## New Keynesian Pricing Behaviour: an Analysis of Micro Data

James Cloyne, Lena Koerber, Martin Weale and Tomasz Wieladek\*
Bank of England,
London EC2R 8AH,
United Kingdom

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#### Abstract

A survey conducted by the Conferderation of British Industry collects a range of data including firms' responses to questions about price increases in the previous twelve months and expected price increases in the coming twelve month. We use these data to estimate a new Keynesian pricing equation in which price changes depend on expected price changes and nominal prices relative to marginal costs. Unit wage costs provides a measure of costs which satisfies the restrictions imposed by price homogeneity, but is most appropriate when firms produce with constant returns to scale. We find a large subgroup of the sample does produce with constant returns to scale and, for this subgroup we find a coefficient on expected price changes of 0.985( 0.675 to 1.30) in the pricing equation entirely consistent with new Keynesian theory.

<sup>\*</sup>We are most grateful to the Confederation of Birtish Industry for making available the data used in this study. The views expressed in this paper are those of the authors and not of the Bank of England or the Monetary Policy Committee.

#### 1 Introduction

An understanding of price dynamics is core to monetary policy-making, and in this paper we explore the link between price changes, expected price changes and costs, making novel data supplied by individual firms. The theoretical framework we adopt is that provided by Rotemberg (1982). His assumption that monopolistic firms face costs to changing prices is used to derive a relationship between price changes, expected future price changes and marginal costs of production. If there are fixed costs to changing prices, as well as quadratic costs associated with price changes, then prices will remain fixed for some interval; the fixed element means that firms will not make very small changes (Rotemberg 1983). The implications of this framework are, in broad terms at least, not very different from the assumption, due to Calvo (1983), that only a proportion of firms can change their prices at any particular time, and it underpins the New Keynesian Phillips curve which is at the core of modern macro-economic analysis (Gali 2008). It has been used widely in the estimation of dynamic stochastic general equilibrium models, usually by means of Bayesian updating of prior estimates of the parameters.

There have been a number of studies looking at individual prices or prices set by individual firms, tending to focus on the degree of price stickiness rather than fitting Rotemberg's model. Lach & Tsiddon (1992) studied prices in Israel between 1978 and 1984, a period of rapid inflation. They found that, even with monthly inflation of 6.6 per cent, on average prices were fixed for six weeks supporting the idea that there were costs to price changes. Levy, Bergen, Dutt & Venable (1997), among others, suggested that in more normal times, retail prices remain fixed for many months. Blinder, Canetti, Lebow & Rudd (1998) surveyed firms in the United States, finding that the median firm changed its prices 1.4 times per year, and that the most important reason for price stickiness seemed to be co-ordination failure, an explicit concern about the responses of competitors, suggesting that, rather than being imperfectly competitive, firms saw themselves in an olgipolistic structure. Cost-based pricing was the second-most important reason for price-sickness; firms tended to change prices when they observed changes in costs. Bils & Klenow (2004) cast doubt on the degree of price stickiness identified by Blinder et al. (1998), finding that, for half of a range of three hundred and fifty consumer goods, price remained fixed for 4.3 months or fewer, even during the stable environment of the United States in the late 1990s. Apel, Friberg & Hallsten (2005) find results more consistent with authors such as Levy et al. (1997), suggesting that in Sweden, the median firm changes prices only once a year, with some firms reviewing prices after a given interval, consistent with Taylor (1980) or, in some sense Calvo (1983) while others reacted to circumstances, more in keeping with Rotemberg (1982). Alvarez, Dhyne, Hoeberichts, Kwapoil, Le Bihan, Lünneman, Martins, Sabbatini, Stahl, Vermeulen & Vilmunen (2006) used price data underlying consumer price and producer price indices in ten countries of the euro area, together with surveys, finding greater stickiness than in the United States, but also that nearly half of firms used both time-dependent and state-dependent price setting. About half of the firms were found to set prices with reference to expected future developments, consistent with the New Keynesian Phillips curve.

The most important reason for price stickiness given was that firms wanted to maintain long-standing customer relationships (implicit contracts) with explicit contracts and a direct relationship of prices to costs coming second and third. This gave higher prominence to implicit and explicit contracts than Blinder et al. (1998) found in the United States. Menu costs and oligipolistic effects were found to be less important, however. Gautier (2009), in a study of producer price data for France for the period 1994 to 2005 confirmed earlier evidence for both time and state dependence, but suggested that timedependence was more important. Loupias & Sevestre (2013), again looking at France found that firms responded more readily to costs than to demand, although of course to the extent that both of these affect marginal cost, it is not clear how far they should be distinguished, at least when looking through the lens provided by Rotemberg. A common feature of these studies is, however, that none of them address the relationship between expected price changes and actual price changes; as far as we are aware none of the surveys used has asked firms about their expectations, while studies based on data collected to compute consumer or producer price indices cannot be expected also to use specific information on expectations. For this reason estimation of the New Keynesian Phillips curve has relied on aggregate data.

At the same time, estimation of New Keynesian Phillips curves from macro-economic data is not straightforward. The standard model consists of an inflation expectations and real marginal cost term as the two main determinants of inflation, together with a cost-push shock. Because inflation expectations are endogenous with respect to inflation, it is necessary to instrument for inflation expectations and marginal costs. A large number of papers have attempted to estimate the parameters of this equation from macroeconomic data with either a GIVE/GMM or VAR approach. For example, Gali

(1999) introduce a lagged inflation term by assuming price indexation into the standard model, which is also referred to as the hybrid New Keynesian Phillips curve. Their estimates of the coefficient on the forward looking inflation term are close .99, the value implied by economic theory. An alternative approach is to use VAR models to extract inflation expectations from the data and estimate the parameters by minimizing the distance between the model predicted and actual inflation out-turns. Shordone (2002) and Shordone (2005) use this technique to estimate the hybrid Phillips Curve on US data and broadly confirms the findings of Gali (1999).

