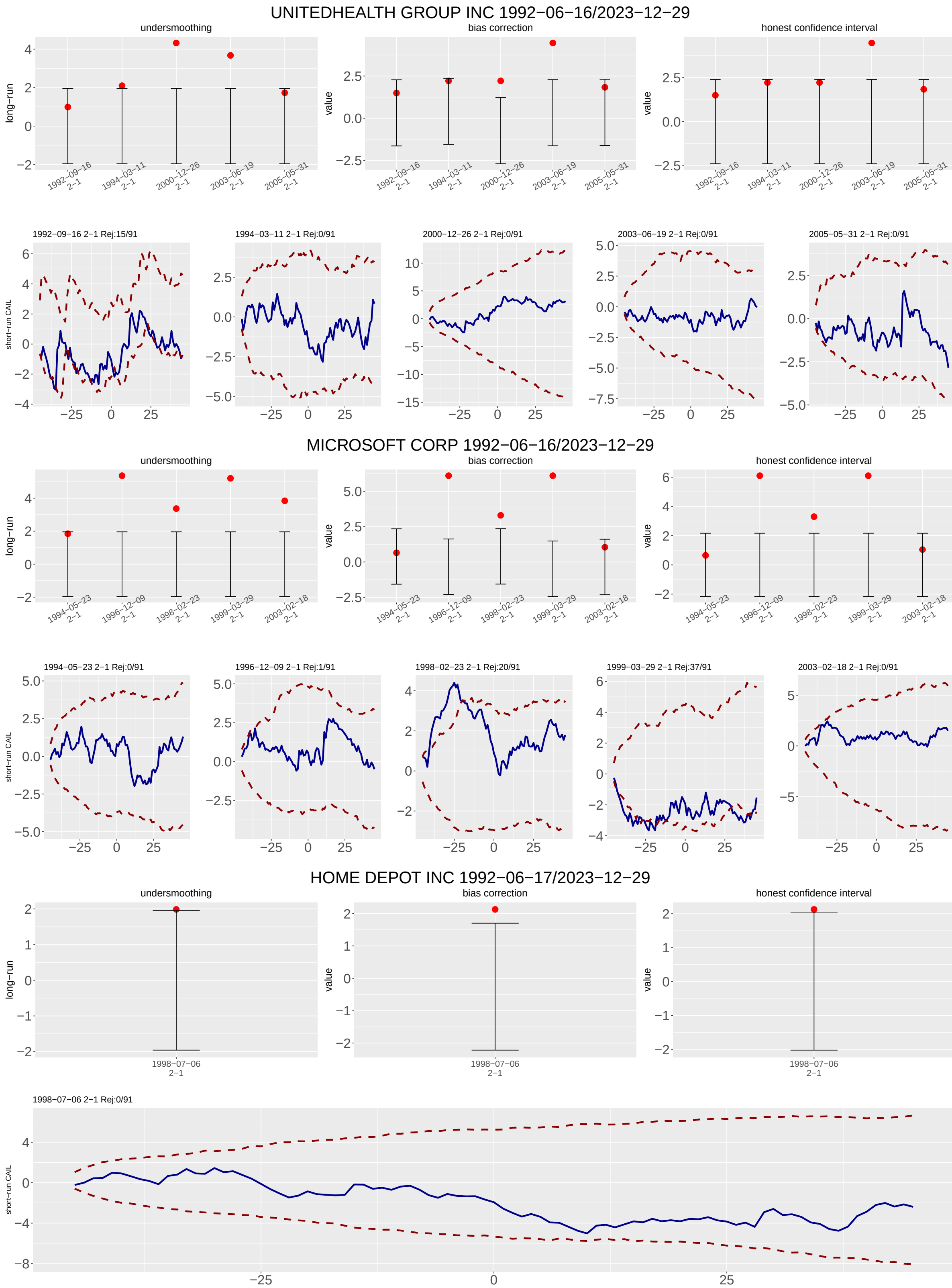

Figure 4: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

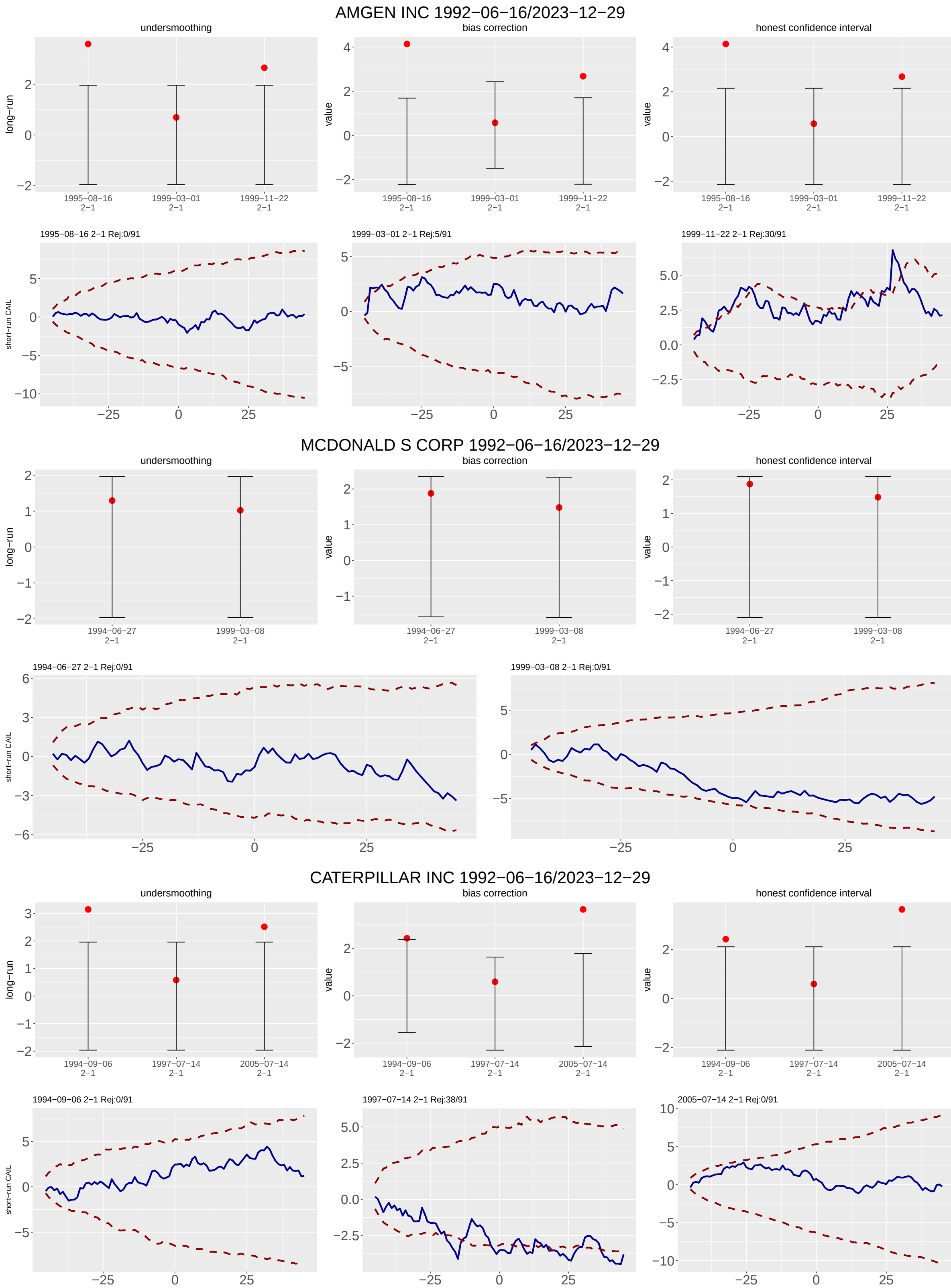

Figure 5: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