All of the this work is subject to one particular econometric challenge: identification, and in particular with respect to the inflation expectations term. A good instrument needs to be both exogenous and strong (highly correlated with the inflation expectations term). But, because changes in inflation are typically hard to forecast in practice (Stock & Watson 2007), it is difficult to obtain plausible instruments which satisfy the second condition. In practice this means that the results will be dependent both upon the exact econometric specification and choice of instruments. For example, Rudd & Whelan (2007) argue that the approach of Gali (1999) yields spurious results. In particular, they argue that the use of particular instruments (commodity price and wage inflation) pushes the coefficient closer to .99 and that alternative econometric approaches yield smaller estimates. Gali, Gertler & Lopez-Salido (2005) show that, so long their original specification and choice of instruments is used, their finding is robust to alternative econometric estimators. With respect to the VAR approach, Mavroedis, Plagborg-Moller & Stock (2014) show that the presence of the weak instrument problem tends to push the coefficient spuriously closer to unity. Based on their survey of over one hundred papers, which attempt to estimate the NKPC, they conclude that economists have learned all that they can from macroeconomic time series. Their suggestion is to instead focus on either sectoral or microeconomic data to address the weak instrument problem.

In this paper we follow their prescription and use firm-level inflation expectations to estimate the Neo-Keynesian Phillips Curve at level of each individual firm. This granular level of detail allows us to use sector-level instrumental variables, which, unlike those used in previous studies, pass all of the weak identification tests. Our data are taken from the *Industrial Trends Survey (ITS)*, a survey of manufacturing firms in the United Kingdom carried out quarterly by the Confederation of British Industry. The survey collects information on a wide range of variables, including the percentage change in firms' prices over the last twelve months and the expected change in prices over the

next twelve months.

In the next section we describe the survey. Section 3 offers a account of the data. In section 4 we set out the New Keynesian model we use to interpret the data. The results of this are to be found in section 5 while section 6 concludes.

# 2 Data: the Confederation of British Industry Survey

To study the New Keynesian Phillips curve from a micro-econometric perspective, we require panel data providing information on price changes, expected future prices changes, movements in costs and capacity utilisation. The Confederation of British Industry (CBI) has collected such data since 2008Q1 although it has a much longer history in collecting other data on business experience and business expectations. In fact very few responses were collected in 2008Q1 and 2008Q2 so, for practical purposes, the data begin in 2008Q3. The CBI runs a number of surveys; only the ITS, however, provides information in the form that is required. This part of the survey provide around three to four hundred returns. Lui, Mitchell & Weale (2011) examined the data collected in this survey in the period before the economic crisis, and showed that the qualitative answers firms gave to questions about output movements were coherent with the answers the same firms provided in quantitative returns to the Office for National Statistics. This provides an element of underlying confidence in the data.

#### Questions of Interest. The question we are primarily interested in is:

1. What has been the percentage change over the past 12 months in your firm's own average output price for goods sold into UK markets and what is expected to occur over the next 12 months?

Firms can answer these questions by choosing one of 11 buckets or by entering their own answer manually. The midpoints of the buckets range from -9% to +9%. We put the manual answers into the corresponding buckets. If the manual answers lie outside the bucket ranges, they are allocated to the largest bucket on either side.<sup>1</sup> Thus we have, in effect Winsorised the price data, with responses below -9% or above 9% entered as -9% and 9% respectively.

We also use in our estimation the data collected by a question on capacity utilisation:

<sup>&</sup>lt;sup>1</sup>This treatment does not affect our results as there is only a small number of firms that enter their answers manually.

- 2. What is your capacity utilisation, measured as a percentage of full capacity?
- On top of this we use the responses to the following questions to provide instruments for our subsequent estimation:
- 3. Excluding seasonal variations, what has been the trend over the past three months with regard to average costs per unit of output?
- 4. What factors are likely to limit (wholly or partly) your capital expenditure authorizations over the next twelve months? To answer this question, firms can select multiple factors out of: inadequate net return on investment; uncertainty about demand; shortage of internal finance; shortage of labour, including managerial and technical staff; inability to raise external finance; cost of finance; other; n/a. From these answers, we only use cost of finance.

Sample Demographics In principle the survey is a panel, with firms approached repeatedly. As table 1 shows, however, the number of exits and re-entrants is large relative to the sample size, pointing to substantial, if sometimes temporary non-response. In part the reason behind this is that the Survey is intended to provide a rapid snapshot of the state of the economy; the CBI does not follow up very late respondents and does not revise the figures after they have been published. Unlike official statistics, its output is not intended to provide a historical record. Table 1 also shows the number of respondents in each quarter, classified by employment size. About half of the respondents have between 75 and 149 employees.

Rapid rotation means that the survey does not remotely resemble a balanced panel. Our data set covers twenty-one quarters; the average number of responses from each respondent is 5.7. Out of the 2,738 firms which reply to the survey at some point, only twenty firms provide complete records. This feature of the data set obviously imposes a major limitation on the way in which we analyse its contents.

Descriptive statistics. Expected own price increases are about 1% on average which is significantly below realized consumer price inflation rates during the period in question. The largest single factor accounting for this difference is probably that output prices were less affected than consumer prices by the sharp increase in the price of imports which followed sterling's depreciation in 2007-8 and the subsequent increase in raw material prices. Output prices are also net of VAT; the VAT rate increased from 17.5 per cent at the start of the period to 20 per cent at the end. It is also possible that some respondents may misinterpret the questions by answering "no change" when they mean

						Employees		
date	Enter	Exit	Re-enter	Total	<25	25 - 149	150 - 749	750 +
2008q3	327		0	327		262	44	21
2008q4	291	128	0	490		392	66	32
2009q1	121	204	49	456	27	335	63	31
2009q2	88	173	95	466	21	356	56	33
2009q3	56	206	100	416	15	308	62	31
2009q4	45	160	122	423	19	306	67	31
2010q1	40	165	101	399	19	291	59	30
2010q2	37	163	135	408	27	288	61	32
2010q3	46	182	125	397	19	279	64	35
2010q4	31	172	111	367	19	264	54	30
2011q1	49	143	149	422	21	309	60	32
2011q2	36	187	162	433	29	317	50	37
2011q3	40	171	128	430	23	323	50	34
2011q4	45	213	108	370	12	273	49	36
2012q1	37	178	148	377	21	273	47	36
2012q2	27	156	136	384	18	280	51	35
2012q3	24	165	141	384	18	279	51	36
2012q4	23	169	134	372	20	260	53	39
2013q1	36	158	112	362	24	255	53	30
2013q2	27	138	125	376	19	281	43	33
2013q3	18	146	110	358	16	262	48	32

Table 1: The Dynamics of the Panel of Respondents to the *Industrial Trends Survey* 

that the rate of inflation rather than the price level has not changed. But a recent answering practices survey conducted by the CBI suggests that this is not the case.<sup>2</sup> An alternative explanation is that the "no change" bucket is placed in the middle of all possible answers which may bias respondents to select that bucket (Bell and Macallan, 2011).<sup>3</sup>

Figure 1 shows the distribution of past and expected future price increases of the Winsorised data. The graph shows that the distributions of the two variables are very similar. In particular they show clustering at values of nought, three, five and seven which are also characteristic of the Michigan Survey of inflation expectations in the United States (Bryan & Palqvist 2004). It should be noted that this, although accentuated by, is not an artefact of the data being rounded. As we noted above, most respondents used the integer buckets rather than the writing in their own response. For own inflation expectations, the distribution is centred around zero, but there is a second

<sup>&</sup>lt;sup>2</sup>We thank Garry Young for pointing this out.

<sup>&</sup>lt;sup>3</sup>For households' inflation expectations, Klaauw et al. (2012) have analyzed how the wording of questions can affect the responses.

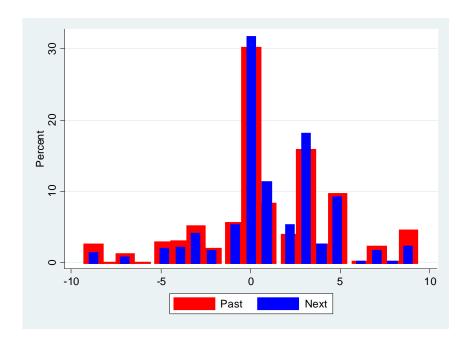


Figure 1: The Density of Past and Expected Own Price Increases

mode at 3%. The clustering of past and future own price increases may, of course, reflect the way in which firms actually behave. It may, however, also reflect a preference for reporting numbers like three and five.

There is a question of how seriously respondents take the survey. One means of exploring this is to count how many always give the same answer<sup>4</sup>. Of the 1412 firms which respond three or more times, 89 give the same answer to the question about past price increases on every occasion. Out of the 922 which give six or more answers, 40 provide the same answer to the question each time. 67 of the 89 respondents in the first case and 35 in the second case reported zero on each occasion. This summary of the pattern of answers suggests that, while there is evidence for clustering of responses about price changes, but not wage increases, around popular numbers, there is little evidence that the survey is abused by firms which provide formulaic responses.

The aggregate properties of the survey results can be seen in figure ??. This shows, for each quarter in the survey, the unweighted mean of the actual figure for the price increase over the last twelve months from the firms who actually answered the survey at the time, together with the aggregate official data. These are calculated by allocating

<sup>&</sup>lt;sup>4</sup>A study of the qualitative survey of output in the Netherlands found that about fifteen per cent of firms always gave the same answer. On discovering this, the Netherlands Bureau of Statistics approached respondents to ask why that was the case.

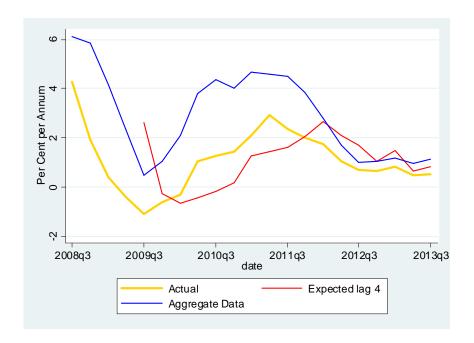


Figure 2: Time Series of Official and Survey Data

each respondent to a 2-digit SIC class, and using the relevant producer price output index; again the graph shows the unweighted average. Finally, the graph also shows the unweighted men of the price increase expected over the coming twelve months, lagged by four quarters. Had firms expectations been correct, the actual and expected lines would co-incide. As it is, the graph suggests a strong retrospective element to expectations formation.

## 3 Data Properties

In this section we elaborate on what is shown in the graphs First of all we look at the relationship between the price changes reported by firms and the published output price indices. Secondly we explore the connection between firms' expectations of price changes and the subsequent out-turns, and finally we examine influences on expectations.

## 3.1 The Relationship between Producer Price Indices and the ITS Data

To the extent that the respondents have the same understanding of sector as does the ONS, we would expect a close correspondence price changes reported by firms and the movements in output prices deduced from the producer price indices for the correspond-

		Employees				
	All Firms	<25	25-149	150 - 749	750 +	
SIC Price Change	.255	.123	.216	.490	.405	
	(.032)***	(.099)	(.035)***	(.116)***	(.086)***	
Const.	011	152	.104	741	.235	
	(.126)	(.436)	(.149)	(.319)**	(.369)	
Restriction	F(2,718) = 388	F(2,60) = 76	F(2,499)=316	F(2,98)=30	F(2,58)=27	
Observations	1688	126	1211	227	124	
Groups	719	60	500	99	59	
Dor	ondont Variable	. Price Chance	ro over I act 19 ]	Montha		

Dependent Variable: Price Change over Last 12 Months Significant levels \* 10% \*\* 5% \*\*\*1%

Table 2: The Relationship between reported Price Changes and the Corresponding 2-digit SIC Producer Price Changes

ing industrial sectors. We might separately expect to see a coefficient close to unity and a constant term close to zero in a pooled regression equation explaining firms' responses to the CBI by means of the relevant ONS price index. We present in table 2 the results of this regression, together with a test statistic for the hypothesis of unit coefficient and zero constant. The results are presented for all respondents taken together, and also for firms classified by size.