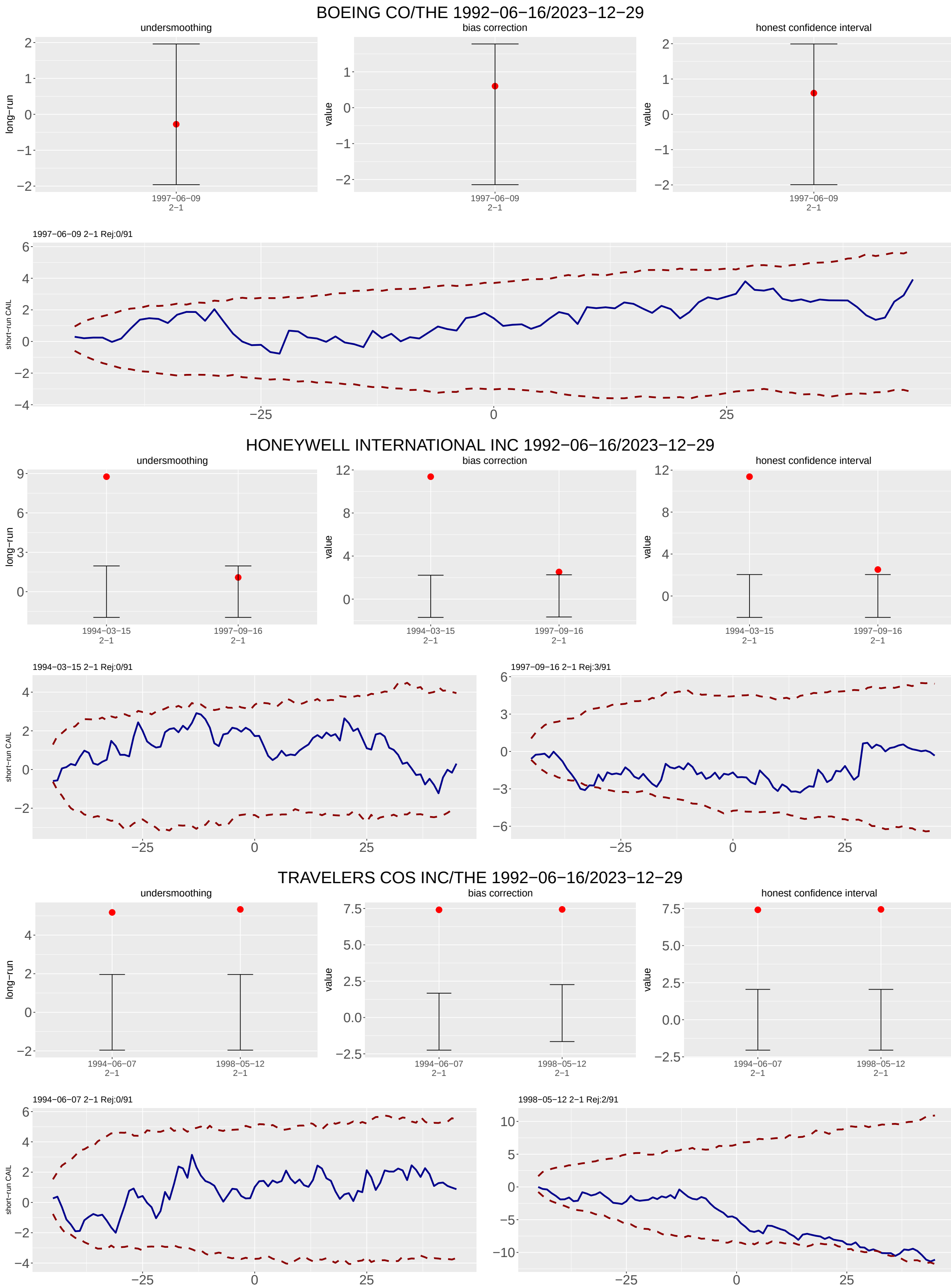

Figure 6: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

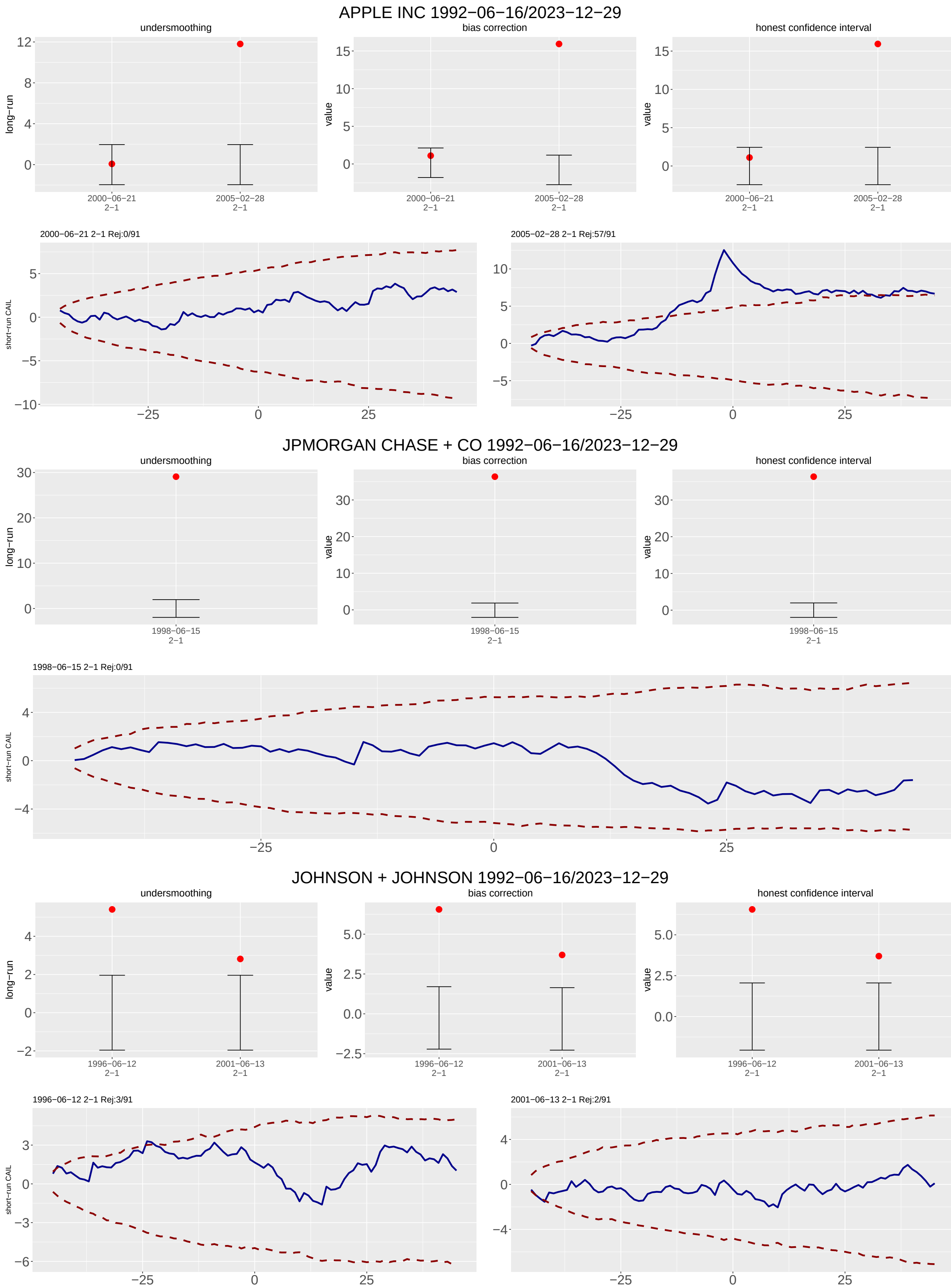

Figure 7: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

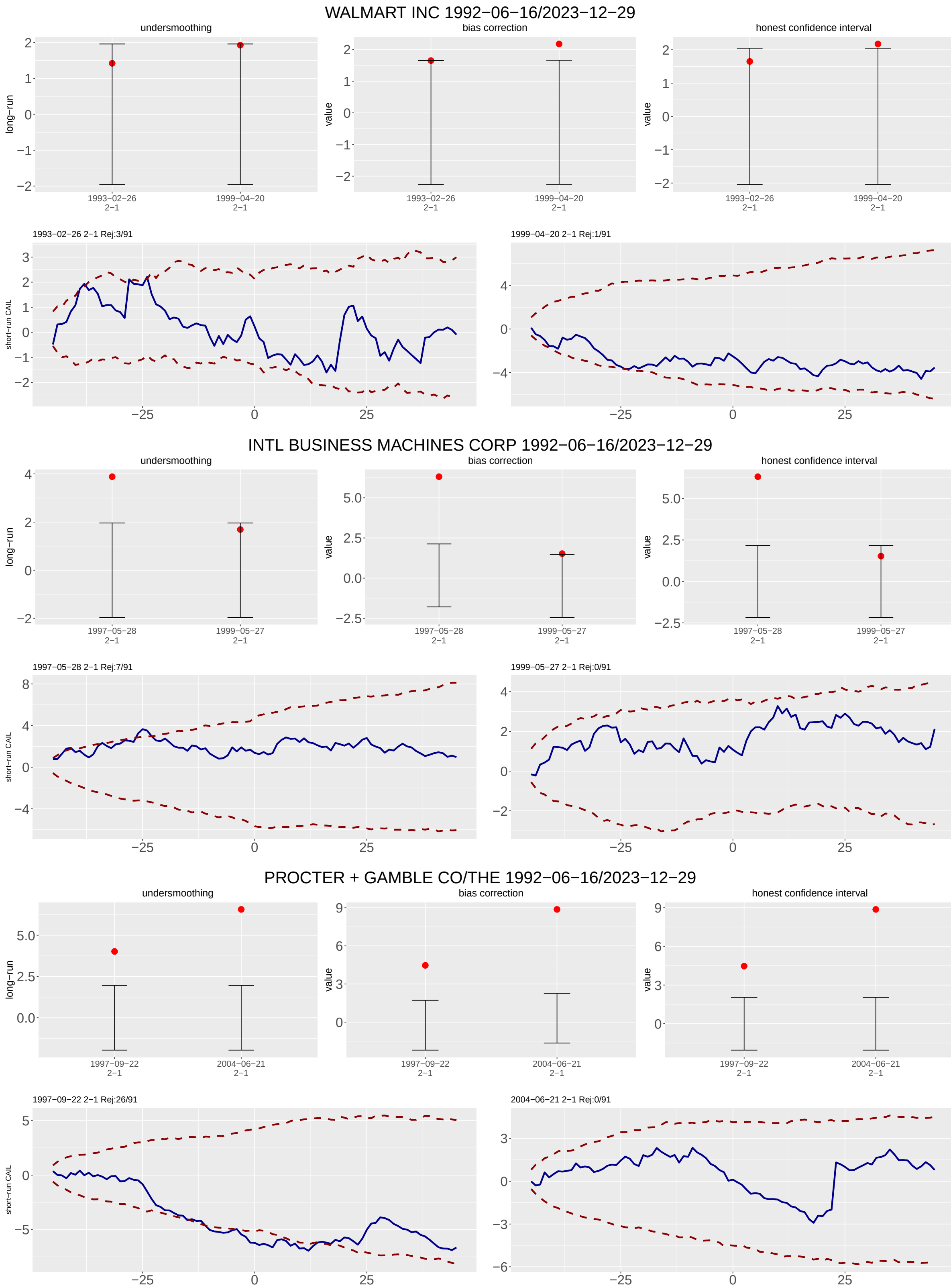