The results are very striking. For the respondents as a whole, and for each individual size category, we firmly reject the hypothesis that firms' responses track the published data. It is, however, clear that, while the relationship is, on average, closer for firms with at least 150 employees, it is not good. There are a number of possible reasons for this poor match. Perhaps the most obvious reason is that the match between firm products and the SIC 2-digit sector to which the firms are classified is not very good. It is also possible, of course, that firms are quoting prices which reflect ad hoc contractual arrangements made with their customers, and these are not reflected in the official price indices.

## 3.2 The Relationship between Expectations and Out-turns

We now turn to the relationship between businesses expectations of their own pricesetting behaviour, and the subsequent out-turns. We use a pooled regression equation, explaining the reported change in prices over the past four quarters, both for the firm's industry and the firm itself, in terms of the expectation it reported four quarters previously. A standard test for rationality of expectations is that the coefficient on expectations is not significantly different from 1, while the constant term is not significantly

		$\operatorname{Employees}$				
	All Firms	< 25	25-149	150-749	750 +	
4-Period Expectation	.190	.265	.090	.506	.619	
	(.032)***	(.094)***	(.038)**	(.096)***	(.088)***	
Constant	.629	.179	.793	.139	.523	
	(.083)***	(.199)	(.101)***	(.223)	(.241)**	
	F(2,992)=323	F(2,67)=32	F(2,723)=30	F(2,132)=14	F(2,67)=9.4	
Observations	1716	130	1226	233	127	
Groups	723	62	502	100	59	

Dependent Variable: Price Change over Last 12 Months Significant levels \* 10% \*\* 5% \*\*\*1%

Table 3: The Relationship between Expected Price Changes and Subsequent Out-turns

different from zero. It should be noted that, while the observations are quarterly, firms are asked to report price changes and expected price changes over four quarters. This would lead to serial correlation if observations fewer than four quarters apart were used. We therefore limit ourselves to those observations which are at least four quarters apart in our subsequent analysis. These tests are, of course, tests for rationality only if firms have quadratic loss functions with respect to the deviation of the out-turns from their expectations. An symmetric loss function will result in bias while non-quadratic losses will, even with symmetry, lead to a coefficient on *Expectations* different from one.

We show, in table 3, the results of tests unit coefficient and zero constant for firms in different size bands. The tests give a very clear picture. Either i)firms' expectations of their own future price increases are not rational, ii) they mis-report either their expectations or iii) their loss functions are very different from what might be represented by a quadratic loss function. The pattern is also consistent with what would be expected if there were a substantial noise element in the expectational data; that would lead the coefficient on it to be biased towards zero with a corresponding positive constant<sup>5</sup>.

### 3.3 Influences on Expectations

As a background to our subsequent work it is helpful to provide some analysis of influences on expectations. Table 4 shows the role that past movements in costs and prices play as drivers of firms' expectations of their own price increases. The rate of consumer price inflation reported most recently is also included as an explanatory variables. It

<sup>&</sup>lt;sup>5</sup>It should be noted that, of the data are not Winsorised, the hypothesis of rationality can be accepted for the largest group of firms. This is a consequence of just two observations. One shows a forecast of -15 per cent with a reported out-turn of -50 per cent, while the other shows a forecast of 12.5 per cent and an out-turn of 23 per cent. The same participant provides both observations.

is possible that current prices, which affect past price movements, and capacity utilisation, which is again a consequence of business decisions, are determined jointly with expected future price increases. We show both OLS and IV unbalanced panel fixed effects estimates, in which the three firm-specific variables were instrumented by their lagged values, to address possible endogeneity. A Hausman test suggested that we were able to accept the hypothesis of exogeneity ( $\chi_4^2 = 8.2$ ) although only at a relatively low level of significance. We nevertheless present here the results estimated by ordinary regression with fixed effects, again distinguishing firms of different sizes. In interpreting these coefficients, it must be remembered that past changes in costs are measured by a trichotomous variable which takes values of -1, 0 or 1. This means it is not possible to test hypotheses such as homogeneity.

Some generalisations are, nevertheless, possible. First of all past increases in output prices have an important but not substantial role in influencing expectations. Firms do not have classic adaptive expectations. Secondly, the influence of past consumer inflation is similar for the three smaller size groups, even though it significance is not. Thirdly, past costs have a fairly substantial impact on expected price increases, notwithstanding again that their significance varies. Finally, there is a material difference between the impact of capacity utilisation in the OLS and IV estimates; the latter suggest that expected future price increases are not affected by capacity. It should be noted that the instruments used pass the Sargan and underidentification tests. The Cragg-Donald statistic is just below the 10 maximal size but suggests an IV bias of below 5 per cent. We not present values for different sized of firms because our instruments appeared weak in sub-samples. We attribute this to the reduction in sample size.

With this background, we now proceed to the core of the paper, an examination of the relationship between past and expected future price increases in the data reported by respondents to the survey.

## 4 Adjustment Costs and Sticky Prices

### 4.1 The Standard NKPC with Rotemberg pricing

We consider the problem faced by a firm maximises its expected profits subject to quadratic price adjustment costs. With Rotemberg pricing, the problem of the individual

	OLS	IV
Past own price increase (last 12 months)	.314 (.023)***	.168 (.073)**
its-CurrRateOper	.020 $(.006)***$	.001 (.025)
CPI inflation	.167 (.073)**	.292 (.105)***
its-PstCostPerUnit	.563 (.130)***	.584 (.196)***
Const.	-1.332 (.509)***	
Observations	1679	752
Groups	718	262
J-test		3.777
d.f		2
J-test p-value		0.151
Under-identification test		58.723
d.f.		3
Under-identification p-value		0
Cragg-Donald Weak Identification		16.476

Dependent Variable: Expected Price Increase (next 12 months) Significant levels \* 10% \*\* 5% \*\*\*1%

Table 4: Influences on Firms' Expectations of Price Changes

firm can be written as

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[ p_t^f y^f - P_t \Psi_t^f - \frac{\gamma}{2} \left( \frac{p_t^f}{p_{t-1}^f} - 1 \right)^2 P_t y_t \right] / P_t$$
 (1)

subject to the demand function

$$y_t^f(d) = \left(\frac{p_t^f}{P_t}\right)^{-\theta} y_t \tag{2}$$

Here  $p_t^f$  is the price firm f charges for its output, and  $y_t^f$  is the quantity produced.  $P_t$  is the price of consumption goods and  $y_t$  is aggregate output.  $\Psi_t^f$  is the cost of production measured in terms of consumption goods, so that  $P_t\Psi_t^f$  is the nominal cost of production.