Figure 8: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

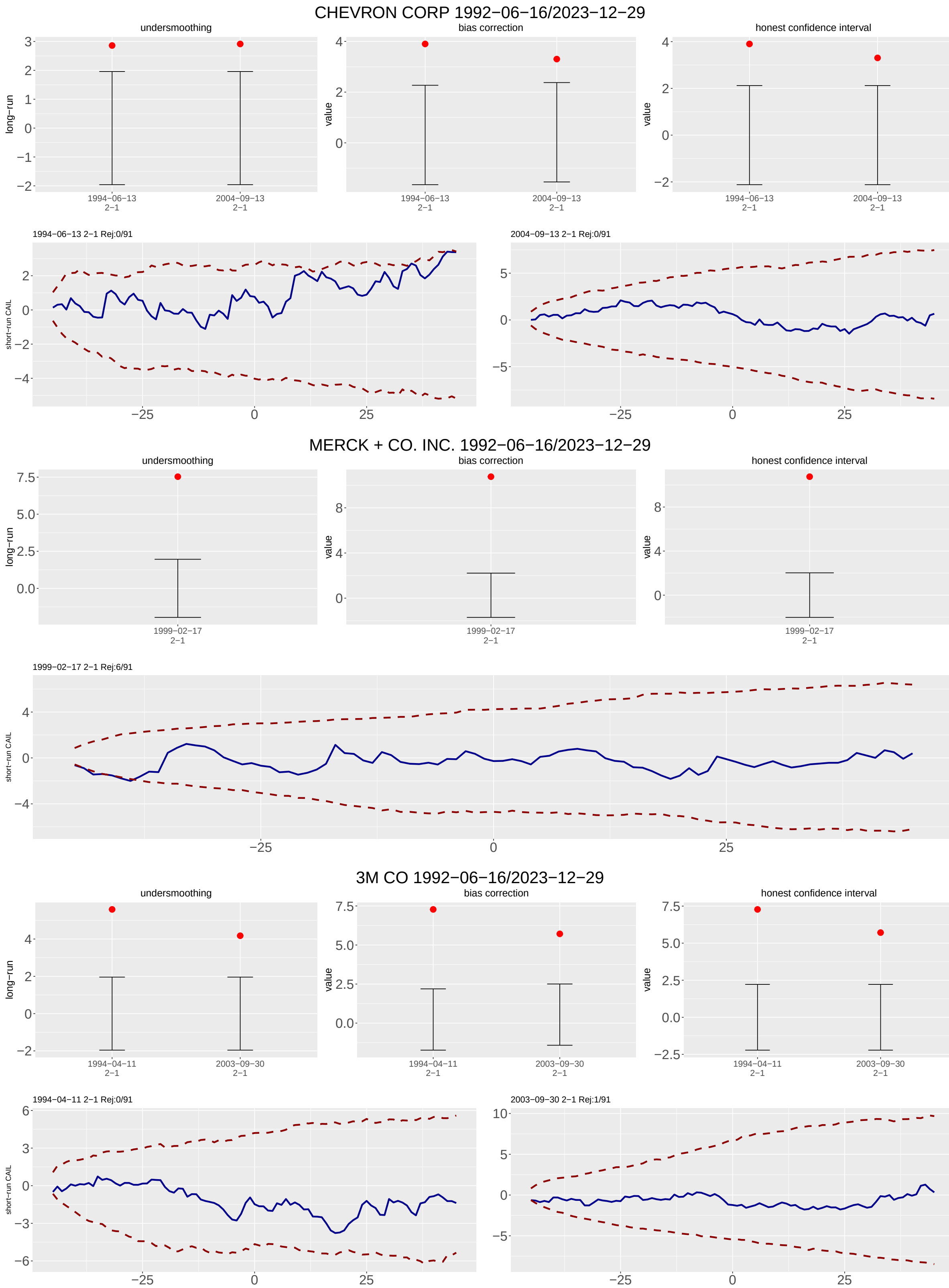

Figure 9: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

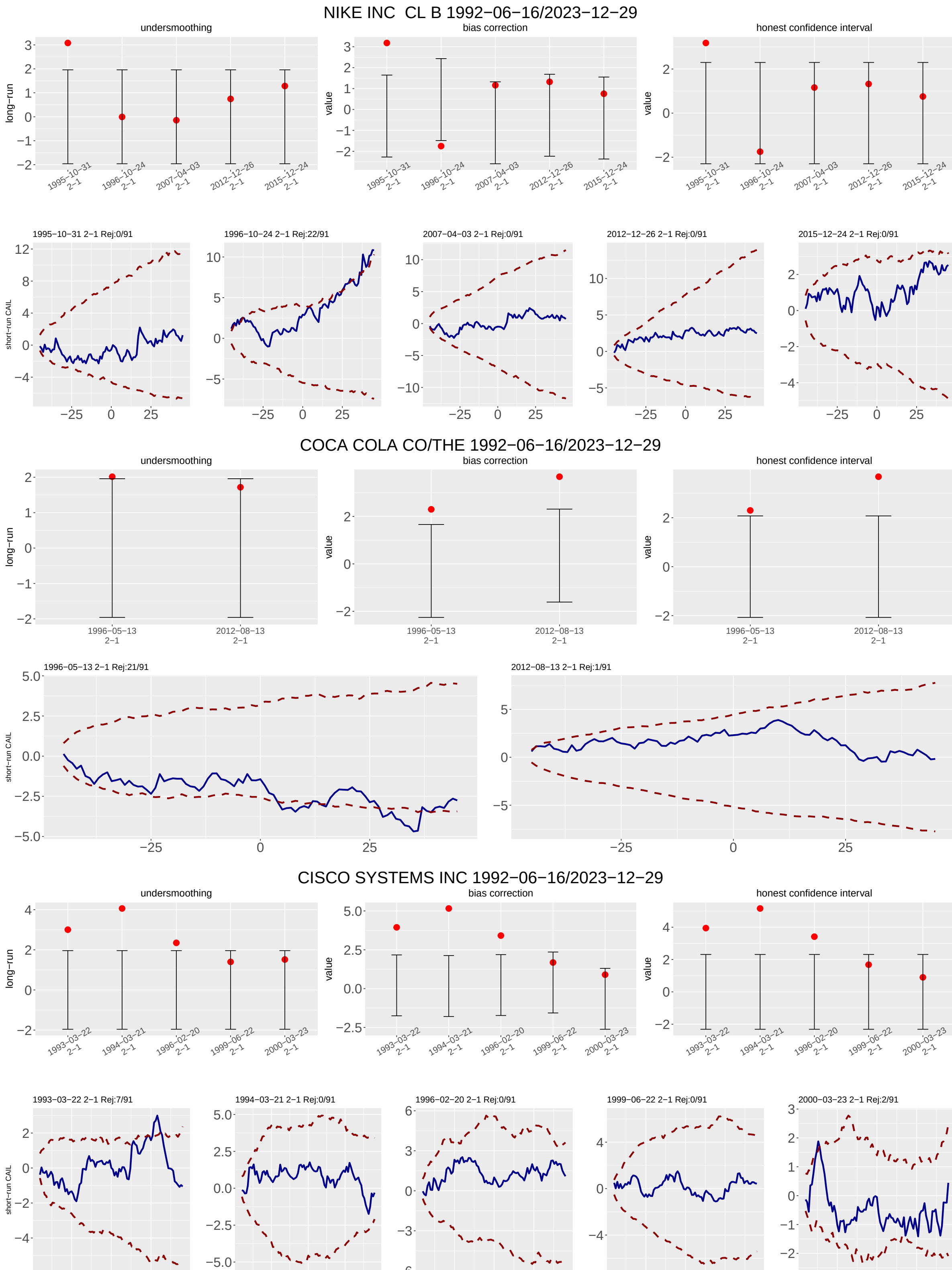

Figure 10: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

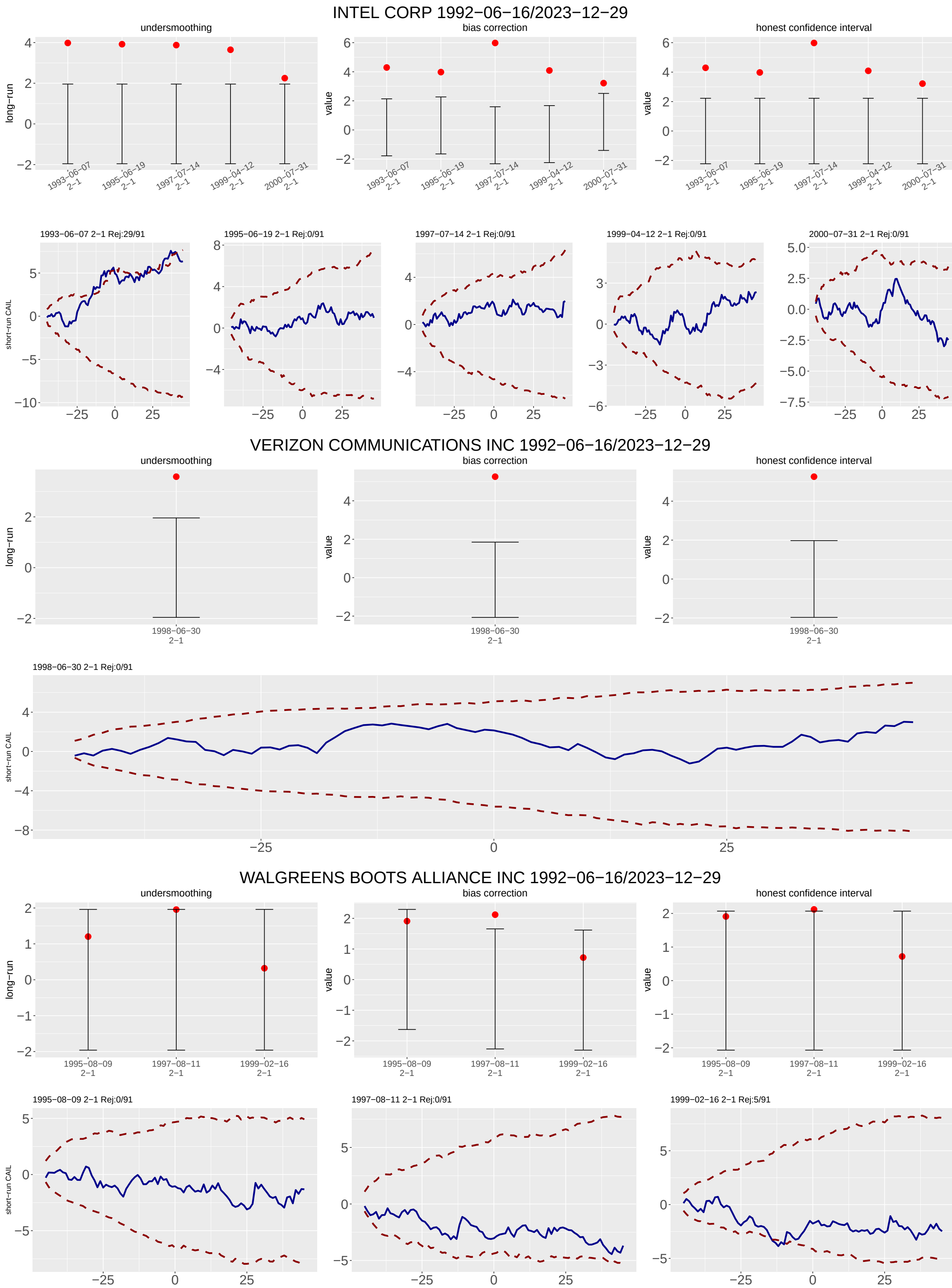

Figure 11: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the test statistic and the red lines are the 2.5% and 97.5% quantiles.