The first order condition is, with  $\psi_t^f = \partial \Psi_t^f/\partial y_t^f$ , the marginal cost of production,

$$0 = y_t^f (1 - \theta) + \psi_t^f \theta y_t^f \tilde{p}_t^f - \gamma \pi_t^f \tilde{p}_{t|t-1}^f y_t + \beta E_t \left( \lambda_t \gamma \frac{(1 + \pi_{t+1}^f)}{(1 + \pi_{t+1})} \pi_{t+1}^f \tilde{p}_{t+1|t}^f y_{t+1} \right)$$
(3)

where 
$$\tilde{p}_t^f \equiv \frac{P_t}{p_t^f}, \tilde{p}_{t|t-1}^f \equiv \frac{P_t}{p_{t-1}^f}$$
.

#### 4.2 Linearisation

The linearisation is set out in appendix A. The linearised first-order condition is:

$$\hat{\pi}_t^f = \beta E_t \hat{\pi}_{t+1}^f + \frac{\theta \psi}{\gamma} \left( \hat{\psi}_t^f + \hat{p}_t - \hat{p}_t^f \right) \tag{4}$$

This is essentially the NKPC type equation, with marginal cost on the RHS but with the additional difference between the aggregate price level and the firm level price level - so this is the complication. However, since costs,  $\hat{\psi}_t^f$  are measured in units of consumption, an alternative interpretation of this equation is that the firm adjusts its price with respect both to the expected price change in the next period and the deviation of its mark-up, that is its costs relative to its sale price, from the base-line.

### 4.3 Temporal Aggregation

In exploring our model, we face the problem that the survey asks for past changes over the past twelve months and expected future price changes over the coming twelve months. To treat these data as though they related to quarterly changes would run the risk of introduction serial correlation and thus seriously biasing the estimates. We can, however, address the problem of temporal aggregation by adding up four successive equations for the change in the price level, as we now make clear.

We write equation (4) together with three lags as

$$\hat{\pi}_{t}^{f} = \beta E_{t} \hat{\pi}_{t+1}^{f} + \frac{\theta \psi}{\gamma} \left( \hat{\psi}_{t}^{f} + \hat{p}_{t} - \hat{p}_{t}^{f} \right)$$

$$\hat{\pi}_{t-1}^{f} = \beta E_{t-1} \hat{\pi}_{t}^{f} + \frac{\theta \psi}{\gamma} \left( \hat{\psi}_{t-1}^{f} + \hat{p}_{t-1} - \hat{p}_{t-1}^{f} \right)$$

$$\hat{\pi}_{t-2}^{f} = \beta E_{t-2} \hat{\pi}_{t-1}^{f} + \frac{\theta \psi}{\gamma} \left( \hat{\psi}_{t-2}^{f} + \hat{p}_{t-2} - \hat{p}_{t-2}^{f} \right)$$

$$\hat{\pi}_{t-3}^{f} = \beta E_{t-3} \hat{\pi}_{t-2}^{f} + \frac{\theta \psi}{\gamma} \left( \hat{\psi}_{t-3}^{f} + \hat{p}_{t-3} - \hat{p}_{t-3}^{f} \right)$$

If we add these equations together, the result is an equation in the four-quarter growth in prices, explained by the four-quarter growth in expected prices

$$\hat{\pi}_{t}^{4f} = \beta E_{t-3} \hat{\pi}_{t+1}^{4f} + \frac{\theta \psi}{\gamma} \left( \hat{\psi}_{t}^{4f} + \hat{p}_{t}^{4} - \hat{p}_{t}^{4f} \right) + u_{t}$$

$$u_{t} = \beta \left( E_{t} \hat{\pi}_{t+1}^{f} + E_{t-1} \hat{\pi}_{t}^{f} + E_{t-2} \hat{\pi}_{t-1}^{f} \right) - \beta \left( E_{t-3} \hat{\pi}_{t+1}^{f} + E_{t-3} \hat{\pi}_{t}^{f} + E_{t-3} \hat{\pi}_{t-1}^{f} \right)$$
(5)

Here the superscript 4 indicates that the variable, if a change, relates to the change over the preceding four quarters while if level it relates to the sum of the quarterly variables up to, and including the quarter indexed.  $u_t$  is the error introduced in the equation because four-quarter expectations are formed at quarter t-3 instead of being the sum of the quarterly expectations formed one quarter earlier. If firms forecasts of their own expectations are rational, then  $u_t$  is orthogonal to information available at t-3. Similarly, of course, at time t-3, the values of the price and cost variables in the subsequent three quarters are not known. Data observed for the economy as a whole in t-3 should be valid instruments for these. Firm-specific data at t-3 may, however, be endogenous to the pricing process; thus only firm-specific data observed in t-4 or earlier are valid instruments. This choice of instruments should also address the effects of reporting error on the expectations data, at least if that reporting error is uncorrelated across firms and with events in the broader economy.

In order to avoid the serial correlation which would result from estimating equation (5) from quarterly data, we take care to ensure that only non-overlapping observations are used. There is, however, no requirement that all the observations for the different firms relate to exactly the same periods.

#### 5 Estimation

We describe here the way in which we estimate the model set out in section 4 and present the results of that estimation.

#### 5.1 Variable Specification

Prices We use the consumer price index as a general price index, in common with much work on New Keynesian Phillips curves. The price series for the individual firms are compiled from the returns they have provided to the CBI. There are two practical problems. First of all, the responses relate to changes over four quarters. This means that we cannot simply cumulate reported price changes forward from the start of the data set in 2008Q3. Each firm is located in its 2-digit sector (of which twenty-four relate to manufacturing); and we assume that prices in 2007Q4-2008Q2 are given by the price index for the output of the relevant 2-digit industry. If four quarter price changes were available for all quarters of our dataset for each firm, rolling forward would provide a price index for each firm. Estimation of equation (5) allowing for the presence of firm fixed effects means that any deviation of the actual starting price of each firm from that given by the output price index is absorbed into the fixed effect.

There is, however, a separate and more substantial problem that the panel is incomplete. In order to avoid a loss of data which would be almost complete, we replace missing entries by the average price growth reported by other firms in the same SIC category.

Costs The pricing equation requires marginal costs, which are of course equal to average costs with constant returns to scale. We explore a number of possible cost measures. The survey does not ask firms how much their costs have increased; instead there is a question about whether costs have changed in the previous quarter, with a trichotomous response, up, no change or down. The first measure of costs we construct attempts to turn this into an index. We construct an index of costs for each SIC sector; material costs are provided by the quarterly value of the producer price index, and labour costs are provided by the index of unit wage costs in manufacturing<sup>6</sup>. The logarithms of indices of each are weighted together using the 2-digit sectoral ratios of expenditure on materials

<sup>&</sup>lt;sup>6</sup>Measures of wage rates are calculated from the Monthly Wages and Salaries Survey. The sample is too small to allow calculation of wage rates at a level of disaggregation finer than manufacturing as a whole.

to expenditure on labour shown in the supply-use table for 2008. For each SIC sector separately the responses of the individual firms to the question on costs are aggregated, giving a value of 1 to a firm which reports a rise in costs, 0 to one which reports no change and -1 to one which reports a fall. The quarterly growth in the computed log cost index is regressed on the mean sectoral response, and the pooled mean group estimator is used to convert answers to the survey to actual changes in costs.

For each firm then, we cumulate the quantified trichotomous responses to the question on unit costs, using the average of responses across the 2-digit sector where responses are missing. The resulting cumulated series is then multiplied by the pooled mean group estimator to give a firm-specific indicator of costs, which we denote *OwnCost*.

The assumptions involved in this calculation are very substantial and we also look at two wage-based measured of costs. The first is the log of Average Weekly Earnings (lAWE), the ONS measure of wage rates, and the second is the log of unit wage costs in manufacturing (lUWC).

All of these are measures of average costs rather than marginal costs. If firms produce with constant returns to scale the two are, of course, equivalent. But strictly the New Keynesian model is specified in terms of marginal costs, and this issue has faced various papers attempting to study the empirical performance of the model and its implications. Gali (1999) consider the performance of the New Keynesian Phillips Curve using the labour income share in the non-farm business sector for real marginal cost, relying on the Cobb-Douglass production function with constant returns to scale in capital and labour. Gali, Gertler & Lopez-Salido (2007) examine the welfare costs of business cycles by examining markup dynamics where price markups (and real marginal cost) are derived from a model with constant returns to scale. However, Gali et al. (2007) also consider generalisations, following Rotemberg & Woodford (1999), where average and marginal cost can diverge (for example in models with overhead labour). Nekarda & Ramey (2013) examine the cyclical properties markups but convert average cost measures into marginal cost measures using insights from Bils (1987). This latter paper argues that, contrary to the predictions of the New Keynesian model, markups are pro-cyclical, although a detailed analysis of this issue is beyond the scope of our paper. Instead, therefore, we follow Gali et al. (2007) in using our average cost measures; we explore separately the issue of returns to scale.

Capacity The survey also asks firms to report their capacity utilisation, as a proportion of maximum capacity. While we do not set out formally a model in which capacity

utilisation rather than costs enters, we estimate, for completeness a model in which real domestic costs,  $\hat{\psi}_t^{4f}$  are replaced by capacity utilisation (*Capac*). Once again, in order to maintain the sample, the sector average is used where individual data for capacity utilisation are missing. In the model where price and cost terms enter, we represent both by nominal indices

#### 5.2 Results

The results of fitting equation (5) to the available data are shown in table 5, using the three different measures of costs, and also with capacity utilisation in place of the cost variable. The instruments used are shown in the table. The following points can be drawn from the analysis. All of the models fitted give a role to expected future price increases as an influence on current price increases, and in that sense they support the idea that price-setting is a forward-looking decision. They also all show that firms with high current prices tend to see downward pressure on them; a negative coefficient is found on actual price in all cases. The three measures of costs give, however, rather different results. The measure compiled from responses to the survey has a coefficient very little different from zero, and the adding up constraint, which implies that, in the long run prices and costs move in line, is rejected. This reference is less marked when average wages are used as the indicator of cost; when unit wage costs are used, the adding up constraint is accepted very easily. That is not to say that unit wage costs are the most appropriate measure of costs, but rather that data limitations mean that they are the measure most in conformity with theoretical restrictions. With this specification the coefficient on expected price increases is greater than one; it should however, be noted that values just below one are safely within the confidence limits implied by the standard errors.

With the three cost variables, the model is estimated with both costs and prices in nominal terms. If capacity utilisation is to stand proxy for marginal cost that is, however, in real terms, and the appropriate price variable is therefore RealPrice, constructed by deflating the nominal price by the consumer price index. We estimated the model using the Stata command xtivreg2. We use the following variables as instruments in all four equations:  $CapConstrCost\ Finance_{t-4}, NxtOwnPrice\ Avex_{t-3}$ . In the first three equations we use additionally,  $OwnPrice_{t-4}, OwnPriceAvex_{t-3}$  and  $Capac_{t-4}$ . When OwnCost is the measure of cost used we further introduce  $OwnCost_{t-4}$  and  $OwnCostAvex_{t-3}$  as instruments. With lAWE as the cost variable, these are replaced by  $lAWE_{t-3}$ , while

log Price Level (4-quarter sum)	OwnCost 096 (.018)***	1AWE 111 (.016)***	1UWC 088 (.015)***	Capac
log Real Price Level (4-quarter sum)				058 (.015)***
Expected Price Change (3-period Lag)	.907 (.103)***	.675 (.124)***	1.096 (.133)***	.846 (.183)***
Capacity Utilisation (4-quarter sum)				002 (.0008)**
log Level of Costs (4-quarter sum)	.022 (.011)**			
log Average Weekly Wage (4-quarter sum)		.07 (.018)***		
log Unit Wage Costs (4-quarter sum)			.082 $(.033)**$	
Observations	827	827	827	827
Groups	280	280	280	280
J-test	7.609	3.662	7.395	9.778
d.f	4	4	4	3
J-test p-value	.107	.454	.116	.021
Under-identification test	125.741	78.396	92.68	29.247
d.f.	5	5	5	4
Under-identification p-value	1.91e-25	1.82e-15	1.84e - 18	6.97e-06
Cragg-Donald Weak Identification	23.026	12.906	15.737	5.093
Homogeneity $F(1,544)$	33	8.1	0.05	-