A.2 Temporary effects aggregated across splits for each firm

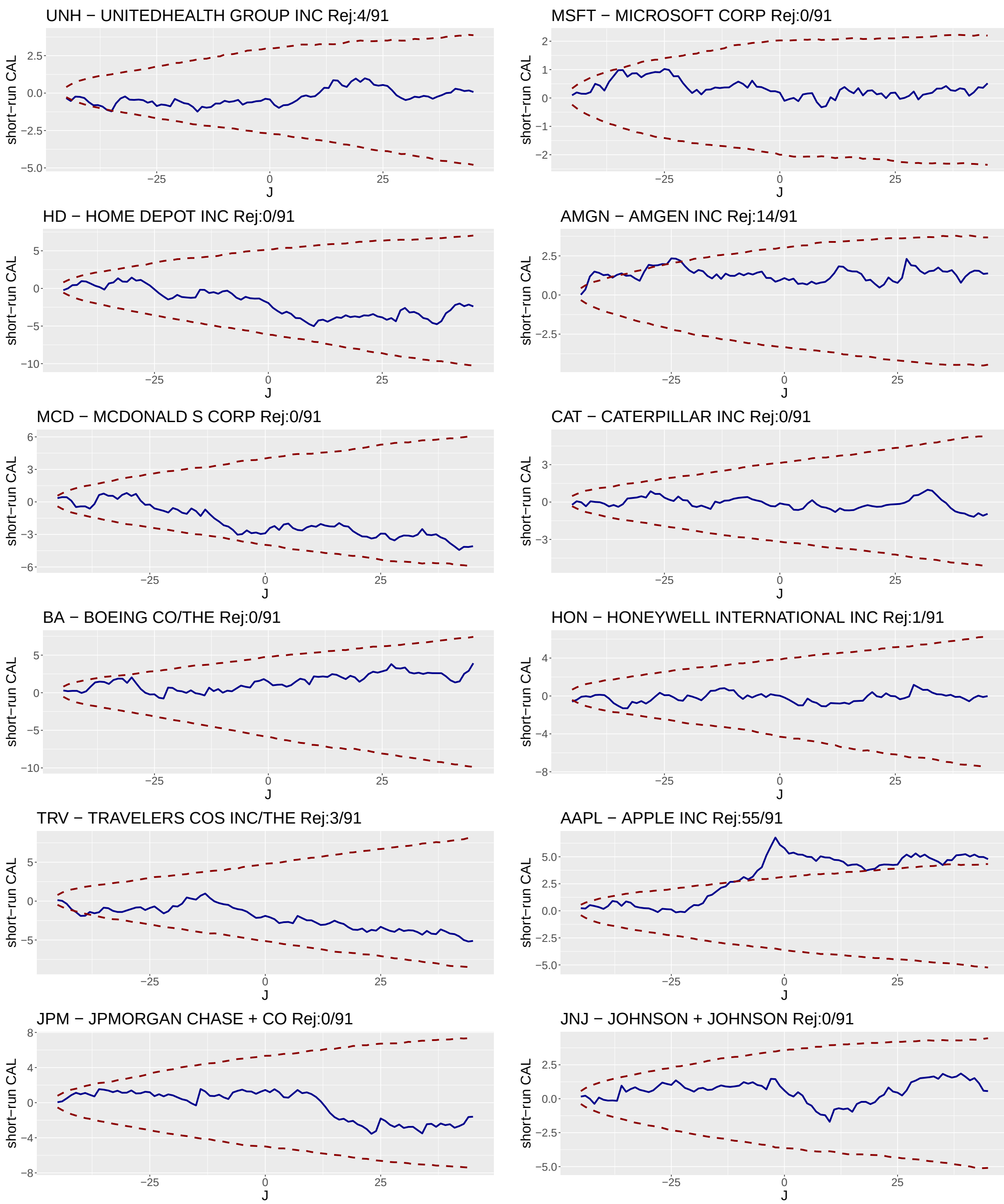

Figure 12: Test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

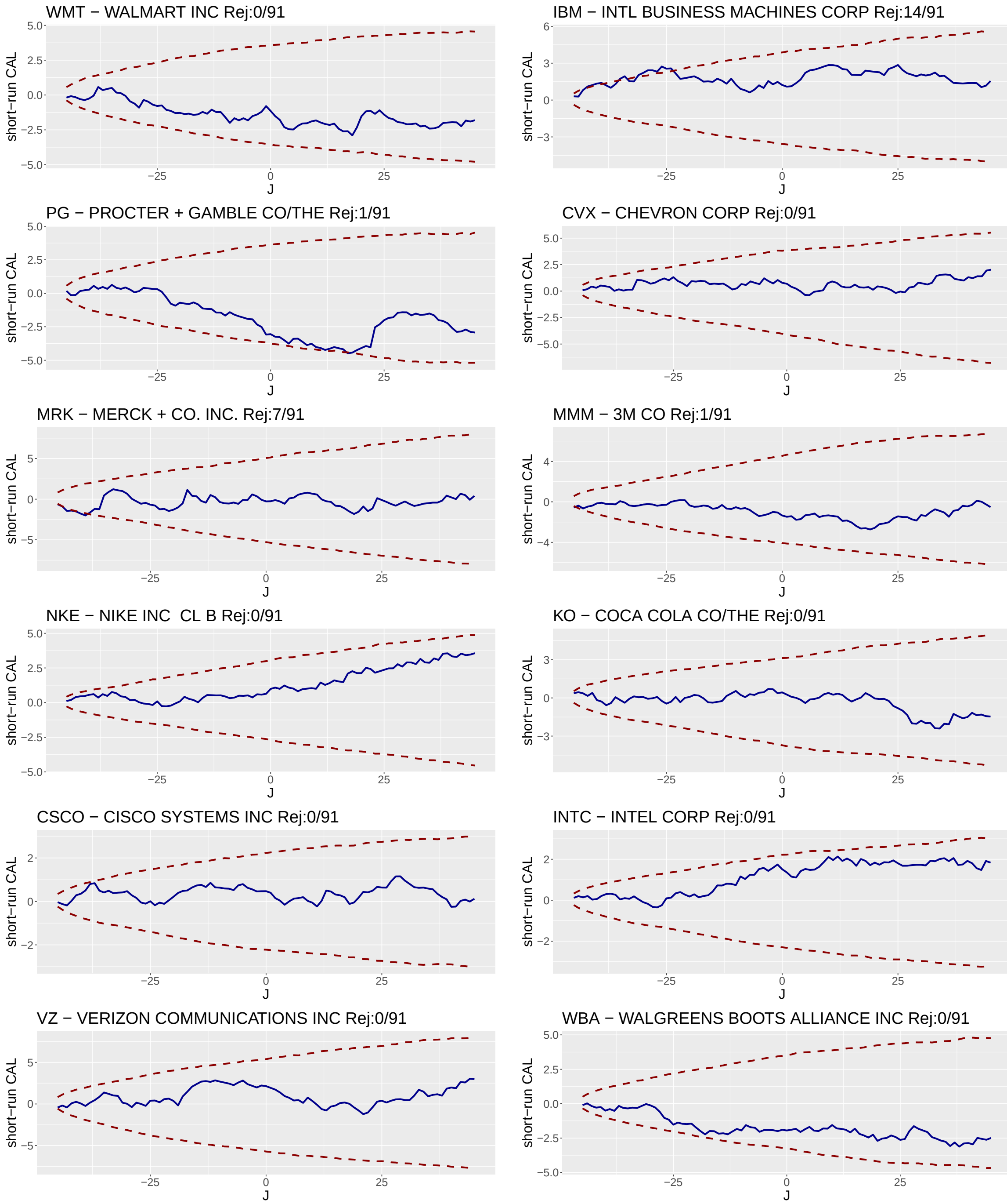

Figure 13: Test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

A.3 Permanent and temporary effects for each reverse split