Dependent Variable: Change in Prices over Last 12 Months Significant levels \* 10% \*\* 5% \*\*\*1%

Table 5: Parameters of the New Keynesian Phillips Curve: All Firms

with IUWC as the cost variable they are replaced by  $IUWC_{t-3}$ . When Capac is used in place of a direct cost measure, they are replaced by  $Capac_{t-4}$  and  $CapacAvex_{t-3}$  and  $OwnPrice_{t-3}$  and  $OwnPrice_{t-3}$  are replaced by  $RealPrice_{t-4}$  and  $RealPriceAvex_{t-3}$ . Thus when we use instruments relating directly to the firm in question we use four lags, so as to avoid the risk of endogeneity affecting the various measures of the firm's operation at any time. On the other hand, when we use macro data or sector averages excluding the firm in question, there is no obstacle to using three lags, because the variables are not directly affected by the decision made by the firm in question. We do not use lags shorter than three periods at all, because these may be correlated with the expectational error term in equation (5) When the three cost measures are used, the tests for instrument validity are very satisfactory. There is no significant evidence of over-identification, while under-identification is safely rejected and the Cragg-Donald

Employees	All Firms	<25	25-149	150-749	750+		
Changes in Capacity Utilisation and Change in Costs							
Polyserial correlation	0.010	0.022	0.021	-0.032	-0.102		
standard error	(0.016)	(0.088)	(0.018)	(0.052)	(0.084)		
Observations	5049	204	3806	639	400		

Table 6: Correlations between Capacity Utilisation and Changes in Unit Costs

test points to a bias of less than five per cent.

Use of capacity utilisation as a proxy for marginal cost does not, however result in a satisfactory equation. The coefficient is very slightly negative, perhaps consistent with declining marginal costs. More importantly, however, the Cragg-Donald statistic is low, pointing to considerable bias. Of course, a negative coefficient is consistent with theory if firms have increasing returns to scale so that marginal costs fall as capacity utilisation increases. We next examine the evidence on returns to scale.

The survey collects trichotomous data on both output changes and unit costs as well as the cardinal data on capacity utilisation. If returns to scale are diminishing, we would expect to see a positive correlation between changes in capacity utilisation and changes in costs, while with increasing returns we would expect the correlation to be negative. Table 6 shows the polyserial correlations between the changes in capacity utilisation and changes in unit costs. Both survey responses relate to developments in the previous quarter, so there is no worry about overlapping observations; we use all the available data. Taking all firms together, and using all available records returns seem close to constant. The different size categories suggest a rather different story. Small firms seem to show diminishing costs while large firms seem to show increasing costs, although none of the correlations are statistically significant.

If we look at the correlations between the change in costs and the change in output, the picture shown in table 6 appears more strongly. In table 7 we can see that there is weak evidence of diminishing returns for the smallest firms, constant returns for the next group and significantly increasing returns for the next group; the largest group the correlation is again negative, but is not significant even at 10 per cent level.

In view of this, we explore whether the firms with fewer than 150 employees, which show constant returns or insignificantly diminishing returns, are different from those with at least 150 employees, some of which show statistically significant evidence of increasing returns. We explore this with log unit wage costs as the cost variable, and also with capacity utilisation as an indicator of marginal cost. Table 8 points to a picture

Employees	All Firms	<25	25-149	150-749	750+
Polychoric correlation	-0.024	0.110	-0.002	-0.130	-0.025
standard error	(0.014)*	(0.067)	(0.017)	(0.038)***	(0.054)
Observations	8790	395	6342	1183	715

Significant levels \* 10% \*\* 5% \*\*\*1%

Table 7: Correlations between Output Changes and Changes in Unit Costs

which is, in fact, not very different from that of table 5. Once again unit wage costs provide a measure of cost which is consistent with the homogeneity restriction. In this case the coefficient on expected price changes for small firms<sup>7</sup> is, at 0.985, very close to what is normally used in New Keynesian models, and easily interpreted as a discount rate; nevertheless the breadth of the confidence interval around it needs to be stressed. The same model when estimated for larger firms gives a higher coefficient on expected future prices which is, nevertheless still not significantly above one. Possibly on account of the small size of the sample, the instruments are weak, and it is not clear how much weight can be put on the results. It might be hoped that the capacity measure would be a good alternative to an explicit measure of costs for the smaller firms, since there is weak evidence for diminishing returns for these. That is,

<sup>&</sup>lt;sup>7</sup>We originally estimated the model for all four size categories. The very small number of firms with fewer than twenty-five employees and the small number for firms with more than seven hundred and fifty employees meant that the parameters were very poorly determined. The results for the category 25-150 employees were not materially different from those for all firms with fewer than one hundred and fifty employees.

	Unit Wa	ge Costs	Capacity	
Employees	<150	150 +	<150	150 +
log Price Level (4-quarter sum)	09	067		
	(.017)***	(.03)**		
log Real Price Level (4-quarter sum)			061	041
			(.017)***	(.033)
Expected Price Change (3-period Lag)	.983	1.346	.782	.93
	(.154)***	(.197)***	(.228)***	(.195)***
Capacity Utilisation (4-quarter sum)			002	-1.00e-05
			(.001)*	(.0009)
log Unit Wage Costs (4-quarter sum)	.074	.079		
	(.037)**	(.061)		
Observations	664	163	664	163
Groups	221	59	221	59
J-test	1.086	.977	7.411	7.37
d.f	3	3	3	3
J-test p-value	.78	.807	.06	.061
Under-identification test	70.275	29.525	20.041	15.428
d.f.	4	4	4	4
Under-identification p-value	1.99e-14	6.11e-06	.0005	.004
Cragg-Donald Weak Identification	13.732	6.475	3.451	2.845
Homogeneity	F(1,440)=0.3	F(1,101)=0.0		

Dependent Variable: Change in Prices over Last 12 Months
Significant levels \* 10% \*\* 5% \*\*\*1%

Table 8: Parameters of the New Keynesian Phillips Curve: Small and Large Firms

#### 6 Conclusions

The New Keynesian Phillips curve underpins the dynamic stochastic general equilibrium models which nowadays form the core of macro-economics. Such models are typically estimated from macro-economic data; parameter estimates, and in particular those of expected future price changes are usually not well identified. In this paper we have explored New Keynesian price setting using micro-economic data which record both firms' past price changes and their expected future price changes. Measures of costs faced by individual firms are not recorded with the same precision. We explore three measures of costs, one calculated for individual respondents by taking the qualitative responses they provide to questions about past movements in costs, the second is provided by the Office for National Statistics measure of average weekly earnings in manufacturing, and the third is unit wage costs i manufacturing, that is average weekly earnings adjusted for changes in output per worker. All three measures result in clear roles for expectational effects in price setting. The second and third measures are strictly appropriate as indicators of costs only with constant returns to scale. We find evidence consistent with the hypothesis that a subgroup of our sample, those with fewer than a hundred and fifty employees, produces with constant returns to scale. estimation over this subsample yields parameter estimates which satisfy homogeneity requirements and deliver a coefficient on expected prices very close to that suggested by economic theory. Models in which marginal costs are represented by reported capacity utilisation are, however, much less satisfactory; we do not find any significant capacity effects, perhaps because much of our sample seems to produce with constant returns to scale. The relationship between changes in costs and changes in output suggests that many large firms produce with increasing returns to scale; for such firms a satisfactory equation estimation of a satisfactory equation might require a more appropriate marginal cost variable. It is also possible, however, that for large firms the classic model of imperfect competition is inappropriate. It is perfectly possible that they react to each others' behaviour, instead of just taking the general price level as given.

Nevertheless, even for small firms, the absence of a role for capacity effects suggests that the link between demand effects and inflation may not lie primarily in the goods market. An obvious alternative source is that demand influences the price level through its effects on wage-setting in the labour market, as set out in a formal New Keynesian framework by Gali (2011) but also consistent with the original analysis presented by Phillips (1958). Of course, this may not be true in other countries but it is probably

consistent with the way policy-makers and others who play close attention to economic developments see things in the United Kingdom.

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### A Linearisation

For ease of derivation re-define any inflation term as the gross inflation rate,  $\tilde{\pi} = 1 + \pi$  above. Also, divide through by  $y_t^f$ 

$$\theta \left( 1 - \psi_t^f \frac{P_t}{p_t^f} \right) = 1 - \gamma (\tilde{\pi}_t^f - 1) \frac{P_t}{p_{t-1}^f} \frac{y_t}{y_t^f} + \beta \gamma E_t \left( \frac{\lambda_{t+1}}{\lambda_t} \frac{\tilde{\pi}_{t+1}^f}{\tilde{\pi}_{t+1}} (\tilde{\pi}_{t+1}^f - 1) \frac{P_{t+1}}{p_t^f} \frac{y_{t+1}}{y_t^f} \right)$$
(6)

Taking the left-hand side first:

$$-\theta w \left( \hat{\psi}_t^f + \hat{p}_t - \hat{p}_t^f \right) \tag{7}$$

It is assumed that the steady-state inflation rate is of size similar to the linearised deviations of the other variables, so that the product of it and any other variables is second-order and can be neglected. The first term on the right-hand side simplifies to:

$$-\gamma \tilde{\pi}_{t}^{f} \frac{P_{t}}{p_{t-1}^{f}} \frac{y_{t}}{y_{t}^{f}} + \gamma \frac{P_{t}}{p_{t-1}^{f}} \frac{y_{t}}{y_{t}^{f}} \tag{8}$$

which linearized (and imposing  $\pi = 1$  and symmetry in the steady state<sup>8</sup>) becomes, with ^indicating deviations, and with the deviations of both gross and net price changes equal to  $\hat{\pi}_t^f$ :

$$-\gamma \left(\hat{\pi}_t^f + \hat{p}_t - \hat{p}_t^f + \hat{y}_t - \hat{y}_t^f\right) + \gamma \left(\hat{p}_t - \hat{p}_t^f + \hat{y}_t - \hat{y}_t^f\right) \tag{9}$$

which neatly reduces to

$$-\gamma \hat{\pi}_t^f \tag{10}$$

Expanding and linearizing the second term on the right-hand side gives

$$\gamma \beta \left( \hat{\lambda}_{t+1} - \hat{\lambda}_t + 2\hat{\pi}_{t+1}^f - \hat{\pi}_{t+1} + \hat{p}_{t+1}^f - \hat{p}_t^f + \hat{y}_{t+1} - \hat{y}_t^f \right)$$
 (11)

plus

$$-\gamma \beta \left( \hat{\lambda}_{t+1} - \hat{\lambda}_t + \hat{\pi}_{t+1}^f - \hat{\pi}_{t+1} + \hat{p}_{t+1}^f - \hat{p}_t^f + \hat{y}_{t+1} - \hat{y}_t^f \right)$$
 (12)

which reduces to

$$\gamma \beta \hat{\pi}_{t+1}^f \tag{13}$$

Putting all this together:

$$\hat{\pi}_t^f = \beta E_t \hat{\pi}_{t+1}^f + \frac{\theta \psi}{\gamma} \left( \hat{\psi}_t^f + \hat{p}_t - \hat{p}_t^f \right) \tag{14}$$

<sup>&</sup>lt;sup>8</sup>Formally this is linearisation around a zero inflation rate. But it is better thought of as linearising around a positive rate of inflation which is first order of smallness. This means that cross terms between the baseline rate of inflation and other first differences are second order of smallness and can be neglected.