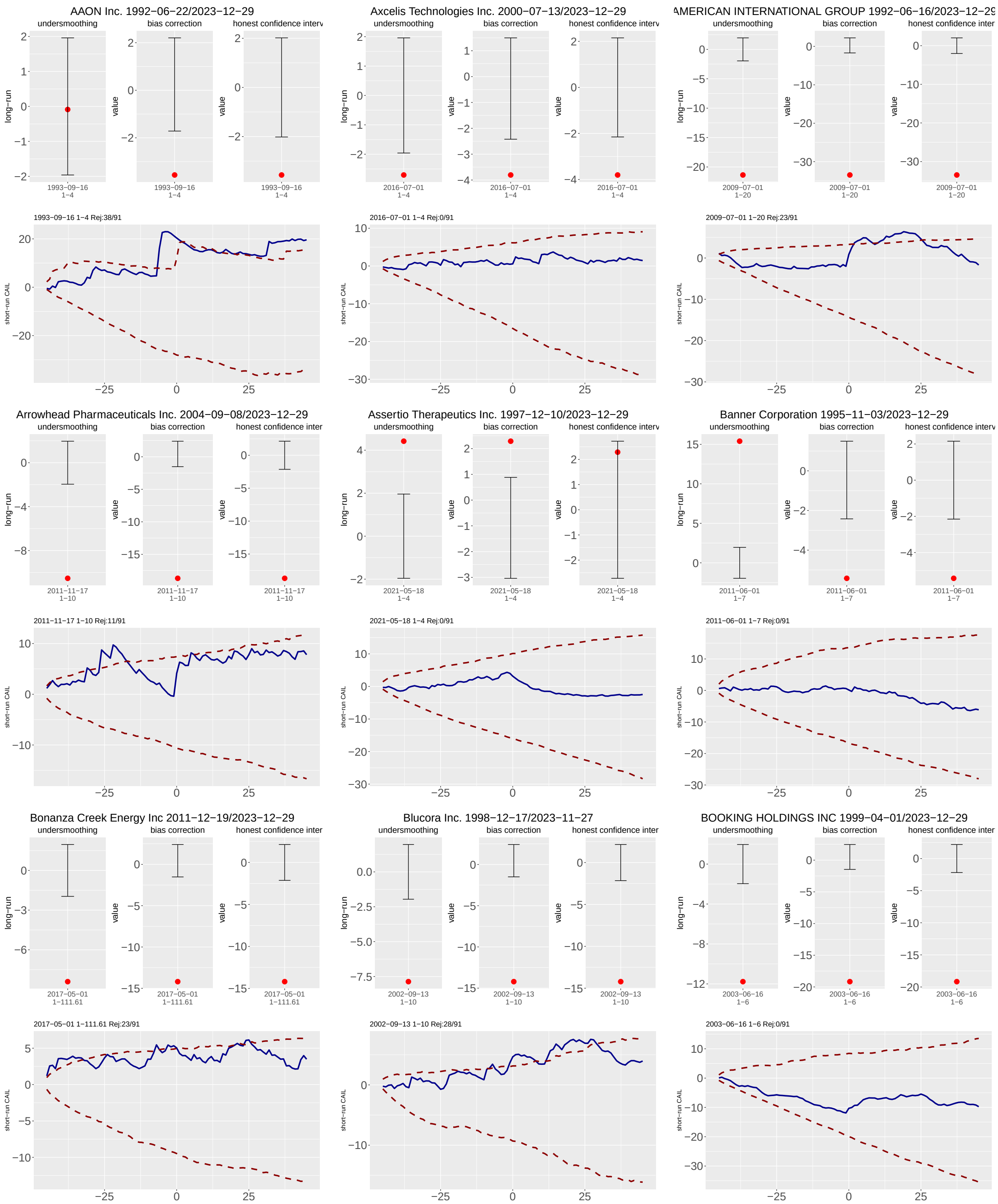

Figure 14: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

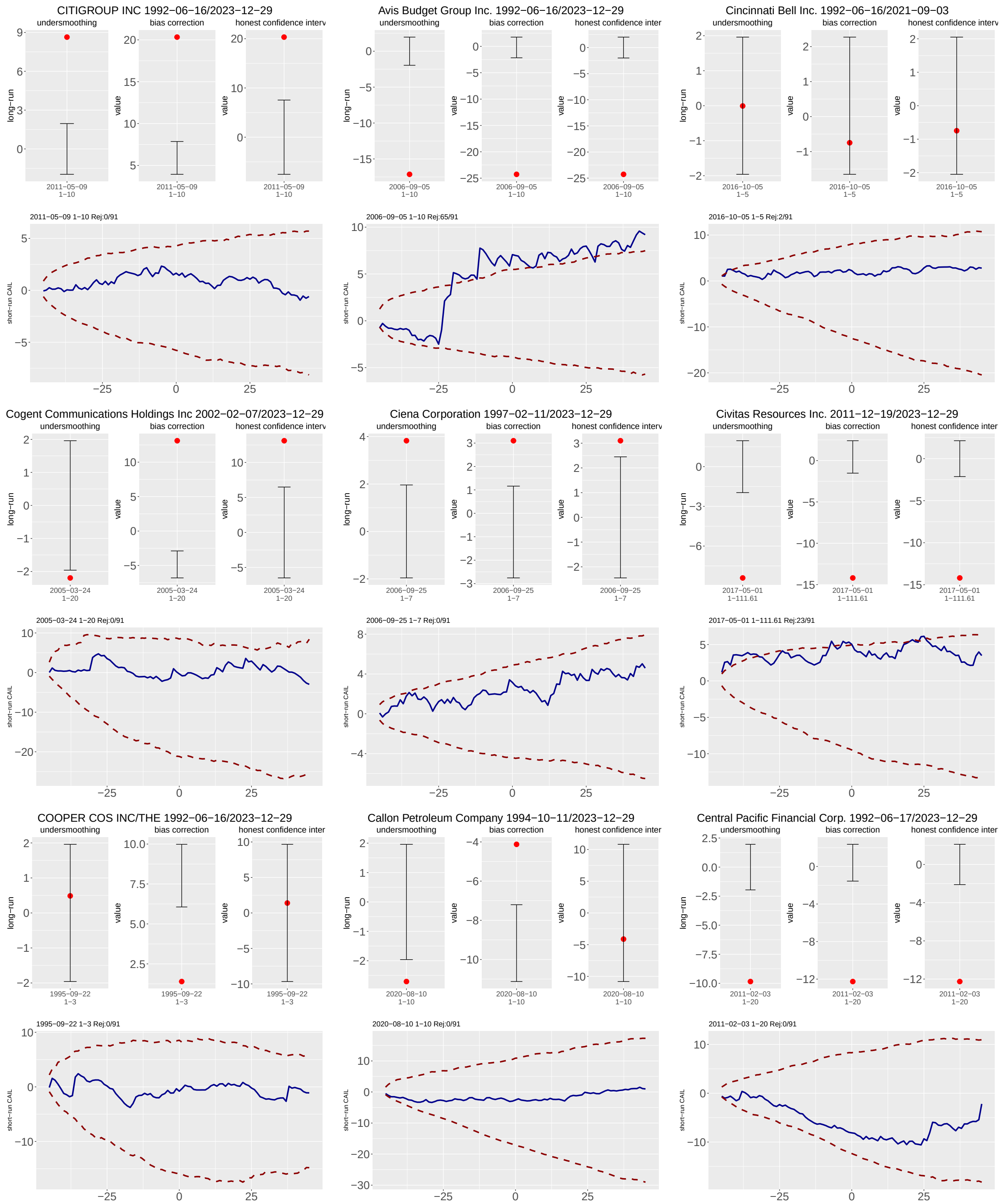

Figure 15: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

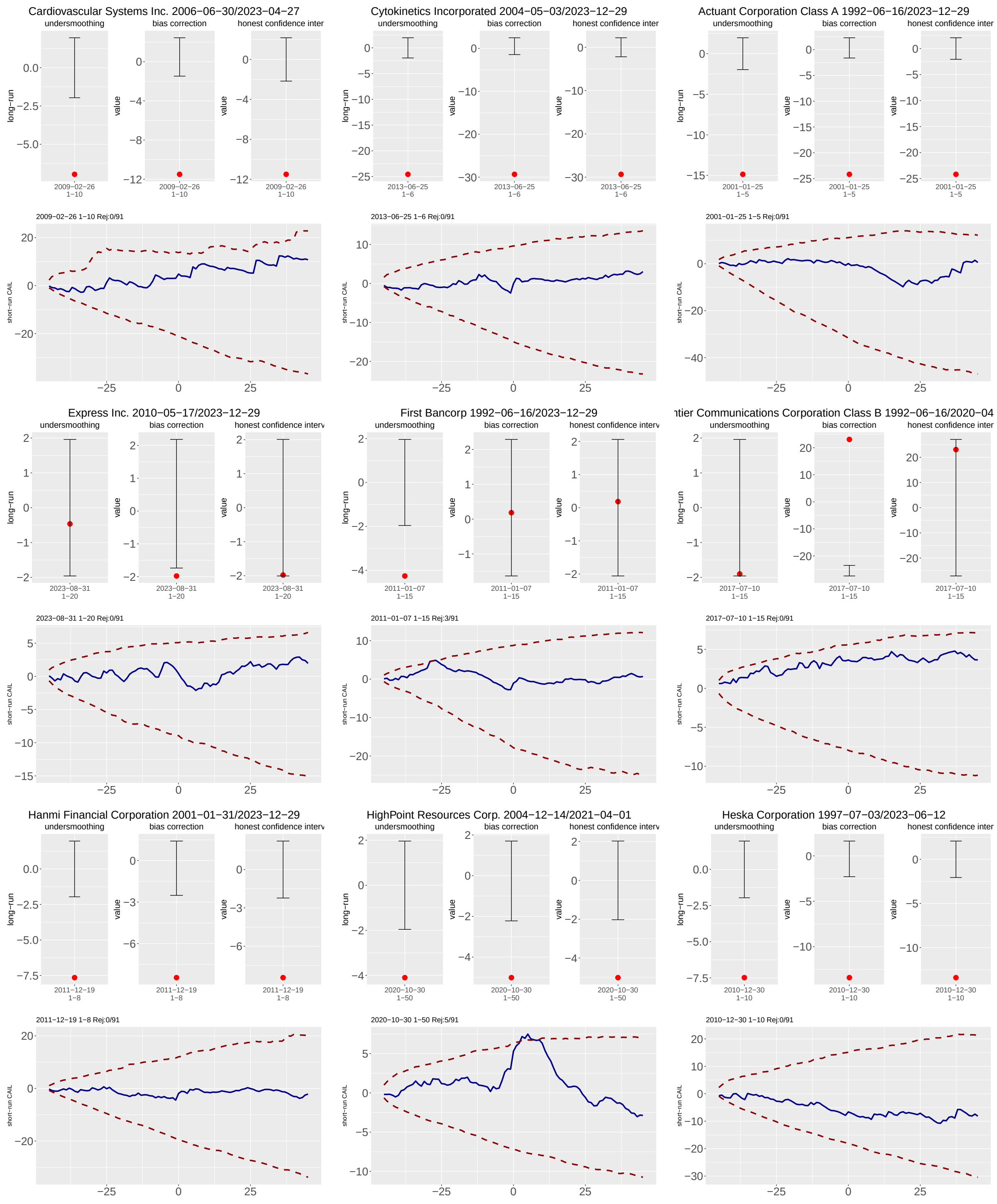

Figure 16: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

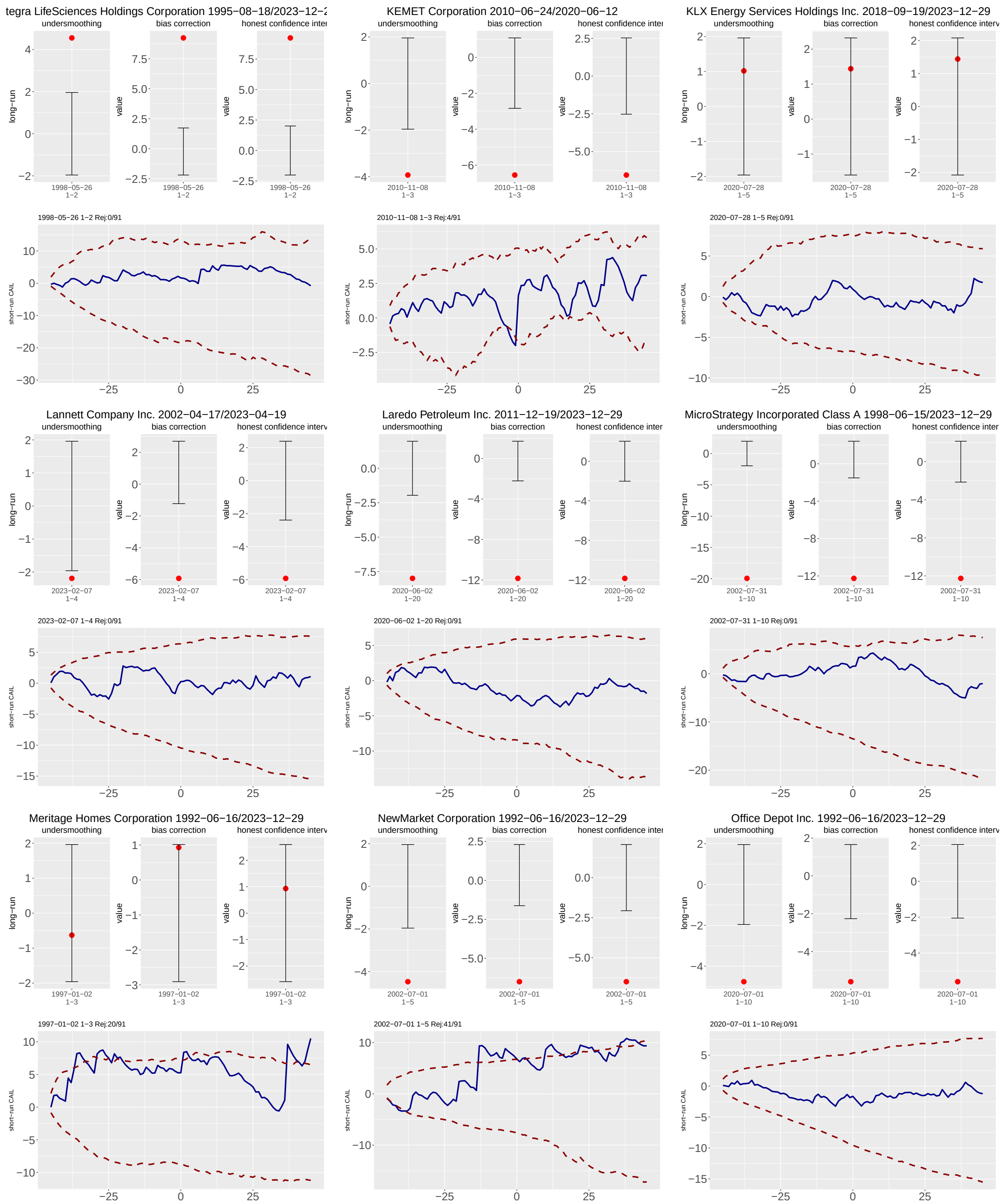

Figure 17: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

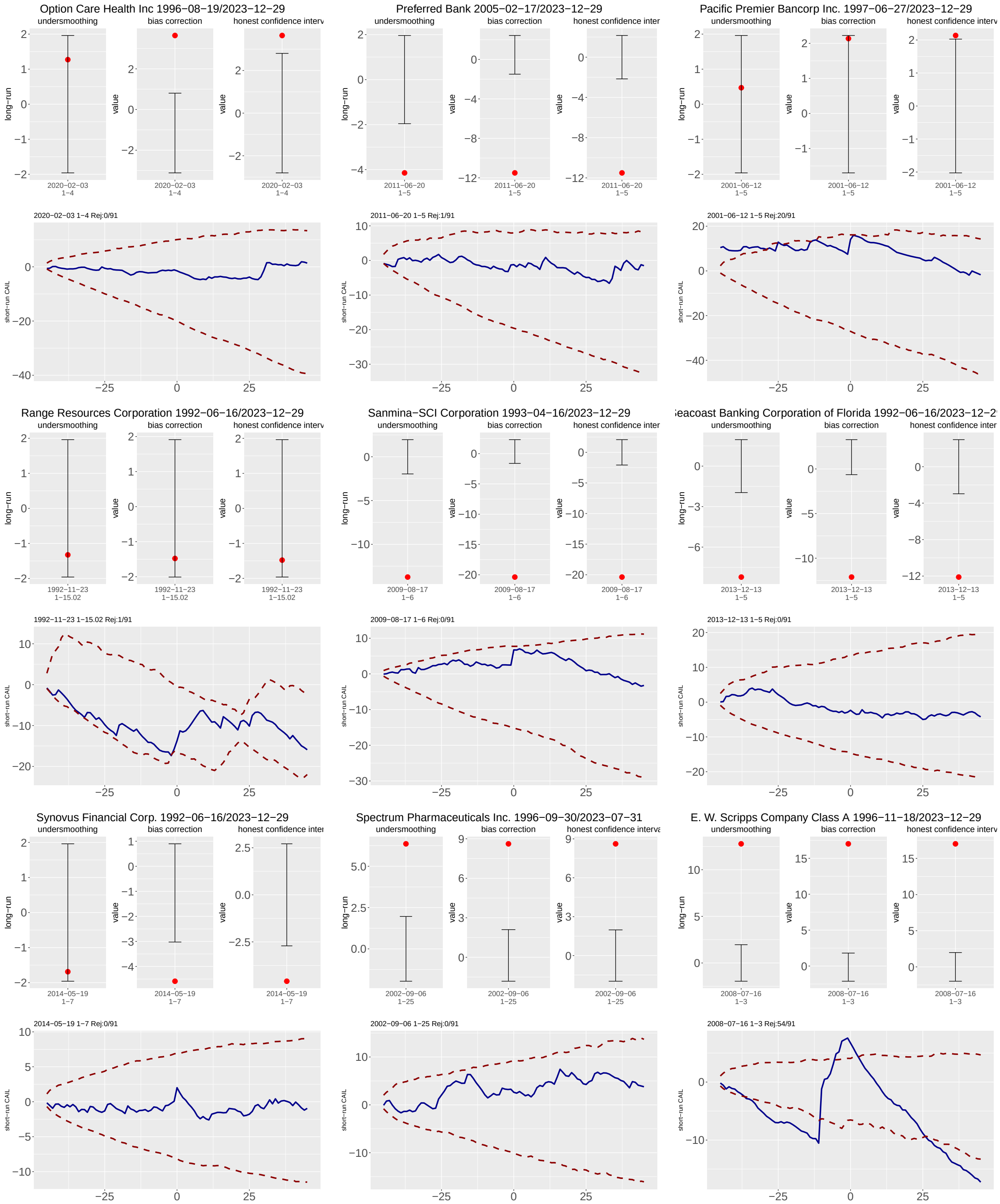

Figure 18: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.

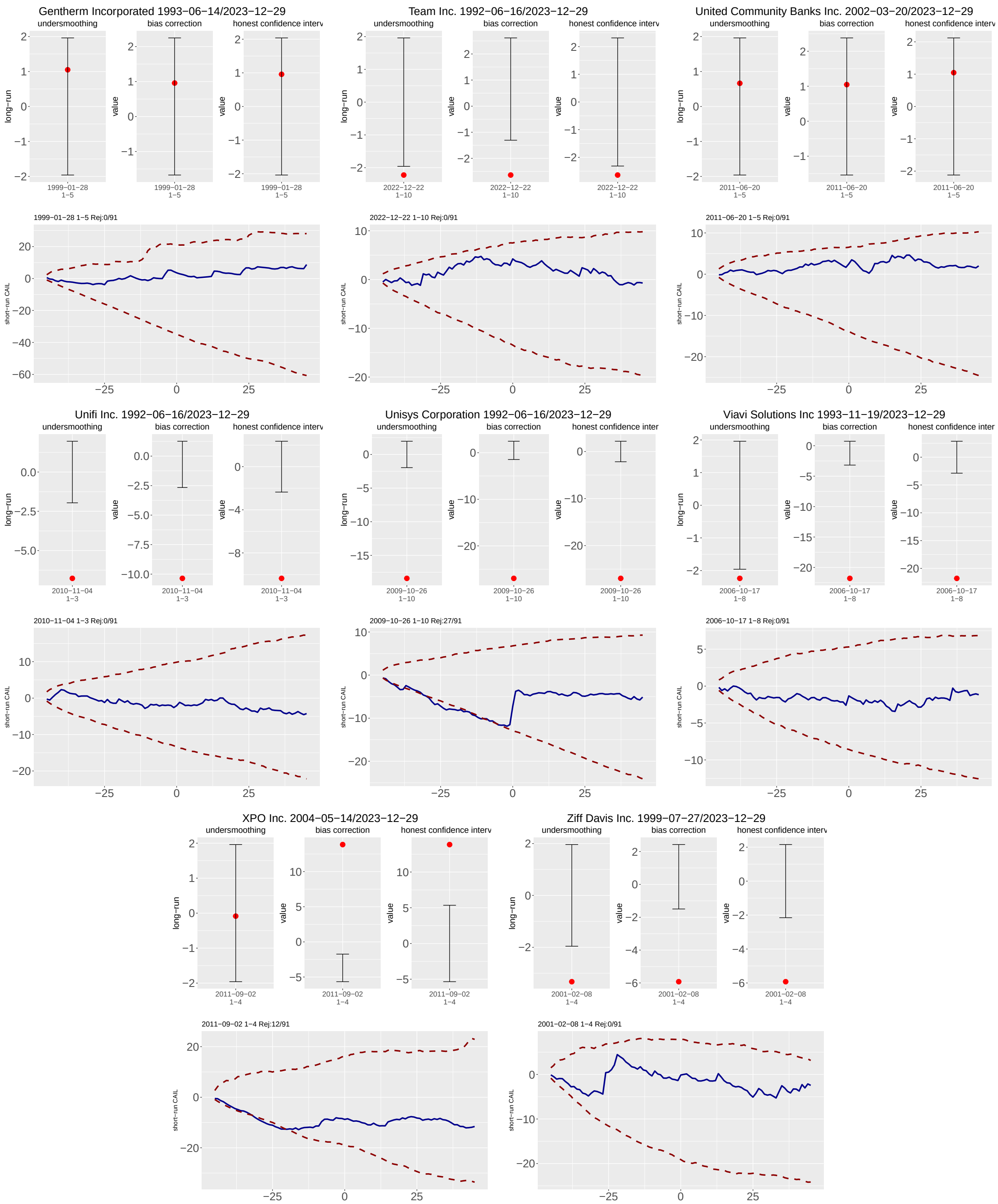

Figure 19: Upper panel: test for permanent effects. The red dot is the test statistic with the two bars marking the two-sided critical values with $\alpha = 5\%$. Lower panel: test for temporary effects. The blue line is the aggregated test statistic and the red lines are the 2.5% and 97.5% quantiles estimated based on data excluding selected window around splits.